BE2 _ Optimal Linear Filtering _ TRAN Gia Quoc Bao, ASI 2nd year

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I. Introduction

In this BE we will learn how to get the fetus' cardio signal out of the combination fetus + mother cardio signal by applying the Widrow filter and the "noise only" signal (the mother's).

Default commands

```
close all;
clear all;
clc;
```

II. The Widrow experiment:

The major difficulty when estimating the fetal ECG lies in the fact that its amplitude is 2-1000 times lower than maternal ECG (smaller heart / environments to cross). Direct processing of records is therefore difficult; it is necessary to eliminate interference from the maternal ECG. We'll do that using the Widrow algorithm (LMS filter).

Data loading

```
% The description :
% The data include the ECG signals recorded on a pregnant woman (8
 electrodes)
% Sampling: Fe = 250 Hz
```

```
% Nb of samples: 2500, i.e. an observation time of 10s
% Formatting:
% Each line of the 8 x 2500 fetal_ecg matrix corresponds to
% one of the 8 electrodes:
% - electrodes 1-5: abdomen, ECG of the mother + fetus -> channel
 signal
% - electrodes 6-8: chest, only the mother's heart rate is measurement
 -> "noise only" references

load foetal_ecg.mat;   % load data
Fs = 250; % sampling frequency in Hz
N = size(foetal_ecg, 2); % number of samples
time = (0 : N - 1)/Fs; % discrete time
```

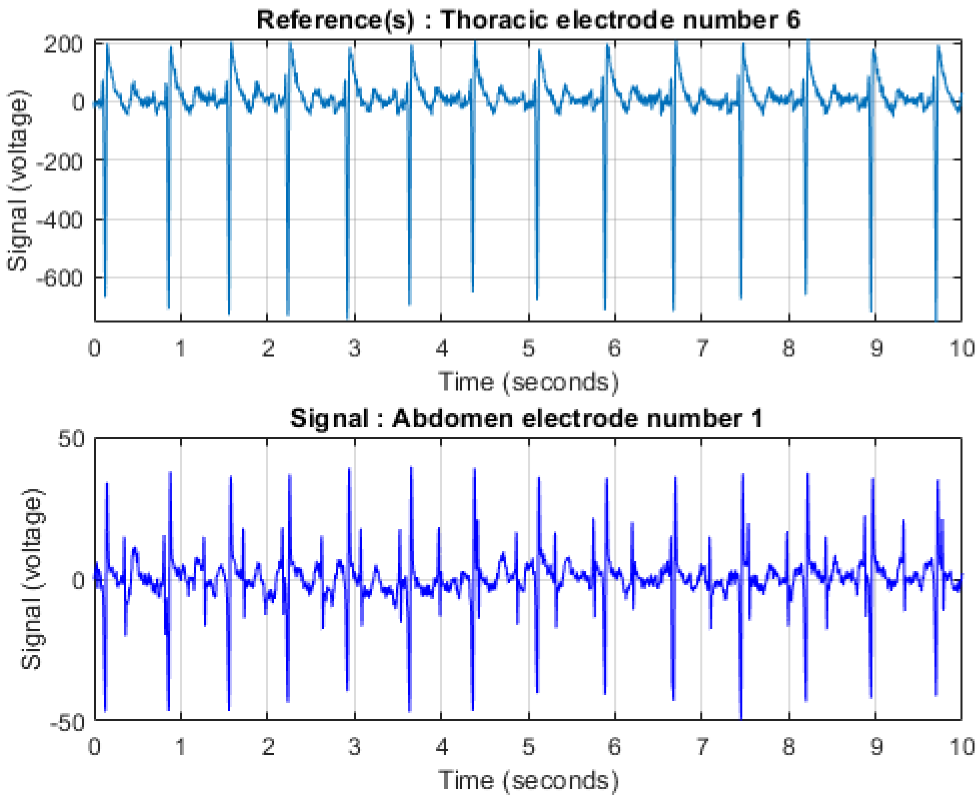
Signal shaping

```
% Signal to denoise: abdomen electrode number 'elec_abdo'
% Rq: the signals are already centered

elec_abdo = 1;
abdomen_ecg = foetal_ecg(elec_abdo, :); % line vector 1 x N

% "Noise only" references (secondary channel)
% Remark: the signals are already centered

ireferences = [6]; % the reference electrode (s) / signals
nRef = length(ireferences) ; % number of reference signals
thorax_ecg = foetal_ecg(ireferences, 1 : N) ; % nRef x N <- multi-ref

clear foetal_ecg ;
```

Visualization of signals

```
figure('Name', 'Visualization of waveforms', 'NumberTitle', 'off');
subplot(211);
plot(time, thorax_ecg');
grid on ;
xlim([0 10]);
xlabel('Time (seconds)');
ylabel('Signal (voltage)');
title(['Reference(s) : Thoracic electrode number ',
 int2str(ireferences)]);
subplot(212);
plot(time, abdomen_ecg, '-b') ;
grid on ;
xlim([0 10]);
xlabel('Time (seconds)') ;
ylabel('Signal (voltage)');
title(['Signal : Abdomen electrode number ' int2str(elec_abdo)]);
% From here we see that the abdomen contains a lot of background
 noises
% while for the thoracic it's almost purely the mother's ECG. So the
```

```
% thoracic is a "noise only" signal that serves as a reference to
 denoise
% the abdomen signal.

% And the signal/noise ratio differs from one abdomen electrode to
 another.
% It depends on the position/distance electrode-baby and electrode-
mother's
% heart. So if the electrode lies closer to the mother's heart its
 signal
% will have more noise. It also depends on the environment the wave
 has
% to cross to get to the electrode.
```



III. Widrow algorithm (LMS)

```
% Algo inputs:
% - a signal channel: abdomen_ecg
% - one, or more (multi-references version), reference: thorax_ecg
% Algo outputs:
% - maternal ECG
% - ECG of the fetus

% LMS filter settings

L = 8; % RIF filter order associated with the reference signal
```

```
bound_mu = 2/(L*mean(thorax_ecg.^2)); % Upper terminal of the
 adaptivity step in order to guarantee covergence (CS)

% the hypotheses from which the sufficient conditions which ensure the
% cvgce are not necessarily verified ... We can have an approximation
 an
% then experiment until we have a good convergence.

% In order to have the necessary adaptability and convergence, we take
 a
% value below the limit for the adaptability step of the LMS algo
% There are 3 cases, with d the denominator:
mu = bound_mu/1000; mu2 = bound_mu/18; mu3 = bound_mu/10;

% After some experiments, I found out the choice mu = bound_mu/18 was
 one
% of the best choices. If we go further to bound_mu/18 the algorithm
 will diverge.
% And there's no visible difference between bound_mu/18 and
 bound_mu/15 anyway.
% The first choice of bound_mu/1000 was also not good because when the
% filter converges that slowly, we do not have enough data for it to
 finish
% converging. Even between /18 and /100 there is no clear difference.

% In short, if mu is too big, it causes instability in the algorithm
% (fluctuations of coefficients, divergence). If mu is too small, we
 cannot
% converge with the data we have.

% So we can use this method of experimenting many times combined with
 the
% mathematical approximations for the best step size mu.

% Declaration / pre-allocation
ecg_maternal_estim = zeros(1, N); ecg_maternal_estim2 = zeros(1, N);
 ecg_maternal_estim3 = zeros(1, N); % maternal cardio estimation
error = zeros(1, N); error2 = zeros(1, N); error3 = zeros(1, N); %
 prediction error
w = zeros(L, N); w2 = zeros(L, N); w3 = zeros(L, N); % dimensions

% Initilization
error(1) = abdomen_ecg(1); error2(1) = abdomen_ecg(1); error3(1) =
 abdomen_ecg(1);
w(1, 1) = mu*error(1)*thorax_ecg(1); w2(1, 1) =
 mu2*error2(1)*thorax_ecg(1); w3(1, 1) = mu3*error3(1)*thorax_ecg(1);
ecg_maternal_estim(1) = w(1, 1)*thorax_ecg(1); ecg_maternal_estim2(1)
 = w2(1, 1)*thorax_ecg(1); ecg_maternal_estim3(1) = w3(1,
 1)*thorax_ecg(1);

% Iterations
for k = 2 : N
    if k < L
```

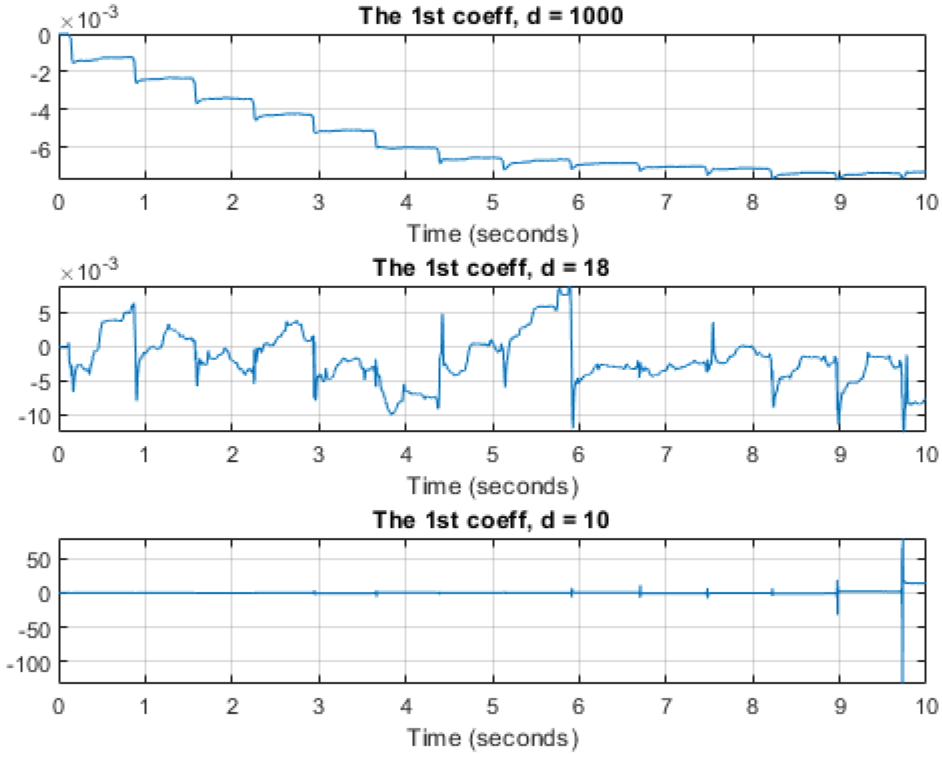
```
        y_k = 0; y2_k = 0; y3_k = 0;
    else
        y_k = thorax_ecg(k - L + 1 : k)'; y2_k = thorax_ecg(k - L +
 1 : k)'; y3_k = thorax_ecg(k - L + 1 : k)';

        error(k) = abdomen_ecg(k) - w(:, k - 1)'*y_k; error2(k) =
 abdomen_ecg(k) - w2(:, k - 1)'*y2_k; error3(k) = abdomen_ecg(k) -
 w3(:, k - 1)'*y3_k; % prediction error
        w(:, k) = w(:, k - 1) + mu*error(k)*y_k; w2(:, k) =
 w2(:, k - 1) + mu2*error2(k)*y2_k; w3(:, k) = w3(:, k - 1) +
 mu3*error3(k)*y3_k; % update filter
        ecg_maternal_estim(k) = w(:, k)'*y_k; ecg_maternal_estim2(k)
 = w2(:, k)'*y2_k; ecg_maternal_estim3(k) = w3(:, k)'*y3_k; % find
 maternal ECG at k
    end
end

ecg_fetal_estim = abdomen_ecg - ecg_maternal_estim; ecg_fetal_estim2 =
 abdomen_ecg - ecg_maternal_estim2; ecg_fetal_estim3 = abdomen_ecg -
 ecg_maternal_estim3;
```

Visualization of results

The filter's first coefficients

```
figure();
subplot(311);
plot(time, w(1, :)'); % change 1 to other numbers to see other
 coefficients
grid on ;
xlabel('Time (seconds)');
title('The 1st coeff, d = 1000');
subplot(312);
plot(time, w2(1, :)'); % change 1 to other numbers to see other
 coefficients
grid on ;
xlabel('Time (seconds)');
title('The 1st coeff, d = 18');
subplot(313);
plot(time, w3(1, :)'); % change 1 to other numbers to see other
 coefficients
grid on ;
xlabel('Time (seconds)');
title('The 1st coeff, d = 10');

% Looking at this we see that the 1st coeff for small mu doesn't have
% enough data to converge. The 2nd one seems the best: it still has
 some
% small fluctuations but it is able to converge to the right value.
 The 3rd
% one clearly diverges as the step was too big. For d = 18 we have
 arrived
% at a minima which gives good convergence so we should stay there.
```

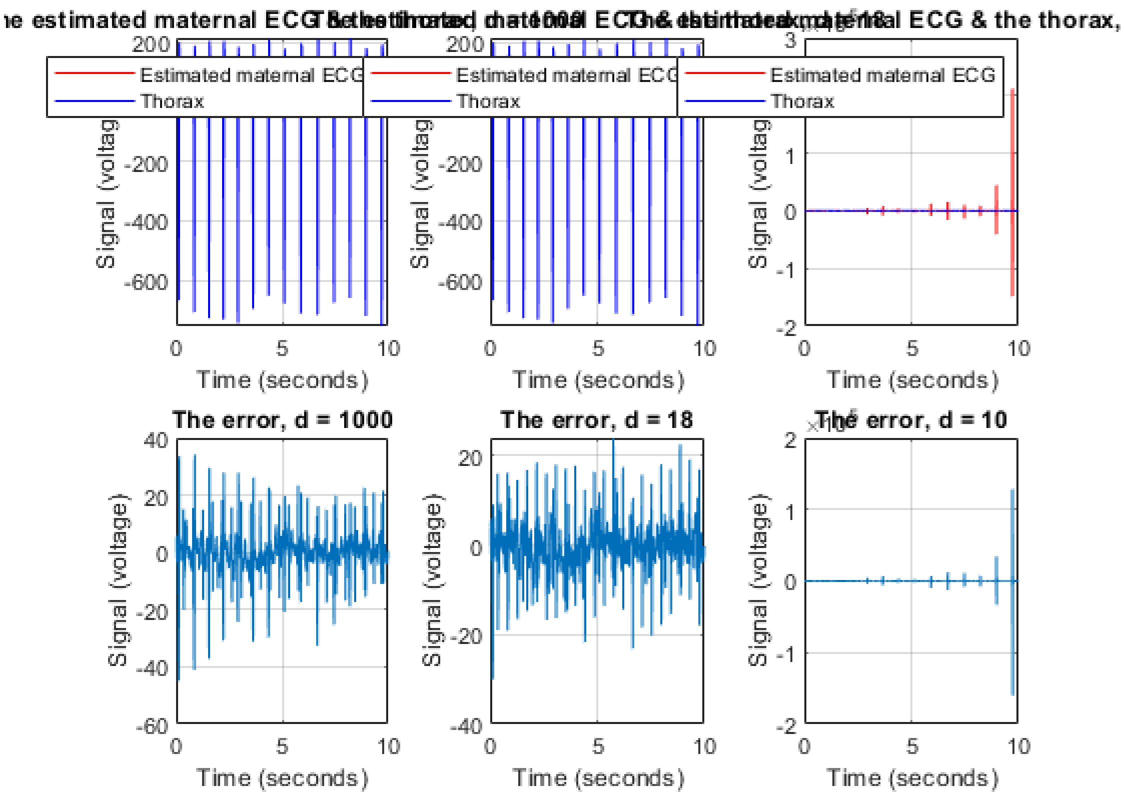
```
% So the lesson here is we should always check the coefficients to
 verify
% if the algorithm works well with the chosen mu or not. We should aim
 to
% have the fastest convergence possible without much fluctuations so
 this
% can be applied in real-time. Clearly the 1st case could not converge
% within 10 seconds.

% I verified all the 8 coeff and noticed the same properties for them.
 A
% lesson is if we see that a coeff keeps increasing or decreasing
 (like the
% 1st case: each of the 8 coeff either goes up or down all the time),
 it
% means that the chosen mu is not large enough to converge. The sign
 of
% divergence is very clear to see. So if we don't have these 2 it
 means the
% convergence is OK.
```



The error and the signals

```
figure();
subplot(231);
```

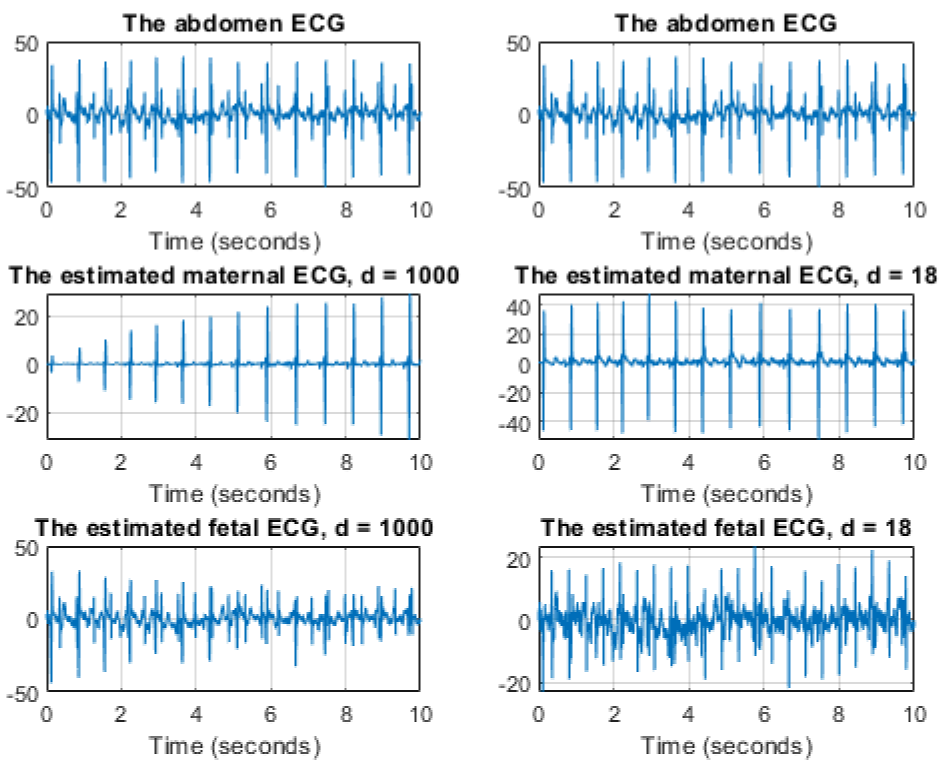
```
plot(time, ecg_maternal_estim', 'r', time, thorax_ecg', 'b');
grid on ;
xlabel('Time (seconds)');
ylabel('Signal (voltage)');
legend('Estimated maternal ECG', 'Thorax');
title('The estimated maternal ECG & the thorax, d = 1000');
subplot(232);
plot(time, ecg_maternal_estim2', 'r', time, thorax_ecg', 'b');
grid on ;
xlabel('Time (seconds)');
ylabel('Signal (voltage)');
legend('Estimated maternal ECG', 'Thorax');
title('The estimated maternal ECG & the thorax, d = 18');
subplot(233);
plot(time, ecg_maternal_estim3', 'r', time, thorax_ecg', 'b');
grid on ;
xlabel('Time (seconds)');
ylabel('Signal (voltage)');
legend('Estimated maternal ECG', 'Thorax');
title('The estimated maternal ECG & the thorax, d = 10');
subplot(234);
plot(time, error');
grid on ;
xlabel('Time (seconds)');
ylabel('Signal (voltage)');
title('The error, d = 1000');
subplot(235);
plot(time, error2');
grid on ;
xlabel('Time (seconds)');
ylabel('Signal (voltage)');
title('The error, d = 18');
subplot(236);
plot(time, error3');
grid on ;
xlabel('Time (seconds)');
ylabel('Signal (voltage)');
title('The error, d = 10');

% For the error, it is overall decreased given a large enough mu (like
 for
% d = 1000 it went to around 30 but for d = 18 it stayed at around
 less
% than 20).
% It varies around 0 but we have a peak at the estimation of peaks in
 the
% maternal ECG. The higher the mu the more frequent the peaks.
```



The signals before and after filtering

```
figure();
subplot(321);
plot(time, abdomen_ecg');
grid on ;
xlabel('Time (seconds)');
title('The abdomen ECG');
subplot(322);
plot(time, abdomen_ecg');
grid on ;
xlabel('Time (seconds)');
title('The abdomen ECG');
subplot(323);
plot(time, ecg_maternal_estim');
grid on ;
xlabel('Time (seconds)');
title('The estimated maternal ECG, d = 1000');
subplot(324);
plot(time, ecg_maternal_estim2');
grid on ;
xlabel('Time (seconds)');
title('The estimated maternal ECG, d = 18');
subplot(325);
plot(time, ecg_fetal_estim');
```

```
grid on ;
xlabel('Time (seconds)');
title('The estimated fetal ECG, d = 1000');
subplot(326);
plot(time, ecg_fetal_estim2');
grid on ;
xlabel('Time (seconds)');
title('The estimated fetal ECG, d = 18');

% This figure helps us see clearer the effect of mu on convergence
 through
% the estimation of peaks. For very small mu, the estimation of peaks
 at
% the first few seconds was not fine. The quality of estimation
 increases
% after some time but the 2nd case it took much less time. The
 estimation
% of peaks was correct right at the 1st peak.
```



Experiment with different order L

From here on, we will use mu = Bu/18 as we already saw that it give a fast convergence. I experimented with different values of L by changing it. I put the results at the end of this report.

```
% What I observed: the bigger the L, the smaller the step size mu
 allowed,
```

```
% but if the mu is small enough the convergence should still be
 guaranteed.
% So the algorithm has more 0 at the beginning and we lose some
% information. But we will have more weights so more filtering (the
 filter
% is more complex). On the other hand if we don't have enough weights
 we
% won't be able to capture information with high variations (in DSP).
 The
% algorithm will diverge if L is too low.

% However we need to store and perform calculations on the weights to
 this
% would consume memory. There's no point in increasing the complexity
 if
% the results are already good enough in terms of visibility. We stop
 at
% the point where further increase in complexity gives almost no
 further
% increase in the clarity of results.

% In this case L = 8 is OK.
```

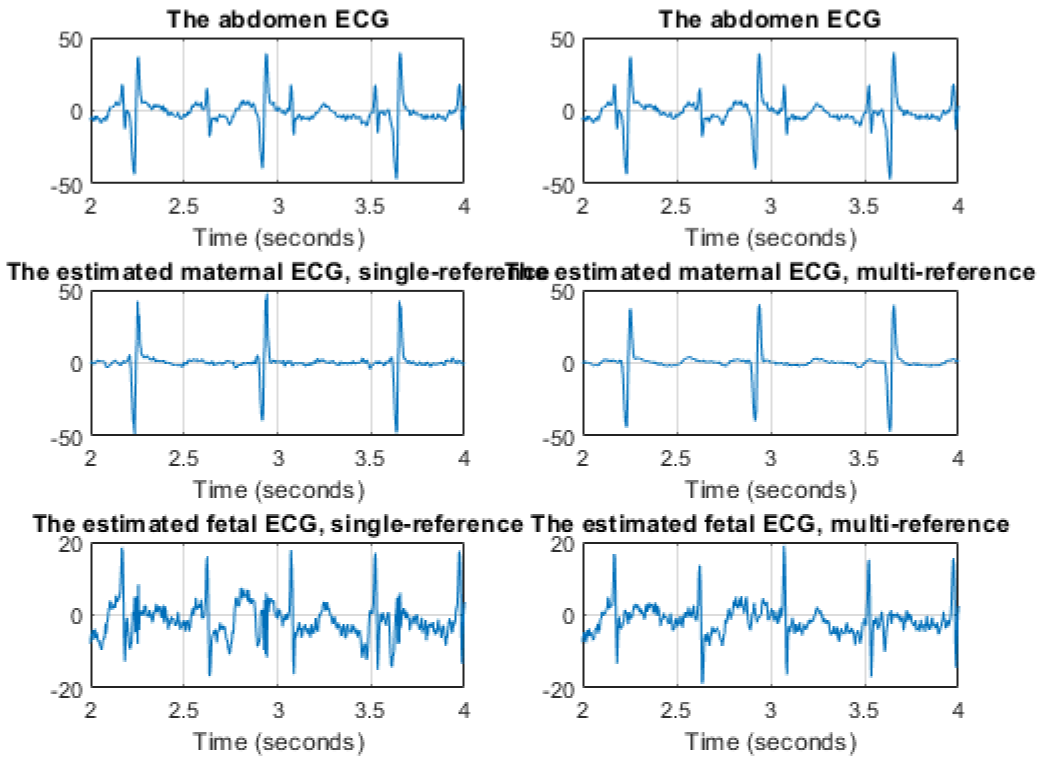
IV. Multi-reference Widrow algorithm (LMS)

In this interesting part we will filter the signal using 3 "nois only" references. All the 3 thoracic signals are taken, giving us a multi-reference filter.

```
load foetal_ecg.mat;   % load data
Fs = 250 ; % sampling frequency in Hz
N = size(foetal_ecg, 2) ; % number of samples
time = (0 : N - 1)/Fs ; % discrete time

ireferences = [6 7 8]; % the reference electrode (s) / signals
nRef = length(ireferences) ; % number of reference signals
thorax_ecg_mult = foetal_ecg(ireferences, 1 : N) ; % nRef x N <-
 multi-ref

% Signal to denoise
elec_abdo = 1;
abdomen_ecg = foetal_ecg(elec_abdo, :);

clear foetal_ecg ;

L_mult = 8;

% choose the smallest bound which corresponds to
bound_mu_mult = 2/(L_mult*max(mean(thorax_ecg_mult.^2)));
% Overall the multi-ref gives us a wider choice of mu.

mu_mult = bound_mu_mult/2;
% Overall the multi-ref gives us a wider choice of mu.
```

```
ecg_maternal_estim_mult = zeros(1, N); % maternal cardio estimation
error_mult = zeros(1, N); % prediction error
w_mult = zeros(L_mult, length(ireferences), N); % dimensions

% There is now a response by reference electrode
% The LMS filter being the sum of each of these "nRef" filters
% The responses of the (sub) filters at time k will be stored in the
% vectors w_mult (:, 1, k), w_mult (:, 2, k), ...

% Initilization
error_mult(1) = abdomen_ecg(1);
w_mult(1, :, 1) = mu*error_mult(1)*thorax_ecg_mult(:, 1)';
ecg_maternal_estim_mult(1) = w_mult(1, :, 1)*thorax_ecg_mult(:, 1);

% Prediction error: the prediction is now the sum of the predictions
 obtained
% for each of the nRef = 3 abdominal electrodes using subdose
 responses are
% stored in w_mult(:, 1, k-1), w_mult(:, 2, k-1) and w_mult(:, 3, k-1)

for k = 2 : N
    if k < L
        y_k = 0;
    else
        y_k = thorax_ecg_mult(:, k - L + 1 : k)';

        sum = 0;
        for j = 1 : length(ireferences)
            sum = sum + w_mult(:, j, k - 1)'*y_k(:, j);
        end

        error_mult(k) = abdomen_ecg(k) - sum; % prediction error
        w_mult(:, :, k) =  w_mult(:, :, k - 1) +
 mu_mult*error_mult(k)*y_k; % update filter

        ecg_maternal_estim_mult(k) = 0;
        for i = 1 : length(ireferences)
            ecg_maternal_estim_mult(k) = ecg_maternal_estim_mult(k) +
 w_mult(:, i, k)'*y_k(:, i);
        end
    end
end

ecg_fetal_estim_mult = abdomen_ecg - ecg_maternal_estim_mult;

% Comparison with the single-reference case

figure();
subplot(321);
plot(time, abdomen_ecg');
grid on ;
xlabel('Time (seconds)');
xlim([2 4]);
title('The abdomen ECG');
```

```
subplot(322);
plot(time, abdomen_ecg');
grid on ;
xlabel('Time (seconds)');
xlim([2 4]);
title('The abdomen ECG');
subplot(323);
plot(time, ecg_maternal_estim2');
grid on ;
xlabel('Time (seconds)');
xlim([2 4]);
title('The estimated maternal ECG, single-reference');
subplot(324);
plot(time, ecg_maternal_estim_mult');
grid on ;
xlabel('Time (seconds)');
xlim([2 4]);
title('The estimated maternal ECG, multi-reference');
subplot(325);
plot(time, ecg_fetal_estim2');
grid on ;
xlabel('Time (seconds)');
xlim([2 4]);
title('The estimated fetal ECG, single-reference');
subplot(326);
plot(time, ecg_fetal_estim_mult');
grid on ;
xlabel('Time (seconds)');
xlim([2 4]);
title('The estimated fetal ECG, multi-reference');

% If we look closely, especially the 2nd figure each column, we see
 that the
% estimation is much better with multi-ref: the estimated maternal ECG
 is
% smoother which means it contains only the mother's ECG. If we look
 in the
% results, where the maternal ECG used to be, we'll see that the
 multi-ref
% case has much fewer peaks (or variations). So the signal was better
% filtered and we can better detect the heartbeat of the foetus.
% This can be explained by the fact that here the multiple
% references can 'help' each other through the cross-effect among
 them.

% After all, the objective of this is to help the doctors measure the
% heartbeat of the foetus, so having multiple thoracic electrodes will
 enable
% the multi-ref algorithm and improve visual the quality.
```



V. Conclusion

After this Lab, I understood the use of the Widrow filter to denoise signals, especially in the case the noise is bigger in amplitude than the useful signal and we can measure the "noise only" reference. Besides, the version with multi-reference gaves smoother results than the one with a single reference.

```
% Below I put the extra figures for demonstration.
```

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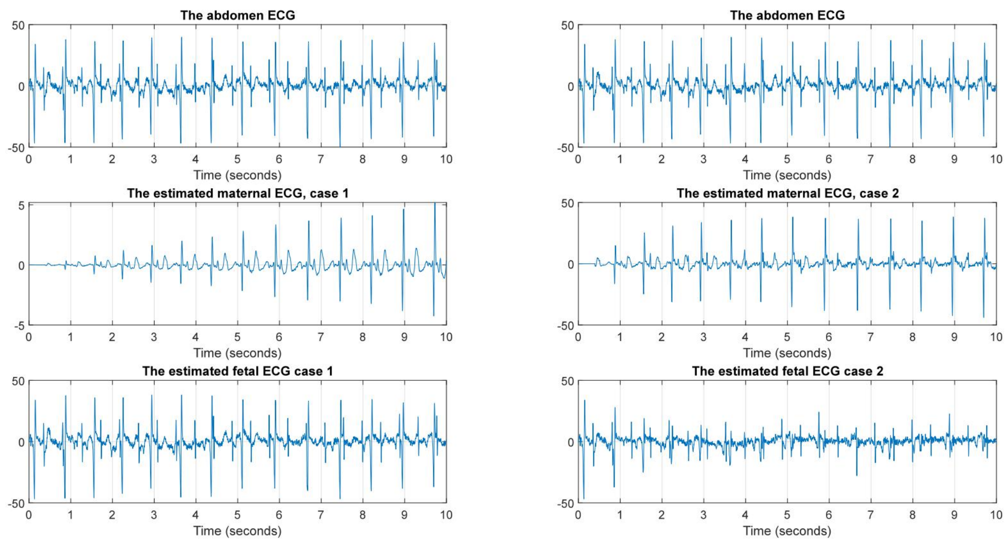
Some extra figures…

So I tried with different L by changing it and run everything again.

For small L, we get a divergence. The filter has to be more complex.

But for large L, we need to store more coefficients and at the same time we have more 0 at the beginning so the first few seconds of the signals are not well filtered: we fail to estimate the first peaks of the maternal ECG. In this case with L = 100, we can see that clearly:



So the lesson is to stop increasing L when any further increase in L results in no significant improvement of quality. Doing this we will avoid wasting memory. In this case L = 8 is already a good value.