Spatial Data Analysis, Spring 2021

Lab Exercise 5:

1. Let Y be the distance from an arbitrary event in a study area A to its nearest neighbor, and suppose the spatial point process is a homogeneous Poisson process with intensity λ. Recall from lecture that the cdf of Y is $F(y) = 1 - \exp(-\lambda\pi y^2)$ for $y > 0$ if edge effects are ignored.

a) Show that, if edge effects are ignored, $\mathrm{E}(Y) = 1/(2\sqrt{\lambda})$ and $\mathrm{Var}(Y) = (4-\pi)/4\lambda\pi$.

b) Now suppose that events are not points but are instead circles of equal radius r_0. Suppose further that the locations of the centers of these circles are generated according to a simple sequential inhibition process with $\delta = r_0$. Let Y be the distance from the center of an arbitrary event in A to the center of its nearest neighbor. Show that the pdf of Y, if edge effects are ignored, is

$$\begin{aligned} f(y) &= 2\lambda\pi y \exp\{-\lambda\pi(y^2 - {r_0}^2)\} \ \text{ if } y \geq r_0 \\ &= 0 \text{ if } 0 < y < r_0. \end{aligned}$$

c) Under the same assumptions as in part (b), show that, if edge effects are ignored, $\mathrm{E}(Y) = r_0 + \{1 - \Phi(\sqrt{2\lambda\pi}r_0\}(1/\sqrt{\lambda})\exp(\lambda\pi {r_0}^2)$ and $\mathrm{Var}(Y) = {r_0}^2 + (1/\lambda\pi) - \{\mathrm{E}(Y)\}^2$, where Φ is the cdf of a N(0,1) random variable.

R problems are provided in the moodle.