IT Tools & Optimization BE 4 Report – 02/10/2019
Constrained Optimization Optimal Drug Injection for Cancer Treatment

Students: BERG Agnes & TRAN Gia Quoc Bao

Introduction: In this report we tried to find out the optimal strategy for drug injection in order to treat cancer while preserving the patient's health. For this, the MATLAB FMINCON function was used under some constraints.

First, we declared globally all model coefficients and constants in a MATLAB function:

```
clear all
global gamma alpha lambda delta beta T rho nu P0 Q0
gamma = 1.46; alpha = 5.63; lambda = 0.16; delta = 0; beta = 0.48;
T = 30; rho = 0.35; nu = 0.001; P0 = 0.5; Q0 = 0.5;
```

Our drug injection strategy follows a step function lesu(i) over a time period T divided into N control actions. We wrote a code to select the control value u(t):

```
function u = udet(t,lesu)
global T
N = length(lesu);
if t == T
        u = lesu(N);
end
for i = 1:1:N
    if t >= (i-1)*T/N && t < i*T/N
        u = lesu(i);
end
end
```

Which satisfies the condition:

$$u(t) = lesu(i), t \in [(i-1)\frac{T}{N}, i\frac{T}{N})$$

And the first "if" is for the value at the right extreme.

We then wrote a function implementing the two differential equations governing the numbers of cells; the "u" was added for later use:

```
function xdot = myode(t,x,lesu)
global gamma alpha lambda delta beta
u = udet(t, lesu);
xdot = [(gamma-delta-alpha-u)*x(1) + beta*x(2); alpha*x(1) - (lambda + beta)*x(2); u];
end
```

As we defined the third component of xdot to be u, the third component of xx is the integral of u(t) over time. We want P(T) and integral of u(t) from 0 to T so we take the final value of xx by putting "end". The cost function J to be minimized is defined as follow:

```
function J = cost(lesu)
global gamma alpha lambda delta beta nu P0 Q0 T
% u = udet(t, lesu);
% xdot = [(gamma-delta-alpha-u)*x(1) + beta*x(2); alpha*x(1) - (lambda + beta)*x(2); u];
[tt,xx]=ode45(@myode,[0 T],[P0 Q0 0],[],lesu);
J = xx(end,1) + nu*xx(end,3);
end
```

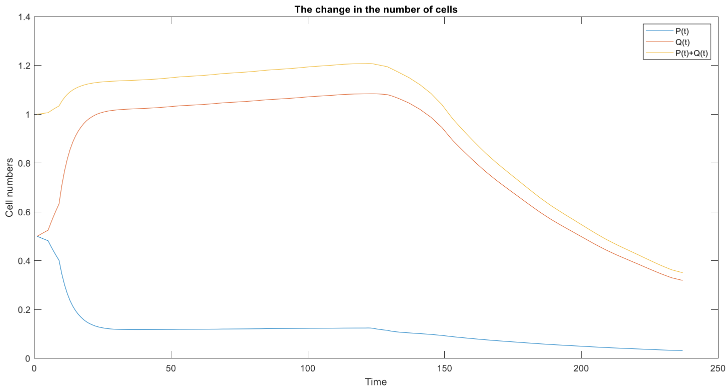
In order to find out the optimal injection strategy we must declare the constraints on the outputs as well as for the value of u(t).

```
function [C, Ceq] = constraints (lesu)
global P0 Q0 T rho
Ceq = [];
[~,xx]=ode45(@myode,[0 T],[P0 Q0 0],[],lesu);
C = max(-xx(:,1)-xx(:,2)+rho);
end
```

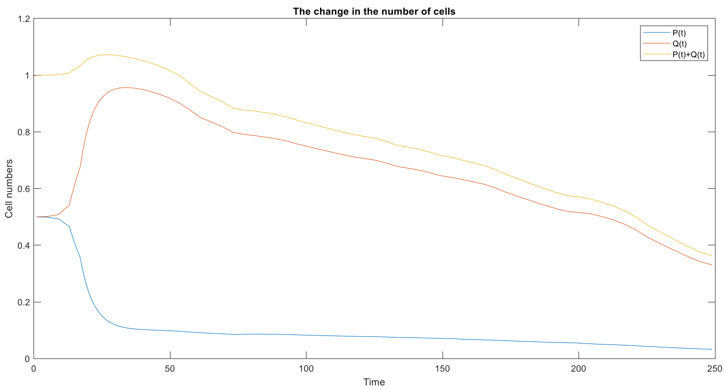
```
lb = zeros(1,length(lesu));
ub = ones(1,length(lesu));
```

For the *initial quess* lesu = (0,0,...0) we used FMINCON with different values of N that represent different injection strategies and found these results:

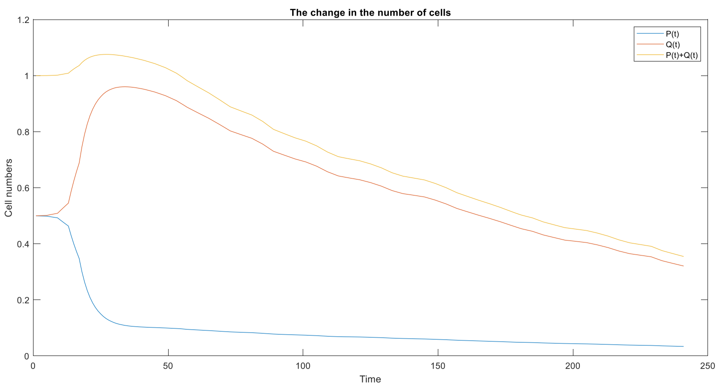
- N = 2: lesuopt = $(0.0196, 0.7618)^T$



- N = 5: lesuopt = $(0.3540, 0.1923, 0.1983, 0.3272, 0.6982)^T$



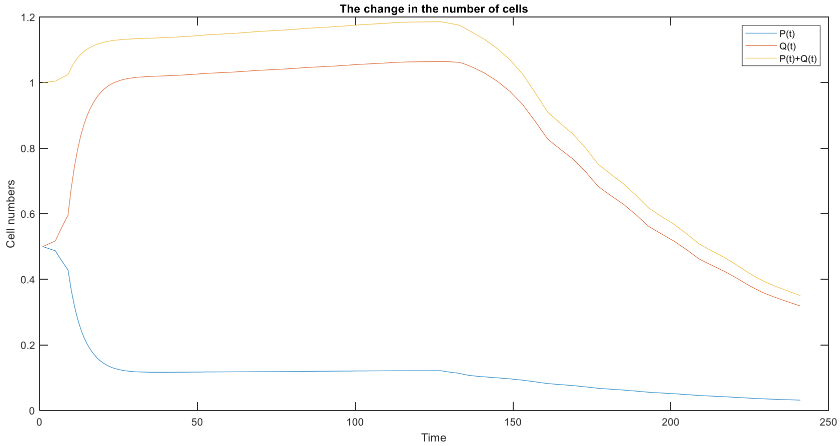
- N = 10: lesuopt = $(0.3277, 0.3590, 0.3552, 0.3497, 0.3179, 0.3637, 0.3672, 0.3290, 0.3858, 0.3989)^T$



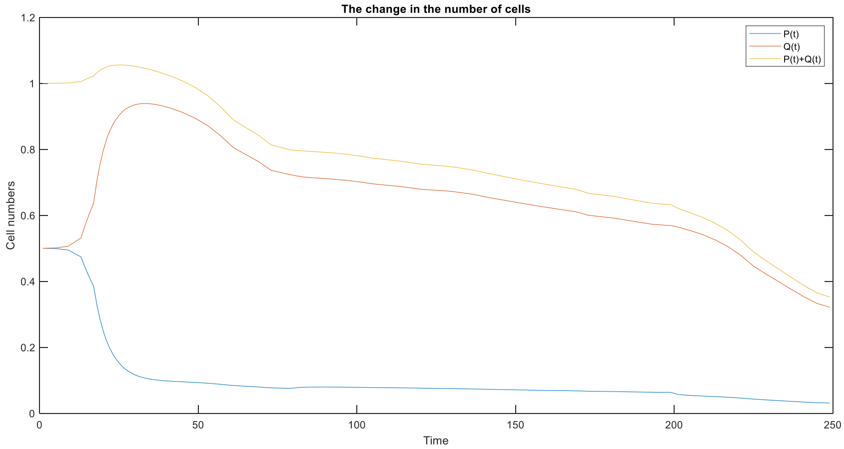
We notice that the more components we have in the lesu, the smoother the change in cell numbers. That is because the change in cell number is noticeable whenever the injection rule changes.

We repeated the procedure for randomly initiated lesu:

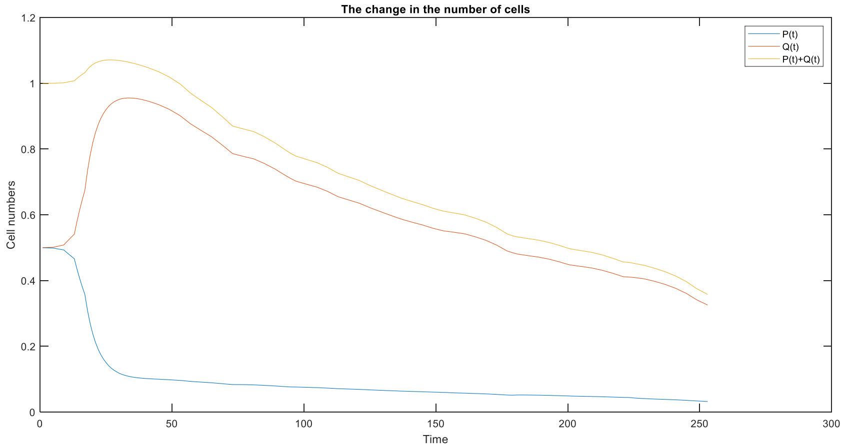
- N = 2: lesuopt = $(0.0285, 0.7489)^T$



- N = 5: lesuopt = $(0.4801, 0.1245, 0.1701, 0.1604, 0.9242)^T$



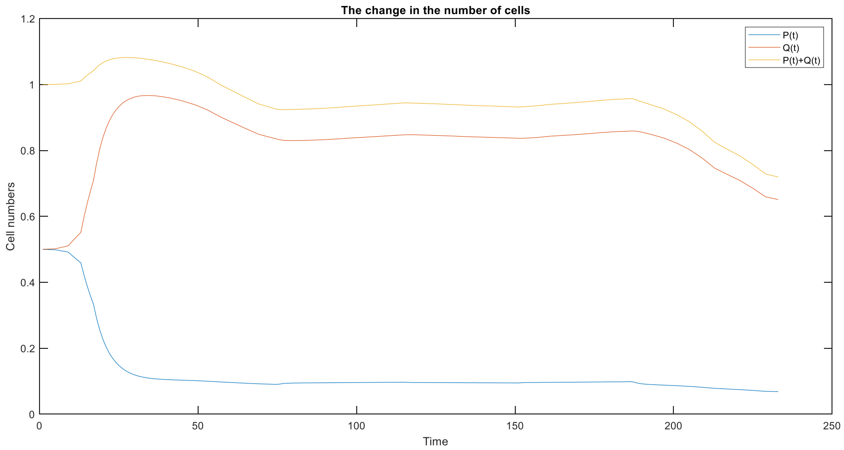
- N = 10: lesuopt = $(0.3600, 0.3767, 0.3271, 0.2707, 0.2730, 0.2702, 0.4019, 0.2483, 0.2886, 0.7454)^T$



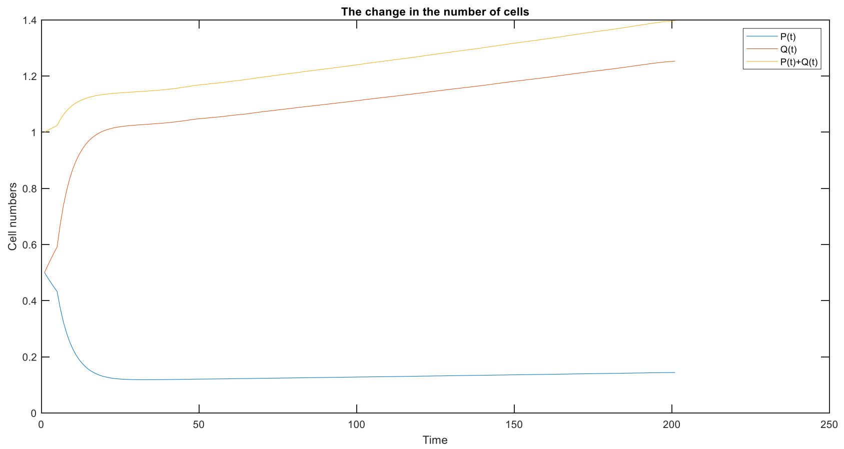
We see that for each initiated lesu, fmincon gives different results. The final values are not so different: with different initiated lesu the program always tries to achieve to same result.

We also tried different values of μ and for lesu of 5 components:

- $\mu = 0.01$: lesuopt = $(0.2873, 0.0230, 0.0701, 0.0184, 0.4481)^T$



- $\mu = 0.1$: lesuopt = $10^{-4}\,(0.3571, 0.0034, 0.0065, 0.0184, 0.4481)^T$



We notice that this time more healthy cells have survived because when we increase the weight of u in the cost we are giving it more attention and importance.
Also when the coefficient of u is too high the injection fails to kill the proliferating cells as we care so much for the patient's health that we do not kill the bad cells effectively (u is now too small). The problem therefore has not converged to the desired result.

Our observations concerning the experiment:

- This method could help minimize the number of proliferating cells while preserving the patient's health, and it would help reduce the treatment cost by optimizing the drug injection strategy. So we can balance between effectiveness and cost efficiency.
- We should try to divide the injection period into more parts because this way the bad cells will be diminished gradually, avoiding shock for the body.

Conclusion: In this experiment we used the MATLAB FMINCON function to try to optimize the injection strategy under constraints in order to treat cancer. Through it we understood better non linear optimization and its application in real life.

Thank you for reading.