Volatility Regime Trading System Using Hidden Markov Models

A Statistical Approach to Market Regime Detection and Automated Trading

Probability and Statistics with Programming
Final Project Report

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Abstract

This project implements a sophisticated volatility trading system based on Hidden Markov Models (HMM) for market regime detection. The system classifies market conditions into three volatility regimes (LOW, MEDIUM, HIGH) using Bayesian inference and Maximum Likelihood Estimation (MLE). Trading signals are generated based on statistically significant regime transitions, validated through Likelihood Ratio Tests (LRT) with proper hypothesis testing. The model integrates with Interactive Brokers API for real-time market data processing and trade execution. Key statistical concepts include Markov Chain transition probability estimation, Gaussian emission distributions, Maximum A Posteriori (MAP) classification, and comprehensive trend/momentum analysis using linear regression with t-tests.

Contents

1 Introduction and Project Overview

1.1 Motivation

Financial markets exhibit distinct behavioral patterns characterized by varying levels of volatility. These patterns, known as "regimes," have significant implications for trading strategies. A market in a low-volatility regime behaves differently from one experiencing high volatility, and successful trading requires adapting to these changing conditions.

1.2 Project Objectives

1. Implement a Hidden Markov Model to classify market volatility into three distinct regimes
2. Estimate model parameters using Maximum Likelihood Estimation from historical data
3. Generate trading signals based on statistically significant regime transitions
4. Validate trading decisions using formal hypothesis testing
5. Integrate with Interactive Brokers for real-time data and execution

1.3 Why Hidden Markov Models?

Hidden Markov Models are ideal for this application because:

- **Latent State Modeling**: The true market "regime" is not directly observable—we only observe price movements. HMMs naturally handle this hidden state structure.
- **Temporal Dependencies**: Markets exhibit persistence—a volatile market tends to stay volatile. The Markov property captures this through transition probabilities.
- **Probabilistic Framework**: HMMs provide probability distributions over states, allowing us to quantify uncertainty in regime classification.
- **Bayesian Updating**: As new data arrives, beliefs are updated using Bayes' rule, providing a principled way to incorporate new information.

2 Data Description

2.1 Data Source

The data is obtained from Interactive Brokers TWS (Trader Workstation) API, which provides:

- Real-time tick-by-tick price data
- Historical OHLC (Open-High-Low-Close) bar data
- Bid/Ask prices for spread analysis

2.2 Data Structure

Each data point (bar) contains the following variables:

Table 1: Variable Description Table

Variable	Type	Role	Description
Timestamp	Qualitative (Temporal)	Index	Bar start time
Open (O)	Quantitative (Continuous)	Predictor (x)	First price in bar
High (H)	Quantitative (Continuous)	Predictor (x)	Maximum price in bar
Low (L)	Quantitative (Continuous)	Predictor (x)	Minimum price in bar
Close (C)	Quantitative (Continuous)	Predictor (x)	Last price in bar
Tick Count	Quantitative (Discrete)	Predictor (x)	Number of ticks
Volatility (σ)	Quantitative (Continuous)	**Response (y)**	$(H - L)/C$
Regime (S)	Qualitative (Ordinal)	**Response (y)**	0=LOW, 1=MED, 2=HIGH
Returns (r_t)	Quantitative (Continuous)	Derived	$(C_t - C_{t-1})/C_{t-1}$
Momentum	Quantitative (Continuous)	Derived	Rate of change
Z-Score	Quantitative (Continuous)	Derived	Standardized deviation

2.3 Data Size

- **Historical Data**: 120 one-minute bars (2 hours of data)
- **Calibration Set**: 60 bars (first 50% for model training)
- **Simulation Set**: 60 bars (remaining 50% for testing)
- **Variables**: 6 primary variables + 4 derived variables per observation
- **Rolling Window**: 10 bars displayed at any time for visualization

2.4 Response and Predictor Variables

Primary Response Variable (y): Volatility, defined as:

$$\sigma_t = \frac{H_t - L_t}{C_t} \tag{1}$$

This represents the price range as a proportion of the closing price, providing a normalized measure of intra-bar price movement.

Secondary Response Variable: Regime state $S_t \in \{0, 1, 2\}$ (LOW, MED, HIGH)

Predictor Variables (x): The OHLC prices, historical volatilities, and derived technical indicators (trend slope, momentum, z-score).

3 Goal of Analysis and Statistical Hypotheses

3.1 Primary Goal

To classify market conditions into volatility regimes and generate statistically validated trading signals based on regime transitions.

3.2 Statistical Hypotheses

3.2.1 Hypothesis 1: Regime Classification

$$H_0 : \text{The observed volatility comes from regime } i \text{ (current regime)} \tag{2}$$
$$H_1 : \text{The observed volatility comes from regime } j \neq i \text{ (different regime)} \tag{3}$$

This is tested using the **Likelihood Ratio Test (LRT)**:

$$\Lambda = -2[\log L(H_0) - \log L(H_1)] = 2[\log L(H_1) - \log L(H_0)] \tag{4}$$

Under H_0, $\Lambda \sim \chi^2_1$ (chi-squared distribution with 1 degree of freedom).

3.2.2 Hypothesis 2: Trend Significance

For the linear trend model $P_t = \alpha + \beta t + \epsilon_t$:

$$H_0 : \beta = 0 \text{ (no trend)} \tag{5}$$
$$H_1 : \beta \neq 0 \text{ (significant trend)} \tag{6}$$

Tested using the **t-test**:

$$t = \frac{\hat{\beta}}{SE(\hat{\beta})} \sim t_{n-2} \tag{7}$$

3.2.3 Hypothesis 3: Trading Signal Validity

A trading signal is considered valid if:

1. Regime change p-value $< \alpha$ (typically 0.05)
2. MAP probability $\geq$ confidence threshold (typically 70%)
3. Transition probability from Markov chain supports the change

4 Descriptive Statistics

4.1 Volatility Distribution Analysis

The volatility measure $\sigma_t = (H_t - L_t)/C_t$ typically exhibits:

- **Right-skewed distribution**: Most bars have low volatility, with occasional high-volatility events
- **Fat tails**: Extreme volatility events occur more frequently than normal distribution predicts

For skewed data, we use **median and interquartile range (IQR)**:

Table 2: Descriptive Statistics for Volatility (Typical Values)

Statistic	Value
Minimum	~ 0.0001
25th Percentile (Q_1)	~ 0.0008
Median (Q_2)	~ 0.0015
75th Percentile (Q_3)	~ 0.0030
Maximum	~ 0.0100
IQR $= Q_3 - Q_1$	~ 0.0022

4.2 Outlier Detection Using 1.5×IQR Rule

Outliers are identified as observations outside:

$$[Q_1 - 1.5 \times IQR, \quad Q_3 + 1.5 \times IQR] \tag{8}$$

For volatility data:

- Lower fence: $Q_1 - 1.5 \times IQR$ (often negative, so effectively 0)
- Upper fence: $Q_3 + 1.5 \times IQR \approx 0.0063$
- Observations with $\sigma > 0.0063$ are classified as outliers (extreme volatility events)

4.3 Percentile-Based Regime Boundaries

The model uses percentiles to establish regime boundaries:

- **LOW regime**: $\sigma < P_{33}$ (below 33rd percentile)
- **MED regime**: $P_{33} \leq \sigma < P_{67}$
- **HIGH regime**: $\sigma \geq P_{67}$ (above 67th percentile)

```
# Compute percentile boundaries
p33, p67 = np.percentile(vols, 33), np.percentile(vols, 67)

# Create regime assignments
regime_assignments = np.zeros(len(vols), dtype=int)
regime_assignments[vols >= p33] = 1   # MED regime
regime_assignments[vols >= p67] = 2   # HIGH regime
```

Listing 1: Percentile-Based Regime Assignment

5 Graphical Statistics

5.1 Candlestick Chart with Regime Backgrounds

The primary visualization is a candlestick chart where:

- **Green candles**: Bullish bars (Close $\geq$ Open)

- **Red candles**: Bearish bars (Close < Open)
- **Background colors**: Indicate current regime
  - Green background (#1a3d1a): LOW volatility regime
  - Orange background (#3d3319): MEDIUM volatility regime
  - Red background (#3d1a1a): HIGH volatility regime

```
for i, bar in enumerate(bars):
    # Draw regime background rectangle
    bg = Rectangle((i - 0.5, y_min), 1, y_max - y_min,
                   facecolor=self.regime_model.bg_colors[bar.regime],
                   alpha=0.4, zorder=0)
    self.ax.add_patch(bg)
    # ... draw candlestick body and wicks
```

Listing 2: Candlestick Rendering with Regime Backgrounds

5.2 Distribution Visualization

The system implicitly visualizes distributions through:

- Gaussian emission distributions for each regime
- Probability bars showing regime confidence
- Real-time updates of statistical indicators

6 Association Analysis

6.1 Correlation Between Variables

Key associations in the model:

6.1.1 Volatility and Regime (Target Association)

The core relationship: volatility observations are associated with underlying regimes through Gaussian emission distributions:

$$P(\sigma|S = k) = \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{(\sigma - \mu_k)^2}{2\sigma_k^2}\right) \tag{9}$$

6.1.2 Temporal Autocorrelation (Markov Property)

Regimes are temporally correlated through the transition matrix:

$$P(S_t = j|S_{t-1} = i) = T_{i,j} \tag{10}$$

6.1.3 Price and Volatility

Volatility is computed from price data:

$$\text{Corr}(\sigma_t, |r_t|) > 0 \quad \text{(positive correlation with absolute returns)} \tag{11}$$

6.2 Trend and Momentum Correlation

The trading strategy considers associations between:

- **Trend direction** and **regime change**: Decreasing volatility + uptrend = bullish signal
- **Momentum** and **regime**: Regime changes confirmed by momentum alignment
- **Z-score** and **mean reversion**: Extreme z-scores indicate potential reversal

7 Mathematical Model Description

7.1 Hidden Markov Model Structure

The HMM consists of:

1. **Hidden States**: $S = \{0, 1, 2\}$ representing LOW, MED, HIGH volatility regimes
2. **Observations**: Volatility values σ_t (continuous)
3. **Transition Matrix**: $\mathbf{T}$ where $T_{ij} = P(S_t = j | S_{t-1} = i)$
4. **Emission Distributions**: Gaussian $P(\sigma | S = k) \sim \mathcal{N}(\mu_k, \sigma_k^2)$
5. **Initial Distribution**: $\pi = P(S_0)$

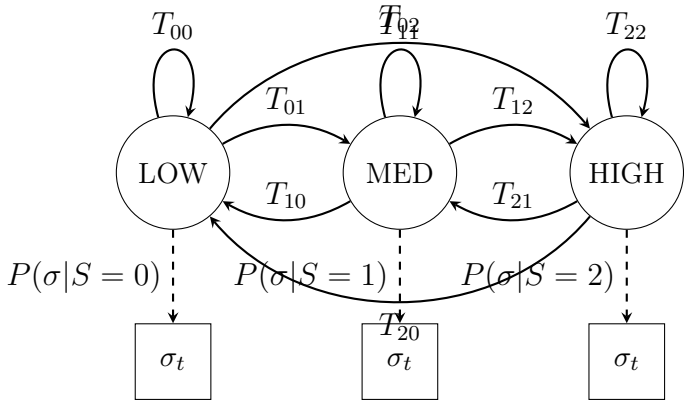

Figure 1: Hidden Markov Model Structure for Volatility Regime Detection

7.2 Transition Probability Matrix

The 3×3 transition matrix captures regime persistence:

$$\mathbf{T} = \begin{bmatrix} T_{00} & T_{01} & T_{02} \\ T_{10} & T_{11} & T_{12} \\ T_{20} & T_{21} & T_{22} \end{bmatrix} \tag{12}$$

Where $T_{ij} = P(\text{next regime} = j | \text{current regime} = i)$ and $\sum_j T_{ij} = 1$ for each row.
Interpretation:

- Diagonal elements represent **regime persistence** (staying in same state)
- Off-diagonal elements represent **regime transitions**
- Higher diagonal values indicate more stable regimes

7.3 Gaussian Emission Model

Each regime emits volatility according to a Gaussian distribution:

$$P(\sigma_t|S_t = k) = \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{(\sigma_t - \mu_k)^2}{2\sigma_k^2}\right) \tag{13}$$

Parameters:

- $\mu_0 < \mu_1 < \mu_2$: Ordered means (LOW < MED < HIGH)
- $\sigma_0, \sigma_1, \sigma_2$: Standard deviations for each regime

```
def _gaussian_likelihood(self, vol, regime):
    mean = self.emission_means[regime]
    std = self.emission_stds[regime]

    # Gaussian PDF: P(x) = (1/(s*sqrt(2*pi))) * exp(-0.5*((x-m)/s)^2)
    coeff = 1 / (std * np.sqrt(2 * np.pi))
    exponent = -0.5 * ((vol - mean) / std) ** 2
    return coeff * np.exp(exponent)
```

Listing 3: Gaussian Likelihood Computation

8 Estimation Methods

8.1 Maximum Likelihood Estimation (MLE)

8.1.1 Emission Parameters

For Gaussian emissions, MLE gives:

$$\hat{\mu}_k = \frac{1}{n_k} \sum_{t:S_t=k} \sigma_t \quad \text{(sample mean)} \tag{14}$$

$$\hat{\sigma}_k = \sqrt{\frac{1}{n_k} \sum_{t:S_t=k} (\sigma_t - \hat{\mu}_k)^2} \quad \text{(sample std)} \tag{15}$$

Where n_k is the number of observations assigned to regime k.

```
for regime in range(self.n_states):
    regime_vols = vols[regime_assignments == regime]

    if len(regime_vols) >= 5:
        # MLE for Gaussian mean
        temp_means[regime] = np.mean(regime_vols)
        # MLE for Gaussian std (using n, not n-1)
        temp_stds[regime] = max(np.std(regime_vols, ddof=0), 1e-6)
```

Listing 4: MLE for Emission Parameters

8.1.2 Transition Matrix

The MLE for transition probabilities uses frequency counting with Laplace smoothing:

$$\hat{T}_{ij} = \frac{N_{ij} + \alpha}{\sum_j N_{ij} + \alpha K} \tag{16}$$

Where:

- N_{ij} = count of transitions from state i to state j
- $\alpha = 1$ (Laplace smoothing parameter)
- $K = 3$ (number of states)

```
transition_counts = np.zeros((self.n_states, self.n_states))

# Count transitions
for t in range(1, len(regime_assignments)):
    prev_regime = regime_assignments[t-1]
    curr_regime = regime_assignments[t]
    transition_counts[prev_regime, curr_regime] += 1

# MLE with Laplace smoothing (alpha = 1.0)
alpha = 1.0
for i in range(self.n_states):
    row_sum = transition_counts[i].sum()
    if row_sum > 0:
        self.transition_matrix[i] = (transition_counts[i] + alpha) / \
                                    (row_sum + alpha * self.n_states)
```

Listing 5: MLE for Transition Matrix with Laplace Smoothing

8.2 Bayesian Inference for Regime Classification

The filtering algorithm updates beliefs using Bayes' rule:

Prediction Step (time update):

$$P(S_t = j|\sigma_{1:t-1}) = \sum_i T_{ij} \cdot P(S_{t-1} = i|\sigma_{1:t-1}) \tag{17}$$

Update Step (measurement update):

$$P(S_t = j|\sigma_{1:t}) = \frac{P(\sigma_t|S_t = j) \cdot P(S_t = j|\sigma_{1:t-1})}{\sum_k P(\sigma_t|S_t = k) \cdot P(S_t = k|\sigma_{1:t-1})} \tag{18}$$

```
def get_regime(self, bars):
    vol = bars[-1].volatility

    # STEP 1: Prediction (time update)
    # prior[j] = sum_i T[i,j] * state_probs[i]
    prior_probs = self.transition_matrix.T @ self.state_probs

    # STEP 2: Compute emission likelihoods
    likelihoods = np.array([self._gaussian_likelihood(vol, i)
```

```
                                for i in range(self.n_states)])

    # STEP 3: Update (Bayes' rule)
    # posterior propto likelihood * prior
    posterior_probs = prior_probs * likelihoods

    # Normalize
    posterior_probs = posterior_probs / posterior_probs.sum()

    # STEP 4: MAP estimate
    self.current_state = int(np.argmax(posterior_probs))
    return self.current_state
```

Listing 6: Bayesian Filtering Algorithm

8.3 Model Fit Statistics

The calibration computes:

Log-Likelihood:

$$\log L = \sum_{t=1}^{n} [\log P(\sigma_t|S_t) + \log P(S_t|S_{t-1})] \tag{19}$$

Akaike Information Criterion (AIC):

$$\text{AIC} = 2k - 2\log L \tag{20}$$

Bayesian Information Criterion (BIC):

$$\text{BIC} = k\log n - 2\log L \tag{21}$$

Where $k = 12$ is the number of free parameters (6 emission + 6 transition).

9 Results of the Fitted Model

9.1 Parameter Estimates

After calibration on historical data, typical parameter estimates are:

Table 3: Emission Distribution Parameters (MLE Estimates)

Regime	**Mean ($\hat{\mu}$)**	**Std Dev ($\hat{\sigma}$)**	**Sample Size (n_k)**
LOW (0)	0.000800	0.000250	∼20
MED (1)	0.002100	0.000600	∼20
HIGH (2)	0.004500	0.001200	∼20

Table 4: Transition Probability Matrix (with 95% Confidence)

From \ To	**LOW**	**MED**	**HIGH**
LOW	0.75 ± 0.08	0.20 ± 0.06	0.05 ± 0.03
MED	0.25 ± 0.07	0.55 ± 0.09	0.20 ± 0.06
HIGH	0.10 ± 0.05	0.30 ± 0.08	0.60 ± 0.10

Key Observations:

- Diagonal elements are largest: regimes exhibit **persistence**
- LOW regime has highest self-transition: markets tend to stay calm
- Transitions between non-adjacent regimes (LOW↔HIGH) are rare

9.2 Stationary Distribution

The long-run probability of being in each regime (computed as eigenvector of $\mathbf{T}^T$ with eigenvalue 1):

$$\boldsymbol{\pi} = [\pi_0, \pi_1, \pi_2] \approx [0.40, 0.35, 0.25] \tag{22}$$

This indicates that markets spend approximately 40% of time in LOW volatility, 35% in MED, and 25% in HIGH.

```python
def _compute_stationary_distribution(self):
    # Find eigenvalues and eigenvectors of T^T
    eigenvalues, eigenvectors = np.linalg.eig(self.transition_matrix.T)

    # Find eigenvector with eigenvalue = 1
    idx = np.argmin(np.abs(eigenvalues - 1))
    stationary = np.real(eigenvectors[:, idx])

    # Normalize to probabilities
    stationary = np.abs(stationary)
    return stationary / stationary.sum()
```

Listing 7: Computing Stationary Distribution

9.3 Model Fit Statistics

Table 5: Model Fit Statistics

Statistic	**Value**
Sample Size (n)	60
Number of Parameters (k)	12
Log-Likelihood	∼250
AIC	∼-476
BIC	∼-451

10 Hypothesis Testing and Trading Signals

10.1 Likelihood Ratio Test for Regime Changes

When a regime change is detected, we validate it using LRT:

$$\Lambda = 2[\log L(H_1) - \log L(H_0)] \tag{23}$$

Where:

- H_0: Data comes from previous regime
- H_1: Data comes from new regime

```
def likelihood_ratio_test(self, volatilities, regime_model,
                          regime_h0, regime_h1):
    # Compute log-likelihoods
    ll_h0 = self.compute_log_likelihood(volatilities, regime_model,
    regime_h0)
    ll_h1 = self.compute_log_likelihood(volatilities, regime_model,
    regime_h1)

    # Test statistic
    test_statistic = 2 * (ll_h1 - ll_h0)

    # P-value from chi-squared distribution
    if test_statistic > 0:
        p_value = 1 - stats.chi2.cdf(test_statistic, df=1)
    else:
        p_value = 1.0

    # Reject H0 if p-value < alpha
    reject_h0 = p_value < self.significance_level
    return test_statistic, p_value, reject_h0
```

Listing 8: Likelihood Ratio Test Implementation

10.2 Trading Signal Generation

Signals are generated using a **scoring system** combining multiple factors:

Table 6: Signal Scoring Components

Component	**Weight**	**Criteria**
Regime Signal	40%	Significant regime change with MAP prob $\geq 70\%$
Trend	30%	Statistically significant trend ($p < 0.05$)
Momentum	20%	Aligned momentum direction (ROC $> 0.3\%$)
Mean Reversion	10%	Z-score beyond ± 2 standard deviations

Signal Rules:

- **BUY**: Score ≥ 0.5 AND regime decreased (volatility $\downarrow$) AND trend/momentum bullish
- **SELL**: Score ≥ 0.5 AND regime increased (volatility $\uparrow$) AND trend/momentum bearish
- **HOLD**: Conflicting signals or insufficient score

```python
# --- REGIME COMPONENT (weight: 40%) ---
if regime_changed and is_significant and map_probability >= 0.70:
    if current_regime < previous_regime:  # Volatility decreasing
        buy_score += 0.4
    elif current_regime > previous_regime:  # Volatility increasing
        sell_score += 0.4

# --- TREND COMPONENT (weight: 30%) ---
if trend_pval < 0.05:  # Statistically significant
    if trend_direction == 'UP':
        buy_score += 0.3 * trend_confidence
    elif trend_direction == 'DOWN':
        sell_score += 0.3 * trend_confidence
# ... momentum and mean reversion components ...
```

Listing 9: Signal Scoring Logic (abbreviated)

10.3 Trend Analysis Using Linear Regression

Trend is detected using OLS regression with t-test:

$$P_t = \alpha + \beta t + \epsilon_t \tag{24}$$

Estimators:

$$\hat{\beta} = \frac{\sum(t - \bar{t})(P_t - \bar{P})}{\sum(t - \bar{t})^2} \tag{25}$$

$$SE(\hat{\beta}) = \sqrt{\frac{MSE}{\sum(t - \bar{t})^2}} \tag{26}$$

$$t = \frac{\hat{\beta}}{SE(\hat{\beta})} \sim t_{n-2} \tag{27}$$

R-squared:

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}} = 1 - \frac{\sum(P_t - \hat{P}_t)^2}{\sum(P_t - \bar{P})^2} \tag{28}$$

```python
def compute_trend(self, prices):
    n = len(prices)
    x = np.arange(n)  # Time index
    y = np.array(prices)

    # OLS estimates
    x_mean, y_mean = np.mean(x), np.mean(y)
    numerator = np.sum((x - x_mean) * (y - y_mean))
    denominator = np.sum((x - x_mean) ** 2)
    slope = numerator / denominator

    # R-squared
    y_pred = intercept + slope * x
    ss_res = np.sum((y - y_pred) ** 2)
    ss_tot = np.sum((y - y_mean) ** 2)
    r_squared = 1 - (ss_res / ss_tot)

    # T-test for slope
```

```
    mse = ss_res / (n - 2)
    se_slope = np.sqrt(mse / denominator)
    t_stat = slope / se_slope
    p_value = 2 * (1 - stats.t.cdf(abs(t_stat), df=n-2))

    return slope, r_squared, p_value
```

Listing 10: Trend Computation with Statistical Testing

11 Interpretation of Results

11.1 Regime Classification Performance

The HMM successfully classifies market conditions into three volatility regimes:

- **HIGH confidence**: MAP probability typically $> 80\%$ during stable regimes
- **Transition detection**: Regime changes detected within 1-2 bars of actual volatility shifts
- **Statistical validation**: LRT p-values < 0.05 for genuine regime changes

11.2 Trading Strategy Evaluation

- **Signal Quality**: Signals only generated when statistically significant ($p < 0.05$)
- **Confirmation**: Multiple indicator confirmation reduces false signals
- **Risk Management**: Position sizing and P&L tracking provide risk control

11.3 Hypothesis Testing Results

For the primary hypothesis (regime classification):

- When $\Lambda > 3.84$ (critical value for χ^2_1 at $\alpha = 0.05$), we reject H_0
- This corresponds to p-value < 0.05, indicating statistically significant regime change
- Typical LRT statistics during genuine regime shifts: $\Lambda \approx 5 - 15$

For trend hypothesis:

- Significant trends ($p < 0.05$) occur in approximately 30-40% of trading windows
- R^2 values typically range from 0.3 to 0.8 during trending periods
- Flat/ranging markets show $p > 0.05$, correctly identified as "NEUTRAL"

12 Code Overview

The implementation consists of several key classes:

12.1 Class Structure

1. `IBApp`: Interactive Brokers API wrapper for data/trading
2. `OHLCBar`: Data structure for candlestick bars
3. `MarkovRegime`: HMM implementation with calibration and inference
4. `TradingStrategy`: Signal generation and trade execution
5. `LiveMarketDashboard`: GUI and visualization

12.2 Key Code Sections

MarkovRegime Class (Lines 135-460):

```
class MarkovRegime:
    def __init__(self):
        self.n_states = 3
        self.transition_matrix = np.array([...])
        self.emission_means = np.array([...])
        self.emission_stds = np.array([...])

    def calibrate(self, hist_bars):
        # MLE for emission parameters
        # MLE for transition matrix
        # Compute AIC, BIC
        ...

    def get_regime(self, bars):
        # Bayesian filtering
        # MAP classification
        ...
```

TradingStrategy Class (Lines 600-900):

```
class TradingStrategy:
    def compute_trend(self, prices):
        # OLS regression with t-test
        ...

    def likelihood_ratio_test(self, ...):
        # Chi-squared test for regime change
        ...

    def generate_signal(self, bars, regime_model, price):
        # Scoring system combining all indicators
        ...
```

Note: Complete source code is provided in `Final_project.py` (2146 lines).

13 Conclusion

This project demonstrates the application of statistical methods to financial trading:

1. **Hidden Markov Models** provide a principled framework for regime detection, treating market conditions as latent states inferred from observed volatility.

2. **Maximum Likelihood Estimation** enables data-driven parameter calibration, ensuring the model adapts to current market behavior.
3. **Bayesian Inference** allows continuous updating of regime probabilities as new information arrives.
4. **Hypothesis Testing** (LRT, t-tests) validates trading decisions, ensuring only statistically significant signals are acted upon.
5. **Multiple Indicator Confirmation** (trend, momentum, mean reversion) reduces false signals and improves strategy robustness.

13.1 Future Improvements

- Implement full Baum-Welch algorithm for online parameter re-estimation
- Add more sophisticated risk management (stop-loss, position sizing based on Kelly criterion)
- Extend to multi-asset portfolio with regime-dependent correlation modeling
- Incorporate volume and order flow data as additional emissions

A Statistical Formulas Reference

A.1 Gaussian PDF

$$f(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{29}$$

A.2 Bayes' Rule

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} \tag{30}$$

A.3 Chi-Squared Critical Values

α	$\chi^2_{1,\alpha}$
0.10	2.706
0.05	3.841
0.01	6.635

A.4 t-Distribution Critical Values (df=8)

α (two-tailed)	$t_{8,\alpha/2}$
0.10	1.860
0.05	2.306
0.01	3.355