ALI ABDULLAH AHMAD
20031246—CWID

HW2

① $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{bmatrix}$$

$$\det(A-\lambda I) = (1-\lambda)(3-\lambda) - (2)(4)$$

$$= 3 - \lambda - 3\lambda - \lambda^2 - 8$$

$$= \lambda^2 - 4\lambda - 5$$

$$= \lambda^2 - 5\lambda + \lambda - 5$$

$$= \lambda(\lambda-5) + 1(\lambda-5)$$

$$= (\lambda+1)(\lambda-5)$$

$$\lambda = 5, -1$$

$$\boxed{\lambda_1 = 5, \lambda_2 = -1}$$

Diagonal Matrix $= \begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}$

for $\lambda_1 = 5$

$$\Rightarrow \begin{bmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 4 & -2 \end{bmatrix}$$

$$B\vec{x} = \vec{0}$$

$$\Rightarrow \begin{bmatrix} -4 & 2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_2 = R_2 + R_1$

$$\left[\begin{array}{cc|c} -4 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$R_1 \Rightarrow \frac{-R_1}{4}$

$$\left[\begin{array}{cc|c} 1 & -1/2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$\therefore x_1 = +1/2\, x_2$

$\cancel{x_2 = x_2}$

Prioritise $x_1 = \frac{+1}{2} t$

$x_2 = t$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \end{bmatrix} = \vec{v}_1$$

for $\lambda_2 = -1$

$$\Rightarrow \begin{bmatrix} 1+1 & 2 \\ 4 & 3+1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{cc|c} 2 & 2 & 0 \\ 4 & 4 & 0 \end{array}\right]$$

$R_2 = R_2 - 2R_1$

$$\Rightarrow \left[\begin{array}{cc|c} 2 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$R_1 = \frac{R_1}{2}$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow x_1 = -x_2$

$\cancel{x_2 = x_2}$

Prioritise

$x_1 = -t$

$x_2 = t$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \vec{v}_2$$

$$P = \begin{bmatrix} 1/2 & -1 \\ 1 & 1 \end{bmatrix}$$

2

$$A = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix} \quad \text{find } \underline{A = U \Sigma V^T}$$

<u>Step I</u>

$$A^T A = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 12 \\ 12 & 13 \end{bmatrix}$$

$$\cancel{\det(A^T \cdot A)} \; (A^T A - \lambda I) = \begin{pmatrix} 13 - \lambda & 12 \\ 12 & 13 - \lambda \end{pmatrix}$$

$$\det(A^T \cdot A - \lambda I) = (13-\lambda)(13-\lambda) - (12)(12)$$

$$= 169 - 13\lambda - 13\lambda + \lambda^2 - 144$$

$$= \lambda^2 - 26\lambda - 25$$

$$\cancel{\lambda(\lambda - 25)}$$

$$= (\lambda - 25)(\lambda - 1)$$

$$\therefore \boxed{\lambda_1 = 25, \; \lambda_2 = 1}$$

(P.T.O)

for λ_1

$$\left[\begin{array}{cc|c} 13-25 & 12 & 0 \\ 12 & 13-25 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cc|c} -12 & 12 & 0 \\ 12 & -12 & 0 \end{array}\right]$$

$R_2 = R_2 + R_1$

$$\Rightarrow \left[\begin{array}{cc|c} -12 & 12 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$R_1 = -\frac{R_1}{12}$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$x_1 = +x_2$

$$\vec{v}_1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} +1 \\ 1 \end{bmatrix}$$

for λ_2

$$\left[\begin{array}{cc|c} 13-1 & 12 & 0 \\ 12 & 13-1 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cc|c} 12 & 12 & 0 \\ 12 & 12 & 0 \end{array}\right]$$

$R_2 = R_2 - R_1$

and $R_1 = \frac{R_1}{12}$

$$\Rightarrow \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$x_1 = -x_2$

$$\vec{v}_2 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

We know, that $\Rightarrow \Sigma^2 = \begin{bmatrix} 25 & 0 \\ 0 & 1 \end{bmatrix} \therefore \Sigma$ (Diagonal Matrix) $= \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$

Step II) unit vectors v_1 & v_2

$\|v_1\| = \sqrt{(-1)^2 + (1)^2}$

$= \sqrt{2}$

$\therefore v_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\|v_2\| = \sqrt{(-1)^2 + (1)^2}$

$= \sqrt{2}$

$\therefore v_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Hence, $V = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

Step III)

$A V \Sigma^{-1} = U$

Finding Σ^{-1}

$\Sigma^{-1} = \frac{1}{|\Sigma|} \cdot \operatorname{adj}(\Sigma)$

$= \frac{1}{(5 \times 1) - 0} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$

$= \frac{1}{5} \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1/5 & 0 \\ 0 & 1 \end{bmatrix}$

$\cancel{= \begin{matrix} 5 & 0 \\ 0 & 25 \end{matrix}}$

Now, $U = A.V.\Sigma^{-1}$

$$= \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \cdot \begin{bmatrix} 1/5 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2/5 & 3 \\ 3/5 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{17\sqrt{2}}{10} & \frac{13\sqrt{2}}{10} \\ \frac{13\sqrt{2}}{10} & \frac{7\sqrt{2}}{10} \end{bmatrix}$$

$$A = U \Sigma V^T$$

$$U = \begin{bmatrix} \frac{17\sqrt{2}}{10} & \frac{13\sqrt{2}}{10} \\ \frac{13\sqrt{2}}{10} & \frac{7\sqrt{2}}{10} \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$$

$$V^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(5) $\mu = 180g$, $\sigma = 60g$

(1) Jane does not like ($\leq 100g$)

using z score

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{100 - 180}{60}$$

$$= \frac{-80}{60}$$

$$= -1.33$$

z score $= -1.33 = \underline{\underline{0.0918}}$

$\therefore$ 9.18 % of apples Jane picks she does not like.

(2) q_3 = upper quartile

$$q_3 = \mu + \underset{\text{75th percentile}}{z}\sigma = 180 + 0.6745 \times 60 = \underline{\underline{220.47g}}$$

$$\therefore P(X \geq 220.47) = \frac{220.47 - 180}{60}$$

$$= 0.6745$$

z score $= 0.2514$

OR

$\underline{\underline{25.14\%}}$ Chances.

	frequency	#	~~frequency~~ PROBABILITY
6 (a) Red	= 9 balls	⇒ 300	9/39 = 3/13
Blue	= 9 balls	⇒ 200	9/39 = 3/13
Green	= 9 balls	⇒ 0	9/39 = 3/13
Yellow	= 9 balls	⇒ 0	9/39 = 3/13
Orange	= 3 balls	⇒ 500	3/39 = 1/13

$$E(X) = \frac{3 \times 300}{13} + \frac{200 \times 3}{13} + \frac{0 \times 3}{13} + \frac{0 \times 3}{13} + \frac{500 \times 1}{13}$$

$$= \frac{900 + 600 + 500}{13}$$

$$= \underline{\underline{153.84}}$$

(b) $\underline{Var(X) = E(X^2) - E(X)^2}$

$$E(X^2) = \frac{3 \times (300)^2}{13} + (200)^2 \times \frac{3}{13} + 0^2 \times \frac{3}{13} + 0^2 \times \frac{3}{13} + \frac{(500)^2}{13}$$

$$= \frac{270{,}000 + 120000 + 0 + 0 + 250000}{13}$$

$$= \underline{\underline{49{,}230.76}}$$

$$Var(x) = 49230.76 - (153.84)^2$$
$$= 49230.76 - 23666.74$$
$$= \underline{\underline{25,564.02}}$$

$$\text{Standard Deviation } (\sigma) = \sqrt{25,564.02}$$
$$= \underline{\underline{159.88}}$$

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(7) $P(K)$ = Probability of knows the answer is correct $= \underline{\underline{x}}$

$P(K')$ = Probab. of ~~not know~~ doesn't know the correct answer $= \underline{\underline{1-x}}$

$P(C|K)$ = Correct answer given he knows the correct answer $= \underline{\underline{1}}$

$P(C|K')$ = Correct answer given doesn't ~~know~~ know the correct answer $= \frac{1}{y}$

$P(K|C) = ??$

$$\therefore P(K|C) = \frac{P(C|K) \cdot P(K)}{P(C)}$$

Now, $P(C) = P(C|K) \cdot P(K) + P(C|K') \cdot P(K')$

$$P(C) = (1)(x) + (1-x)\left(\frac{1}{y}\right)$$

$$= x + \frac{1-x}{y}$$

$$= \frac{xy - x + 1}{y}$$

Now, $P(K|C) = \dfrac{(x)(1)}{\frac{xy+1-x}{y}}$

$$P(K|C) = \frac{xy}{xy+1-x}$$

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8 $P(S)$ = ~~Detected~~ Spam = 0.40

$P(NS)$ = ~~Detected~~ Not Spam = 0.60

$P(D|S)$ = Detected and Spam = 0.98

$P(D|NS)$ = Detected and Not Spam = 0.05

$P(NS|D) = ??$

$$P(NS|D) = \frac{P(D|NS) \cdot P(NS)}{P(D)}$$

Now, $P(D) = P(D|S) \cdot P(S) + P(D|NS) \cdot P(NS)$

$$P(D) = (0.98 \times 0.4) + (0.05 \times 0.60)$$

$$= \underline{\underline{0.422}}$$

Now, $P(NS|D) = \dfrac{(0.05).(0.60)}{(0.422)}$

$$= \frac{0.03}{0.422}$$

$$= \underline{\underline{0.0711}}$$

Hence, 7.11% probab that it was detected as spam but was not spam.

~~9) D = has disease | D' = No disease~~
~~Y = test +ve | N = test -ve~~

~~$P(D) = \frac{1}{10,000} = \underline{0.0001}$~~

~~$P(D') = 1 - 0.0001 = 0.9999$~~

9) D = has disease , D' = no disease
Y = test positive , N = test negative

$P(D) = \frac{1}{10,000} = \underline{0.0001}$

$P(D') = 1 - 0.0001 = \underline{0.9999}$

$P(Y|D') = \underline{\underline{0.02}}$ and $P(N|D) = \underline{\underline{0.01}}$

$P(Y|D) = 1 - \cancel{P(Y|D')}\ P(N|D)$

$= 1 - 0.01$

$= \underline{\underline{0.99}}$

$P(N|D') = 1 - P(Y|D')$

$= 1 - 0.02$

$= \underline{\underline{0.98}}$

<u>Now,</u> $P(D|Y) = ??$

$$P(D|Y) = \frac{P(Y|D).P(D)}{P(Y)}$$

where, $P(Y) = P(Y|D).P(D) + P(Y|D').P(D')$

$= (0.99 \times 0.0001) + (0.02 \times 0.9999)$

$= \underline{\underline{0.020097}}$

Now, $P(D|Y) = \frac{0.99 \times 0.0001}{0.020097}$

$= \underline{\underline{0.0049}}$

(10) $F(x) = \frac{f(x)}{g(x)} \longrightarrow \frac{g(x).f'(x) - f(x).g'(x)}{[g(x)]^2}$

(a)

taking $\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\}$ using $f'(x) = \frac{f(x+h) - f(x)}{h}$

$$\Rightarrow \lim_{h\to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{f(x+h).g(x) - f(x).g(x+h)}{g(x+h).g(x).h}$$

$$\Rightarrow \lim_{h\to 0} \frac{1}{g(x+h).g(x)} \cdot \lim_{h\to 0} \frac{f(x+h).g(x) - f(x).g(x+h)}{h}$$

$$\Rightarrow \frac{1}{[g(x)]^2} \cdot \lim_{h\to 0} \frac{f(x+h).g(x) - f(x).g(x+h)}{h}$$

* Adding $[f(x).g(x) - f(x).g(x)]$

$$\Rightarrow \frac{1}{[g(x)]^2} \cdot \lim_{h\to 0} \frac{f(x+h).g(x) - f(x).g(x+h) + f(x).g(x) - f(x).g(x)}{h}$$

$$\Rightarrow \frac{1}{[g(x)]^2} \cdot \lim_{h\to 0} \frac{f(x+h).g(x) - f(x).g(x) - f(x).g(x+h) + f(x).g(x)}{h}$$

$$\Rightarrow \frac{1}{[g(x)]^2} \lim_{h \to 0} \frac{g(x)[f(x+h) - f(x)] - f(x)[g(x+h) - g(x)]}{h}$$

$$\Rightarrow \frac{1}{[g(x)]^2} \left(g(x) \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} - f(x) \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} \right)$$

$$\Rightarrow \frac{1}{[g(x)]^2} \left(g(x) \cdot f'(x) - f(x) \cdot g'(x) \right)$$

$$\Rightarrow \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

~~(b) $f(x) = \sin x \Rightarrow \cos(x)$~~

(b) $f(x) = \sin x$ To Prove : $f'(x) = \cos(x)$

$$\Rightarrow \frac{d \sin(x)}{dx} \Rightarrow \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

* using, $\sin(a+b) = \sin(a) \cdot \cos(b) + \cos(a) \cdot \sin(b)$

$$\Rightarrow \lim_{h \to 0} \frac{\sin(x) \cdot \cos(h) + \cos(x) \cdot \sin(h) - \sin(h)}{h}$$

$$\Rightarrow \lim_{h \to 0} \frac{\sin x(\cos h - 1) + \cos(x) \cdot \sin(h)}{h}$$

$$= \lim_{h \to 0} \left(\sin x \cdot \frac{(\cos h - 1)}{h} + \cos(x) \cdot \frac{\sin h}{h} \right)$$

$*$ using, $\lim_{h \to 0} \frac{\cos h - 1}{h} = 0$

and $\lim_{h \to 0} \frac{\sin h}{h} = 1$

$$\Rightarrow \sin(x) \cdot 0 + \cos(x) \cdot (1)$$

$$\Rightarrow \underline{\underline{\cos(x)}}$$

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(11) $f(x) = x^4 + 5x^3 + 9x^2 - 3$

at $x = 1$ & $y = 12$

$\therefore f'(x) = 4x^3 + 15x^2 + 18x$

$m = 4 + 15 + 18$ at $x = 1$

$m = \underline{\underline{37}}$

$\therefore m = \frac{y - y_1}{x - x_1}$

$$\Rightarrow \boxed{\underline{y = 37x - 25}}$$

(12) $f(x) = e^{2x}$

$g(x) = \dfrac{x^2}{x-1}$

$F(x) = f(g(x))$

$= f\left(\dfrac{x^2}{x-1}\right)$

$= e^{2\left(\frac{x^2}{x-1}\right)}$

$F'(x) = e^{2\left(\frac{x^2}{x-1}\right)}$ using chain rule.

$\Rightarrow e^{\frac{2x^2}{x-1}} \cdot \dfrac{d}{dx}\left(\dfrac{2x^2}{x-1}\right)$ using exponential diff.

$\Rightarrow e^{\frac{2x^2}{x-1}} \cdot 2 \cdot \dfrac{d}{dx}\left(\dfrac{x^2}{x-1}\right)$

$\Rightarrow e^{\frac{2x^2}{x-1}} \cdot 2 \cdot \left[\dfrac{2x(x-1) - x^2 \cdot (1-0)}{(x-1)^2}\right]$

using quotient Rule.

$\Rightarrow e^{\frac{2x^2}{x-1}} \cdot 2 \cdot \left[\dfrac{2x^2 - 2x - x^2}{(x-1)^2}\right]$

$$\Rightarrow e^{\frac{2x^2}{x-1}} \cdot 2 \cdot \left[\frac{x^2 - 2x}{(x-1)^2} \right]$$

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(13) (a) $F(x) = \dfrac{\sqrt{x} + 2x}{7x - 4x^2}$

$$f'(x) = \frac{(7x - 4x^2)(\sqrt{x} + 2x)' - (\sqrt{x} + 2x)(7x - 4x^2)'}{(7x - 4x^2)^2}$$

Using quotient Rule

$$\Rightarrow \frac{(7x - 4x^2)\left(\frac{1}{2\sqrt{x}} + 2\right) - (\sqrt{x} + 2x)(7 - 8)}{(7x - 4x^2)^2}$$

(b) $F(x) = (1 + \sqrt{x^3})\left(\frac{1}{x^3} - 2\sqrt[3]{x}\right)$

applying Chain Rule, ~~by~~ using product Rule

$$F'(x) = (1 + \sqrt{x^3})(x^{-3} - 2.x^{1/3})' + (1 + x^{3/2})'(x^{-3} - 2x^{1/3})$$

$$= (1 + x^{3/2})\left(-3x^{-4} - \frac{2}{3}x^{-\frac{2}{3}}\right) + \left(\frac{3}{2}x^{1/2}\right)(x^{-3} - 2x^{1/3})$$

(c) $F(x) = (2x^2+1)^3 . (3x^3-2)^2$

$$F'(x) = (2x^2+1)^{3'}(3x^3-2)^2 + (2x^2+1)^3(3x^3-2)^{2'}$$

Using product Rule.

$$= 3(2x^2+1)^2.(4x+0)(3x^3-2)^2 + (2x^2+1)^3 . 2.(3x^3-2).9x^2$$

$$= 12x(2x^2+1)^2(3x^3-2)^2 + (2x^2+1)^3.(3x^3-2).18x^2$$

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(14) Partial derivative with respect to each variable.

(a) $F(x,y) = \dfrac{x^3y^2 - 2x^2 + 5y}{e^x}$

$$f_x(x,y) = \frac{\partial}{\partial x}\left(\frac{x^3y^2 - 2x^2 + 5y}{e^x}\right)$$

Using quotient Rule.

$$\Rightarrow \frac{(e^x)[x^3y^2 - 2x^2 + 5y]' - (e^x)'[x^3y^2 - 2x^2 + 5y]}{(e^x)^2}$$

$$= \frac{(e^x)[3x^2y^2 - 4x + 0] - (e^x)[x^3y^2 - 2x^2 + 5y]}{(e^x)^2}$$

(b) $F(x,y,z) = y^2 \lg(x+2y) - \lg(3z)(x^3+y^2-4z)$

For $F_x(x,y,z) = \dfrac{\partial\left[y^2 \lg(x+2y) - \lg(3z)(x^3+y^2-4z)\right]}{\partial x}$

$$= \frac{\partial}{\partial x}\left(y^2 \lg(x+2y)\right) - \frac{\partial}{\partial x}\left[\lg(3z)(x^3+y^2-4z)\right]$$

$$= y^2 \frac{\partial}{\partial x}\left[\lg(x+2y)\right] - \lg(3z)\frac{\partial}{\partial x}\left[x^3+y^2-4z\right]$$

$$= y^2 \cdot \frac{1}{x+2y} \cdot (1+0) - \lg(3z) \cdot \left[3x^2+0-0\right]$$

$$= \frac{y^2}{x+2y} - \lg(3z) \cdot \left[3x^2\right]$$

For $F_y(x,y,z) = \dfrac{\partial}{\partial y}\left[y^2 \lg(x+2y)\right] - \dfrac{\partial}{\partial y}\left[\lg(3z)(x^3+y^2-4z)\right]$

using product Rule

$$= 2y \cdot \lg(x+2y) + y^2 \cdot \frac{1}{x+2y} \cdot (0+2) - \lg(3z)\frac{\partial(x^3+y^2-4z)}{\partial y}$$

$$= 2y \cdot \lg(x+2y) + \frac{2y^2}{x+2y} - \lg(3z) \cdot 2y$$

$$= 2y \cdot \lg(x+2y) + \frac{2y^2}{x+2y} - 2y \cdot \lg(3z)$$

for, $F_y(x,y) = \dfrac{\partial}{\partial y}\left(\dfrac{x^3y^2 - 2x^2 + 5y}{e^x}\right)$

Using quotient rule.

$$\cancel{= \frac{(e^x)'[x^3y^2 - 2x^2 + 5y] - (e^x)[x^3y^2 - 2x^2 + 5y]'}{(e^x)^2}}$$

$$= \frac{(e^x)[x^3y^2 - 2x^2 + 5y]' - (e^x)'[x^3y^2 - 2x^2 + 5y]}{(e^x)^2}$$

$$= \frac{(e^x)[2yx^3 - 0 + 5] - 0}{(e^x)^2} \qquad \because \frac{\partial(e^x)}{\partial y} = 0$$

$$= \frac{(e^x)[2yx^3 + 5]}{(e^x)^2}$$

For $F_z(x,y,z) = \frac{\partial[y^2 \lg(x+2y)]}{\partial z} - \frac{\partial(\lg(3z)(x^3+y^2-4z))}{\partial z}$

using product Rule

$= 0 - \{\lg(3z)' \cdot [x^3+y^2-4z] + \lg(3z)[x^3+y^2-4z]'\}$

$= 0 - \{\frac{1}{\cancel{3}z} \cdot \cancel{3}[x^3+y^2-4z] + \lg(3z) \cdot [0+0-4]\}$

$= 0 - \{\frac{(x^3+y^2-4z)}{z} + (-4 \lg(3z))\}$

$= -\frac{x^3+y^2-4z}{z} + 4\lg(3z)$

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15 a $f'(22.5) = \frac{f(22.5+2.5) - f(22.5-2.5)}{2(2.5)}$

$= \frac{f(25) - f(20)}{5}$

$= \frac{400-325}{5}$

$= \frac{75}{5} = 15$

15

(a) $$f'(12.5) = \frac{f(12.5+2.5) - f(12.5-2.5)}{2(2.5)}$$

$$= \frac{f(15) - f(10)}{5}$$

$$= \frac{370 - 300}{5}$$

$$= \frac{70}{5} = \underline{\underline{14}}$$

(b) $$f'(15) = \frac{f(15+5) - f(15)}{5}$$

$$= \frac{f(20) - f(15)}{5}$$

$$= \frac{325 - 370}{5}$$

$$= \frac{-45}{5} = \underline{\underline{-9}}$$

(c) $$\cancel{f'(15) = \frac{f(15) - f(15+5)}{5}}$$

$$\cancel{= \frac{370 - 325}{5}}$$

$$\cancel{= \frac{45}{5} = 9}$$

$$f'(15) = \frac{f(15) - f(15-5)}{5}$$

$$= \frac{f(15) - f(10)}{5}$$

$$= \frac{370 - 300}{5}$$

$$= \frac{70}{5} = \underline{14}$$