

HW7

1. Recommended Dynamic Programming Data Structure

The best dynamic programming data structure for MysteryRecursion is a HashMap where:

- The key is a combination of the two input values n and m (e.g., as a string " n,m ").
- The value is the result of $\text{MysteryRecursion}(n, m)$

2. Pseudocode for a Memoized Dynamic Programming Algorithm

Input: n : positive integer

Input: m : positive integer

Input: memo: a map to store computed results

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1 Algorithm: MysteryRecursionMemoized( $n, m, \text{memo}$ )
2 if  $n = 1$  and  $m = 1$  then
3   return 1
4 else if  $(n, m)$  exists in memo then
5   return memo[( $n, m$ )]
6 else if  $n = 1$  then
7   result  $\leftarrow m \cdot \text{MysteryRecursionMemoized}(n, \lfloor m/2 \rfloor, \text{memo})$ 
8 else if  $m = 1$  then
9   result  $\leftarrow n \cdot \text{MysteryRecursionMemoized}(\lfloor n/2 \rfloor, m, \text{memo})$ 
10 else
11   result  $\leftarrow n \cdot \text{MysteryRecursionMemoized}(\lfloor n/2 \rfloor, m, \text{memo}) +$ 
         $m \cdot \text{MysteryRecursionMemoized}(n, \lfloor m/2 \rfloor, \text{memo})$ 
12 end
13 memo[( $n, m$ )]  $\leftarrow$  result
14 return result
```

3. Iterative Dynamic Programming Algorithm

Input: n: positive integer

Input: m: positive integer

1 Algorithm: MysteryRecursionIterative(n, m)

2 Initialize $dp[n+1][m+1]$ to 0

3 $dp[1][1] \leftarrow 1$

4 for i from 1 to n do

5 for j from 1 to m do

6 if $i = 1$ and $j > 1$ then

7 $dp[i][j] \leftarrow j \cdot dp[i][\lfloor j/2 \rfloor]$

8 else if $j = 1$ and $i > 1$ then

9 $dp[i][j] \leftarrow i \cdot dp[\lfloor i/2 \rfloor][j]$

10 else if $i > 1$ and $j > 1$ then

11 $dp[i][j] \leftarrow i \cdot dp[\lfloor i/2 \rfloor][j] + j \cdot dp[i][\lfloor j/2 \rfloor]$

12 end

13 end

14 end

15 return $dp[n][m]$