DATA.STAT.770

Exercise set 9.

E8.

Curvilinear Component Analysis cost function:

$$\sigma_r = \sum_{i<j} \left[d(x_i, x_j) - d(y_i, y_j)\right]^2 F(d(y_i, y_j), \lambda_y) = \frac{1}{2} \sum_i \sum_j \left[d(x_i, x_j) - d(y_i, y_j)\right]^2 F(d(y_i, y_j), \lambda_y)$$

<u>High-dimensional characteristics:</u>

$char_\chi : S(\mathbb{R}^N) \times \mathbb{R}^N \to S(\mathbb{R})$

In the case of CCA, the set $S(\mathbb{R}^N)$ corresponds to the all points measured in the training set of high dimensional data X and the second parameter corresponding to $\mathbb{R}^N$ is some $x_i \in X$. Therefore the domain of the characteristic function is the set of tuples $\{(x_1, x_i), \cdots, (x_n, x_i)\} \subseteq S(\mathbb{R}^N) \times \mathbb{R}^N$. Respectively the codomain of the function is the sequence of real numbers $\{r_1, \cdots, r_n\}$

The actual characteristic function of high dimensional data is then a mapping from $\{(x_1, x_i), \cdots, (x_n, x_i)\}$ to $\{r_1, \cdots, r_n\}$ defined as

$char_\chi(X, x_i) = \{d(x_1, x_i), \cdots, d(x_n, x_i)\}$

<u>Low-dimensional characteristics:</u>

$char_\xi : S(\mathbb{R}^M \times \mathbb{R}^N) \times (\mathbb{R}^M \times \mathbb{R}^N) \to S(\mathbb{R})$

In the case of CCA, the low dimensional characteristic function can be defined as

$char_\xi \equiv char_Y : S(\mathbb{R}^M) \times (\mathbb{R}^M) \to S(\mathbb{R})$.

Here, as in the case of high-dimensional characteristics, the domain is the set of tuples $\{(y_1, y_i), \cdots, (y_n, y_i)\} \subseteq S(\mathbb{R}^M) \times \mathbb{R}^M$, and the actual mapping is defined as

$char_Y(Y, y_i) = \{d(y_1, y_i), \cdots, d(y_n, y_i)\}$.

<u>Error function:</u>

$error : S(\mathbb{R}) \times S(\mathbb{R}) \to \mathbb{R}$

In the case of CCA, the domain of error function is $\{d(x_1, x_i), \cdots, d(x_n, x_i)\} \times \{d(y_1, y_i), \cdots, d(y_n, y_i)\} \subseteq S(\mathbb{R}) \times S(\mathbb{R})$ and the codomain is a set of real numbers. The actual error function is then defined as

$$error(char_\chi(X, x_i), char_Y(Y, y_i)) = \sum_j \left[d(x_i, x_j) - d(y_i, y_j)\right]^2 F(d(y_i, y_j), \lambda_y).$$