Exam answers

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1.

a)

1. Random process: $\{Y(s)\}$ a)

- $Cov(Y(s), Y(t)) = \sigma^2(|s| + |t| - |s-t|)$
- $E(Y(s)) = 0$

Intrinsic stationary:

i) Ok, mean is constant

ii) Ok, see (*)

i), ii) Corresponding requirements for specific stationarity (in lecture slides

Second-order stationarity:

i) Ok, mean is constant

ii) not fulfilled, since

$$Cov(Y(s+h), Y(t+h)) = \sigma^2(|s+h| + |t+h| - |s-t|)$$
$$\neq \sigma^2(|s| + |t| - |s-t|)$$

$$\mathrm{Cov}(Y(s), Y(t)) = E(Y(s)Y(t)) - E(Y(s))E(Y(t))$$
$$= E(Y(s)Y(t))$$
$$= \sigma^2(|s| + |t| - |s-t|) \qquad (*)$$

$$\mathrm{Var}(Y(s)-Y(t)) = E((Y(s)-Y(t))^2) - E(Y(s)-Y(t))^2$$
$$= E((Y(s)-Y(t))^2)$$
$$= E(Y(s)^2) - 2E(Y(s)Y(t)) + E(Y(t)^2)$$
$$= \mathrm{Var}(Y(s)) - 2E(Y(s)Y(t)) + \mathrm{Var}(Y(t))$$

$$= 2\sigma^2|s| - 2\sigma^2(|s| + |t| - |s-t|) + 2\sigma^2|t|$$

$$= 2\sigma^2[|s| - |s| - |t| + |s-t| + |t|$$

$$= 2\sigma^2|s-t|$$

$\mathrm{Var}(Y(s)-Y(t))$ depends only on lag difference

$\Rightarrow \{Y(s)\}$ is intrinsically stationary b)

The empirical semivariogram might not satisfy the negative-definite property and hence it can not be used directly as a semivariogram. Additionally, if the interest is to predict/krige new observations, then the parametrized theoretical semivariogram model is necessery. For example, if we want to predict a new value $Z(s_0)$ at unobserved site s_0, then the empirical semivariogram doesn't have an estimate for $\gamma(s_i - s_0)$.

c)

OLS method is not appropriate method for estimating the semivariogram parameters, since OLS assumes that the observations $\hat{\gamma}(h)$ are independent. This assumption does not hold with empirical semivariogram values. First of all Z_i's are autocorrelated and since same Z_i's are used to calculate $\hat{\gamma}(h)$'s, the $\hat{\gamma}(h)$'s are correlated as well. Due to this dependence property, the obtained parameter estimates $\hat{\theta}$ from OLS are not BLUE (Best Linear Unbiased Estimates) to ground truth ones.

2.

2.

Z_1	Z_2	Z_3
Z_4	Z_5	Z_6

Proximity matrix:

	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
Z_1	0	1	0	1	0	0
Z_2	1	0	1	0	1	0
Z_3	0	1	0	0	0	1
Z_4	1	0	0	0	1	0
Z_5	0	1	0	1	0	1
Z_6	0	0	1	0	1	0

4.

a)

Quadrat based methods

$\sum(n_i - \bar{n})^2/\bar{n}$