Computer exercises
Nonlinear systems

PART II : Robot control (4h)

Let us consider the dynamics of a robot arm as depicted in figure 1, made of a drive actuating a shaft through a flexible joint. The corresponding dynamical equations can be written as follows :

$$\begin{aligned} J_l\ddot{\theta}_l + B_l\dot{\theta}_l + k(\theta_l - \theta_m) + mgl\cos(\theta_l) &= 0 \\ J_m\ddot{\theta}_m + B_m\dot{\theta}_m - k(\theta_l - \theta_m) &= u. \end{aligned}$$

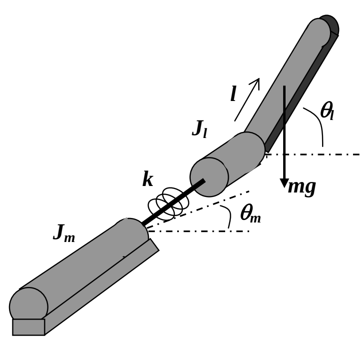

FIGURE 1 – Robot arm with flexible joint.

$$with : k = 0.8Nm/rad; J_m = J_l = 4e-4Nms^2/rad; B_m = 0.015Nms/rad; B_l = 0.0;$$
$$m = 0.3kg; l = 0.3m; g = 9.8ms^{-2}.$$

1. Give a state space representation of this system.
   If we consider a purpose of stabilization at $\theta_l = 0°$ on the basis of approximate linearization, what would be the linear model to be considered ? What could be a design methodology (without expanding it) ?

2. From now on, the purpose we consider is to vertically stabilize the arm at $\theta_l = 90°$. Check that the system is fully I/O linearizable by state feedback with $\frac{\pi}{2} - \theta_l$ as the output, and propose a stabilizing state feedback law on this basis.

3. Verify the results in simulation, distinguishing between the system model, and its control.