The details of the numerical setup are presented in Section ??.

Each element has $m_V = 9$ vertices so in total $ndof_V \times m_V = 18$ velocity dofs and $ndof_P * m_P = 4$ pressure dofs. The total number of velocity dofs is therefore $NfemV = nnp \times ndofV$ while the total number of pressure dofs is $NfemP = nel$. The total number of dofs is then $Nfem = NfemV + NfemP$.

As a consequence, matrix $\mathbb{K}$ has size $NfemV, NfemV$ and matrix $\mathbb{G}$ has size $NfemV, NfemP$. Vector f is of size $NfemV$ and vector h is of size $NfemP$.

[redacted]

renumber all nodes to start at zero!! Also internal numbering does not work here

The velocity shape functions are given by:

$$
\begin{aligned}
N_0^V &= \frac{1}{2}r(r-1)\frac{1}{2}s(s-1) \\
N_1^V &= \frac{1}{2}r(r+1)\frac{1}{2}s(s-1) \\
N_2^V &= \frac{1}{2}r(r+1)\frac{1}{2}s(s+1) \\
N_3^V &= \frac{1}{2}r(r-1)\frac{1}{2}s(s+1) \\
N_4^V &= (1-r^2)\frac{1}{2}s(s-1) \\
N_5^V &= \frac{1}{2}r(r+1)(1-s^2) \\
N_6^V &= (1-r^2)\frac{1}{2}s(s+1) \\
N_7^V &= \frac{1}{2}r(r-1)(1-s^2) \\
N_8^V &= (1-r^2)(1-s^2)
\end{aligned}
$$

and their derivatives:

$$
\begin{aligned}
\frac{\partial N_0^V}{\partial r} &= \frac{1}{2}(2r-1)\frac{1}{2}s(s-1) \\
\frac{\partial N_1^V}{\partial r} &= \frac{1}{2}(2r+1)\frac{1}{2}s(s-1) \\
\frac{\partial N_2^V}{\partial r} &= \frac{1}{2}(2r+1)\frac{1}{2}s(s+1) \\
\frac{\partial N_3^V}{\partial r} &= \frac{1}{2}(2r-1)\frac{1}{2}s(s+1) \\
\frac{\partial N_4^V}{\partial r} &= (-2r)\frac{1}{2}s(s-1) \\
\frac{\partial N_5^V}{\partial r} &= \frac{1}{2}(2r+1)(1-s^2) \\
\frac{\partial N_6^V}{\partial r} &= (-2r)\frac{1}{2}s(s+1) \\
\frac{\partial N_7^V}{\partial r} &= \frac{1}{2}(2r-1)(1-s^2) \\
\frac{\partial N_8^V}{\partial r} &= (-2r)(1-s^2)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial N_0^V}{\partial s} &= \frac{1}{2}r(r-1)\frac{1}{2}(2s-1) \\
\frac{\partial N_1^V}{\partial s} &= \frac{1}{2}r(r+1)\frac{1}{2}(2s-1) \\
\frac{\partial N_2^V}{\partial s} &= \frac{1}{2}r(r+1)\frac{1}{2}(2s+1) \\
\frac{\partial N_3^V}{\partial s} &= \frac{1}{2}r(r-1)\frac{1}{2}(2s+1) \\
\frac{\partial N_4^V}{\partial s} &= (1-r^2)\frac{1}{2}(2s-1) \\
\frac{\partial N_5^V}{\partial s} &= \frac{1}{2}r(r+1)(-2s) \\
\frac{\partial N_6^V}{\partial s} &= (1-r^2)\frac{1}{2}(2s+1) \\
\frac{\partial N_7^V}{\partial s} &= \frac{1}{2}r(r-1)(-2s) \\
\frac{\partial N_8^V}{\partial s} &= (1-r^2)(-2s)
\end{aligned}
$$

features

- $Q_2 \times Q_1$ element
- incompressible flow
- mixed formulation
- Dirichlet boundary conditions (no-slip)
- isothermal
- isoviscous
- analytical solution

[redacted]