

Hash Table

Introduction to Algorithms

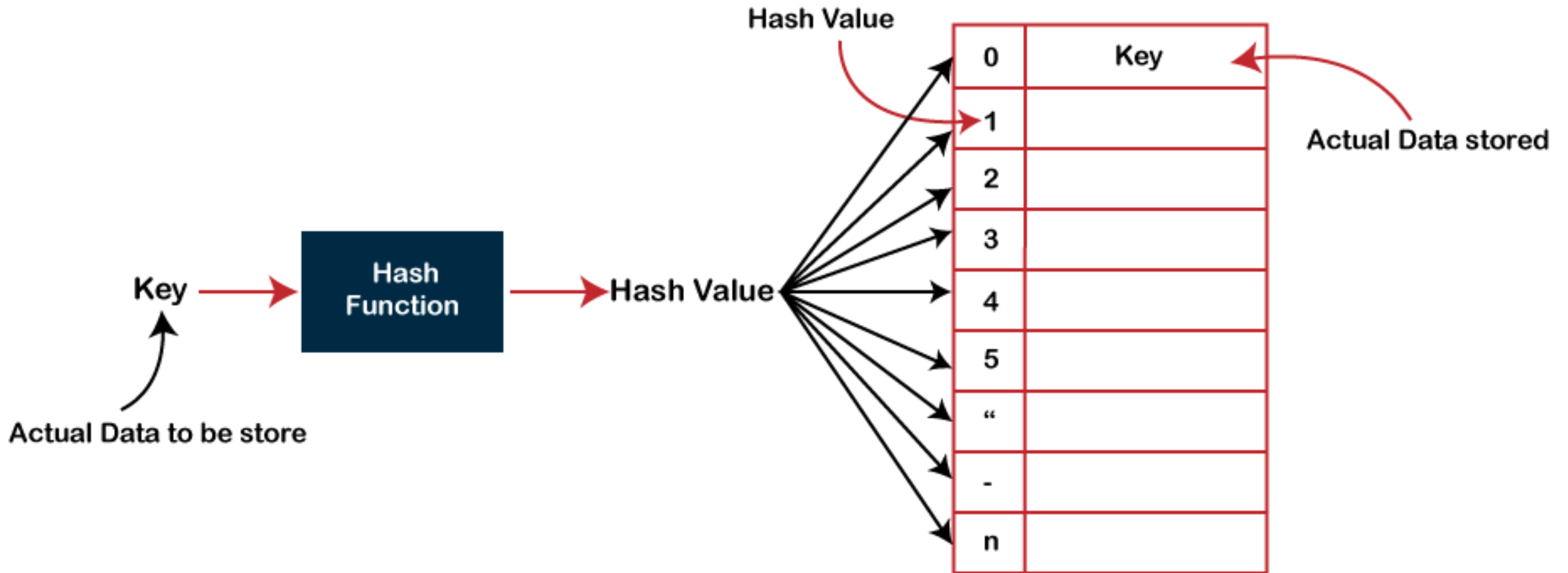
Overview

1. Hashing
2. Collision problem
3. Collision Hashing Techniques
4. Demonstrate with Python

What is Hashing?

- Hashing is a data structure technique or process of **mapping keys, values** into the hash table by using a hash function.
- It is done for **faster access to elements**.
- The efficiency of mapping depends on the efficiency of the hash function used.

What is Hashing?

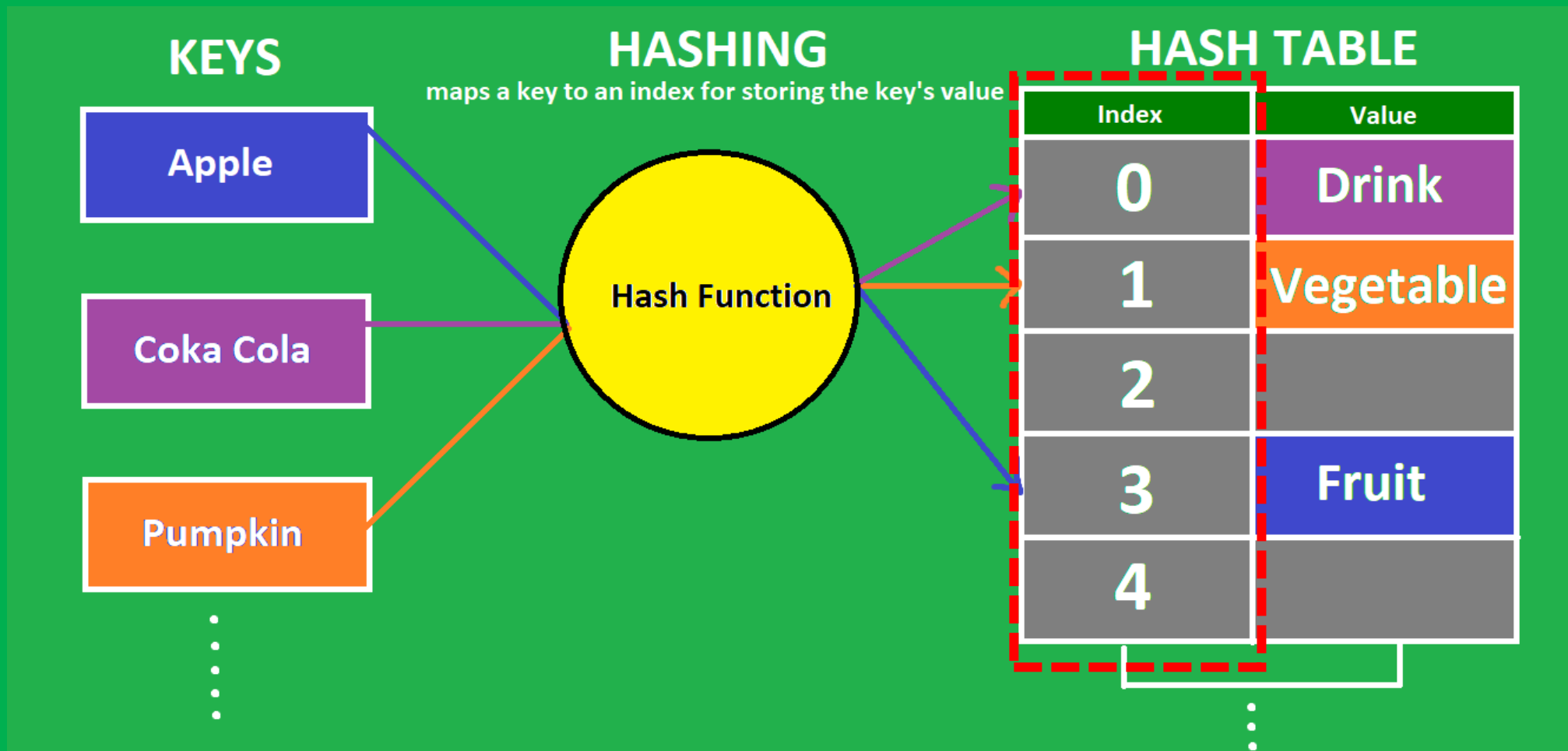


Key : integer, string, ...

How we use Hash table ?

Use a key (arbitrary string or number) to index directly into an array

- $O(1)$ time to access records



How we find the hash key (index) ?

Example: integer as key

Calculate with *Hash Function*

Hash Function

$$H(x) = X \% 10$$

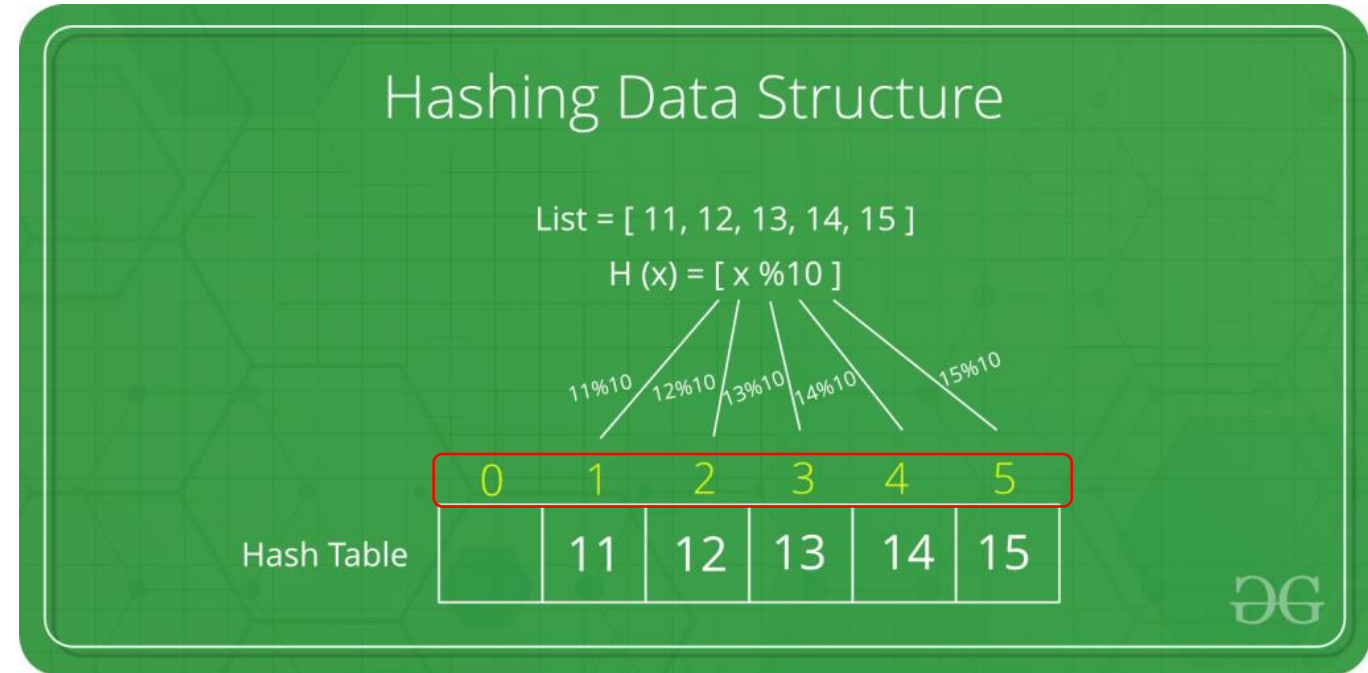
Input : Integer

[11 12 13 14 15]



hash key

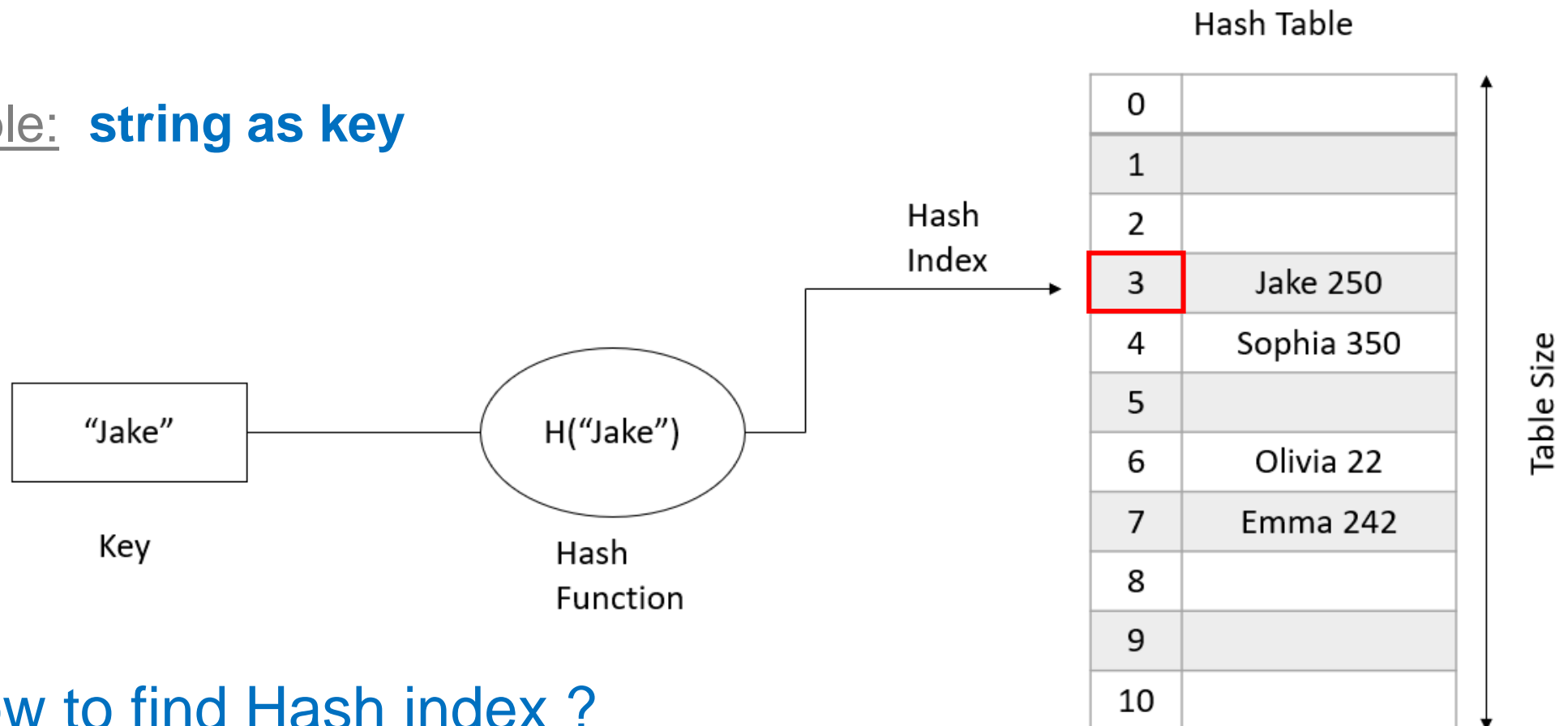
[1 2 3 4 5]



Hasing

We generate a hash for the input using the hash function and then store the element using the generated hash as the key in the hash table.

Example: **string as key**



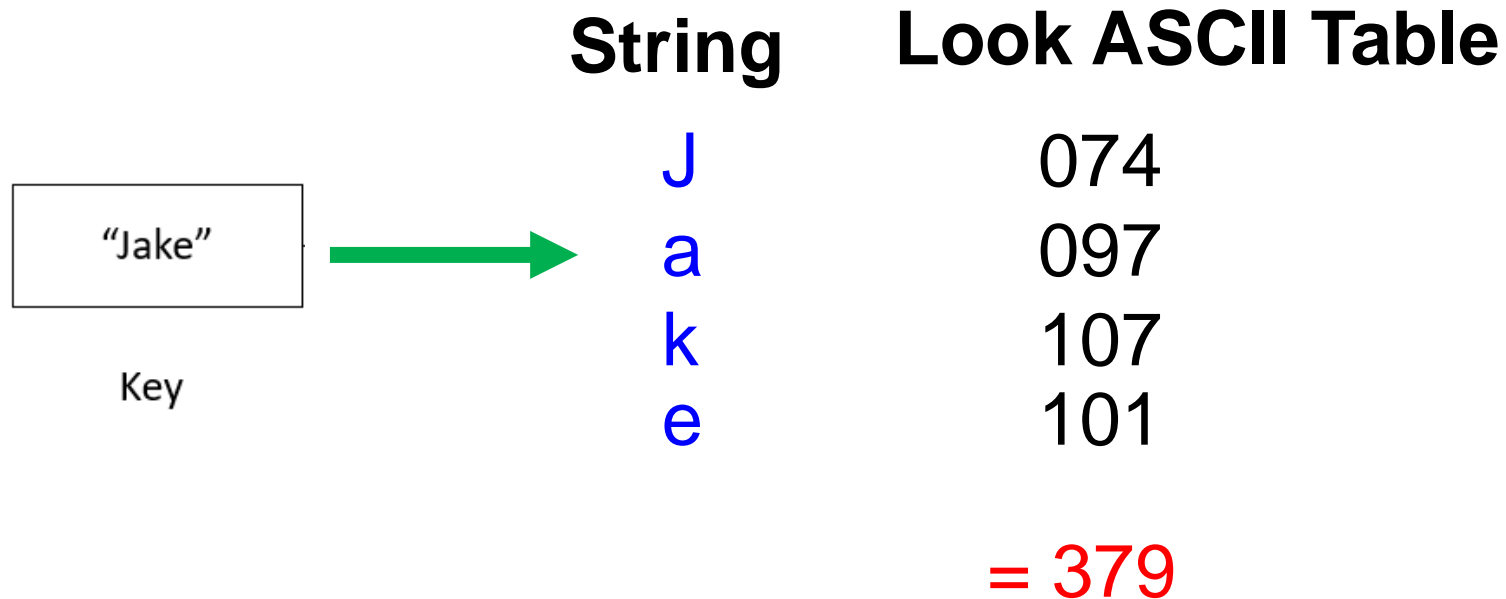
How to find Hash index ?

ASCII Table

Dec	Hex	Oct	Char	Dec	Hex	Oct	Char	Dec	Hex	Oct	Char	Dec	Hex	Oct	Char
0	0	0		32	20	40	[space]	64	40	100	@	96	60	140	`
1	1	1		33	21	41	!	65	41	101	A	97	61	141	a
2	2	2		34	22	42	"	66	42	102	B	98	62	142	b
3	3	3		35	23	43	#	67	43	103	C	99	63	143	c
4	4	4		36	24	44	\$	68	44	104	D	100	64	144	d
5	5	5		37	25	45	%	69	45	105	E	101	65	145	e
6	6	6		38	26	46	&	70	46	106	F	102	66	146	f
7	7	7		39	27	47	'	71	47	107	G	103	67	147	g
8	8	10		40	28	50	(72	48	110	H	104	68	150	h
9	9	11		41	29	51)	73	49	111	I	105	69	151	i
10	A	12		42	2A	52	*	74	4A	112	J	106	6A	152	j
11	B	13		43	2B	53	+	75	4B	113	K	107	6B	153	k
12	C	14		44	2C	54	,	76	4C	114	L	108	6C	154	l
13	D	15		45	2D	55	-	77	4D	115	M	109	6D	155	m
14	E	16		46	2E	56	.	78	4E	116	N	110	6E	156	n
15	F	17		47	2F	57	/	79	4F	117	O	111	6F	157	o
16	10	20		48	30	60	0	80	50	120	P	112	70	160	p
17	11	21		49	31	61	1	81	51	121	Q	113	71	161	q
18	12	22		50	32	62	2	82	52	122	R	114	72	162	r
19	13	23		51	33	63	3	83	53	123	S	115	73	163	s
20	14	24		52	34	64	4	84	54	124	T	116	74	164	t
21	15	25		53	35	65	5	85	55	125	U	117	75	165	u
22	16	26		54	36	66	6	86	56	126	V	118	76	166	v
23	17	27		55	37	67	7	87	57	127	W	119	77	167	w
24	18	30		56	38	70	8	88	58	130	X	120	78	170	x
25	19	31		57	39	71	9	89	59	131	Y	121	79	171	y
26	1A	32		58	3A	72	:	90	5A	132	Z	122	7A	172	z
27	1B	33		59	3B	73	;	91	5B	133	[123	7B	173	{
28	1C	34		60	3C	74	<	92	5C	134	\	124	7C	174	
29	1D	35		61	3D	75	=	93	5D	135]	125	7D	175	}
30	1E	36		62	3E	76	>	94	5E	136	^	126	7E	176	~
31	1F	37		63	3F	77	?	95	5F	137	_	127	7F	177	

Example : Find the Hash index

ASCII values of characters in *key*



Dec	Hex	Oct	Char
64	40	100	@
65	41	101	A
66	42	102	B
67	43	103	C
68	44	104	D
69	45	105	E
70	46	106	F
71	47	107	G
72	48	110	H
73	49	111	I
74	4A	112	J
75	4B	113	K

Dec	Hex	Oct	Char
96	60	140	`
97	61	141	a
98	62	142	b
99	63	143	c
100	64	144	d
101	65	145	e
102	66	146	f
103	67	147	g
104	68	150	h
105	69	151	i
106	6A	152	j
107	6B	153	k

What is the table size ?

What the value of this string ?

John

$$74+111+104+156 = ???$$

Peter

$$80+101+116+101+114 = ???$$

Robert

???

Jake

$$= 379$$

Apple

$$= 530$$

Dec	Hex	Oct	Char	Dec	Hex	Oct	Char
64	40	100	@	96	60	140	`
65	41	101	A	97	61	141	a
66	42	102	B	98	62	142	b
67	43	103	C	99	63	143	c
68	44	104	D	100	64	144	d
69	45	105	E	101	65	145	e
70	46	106	F	102	66	146	f
71	47	107	G	103	67	147	g
72	48	110	H	104	68	150	h
73	49	111	I	105	69	151	i
74	4A	112	J	106	6A	152	j
75	4B	113	K	107	6B	153	k
76	4C	114	L	108	6C	154	l
77	4D	115	M	109	6D	155	m
78	4E	116	N	110	6E	156	n
79	4F	117	O	111	6F	157	o
80	50	120	P	112	70	160	p
81	51	121	Q	113	71	161	q
82	52	122	R	114	72	162	r
83	53	123	S	115	73	163	s
84	54	124	T	116	74	164	t
85	55	125	U	117	75	165	u
86	56	126	V	118	76	166	v
87	57	127	W	119	77	167	w
88	58	130	X	120	78	170	x
89	59	131	Y	121	79	171	y
90	5A	132	Z	122	7A	172	z
91	5B	133	[123	7B	173	{
92	5C	134	\	124	7C	174	
93	5D	135]	125	7D	175	}
94	5E	136	^	126	7E	176	~
95	5F	137	_	127	7F	177	

How to reduce the table size ?

John

Peter

Robert

Jake

Apple

...

If the array has size = **10** , modulus operation (%)

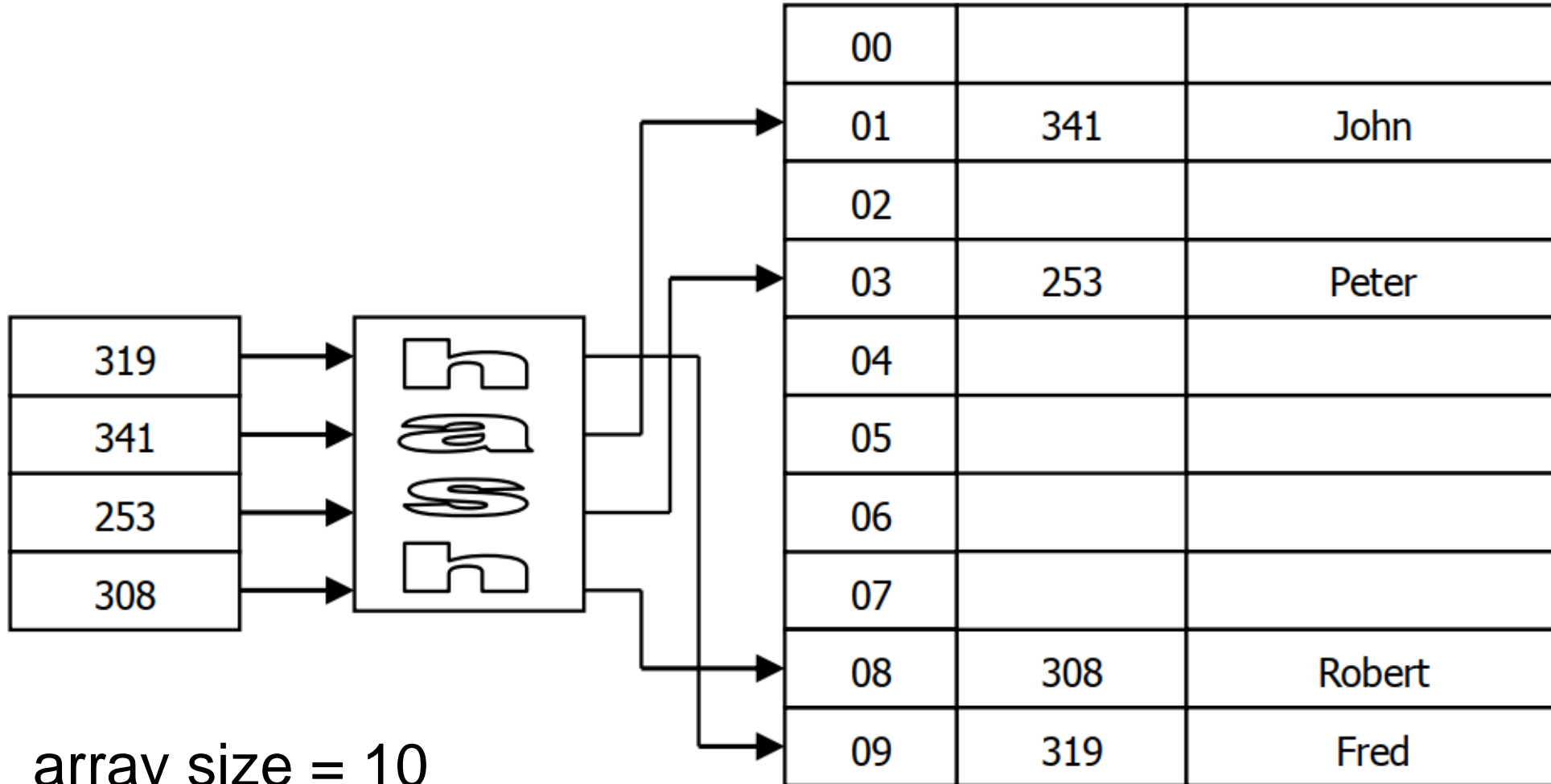
Ex.

$$379 \% 10 = 9$$

$$530 \% 10 = 0$$

Hash key

Example of the table size (input string)



Collision

hash table

What is Collision ?

- When two keys have the same hash value.
 - $312 \% 10 = 2$
 - $322 \% 10 = 2$!!!
- Because the hash method is not guaranteed to return a unique integer for each key

Collision Hashing Techniques

There are several ways to handle collisions :

1. Linear Probing
2. Quadratic Probing
3. Double Hashing
4. Linked-list

1. Linear Probing

- **Linear Probing**: search sequentially for an unoccupied position
 - uses a wraparound (circular) array

Algorithm : forward by 1, until not occur collision

- Probe sequence:

$$0^{\text{th}} \text{ probe} = h(k) \bmod \text{TableSize}$$

$$1^{\text{th}} \text{ probe} = (h(k) + \mathbf{1}) \bmod \text{TableSize}$$

$$2^{\text{th}} \text{ probe} = (h(k) + \mathbf{2}) \bmod \text{TableSize}$$

...

$$i^{\text{th}} \text{ probe} = (h(k) + \mathbf{i}) \bmod \text{TableSize}$$

A hash table after three insertions

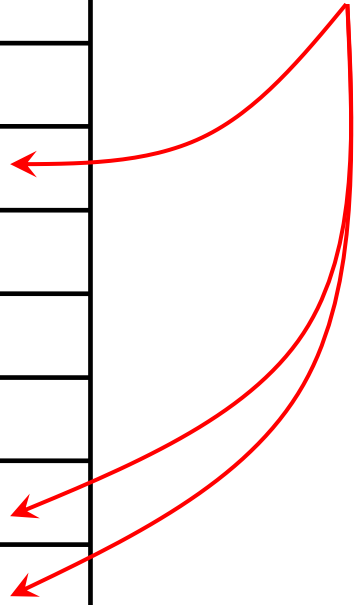
using the too simple (lousy) hash method

insert objects
with these
three keys:

"abba"
"abcd"
"abce"

0	
...	
80	
81	"abba"
82	
83	
84	
85	"abcd"
86	"abce"
...	
308	

Keys



Collision occurs while inserting "baab"

can't insert
"baab" where it
hashes to
same slot as
"abba"

Linear probe
forward by 1,
inserting it
at the next
available slot

0	
...	
80	
81	"abba"
82	"baab"
83	
84	
85	"abcd"
86	"abce"
...	
308	

"baab"
Try [81]
Put in [82]

Wrap around when collision occurs at end

Insert
"KLMP" and
"IKLT"
both of which
have a hash
value of 308

0	"IKLT"
...	
80	
81	"abba"
82	"baab"
83	
84	
85	"abcd"
86	"abce"
...	
308	"KLMP"

Wrap around
(circular)

How to Find object with key "baab"

"baab" still hashes to 81, but since [81] is occupied, linear probe to [82]

At this point, you could return a reference or remove baab

0	"IKLT"
...	
80	
81	"abba"
82	"baab"
83	
84	
85	"abcd"
86	"abce"
...	
308	"KLMP"

2. Quadratic Probing

Quadratic probing eliminates the primary clustering problem

Algorithm: add index values in increments of powers of 2

- Probe sequence:

$$0^{\text{th}} \text{ probe} = h(k) \bmod \text{TableSize}$$

$$1^{\text{th}} \text{ probe} = (h(k) + 1) \bmod \text{TableSize}$$

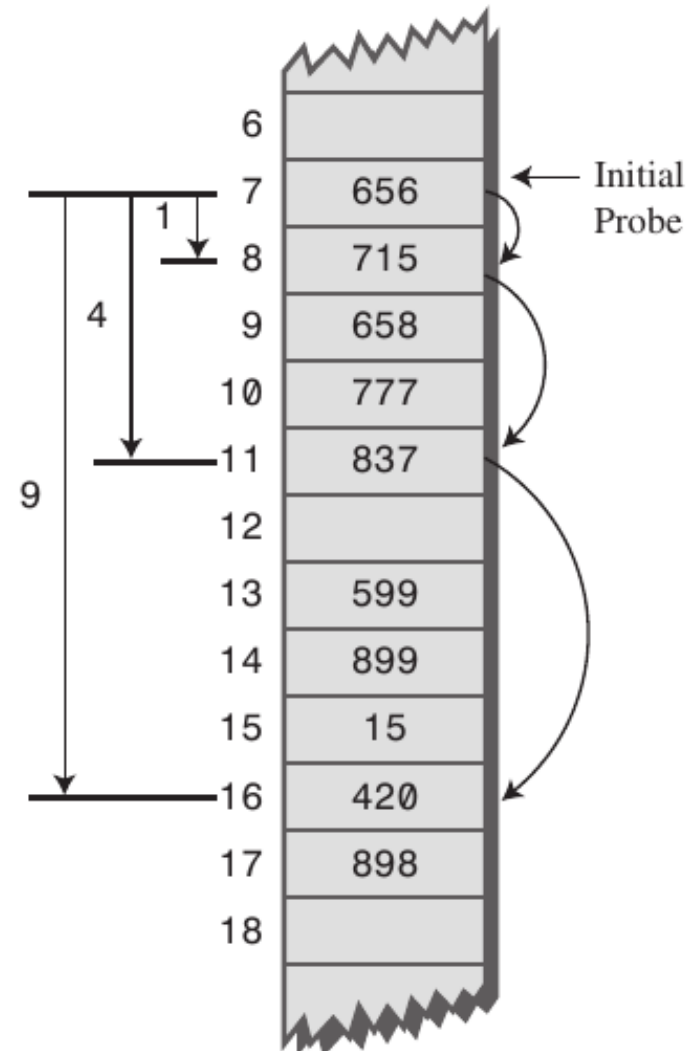
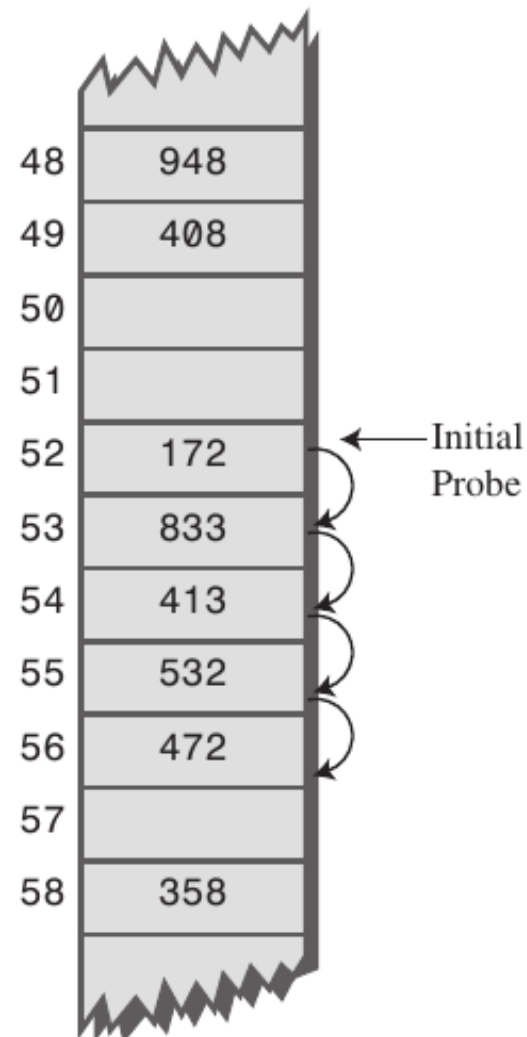
$$2^{\text{th}} \text{ probe} = (h(k) + 4) \bmod \text{TableSize}$$

$$3^{\text{th}} \text{ probe} = (h(k) + 9) \bmod \text{TableSize}$$

...

$$i^{\text{th}} \text{ probe} = (h(k) + i^2) \bmod \text{TableSize}$$

Linear vs Quadratic probing



3. Double Hashing

$$f(i) = i * g(k)$$

where g is a second hash function

Probe sequence:

$$0^{\text{th}} \text{ probe} = h(k) \bmod \text{TableSize}$$

$$1^{\text{th}} \text{ probe} = (h(k) + g(k)) \bmod \text{TableSize}$$

$$2^{\text{th}} \text{ probe} = (h(k) + 2 * g(k)) \bmod \text{TableSize}$$

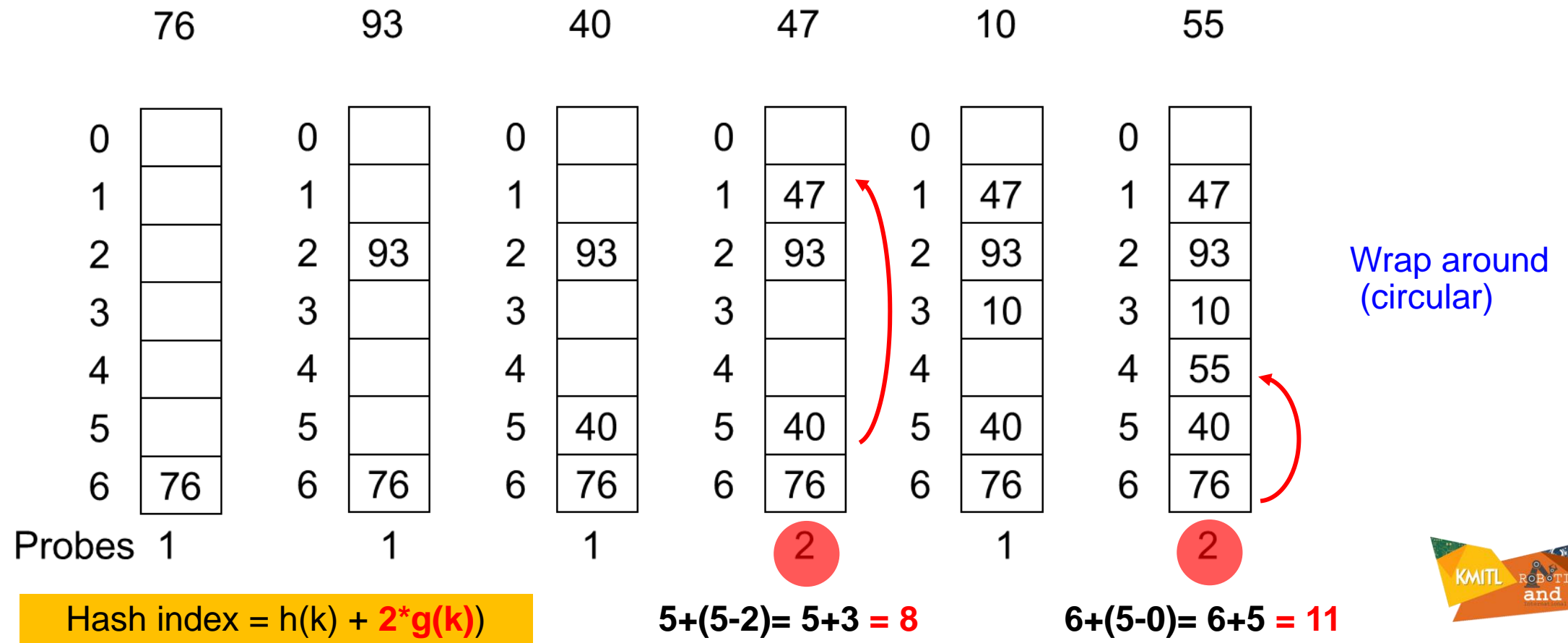
$$3^{\text{th}} \text{ probe} = (h(k) + 3 * g(k)) \bmod \text{TableSize}$$

...

$$i^{\text{th}} \text{ probe} = (h(\underline{k}) + i * g(\underline{k})) \bmod \text{TableSize}$$

Double Hashing Example

$$h(k) = k \bmod 7, \quad g(k) = 5 - (k \bmod 5)$$



4. **Linked-list** (Separate Chaining)

The idea is to make each cell of hash table point to a linked list of records that have same hash function value

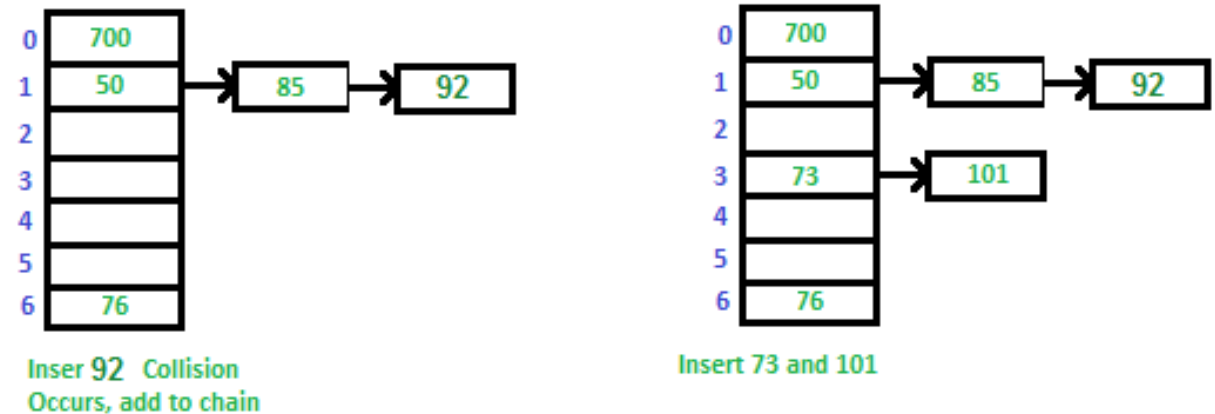
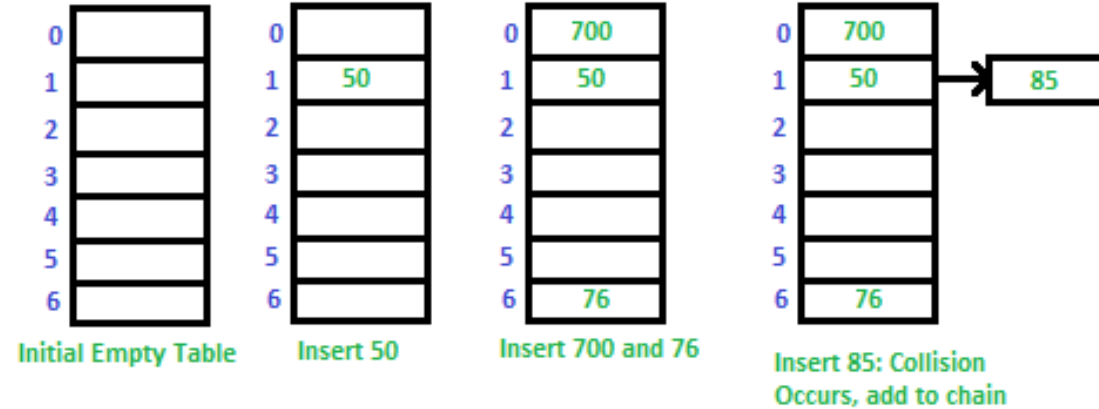
Advantages:

- 1) Simple to implement.
- 2) Hash table never fills up, we can always add more elements to the chain.
- 3) Less sensitive to the hash function or load factors.
- 4) It is mostly used when it is unknown how many and how frequently keys may be inserted or deleted.

4. Linked-list (Separate Chaining)

Let us consider a simple hash function as “**key mod 7**” and sequence of keys as

50, 700, 76, 85, 92, 73, 101.



Real-world Applications

- Databases
- Associative arrays
- Sets
- Memory cache

Advantages of hash tables

- Here, are benefits of using hash tables:
 - Hash tables have high performance when looking up data, inserting, and deleting existing values.
 - The time complexity for hash tables is constant regardless of the number of items in the table.
 - They perform very well even when working with large datasets.

Disadvantages of hash tables

- Here, are cons of using hash tables:
 - You **cannot use a null value** as a key.
 - Collisions cannot be avoided when generating keys using. hash functions. Collisions occur when a key that is already in use is generated.
 - If the hashing function has many collisions, this can lead to performance decrease.

Demonstrate with Python

- In Python, the **Dictionary data types** represent the implementation of hash tables. The Keys in the dictionary satisfy the following requirements.
 - The keys of the dictionary are hashable i.e. they are generated by hashing function which **generates unique result** for each unique value supplied to the hash function.
 - The order of data elements in a dictionary is not fixed.
- So we see the implementation of hash table by using the dictionary data types as b

Accessing Values in Dictionary

To access dictionary elements, you can use the familiar square brackets along with the key to obtain its value.

Example

```
# Declare a dictionary
dict = {'Name': 'Zara', 'Age': 7, 'Class': 'First'}
# Accessing the dictionary with its key
print (dict['Name']: ", dict['Name'] )
print (dict['Age']: ", dict['Age'])
```

Output

```
dict['Name']: Zara
dict['Age']: 7
```

Updating Dictionary

You can update a dictionary by adding a new entry or a key-value pair, modifying an existing entry, or deleting an existing

Example

```
# Declare a dictionary
dict = {'Name': 'Zara', 'Age': 7, 'Class': 'First'}
dict['Age'] = 8;
# update existing entry
dict['School'] = "DPS School";
# Add new entry
print ("dict['Age']: ", dict['Age'])
print ("dict['School']: ", dict['School'])
```

Output

```
dict['Age']: 8
dict['School']: DPS School
```


Delete Dictionary Elements

You can either remove individual dictionary elements or clear the entire contents of a dictionary.

Example

```
dict = {'Name': 'Zara', 'Age': 7, 'Class': 'First'}  
del dict['Name'];  
# remove entry with key 'Name'  
dict.clear();  
# remove all entries in dict  
del dict ;  
# delete entire dictionary  
print ("dict['Age']: ", dict['Age'])  
print ("dict['School']: ", dict['School'])
```

Problems Assignment 9

* Coding with **Colab**

1. Demonstrate declare a dictionary elements

Key	Value
'name'	'Jack'
'age'	18
'class'	Algorithms

3. Demonstrate delete a dictionary elements

Key	Value
'age'	20
'class'	Introduction

2. Demonstrate update a dictionary and insert the elements

Key	Value
'name'	'RAI'
'age'	20
'year'	2021
'class'	Introduction

Submitted before 11.59 (Sunday)

Q & A