Computer exercises
Nonlinear systems

PART I : Blending control (4h)

Let us consider a blender made of a tank as in figure 1, with section S, fed with both warm and cold water, with flow rates respectively equal to Q_w and Q_c, and with an output flow rate $Q_{out} = a\sqrt{H}$, where H stands for the water level in the tank, and $a > 0$. Let T_w, T_c, T respectively denote temperatures for warm water, cold water and tank water.
In steady-state, water level and temperature in the tank are H_0, T_0, for input flow rates both equal to Q_0. Let h and θ denote level and temperature variations in the tank, and q_w, q_c the variations for input flow rates.

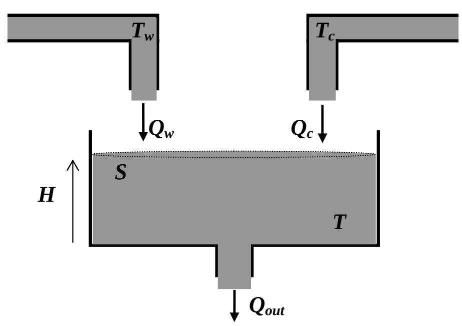

FIGURE 1 – Blender.

We will assume that the input flows are controlled by opening or closing input valves, and that the resulting flows are proportional to the opening ratios, with a coefficient k_v. For numerical applications, let us consider the following values :

$$k_v = 1.2cm^3/s/\%; a = 24cm^{5/2}/s; T_w = 70°C; T_c = 20°C; S = \pi * 5^2cm^2;$$

$$H_0 = 16cm; T_0 = 45°C; Q_0 = 48cm^3/s.$$

1. Recall the equations which are ruling the evolution of h and θ under the action of q_w, q_c.

2. By linearizing the equations around the equilibrium corresponding to steady state (H_0, T_0, Q_0), check that h and θ can directly be controlled respectively by $v_h = \frac{q_w+q_c}{S}$ and $v_\theta = \frac{(T_w-T_0)q_w+(T_c-T_0)q_c}{SH_0}$.

3. Choose v_h and v_θ proportional to h and θ respectively, with two numerical applications :
   — First to increase by 4 the time response ;
   — Then to decrease by 4 the time response.

   Simulate the resulting temperature and level evolutions in each case, both on the linearized model and on the nonlinear one (e.g. for step variations of few units). Conclusions ?

4. Coming to the nonlinear model, show that $\bar{v}_h = v_h$ and $\bar{v}_\theta = \frac{(T_w-T_0-\theta)q_w+(T_c-T_0-\theta)q_c}{S(H_0+h)}$ can be used to exactly linearize the system by state feedback, and give the state feedback laws based on the resulting linear system so as to impose the same dynamic as before.
   Provide the expressions of the resulting full control laws, and compare the results obtained in simulation with this approach and the nonlinear model, with those previously obtained.