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Multistage Stochastic Programming Model for Electric Power Capacity Expansion Problem

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This paper is concerned with power system expansion planning under uncertainty. In our approach, integer programming and stochastic programming provide a basic framework. We develop a multistage stochastic programming model in which some of the variables are restricted to integer values. By utilizing the special property of the problem, called block separable recourse, the problem is transformed into a two-stage stochastic program with recourse. The electric power capacity expansion problem is reformulated as the problem with first stage integer variables and continuous second stage variables. We propose an L-shaped algorithm to solve the problem.

Key words: stochastic programming, optimization under uncertainty, electric power capacity expansion problem, L-shaped method, block separable recourse

1. Introduction

This paper is concerned with the capacity expansion planning of power systems under uncertainty. The basic objective of the capacity expansion planning is to determine an investment schedule for the installation of new generation plants and economic operations which ensure a reliable supply to the electricity demand.

Since the long-term planning problems involve uncertain data, methods which can deal with stochastic factors have been developed. In Murphy et al. [14], the uncertainty in demand is incorporated into stochastic programming with recourse. Dapkus and Bowe [6] developed a stochastic dynamic programming method for expansion planning. Gorenstin et al. [10] examined an approach to expansion planning based on stochastic programming and decision analysis. Shiina [15] developed a chance-constrained programming model for capacity expansion, where electricity demand is defined by a random variable. Electric power industry is undergoing restructuring and deregulation. It is necessary for electric power utilities or power generators to incorporate uncertainty into power generation planning. Hence, the development of an effective algorithm to solve the capacity expansion problem under uncertainty is required.

Stochastic programming (Birge and Louveaux [4]) deals with optimization under uncertainty. Birge [3] is a state-of-the-art survey in this field. A stochastic programming problem with recourse is referred to as a two-stage stochastic problem. In the first stage, a decision has to be made without complete information on random factors. After the value of random variables are known, recourse action can be taken in the second stage. For the continuous stochastic programming problem with recourse, an L-shaped method (Van Slyke and Wets [18]) is well-known. It came from the shape of the non-zeros in the constraint matrix. The L-shaped method was used to solve the stochastic concentrator location problems (Shiina [16, 17]). For a multistage stochastic programming with recourse, nested decomposition methods have been studied by Birge [2] in the linear case and Louveaux [12] in the quadratic case. Louveaux [13] introduced the concept of block-separable recourse. This property is essential for capacity expansion as described later. Birge et al. [5] explored a parallel implementation of the nested decomposition algorithm.

An important problem has been left unsolved in modeling the capacity expansion problem. The capacity expansion models we mentioned above are based on a stochastic linear programming problem with continuous decision variables. However, decision variables are restricted to integer values in some real problems. For example, the decision to build new plants or not is represented by a binary variable. To evaluate the exact investment and operation cost of power generation, we develop a multistage stochastic programming problem in which some of the decisions are binary variables.

A stochastic integer programming problem is a difficult problem to solve. If integer variables are involved in a recourse problem, optimality cuts do not provide facets of the epigraph of recourse function. It is difficult to approximate the recourse function of a multistage problem, since it is necessary to consider the nesting of integer programming problems. In the capacity expansion problem, integer variables are involved only in the decisions about the investment of new technology. By utilizing the special property called block separable recourse (Louveaux [13]), the decision variables are classified into two categories, the aggregate level decision and the detailed level decision. Binary decision variables are involved only in aggregate level decision. The algorithm we develop exploits this property and solves the problem efficiently.

In Section 2, we review the definition of multistage stochastic programming problems and block separable recourse. In Section 3, we show that the electric power capacity expansion problem is reformulated as the problem with first stage integer variables and continuous second stage variables. In Section 4, we present numerical results obtained from applying our solution approach to a power generating system.

2. Multistage Stochastic Programming Problem

We consider a multistage stochastic linear programming problem with stages $t = 0, 1, \ldots, H$ as

(Multistage Stochastic Linear Programming Problem)

$$\begin{array}{ll} \min & c^0 x^0 + E_{\xi^1}\left[\min \boldsymbol{c}^1 \boldsymbol{x}^1 + \cdots + E_{\xi^H|\xi^1\cdots\xi^{H-1}}[\min \boldsymbol{c}^H \boldsymbol{x}^H] \cdots\right] \\ \text{subject to} & W^0 x^0 = h^0 \\ & \boldsymbol{T}^0 x^0 + W^1 \boldsymbol{x}^1 = \boldsymbol{h}^1, \text{ a.s.} \\ & \vdots \\ & \boldsymbol{T}^{H-1} \boldsymbol{x}^{H-1} + W^H \boldsymbol{x}^H = \boldsymbol{h}^H, \text{ a.s.} \\ & x^0 \geq 0, \ \boldsymbol{x}^t \geq 0, \ t = 1, \ldots, H, \text{ a.s.,} \end{array}$$

where c^0 is a known vector in $\Re^{n_0}$, h^0 is a known vector in $\Re^{m_0}$, and each W^t is a known $m_t \times n_t$ matrix. Bold face vectors and matrices are possibly stochastic, where $\boldsymbol{c}^t$, $\boldsymbol{h}^t$, $\boldsymbol{T}^t$ are in $\Re^{n_t}$, $\Re^{m_t}$, $\Re^{m_t} \times \Re^{n_t}$, respectively. Let $\boldsymbol{x}^t$ denote decision vector in $\Re^{n_t}$ for stage t, $t = 1, \ldots, H$. They are chosen so that the constraints hold almost surely (denoted a.s.).

We assume the stochastic elements are defined over a finite discrete probability space $(\Xi, \sigma(\Xi), P)$, where $\Xi = \Xi^1 \times \cdots \times \Xi^H$ is the support of the random data in each stage with $\Xi^t = \{\xi^t_s = (T^t_s, h^t_s, c^t_s),\ s = 1, \ldots, k_t\}$ and (T^t_s, h^t_s, c^t_s) is a realization of $(\boldsymbol{T}^t, \boldsymbol{h}^t, \boldsymbol{c}^t)$. The possible sequences of the realization of random variables $(\xi^1, \ldots, \xi^H)$ are called scenario. The scenarios are often described using a scenario tree as shown in Fig. 1. In stages $t \leq H$, we have limited number of possible realizations which we call the stage t scenarios.

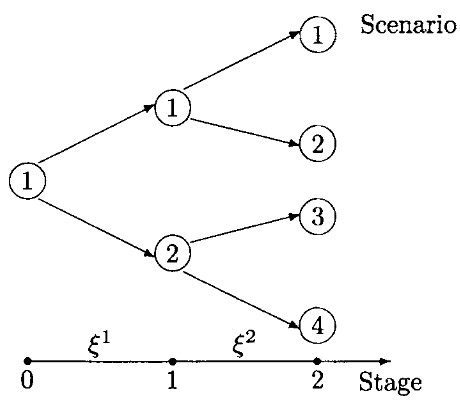

Fig. 1. Scenario tree.

In a scenario tree, the stage t scenario connected to the stage $t-1$ scenario s is referred to as a successor of stage $t-1$ scenario s. The set of all successors of stage $t-1$ scenario s is denoted by $D^t(s)$. Similarly, the predecessor of stage t

scenario s is denoted by $\alpha(s,t)$. These relationships are illustrated in Fig. 2.

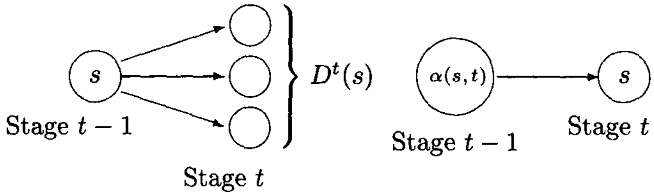

Fig. 2. Successor and predecessor.

We can formulate a deterministic equivalent problem for the multistage stochastic linear programming problem by replicating the constraints for each possible event in Ξ^t, $t = 1, \ldots, H$ since the probability space is finite and discrete. To solve the deterministic equivalent problem, the nested decomposition method (Birge [2], Birge et al. [5]) can be used. But this problem becomes large as the number of stages or the number of realizations increase. However in some problems, we can avoid difficulty in solving the problem if the problem has a special structure. The concept of block separable recourse was introduced by Louveaux [13].

DEFINITION 2.1. *A multistage stochastic linear program has* block separable recourse *if for all stages $t = 1, \ldots, H$ the decision vectors $\boldsymbol{x}^t$ can be written as $\boldsymbol{x}^t = (\boldsymbol{w}^t, \boldsymbol{y}^t)$ where $\boldsymbol{w}^t$ represents aggregate level decisions and $\boldsymbol{y}^t$ represents detailed level decisions. The constraints also follow these partitions:*

1. *The stage t objective contribution is $\mathbf{c}^t\boldsymbol{x}^t = \boldsymbol{r}^t\boldsymbol{w}^t + \boldsymbol{q}^t\boldsymbol{y}^t$.*
2. *The constraint matrix W^t is block diagonal: $W^t = \begin{pmatrix} A^t & 0 \\ 0 & B^t \end{pmatrix}$, where A^t is associated to the vector $\boldsymbol{w}^t$ and B^t to the vector $\boldsymbol{y}^t$.*
3. *The other components of the constraints are random but we assume that $\boldsymbol{T}^t$ and $\boldsymbol{h}^t$ can be writte: $\boldsymbol{T}^t = \begin{pmatrix} \boldsymbol{R}^t & 0 \\ \boldsymbol{S}^t & 0 \end{pmatrix}$ and $\boldsymbol{h}^t = \begin{pmatrix} \boldsymbol{b}^t \\ \boldsymbol{d}^t \end{pmatrix}$ to conform with the $(\boldsymbol{w}^t, \boldsymbol{y}^t)$ separation.*

If the problem has block separable recourse, the problem can be rewritten as follows.

(Multistage Stochastic Linear Programming Problem with Block Separable Recourse)

$$\begin{aligned} \min \quad & r^0 w^0 + q^0 y^0 \\ & + E_{\xi^1}\left[\min(\boldsymbol{r}^1\boldsymbol{w}^1 + \boldsymbol{q}^1\boldsymbol{y}^1)\right. \\ & \vdots \\ & + E_{\xi^H|\xi^1\ldots\xi^{H-1}}[\min(\boldsymbol{r}^H\boldsymbol{w}^H + \boldsymbol{q}^H\boldsymbol{y}^H)]\cdots] \end{aligned}$$

$$
\begin{aligned}
\text{subject to}\quad & A^0 w^0 = b^0 \\
& B^0 y^0 = d^0 \\
& \boldsymbol{R}^0 w^0 + A^1 \boldsymbol{w}^1 = \boldsymbol{b}^1, \text{ a.s.} \\
& \boldsymbol{S}^0 w^0 + B^1 \boldsymbol{y}^1 = \boldsymbol{d}^1, \text{ a.s.} \\
& \vdots \\
& \boldsymbol{R}^{H-1} \boldsymbol{w}^{H-1} + A^H \boldsymbol{w}^H = \boldsymbol{b}^H, \text{ a.s.} \\
& \boldsymbol{S}^{H-1} \boldsymbol{w}^{H-1} + B^H \boldsymbol{y}^H = \boldsymbol{d}^H, \text{ a.s.} \\
& w^0 \geq 0,\ \boldsymbol{w}^t \geq 0,\ t = 1, \ldots, H, \text{ a.s.} \\
& y^0 \geq 0,\ \boldsymbol{y}^t \geq 0,\ t = 1, \ldots, H, \text{ a.s.}
\end{aligned}
$$

The equivalence of multistage programs with block-separable recourse and two-stage programs was shown in Louveaux [13].

PROPOSITION 2.1 (Louveaux [13]). *A multistage stochastic program with block separable recourse is equivalent to a two stage stochastic program, where the first-stage is the extensive form of the aggregate level problems, and the value function of the second stage is the sum (weighted by the appropriate probabilities) of the detailed level recourse functions for all stage t scenarios, $t = 1, \ldots, H$.*

From the proposition, the multistage stochastic program with block separable recourse can be transformed into the two stage stochastic program with recourse. The electric power capacity expansion problem turns out to be the problem that has first stage integer variables and continuous second stage variables.

The deterministic equivalent problem for the multistage stochastic programming problem with block-separable recourse can be written as

(Deterministic Equivalent for Multistage Stochastic Linear Programming Problem with Block Separable Recourse)

$$
\begin{aligned}
\min\quad & r^{1,0} w^{1,0} + q^{1,0} y^{1,0} \\
& + \sum_{s=1}^{K_1} p_s^1 r^{s1} w^{s1} + \cdots + \sum_{s=1}^{K_H} p_s^H r^{sH} w^{sH} \\
& + \sum_{s=1}^{K_1} p_s^1 Q_s^1 (w^{1,0}) + \cdots + \sum_{s=1}^{K_H} p_s^H Q_s^H (w^{\alpha(s,H),H-1}) \\
\text{subject to}\quad & A^0 w^{1,0} = b^0 \\
& B^0 y^{1,0} = d^0 \\
& R^{\alpha(s,1),0} w^{s0} + A^1 w^{s1} = b^{s1},\ s = 1, \ldots, K_1 \\
& \vdots \\
& R^{\alpha(s,H),H-1} w^{s,H-1} + A^H w^{sH} = b^{sH},\ s = 1, \ldots, K_H \\
& w^0 \geq 0,\ w^{st} \geq 0,\ s = 1, \ldots, K_t,\ t = 1, \ldots, H \\
& Q_s^t (w^{\alpha(s,t),t-1}) = \min\{ q^{st} y^{st} \mid S^{\alpha(s,t),t-1} w^{\alpha(s,t),t-1} + B^t y^{st} = d^{st}, \\
& \qquad\qquad y^{st} \geq 0 \},\ s = 1, \ldots, K_t,\ t = 1, \ldots, H,
\end{aligned}
$$

where:

K_t	=	Cumulative number of realizations through stage t, $K_t = k_0 \times k_1 \times \cdots \times k_t$, $t = 1, \ldots, H$, $k_0 = 1$,
(r^{st}, q^{st})	=	Realization of random objective coefficents $(\boldsymbol{r}^t, \boldsymbol{q}^t)$ for stage t scenario s, $s = 1, \ldots, K_t$, $t = 0, 1, \ldots, H$, $(r^{1,0}, q^{1,0}) \equiv (r^0, q^0)$ for the sake of simplicity,
(R^{st}, S^{st})	=	Realization of random technology matrix $(\boldsymbol{R}^t, \boldsymbol{S}^t)$ for stage t scenario s, $s = 1, \ldots, K_t$, $t = 0, 1, \ldots, H-1$,
w^{st}	=	Aggregate decision vector to take in stage t for stage t scenario s, $s = 1, \ldots, K_t$, $t = 0, 1, \ldots, H$,
y^{st}	=	Detailed level decision vector to take in stage t for stage t scenario s, $s = 1, \ldots, K_t$, $t = 0, 1, \ldots, H$,
p_s^t	=	Probability that stage t scenario s occurs, $s = 1, \ldots, K_t$, $t = 1, \ldots, H$,
$\alpha(s,t)$	=	The predecessor or parent of stage t scenario s, $s = 1, \ldots, K_t$, $t = 1, \ldots, H$.

For the scenario tree of Fig. 1, the matrix structure of block separable recourse and the transformed two-stage stochastic programming problem are illustrated in Fig. 3 and Fig. 4, respectively.

Stage 0		Stage 1				Stage 2								
w^{10}	y^{10}	w^{11}	y^{11}	w^{21}	y^{21}	w^{12}	y^{12}	w^{22}	y^{22}	w^{32}	y^{32}	w^{42}	y^{42}	
A^0														b^0
	B^0													d^0
R^{10}		A^1												b^{11}
S^{10}			B^1											d^{11}
R^{10}				A^1										b^{21}
S^{10}					B^1									d^{21}
		R^{11}				A^2								b^{12}
		S^{11}					B^2							d^{12}
		R^{11}						A^2						b^{22}
		S^{11}							B^2					d^{22}
				R^{21}						A^2				b^{32}
				S^{21}							B^2			d^{32}
				R^{21}								A^2		b^{42}
				S^{21}									B^2	d^{42}

Fig. 3. Matrix structure of block separable recourse.

In Fig. 4, it is shown that the aggregate level decision w and the detailed level decision y are divided into stage 0 and stage 1, respectively.

			Stage 0							Stage 1				
w^{10}	w^{11}	w^{21}	w^{12}	w^{22}	w^{32}	w^{42}	y^{10}	y^{11}	y^{21}	y^{12}	y^{22}	y^{32}	y^{42}	
A^0														b^0
R^{10}	A^1													b^{11}
R^{10}		A^1												b^{21}
	R^{11}		A^2											b^{12}
	R^{11}			A^2										b^{22}
		R^{21}			A^2									b^{32}
		R^{21}				A^2								b^{42}
							B^0							d^0
S^{10}								B^1						d^{11}
S^{10}									B^1					d^{21}
	S^{11}									B^2				d^{12}
	S^{11}										B^2			d^{22}
		S^{21}										B^2		d^{32}
		S^{21}											B^2	d^{42}

Fig. 4. Matrix structure of transformed two-stage problem.

3. Electric Power Capacity Expansion

We consider the application of the multistage stochastic programming problem to the electric power capacity expansion problem. The basic objective of the problem is to determine an investment schedule of new technology and to operate power plants to ensure an economic and reliable supply to electricity demand. As an aid in long range system planning, production cost models are widely used to calculate a generation system's costs, requirements for new generation plants and fuel consumption. In expansion planning, load models which cover weeks, months or years are required. Wood and Wollenberg [19] classified the energy cost programs and load models as shown in Table 1.

Table 1. Energy production cost programs.

Load Model	Total Energy or Load Duration	Load Duration or Load Cycles	Load Duration or Load Cycles	Load Cycle
Long-Range Planning	Seasons or Years	Months or weeks	Months, Weeks or Days	Weeks or Days
Operation Planning	———	Months or weeks	Months, Weeks or Days	Weeks or Days

The load patterns are modeled by load cycles or load duration curves. For long-range planning, block approximations of load duration curves are used. A load duration curve represents the number of hours in which the load equals or exceeds the given load value. A typical load duration curve is illustrated in Fig. 5.

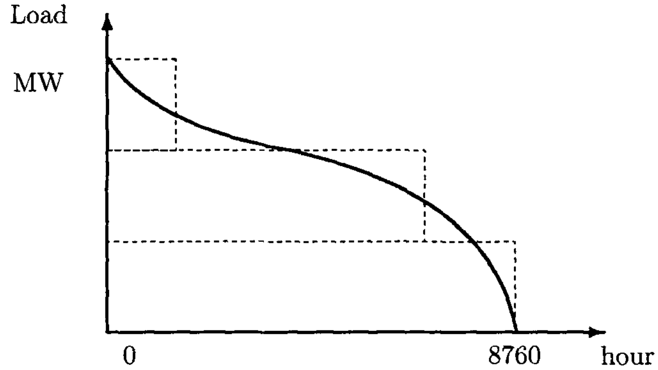

Fig. 5. Yearly load duration curve.

This types of problems have been considered many times over the past years. Anderson [1] is a review on the models used in electric power industry for determining investments. Delson and Shahidehpour [7] reviews how linear programming and mathematical programming methods are applied to the planning of capital investments in electric power generation. Kleinpeter [11] is a survey of the different energy models and energy planning methods. Frauendorfer [9] covers modeling subjects and mathematical programming methodologies in electric power systems.

In this problem, the investment cost, the operation cost or the electricity demand are regarded as random. The problem is formulated as a multistage stochastic programming problem in which the decision to install new technology or not is represented by a binary variable. Let

- $t = 0, 1, \ldots, H$ index the period of stages;
- $i = 1, \ldots, n$ index the available types of plants;
- $j = 1, \ldots, m$ index the operating modes in the load duration curve.

We also define the following:

- $a_i =$ availability factor of plant i;
- $g_i^t =$ existing capacity of plant i at stage t, decided before $t = 0$;
- $C_i^t =$ maximum capacity of plant i that can be installed at stage t;
- $\boldsymbol{r}_i^t =$ unit investment cost of plant i at stage t;
- $\boldsymbol{f}_i^t =$ fixed investment cost of plant i at stage t;
- $\boldsymbol{q}_i^t =$ unit production cost of plant i at stage t;
- $\boldsymbol{d}_j^t =$ maximal power demanded in mode j at stage t;
- $\tau_j^t =$ duration of mode j at stage t.

We consider the set of decisions as:

- $\boldsymbol{x}_i^t$ = new capacity made available for plant i at stage t;
- $\boldsymbol{w}_i^t$ = total capacity of plant i available at stage t;
- $\boldsymbol{v}_i^t = \begin{cases} 1, & \text{if new capacity for technology } i \text{ is installed at stage } t; \\ 0, & \text{otherwise;} \end{cases}$
- $\boldsymbol{y}_{ij}^t$ = generation level of plant i at stage t in mode j.

The multistage stochastic electric power capacity expansion problem is formulated as follows.

(Multistage Stochastic Electric Power Capacity Expansion Problem)

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{n} (r_i^0 w_i^0 + f_i^0 v_i^0) \\
& + E_{\xi^1}\left[\min \sum_{i=1}^{n}\left(\boldsymbol{r}_i^1 \boldsymbol{w}_i^1 + \boldsymbol{q}_i^1 \sum_{j=1}^{m} \boldsymbol{\tau}_j^1 \boldsymbol{y}_{ij}^1 + \boldsymbol{f}_i^1 \boldsymbol{v}_i^1\right)\right. \\
& \vdots \\
& + E_{\xi^H|\xi^1 \ldots \xi^{H-1}}\left[\min \sum_{i=1}^{n}\left(\boldsymbol{r}_i^H \boldsymbol{w}_i^H + \boldsymbol{q}_i^H \sum_{j=1}^{m} \boldsymbol{\tau}_j^H \boldsymbol{y}_{ij}^H + \boldsymbol{f}_i^H \boldsymbol{v}_i^H\right)\right] \cdots \Bigg] \\
\text{subject to} \quad & w_i^0 = x_i^0, \; i = 1, \ldots, n \\
& \boldsymbol{w}_i^1 = w_i^0 + \boldsymbol{x}_i^1, \; i = 1, \ldots, n, \; \text{a.s.} \\
& \boldsymbol{w}_i^t = \boldsymbol{w}_i^{t-1} + \boldsymbol{x}_i^t, \; i = 1, \ldots, n, \; t = 2, \ldots, H, \; \text{a.s.} \\
& x_i^0 \le C_i^0 v_i^0, \; i = 1, \ldots, n \\
& \boldsymbol{x}_i^t \le C_i^t \boldsymbol{v}_i^t, \; i = 1, \ldots, n, \; t = 1, \ldots, H, \; \text{a.s.} \\
& \sum_{i=1}^{n} \boldsymbol{y}_{ij}^t = \boldsymbol{d}_j^t, \; j = 1, \ldots, m, \; t = 1, \ldots, H, \; \text{a.s.} \\
& \sum_{j=1}^{m} \boldsymbol{y}_{ij}^t \le a_i(g_i^t + \boldsymbol{w}_i^{t-1}), \; i = 1, \ldots, n, \; t = 1, \ldots, H, \; \text{a.s.} \\
& v_i^0 \in \{0, 1\}, \; \boldsymbol{v}_i^t \in \{0, 1\}, \; i = 1, \ldots, n, \; t = 1, \ldots, H, \; \text{a.s.} \\
& w_i^0 \ge 0, \; \boldsymbol{w}_i^t \ge 0, \; i = 1, \ldots, n, \; t = 1, \ldots, H, \; \text{a.s.} \\
& x_i^0 \ge 0, \; \boldsymbol{x}_i^t \ge 0, \; i = 1, \ldots, n, \; t = 1, \ldots, H, \; \text{a.s.} \\
& \boldsymbol{y}_{ij}^t \ge 0, \; i = 1, \ldots, n, \; j = 1, \ldots, m, \; t = 1, \ldots, H, \; \text{a.s.}
\end{aligned}
$$

Since the problem has the property of block separable recourse from Proposition 1, the decision variables v_i^0, $\boldsymbol{v}_i^t$, w_i^0, $\boldsymbol{w}_i^t$, x_i^0, $\boldsymbol{x}_i^t$, $i = 1, \ldots, n$, $t = 1, \ldots, H$ and $\boldsymbol{y}_{ij}^t \ge 0$, $i = 1, \ldots, n$, $j = 1, \ldots, m$, $t = 1, \ldots, H$ correspond to the aggregate level decisions and the detailed level decisions, respectively. The structure of constraint matrix at stage t is shown in Fig. 6, where I denotes an identity matrix and $e^\top = (1, \ldots, 1)$. From Fig. 6, it is evident that the problem has block separable recourse.

$$
\left(\begin{array}{c|c|c|c|c|c}
I & 0 & 0 & 0 & \cdots & 0 \\ \hline
0 & 0 & 0 & 0 & \cdots & 0 \\ \hline
0 & 0 & 0 & 0 & \cdots & 0 \\ \hline
\begin{matrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_n \end{matrix} & 0 & 0 & 0 & \cdots & 0
\end{array}\right)
\left(\begin{array}{c}
\boldsymbol{w}^{t-1} \\ \boldsymbol{x}^{t-1} \\ \boldsymbol{v}^{t-1} \\ \hline \boldsymbol{y}_1^{t-1} \\ \vdots \\ \boldsymbol{y}_n^{t-1}
\end{array}\right)
\begin{array}{l}
\left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{aggregate level decisions} \\
\left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{detailed level decisions}
\end{array}
$$

$$
+\left(\begin{array}{c|c|c|c|c|c}
-I & I & 0 & 0 & \cdots & 0 \\ \hline
0 & -I & \begin{matrix} C_1^t & & 0 \\ & \ddots & \\ 0 & & C_n^t \end{matrix} & 0 & \cdots & 0 \\ \hline
0 & 0 & 0 & I & \cdots & I \\ \hline
0 & 0 & 0 & \begin{matrix} -e^{\top} \\ 0 \end{matrix} & \cdots & \begin{matrix} 0 \\ -e^{\top} \end{matrix}
\end{array}\right)
\left(\begin{array}{c}
\boldsymbol{w}^{t} \\ \boldsymbol{x}^{t} \\ \boldsymbol{v}^{t} \\ \hline \boldsymbol{y}_1^{t} \\ \vdots \\ \boldsymbol{y}_n^{t}
\end{array}\right)
\begin{array}{c} = \\ \geq \\ = \\ \\ \geq \\ \\ \end{array}
\left(\begin{array}{c}
0 \\ \hline 0 \\ \hline \boldsymbol{d}^t \\ \hline -a_1 g_1^t \\ \vdots \\ -a_n g_n^t
\end{array}\right)
$$

Fig. 6. Structure of constraint matrix at stage t.

We assume the stochastic elements in bold face are defined over a finite discrete probability space $(\Xi, \sigma(\Xi), P)$, where $\Xi = \Xi^1 \times \cdots \times \Xi^H$ is the support of the random data in each period with $\Xi^t = \{\xi_s^t = (r_i^t, f_i^t, q_i^t, d_j^t, \tau_j^t),\ s = 1, \ldots, k_t\}$ and $(r_i^t, f_i^t, q_i^t, d_j^t, \tau_j^t)$ is a realization of $(\boldsymbol{r}_i^t, \boldsymbol{f}_i^t, \boldsymbol{q}_i^t, \boldsymbol{d}_j^t, \boldsymbol{\tau}_j^t)$.

The deterministic equivalent problem for the multistage stochastic electric power capacity expansion problem can be written as

(Deterministic Equivalent for
Multistage Stochastic Electric Power Capacity Expansion Problem)

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{n} (r_i^{1,0} w_i^{1,0} + f_i^{1,0} v_i^{1,0}) \\
& + \sum_{s=1}^{K_1} p_s^1 \sum_{i=1}^{n} (r_i^{s1} w_i^{s1} + f_i^{s1} v_i^{s1}) \cdots + \sum_{s=1}^{K_H} p_s^H \sum_{i=1}^{n} (r_i^{sH} w_i^{sH} + f_i^{sH} v_i^{sH}) \\
& + \sum_{s=1}^{K_1} p_s^1 Q_s^1(w^{1,0}) + \cdots + \sum_{s=1}^{K_H} p_s^H Q_s^H(w^{\alpha(s,H),H-1}) \\
\text{subject to} \quad & w_i^{1,0} = x_i^{1,0},\ i = 1, \ldots, n \\
& w_i^{st} = w_i^{\alpha(s,t),t-1} + x_i^{st},\ i = 1, \ldots, n,\ s = 1, \ldots, K_t,\ t = 1, \ldots, H \\
& x_i^{st} \leq C_i^t v_i^{st},\ i = 1, \ldots, n,\ s = 1, \ldots, K_t,\ t = 0, 1, \ldots, H \\
& v_i^{st} \in \{0, 1\},\ i = 1, \ldots, n,\ s = 1, \ldots, K_t,\ t = 0, 1, \ldots, H
\end{aligned}
$$

$$w_i^{st} \geq 0, \; i = 1, \ldots, n, \; s = 1, \ldots, K_t, \; t = 0, 1, \ldots, H$$
$$x_i^{st} \geq 0, \; i = 1, \ldots, n, \; s = 1, \ldots, K_t, \; t = 0, 1, \ldots, H$$
$$\begin{aligned} Q_s^t(w^{\alpha(s,t),t-1}) = \min \Big\{ & \sum_{i=1}^{n} \sum_{j=1}^{m} q_i^{st} \tau_j^{st} y_{ij}^{st} \mid \\ & \sum_{i=1}^{n} y_{ij}^{st} = d_j^{st}, \; j = 1, \ldots, m \\ & \sum_{j=1}^{m} y_{ij}^{st} \leq a_i(g_i^t + w_i^{\alpha(s,t),t-1}), \; i = 1, \ldots, n \\ & y_{ij}^{st} \geq 0, \; i = 1, \ldots, n, \; j = 1, \ldots, m \Big\}, \\ & s = 1, \ldots, K_t, \; t = 1, \ldots, H, \end{aligned}$$

where:

K_t = Cumulative number of realizations through stage t, $K_t = k_0 \times k_1 \times \cdots \times k_t$, $t = 1, \ldots, H$, $k_0 = 1$,

(r^{st}, q^{st}, f^{st}) = Realization of random objective coefficent vectors $(\boldsymbol{r}^t, \boldsymbol{q}^t, \boldsymbol{f}^t)$ for stage t scenario s, $s = 1, \ldots, K_t$, $t = 0, 1, \ldots, H$, $(r^{1,0}, f^{1,0}) \equiv (r^0, f^0)$ for the sake of simplicity,

(τ^{st}, d^{st}) = Realization of random duration and demand vectors $(\boldsymbol{\tau}^{st}, \boldsymbol{d}^t)$ for stage t scenario s, $s = 1, \ldots, K_t$, $t = 1, \ldots, H$,

w^{st} = Total capacity decision vector to take in stage t for stage t scenario s, $s = 1, \ldots, K_t$, $t = 0, 1, \ldots, H$,

x^{st} = New capacity decision vector to take in stage t for stage t scenario s, $s = 1, \ldots, K_t$, $t = 0, 1, \ldots, H$,

v^{st} = Binary decision vector to take in stage t for stage t scenario s, $s = 1, \ldots, K_t$, $t = 0, 1, \ldots, H$,

y^{st} = Generation level decision vector to take in stage t for stage t scenario s, $s = 1, \ldots, K_t$, $t = 1, \ldots, H$,

p_s^t = Probability that stage t scenario s occurs, $s = 1, \ldots, K_t$, $t = 1, \ldots, H$,

$\alpha(s,t)$ = The predecessor or parent of stage t scenario s, $s = 1, \ldots, K_t$, $t = 1, \ldots, H$.

The electric power capacity expansion problem can be transformed to the problem that has first stage integer variables and continuous second stage variables. To solve the problem, the following master problem is formulated.

(Master Problem for Multistage Stochastic Electric Power Capacity Expansion Problem)

$$\begin{aligned}
\min \quad & \sum_{i=1}^{n}(r_i^{1,0}w_i^{1,0} + f_i^{1,0}v_i^{1,0}) \\
& + \sum_{s=1}^{K_1} p_s^1 \sum_{i=1}^{n}(r_i^{s1}w_i^{s1} + f_i^{s1}v_i^{s1}) \cdots + \sum_{s=1}^{K_H} p_s^H \sum_{i=1}^{n}(r_i^{sH}w_i^{sH} + f_i^{sH}v_i^{sH}) \\
& + \sum_{s=1}^{K_1} p_s^1\theta_s^1 + \cdots + \sum_{s=1}^{K_H} p_s^H\theta_s^H \\
\text{subject to} \quad & w_i^{1,0} = x_i^{1,0},\ i = 1,\ldots,n \\
& w_i^{st} = w_i^{\alpha(s,t),t-1} + x_i^{st},\ i = 1,\ldots,n,\ s = 1,\ldots,K_t,\ t = 1,\ldots,H \\
& x_i^{st} \le C_i^t v_i^{st},\ i = 1,\ldots,n,\ s = 1,\ldots,K_t,\ t = 0,1,\ldots,H \\
& v_i^{st} \in \{0,1\},\ i = 1,\ldots,n,\ s = 1,\ldots,K_t,\ t = 0,1,\ldots,H \\
& w_i^{st} \ge 0,\ i = 1,\ldots,n,\ s = 1,\ldots,K_t,\ t = 0,1,\ldots,H \\
& x_i^{st} \ge 0,\ i = 1,\ldots,n,\ s = 1,\ldots,K_t,\ t = 0,1,\ldots,H \\
& \theta_s^t \ge Q_s^t(w^{\alpha(s,t),t-1}),\ s = 1,\ldots,K_t,\ t = 1,\ldots,H
\end{aligned}$$

In this formulation, the recourse functions $Q_s^t(w^{\alpha(s,t),t-1})$, $s = 1,\ldots,K_t$, $t = 1,\ldots,H$ are not known explicitly in advance. Therefore, the optimality cuts are added to approximate $\theta_s^t \ge Q_s^t(w^{\alpha(s,t),t-1})$. The optimal solution to the master problem is obtained by solving the mixed integer programming problem. Let v_i^{*st}, w_i^{*st}, x_i^{*st}, $i = 1,\ldots,n$, $s = 0,1,\ldots,K_t$, $t = 1,\ldots,H$, θ_s^{*t}, $s = 1,\ldots,K_t$, $t = 1,\ldots,H$, be the optimal solution to the master problem. Then the recourse problem for stage t scenario s is solved at the optimal solution of the master problem, $w_i^{*\alpha(s,t),t-1}$, $i = 1,\ldots,n$.

(Recourse Problem for Stage t Scenario s)

$$\begin{aligned}
& Q_s^t(w^{\alpha(s,t),t-1}) \\
& \quad = \min\Big\{ \sum_{i=1}^{n}\sum_{j=1}^{m} q_i^{st}\tau_j^{st}y_{ij}^{st} \ \Big|\ \sum_{i=1}^{n} y_{ij}^{st} = d_j^{st},\ j = 1,\ldots,m \\
& \qquad\qquad \sum_{j=1}^{m} y_{ij}^{st} \le a_i(g_i^t + w_i^{\alpha(s,t),t-1}),\ i = 1,\ldots,n \\
& \qquad\qquad y_{ij}^{st} \ge 0,\ i = 1,\ldots,n,\ j = 1,\ldots,m\Big\}, \\
& \qquad\qquad s = 1,\ldots,K_t,\ t = 1,\ldots,H
\end{aligned}$$

If the recourse problem is infeasible, the feasibility cut is added to the formulation of the master problem. If the solution of the master problem and the recourse problem do not satisfy the inequality $\theta_s^{*t} \ge Q_s^t(w^{*\alpha(s,t),t-1})$, the optimality cut which approximates $Q_s^t(w^{\alpha(s,t),t-1})$ is added to the master problem. The dual problem to the recourse problem for stage t scenario s is described as follows.

(Dual Problem to Recourse Problem for Stage t Scenario s)

$$\begin{aligned} \max \quad & \sum_{j=1}^{m} d_j^{st}\lambda_j^{st} - \sum_{i=1}^{n} a_i(g_i^t + w_i^{\alpha(s,t),t-1})\mu_i^{st} \\ \text{subject to} \quad & \lambda_j^{st} - \mu_i^{st} \le q_i^{st}\tau_j^{st},\ j=1,\dots,m,\ i=1,\dots,n \\ & \mu_i \ge 0,\ i=1,\dots,n \end{aligned}$$

If $Q_s^t(w^{*\alpha(s,t),t-1}) = +\infty$, then $w^{*\alpha(s,t),t-1}$ is not feasible with respect to all constraints of the multistage stochastic electric power capacity expansion problem. By the duality theory, we have $\tilde\lambda_j^{st}$, $j = 1,\dots,m$, $\tilde\mu_i^{st}\,(\ge 0)$, $i = 1,\dots,n$ so that $\sum_{j=1}^m d_j^{st}\tilde\lambda_j^{st} - \sum_{i=1}^n a_i(g_i^t + w_i^{*\alpha(s,t),t-1})\tilde\mu_i^{st} > 0$ and $\tilde\lambda_j^{st} - \tilde\mu_i^{st} \le 0$. For any feasible $w^{\alpha(s,t),t-1}$, there must exist a $y^{st} \ge 0$ such that $\sum_{i=1}^n y_{ij}^{st} = d_j^{st}$, $j = 1,\dots,m$, $\sum_{j=1}^m y_{ij}^{st} \le a_i(g_i^t + w_i^{\alpha(s,t),t-1})$, $i = 1,\dots,n$. Scalar multiplication of these constraints by $\tilde\lambda_j^{st}$, $j = 1,\dots,m$, $\tilde\mu_i^{st}$, $i = 1,\dots,n$ yields

$$\begin{aligned} \sum_{j=1}^{m} d_j^{st}\tilde\lambda_j^{st} - \sum_{i=1}^{n} a_i(g_i^t + w_i^{\alpha(s,t),t-1})\tilde\mu_i^{st} &\le \sum_{j=1}^{m}\sum_{i=1}^{n}\tilde\lambda_j^{st}y_{ij}^{st} - \sum_{i=1}^{n}\sum_{j=1}^{m}\tilde\mu_i^{st}y_{ij}^{st} \\ &= \sum_{j=1}^{m}\sum_{i=1}^{n}(\tilde\lambda_j^{st} - \tilde\mu_i^{st})y_{ij}^{st} \le 0 \end{aligned}$$

which cuts off $w^{*\alpha(s,t),t-1}$, since $\sum_{j=1}^m d_j^{st}\tilde\lambda_j^{st} - \sum_{i=1}^n a_i(g_i^t + w_i^{*\alpha(s,t),t-1})\tilde\mu_i^{st} > 0$.

$$\textbf{Feasibility Cut:}\quad \sum_{j=1}^{m} d_j^{st}\tilde\lambda_j^{st} - \sum_{i=1}^{n} a_i(g_i^t + w_i^{\alpha(s,t),t-1})\tilde\mu_i^{st} \le 0 \qquad (1)$$

If $Q_s^t(w^{*\alpha(s,t),t-1})$ is finite, we have the optimal primal solution y_{ij}^{*st}, $i = 1,\dots,n$, $j = 1,\dots,m$ and the optimal dual solution λ_j^{*st}, $j = 1,\dots,m$, μ^{*st}, $i = 1,\dots,n$. The following inequality holds.

$$\begin{aligned} \theta_s^t &\ge Q_s^t(w^{*\alpha(s,t),t-1}) \\ &= \max\left\{ \sum_{j=1}^{m} d_j^{st}\lambda_j^{st} - \sum_{i=1}^{n} a_i(g_i^t + w_i^{*\alpha(s,t),t-1})\mu_i^{st} \,\middle|\, \begin{array}{ll} \lambda_j^{st} - \mu_i^{st} \le q_i^{st}\tau_j^{st}, & j=1,\dots,m, \\ & i=1,\dots,n \\ \mu_i \ge 0, & i=1,\dots,n \end{array} \right\} \\ &= \sum_{j=1}^{m} d_j^{st}\lambda_j^{*st} - \sum_{i=1}^{n} a_i(g_i^t + w_i^{*\alpha(s,t),t-1})\mu_i^{*st} \end{aligned}$$

To approximate the recourse function, the optimality cut which cuts off $(w^{*\alpha(s,t),t-1}, \theta_s^{*t})$ so that $\theta_s^{*t} < Q_s^t(w^{*\alpha(s,t),t-1})$ is added to the formulation of the master problem.

$$\textbf{Optimality Cut:} \quad \theta_s^t \geq \sum_{j=1}^{m} d_j^{st} \lambda_j^{*st} - \sum_{i=1}^{n} a_i (g_i^t + w_i^{\alpha(s,t),t-1}) \mu_i^{*st} \tag{2}$$

The algorithm of the L-shaped method for multistage stochastic electric power capacity expansion problem is shown in Fig. 7. Since all of the integer variables are aggregate level decision variables, the master problem becomes a mixed integer programming problem. The upper bound for the optimal objective value is calculated as follows.

$$\begin{aligned}
&\text{Upper Bound} \\
&= \sum_{i=1}^{n} (r_i^{1,0} w_i^{*1,0} + f_i^{1,0} v_i^{*1,0}) \\
&+ \sum_{s=1}^{K_1} p_s^1 \sum_{i=1}^{n} (r_i^{s1} w_i^{*s1} + f_i^{s1} v_i^{*s1}) \cdots + \sum_{s=1}^{K_H} p_s^H \sum_{i=1}^{n} (r_i^{sH} w_i^{*sH} + f_i^{sH} v_i^{*sH}) \\
&+ \sum_{s=1}^{K_1} p_s^1 Q_s^1 (w^{*1,0}) + \cdots + \sum_{s=1}^{K_H} p_s^H Q_s^H (w^{*\alpha(s,H),H-1})
\end{aligned} \tag{3}$$

The value of upper bound for the optimal objective value can be adopted as the approximate optimal objective value.

- **Step 1. Solve Master Problem**
  Solve the mixed integer programming master problem by branch and bound method. Let w_i^{*st}, $i = 1, \ldots, n$, $s = 0, 1, \ldots, K_t$, $t = 1, \ldots, H$, θ_s^{*t}, $s = 1, \ldots, K_t$, $t = 1, \ldots, H$ be the optimal solution to the master problem.
- **Step 2. Solve Recourse Problem**
  Solve the recourse problem for stage t scenario s, $s = 1, \ldots, K_t$, $t = 1, \ldots, H$.
- **Step 3. Add Feasibility Cuts**
  If the recourse problem for stage t scenario s is infeasible, the feasibility cut (1) is added to the formulation of the master problem. Go to Step 1.
- **Step 4. Add Optimality Cuts**
  Calculate $Q_{s'}^t(w^{*s,t-1})$, $\forall s' \in D^t(s)$, $s = 1, \ldots, K_t$, $t = 0, \ldots, H-1$. If $\theta_{s'}^{*t} < (1-\varepsilon) Q_{s'}^t(w^{*s,t-1})$, $s' \in D^t(s)$, the optimality cut (2) is added to the formulation of the master problem ($\varepsilon > 0$: tolerance). Go to Step 1.
- **Step 5. Convergence Check**
  If no optimality cuts are added, then stop.

Fig. 7. Algorithm of the L-shaped method for multistage stochastic electric power capacity expansion problem.

4. Numerical Experiments

The L-shaped method for the multistage stochastic electric power capacity expansion problem was implemented using C language on SUN Enterprise 420R. The whole framework of the algorithm was coded in C. The mathematical programming problems were described using AMPL [8] and solved by linear programming/branch-and-bound solver CPLEX 7.0. We consider the following two problems.

- **Problem 1.** $n = 20,\ m = 4,\ H = 4,\ K_4 = 8.$
- **Problem 2.** $n = 60,\ m = 4,\ H = 6,\ K_6 = 16.$

In both problems, the load patterns are modeled by yearly load duration curves. The scenarios are generated from the power demand of stage 1 scenario 1 by adding an increase in demand as shown in Table 2.

Table 2. Demand Increases for different scenarios.

Problem 1		stage					
Scenario	Probability	1	2	3	4		
1	0.125	0	0	0	+10%		
2	0.125	0	0	+10%	+20%		
3	0.125	0	+10%	0	+10%		
4	0.125	0	+10%	+10%	+20%		
5	0.125	+10%	0	0	+10%		
6	0.125	+10%	0	+10%	+20%		
7	0.125	+10%	+10%	0	+10%		
8	0.125	+10%	+10%	+10%	+20%		
Problem 2		stage					
Scenario	Probability	1	2	3	4	5	6
1	0.0625	0	0	0	+10%	+10%	+20%
2	0.0625	0	0	0	+10%	+30%	+30%
3	0.0625	0	0	+10%	+20%	+10%	+20%
4	0.0625	0	0	+10%	+20%	+30%	+30%
5	0.0625	0	+10%	0	+10%	+10%	+20%
6	0.0625	0	+10%	0	+10%	+30%	+30%
7	0.0625	0	+10%	+10%	+20%	+10%	+20%
8	0.0625	0	+10%	+10%	+20%	+30%	+30%
9	0.0625	+10%	0	0	+10%	+10%	+20%
10	0.0625	+10%	0	0	+10%	+30%	+30%
11	0.0625	+10%	0	+10%	+20%	+10%	+20%
12	0.0625	+10%	0	+10%	+20%	+30%	+30%
13	0.0625	+10%	+10%	0	+10%	+10%	+20%
14	0.0625	+10%	+10%	0	+10%	+30%	+30%
15	0.0625	+10%	+10%	+10%	+20%	+10%	+20%
16	0.0625	+10%	+10%	+10%	+20%	+30%	+30%

The structure of the scenario trees of Problem 1 and 2 are shown in Fig. 8.

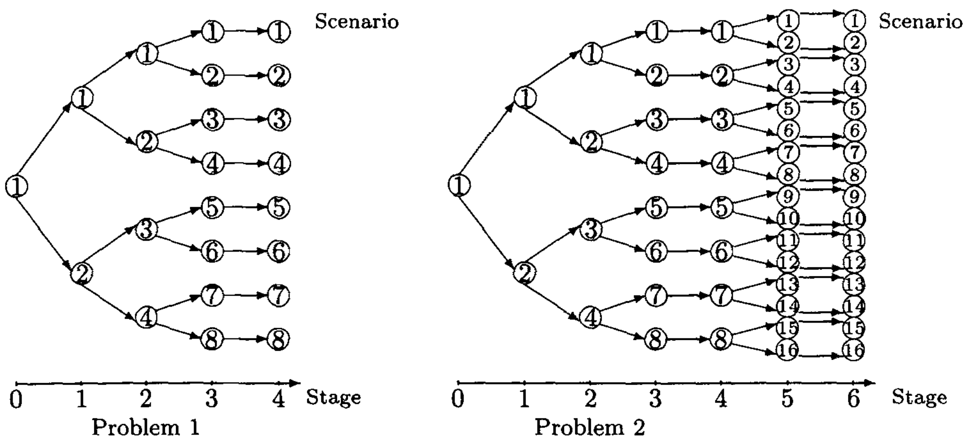

Fig. 8. Scenario tree of Problem 1 and 2.

We compare the L-shaped method with the branch-and-bound method for the deterministic equivalent problem. The L-shaped method is applied to the master problem. We set $\varepsilon = 0.01(\%)$ in step 4 of the L-shaped method. In solving the master problem or the deterministic equivalent problem using branch-and-bound, the search is terminated when the best value of lower bound times $(1 + 10^{-4})$ is greater than or equal to the best integer objective value. From the optimal solution of the master problem, the upper bound for the optimal objective value can be calculated. The results of the numerical experiments are shown in Table 3 and 4. From the numerical experiments, it appears that sufficient results were obtained. It is noticed that the L-shaped method requires far less solution time than the exact branch-and-bound method, and the same objective value is obtained. The results show that the integer L-shaped method performs reasonably well on relatively large problems.

Table 3. Results for Problem 1.

Iteration Number	Master Problem Optimal Objective Value	Number of Added Cuts
1	66375.0	4 feasibility cuts
2	69610	22 optimality cuts
3	98043	11 optimality cuts
4	98099	0 cuts
Approximate Cost by L-shaped	98103	CPU time 152 (sec)
Optimal Cost by branch-and-bound	98103	CPU time 355 (sec)

Table 4. Results for Problem 2.

Iteration Number	Master Problem Optimal Objective Value	Number of Added Cuts
1	118799	28 feasibility cuts
2	142859	54 optimality cuts
3	170384	2 optimality cuts
4	170391	0 cuts
Approximate Cost by L-shaped	170393	CPU time 322 (sec)
Optimal Cost by branch-and-bound	170393	CPU time 1898 (sec)

As for the total capacity, the same solution is obtained by branch-and-bound and L-shaped method as shown in Table 5.

Table 5. Total capacity.

Stage	Scenario	Problem 1 Optimal Capacity by B & B	Problem 1 Approximate Capacity by L-shaped	Problem 1 Demand	Problem 2 Optimal Capacity by B & B	Problem 2 Approximate Capacity by L-shaped	Problem 2 Demand
1	1	255	255	230	315	315	280
1	2	255	255	253	315	315	308
2	1	255	255	230	315	315	280
2	2	255	255	253	315	315	308
2	3	255	255	230	315	315	280
2	4	255	255	253	315	315	308
3	1	255	255	230	315	315	280
3	2	255	255	253	315	315	308
3	3	255	255	230	315	315	280
3	4	255	255	253	315	315	308
3	5	255	255	230	315	315	280
3	6	255	255	253	315	315	308
3	7	255	255	230	315	315	280
3	8	255	255	253	315	315	308
4	1	255	255	253	335	335	308
4	2	276	276	276	339	339	336
4	3	255	255	253	335	335	308
4	4	276	276	276	339	339	336
4	5	255	255	253	335	335	308
4	6	276	276	276	339	339	336
4	7	255	255	253	335	335	308
4	8	276	276	276	339	339	336
5	1				364	364	308
5	2				364	364	364
5	3				364	364	308
5	4				364	364	364
5	5				364	364	308
5	6				364	364	364
5	7				364	364	308
5	8				364	364	364
5	9				364	364	308
5	10				364	364	364
5	11				364	364	308
5	12				364	364	364
5	13				364	364	308
5	14				364	364	364
5	15				364	364	308
5	16				364	364	364
6	1				364	364	336
6	2				364	364	364
6	3				364	364	336
6	4				364	364	364
6	5				364	364	336
6	6				364	364	364
6	7				364	364	336
6	8				364	364	364
6	9				364	364	336
6	10				364	364	364
6	11				364	364	336
6	12				364	364	364
6	13				364	364	336
6	14				364	364	364
6	15				364	364	336
6	16				364	364	364

5. Concluding Remarks

We developed a multistage stochastic programming model for the electric power capacity expansion problem. By utilizing the property of block separable recourse, the L-shaped method solves the capacity expansion problem effectively. The following point is left as a future problem. In the short term planning of power generation, it is necessary to incorporate a setup cost for the operation of a power generating unit. To model such a system, we have to consider the stochastic programming problem in which binary decision variables are involved in the detailed level decisions. It is a worthwhile problem to investigate.

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