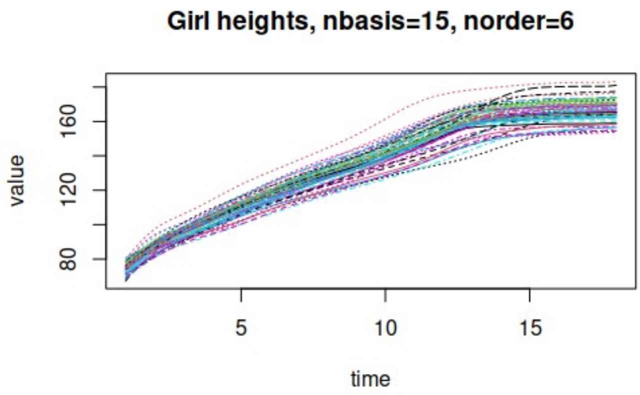
DATA.STAT.750

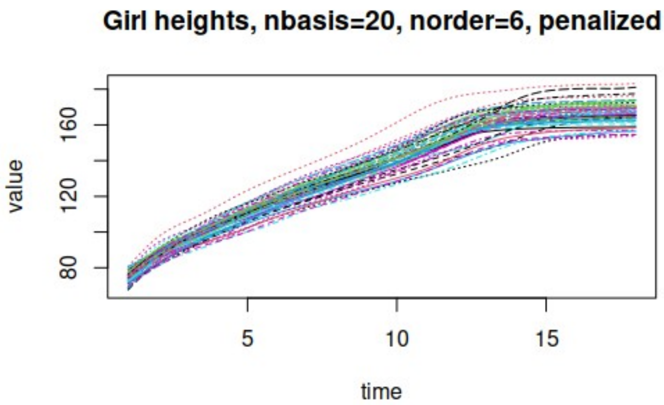
Exercise set 5

1.

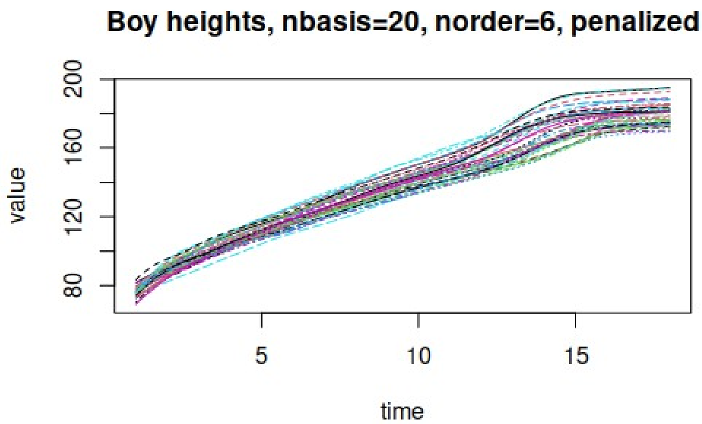
a)

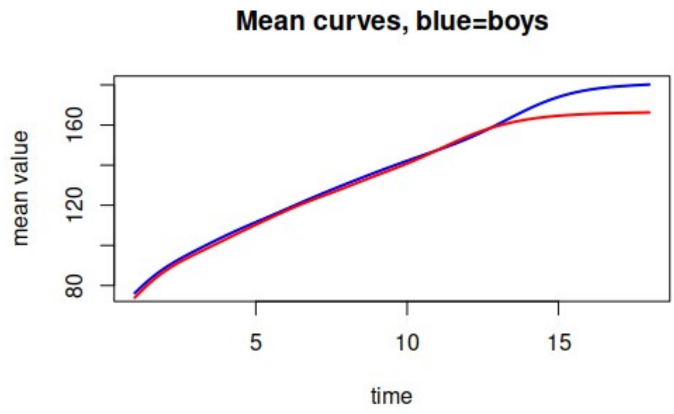


b)



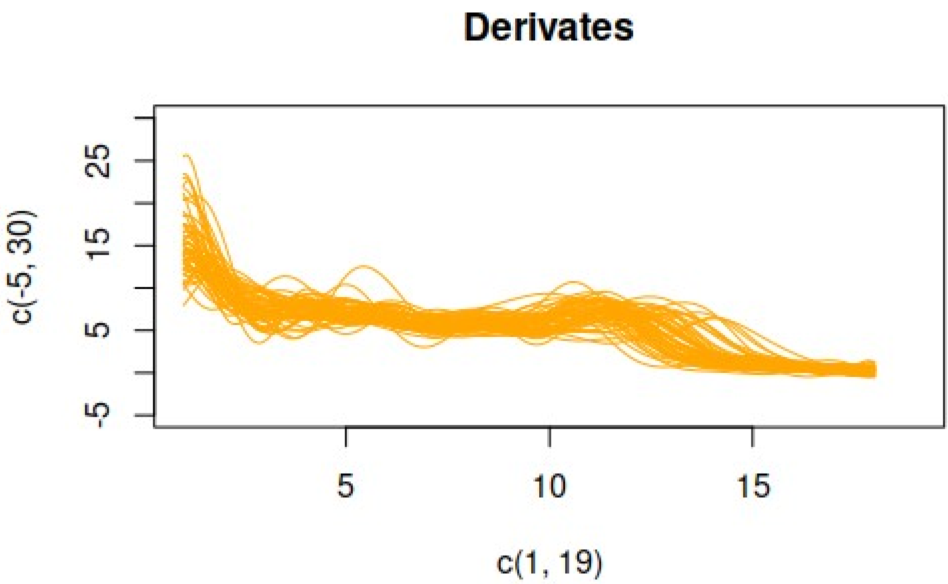
c)





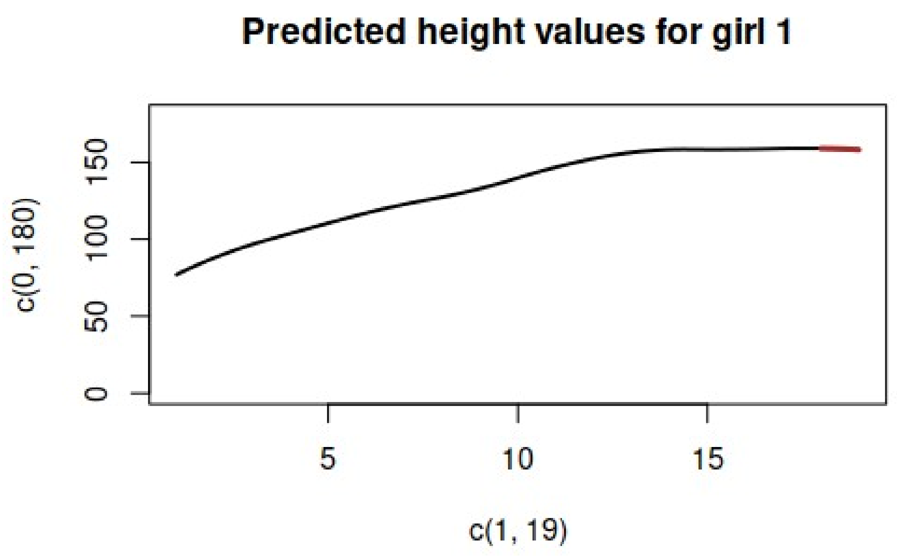

```
> eval.fd(15, Y2fd.mean)
         mean
[1,] 173.9688
> # Girls:
> eval.fd(15, Y1fd.mean)
        mean
[1,] 164.605
```

d)

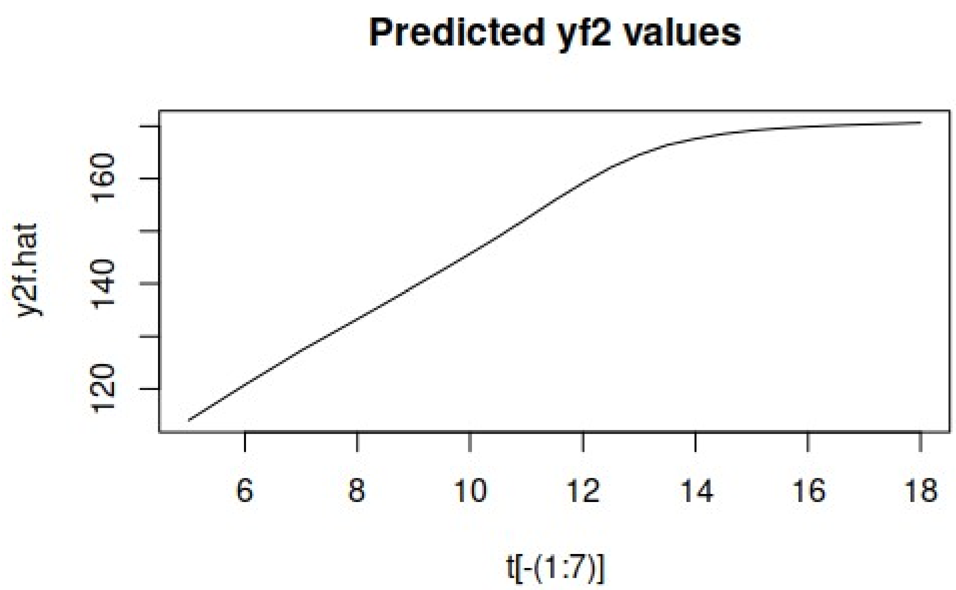


The derivatives tend to shrink to 0 when girls are older than 16 ($t \geq 16$).

e)

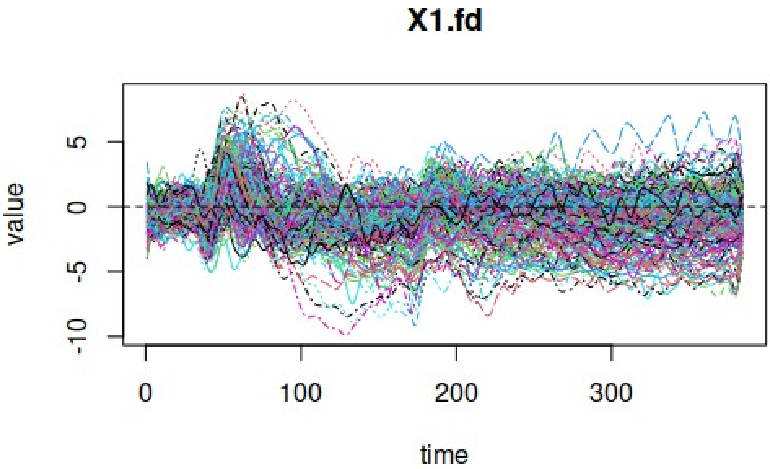

Predicted height values for girl 1
c(0, 180)
0
50
100
150
5
10
15
c(1, 19)

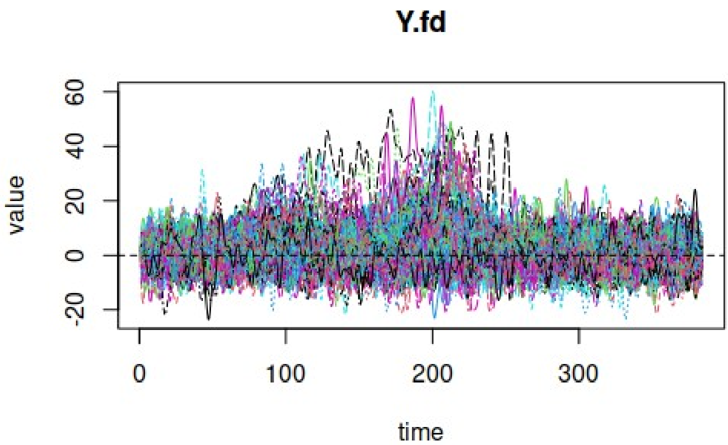
f)


Predicted yf2 values
y2f.hat
120
140
160
6
8
10
12
14
16
18
t[-(1:7)]

2.

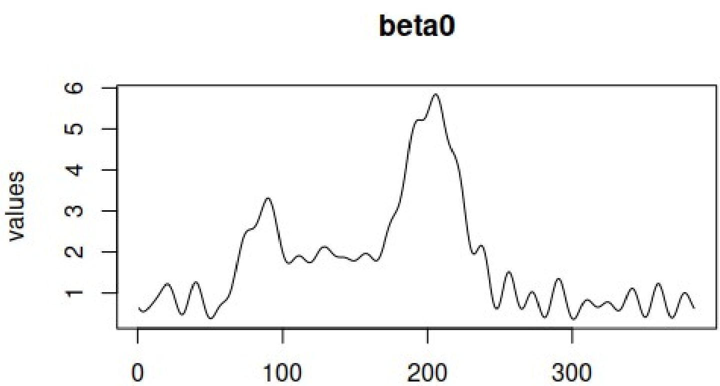
a)

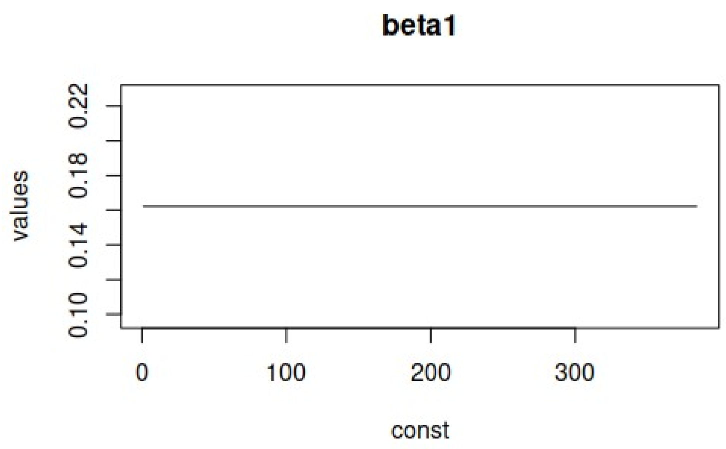


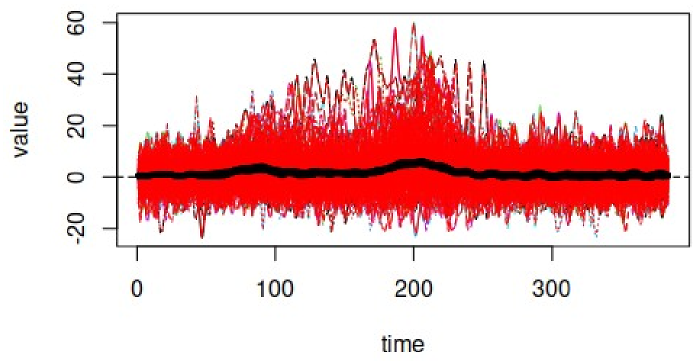


Coefficient estimates can be found from the code.

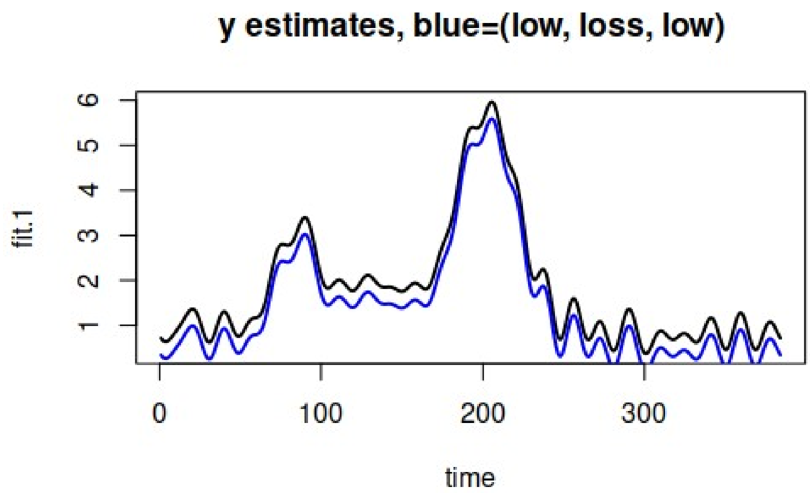
b)





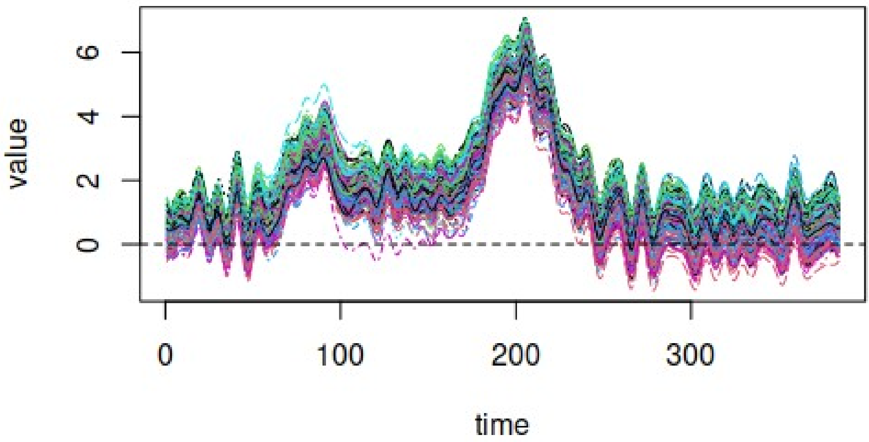


c)



The difference between functional estimates is quite small since I used constant functional models for $\beta_2(t), \beta_3(t), \beta_4(t)$.

d)



Quite poor fit compared to the observed y_i's. Didn't have time to make better modelling.

3.

a)

$$\begin{aligned}\frac{d}{dt}y_i(t) &= \frac{d}{dt}\phi(t)'\mathbf{c_i} \\ &= \frac{d}{dt}c_1 + \frac{d}{dt}c_2 sin(\omega t) + \frac{d}{dt}c_3 cos(\omega t) \\ &= c_2 cos(\omega t) - c_3 sin(\omega t)\end{aligned}$$

b)

$$\begin{aligned}\bar{y}(t) &= \hat{y_i}(t)'\frac{1}{n}\mathbf{1} \\ &= \left[\hat{y_1}(t), \hat{y_2}(t), ..., \hat{y_N}(t)\right]\frac{1}{n}\mathbf{1}\end{aligned}$$

$$\begin{aligned}\left[\hat{y_1}(t), \hat{y_2}(t), ..., \hat{y_N}(t)\right] &= \left[\phi(t)'\hat{\mathbf{c}}_1, \phi(t)'\hat{\mathbf{c}}_2, ..., \phi(t)'\hat{\mathbf{c}}_N\right] \\ &= \phi(t)'\left[\hat{\mathbf{c}}_1, \hat{\mathbf{c}}_2, ..., \hat{\mathbf{c}}_N\right] \\ &= \phi(t)'\left[(\Phi'\Phi)^{-1}\Phi'\mathbf{y}_1, (\Phi'\Phi)^{-1}\Phi'\mathbf{y}_2, ..., (\Phi'\Phi)^{-1}\Phi'\mathbf{y}_N,\right] \\ &= \phi(t)'(\Phi'\Phi)^{-1}\Phi'\left[\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_N,\right] \\ &= \phi(t)'(\Phi'\Phi)^{-1}\Phi'Y\end{aligned}$$

$$\begin{aligned}\bar{y}(t) &= \hat{y_i}(t)'\frac{1}{n}\mathbf{1} \\ &= \left[\hat{y_1}(t), \hat{y_2}(t), ..., \hat{y_N}(t)\right]\mathbf{1} \\ &= \frac{1}{n}\phi(t)'(\Phi'\Phi)^{-1}\Phi'Y\mathbf{1}\end{aligned}$$

c)

$$\frac{d}{d\mathbf{c}_i}(\mathbf{y}_i - \Phi\mathbf{c}_i)'(\mathbf{y}_i - \Phi\mathbf{c}_i) = -2\Phi'\mathbf{y}_i + 2\Phi'\Phi\mathbf{c}_i$$

$$\lambda\frac{d}{d\mathbf{c}_i}\mathbf{c}_i'R\mathbf{c}_i = 2\lambda R\mathbf{c}_i$$

$$\begin{aligned}-2\Phi'\mathbf{y}_i + 2\Phi'\Phi\hat{\mathbf{c}}_i + 2\lambda R\hat{\mathbf{c}}_i &= 0 \\ -\Phi'\mathbf{y}_i + (\Phi'\Phi + \lambda R)\hat{\mathbf{c}}_i &= 0 \\ (\Phi'\Phi + \lambda R)\hat{\mathbf{c}}_i &= \Phi'\mathbf{y}_i \\ \hat{\mathbf{c}}_i &= (\Phi'\Phi + \lambda R)^{-1}\Phi'\mathbf{y}_i\end{aligned}$$