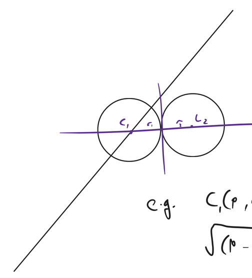


collide when $|c_1 - c_2| = r_1 + r_2$

e.g. $c_1(p,q)$ $c_2(m,n)$

$\sqrt{(p-m)^2 + (q-n)^2} = r_1 + r_2$

c_1 follows $a + \lambda b$ — c_2 follows $c + \mu d$

$p = a_1 + \lambda b_1$ — $m = c_1 + \mu d_1$

$q = a_2 + \lambda b_2$ — $n = c_2 + \mu d_2$

need $\lambda = \mu$ and smallest value
so both represent fraction of a tick

$$\sqrt{(a_1 + \lambda b_1 - (c_1 + \lambda d_1))^2 + (a_2 + \lambda b_2 - (c_2 + \lambda d_2))^2} = r_1 + r_2$$

$$((a_1 - c_1) + \lambda(b_1 - d_1))^2 + ((a_2 - c_2) + \lambda(b_2 - d_2))^2 = (r_1 + r_2)^2$$

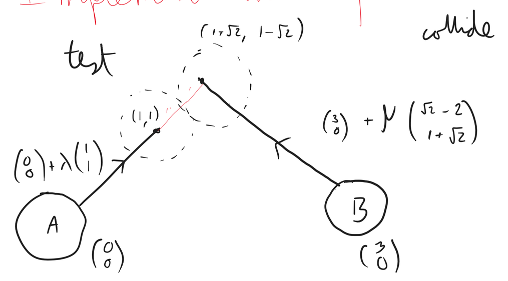
$$(a_1 - c_1)^2 + 2\lambda(a_1 - c_1)(b_1 - d_1) + \lambda^2(b_1 - d_1)^2$$
$$+ (a_2 - c_2)^2 + 2\lambda(a_2 - c_2)(b_2 - d_2) + \lambda^2(b_2 - d_2)^2 = (r_1 + r_2)^2$$

$$((b_1 - d_1)^2 + (b_2 - d_2)^2)\lambda^2 + 2((a_1 - c_1)(b_1 - d_1) + (a_2 - c_2)(b_2 - d_2))\lambda + ((a_1 - c_1)^2 + (a_2 - c_2)^2 - (r_1 + r_2)^2) = 0$$

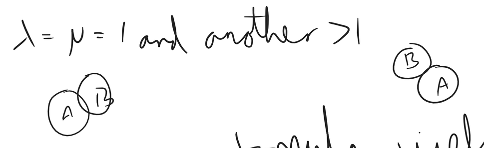
$$\boxed{\lambda = \frac{-2((a_1 - c_1)(b_1 - d_1) + (a_2 - c_2)(b_2 - d_2)) \pm \sqrt{(2((a_1 - c_1)(b_1 - d_1) + (a_2 - c_2)(b_2 - d_2)))^2 - 4((b_1 - d_1)^2 + (b_2 - d_2)^2)((a_1 - c_1)^2 + (a_2 - c_2)^2 - (r_1 + r_2)^2)}}{2((b_1 - d_1)^2 + (b_2 - d_2)^2)}}$$

- Test with an example
- Implement with fewest calculations

test



collide when $\lambda = \mu = 1$ and another > 1



formula yields 1 ✓

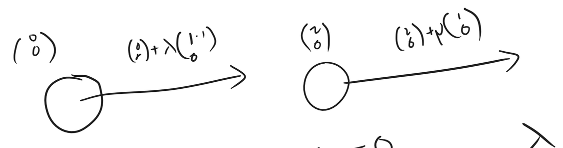
$a_1 = 0$ $a_2 = 0$ $b_1 = 1$ $b_2 = 1$ $c_1 = 3$ $c_2 = 0$ $d_1 = \sqrt{2} - 2$ $d_2 = 1 + \sqrt{2}$

$A = (b_1 - d_1)^2 + (b_2 - d_2)^2$
$= 13 - 6\sqrt{2}$

$B = 2((a_1 - c_1)(b_1 - d_1) + (a_2 - c_2)(b_2 - d_2))$
$= (-9 + 3\sqrt{2})2$

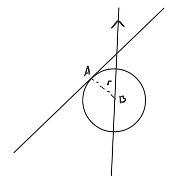
$C = (a_1 - c_1)^2 + (a_2 - c_2)^2 - (r_1 + r_2)^2$
$= 5$

test 2



$a_1 = 0$ $a_2 = 0$
$b_1 = 1.1$ $b_2 = 0$
$c_1 = 2$ $c_2 = 0$
$d_1 = 1$ $d_2 = 0$

$\lambda =$



collide when shortest to distance $= r$

$\lambda \neq \mu$

wall $= a + \lambda b$
object $= c + \mu d$

take point B $\begin{pmatrix} c_1 + \mu d_1 \\ c_2 + \mu d_2 \end{pmatrix}$ for a $0 \le \mu < 1$

and line $\binom{a_1}{a_2} + \lambda\binom{b_1}{b_2}$ for $\lambda \in [\text{start}, \text{end}]$

The distance d from a point (x_0, y_0) to the line $ax + by + c = 0$ is

$$d = \frac{|a(x_0) + b(y_0) + c|}{\sqrt{a^2+b^2}}.$$

line $a + \lambda b$ — point $c + \mu d$ — $d = r$

$x = a_1 + \lambda b_1$, $\lambda = \frac{x - a_1}{b_1}$ — $(c_1 + \mu d_1, c_2 + \mu d_2)$

$y = a_2 + \lambda b_2$

$y = a_2 + \frac{x - a_1}{b_1} \times b_2$

$b_1 y = a_2 b_1 + b_2 x_2 - a_1 b_2$

$\underset{a}{b_2} x_2 - \underset{b}{b_1} y + \underset{c}{(a_2 b_1 - a_1 b_2)} = 0$

$\therefore r = \frac{|b_2(c_1 + \mu d_1) - b_1(c_2 + \mu d_2) + (a_2 b_1 - a_1 b_2)|}{\sqrt{b_2^2 + b_1^2}}$

$r\sqrt{b_2^2 + b_1^2} = |b_2(c_1 + \mu d_1) - b_1(c_2 + \mu d_2) + (a_2 b_1 - a_1 b_2)|$

① $r\sqrt{b_2^2 + b_1^2} = b_2(c_1 + \mu d_1) - b_1(c_2 + \mu d_2) + (a_2 b_1 - a_1 b_2)$

$r\sqrt{b_2^2 + b_1^2} = b_2 c_1 + b_2 d_1 \mu - b_1 c_2 - b_1 d_2 \mu + a_2 b_1 - a_1 b_2$

$r\sqrt{b_2^2 + b_1^2} + b_1 c_2 - b_2 c_1 + a_1 b_2 - a_2 b_1 = \mu(b_2 d_1 - b_1 d_2)$

$\mu = \frac{r\sqrt{b_2^2 + b_1^2} + b_1 c_2 - b_2 c_1 + a_1 b_2 - a_2 b_1}{b_2 d_1 - b_1 d_2}$

② $r\sqrt{b_2^2 + b_1^2} = -b_2(c_1 + \mu d_1) + b_1(c_2 + \mu d_2) - (a_2 b_1 - a_1 b_2)$

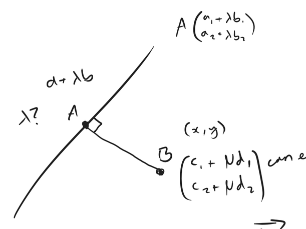
$r\sqrt{b_2^2 + b_1^2} + a_2 b_1 - a_1 b_2 = b_1 c_2 + b_1 d_2 \mu - b_2 c_1 - b_2 d_1 \mu$

$r\sqrt{b_2^2 + b_1^2} + a_2 b_1 - a_1 b_2 + b_2 c_1 - b_1 c_2 = \mu(b_1 d_2 - b_2 d_1)$

$\frac{r\sqrt{b_2^2 + b_1^2} + (a_2 b_1 - a_1 b_2) + (b_2 c_1 - b_1 c_2)}{b_1 d_2 - b_2 d_1} = \mu$

$\mu = \frac{r\sqrt{b_2^2 + b_1^2} \pm (b_1 c_2 - b_2 c_1 + a_1 b_2 - a_2 b_1)}{\pm(b_2 d_1 - b_1 d_2)}$

return the smaller



$\overrightarrow{AB} = \begin{pmatrix} c_1 - a_1 + \mu d_1 - \lambda b_1 \\ c_2 - a_2 + \mu d_2 - \lambda b_2 \end{pmatrix}$

$\overrightarrow{AB} \cdot \binom{b_1}{b_2} = 0$

$b_1(c_1 - a_1) + b_1 d_1 \mu - b_1^2 \lambda + b_2(c_2 - a_2) + b_2 d_2 \mu - b_2^2 \lambda = 0$

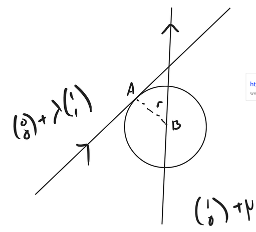
$\frac{b_1(c_1 - a_1) + b_2(c_2 - a_2) + (b_1 d_1 + b_2 d_2)\mu}{b_1^2 + b_2^2} = \lambda$ check is within bounds

$$\boxed{\mu = \frac{r\sqrt{b_2^2 + b_1^2} \pm (b_1 c_2 - b_2 c_1 + a_1 b_2 - a_2 b_1)}{\pm(b_2 d_1 - b_1 d_2)}}$$

use smaller μ, $0 \le \mu < 1$

$$\boxed{\lambda = \frac{b_1(c_1 - a_1) + b_2(c_2 - a_2) + (b_1 d_1 + b_2 d_2)\mu}{b_1^2 + b_2^2}}$$

check is within bounds



$x = 1 + t$ — $t = x - 1$
$y = 2t$
$y = 2(x - 1)$

https://www.desmos.com/calculator/ahshs2tp
www.desmos.com

test:

$r = 1$

$a_1 = 0$ $a_2 = 0$ $b_1 = 1$ $b_2 = 1$ $c_1 = 1$ $c_2 = 0$
$d_1 = 1$ $d_2 = 2$

$\mu = 1 - \sqrt{2}$, $1 + \sqrt{2}$ ✓

$\lambda = \frac{4 - 3\sqrt{2}}{2}$, $\frac{4 + 3\sqrt{2}}{2}$ ✓