1. Consider the word `unusual`. How many unique subsets of 5 letters (of the 7) exist? How many different strings could be made from 5 of those 7 letters?

#of u's — non-'u' letters

$7-3=4$, $5-4=1$, So at least 1 'u' is present so choose out of 4 non-'u' letters to fill spaces

<u>Unique subsets of 5 letters:</u>

Case 1: 1'u', $\binom{4}{4}=1$ subsets (4 spaces)

Case 2: 2'u', $\binom{4}{3}=4$ subsets (3 spaces)

$\therefore$ Total unique subsets $=11$

Case 3: 3'u', $\binom{4}{2}=6$ subsets (2 spaces)

\# of unique subsets

<u>Different strings:</u>

Case 1: $(5!)\times\binom{4}{4}=120$

Case 2: $(5!/2!)\times\binom{4}{3}=240$

Case 3: $(5!/3!)\times\binom{4}{2}=120$

Total different strings $=480$

2. Using a standard deck of playing cards, how many ways are to form a 5-card hand with 2 pairs (i.e. pair of one value, a pair of a different value, and a fifth card of some other value)?

52 Total cards

4 suits, 13 values for each suit

5 card hand with 2 pairs = choose 2 suits (from 4) for each of the pairs,
then choose 2 values from 13 values (i.e. 1 value for each pair),
then choose 1 card from remaining cards $(52-4-4=44)$

$$=\binom{4}{2}\times\binom{4}{2}\times\binom{13}{2}\times 44$$

3. A violinist serenades couples at a romantic restaurant. She <u>will play 16 songs</u> in an hour and <u>there are 7 couples</u>. <u>One couple</u> is having a fight and <u>will allow at most 1 song to be played</u> to them before they ask the violist not to return to their table. If we care only about the number of songs each couple receives, how many ways can the songs be distributed amongst the couples.

Not including fighting couple

1 2 3 4 5 6

** | * | * | * | ** | ** (* x16)

$$\binom{5+16}{16}=\binom{21}{16}$$

1 song played for fighting couple

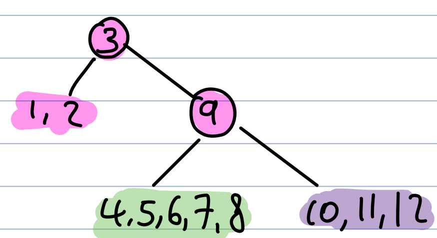
1 2 3 4 5 6

** * | | * | | * (* x15)

$$\binom{5+15}{15}=\binom{20}{15}$$

$$\text{Total ways}=\binom{21}{16}+\binom{20}{15}$$

4. There is a Binary Search Tree with 12 nodes. Each node has a distinct value between 1 and 12. The root has value 3, and its right child has value 9. How many possible Binary Search Trees could this be? Tip: *Try to define how many ways there are to form a BST of 2 nodes. Then try to define how many ways there are to form a BST of 3 nodes (think about the possible insertion order based on rank: smallest, medium, largest)* ***in terms of 2 node trees*** *for certain cases. Continue to do this for 4 node trees (in terms of 3- and 2-node trees for various cases of insertion ordering based on rank) and 5 node trees.*




2 node BST:

0 1 = 1 way

1 0 = 1 way

= 2 ways

3 node BST:

2 0 = 2 ways

1    1 = 1 way
0    2 = 2 ways } = 5 ways

4 node BST:

| | | | |
|---|---|---|---|
| 3 | 0 | = 5 ways | |
| 2 | 1 | = 2 ways | |
| 1 | 2 | = 2 ways | = 14 ways |
| 0 | 3 | = 5 ways | |

5 node BST:

| | | | |
|---|---|---|---|
| 4 | 0 = 14 | | |
| 3 | 1 = 5 | | |
| 2 | 2 = 4 | = 42 ways | |
| 1 | 3 = 5 | | |
| 0 | 4 = 14 | | |

Total ways = 2x5 x42 = 420

5. 10 friends arrive to get their COVID vaccine during a particular time slot. During that time slot there are 4 identical nurses administering shots, but 1 of the nurses **may** (or **may not**) be scheduled for a break during the time slot in which the friends arrive. Also, how long it takes the nurses to administer a shot varies wildly, so the nurses working during the time slot are guaranteed to serve at least 1 person, but how many additional people they are able to serve is arbitrary. How many different combinations are there for the number of patients served by the nurses?

Case 1: 4 nurses

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 7 |
| 1 | 1 | 2 | 6 |
| 1 | 1 | 3 | 5 |
| 1 | 2 | 2 | 5 |
| 1 | 1 | 4 | 4 |
| 1 | 2 | 3 | 4 |
| 1 | 3 | 3 | 3 |
| 2 | 2 | 2 | 4 |
| 2 | 2 | 3 | 3 |

9 Total

Case 2: 3 nurses (1 on break)

| | | |
|---|---|---|
| 1 | 1 | 8 |
| 1 | 2 | 7 |
| 1 | 3 | 6 |
| 2 | 2 | 6 |
| 1 | 4 | 5 |
| 2 | 3 | 5 |
| 2 | 4 | 4 |
| 3 | 3 | 4 |

8 Total

Total = Case 1 + Case 2 = 17