Spatial Data Analysis I, SPRING 2021

Lab Exercise 2:

*** You will get the additional extra credit from the first problem submitted by the due date (March 31)

1. Consider a one-dim random field $\{Z(s) : s \geq 0\}$, which satisfies $\mathrm{E}[Z(s)] = 0$ and $\mathrm{Cov}[Z(s), Z(t)] = \sigma^2 min(s,t)$. Note that $min(s,t) = s$ if $s < t (= t$ otherwise).

   a) Show that $Z(s)$ is intrinsic stationary and that intrinsic stationarity does not imply the second-order stationarity.

   b) Define $Y(s) = Z(s+1) - Z(s) : s \in \mathbb{R}$. Show that $\{Y(s) : s \in \mathbb{R}\}$ is second-order stationary.

   c) Derive the covariance function $C_Y(h)$ of $\{Y(s) : s \in \mathbb{R}\}$. Find also its semivariogram and identify its nugget effect, sill and range.

2. Suppose that $\epsilon(\boldsymbol{s}) = \epsilon_{ME}(\boldsymbol{s}) + \epsilon_{NE}(\boldsymbol{s})$ where $\epsilon_{ME}(\boldsymbol{s})$ is an error process due to measurement errors and $\epsilon_{NE}(\boldsymbol{s})$ is locally (smooth) stationary, independent of $\epsilon_{ME}(\boldsymbol{s})$. Let θ_0 represent the measurement error variance and let $C_{NE}(\boldsymbol{h}; \theta)$ represent the covariance function of $\epsilon_{NE}(\boldsymbol{s})$. Derive the covariance function of $\epsilon(\boldsymbol{s})$ in terms of $C_{NE}(\boldsymbol{h}; \theta)$ or $C_{ME}(\boldsymbol{h}; \theta)$, etc.

***The first (R) problem posted on the moodle should be submitted.