B.E 1 : Optimal planning of agricultural production under water saving legislation

A company, which owns three farms located near a desert, asked us to find the proportion of sugar beets, cotton and corn to plant on each farm that will allow them to have the maximum total net return. We will consider the quantity of water available and the amount of usable land in each farm limited, the price of each product constant and all produced units as sold and the water consumption depending only on product. Moreover, the company enforces a maximum quota of usable land for each product and the same proportion of cultivated land for the different farms.

To answer that demand we begin by defining the variables of the problems. We consider nine variables named $x_{i,1\le i\le 9}$ that will represent the number of acres used to grow one product in particular in one specific farm.

From those variables and the net return in £/Acres for each product , we find the function that we want to optimize. We want the maximum total net return which is equal to $max\,1100(x_1+x_2+x_3)+800(x_4+x_5+x_6)+165(x_7+x_8+x_9)(£)$. However, with Matlab, we can only find the minimum of a function that we want to optimize, that's why, we will try to find $min-[1100(x_1+x_2+x_3)+800(x_4+x_5+x_6)+165(x_7+x_8+x_9)](£)$ with the function LINPROG of Matlab. We are now looking for all the constraints that we have to take into account according to the legislation and to the entreprise enforcements :

-The constraints on the usable land of each farm:

$$\begin{cases} g_1(x)=x_1+x_4+x_7\le 420 \\ g_2(x)=x_2+x_5+x_8\le 620\,(acres) \\ g_3(x)=x_3+x_6+x_9\le 330 \end{cases}$$

-The constraints on the water available at each farm:

$$\begin{cases} g_4(x)=3x_1+2x_4+1.5x_7\le 650 \\ g_5(x)=3x_2+2x_5+1.5x_8\le 900\,(acres\,feet) \\ g_6(x)=3x_3+2x_6+1.5x_9\le 370 \end{cases}$$

-The constraints on maximum quota (in acres) of each product:

$$\begin{cases} g_7(x)=x_1+x_2+x_3\le 590 \\ g_8(x)=x_4+x_5+x_6\le 510\,(acres) \\ g_9(x)=x_7+x_8+x_9\le 335 \end{cases}$$

-The constraints on the equal proportion of land planted on:

$$\frac{x_1+x_4+x_7}{420}=\frac{x_2+x_5+x_8}{620}=\frac{x_3+x_6+x_9}{330}$$

Which simplify to:

$$\begin{cases} h_1(x)=31x_1-21x_2+31x_4-21x_5+31x_7-21x_8=0 \\ h_2(x)=11x_1-14x_3+11x_4-14x_6+11x_7-14x_9=0 \end{cases}$$

-The non-negativity constraints on the x_i variables :

$x_i \geq 0, i=1,\ldots,9.$ We name them from $g_{10}(x)$ to $g_{18}(x)$.

Once we had the cost function and the constraints, we defined the matrices we need to put in the syntax of the LINPROG solver of MATLAB :

```
%define the cost function that must be minimized
f=[ -1100 -1100 -1100 -800 -800 -800 -165 -165 -165];
%define inequality constraints
A=[1 0 0 1 0 0 1 0 0
   0 1 0 0 1 0 0 1 0
   0 0 1 0 0 1 0 0 1
   3 0 0 2 0 0 1.5 0 0
   0 3 0 0 2 0 0 1.5 0
   0 0 3 0 0 2 0 0 1.5
   1 1 1 0 0 0 0 0 0
   0 0 0 1 1 1 0 0 0
   0 0 0 0 0 0 1 1 1];
b=[420 620 330 650 900 370 590 510 335];
%define equality constraint
Aeq=[31 -21 0 31 -21 0 31 -21 0
     11 0 -14 11 0 -14 11 0 -14];
beq=[ 0 0 ];
%define lower bounds
lb=[ 0 0 0 0 0 0 0 0 0];
```

According to Matlab, we found $x^*=\begin{pmatrix} 179.0909 \\ 204.8485 \\ 0 \\ 56.3636 \\ 142.7273 \\ 185.0000 \\ 0 \\ 0 \\ 0 \end{pmatrix}(acres)$ as the optimal vector for a total net return of $f_{max}=7.2961\times 10^5(£)$.

Now that we have found the solution, the company asked us to find all the active constraints. To do so, we seek to find which inequality constraints give equality at the optimal point, we found a total of seven active constraints, the three inequalities that concerns water limitations and four related to the non-negativity of the variables.

As there were no acres of corn, we were asked to find the minimal increase of the net return of corn that would make it profitable enough to be planted in at least one farm. By using a while loop on Matlab, we found that the price of the corn has to increased by 170%.

For the next year we added a few lines to calculate the drop in profit with respect to a 30% decrease in water. We therefore modify the inequality constraints that deal with water restrictions by multiplying each term by 0.7. The result is that the total net return is reduced by 30%.

To conclude, we can see that, by optimizing the total net return and defining all the constraints, we can also allow the company to see which parameters are the most restrictive. For example, it would be useless to buy more usable land for the farms, as the resolution of the optimization problem shows that the limiting factor is the water available. Moreover, we can also reconsider some hypothesis to be more realistic, for example, we do not take into account the effect of the weather that could destroy a part of the production or change the need of water of products.