This benchmark deals with the 2-D thermal convection of a fluid of infinite Prandtl number in a rectangular closed cell. In what follows, I carry out the case 1a, 1b, and 1c experiments as shown in [?]: steady convection with constant viscosity in a square box.

The temperature is fixed to zero on top and to ΔT at the bottom, with reflecting symmetry at the sidewalls (i.e. $\partial_x T = 0$) and there are no internal heat sources. Free-slip conditions are implemented on all boundaries.

The Rayleigh number is given by

$$Ra = \frac{\alpha g_y \Delta T h^3}{\kappa \nu} = \frac{\alpha g_y \Delta T h^3 \rho^2 c_p}{k \mu} \tag{1}$$

In what follows, I use the following parameter values: $L_x = L_y = 1$,$\rho_0 = c_P = k = \mu = 1$, $T_0 = 0$, $\alpha = 10^{-2}$, $g = 10^2 Ra$ and I run the model with $Ra = 10^4, 10^5$ and 10^6.

The initial temperature field is given by

$$T(x,y) = (1-y) - 0.01\cos(\pi x)\sin(\pi z) \tag{2}$$

The perturbation in the initial temperature fields leads to a perturbation of the density field and sets the fluid in motion.

Depending on the initial Rayleigh number, the system ultimately reaches a steady state after some time.

The Nusselt number (i.e. the mean surface temperature gradient over mean bottom temperature) is computed as follows [?]:

$$Nu = L_y \frac{\int \frac{\partial T}{\partial y}(y = L_y) dx}{\int T(y=0) dx} \tag{3}$$

Note that in our case the denominator is equal to 1 since $L_x = 1$ and the temperature at the bottom is prescribed to be 1.

Finally, the steady state root mean square velocity and Nusselt number measurements are indicated in Table **??** alongside those of [?] and [?]. (Note that this benchmark was also carried out and published in other publications [?, ?, ?, ?, ?] but since they did not provide a complete set of measurement values, they are not included in the table.)

		Blankenbach et al	Tackley [?]
$Ra = 10^4$	V_{rms}	42.864947 ± 0.000020	42.775
	Nu	4.884409 ± 0.000010	4.878
$Ra = 10^5$	V_{rms}	193.21454 ± 0.00010	193.11
	Nu	10.534095 ± 0.000010	10.531
$Ra = 10^6$	V_{rms}	833.98977 ± 0.00020	833.55
	Nu	21.972465 ± 0.000020	21.998

Steady state Nusselt number Nu and V_{rms} measurements as reported in the literature.

features

- $Q_1 \times P_0$ element
- incompressible flow
- penalty formulation
- Dirichlet boundary conditions (free-slip)
- Boussinesq approximation
- direct solver
- non-isothermal
- buoyancy-driven flow
- isoviscous
- CFL-condition

ToDo:
implement steady state criterion
reach steady state
do Ra=1e4, 1e5, 1e6
plot against blankenbach paper and aspect
look at critical Ra number