**DATA.STAT.770**

**Exercise set 3**

**1.**

Solution for the first principal component:

$Var(w_1^T X) = w_1^T Cov(X) w_1 = w_1^T C w_1 =$

Consider an optimization problem

$$
\begin{aligned}
&max\, w_1^T C w_1 \\
&s.t \\
&w_1^T w_1 = 1
\end{aligned}
$$

This can be solved by introducing Lagrange function $L(w_1, \lambda)$ and solving it's critical points.

$L(w_1, \lambda) = w_1^T C w_1 - \lambda(w_1^T w_1 - 1)$

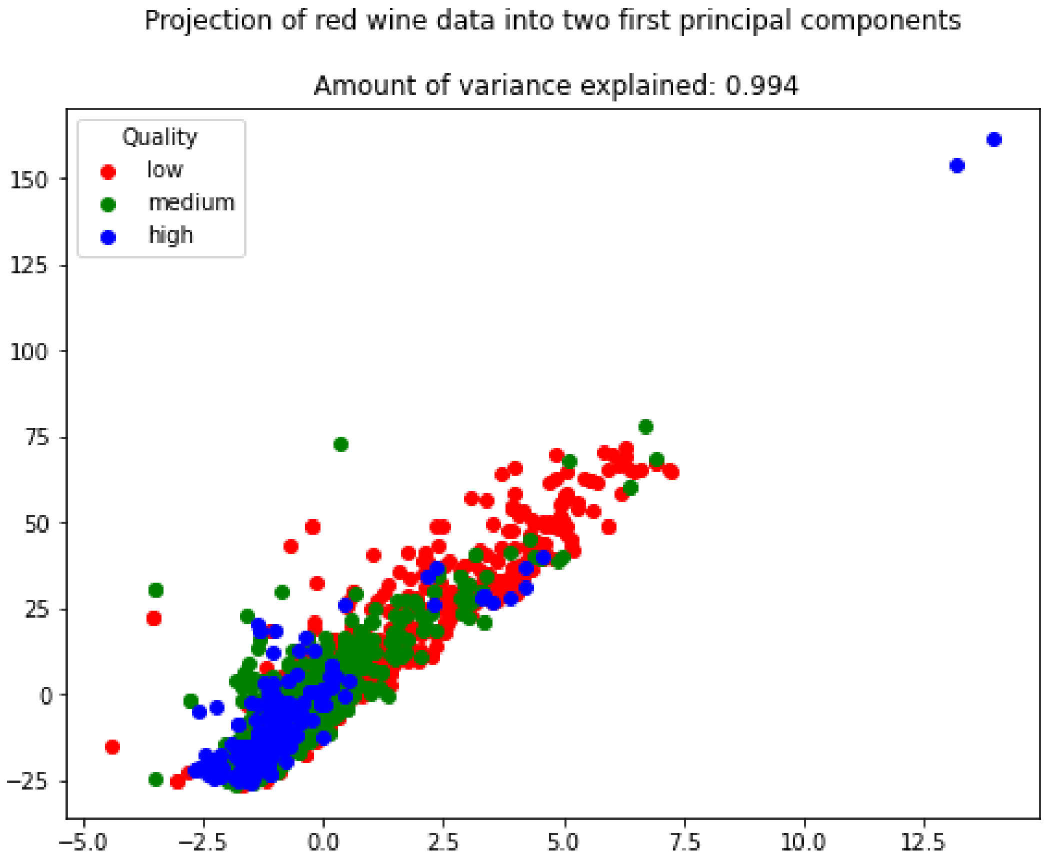
$$
\begin{aligned}
\nabla L &= \begin{pmatrix} \nabla_{w_1} L \\ \nabla_\lambda L \end{pmatrix} \\
&= \begin{pmatrix} 2w_1^T C - 2\lambda w_1^T \\ w_1^T w_1 - 1 \end{pmatrix} \\
&= \begin{pmatrix} 0 \\ 0 \end{pmatrix}
\end{aligned}
$$

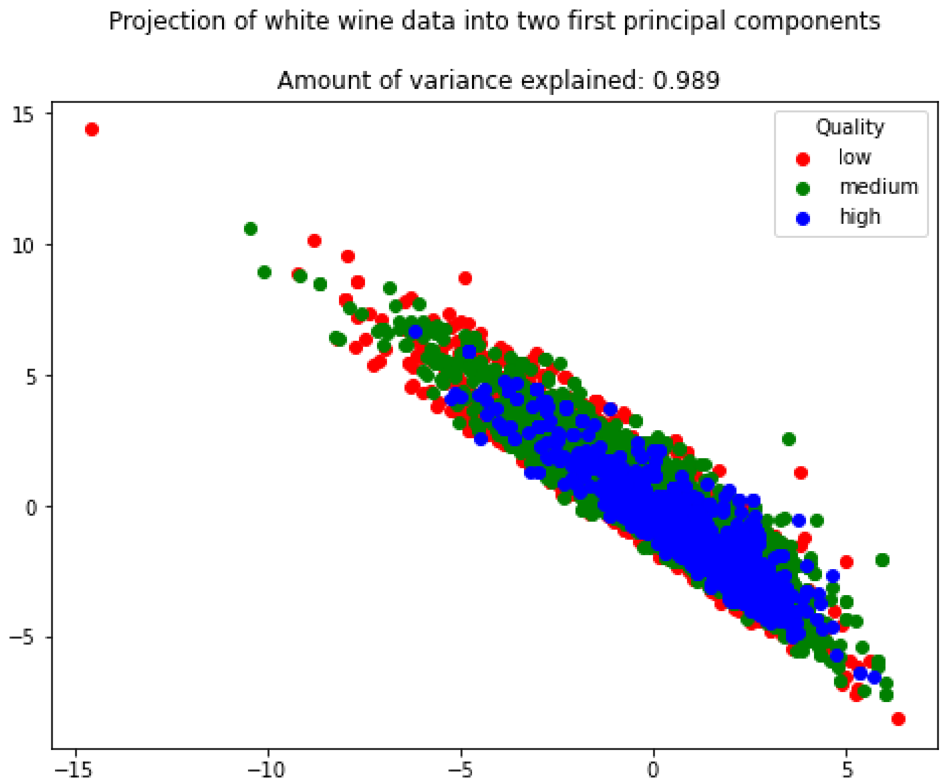
This reduces to

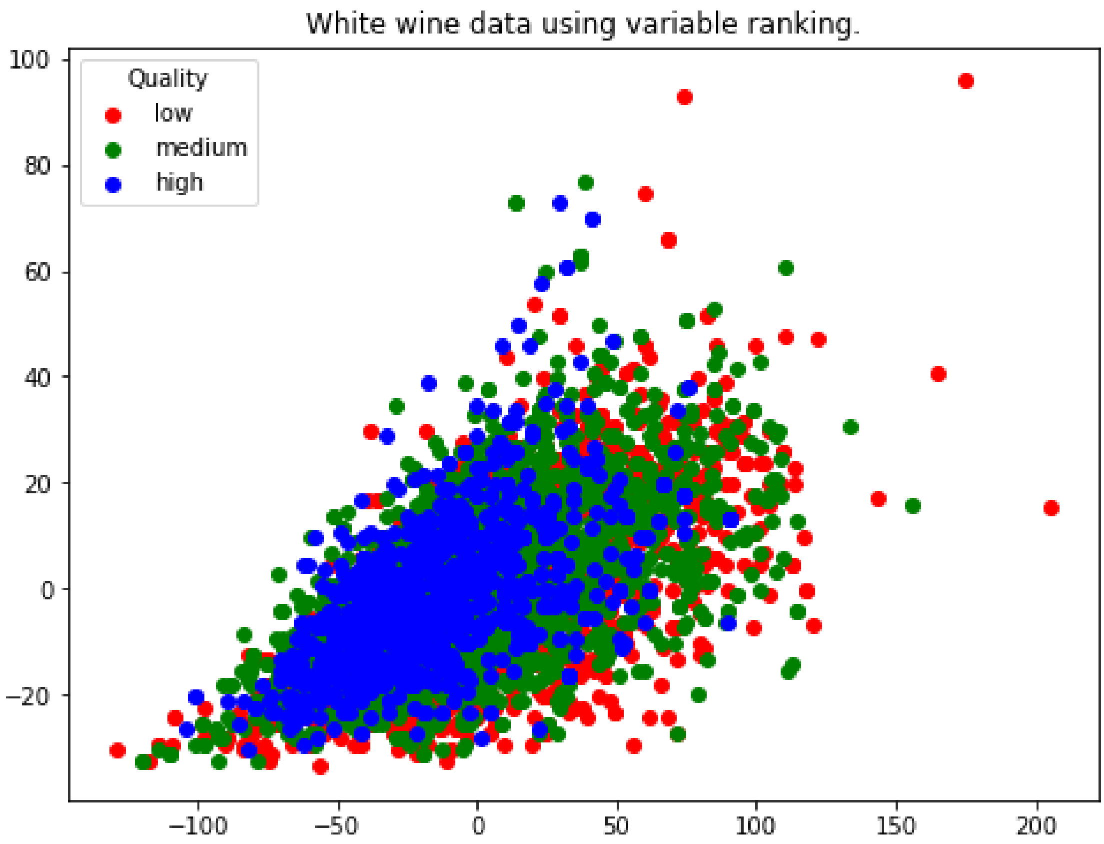
$$
\begin{aligned}
2w_1^T C - 2\lambda w_1^T &= 0 \\
C w_1 &= \lambda w_1
\end{aligned}
$$

Thus $w_1$ is the eigenvector of $C$ with eigenvalue $\lambda$. Because we want to maximize the $w_1^T C w_1 = w_1^T \lambda w_1 = \lambda w_1^T w_1$, we want to select the eigenvector with the largest eigenvalue.

**2.**


Projection of red wine data into two first principal components
Amount of variance explained: 0.994
Quality
low
medium
high
150
125
100
75
50
25
0
−25
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
12.5


Projection of white wine data into two first principal components
Amount of variance explained: 0.989
Quality
low
medium
high
15
10
5
0
−5
−15
−10
−5
0
5


White wine data using variable ranking.
Quality
low
medium
high
100
80
60
40
20
0
−20
−100
−50
0
50
100
150
200

**4.**

Time series of the first five seconds of audio