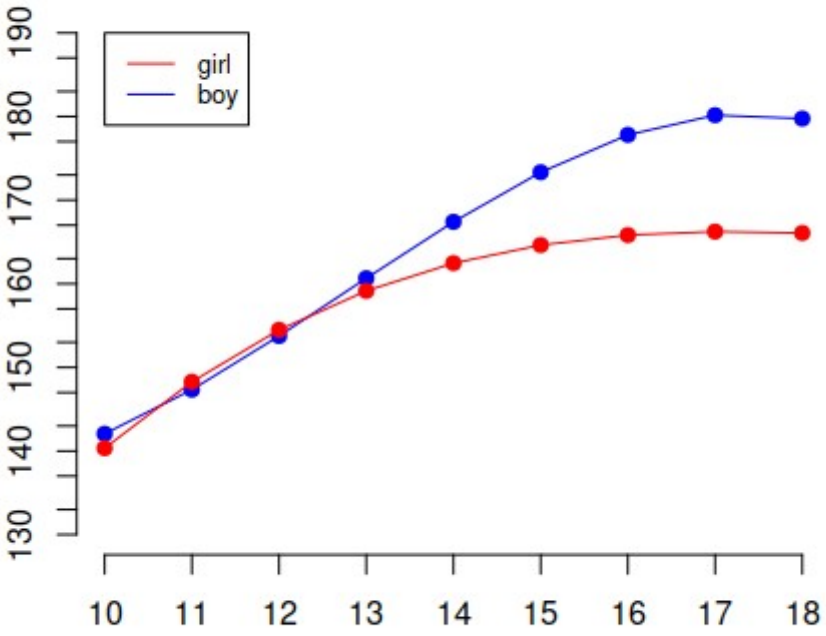
**Statistical Modelling 2**

**Weekly assignments 4**

**1.**

```
# a)
> Theta
                         t
factor(gender)boy  295.3 -46.0  4.15 -0.1085
factor(gender)girl -76.3  37.6 -1.92  0.0319

# b)
> mu.10
    [,1]
[1,]  166

# c) Plot given below
> BP = T%*%t(Theta)
> BP
     factor(gender)boy factor(gender)girl
 [1,]               142                140
 [2,]               147                148
 [3,]               154                154
 [4,]               161                159
 [5,]               167                162
 [6,]               173                165
 [7,]               178                166
 [8,]               180                166
 [9,]               180                166

# e)
> anova(model.H0, model.H1)
Analysis of Variance Table

Model 1: Y.star ~ 1
Model 2: Y.star ~ factor(gender) - 1
  Res.Df Df Gen.var. Pillai approx F num Df den Df Pr(>F)
1     92       0.192
2     91 -1    0.133  0.781     78.6      4     88 <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

→ Gender should be used in the model.
```

```
# e)
> mean(residuals(model.H0)^2)
[1] 56.6
> mean(residuals(model.H1)^2)
[1] 46.7
>
> AIC(model.H0)
[1] 5764
> AIC(model.H1)
[1] 5611
→ model H1 fits better

# f)
> lb
    [,1]
Y13  168
Y14  169
Y15  169
Y16  170
Y17  170
Y18  169
> BP
    [,1]
Y13  170
Y14  173
Y15  174
Y16  174
Y17  174
Y18  174
> ub
    [,1]
Y13  172
Y14  177
Y15  178
Y16  178
Y17  179
Y18  179
```


girl
boy
130
140
150
160
170
180
190
10
11
12
13
14
15
16
17
18

**3.**

a)

```
> mu.hat
    1
0.172
```

b)

Based on the classilac LM's OLS $\hat{\boldsymbol{\beta}} = (X'X)^{-1}X'\mathbf{y}$, we can deduce the following:

$$\begin{aligned}
\hat{vec}(\Theta) &= [(T \otimes X)'(T \otimes X)]^{-1}(T \otimes X)'vec(Y) \\
&= [(T' \otimes X')(T \otimes X)]^{-1}(T' \otimes X')vec(Y) \\
&= [(T'T) \otimes (X'X)]^{-1}vec(X'YT) \\
&= (T'T)^{-1} \otimes (X'X)^{-1}vec(X'YT) \\
&= vec((X'X)^{-1}X'YT(((T'T)^{-1})') \\
&= vec((X'X)^{-1}X'YT((T'T)')^{-1}) \\
&= vec((X'X)^{-1}X'YT(T'T)^{-1})
\end{aligned}$$