Constrained Optimization
Necessary conditions of optimality (2h maximum)

Objective

The use of the necessary conditions of constrained optimality. As usual the report must be only two pages and submitted to Chamilo in the corresponding folder.

Questions

1. Consider the following quadratic constrained problem (with $\beta_i > 0$ for all i) :

$$\min_{x\in\mathbb{R}^n} \sum_{i=1}^{n} \left[\beta_i x_i^2\right] \quad \text{under the constraint } \sum_{i=1}^{n} x_i = \alpha$$

   (a) Write the first order KKT necessary condition of optimality

   (b) Solve by hand the resulting equations to obtain the solution $x^*(\alpha, \beta_1, \ldots, \beta_n)$.

   (c) Use the Matlab function QUADPROG to solve the optimization problem in the particular case :

$$n = 4 \quad ; \quad \alpha = 2 \quad ; \quad \beta_i = 2i$$

2. Consider the following optimization problem :

$$\min_{x\in\mathbb{R}^2} -x_1^2 - 2x_2^2 \quad \text{under the constraint} \quad \frac{1}{16}(x_1^2 + x_2^2) \leq 4$$

   (a) Write the first order KKT optimality conditions

   (b) Represent graphically the optimization problem and determine the solution(s) of the problem. You can use the Matlab functions CONTOUR, SURFC, OR CONTOUR3.

   (c) Check the first order necessary condition at the optimal points.

3. Consider the optimization problem :

$$\min_{x\in\mathbb{R}^2} x_2 \quad \text{under} \quad \begin{array}{l} (x_1+1)^2 + x_2^2 \geq 1 \\ (x_1-2)^2 + x_2^2 \geq 4 \\ 4x_2 \geq x_1 + 2 \\ x_1 + 1 \geq 0 \end{array} \tag{1}$$

1. Write the KKT first order necessary conditions of optimality
2. Solve the problem graphically as before.
3. Check that the solution satisfies the KKT condition and give the corresponding value of the Lagrange multiplier $\mu \in \mathbb{R}^4$ (all the cases).