The details of the numerical setup are presented in Section ??.

Each element has $m_V = 16$ vertices so in total $ndof_V \times m_V = 32$ velocity dofs and $ndof_P * m_P = 9$ pressure dofs. The total number of velocity dofs is therefore $NfemV = nnp \times ndofV$ while the total number of pressure dofs is $NfemP = nel$. The total number of dofs is then $Nfem = NfemV + NfemP$.

As a consequence, matrix $\mathbb{K}$ has size $NfemV, NfemV$ and matrix $\mathbb{G}$ has size $NfemV, NfemP$. Vector f is of size $NfemV$ and vector h is of size $NfemP$.

```
60===61===62===63===64===65===66===67===68===70
||             ||             ||             ||
50   51   52   53   54   55   56   57   58   59
||             ||             ||             ||
40   41   42   43   44   45   46   47   48   49
||             ||             ||             ||
30===31===32===33===34===35===36===37===38===39
||             ||             ||             ||
20   21   22   23   24   25   26   27   28   29
||             ||             ||             ||
10   11   12   13   14   15   16   17   18   19
||             ||             ||             ||
00===01===02===03===04===05===06===07===08===09

Example of 3x2 mesh. nnx=10, nny=7, nnp=70, nelx=3, nely=2, nel=6

12===13===14===15            06=====07=====08
||   ||   ||   ||            ||     ||     ||
08===09===10===11            ||     ||     ||
||   ||   ||   ||            03=====04=====05
04===05===06===07            ||     ||     ||
||   ||   ||   ||            ||     ||     ||
00===01===02===03            00=====01=====02

Velocity (Q3)                Pressure (Q2)

(r,s)_{00}=(-1,-1)           (r,s)_{00}=(-1,-1)
(r,s)_{01}=(-1/3,-1)         (r,s)_{01}=(0,-1)
(r,s)_{02}=(+1/3,-1)         (r,s)_{02}=(+1,-1)
(r,s)_{03}=(+1,-1)           (r,s)_{03}=(-1,0)
(r,s)_{04}=(-1,-1/3)         (r,s)_{04}=(0,0)
(r,s)_{05}=(-1/3,-1/3)       (r,s)_{05}=(+1,0)
(r,s)_{06}=(+1/3,-1/3)       (r,s)_{06}=(-1,+1)
(r,s)_{07}=(+1,-1/3)         (r,s)_{07}=(0,+1)
(r,s)_{08}=(-1,+1/3)         (r,s)_{08}=(+1,+1)
(r,s)_{09}=(-1/3,+1/3)
(r,s)_{10}=(+1/3,+1/3)
(r,s)_{11}=(+1,+1/3)
(r,s)_{12}=(-1,+1)
(r,s)_{13}=(-1/3,+1)
(r,s)_{14}=(+1/3,+1)
(r,s)_{15}=(+1,+1)
```

The velocity shape functions are given by:

$$
\begin{aligned}
N_1(r) &= (-1 + r + 9r^2 - 9r^3)/16 \\
N_2(r) &= (+9 - 27r - 9r^2 + 27r^3)/16 \\
N_3(r) &= (+9 + 27r - 9r^2 - 27r^3)/16 \\
N_4(r) &= (-1 - r + 9r^2 + 9r^3)/16
\end{aligned}
$$

$$
\begin{aligned}
N_1(t) &= (-1 + t + 9t^2 - 9t^3)/16 \\
N_2(t) &= (+9 - 27t - 9t^2 + 27t^3)/16 \\
N_3(t) &= (+9 + 27t - 9t^2 - 27t^3)/16 \\
N_4(t) &= (-1 - t + 9t^2 + 9t^3)/16
\end{aligned}
$$

$$N_01(r,t) = N_1(r)N_1(t) \tag{1}$$
$$N_02(r,t) = N_2(r)N_1(t) \tag{2}$$
$$N_03(r,t) = N_3(r)N_1(t) \tag{3}$$
$$N_04(r,t) = N_4(r)N_1(t) \tag{4}$$
$$N_05(r,t) = N_1(r)N_2(t) \tag{5}$$
$$N_06(r,t) = N_2(r)N_2(t) \tag{6}$$
$$N_07(r,t) = N_3(r)N_2(t) \tag{7}$$
$$N_08(r,t) = N_4(r)N_2(t) \tag{8}$$
$$N_09(r,t) = N_1(r)N_3(t) \tag{9}$$
$$N_10(r,t) = N_2(r)N_3(t) \tag{10}$$
$$N_11(r,t) = N_3(r)N_3(t) \tag{11}$$
$$N_12(r,t) = N_4(r)N_3(t) \tag{12}$$
$$N_13(r,t) = N_1(r)N_4(t) \tag{13}$$
$$N_14(r,t) = N_2(r)N_4(t) \tag{14}$$
$$N_15(r,t) = N_3(r)N_4(t) \tag{15}$$
$$N_16(r,t) = N_4(r)N_4(t) \tag{16}$$

and their derivatives:
Write about 4 point quadrature.

features

- $Q_3 \times Q_2$ element
- incompressible flow
- mixed formulation
- isothermal
- isoviscous
- analytical solution

velocity error rate is cubic, pressure superconvergent since the pressure field is quadratic and therefore lies into the Q2 space.