Mobile Robotics – Perception
Lab preparation: Attitude Estimation using Indirect Kalman Filtering

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For this lab, we rely on the method proposed in the article "Orientation Estimation Using a Quaternion-Based Indirect Kalman Filter With Adaptive Estimation of External Acceleration" by Young So Suh. This preparatory work summarizes the mentioned paper.
Brief summary: In this paper, the author presents an adaptive orientation filter following the quaternion-based indirect Kalman filter structure that has two-step measurement updates, and that compensates external acceleration by estimating the direction of that acceleration and decreasing the weight corresponding to accelerometer values affected by it, and then validates the method.

1 Introduction

Orientation estimation, i.e., estimating the pitch, roll, and yaw angles, has many applications in various fields such as robotics and biomedicine. This is normally done using three triaxial sensors: a gyroscope (to measure angular velocity), an accelerometer (to measure gravity), and a magnetic sensor (to measure the Earth's magnetic field). The problem with the gyroscope is bias and numerical integration errors, while that of using the accelerometer and the magnetic sensor output is external acceleration and magnetic disturbance. This paper discusses how to combine the outputs from the three sensors to use in an orientation estimation algorithm.
The chosen structure is a quaternion-based indirect Kalman filter, as quaternion has a singularity-free orientation representation, and the orientation error is estimated instead of the orientation (indirect). The two contributions of this method are:

- A new method of orientation computation from the magnetic sensor: The standard Kalman filter is modified so that the magnetic sensor output is only used for yaw estimation error compensation.)
- An adaptive method compensating the external: When there is external acceleration, smaller weights are given to the accelerometer output by increasing the corresponding measurement noise covariance. The direction of external acceleration is estimated for compensation.

2 Quaternion-based indirect Kalman filter

The points p_b in the body frame and p_n in the navigation frame are related as $p_b = C(q)p_n$, where $q = \begin{bmatrix} q_0 & q_1 & q_2 & q_3 \end{bmatrix}^T \in \mathbb{R}^4$ is the orientation quaternion in the navigation frame and $C(q)$ is the rotation matrix. The accelerometer output y_a, the gyroscope output y_g, and the magnetic sensor output y_m are:

$$\begin{cases} y_a = C(q)\tilde{g} + b_a + v_a + a_b \\ y_g = \omega + b_g + v_g \\ y_m = C(q)\tilde{m} + v_m \end{cases} \tag{1}$$

where $\tilde{g} = \begin{bmatrix} 0 & 0 & g \end{bmatrix}^T$ and $\tilde{m} = \begin{bmatrix} cos(\alpha) & 0 & -sin(\alpha) \end{bmatrix}^T$ where g is the gravitational acceleration, α is the dip angle, b_a and b_g are (nearly constant) accelerometer biases, a_b is unknown external acceleration, and v_a, v_b, and v_m are zero-mean white Gaussian sensor noises.

2.1 Indirect Kalman filter

The objective is to estimate q with $\dot{q} = \frac{1}{2} q \otimes \omega$ where $\omega \in \mathbb{R}^3$ is the angular velocity, from the three outputs, using $\dot{\hat{q}} = \frac{1}{2}\hat{q} \otimes y_g$. Under these assumptions that the the error $\tilde{q}_e$ in $\hat{q}$, $\omega - y_g$, the gyroscope noise v_g and bias b_g, and the second-order terms in the rotation matrix are small, we define the state $x = \begin{bmatrix} q_e & b_g & b_a \end{bmatrix}^T \in \mathbb{R}^9$ and we have:

$$\begin{cases} \dot{x}(t) = Ax(t) + \begin{bmatrix} -0.5v_g \\ v_{b_g} \\ v_{b_a} \end{bmatrix} \\ y_a - C(\hat{q})\tilde{g} = 2[C(\hat{q})\tilde{g}\times]q_e + a_b + v_a + b_a \\ y_m - C(\hat{q})\tilde{m} = 2[C(\hat{q})\tilde{m}\times]q_e + v_m \end{cases} \tag{2}$$

where $A = \begin{bmatrix} -[y_g\times] & -0.5I & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (time varying) and $[p\times] = \begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix}$ for a vector $p \in \mathbb{R}^3$. That gives the system model for the indirect Kalman filter, illustrated as:

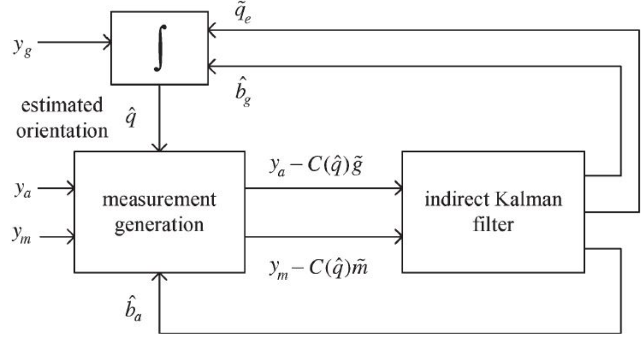

Figure 1: The indirect Kalman filter.

2.2 Discretization

In (2), the output (last two equations) is assumed to be periodically sampled with the sampling period T, while the first equation is discretized as:

$$x_{k+1} = \phi_k x_k + w_k \tag{3}$$

where in real-time application $\phi_k = e^{AT} \approx I + A(kT)T + 0.5A(kT)^2T^2$ and $Q_{d,k} = E\{w_k w_k'\} = \int_{kT}^{(k+1)T} e^{At} Q e^{(At)'}\, dt \approx QT + \frac{1}{2}A(kT)Q + \frac{1}{2}QA(kT)'$. Based on the discrete model, a standard project ahead algorithm is used, i.e.,

$$\begin{cases} \hat{x}_{k+1}^- = \phi_k \hat{x}_k \\ P_{k+1}^- = \phi_k P_k \phi_k' + Q_d \end{cases} \tag{4}$$

where $\hat{x}_k^-$ and $\hat{x}_k$ are a state estimate before and after a measurement update and P_k^- and P_k are the estimation error covariances. Then the quaternion is calculated using equation (18) in the paper, and it is then normalized.

3 Two-step measurement updates

The magnetic disturbance could be considerable, and it affects not only yaw but also pitch and roll, so normally, we use the magnetic sensor output only for yaw estimation, but doing so, we risk losing information. Here, the author proposes a two-step algorithm consisting of an accelerometer measurement ($y_a - C(\hat{q})\tilde{m}$ is used to update $\hat{x}_k^-$, where the updated state is denoted by $\hat{x}_{k,a}$) and a magnetic sensor measurement update ($\hat{x}_{k,a}$ is updated to $\hat{x}_k$ using $y_m - C(\hat{q})\tilde{m}$ while q_e is constrained so that the magnetic sensor output only affects the yaw angle). Between these step, all nine states of x_k^- are updated. The author details in each step the equations used for measurement update.

4 Adaptive algorithm compensating external acceleration

An adaptive algorithm to estimate external acceleration from the residual $r_{a,k}$ is proposed. $E\{r_{a,k}r_{a,k}'\}$ is a function of $Q_{a_b,k}$, the time-varying covariance of the external acceleration a_b (see equation (29)). Since $E\{r_{a,k}r_{a,k}'\}$ cannot be obtained, the author approximates it and then the estimated $\hat{Q}_{a_b,k}$. To guarantee that $\hat{Q}_{a_b,k} \geq 0$, he proposes an adaptive estimation algorithm of $Q_{a_b,k}$ in pseudocode (equations (34) and (35)), by defining two modes to be used in the case of no external acceleration and there is external acceleration. Doing this, we can also prevent $\hat{Q}_{a_b,k}$ from being affected by normal fluctuation of accelerometer noises. The variable M_2 is introduced in the condition of the first mode to prevent falsely entering this mode when there is external acceleration. There is no delay in the transition from mode 1 to mode 2, so external acceleration is quickly estimated.
The author also points out that the state estimation error is usually the largest at the initial time and tends to decrease until it converges to some values. This is because bias estimation is not accurate immediately after the initial time, making the accurate estimation of $Q_{a_b,k}$ difficult immediately after this time. In the numerical examples, sensor biases can be estimated after a few minutes, after which the sensor bias effect on $Q_{a_b,k}$ becomes small.

5 Numerical examples and results

In this part, some numerical examples are presented. Three different quaternion-based indirect Kalman filters are compared: The standard Kalman filter (not adaptive), the accelerometer norm-based adaptive algorithm (where the second mode is constant instead of a function like the proposed method), and the proposed method.
In the first experiment, external acceleration is given on all three axes. The purpose here is to compare the three filters. We see that the average estimation error of 100 experiments is significantly smaller for the proposed filter than the other two (Table I). Figure 3 shows that orientation estimation by the proposed filter is highly accurate, and by Figures 4 and 5, we see that bias estimation for gyroscope and accelerometer needs around one or two minutes.
Real sensor data are used in the second experiment, and external acceleration is only on the x-axis. The purpose here is to show the effectiveness of the adaptive method for $\hat{Q}_{a_b,k}$. Both adaptive filters are robust with respect to external acceleration compared to the standard Kalman filter. Looking at Figures 8 and 9, we can see that the proposed method works better than the accelerometer norm-based adaptive algorithm. Because in the earlier method, $\hat{Q}_{a_b,k} = sI$ (here $s = 10$) is equivalent to giving less weights to all three-axis accelerometer output,. If we do so when external acceleration is only applied to the x-axis, giving smaller weights on the y- and z-axis accelerometer output will make us lose the information stored in these. If we look at $\hat{Q}_{a_b,k}$ of the proposed method, we see that only the x-direction element of it is increased, so we keep the information in the other two axes. So the proposed method for $\hat{Q}_{a_b,k}$ has proven its effectiveness. Lastly, Table III shows that there is a trade-off between robustness to external acceleration and use of the nonexternal-acceleration-direction accelerometer output in the accelerometer norm-based adaptive algorithm, which can be solved by the proposed method.

One limitation of this paper is that the magnetic disturbances are not adaptively compensated.