# Automotive Control Using Surface Electromyography

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## 1 Optimal linear filtering background theories

This part presents the background theories of optimal linear filtering, in particular the Wiener filter which is used to process periodic noises. We consider the following model in which the observed signal is considered to be composed of two components:

$$y(t) = x(t) + v_p(t) \tag{1}$$

In which $y(t)$ is the observed signal, $x(t)$ is the useful signal (centered, large-band, and stationary up to second order), and $v_p(t)$ is a periodic noise uncorrelated with $x(t)$. The Wiener approach is to estimate this periodic noise $s(t) = v_p(t)$ and then the estimated useful signal is calculated from $x(t) = y(t) - v_p(t)$. The filter's impulse response $w$ is the solution of the Wiener-Hopf equation:

$$\Gamma_{sy} = \Gamma_y \times w \tag{2}$$

Which means:

$$\begin{pmatrix} \Gamma_{sy}[0] \\ \Gamma_{sy}[1] \\ ... \\ \Gamma_{sy}[N] \end{pmatrix} = \begin{pmatrix} \Gamma_y[0] & \Gamma_y[1] & ... & \Gamma_y[N] \\ \Gamma_y[1] & \Gamma_y[0] & ... & \Gamma_y[N-1] \\ ... & ... & ... & ... \\ \Gamma_y[N] & \Gamma_y[N-1] & ... & \Gamma_y[0] \end{pmatrix} \begin{pmatrix} w_0 \\ w_1 \\ ... \\ w_N \end{pmatrix} \tag{3}$$

The matrix $\Gamma_{sy}$ is obtained from the cross-correlation between $s(t)$ and $y(t)$ and $\Gamma_y$ is obtained from the autocorrelation of $y(t)$.
Because all we know is $y(t)$, we estimate $\Gamma_{sy}$ to be equal to $\Gamma_{yy_d}$, in which $y_d(t) = y(t - \tau')$ with a large enough delay $\tau'$ (a large enough number of samples). This delay is determined as follows. We obtain the autocorrelation of $y(t)$, then we find the time $t_c$ from which the amplitude of this autocorrelation remains unchanged, and $\tau' = t_c \times f_s$ in which $f_s$ is the sampling frequency. For brevity, the proof is not shown here.
The procedure for applying this filter is as follows:

- Obtain the observed signal's PSD to detect the noises and the width of the peaks corresponding to them and then estimate the filter's order
- Perform autocorrelation of $y(t)$ to estimate the delay
- Perform cross-correlation between $y(t)$ and $y_d(t)$
- Form the Wiener-Hopf equation and solve it to obtain the filter's impulse response
- Filter the observed signal to have the estimated periodic noise
- Subtract this estimated noise from the observed signal to obtain the estimated useful signal

Note that this type of filtering does not result in a delay in signals like the case of a classical filter, so it can be applied more safely in our case. Another advantage compared to ICA is that, we only need the individual signals for this method, instead of all the signals as in the case of ICA. This can simplify the calculations: in application this is as simple as a classical filter.
In MATLAB, we use the *xcorr* command to find the cross-correlation and autocorrelation of signals. The mode to be selected is *unbiased*. To construct the matrix $\Gamma_y$ from the autocorrelation of $y(t)$, we use the command *toeplitz*. Then if we use *filter* on the delayed signal we will get the estimated periodic noise.