3.

Most likely state sequence is calculated by Viterbi algorithm as follows:

$N = \text{Number of timesteps } = 3$
$N_s = \text{Number of state values } = 3$
$\mathbf{x} = [x_1, x_2, x_3] = [\text{one, light, can}]$
$z_t^* = \text{most likely state at timestep } t$

For $t = 1$

$Best(1,1) = \pi_1 \beta_1(x_1) = 0.5 * 0.2 = 0.1$
$Best(1,2) = 0 * 0 = 0$
$Best(1,3) = 0.5 * 0.1 = 0.05$

For $t = 2$

$$\begin{aligned}
Best(2,1) &= max_j \; Best(1,j)\theta_{1|j}\beta_1(x_2) \\
&= Best(1,1)\theta_{1|1} * 0.25 \\
&= 7.5 * 10^{-3}
\end{aligned}$$

$BestZ(2,1) = 1$

$$\begin{aligned}
Best(2,2) &= max_j \; Best(1,j)\theta_{2|j}\beta_2(x_2) \\
&= Best(1,3)\theta_{2|3} * 0.3 \\
&= 0.05 * 1 * 0.3 \\
&= 0.015
\end{aligned}$$

$BestZ(2,2) = 3$

$$\begin{aligned}
Best(2,3) &= max_j \; Best(1,j)\theta_{3|j}\beta_3(x_2) \\
&= Best(1,1)\theta_{3|1} * 0.2 \\
&= 0.01
\end{aligned}$$

$BestZ(2,3) = 1$

For $t = 3$

$Best(3,1) = max_j \; Best(3,j)\theta_{1|j}\beta_1(x_3) = 0$

$$\begin{aligned}
Best(3,2) &= max_j \; Best(2,j)\theta_{2|j}\beta_2(x_3) \\
&= Best(2,2)\theta_{2|2} * 0.3 \\
&= 1.35 * 10^{-3}
\end{aligned}$$

$BestZ(3,2) = 2$

$$\begin{aligned}
Best(3,3) &= max_j \; Best(2,j)\theta_{3|j}\beta_3(x_3) \\
&= Best(2,1)\theta_{3|1} * 0.2 \\
&= 7.5 * 10^{-3} * 0.5 * 0.2 \\
&= 7.5 * 10^{-4}
\end{aligned}$$

$BestZ(3,3) = 1$

Therefore the most likely state sequence is: z_1, z_2, z_3 = 1, 3, 2