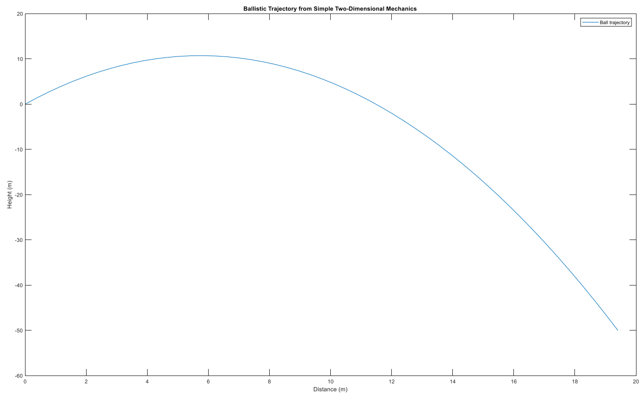
BE IT Tools & Optimization : Hands on Matlab, Graphical Optimization

Group 3. Names : TRAN Gia Quoc Bao, BERG Agnes

Question 1 :

We use a MATLAB function to plot this trajectory from 0 to 5 seconds with 0.1 second increment.



Question 2 :

(a) On paper we derive the 1st derivative of this single parameter function :

$$\frac{df}{dx} = 2x - 2$$

Use Theorem 1 to find the (possible) local minimum :

$$\frac{df}{dx} = 2x - 2 = 0 \leftrightarrow x^* = 1$$

Using Theorem 2 we have :

$$\frac{d^2 f}{dx^2} = 2 > 0, \forall x$$

Which shows that $x^* = 1$ is indeed a local minimum. The minimum value of f is :

$$f(x)|_{x=1} = -1$$

Also as this function and its derivatives are continuous we can conclude that this is the global minimum.

(b) We used a script connected to a function. A function called f.m is first declared :

```
function [y] = f(x)
y=x^2 - 2*x;
end
```

Then we created another script calling for the f.m function and minimize it using fminunc :

```
x0=-10;                    [x,fval] = fminunc(@f,x0)
```

And it gave the same result as (a) :   x = 1.0000   fval = -1

Question 3 :

(a) The gradient of the given function :

$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix} = \begin{pmatrix} x - 1 - 200x(y - x^2) \\ 100(y - x^2) + y \end{pmatrix}$$

Its Hessian matrix is :

$$H = \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix} = \begin{pmatrix} 1 + 600x^2 - 200y & -200x \\ -200x & 101 \end{pmatrix}$$

(b) Using MATLAB we wrote this script :

```
syms x y
%Declaring function
f = 0.5*(x-1)^2 + 50*(y-x^2)^2 + 0.5*y^2;
%Find Gradient and Hessian
gradient = gradient(f, [x, y])
Hessian = hessian(f,[x,y])
```

Which gave the same results :

gradient = (x - 200*x*(- x^2 + y) – 1 ; - 100*x^2 + 101*y)^T

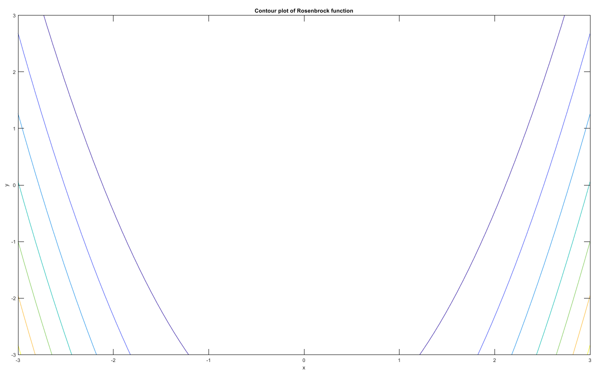
Hessian = [600*x^2 - 200*y + 1   -200*x ;   -200*x   101]

Question 4 :

```
%Declaring function
f = 100*(y - x^2)^2 + (1 - x)^2;
gradf = gradient(f, [x, y]);
subs(gradf,{x,y},{1,2})
```

The result is : $(\mathbf{-400\ 200})^{\boldsymbol{T}}$. To draw contour lines of this function :

```
x = linspace(-3,3);y = linspace(-3,3);
[X,Y] = meshgrid(x,y);
Z = 100*(Y - X.^2).^2 + (1 - X).^2;
contour(X,Y,Z)
xlabel('x');ylabel('y');title('Contour plot of Rosenbrock function');
```



Question 5 :

(a) The problem is to :

$$\min_{a,b,c}\big(1.5 \times 2 \times (ab + bc + ac)\big)\ (dollars)$$

Subject to :

$$abc = 0.032\ (m^3)$$
$$2 \times (a + b) \leq 1.5\ (m)$$
$$b \leq 3\ (m)$$
$$0 < c < \frac{2}{3} b\ (m)$$
$$0 < a < 0.5\ (m)$$
$$0 < b < 0.5\ (m)$$

(b) MATLAB code :

```
a=linspace(0,0.5,50);b=linspace(0,0.5,50);c=linspace(0,0.33,50);
[A,B,C] = meshgrid(a,b,c);
C1 = A.*B.*C - 0.032; C2 = 0.5 - A; C3 = 0.5 - B;
C4 = 3*A - B; C5 = 2*B - 3*C; C6 = 1.5 - 2*(A + B);
p1 = patch(isosurface(a,b,c,C1,0));p1.FaceColor = 'red';
p1.EdgeColor = 'none';
p2 = patch(isosurface(a,b,c,C2,0));p2.FaceColor = 'g';
p2.EdgeColor = 'none';
p3 = patch(isosurface(a,b,c,C3,0));p3.FaceColor = 'blue';
p3.EdgeColor = 'none';
p4 = patch(isosurface(a,b,c,C4,0));p4.FaceColor = 'y';
p4.EdgeColor = 'none';
p5 = patch(isosurface(a,b,c,C5,0));p5.FaceColor = 'm';
p5.EdgeColor = 'none';
p6 = patch(isosurface(a,b,c,C6,0));p6.FaceColor = 'w';
p6.EdgeColor = 'none';
view(3); axis tight
camlight
lighting gouraud
```

We see that it is very difficult to solve the problem graphically due to the complexity of the plots.

Question 6 :

(a) We made substitutions and finally found :

$$D = C_{D0}x + K\frac{W^2}{x}$$

With W being the constant weight of the plane and $x = \frac{1}{2}\rho S V^2$. For simplicity we write :

$$D = AV^2 + \frac{B}{V^2}$$

Taking the 1st derivative and make it 0 to find local minimum :

$$\frac{dD}{dV} = 2AV - 2\frac{B}{V^3} = 0 \leftrightarrow V = \sqrt[4]{\frac{B}{A}}$$

(b) Similarly, we have :

$$P = MV^3 + \frac{N}{V}$$

Taking the 1st derivative and make it 0 to find local minimum :

$$\frac{dP}{dV} = 3MV^2 - \frac{N}{V^2} = 0 \leftrightarrow V = \sqrt[4]{\frac{N}{3M}}$$

(c) We want to maximize :

$$E = \frac{C_L}{C_D} = \frac{C_L}{C_{D0} + K{C_L}^2} = \frac{1}{\frac{C_{D0}}{C_L} + KC_L}$$

So we minimize :

$$f(V) = \frac{C_{D0}}{C_L} + KC_L = PV^2 + \frac{Q}{V^2}$$

Taking the 1st derivative and make it 0 to find local minimum :

$$\frac{df}{dV} = 2PV - 2\frac{Q}{V^3} = 0 \leftrightarrow V = \sqrt[4]{\frac{Q}{P}}$$

Note : A, B, M, N, P, and Q are just constants.