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VECTORS

(1) MAGNITUDE OF VECTOR $x = \begin{bmatrix} 1 \\ -3 \\ 8 \\ -5 \\ -1 \end{bmatrix}$

$$|x| = \sqrt{(1)^2 + (-3)^2 + (8)^2 + (-5)^2 + (-1)^2}$$

$$= \sqrt{1+9+64+25+1}$$

$$= \sqrt{100}$$

$$= \underline{\underline{10}}$$

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(2) (a) $S = \left\{ \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \end{bmatrix} \right\}$ vector space $\mathbb{R}^2$

$v_1 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

We can see, $v_2 = 2 \times v_1$

Hence, linearly dependent

$\therefore$ Span $(S) = \left\{ \alpha \begin{bmatrix} 2 \\ 4 \end{bmatrix},\right.$ WHERE $\left.\alpha \in \mathbb{R} \right\}$

(b) Vector $\begin{bmatrix} 20 \\ 4 \end{bmatrix}$ would NOT BE IN THE SPAN AS IT CANNOT BE EXPRESSED AS A SCALAR MULTIPLE OF $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$

Hence, NOT IN SPAN.

DOT PRODUCT

(3) $|a| = 10$

$|b| = 6$

$\theta = \frac{\pi}{3}$

$$\vec{a}.\vec{b} = |a|.|b|.\cos\theta$$

$$\vec{a}.\vec{b} = (10)(6)\left(\cos\frac{\pi}{3}\right)$$

$$= (60)\left(\frac{1}{2}\right)$$

$$\vec{a}.\vec{b} = \underline{\underline{30}}$$

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(4) $u = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$$\vec{u}.\vec{v} = |u|.|v|.\cos\theta$$

(a) $\vec{u}.\vec{v} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} \cancel{= (5\times 2) + (-3\times 1)}$

$$= (5\times 2) + (-3\times 1)$$

$$= 10 + (-3)$$

$$= 10 - 3$$

$$\vec{u}.\vec{v} = \underline{\underline{7}}$$

(b) $\cos\theta = \dfrac{\vec{u}.\vec{v}}{|u|\,|v|} = \dfrac{7}{\sqrt{170}} = \dfrac{7\sqrt{170}}{170}$

$$\therefore \theta = \cos^{-1}\frac{7\sqrt{170}}{170} = 57.53^\circ$$

LINEAR INDEPENDENCE

(5) $x = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}, \quad y = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \quad z = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$

$$a\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}, \; b\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \; c\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$$

$$\rightarrow \underset{3\times 3}{\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$R_2 = R_2 - 4R_1$

$R_3 = R_3 - 3R_1$

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -5 \\ 0 & 2 & 0 \end{bmatrix}$$

$R_2 = -R_2$

$$\rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 5 \\ 0 & 2 & 0 \end{bmatrix}$$

$R_1 = R_1 - R_2$

$$\rightarrow \begin{bmatrix} 1 & 0 & -4 \\ 1 & 1 & 5 \\ 0 & 2 & 0 \end{bmatrix}$$

$R_3 = R_3 - 2R_2$

$$\rightarrow \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 5 \\ 0 & 0 & -10 \end{bmatrix}$$

$$R_3 = \frac{-R_3}{10}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 = R_1 + 4R_3$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 = R_2 - 5R_3$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$\therefore \quad ax + by + cz = 0$

where $a = b = c = 0$

Hence, THEY ARE LINEARLY INDEPENDENT.

(6) Suppose that a subset S of a vector space V is linearly independent.

We need to prove that 0 cannot be expressed as a linear combination of elements of S with non-zero coefficients.

By Contradiction,

(a) suppose that

$$\lambda_1 x_1 + \lambda_2 x_2 \cdots + \lambda_k x_k = 0$$

where, λ_i be non-zero coefficient and $x_1, \ldots x_k$ are elements of S

→ By substracting $\lambda_i x_i$ from both sides and dividing the resulting equality by $-\lambda_i$ we get:

$$\frac{1}{\lambda_i}(\lambda_1 x_1 + \cdots + \lambda_k x_k) = x_i$$

where, x_i is missing on the left hand sum.

→ This equality implies that x_i is a linear combination of other element S.

→ We obtained by contradiction, that S is linearly independent

(b) Now, we need to prove that,

if 0 cannot be expressed as a linear combination of elements of S with non-zero coefficients then S is linearly independent.

→ By Contradiction,

Suppose that S is not linearly independent.

→ Then there exists an element x in S which is equal to a linear combination of other element S:

$$\lambda_1 x_1 + \lambda_2 x_2 + \dots \lambda_k x_k = x$$

→ By substracting x from both sides

we get:

$$\lambda x_1 + \lambda_2 x_2 + \dots + \lambda_k x_k - x = 0$$

Thus we have a linear combination of distinct elements of S which is equal to 0 & not all coefficients of this linear combination are equal to zero.

MATRICES

(7) $A = \begin{bmatrix} -1 & 5 \\ -7 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 \\ 1 & 8 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 7 \\ -4 & 8 \end{bmatrix}$

TO DEMONSTRATE $\Rightarrow$ $\underset{LHS}{A(B+C)} = \underset{RHS}{AB+AC}$

LHS

$\rightarrow A(B+C)$

$\rightarrow A\left(\begin{bmatrix} 5 & 4 \\ -3 & 16 \end{bmatrix}\right)$

$\rightarrow \begin{bmatrix} -1 & 5 \\ -7 & 3 \end{bmatrix} \cdot \begin{bmatrix} 5 & 4 \\ -3 & 16 \end{bmatrix}$

$\rightarrow \begin{bmatrix} -20 & 76 \\ -44 & 20 \end{bmatrix}$

RHS

$\rightarrow AB + AC$

$\rightarrow \begin{bmatrix} -1 & 5 \\ -7 & 3 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 8 \end{bmatrix} + AC$

$\rightarrow \begin{bmatrix} 3 & 43 \\ -11 & 45 \end{bmatrix} + \begin{bmatrix} -1 & 5 \\ -7 & 3 \end{bmatrix}\begin{bmatrix} 3 & 7 \\ -4 & 8 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 3 & 43 \\ -11 & 45 \end{bmatrix} + \begin{bmatrix} -23 & 33 \\ -33 & -25 \end{bmatrix}$

$\rightarrow \begin{bmatrix} -20 & 76 \\ -44 & 20 \end{bmatrix}$

$\therefore$ LHS = RHS

(8) $A = \begin{bmatrix} 1 & -3 & 3 \\ 8 & 5 & -2 \\ -4 & 7 & 2 \end{bmatrix}$, $A^{-1} = ?$

AUGMENT THE MATRIX

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & -3 & 3 & 1 & 0 & 0 \\ 8 & 5 & -2 & 0 & 1 & 0 \\ -4 & 7 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\boxed{\begin{array}{l} R_2 = R_2 - 8R_1 \\ R_3 = R_3 + 4R_1 \end{array}}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & -3 & 3 & 1 & 0 & 0 \\ 0 & 29 & -26 & -8 & 1 & 0 \\ 0 & -5 & 14 & 4 & 0 & 1 \end{array}\right]$$

$$\boxed{R_2 = R_2/29}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & -3 & 3 & 1 & 0 & 0 \\ 0 & 1 & -26/29 & -8/29 & 1/29 & 0 \\ 0 & -5 & 14 & 4 & 0 & 1 \end{array}\right]$$

$$\boxed{R_1 = R_1 + 3R_2}$$

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 9/29 & 5/29 & 3/29 & 0 \\ 0 & 1 & -26/29 & -8/29 & 1/29 & 0 \\ 0 & -5 & 14 & 4 & 0 & 1 \end{array}\right]$$

$$\boxed{R_3 = R_3 + 5R_2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 9/29 & 5/29 & 3/29 & 0 \\ 0 & 1 & -26/29 & -8/29 & 1/29 & 0 \\ 0 & 0 & 276/29 & 76/29 & 5/29 & 1 \end{array}\right]$$

$$\boxed{R_3 = \frac{29R_3}{276}}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 9/29 & 5/29 & 3/29 & 0 \\ 0 & 1 & -26/29 & -8/29 & 1/29 & 0 \\ 0 & 0 & 1 & 19/69 & 5/276 & 29/276 \end{array}\right]$$

$$\boxed{\begin{array}{l} R_1 = R_1 - \dfrac{9R_3}{29} \\ R_2 = R_2 + \dfrac{26R_3}{29} \end{array}}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2/23 & 9/92 & -3/92 \\ 0 & 1 & 0 & -2/69 & 7/138 & 13/138 \\ 0 & 0 & 1 & 19/69 & 5/276 & 29/276 \end{array}\right]$$

Hence,

$$A^{-1} = \begin{bmatrix} 2/23 & 9/92 & -3/92 \\ -2/69 & 7/138 & 13/138 \\ 19/69 & 5/276 & 29/276 \end{bmatrix}$$

(9) $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $A = \begin{bmatrix} 1 & -3 & 3 \\ 8 & 5 & -2 \\ -4 & 7 & 2 \end{bmatrix}$

So, $Ac = x$

$A = \begin{bmatrix} 1 & -3 & 3 \\ 8 & 5 & -2 \\ -4 & 7 & 2 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

To find x_b, $x_b = A^{-1} x$

Minor $= \begin{bmatrix} 24 & 8 & 76 \\ -27 & 14 & -5 \\ -9 & -26 & 29 \end{bmatrix}$, Cofactor $= \begin{bmatrix} 24 & -8 & +76 \\ 27 & 14 & 5 \\ -9 & 26 & 29 \end{bmatrix}$

$A\,dj. = \begin{bmatrix} 24 & 27 & -9 \\ -8 & 14 & 26 \\ 76 & 5 & 29 \end{bmatrix}$

$\det(A) = 1(10+14) - (-3)(16-8) + 3(56+20)$

$= 24 + 24 + 228$

$= 276$

So, $A^{-1} = 1/\det(A) \cdot \operatorname{adj}(A)$

$= 1/276 \begin{bmatrix} 24 & 27 & -9 \\ -8 & 14 & 26 \\ 76 & 5 & 29 \end{bmatrix}$

$\therefore x_b = A^{-1} x.$

$= \frac{1}{276} \begin{bmatrix} 24 & 27 & -9 \\ -8 & 14 & 26 \\ 76 & 5 & 29 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 17/92 \\ 49/138 \\ 173/276 \end{bmatrix}$

(10) $A_1 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow$ NOT IN RREF.

(1) THE 2nd ROW WITH ALL ZERO ELEMENTS SHOULD BE AT THE BOTTOM OF THE MATRIX

(2) EVERY PIVOT SHOULD BE 1, BUT IN ROW 1, THE PIVOT IS 2.

$A_2 = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 8 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow$ YES, IT IS IN RREF

$A_3 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow$ NOT IN RREF

(1) EVERY ELEMENT ABOVE AND BELOW THE PIVOT SHOULD BE ZERO.

(11) FINDING RANK

(a) $A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The A_1 MATRIX IS ALREADY IN ROW ECHELON FORM

$\therefore$ THE No. OF ROWS WITH NON-ZERO ARE THE RANK

$\therefore$ RANK $(A_1) = 3$

(b) $A_2 = \begin{bmatrix} 1 & -1 & 9 \\ 1 & 4 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

$\boxed{R_2 = R_2 - R_1}$

$\begin{bmatrix} 1 & -1 & 9 \\ 0 & 5 & -5 \\ 0 & 0 & 1 \end{bmatrix} \Leftarrow$ ROW ECHELON FORM

$\therefore$ THE RANK OF MATRIX IS THE No. OF NON ZERO ROWS IN THE REDUCED MATRIX

$\therefore$ RANK $(A_2) = 3$

(12) Dimension of Matrix : 5×9
Rank : 3

(1) ROW SPACE: $\dim(\text{row space}) = r = \underline{\underline{3}}$

(2) COLUMN SPACE: $\dim(\text{column space}) = r = \underline{\underline{3}}$

(3) NULL SPACE: $\dim(\text{null space}) = n - r$
$= 9 - 3$
$= \underline{\underline{6}}$

(4) LEFT NULL SPACE: $\dim(\text{left null space}) = m - r$
$= 5 - 3$
$= \underline{\underline{2}}$

SUM OF ALL 4 dimensions $= 3 + 3 + 6 + 2$
$= \underline{\underline{14}}$

13 DIMENSION OF MATRIX: 4×5

RANK $= 4$

- COLUMN SPACE (COLUMN RANK) $= \mathbb{R}^M = \underline{\underline{\mathbb{R}^4}}$
- LEFT NULL SPACE $= \underline{\underline{R^1}}$

(14) $A = \begin{bmatrix} 3 & 3 & 3 & 3 \\ 5 & 4 & 3 & 2 \\ 3 & 3 & 4 & 1 \end{bmatrix}$

REDUCING TO ROW ECHELON FORM

$\downarrow \boxed{R_1 = R_1/3}$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 \\ 5 & 4 & 3 & 2 \\ 3 & 3 & 4 & 1 \end{bmatrix} \xrightarrow{\boxed{R_2 = R_2 - 5R_1}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & -3 \\ 3 & 3 & 4 & 1 \end{bmatrix} \rightarrow$$

$$\xrightarrow{\boxed{R_3 = R_3 - 3R_1}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 1 & -2 \end{bmatrix} \xrightarrow{\boxed{\begin{array}{l} R_2 = -R_2 \\ R_1 = R_1 - R_2 \end{array}}} \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix} \rightarrow$$

$$\xrightarrow{\begin{array}{l} R_1 = R_1 + R_3 \\ R_2 = R_2 - 2R_3 \end{array}} \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

SO, MATRIX EQUATION $\Rightarrow$

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Here, $x_1 = 4t$

$x_2 = -7t$

$x_3 = 2t$

$x_4 = t$

THUS, $\vec{x} = \begin{bmatrix} 4t \\ -7t \\ 2t \\ t \end{bmatrix} = \begin{bmatrix} 4 \\ -7 \\ 2 \\ 1 \end{bmatrix} t.$

$\therefore \text{null}(A) = \text{span}\left\{ \begin{bmatrix} 4 \\ -7 \\ 2 \\ 1 \end{bmatrix} \right\}$

(15)

$$x_1 - 2x_2 - 3x_3 = b_1$$

$$2x_1 - 5x_2 - 4x_3 = b_2$$

$$4x_1 - 9x_2 - 8x_3 = b_3$$

Here, $A = \begin{bmatrix} 1 & -2 & -3 \\ 2 & -5 & -4 \\ 4 & -9 & -8 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

Performing R.E.F

$$\left[\begin{array}{ccc|c} 1 & -2 & -2 & b_1 \\ 0 & 1 & 0 & 2b_1 - b_2 \\ 0 & 0 & 0 & b_2 - 2b_3 \end{array}\right]$$

So,
$$x_1 = b_1 + 2t$$
$$x_2 = 2b_1 - b_2 + t$$
$$x_3 = t$$

t is free parameter.

Now, for $Ax = 0$

$$b_1 = b_2 = b_3 = 0$$

$$x_1 - 2x_2 - 2x_3 = 0$$
$$2x_1 - 5x_2 - 4x_3 = 0$$
$$4x_1 - 9x_2 - 8x_3 = 0$$

Row Reduction

$$\left[\begin{array}{ccc|c} 1 & -2 & -2 & 0 \\ 2 & -5 & -4 & 0 \\ 4 & -9 & -8 & 0 \end{array}\right]$$

$$\Downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Here,
$$x_1 = 4t$$
$$x_2 = t$$
$$x_3 = 0$$

(16) (i) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ $b = \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$A^T . A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

$$\boxed{P = A.(A^T.A)^{-1}. A^T . b}$$

$(A^T.A)^{-1} = \frac{1}{\det(A^TA)} . \operatorname{adj}(A^TA)$

$= \frac{1}{1\times 2 - 1\times 1} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

$= \frac{1}{1} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

$(A^T.A)^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

Now, $A. (A^T.A)^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow A.(A^TA)^{-1}. A^T = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & 0 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow A.(A^TA)^{-1}. A^T.b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

$\therefore$ Projection $\Rightarrow b-p = \begin{bmatrix}2\\5\\7\end{bmatrix} - \begin{bmatrix}2\\5\\0\end{bmatrix} = \begin{bmatrix}0\\0\\7\end{bmatrix}$

Magnitude $= \sqrt{7^2} = \underline{\underline{7}}$

(iii) $A = \begin{bmatrix}1 & 1\\1 & 1\\0 & 1\end{bmatrix}$, $b = \begin{bmatrix}3\\3\\7\end{bmatrix}$

$$P = A.(A^T A)^{-1} A^T . b$$

$\Rightarrow (A^T . A)^{-1} = \left(\begin{bmatrix}1 & 1 & 0\\1 & 1 & 1\end{bmatrix} \begin{bmatrix}1 & 1\\0 & 1\\0 & 1\end{bmatrix} \right)^{-1} = \begin{pmatrix}2 & 2\\2 & 3\end{pmatrix}^{-1}$

$\Rightarrow A.(A^T . A)^{-1} = \begin{bmatrix}1 & 1\\0 & 1\\0 & 1\end{bmatrix} \frac{2}{2} \begin{bmatrix}3/2 & -1\\-1 & 1\end{bmatrix} = \begin{bmatrix}1/2 & 0\\1/2 & 0\\-1 & 1\end{bmatrix}$

$\Rightarrow A.(A^T A)^{-1} A^T . b = \left(\begin{bmatrix}1/2 & 0\\1/2 & 0\\-1 & 1\end{bmatrix} \begin{bmatrix}1 & 1 & 0\\1 & 1 & 1\end{bmatrix} \right) . \begin{bmatrix}3\\3\\7\end{bmatrix}$

$= \begin{bmatrix}1/2 & 1/2 & 0\\1/2 & 1/2 & 0\\0 & 0 & 1\end{bmatrix} . \begin{bmatrix}3\\3\\7\end{bmatrix} = \begin{bmatrix}3\\3\\7\end{bmatrix}$

$\therefore b - p = \begin{bmatrix}3\\3\\7\end{bmatrix} - \begin{bmatrix}3\\3\\7\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

magnitude $= \sqrt{0} = \underline{\underline{0}}$

(17) $M = \begin{bmatrix} 1 & 0 & 4 \\ 2 & 3 & 4 \\ 0 & -1 & 2 \end{bmatrix}$, INDEPENDENT VECTORS $\Rightarrow a = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix}$

$c = \begin{bmatrix} 4 \\ 4 \\ 2 \end{bmatrix}$

3 ORTHOGONAL VECTORS A, B, C:

$$A = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$B = b - \frac{A^T b}{A^T A} \cdot A$$

$$= \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix} - \frac{[1\ 2\ 0]\begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix}}{[1\ 2\ 0]\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}} \cdot \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 3 \\ -1 \end{bmatrix} - \begin{bmatrix} 6/5 \\ 12/5 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -6/5 \\ 3/5 \\ -1 \end{bmatrix}$$

$$C = c - \frac{A^T c}{A^T A} \cdot A - \frac{B^T c}{B^T B} \cdot B$$

$$= \begin{bmatrix} 4 \\ 4 \\ 2 \end{bmatrix} - \frac{[1\ 2\ 0]\begin{bmatrix} 4 \\ 4 \\ 2 \end{bmatrix}}{[1\ 2\ 0]\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}} \cdot \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} - \frac{[-6/5\ \ 3/5\ \ -1]\begin{bmatrix} 4 \\ 4 \\ 2 \end{bmatrix}}{[-6/5\ \ 3/5\ \ -1]\begin{bmatrix} -6/5 \\ 3/5 \\ -1 \end{bmatrix}} \begin{bmatrix} -6/5 \\ 3/5 \\ -1 \end{bmatrix}$$

$$C = \begin{bmatrix} -2/7 \\ 1/7 \\ 3/7 \end{bmatrix}$$

Now, Producing ORTHONORMAL VECTORS →

$$q_1 = \frac{A}{|A|}, \quad q_2 = \frac{B}{|B|}, \quad q_3 = \frac{C}{|C|}$$

$$= \frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad q_2 = \frac{5}{\sqrt{70}}\begin{bmatrix} -6/5 \\ 3/5 \\ -1 \end{bmatrix}, \quad q_3 = \frac{7}{\sqrt{14}}\begin{bmatrix} -2/7 \\ 1/7 \\ 3/7 \end{bmatrix}$$

$$q_1 = \begin{bmatrix} \frac{\sqrt{5}}{5} \\ \frac{2\sqrt{5}}{5} \\ 0 \end{bmatrix}, \quad q_2 = \begin{bmatrix} -\frac{3\sqrt{70}}{35} \\ \frac{3\sqrt{70}}{70} \\ -\frac{\sqrt{70}}{14} \end{bmatrix}, \quad q_3 = \begin{bmatrix} -\frac{\sqrt{14}}{7} \\ \frac{\sqrt{14}}{14} \\ \frac{3\sqrt{14}}{14} \end{bmatrix}$$