Linear Programming
Optimal Planning of Agricultural Production under Water Saving Legislation (3h)

Objective

The aim of this project is to learn how to use a linear programming solver. In particular, the Matlab LINPROG procedures is used. But the first step remains (as in any optimization problem) to put the problem in a standard form.

Problem Statement

A company, which owns three farms at the edge of a desert, is planning agricultural production for the coming year. The agricultural output of each farm is limited by both :

— The amount of available irrigable land and

— the quantity of water available.

The corresponding data are given on the table given below.

farm	Usable land (Acres)	Water available (Acres Feet)
1	420	650
2	620	900
3	330	370

The available irrigable land and the available water at each farm

The product being considered on these farms include :

1. Sugar beets
2. Cotton and
3. Corn

These products differ primarily in their expected net return per acre and their consumption of water. In addition, the company has set a maximum quota for the total acreage that can be devoted to each of these products. The corresponding data are given in the table below.

Product	max Quota (Acres)	Water Consumption (Acre Feet/Acre)	Net Return (£/Acres)
Sugar Beets	590	3	1100
Cotton	510	2	800
Corn	335	1.5	165

The maximum quota, water consumption and the net return for the three product being considered.

Because of the limited water available for irrigation, the company will not be able to use all its irrigable land. To ensure equity between three farms, it has been agreed that each farm will plant the same proportion of its available irrigable land. For example, if farm 1 plants 210 of its 420 acres, then farm 2 must plant 310 of its 620 acres, while farm 3 plants 165 of its 330 acres. However, any combination of products can be grown at any of the farms.

The job facing the company is to plan how many acres to devote to each product on these farms while satisfying the given restrictions. The objective is to maximize the total net return of the company.

In the remainder of this project, the notation depicted on the table below will be used to state the optimization problem at hand.

Product	farm 1 (acres)	farm 2 (acres)	farm 3 (acres)
Sugar Beets	x_1	x_2	x_3
Cotton	x_4	x_5	x_6
Corn	x_7	x_8	x_9

The designation of the problem's unknowns (decision variables). For example, x_4 is the number of acres used in farm 1 to grow cotton while x_9 is the number of acres used in farm 3 to grow corn etc.

Questions

1. Express successively the following in terms of the x_i's variables :
   (a) The cost function to be minimized
   (b) The constraints on the usable land of each farm
   (c) The constraints on the water available at each farm
   (d) The constraints on maximum quota (in acres) of each product
   (e) The constraints on the equal proportion of land planted
   (f) The non negativity constraint on the x_i variables.
   (g) Define the matrices you need to put the problem in the syntaxe of the Linear Programing Solver LINPROG of MATLAB.
   (h) Solve the problem to obtain the optimal vector x^*. What is the maximum net return ?
2. Enumerate the set of active constraints at the solution
3. What is the minimal increase (in %) of the net return of corn that would incite the company to begin growing it in at least one farm.
4. Suppose that the year after, the local environmental services asks the company to use only 70% of the total amount of water used in the preceding solution. Assuming that all the other problem's parameters remain unchanged (Corn net return is unchanged at 275 £), how does this new constraint affect the problem's formulation and the resulting solution. What is the decrease in the net return that results from this environmental constraint.