DATA.ML.200, Exercise set 1.

Task 2.

$p(x;\lambda) = \frac{1}{x!}e^{-\lambda}\lambda^x$

a)

Likelihood function for λ based on independent samples $\mathbf{x} = \{x_0, \cdots, x_{N-1}\}$

$$p(\mathbf{x};\lambda) = \prod_{n=0}^{N-1} p(x_n;\lambda)$$

$$= \prod_{n=0}^{N-1} \frac{1}{x_n!}e^{-\lambda}\lambda^{x_n}$$

$$= e^{-N\lambda}\prod_{n=0}^{N-1} \frac{1}{x_n!}\lambda^{x_n}$$

$$= e^{-N\lambda}\lambda^{\sum_{n=0}^{N-1} x_n}\prod_{n=0}^{N-1} \frac{1}{x_n!}$$

b)

$$l(\mathbf{x};\lambda) = log(p(\mathbf{x};\lambda))$$

$$= log(e^{-N\lambda}\lambda^{\sum_{n=0}^{N-1} x_n}\prod_{n=0}^{N-1} \frac{1}{x_n!})$$

$$= log(e^{-N\lambda}) + log(\lambda^{\sum_{n=0}^{N-1} x_n}) + log(\prod_{n=0}^{N-1} \frac{1}{x_n!})$$

$$= -N\lambda + log(\lambda)\sum_{n=0}^{N-1} x_n - \sum_{n=0}^{N-1} log(x_n!)$$

c)

$$\frac{\partial}{\partial\lambda}l(\mathbf{x};\lambda) = -N\frac{\partial}{\partial\lambda}\lambda + \frac{\partial}{\partial\lambda}log(\lambda)\sum_{n=0}^{N-1} x_n - \frac{\partial}{\partial\lambda}\sum_{n=0}^{N-1} log(x_n!)$$

$$= -N + \frac{1}{\lambda}\sum_{n=0}^{N-1} x_n$$

d)

$$\frac{\partial}{\partial \lambda} l(\mathbf{x}; \lambda) = 0$$

$$-N + \frac{1}{\lambda} \sum_{n=0}^{N-1} x_n = 0$$

$$\frac{1}{\lambda} \sum_{n=0}^{N-1} x_n = N$$

$$\frac{1}{\lambda} = \frac{N}{\sum_{n=0}^{N-1} x_n}$$

$$\lambda = \frac{1}{N} \sum_{n=0}^{N-1} x_n$$

$$\therefore \lambda_{MLE} = \frac{1}{N} \sum_{n=0}^{N-1} x_n = mean(\mathbf{x})$$