1) PROVE THAT $\sqrt{(n+1)^3} = \Omega(n\sqrt{n})$

PROOF: $\sqrt{(n+1)^3} \geq (n+1)^{3/2}$

$\geq (n)^{3/2}$ $\because (n+1) \approx n$ FOR LARGE VALUES OF n

RHS, $n\sqrt{n} = n(n^{1/2})$

$= n^{2+1/2}$

$= n^{3/2}$

THEREFORE, THERE EXISTS +ve CONSTANT $c=1$ AND $n_0 = 1$ such that

$\sqrt{(n+1)^3} \geq c\, n(\sqrt{n})$

for all $n \geq n_0$

2 **PROOF:**

ACC. to the DEFINITION OF BIG O,

$$f_1(n) \leq c_1 \cdot g_1(n)$$
$$f_2(n) \leq c_2 \cdot g_2(n)$$

MULTIPLYING

$$f_1(n) \cdot f_2(n) \leq (c_1 \cdot g_1(n))(c_2 \cdot g_2(n))$$
$$= c_1 \cdot c_2 \cdot g_1(n) \, g_2(n)$$

THUS, $f_1(n) \cdot f_2(n) \leq C \cdot g_1(n) \, g_2(n)$

WHERE, $C = c_1 \cdot c_2$

THUS, $f_1(n) \, f_2(n) = O(g_1(n) \, g_2(n))$

3. PROVE THAT $f(n) = O(h(n))$

PROOF:

USING FORMAL DEFINITION OF BIG-OH,

- $f(n) \leq c_1 . g(n)$
- $g(n) \leq c_2 . h(n)$

SUBSTITUTING $g(n)$ into $f(n)$

- $f(n) \leq c_1 . (c_2 . h(n))$

$= c_1 . c_2 . h(n)$

- $f(n) \leq C . h(n)$

WHERE, $C = c_1 . c_2$

THUS, $f(n) = O(h(n))$

USING the DEFINITION OF BIG-Oh