EE2211 Tutorial 3

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Q1

The random variable N has probability mass function (PMF)

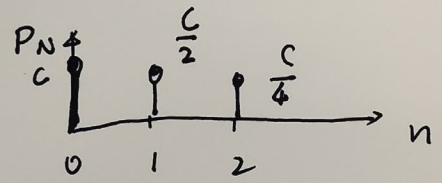
$$P_N(n) = \begin{cases} c\left(\frac{1}{2}\right)^n, & n = 0,1,2 \\ 0, & \text{otherwise} \end{cases}$$

(a) What is the value of the constant *c*?

(b) What is $Pr[N \leq 1]$?

Q1

Answer:



(a) We wish to find the value of c that makes the PMF sum up to one

$$\sum_{n=0}^{2} P_N(n) = c + \frac{c}{2} + \frac{c}{4} = 1 \Rightarrow c = \frac{4}{7}$$

(b) The probability that $N \leq 1$ is

$$Pr[N \leq 1] = Pr[N=0] + Pr[N=1] = c + \frac{c}{2}$$

$$= \frac{4}{7} + \frac{2}{7} = \frac{6}{7}$$

Q2

The random variable X has probability density function (PDF)

$$f_X = \begin{cases} cx, & 0 \leq x \leq 2 \\ 0, & \text{otherwise.} \end{cases}$$

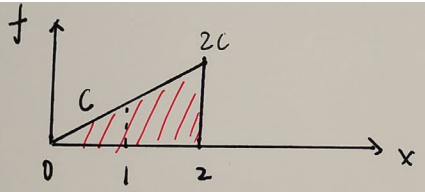
Use the PDF to find

(a) The constant c,

(b) $Pr[0 \leq X \leq 1]$

(c) $Pr\left[-\frac{1}{2} \leq X \leq \frac{1}{2}\right]$

Q2



(a) From the above PDF, we can determine the value of c by integrating the PDF and setting it equal to 1

$$\int_0^2 cx\,dx = \frac{1}{2}cx^2\Big|_0^2 = \frac{1}{2}c\cdot 4 - 0 = 2c = 1$$

$$\Rightarrow \quad c = 1/2$$

(b) $Pr[0 \le X \le 1] = \int_0^1 x/2\,dx = \frac{1}{4}x^2\Big|_0^1 = 1/4$

(c) $Pr[-1/2 \le X \le 1/2] = \int_0^{1/2} x/2\,dx = \frac{1}{4}x^2\Big|_0^{1/2} = \frac{1}{4}(1/2)^2 - 0$

$$= \frac{1}{16}$$

Q3

Let A = {resistor is within 50Ω of the nominal value}. The probability that a resistor is from machine B is Pr[B] = 0.3. The probability that a resistor is acceptable, i.e., within 50 Ω of the nominal value, is Pr[A] = 0.78. Given that a resistor is from machine B, the conditional probability that it is acceptable is Pr[A|B] = 0.6. What is the probability that an acceptable resistor comes from machine B?

Q3

Step 1: Identify the events and given probability.

- Let A be the event that a resistor is acceptable (within 50 Ω of the nominal value)
- Let B be the event that a resistor is from machine B

then we have

$$Pr[B] = 0.3$$

$$Pr[A] = 0.78$$

$Pr[A|B] = 0.6$ (The conditional probability that a resistor is acceptable given that it is from machine B)

Step 2: Use Bayes's Theorem

$$Pr[B|A] = \frac{Pr[A|B] \times Pr[B]}{Pr[A]}$$

$$\Rightarrow Pr[B|A] = \frac{0.6 \times 0.3}{0.78}$$

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Q4

Consider tossing a fair six-sided die. There are only six outcomes possible, Ω = {1, 2, 3, 4, 5, 6}. Suppose we toss two dice and assume that each throw is independent.

(a) What is the probability that the sum of the dice equals seven?

i. List out all pairs of possible outcomes together with their sums from the two throws. (hint: enumerate all the items in range(1,7))

ii. Collect all of the (a, b) pairs that sum to each of the possible values from two to twelve (including the sum equals seven). (hint: use dictionary from collections import default dict to collect all of the (a, b) pairs that sum to each of the possible values from two to twelve)

(b) What is the probability that half the product of three dice will exceed their sum?

Q4

(a)

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Step 1: determine the sample space

- Each die has 6 faces, so when we rolling two dice, the total outcomes is

$$6 \times 6 = 36$$

step 2: list all pairs that sum to 7

(1, 6)
(2, 5)
(3, 4)
(4, 3)
(5, 2)
(6, 1)

There are 6 pairs that satisfy this condition

step 3: calculate probability

$$Pr(\text{sum} = 7) = \frac{6}{36}$$

Q4

(b) Please read thought the code with detailed comments

Q5

Assuming a normal (Gaussian) distribution with mean 30 Ω and standard deviation of 1.8 Ω, determine the probability that a resistor coming off the production line will be within the range of 28 Ω to 33 Ω. (Hint: use stats.norm.cdf function from scipy import stats)

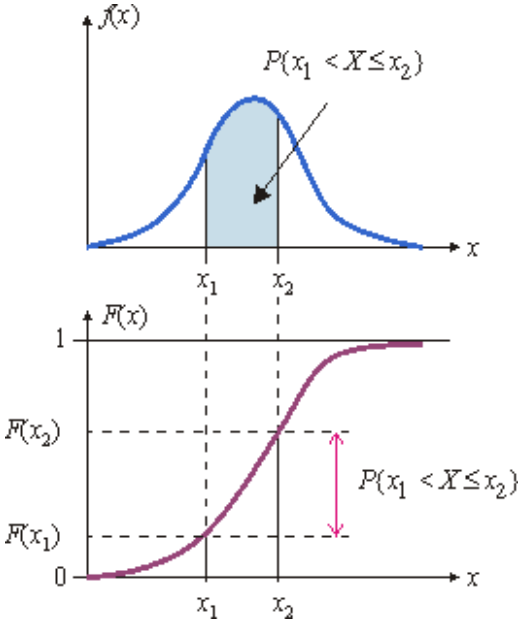
Q5

Answer:
Wiki: The cumulative distribution function of a real-valued random variable X is the function given by

$$F_X(x) = Pr(X \leq x)$$

where the right-hand side represents the probability that the random variable X takes on a value less than or equal to x. The probability that X lies in the semi-closed interval (a, b], where a < b, is therefore

$$Pr(a < X \leq b) = F_X(b) - F_X(a).$$



Please read thought the code with detailed comments

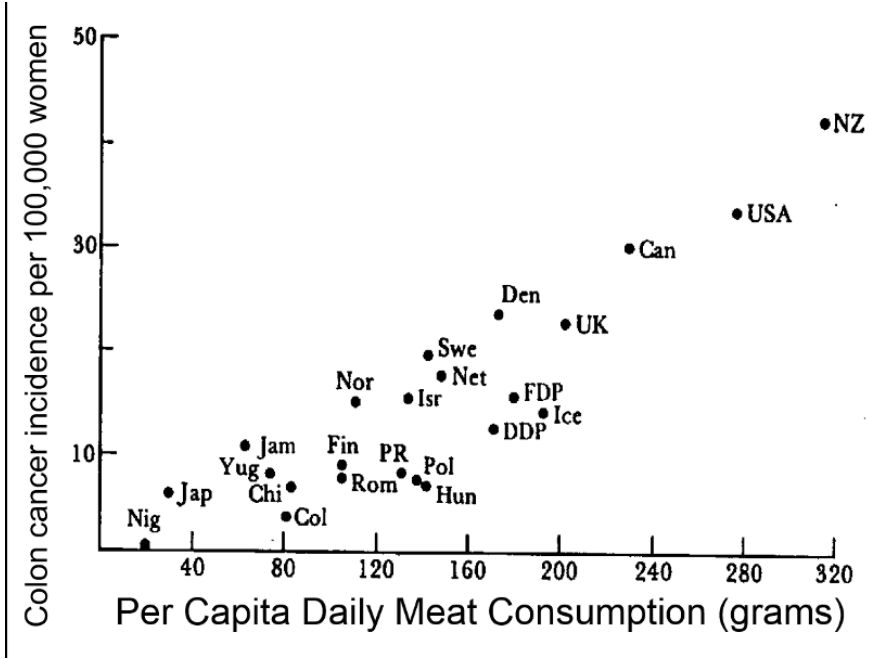
Q6

For each of the following graphs,

(i) State what you think the evidence is trying to suggest.

(ii) Give a reason why you agree or disagree with what the evidence is suggesting.

(iii) Identify whether the variable of the y-axis and the variable of the x-axis are correlated and/or causal?



http://sphweb.bumc.bu.edu/otlt/MPH-Modules/PH717-QuantCore/PH717-Module1BDescriptiveStudies_and_Statistics/PH717-Module1B-DescriptiveStudies_and_Statistics6.html

Q6

Discussion

(i) Colon cancer is correlated to the amount of daily meat consumption.

(ii) There is a clear linear trend; countries with the lowest meat consumption have the lowest rates of colon cancer, and the colon cancer rate among these countries progressively increases as meat consumption increases.

(iii) Probably causal.

Q7

If A and B are correlated, but they're actually caused by C, which of the following statements are correct?

a) A and C are correlated

b) B and C are correlated

c) A causes B to happen

d) A causes C to happen

Q7

- Please allow students to answer the questions

Q8

We toss a coin and observe which side is facing up. Which of the following statements represent valid probability assignments for observing head P[‘H’] and tail P[‘T’]?

a) P[‘H’]=0.2, P[‘T’]=0.9

b) P[‘H’]=0.0, P[‘T’]=1.0

c) P[‘H’]=-0.1, P[‘T’]=1.1

d) P[‘H’]=P[‘T’]=0.5

Q8

- Please allow students to answer the questions

Q9

Two vectors $a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $b = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ are linearly dependent?

Q9

- Ans: False (because a is not a multiply of b.)

Q10

The rank of the matrix $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ is _____

Answer:

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}$$

The rank is 2

Q10

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The rank of matrix $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ is ____

The determinant of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is calculated as:

$$\det(A) = ad - bc$$

$$= 1 \times 4 - 2 \times 3 = -2 \neq 0$$

If the determinant is non-zero, the matrix is full rank, which means the rank of the matrix is 2.

Q11

The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is _____

Q11

Perform Gaussian Elimination to obtain Row Echelon Form

step 1: make the first entry in first column as 1

The first row already has a leading 1 in the first column, so

- To eliminate 4 below the 1, we substract 4 times the firs row from the second row

$$\begin{bmatrix} 1 & 2 & 3 \\ \textcircled{4} & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 7 & 8 & 9 \end{bmatrix} \quad \text{Row2} = \text{Row2} - 4 \times \text{Row1}$$

- Eliminate the 7 below the 1

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ \textcircled{7} & 8 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix} \quad \text{Row3} = \text{Row3} - 7 \times \text{Row1}$$

step 2: make the second entry in the second column a Leading 1

$$\text{Row2} = -\frac{1}{3} \times \text{Row2}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & \textcircled{-3} & -6 \\ 0 & -6 & -12 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -6 & -12 \end{bmatrix}$$

step 3. Elimate the entry below the leading 1 in the seconed column.

$$\text{Row3} = \text{Row3} + 6 \times \text{Row2}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & \textcircled{-6} & -12 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

Rank is 2

THANK YOU