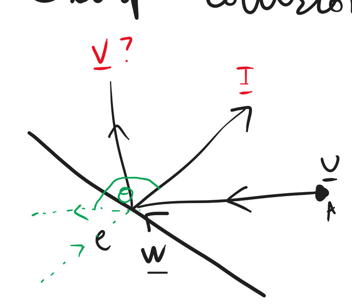
- need to remove:
- replace boundaries with fixed lines so can calculate collisions
  - ↳ to do this need to represent line of movement of object* as a vector and find intersection with boundary
    - *per tick
  - if no collision then update s
  - if collision calc v and update s

need to generalise to allow non-wall collisions

· Oblique collision with wall:



*need obtuse θ

A: mass, u, want v

Wall: $a + \lambda b$, $\underline{w} = b$

let $e = e$

$-e\mathbf{I}\cdot\mathbf{u} = \mathbf{I}\cdot\mathbf{v}$

$\mathbf{u}\cdot\mathbf{w} = \mathbf{v}\cdot\mathbf{w}$

- need to find I:

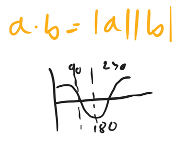
$I \cdot W = 0$

$I = m(v - u)$

to find perp $\underline{w} = \underline{I}$

$\underline{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$ $\underline{I} = \begin{pmatrix} w_2 \\ -w_1 \end{pmatrix}$ or $\begin{pmatrix} -w_2 \\ w_1 \end{pmatrix}$

$a \cdot b = |a||b|\cos\theta$



$(= \Rightarrow //)$

if $u_1 w_2 > u_2 w_1$

$I = \begin{pmatrix} -w_2 \\ w_1 \end{pmatrix}$

else $I = \begin{pmatrix} w_2 \\ -w_1 \end{pmatrix}$

$\therefore \frac{\underline{u}\cdot\underline{I}}{|\underline{u}||\underline{I}|} = \cos\theta \leq 0 \Rightarrow \underline{u}\cdot\underline{I} \leq 0$

$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}\cdot\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \leq 0$

$I_1 = w_2$

$I_2 = w_1$

$u_1 I_1 + u_2 I_2 \leq 0$

⇐ need -ve infront of the larger

" $u_1 w_2 + u_2 w_1 \leq 0$ "

- generate sim. equations:

(1) $-e\underline{I}\cdot u = I \cdot v \Rightarrow -e\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}\cdot\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}\cdot\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$-e(I_1 u_1 + I_2 u_2) = I_1 v_1 + I_2 v_2$ (1)

(2) $u \cdot W = V \cdot W \Rightarrow \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}\cdot\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}\cdot\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$

$u_1 w_1 + u_2 w_2 = w_1 v_1 + w_2 v_2$ (2)

$\underline{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$

$I_1 = \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$

solving: $\begin{pmatrix} w_1 & w_2 \\ I_1 & I_2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} u_1 w_1 + u_2 w_2 \\ -e(I_1 u_1 + I_2 u_2) \end{pmatrix}$

det: $w_1 I_2 - w_2 I_1 \neq 0$

$w_1 I_2 \neq w_2 I_1$

$\frac{w_1}{w_2} \neq \frac{I_1}{I_2}$

$\Rightarrow //$ won't happen ✓

$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}\cdot\begin{pmatrix} I_2 \\ -I_1 \end{pmatrix} \neq 0$ ✓

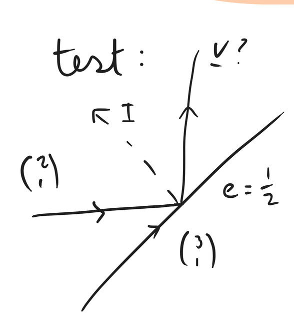
$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \frac{1}{w_1 I_2 - w_2 I_1}\begin{pmatrix} I_2 & -w_2 \\ -I_1 & w_1 \end{pmatrix}\begin{pmatrix} u_1 w_1 + u_2 w_2 \\ -e(I_1 u_1 + I_2 u_2) \end{pmatrix}$

$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \frac{1}{w_1 I_2 - w_2 I_1}\begin{pmatrix} I_2(u_1 w_1 + u_2 w_2) + e w_2(I_1 u_1 + I_2 u_2) \\ -I_1(u_1 w_1 + u_2 w_2) - e w_1(I_1 u_1 + I_2 u_2) \end{pmatrix}$

$\therefore V_1 = \frac{I_2(u_1 w_1 + u_2 w_2) + e w_2(I_1 u_1 + I_2 u_2)}{w_1 I_2 - w_2 I_1}$

$V_2 = \frac{-I_1(u_1 w_1 + u_2 w_2) - e w_1(I_1 u_1 + I_2 u_2)}{w_1 I_2 - w_2 I_1}$

test:



$\underline{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\underline{w} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $\underline{I} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ try $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$-e\underline{u}\cdot\underline{I} = \underline{v}\cdot\underline{I}$

$-\frac{1}{2}\begin{pmatrix} 2 \\ 1 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ $-\frac{1}{2}\begin{pmatrix} 2 \\ 1 \end{pmatrix}\cdot\begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}\cdot\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$-\frac{1}{2}(2-3) = v_1 - 3v_2$ $-\frac{1}{2}(-2+3) = -v_1 + 3v_2$

$\frac{1}{2} = v_1 - 3v_2$ (1) $-\frac{1}{2} = -v_1 + 3v_2$ (1)

$7 = 3v_1 + v_2$ (2)

$v = \begin{pmatrix} 2.15 \\ 0.55 \end{pmatrix}$ doesn't matter ✓

$\underline{u}\cdot\underline{w} = \underline{v}\cdot\underline{w}$

$\begin{pmatrix} 2 \\ 1 \end{pmatrix}\cdot\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}\cdot\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$7 = 3v_1 + v_2$ (2)

$\therefore v = \begin{pmatrix} 2.15 \\ 0.55 \end{pmatrix}$

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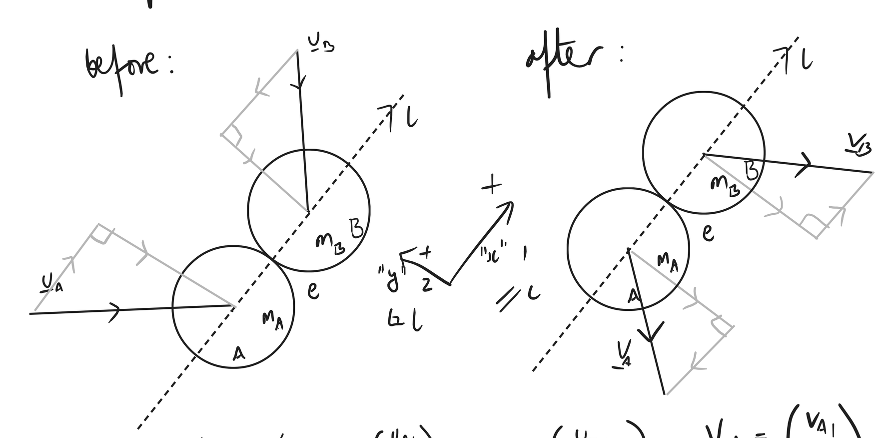
with formula: $I_1 = 1$ $I_2 = -3$

$v = \begin{pmatrix} 2.15 \\ 0.55 \end{pmatrix}$ $v = \begin{pmatrix} 2.15 \\ 0.55 \end{pmatrix}$ ✓

· Oblique collision with other object

line of centres = "x" axis



* need to convert actual v to v relative to line of centres etc.

relative vs: $\underline{u}_A = \begin{pmatrix} u_{A1} \\ u_{A2} \end{pmatrix}$ $\underline{u}_B = \begin{pmatrix} u_{B1} \\ u_{B2} \end{pmatrix}$ $\underline{V}_A = \begin{pmatrix} V_{A1} \\ V_{A2} \end{pmatrix}$ $\underline{V}_B = \begin{pmatrix} V_{B1} \\ V_{B2} \end{pmatrix}$

NLR:

$e = \frac{\text{sep.}}{\text{apr.}}$ $e = \frac{V_{B1} - V_{A1}}{U_{A1} - U_{B1}}$

$e(U_{A1} - U_{B1}) = V_{B1} - V_{A1}$ (1)

CLM:

$m_A \times U_{A1} + m_B \times U_{B1} = m_A \times V_{A1} + m_B \times V_{B1}$ (2)

solving: $\begin{pmatrix} -1 & 1 \\ m_A & m_B \end{pmatrix}\begin{pmatrix} V_{A1} \\ V_{B1} \end{pmatrix} = \begin{pmatrix} e(U_{A1} - U_{B1}) \\ m_A U_{A1} + m_B U_{B1} \end{pmatrix}$

$\begin{pmatrix} V_{A1} \\ V_{B1} \end{pmatrix} = \frac{-1}{m_A + m_B}\begin{pmatrix} m_B & -1 \\ -m_A & -1 \end{pmatrix}\begin{pmatrix} e(U_{A1} - U_{B1}) \\ m_A U_{A1} + m_B U_{B1} \end{pmatrix}$

$= \frac{-1}{m_A + m_B}\begin{pmatrix} m_B e(U_{A1} - U_{B1}) - m_A U_{A1} - m_B U_{B1} \\ -m_A e(U_{A1} - U_{B1}) - m_A U_{A1} - m_B U_{B1} \end{pmatrix}$

$\therefore V_{A1} = \frac{m_A U_{A1} + m_B U_{B1} - m_B e(U_{A1} - U_{B1})}{m_A + m_B}$

$V_{B1} = \frac{m_A U_{A1} + m_B U_{B1} + m_A e(U_{A1} - U_{B1})}{m_A + m_B}$

$\therefore V_A = \begin{pmatrix} U_{A2} \\ \frac{m_A U_{A1} + m_B U_{B1} - m_B e(U_{A1} - U_{B1})}{m_A + m_B} \end{pmatrix}$

$V_B = \begin{pmatrix} U_{B2} \\ \frac{m_A U_{A1} + m_B U_{B1} + m_A e(U_{A1} - U_{B1})}{m_A + m_B} \end{pmatrix}$

$\hat{LOC} = \frac{\underline{s}_1 - \underline{s}_2}{|\underline{s}_1 - \underline{s}_2|}$

$V_t = \begin{pmatrix} v \cdot \hat{LOC} \\ v \cdot \perp \hat{LOC} \end{pmatrix}$

$V_A = \begin{pmatrix} \frac{m_A U_{A1} + m_B U_{B1} - m_B e(U_{A1} - U_{B1})}{m_A + m_B} \\ U_{A2} \end{pmatrix}$

$V_B = \begin{pmatrix} \frac{m_A U_{A1} + m_B U_{B1} + m_A e(U_{A1} - U_{B1})}{m_A + m_B} \\ U_{B2} \end{pmatrix}$