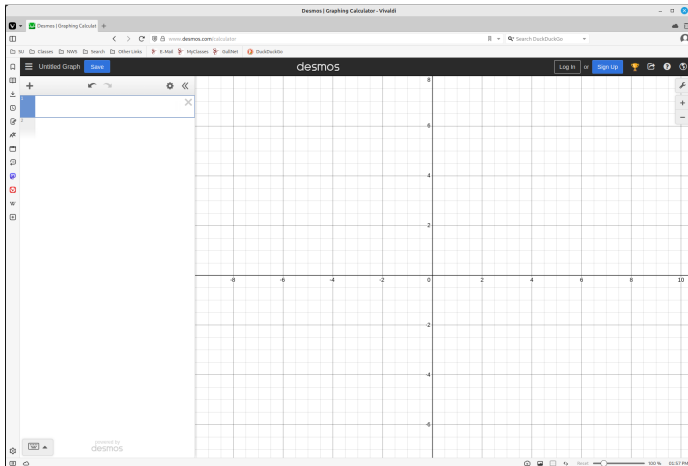- Curve fitting means finding the "best" line that gets close to the data set. For us, the data is usually going to be timing analyses where our independent variable is the size of the problem (usually denoted $n$ or $x$) and our dependent variable is the time it takes the program to run (usually denoted $T$ or $y$).

- The term "best" in this case means that the sum of the squares of the residuals between the predicted values and the actual data values is a minimum.

- If you have taken statistics you might have studied and computed a regression line, which is the best fit linear equation (curve) $y = mx + b$ to the data set. The calculations you did result in values for $m$ and $b$, the parameters for the linear equation.

- Using the same methods, which are sometimes studied in an advanced linear algebra course or a course in numerical linear algebra, you can also fit lines of the form $y = ax^2 + bx + c$, $y = ax^3 + bx^2 + cx + d$, $y = a\sin(bx + c)$, $y = a\ln(bx) + c$ and so on.
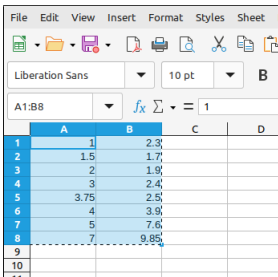
▸ We will mainly be interested in lines of the forms $y = mx + b$, $y = ax^2 + bx + c$, $y = ax^3 + bx^2 + cx + d$, and $y = a\log_2(x) + b$, and $y = ax\log_2(x) + bx + c$. Although, we may see other curves of interest.

▸ These calculations can be done using spreadsheets or we could write programs to do the calculations but the best two tools I have found to do general curve fitting are Desmos and GeoGebra. Both of these programs are free and were developed for mathematics education and visualization.

▸ Desmos is an online (only) exploratory graphing app primarily for middle and high school mathematics education.

▸ GeoGebra has both online and downloadable application versions. It was developed for both high school and college mathematics education. It has substantially more features for exploring higher-level mathematics as well as incorporates a computer algebra system for symbolic Calculus. If you download and install the local application you will want GeoGebra Classic 6. The local application and the online version are identical.

- Using Desmos
  - Go to the following website: `https://www.desmos.com/calculator`
    The app should look like the following.
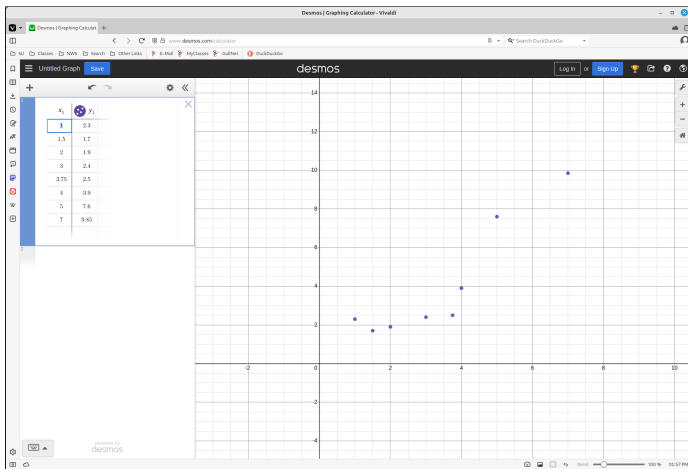
▷ Store data (timing data for us) in a spreadsheet such as LibreOffice Calc or Microsoft Excel. Select the data and copy to the clipboard.
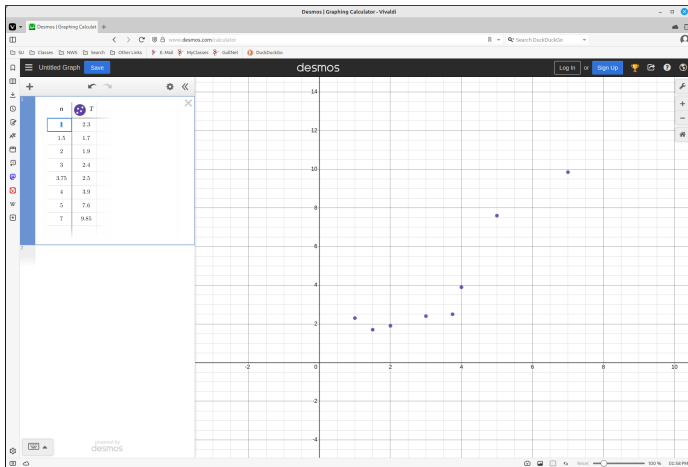
▷ Go back to Desmos, select the input cell on the left, and paste the data into the cell. The usual Ctrl+C and Ctrl+V will work here. It will populate the cell with a table of your data and plot the points on the graph.

▷ Change the headings for the chart to *n* for $x_1$ and *T* for $y_1$. This is not really necessary but it is easier then using the default $x_1$ and $y_1$.

▷ We will now fit a best-line quadratic to this data. In the second cell input the following expression, $T \sim a\, n^2 + b\, n + c$
The exponent is done with the caret symbol and the second character is the tilde.
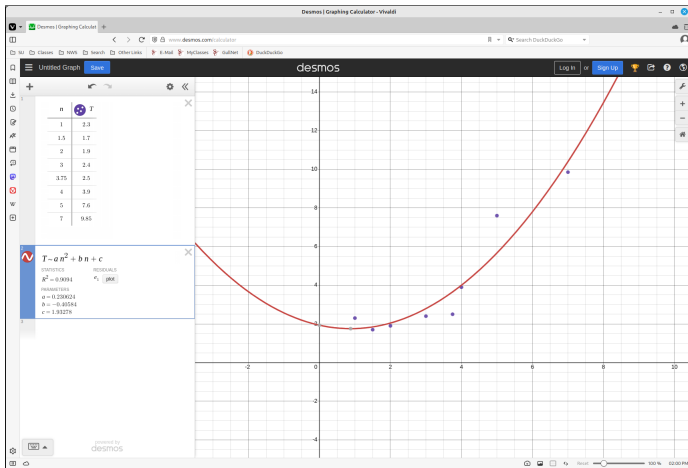
▷ We will now fit a best-line $O(n \lg(n)) + \Theta(n)$. In the next cell input the following expression, $T \sim a \, n \log_2(n) + b \, n + c$
To get the $\log_2(n)$, type in log, then the underscore, then 2, then a right arrow to get out of the subscript.
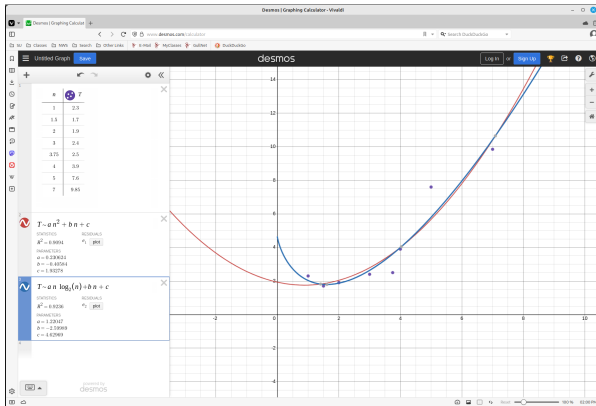
▷ To save an image select the share icon in the upper right, select export image, the following dialog box will appear, set the size to Large Square or Large rectangle, select Download PNG. This will save the image to your downloads folder.
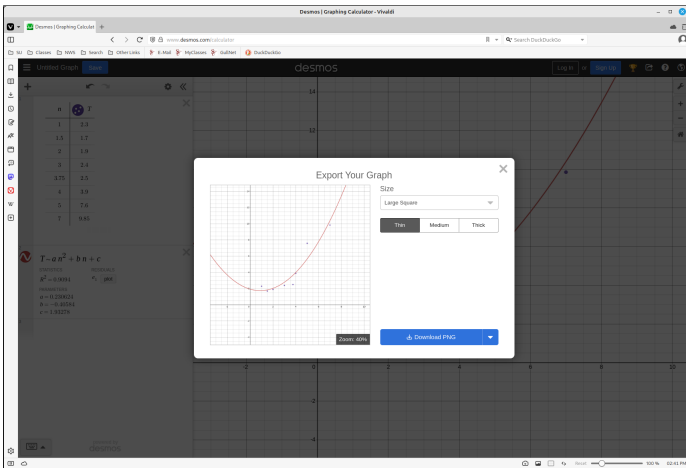
▷ You can copy and paste the parameter values from the object list on the left by selecting them and doing the standard Ctrl+C and Ctrl+V.

▷ A shift click and drag over an axis will scale the axis.

▷ A click and drag will reposition the graph.

▷ The mouse wheel will scale the image keeping the current aspect-ratio.

▷ There is also a help system for using Desmos in the upper right of the app as well.
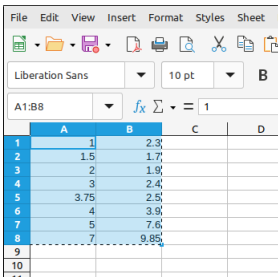
- Using GeoGebra
  - The online app is at https://www.geogebra.org/classic and the installable version can be found at https://www.geogebra.org under App Downloads.
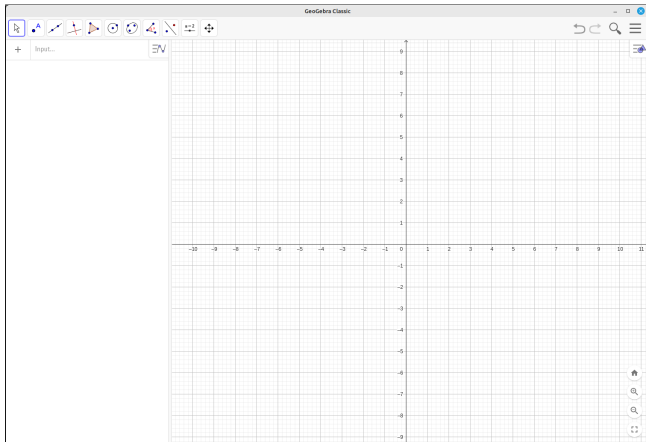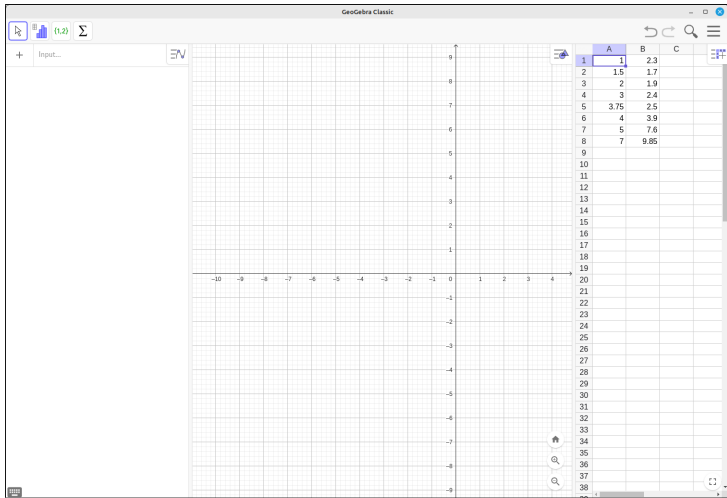
▷ Turn on the spreadsheet tool: Menu, View, Spreadsheet.
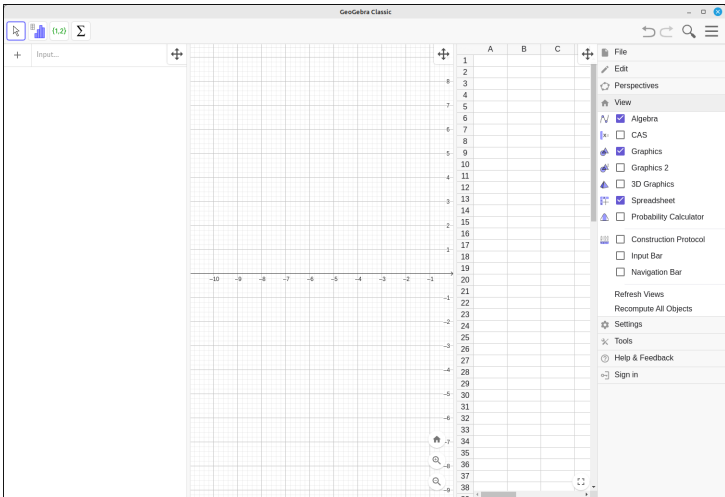
▷ Store data (timing data for us) in a spreadsheet such as LibreOffice Calc or Microsoft Excel. Select the data and copy to the clipboard.
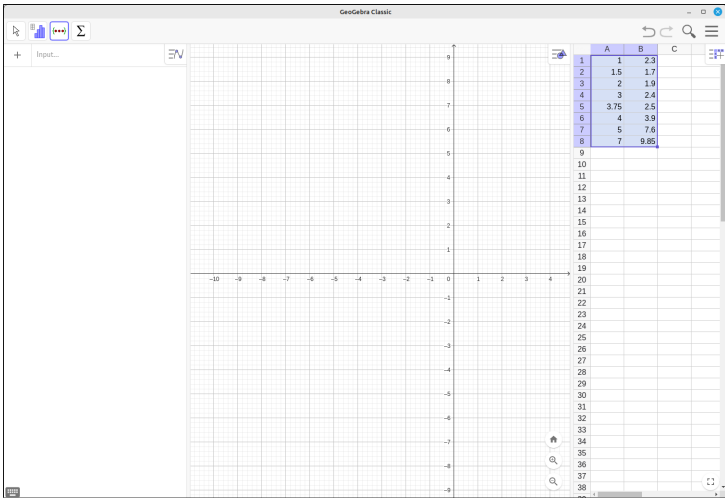
▷ Paste the data into GeoGebra by selecting the A1 cell and typing Ctrl+V.

▷ Select the data on the GeoGebra spreadsheet.

▷ Select List tool and List of Points from the menu.

▷ Choose a list name (we will leave it as l1), keep the Dependent Objects selected and click OK.

▷ Each point will get a name and be plotted on the graph. There will also be a list entry in the objects to the left.
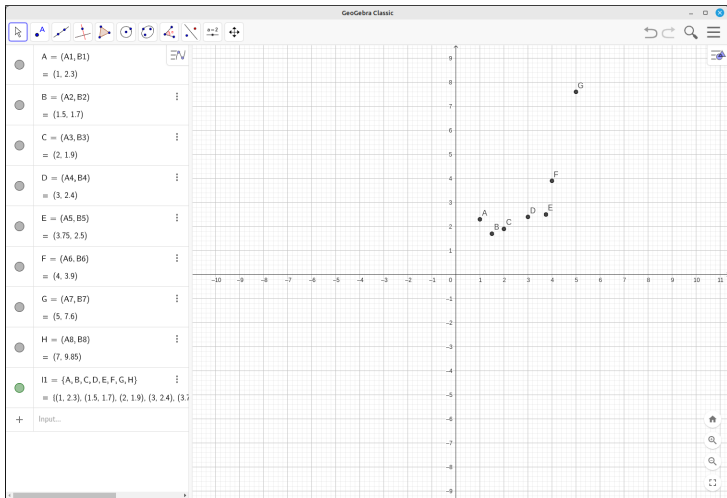
▷ Adjust the points to fit the screen nicely. You can also group the items in the object
list by type and collapse the points entry, this will make it easier to view.
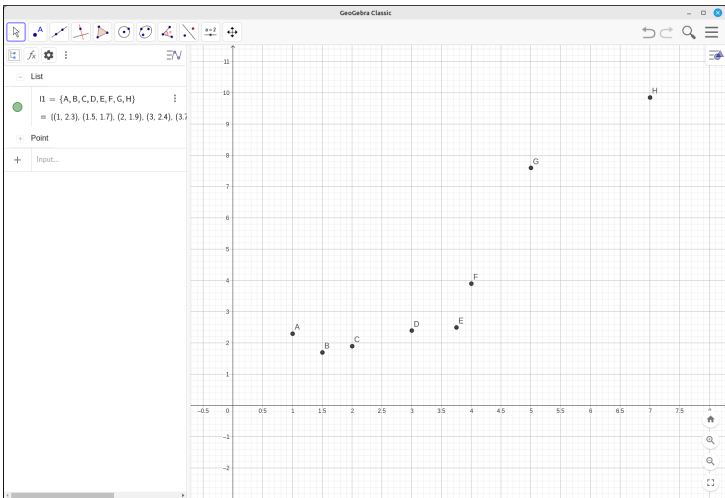
▷ We will fit the quadratic equation to the points. In a new input box type the following: fit(l1, $\{x^2, x, 1\}$)
Note that the caret will put you in the exponent and right arrow will take you out.
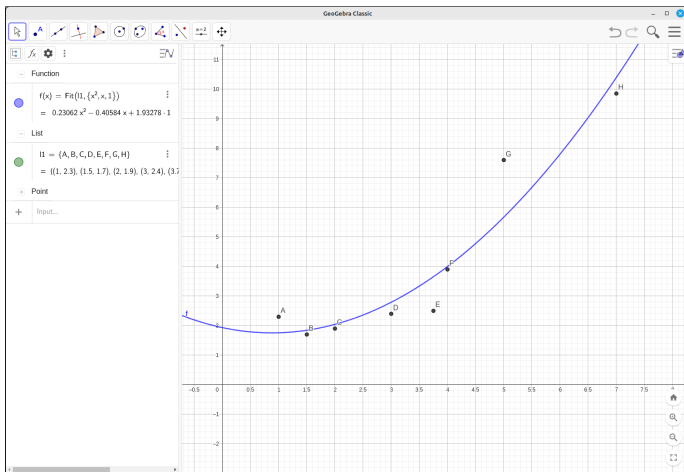
▷ Now add the log fit with fit(l1, $\{x \log 2(x), x, 1\}$)
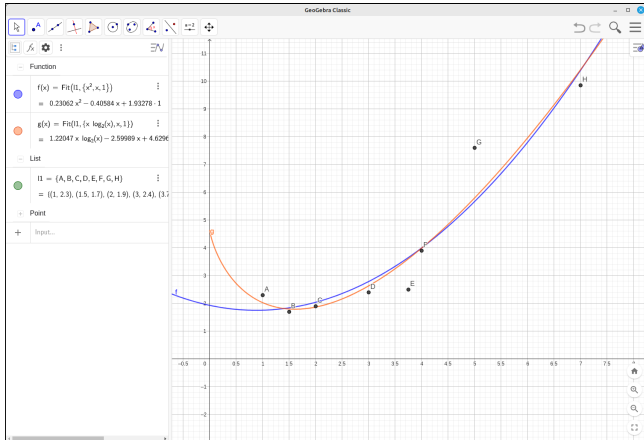Note that you will type log2 but the program will display this as $\log_2$. You can also input this as $\log(2, x)$ or use the underscore to make the subscript.

▷ There are several ways to save the graphics image.
  - From the menu, select File, Export Image. This will bring up a dialog box that will allow you to copy the image to the clipboard or to download (i.e. save) the image to a file.
  - From the menu, select Edit, Graphics View to Clipboard. This will copy the image directly to the clipboard.
▷ To copy and paste the resulting equation of the fit you can do the following. Note that GeoGebra locks out the editing of output, as it should.
  - The three vertical dots on each of the input object cells on the left is a small menu of options that can be done to that input.
  - Click this and select Duplicate Output.
  - This will take the output and copy it as input to the next cell.
  - You can then copy this new input to the clipboard.
▷ A shift click and drag over an axis will scale the axis.
▷ A click and drag will reposition the graph.
▷ The mouse wheel will scale the image keeping the current aspect-ratio.
▷ There are online help manuals of you wish to explore other features of the program.