Lab session of Image Analysis
BE 2. 2-D Fourier Transform

Data Duration: 2h.
The material of this lab session can be found on **Chamilo**.

Instructions Submit a **report** for each group (binome) of the session in a unique **pdf** , name it **LabX_Name1_Name2**, with **X** denoting the number of the lab session and **Name1,2** your surnames. Upload it in the folder corresponding to your group and lab in **Chamilo**

Deadline submission The material report should be submitted within a week from the lab work. The preparation has to be done individually and will be collected at the **beginning** of the lab.

Note you can use the command `addpath('images')` to add the folder `'images'` to the search paths Matlab looks into when calling `imread` and other such commands.

Useful commands To compute the DFT, use **fft2**, combined with **fftshift**, so that the low frequency part will be placed at the image center. So, the command is: **Ft = fftshift(fft2(I));**
To obtain the real and imaginary parts, modulus and phase, use **real**, **imag**, **abs**, **angle**, respectively. Remark: due to the large dynamic range, we use a logarithmic scale to visualize the modulus of the DFT: **imshow(log(1+abs(Ft)),[])**.
To compute the IDFT use **ifft2**, but remind to shift back the low frequency parts to the edges with **ifftshift**; NB: take into consideration that the output will be complex

Objectives The objectives of this lab work are:
- Observe some properties of the 2-D Discrete Fourier Transform (DFT)
- Use the 2D DFT for filtering

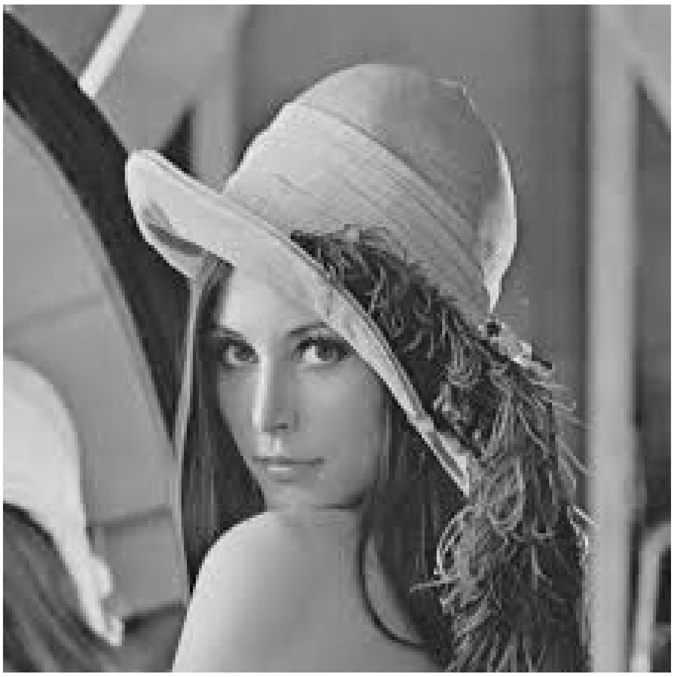

Figure 1: The well known Lena image

Preparation

- Explain how do we extend, mathematically, the concept of monodimensional Fourier transform to two dimensions.
- Explain why the Fourier Spectrum is well suited to describe the directionality of textures.
- Explain why is it useful to filter images.
- Without Matlab, draw the spectrum of the previous image (*Lena*) in the empty square.

1 Fourier analysis of images

1.1 Familiarizing with the Fourier domain

For the following images, visualize the real part, imaginary part, the modulus and the phase of its DFT.

Synthetic images Generate the following synthetic images of size 128x128 and study the changes when you modify the parameters w and r:

- A grid composed of black and white bars. Apply the following code to generate this image, after generating : `[x,y]=meshgrid(0:127,0:127); Igrid=(sign(sin(pi/w*x+1e-8))+1)*127.5;` where the integer w is the width of the bars. Consider the two different cases: 128 is a multiple of w or not.
- A disc: `Idisc=(sqrt((x-64).^2+(y-64).^2) <= r)*255` where r is the radius of the disc.

Real images

- Use the three images *camera*, *Lena* and *ville*. Interpret the frequency content (high and low frequency parts, oriented structures...).

Texture images

- Study the different images of textures available. Do not put the images in your report, only a summary of what you see and your interpretation.

1.2 Amplitude and phase information

Choose one of the real images and perform the following operations:

- Make a periodic shift of the image with the instruction `circshift` and perform the DFT. Compare the results against the DFT of the non-shifted version of the image.

- For the same image, perform the reconstruction by using just the information on the amplitude of the DFT (by imposing the phase to be equally zero) and just the information on the phase (by imposing the amplitude to be flat over all frequencies).

2 Filtering in frequency domain

2.1 Filtering

Perform filtering in Fourier domain, with different filters, over the different real images available. Show their results for different values of the radius and degree parameters. Comment the results and the difference between the different filtering operation.
Use:

- The ideal isotropic lowpass filter. (Note: This is done by creating a flat disc in the Fourier domain; its radius represents the cutoff frequency)
- The ideal isotropic high-pass filter.
- The Butterworth lowpass filter.

For the ideal and Butterworth low-pass filter, take the central row, compute their IDFT and perform the DFT on a zero-padded result. The extra samples allow to simulate the behaviour of the filter in the continuous Fourier domain. Comment the results again taking this result into account.

2.2 Equivalence with convolution in spatial domain

- Apply 3x3 mean filtering to the image *ville*, using periodic (circular) boundary conditions (use `imfilter`).
- Do the equivalent operation in frequency domain: compute the DFT of the image and the DFT of the mask (appropriately extended with zeros. Be careful with the location of the origin, you can use `fftshift` or `ifftshift` for that).
- Take the product of the DFTs and apply the inverse DFT
- Show that the images obtained with the two methods are identical: the mean squared error between the two images should be zero.