(c) Show that the sample mean

$$\hat{\beta}_0 = (\underline{1}'\underline{1})^{-1}\underline{1}'y = \frac{1}{n}\underline{1}'\underline{y} = \bar{y}$$

is the BLUP of y_f

linear predictor $g'y$, where $g' = \frac{1}{n}1'$

Show: $g'(X \vdots VM) = (x_f' \vdots w'M)$, where $x_f = 1$, $w' = \rho 1'$, $M = I - \frac{1}{n}11'$

$g'X = \frac{1}{n}1'1 = \frac{1}{n}n = 1 = x_f$

$$g'VM = \frac{1}{n}1'[(1-\rho)I + \rho 11'][I - \frac{1}{n}11']$$

$$= [\frac{1}{n}1'(1-\rho)I + \frac{1}{n}1'\rho 11'][I - \frac{1}{n}11']$$

$$= \frac{1}{n}1'(1-\rho)I[I - \frac{1}{n}11'] + \frac{1}{n}1'\rho 11'[I - \frac{1}{n}11']$$

$$* = 0 + \frac{1}{n}1'\rho 11'[I - \frac{1}{n}11']$$

$$= \frac{1}{n}1'1\rho 1'[I - \frac{1}{n}11']$$

$$= \rho 1'[I - \frac{1}{n}11']$$

$$= w'M$$

$\therefore g'(X \vdots VM) = (x_f' \vdots w'M)$

$$* \quad \frac{1}{n}1'(1-\rho)I[I - \frac{1}{n}11']$$

$$= (1-\rho)[\frac{1}{n}1'I[I - \frac{1}{n}11']]$$

$$= (1-\rho)[\frac{1}{n}1' - \frac{1}{n}1'\frac{1}{n}11']$$

$$= (1-\rho)[\frac{1}{n}1' - \frac{1}{n^2}1'11']$$

$$= (1-\rho)[\frac{1}{n}1' - \frac{1}{n^2}n1']$$

$$= (1-\rho)[\frac{1}{n}1' - \frac{1}{n}1'] = 0$$