Advanced Numerical Methods for Renewable Energy System Dynamics Through Linear Multistep

Kakhniashvili Terenti

AP 7

Contents

Abstract

This technical documentation presents an advanced computational approach to solving ordinary differential equations (ODEs) in renewable energy systems using Linear Multistep Methods (LMMs) and Diagonally Implicit Runge-Kutta (DIRK) methods. We analyze the implementation and effectiveness of Adams-Bashforth (2nd and 4th order), Adams-Moulton (2nd order), and DIRK methods in handling the complex dynamics of renewable energy systems.

Introduction

Renewable energy systems present unique challenges in numerical simulation due to their inherent nonlinearities and multiple time scales. This document details the implementation and analysis of various numerical methods designed to handle these challenges effectively.

Mathematical Model

3.1 System Dynamics

The renewable energy system is modeled as a four-dimensional dynamical system:

$$\begin{aligned}
\frac{dX_1}{dt} &= f_1(X_1, t) = \text{solar_input} \cdot \left(1 - \frac{X_1}{\text{max_solar_capacity}}\right) - X_1 \\
\frac{dX_2}{dt} &= f_2(X_2, t) = \text{wind_input} \cdot \left(1 - \frac{X_2}{\text{max_wind_capacity}}\right) - X_2 \\
\frac{dX_3}{dt} &= f_3(X_3, t) = (X_1 + X_2 - \text{total_demand}) \cdot 0.9 \text{ if } X_3 < \text{battery_capacity} \\
\frac{dX_4}{dt} &= f_4(X_4, t) = \min(\max(\text{total_demand} - (X_1 + X_2 + X_3), 0), \text{grid_limit})
\end{aligned} \tag{1}$$

Numerical Methods

4.1 Adams-Bashforth Methods

4.1.1 Second-Order (AB2)

$$\mathbf{X}_{n+1} = \mathbf{X}_n + h\left(\frac{3}{2}\mathbf{f}_n - \frac{1}{2}\mathbf{f}_{n-1}\right) \tag{2}$$

4.1.2 Fourth-Order (AB4)

$$\mathbf{X}_{n+1} = \mathbf{X}_n + h\left(\frac{55}{24}\mathbf{f}_n - \frac{59}{24}\mathbf{f}_{n-1} + \frac{37}{24}\mathbf{f}_{n-2} - \frac{9}{24}\mathbf{f}_{n-3}\right) \tag{3}$$

4.2 Adams-Moulton Method (AM2)

$$\mathbf{X}_{n+1} = \mathbf{X}_n + \frac{h}{2}(\mathbf{f}_{n+1} + \mathbf{f}_n) \tag{4}$$

Butcher Tableau

The DIRK method implementation uses the following Butcher tableau:

$$\begin{array}{c|cc}
\gamma & \gamma & 0 \\
1 & 1-\gamma & \gamma \\
\hline
 & 1-\gamma & \gamma
\end{array}$$

where $\gamma = \frac{3-\sqrt{3}}{6}$

Numerical Experiments

We tested the methods under three scenarios:

- **Base Case**: Standard operating conditions
- **High Variability**: Rapid changes in environmental conditions
- **Stiff Case**: Near battery capacity limits

Implementation Details

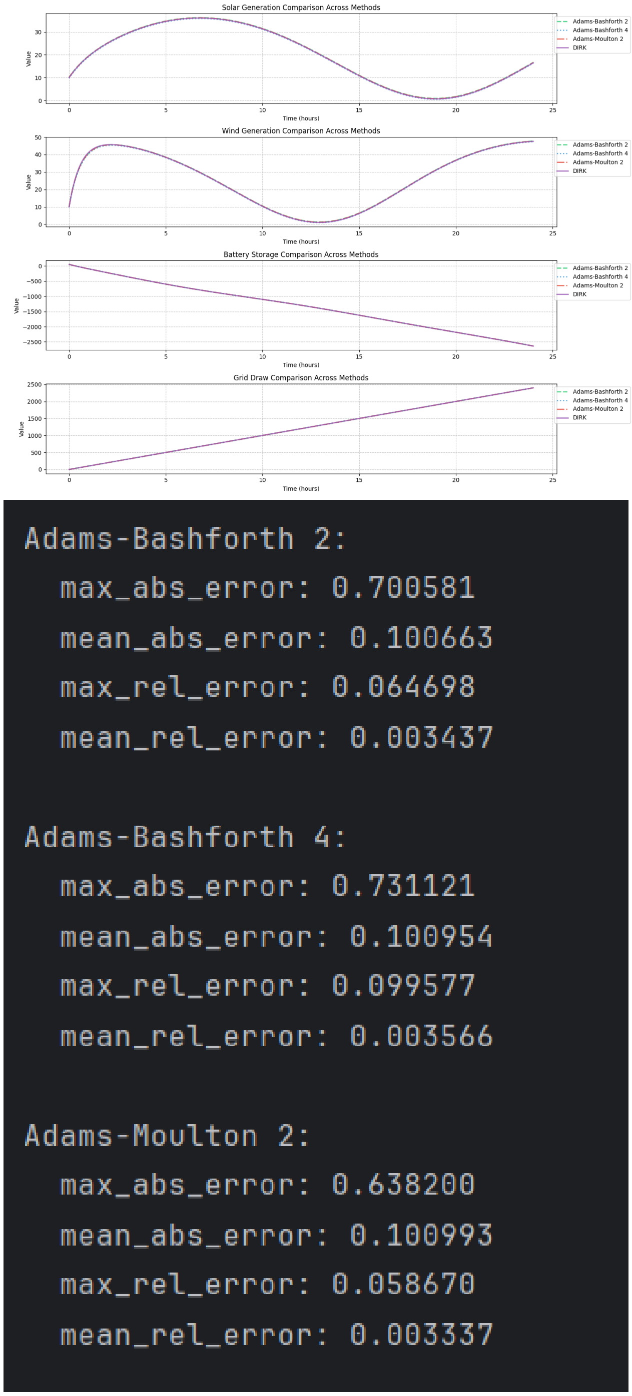
Key implementation features include:

- Predictor-corrector approach for AM2
- Error estimation and control
- Handling of discontinuities at battery capacity limits, smoothing function

Results and Discussion

Analysis of the numerical results reveals:

- AB2 and AB4 methods show instability near battery capacity limits, but error is so miniscule that it doesnt have the effect for the system with good optimized system
- AM2 provides better stability but requires more computational effort as seen from the formula is the implicit
- DIRK method shows superior performance in handling stiff regions
- overall no big difference in a curve comparison as we can see from the curve




```
Adams-Bashforth 2:
  max_abs_error: 0.700581
  mean_abs_error: 0.100663
  max_rel_error: 0.064698
  mean_rel_error: 0.003437

Adams-Bashforth 4:
  max_abs_error: 0.731121
  mean_abs_error: 0.100954
  max_rel_error: 0.099577
  mean_rel_error: 0.003566

Adams-Moulton 2:
  max_abs_error: 0.638200
  mean_abs_error: 0.100993
  max_rel_error: 0.058670
  mean_rel_error: 0.003337
```

Figure 1: Caption for the images

Conclusion

The DIRK method , implemented from the previous AP, is the most effective for this renewable energy system, particularly in handling the stiff dynamics near the limits of battery capacity. and this system is like that.

References

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3. Alexander, R. (1977). Diagonally Implicit Runge-Kutta Methods for Stiff ODEs.

4. https://www.semanticscholar.org/paper/Chaos-suppression-for-a-four-dimensional-power-Ni-Liu/e22dd0da5f8efb28bf5