Spatial Data Analysis, SPRING 2021

Lab Exercise 3: due **14.04**

1. **(For additional extra credit)** Consider the tent covariance function as given in notes. Suppose that a second-order stationary random field is observed at sites $\boldsymbol{s_1}, \ldots, \boldsymbol{s_{48}}$ in $\mathbb{R}^2$ which form a regular 6×8 square grid with spacing $\theta_2/\sqrt{2}$. If site i lies in row k and column l let $a_i = (-1)^{k+1}$ (in the condition for nonnegative definiteness). Use this set-up to show that the tent covariance function is not nonnegative definite in $\mathbb{R}^2$.

2. **Submit this problem** Consider the same configuration of data locations given as a class example for ordinary kriging.

i) To see the effect that semivariogram parameters may have on the ordinary kriging predictor and the ordinary kriging variance, compute the ordinary kriging weights and the kriging variance for each of the following semivariograms at site s0.

a. $\gamma(||\boldsymbol{h}||) = 1 - \exp(-||\boldsymbol{h}||/4)$

b. $\gamma(||\boldsymbol{h}||) = 0.25 + 0.75[1 - \exp(-||\boldsymbol{h}||/2)]$

Based on these results, try to draw some general conclusions about what happens to the kriging weights and kriging variances when either the spatial dependence gets stronger or the nugget effect increases.

ii) Suppose that the underlying random field is not intrinsically stationary. Instead, assume that $\mathrm{E}[Z(\boldsymbol{s})] = \beta_0 + \beta_1 x + \beta_2 y$ where $\boldsymbol{s} = (x, y)$: i.e. the random field satisfies the assumptions required for universal kriging. Also, assume that the residuals of the geostatistical model have the same semivariogram that the data themselves were assumed to have as in the class example (isotropic exponential model).

a. Obtain the universal kriging predictor weights and universal kriging variance for prediction at $\boldsymbol{s}_0$. Compare the results obtained by ordinary kriging and comment. (R computation is recommended.)

b. From part a), do you think that the OK predictor is better than the UK predictor for this spatial configuration and this mean function? Explain why or why not.

3. Using the isotropic model that best fits the omnidirectional empirical semivariogram of the Soil pH data,

i) obtain kriging predictors and kriging variances for the soil pH data at the following three sites: (Columns are numbered from left to right and rows from top to bottom.)

e1. Halfway between the first and second columns and halfway between the first and second rows.

e2. Halfway between the fifth and sixth columns and halfway between the fifth and sixth rows.

e3. In the eighth row, one-fourth of the way from the third column to the fourth column.

ii) Perform cross validation for the soil pH data to compare 2 model estimates based on REML and WNLS (Use your own results from the previous lab). Comment on your findings.

(Assume the 2nd order polynomial trend.)