ENSE3 - 3ème année ASI - 2020/2021

# Tutorials in Advanced Control
# Bureaux d'étude - Robust Control of Mechatronics systems -10h

Olivier Sename & John Martinez

# Lateral control of autonomous vehicles

**Objectives**:

- Understand how to generate a reference for an autonomous car, provide a method and simuate
- Use kynematic and dynamic car models
- Design and analyze LTI and LPV controllers (in state feedback form)

# 1 Review

It is of importance to read few documents on vehicle modelling and on lateral (steering) control of autonomous cars.

Some parts of ones presented in the reference lists will be given but you may read more ....

# 2 Modelling

## 2.1 Kinematic Model

The kinematic model assumes that the tire-slip of the vehicle with the ground is null. This kind of model is mainly used at low speed systems such as mobile-robots as well as for the vehicles in automatic parking tasks, also it is very useful for motion planning ([1]). It is derived geometrically where no forces are considered, as if the vehicle is a point-object at the vehicle's center of mass. It is based on the velocity vector movement in order to compute longitudinal and lateral velocities referenced to the global inertial frame. The kinematic equations are expressed as:

$$\begin{cases} \dot{x} = v_x \cos\psi \\ \dot{y} = v_x \sin\psi \end{cases} \tag{1}$$

where $x$, $y$ and $\psi$ specifies the position in meters ($m$) and the orientation in radians ($rad$) respectively, with respect to the global frame ($x$, $y$). $v_x$ and $\dot{\psi}$ represents the longitudinal velocity in $\frac{m}{s}$ and the yaw rate in $\frac{rad}{s}$, respectively.

## 2.2 Dynamic Model

The lateral vehicle dynamics can be modeled as a two-wheeled bicycle model [2]. Using this model and by parameterizing the longitudinal speed of the vehicle, the yaw dynamics can be decoupled and a steering controller can be designed regardless of the longitudinal dynamics of the vehicle.

Fig. 1 shows the resulting vehicle model which is expressed by the parameters where $v_x$ and $v_y$ are the longitudinal and lateral velocity accordingly. $\dot{\psi}$ is the yaw rate of the car. $\alpha_f$, $\alpha_r$ are the tire side-slip angles of the front and rear wheels respectively. $\beta$ is the side slip angle of the vehicle body. $\delta$ is the steering wheel

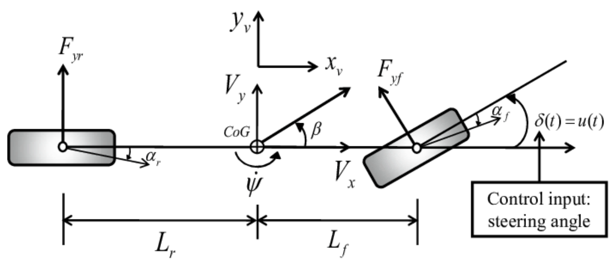


Figure 1: Two-wheeled bicycle model representing the vehicle lateral dynamics.

angle. $L_f$, $L_r$ are the distances of the front and rear wheel from the center of the gravity of the car and $C_f$, $C_r$ the front and rear cornering stiffness. The lateral tire forces are approximated as linear functions:

$$\begin{aligned} F_{yf} &= C_f \alpha_f \\ F_{yr} &= C_r \alpha_r \end{aligned} \tag{2}$$

Using small angles approximations, the side slip angles can be written as:

$$\begin{aligned} \alpha_f &= \delta - \frac{v_y + L_f \dot{\psi}}{v_x} \\ \alpha_r &= -\frac{v_y + L_r \dot{\psi}}{v_x} \end{aligned} \tag{3}$$

Using Newton's second law and the moment balance at the z axis, the next equations are derived:

$$\begin{aligned} m a_y &= m(\dot{v}_y + v_x \dot{\psi}) = F_{yf} + F_{yr} \\ I_z \ddot{\psi} &= L_f F_{yf} - L_r F_{yr} \end{aligned} \tag{4}$$

where $a_y$ is the lateral acceleration, $m$ the mass and $I_z$ the car inertia.

Considering, $v_y$ and $\dot{\psi}$ as state variables and combining the equations (2), (3), (4) the linear vehicle model in state space form is derived:

$$\begin{bmatrix} \dot{v}_y \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} -\dfrac{C_f + C_r}{m v_x} & -v_x + \dfrac{C_r L_r - C_f L_f}{m v_x} \\ \dfrac{-L_f C_f + L_r C_r}{I_z v_x} & \dfrac{L_f^2 C_f + L_r^2 C_r}{I_z v_x} \end{bmatrix} \begin{bmatrix} v_y \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} \dfrac{C_f}{m} \\ \dfrac{L_f C_f}{I_z} \end{bmatrix} \delta \tag{5}$$

# 3 Reference generation

## 3.1 Design

Propose a reference generator based on this kinematical model (suppose that the road curvature is known and given at every timeinstant).

REMEMBER THAT: the path planing should to consider limits of the contact forces and the steering angle

Ideally, the yaw rate follows the road curvature as:

$$\dot{\psi} = \frac{v_x}{R} = \rho v_x \tag{6}$$

And then, if the objective is to minimize the lateral acceleration (so ideally zero), the centrifugal force can be given in terms of the road curvature:

$$|F_c| \quad = \quad |m\dot{v}_y| = |mv_x\dot{\psi}| = |\frac{mv_x^2}{R}| \tag{7}$$

This means that the lateral contact forces compensates the centrifugal ones.

So, we have to design trajectories providing road curvatures compatible with the available contact forces (which depends of the quality of the road/tire adhesion ).

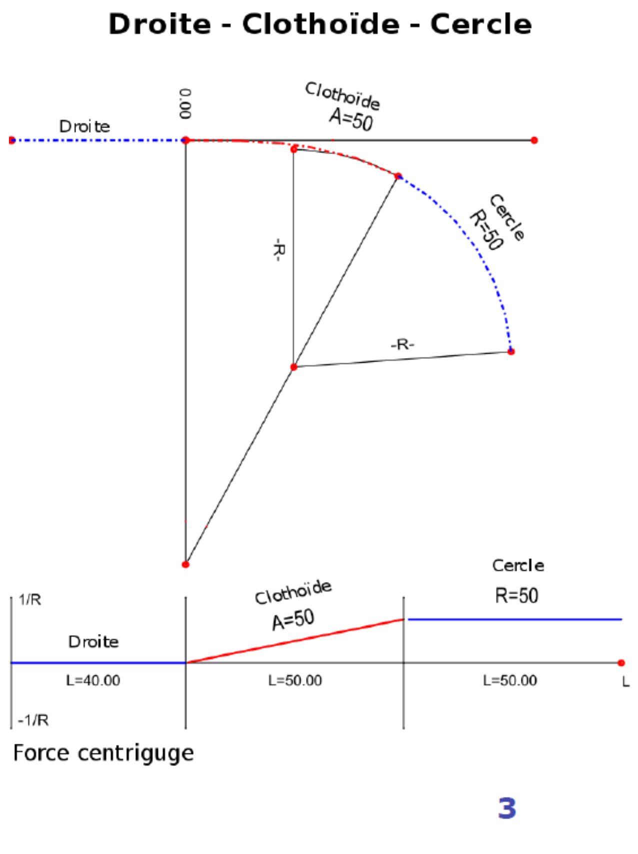


Figure 2: Path

### 3.2 Simulation

Simulate the reference generator in Matlab.

## 4 Control design

In that section, the objectives is to design several type of controllers, from LTI to LPV, in discrete-time.

Some LMIs to design the controllers will be given (you will have to install YALMIP and optimization codes as SEDUMI) to do this in Matlab (cf course slides and `https://yalmip.github.io/`).

### 4.1 LTI case

From the dynamical bicycle model, design a state feedback control law that allows to track a given reference of yaw rate.

The methodology could be to design LQR and then $H_\infty$ state feedback controllers.
So it is important to think about the performance specifications and the weighting functions.
Simulate the trajectory tracking.

### 4.2 LPV case

In that case, after reading carefully the course slides on LPV systems, the tasks to be done are

- Model the bicyle model as an LPV one: so choose the adequate scheduling parameters.
- Determine the convex set to be considered in order to write a polytopic LPV model. Analyse the dimension of the polytopes and think about reducing such a convex set describing the possible evolution of the scheduling parameters.
- Obtain the discrete-time LPV model.
- Write the LPV model as a polytopic model.

## References


[1] González, D., Pérez, J., Milanés, V., Nashashibi, F.: 'A review of motion planning techniques for automated vehicles.' IEEE Transactions on Intelligent Transportation Systems 17.4 (2015): 1135-1145.

[2] Rajamani, R.: 'Vehicle Dynamics and Control', Springer Science & Business Media, 2011.

[3] Uwe Kiencke & Lars Nielsen, "Automotive Control Systems", Springer 2005