Hands on Matlab,
Graphical Optimization (3h)

General introduction

For each computer session, you have to submit an electronic report in pdf format (one page per one work hour). The group number and members' names must clearly. For figures, no screen shots are allowed. Rather use the figure envirnment of Matlab. Don't forget the figures titles and legends. Equations must be typeset using the equation environmental of Latex or word. Essential Matlab code can be included.

Introduction

This computer session aims at introducing Matlab and its powerful visualization features to solve numerical problems.

Questions

1. Using Matlab, plot the ballistic trajectory from simple two-dimensional mechanics.

$$x(t) = u_0 t + x_0 \quad (1)$$
$$y(t) = y_0 + v_0 t - 0.5gt^2 \quad (2)$$
$$u_0 = V\cos\theta \quad (3)$$
$$v_0 = V\sin\theta \quad (4)$$

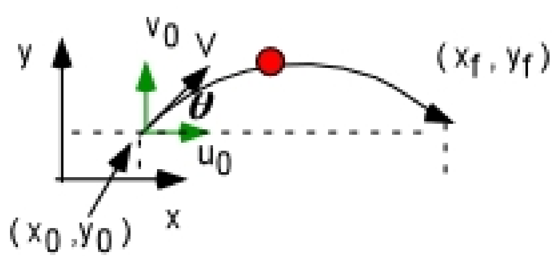

Fig. 1: The ballistic trajectory.

   Take the initial position $(x_0, y_0) = (0, 0)$, initial velocity $v_0 = 15$m/s, the angle of the initial velocity with the horizontal is 75 deg. The flight time is 5 sec.

2. Lets look at the following unconstrained optimization problem

$$\min_x x^2 - 2x$$

   (a) Derive first the optimal value for x on paper.

   (b) Find the solution using Matlab.

3. Consider this nonlinear function :

$$f(x;y) = \frac{1}{2}(x-1)^2 + \frac{1}{2}(10(y-x^2))^2 + \frac{1}{2}y^2$$

   (a) Derive, first on paper, the gradient and Hessian matrix of the function.

   (b) Find the same result using Matlab and its symbolic toolbox. Hint : You can use the Matlab functions "sym", "diff", "subs", "ezplot", and "jacobian".

4. Find the gradient of the Rosenbrock function given by :

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

at the point $x = [1\ 2]^T$.

5. Today's concern over the waste, recycling, and the environment has manufacturers trying to adopt new packaging materials to deliver their products. One such case involves using bio-degradable cartons made from recycled materials. The cost of the material is based on the surface area of the rectangular container that will house the product. The cost per unit area is \$1.5 per square meter. The different products can be accommodated by a single container. The container must hold a volume of 0.032 m^3. The perimeter of the base must be less than or equal to 1.5 m. Its sides are scaled geometrically to hold information labels. The width should not exceed three times the length. Its height must be less than two thirds the width. Its length and width are less than 0.5 m. The design variables are a length of the container, b its width, and c is its height.

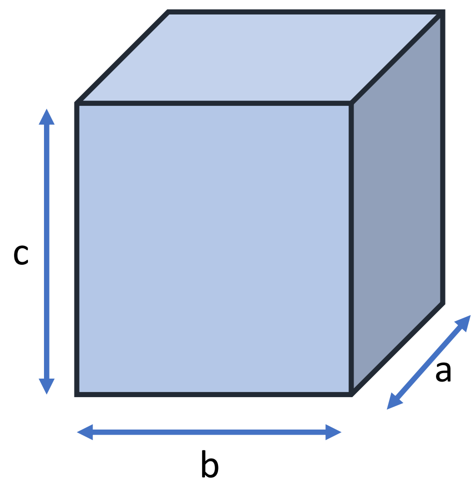

Fig. 2: The container form.

   (a) Write the objective and constraint functions.

   (b) Find graphically the container of minimum cost. Hint : Use the Matlab function "isosurface".

6. Consider a plane in level flight where lift (L) is equal to the weight (W) and thrust (T) is equal drag (D). The forces are shown in the figure below. The lift and drag can be also calculated from the non-dimensional lift and drag coefficients (C_L and C_D) through

$$L = \frac{1}{2}\rho V^2 S C_L, \quad D = \frac{1}{2}\rho V^2 S C_D$$

In standard design, the relation between the coefficients can be expressed through the parabolic drag polar :

$$C_D = C_{D0} + K C_L^2$$

where ρ is the atmospheric density at the flight altitude, V is the speed of flight, S is the reference area of the wing, C_{D0} is the zero lift drag, and K is induced drag factor, all of which are constants for the flight.

   (a) Find the speed for minimum drag.

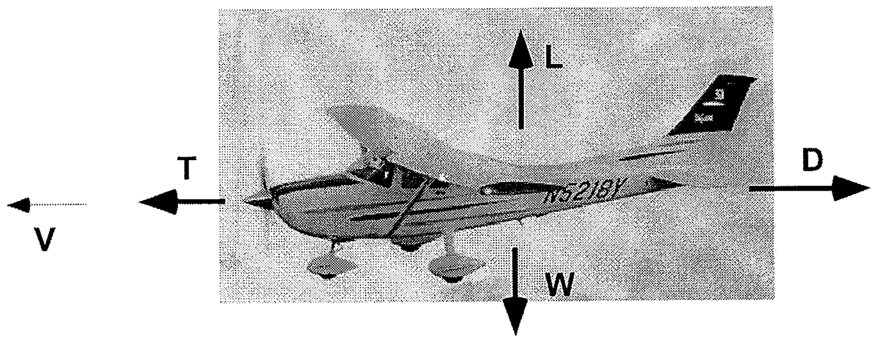

Fig. 3: The plan under study.

(b) Since the aircraft is powered by a piston propeller engine, maybe it is better to fly at the speed for minimum power, where the power is calculated as

$$P = DV$$

. Find the speed for minimum power.

(c) Another way of looking at the problem is to work with the non-dimensional relations. A measure of aircraft performance is aerodynamic efficiency E, which is the ratio of C_L over C_D. Find the maximum aerodynamic efficiency and the speed corresponding to this efficiency.