ROBUST DESIGN OF SMITH PREDICTOR CONTROLLERS UNDER PRE-PRESCRIBED CONSTRAINTS USING GENETIC ALGORITHM

A. H. Jones and A.M. Segayer

Intelligent Control Group
School of Aeronautical and Mechanical Engineering
University of Salford, Salford M5 4WT
E-mail a.h.jones@salford.ac.uk a.m.segayer@pgr.salford.ac.uk

Abstract: In many cases, the standard process control model involving a first order lag plus time-delay can be incorporated as the model in the Smith predictor controller (SPC). In this paper, a genetic algorithms is used to show that for this type of model, the design of optimally tuned (for minimum ISE) proportional plus integral Smith predictor controllers when subject to a broad range of robustness constraints can be simplified by choosing the integral time-constant equal to the dominant process time-constant. This results in a single tuneable parameter Smith predictor controller.(SP-SPC). Controller tuning curves are then produced which facilitate the design of such a SPC which satisfy pre-prescribed gain margins and delay margins constraints. Finally, the results of the SP-SPC are compared to optimally tuned PI controllers when subject to the same robustness constraints. *Copyright ©2000 IFAC*

Keywords: Genetic algorithm, Predictive control,Robust control, Smith predictor control, delay margin

1. Introduction

Dead times between inputs and outputs are common phenomena in many industrial processes and cause considerable difficulties in effective control of such processes. Early researchers addressed the problem of controller design for systems with time-delay by correlating PID controller settings with model gain, time-constant and time-delay (e.g. Cohen and Coon 1953). Low loop gains were required to avoid instability when time-constant were small compared to time-delay, leading to poor system performance. Smith (1957) proposed the PI plus Smith predictor approach which represented an important breakthrough for controlling systems with time-delays. Smith's approach enabled larger loop gains to be used by incorporating a minor feedback loop around a conventional controller to stabilise the system. In the absence of modelling error the SPC has been shown to lead to optimal response to step disturbances and its attractiveness comes from the fact that it eliminates the dead-time from the characteristic equation of the closed loop system, thus seemingly converting the design and tuning problem of a system with time-delays to that for a system without time-delay (Donoghue 1977). However, when mismatches in the model occurs, the SPC can exhibits very poor stability properties. The analysis for computing the stability and robustness of such controllers has been reported by Palmor and Shinnar (1978,81) Morari(1989) and Landua (1995).

The most significant issue for the design of SPC discussed by both Polmar and et al Landua is the realisation that phase margin is not a good measure of robustness . Indeed, both authors fully explained the necessity to measure delay margin as the correct robustness measure instead of phase changes. Moreover the use of low order Pade' approximations in the design process is also shown to give rise to misleading results. Laughlin et al. (1987) used a forth-order Pade' approximation and so did Wang and Skogestad (1993). Generally one would believe that a fourth-order Pade' approximation should be sufficiently accurate, but this appears not to be true as even high order Pade' approximations filter out the multiple intersections of the unit disc thus completely obscuring the intrinsic stability problems present.

Many process control applications involve plants with a monotone output step response which exhibit no overshoot. Historically, the first order plus dead time model has been used to represent such plants, and the approach has received widespread acceptance in industry. In this paper such a model structure has been adopted for the model. It has been shown by Morrari(1989) that for a first order system with dead-time a SPC with a PI controller is equivalent to IMC if the integral time is set equal to the system time-constant. In this paper a genetic algorithm is used to show that if the integral time is chosen equal to the process time-constant, the resulting single parameter controller is also optimally tuned for minimum ISE when subject to robustness constraints involving up to 400% simultaneous changes in gain margin and delay margin. Furthermore it is also possible to produce a set of design curves which map the ratio of time-delay to time-constant to the single controller gain kp for pre-prescribed gain and delay margins. Such curves facilitate the robust design of SPC in the sense that if the plant time-constant and time-delay are both known, and the desired gain margin and delay margin are known, then the design curves can be used to evaluate the value of process gain kp. This feature can be very useful in a variety of computer aided design situations including gain scheduled and adaptive control.

2. Robustness margins

The SPC configuration is shown in Figure 1, where C(s) denotes the transfer function of the controller, P(s) is the actual time-delayed plant, M(s) is the model of the plant without time-delay and D(s)=e^{-sTdm} is the model time-delay. From Figure 1, it can easily be derived that the closed loop transfer function is expressed by:

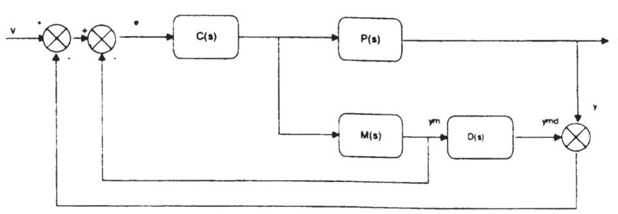

Figure 1, Smith Predictor Control System

$$\frac{y}{v} = \frac{C(s)P(s)}{1 + C(s)[M(s) + P(s) - M(s)D(s)]}$$

An alternative construction of the SPC is shown in Figure 2. By combining the two blocks of the controller in Figure 2 it is evident that the control system can be converted into the classical unity feedback control system where the relationship between C(s) and the standard feedback controller k(s) is given by:

$$k(s) = \frac{C(s)}{1 + C(s)M(s)[1 - D(s)]}$$

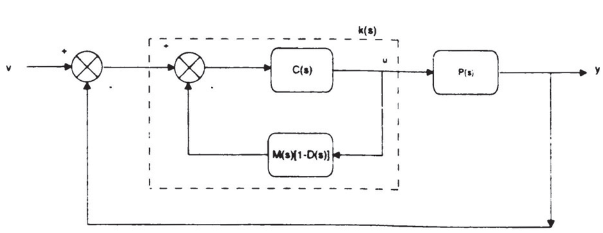

Figure 2, Alternative construction of Smith predictor Control System

Using this controller the loop transfer function L(s)=K(s)P(s) can be derived, and the Nyquist plot for the SPC can be plotted, a typical example is given in Figure 3. The Nyquist plot of the open loop transfer function allows one to assess the influence of modelling errors and to derive appropriate specifications for the controller design in order to assure robust stability of the closed loop system. The following indicators serve for characterising the distance between the Nyquist plot and the -1 point:

- Gain margin (GM)
- Phase margin (PM)
- Delay margin (DM)

SPC can suffer from stability problems when the dc gain, effective time-constant or effective time-delay in the process changes. Such stability problems can be inferred from the Nyquist plot. However, when using Nyquist plots to infer stability margins, care must be exercised when inspecting the plot. This is because for predictive controllers the Nyquist plot can intersect the unit circle at several frequencies and in general the first intersection; which normally defines the phase margin; no longer represents the minimum phase change to induce instability into the closed loop system. Rather, in this case the delay margin has to be computed. The delay margin (DM) is the minimum time delay which when added to the closed loop system causes the system to go unstable and is computed by considering all of the intersections of the Nyquist plot with the unit disc and choosing the minimum delay to cause instability. The concept of delay margin was introduced by Anderson and Moore(1969) and also adopted by Landau, Lozano and M'Saad (1999). Formally, the delay margin is defined as:

For a certain frequency the phase lag introduced by a pure time delay T is:

$$\Phi(w) = w\,T$$

It is thus possible to use the above expression to convert the phase margin into a delay margin

$$\Delta T = \frac{\Phi(w)}{w_{cr}}$$

where w_{cr} is the crossover frequency. If the Nyquist plot intersects the unit disc at several frequencies w_{icr} characterised by the corresponding phase margin $\Phi_i(w)$, then the delay margin is defined as:

$$\Delta T = \min\left(\frac{\Phi i(w)}{w_{icr}}\right)$$

Moreover, in order to ensure the validity of the results, low order Pade' approximations should be avoided as the Pade' approximations introduces additional uncertainty and appears to be very sensitive to the time-delay systems. In obtaining the Nyquist plots no approximations have been used. However, many designs for SPC rely on low order Pade approximations to the pure time delays present in the process. The Pade approximations are good at low frequency, however, at higher frequencies only high order Pade approximations are sufficiently accurate. Moreover, the Pade approximations filter out the multiple intersections to the unit disc and provide incorrect delay margins. An example of this is shown below, where the exact Nyquist plot together with 1st order Pade, and 4th order Pade approximations are used. It is interesting to note that the actual SPC system shown in Figure 3 is marginally stable. However, the 1st order Pade approximation shown in Figure 4 indicates the system is very stable. Only the 4th order Pade approximation shown in Figure 5 start to hint at a stability problem. However, the approximation still indicates the system is stable and it would require a 6th order Pade approximation to give enough accuracy. In the two cases the phase margin gives no hint of any stability problems. As much work on SPC has been done using Pade approximations (Morari et al). It is interesting to further investigate the robustness of SPC by not using Pade approximations and deploying the concept of delay margin as to assess robustness.

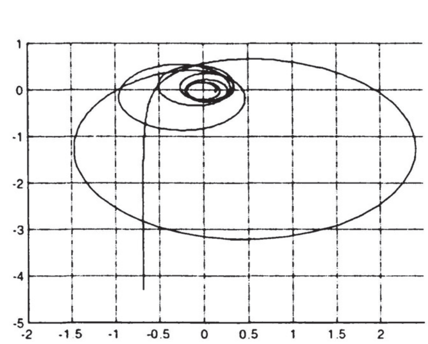

Fig 3, marginally stable Nyquist plot SPC

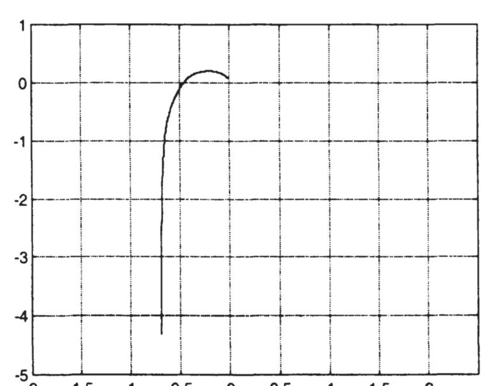

Figure 4, Ist order Pade' approximation.

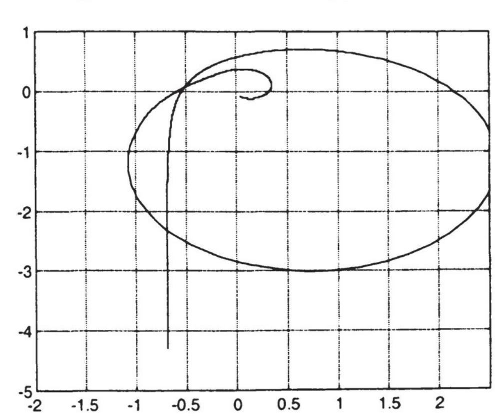

Figure 5, 4th order Pade' approximation.

3.Single parameter Smith predictor tuning curves

Many process control applications involve plants with a monotone output step response which exhibit no overshoot. Historically, the first order plus dead time model has been used to represent such plants, and the approach has received widespread acceptance in industry. It is thus of interest to develop SPC which adopt this form of model. Thus the model of the SPC can be expressed in the following form:

$$M(s) = \frac{K e^{-T_d s}}{1+T_p s}$$

where K is the dc gain, T_d is the pure time delay, and T_p is the time constant. Moreover, in such cases the controller takes the form:

$$C(s) = kp\,(1 + 1 / Ti\, s)$$

where kp and Ti are the controller tuning parameter. The design of the PI Smith predictor controller can be simplified by choosing the integral gain equal to the process time-constant. If it is assumed that in the case of the nominal design the model adequately represents the process. Then, clearly, the controller performance and robustness properties are all dependant on the single parameter kp and it is possible to calculate both the GM & DM given Td, Tp and Kp. Furthermore, by considering the normalised quantity T_d/T_p it is possible to obtain a set of graphs of T_d/T_p against gain margin and T_d/T_p against delay margin/T_d (i.e. as percentage of current delay T_d) as the parameter kp is varied as shown in Figures 6 and Figure 7. These graphs facilitate the selection of the tuning parameter kp once the ratio of T_d/T_p is known, together with the desired GM and the desired DM. The use of the graphs is as follows, once the ratio of T_d/T_p is known and the desired GM & DM are known, both figures 6 and 7 can be used to compute values for kp, the maximum value of kp is then chosen as this guarantees both robustness constraints are met. In general, when a controller is simplified, some loss of performance in the controller will occur. In order to quantify this loose in performance when Ti = Tp, the optimal time-domain performance of the SPC when subject to a set of GM & DM constraints can be evaluated for a range of the ratio of time-delay to time-constant (Td/Tp). The evaluation of this optimal performance under such constraints does not have a closed form analytical solution. However, such an optimisation task can be readily achieved by using a genetic algorithm.

4.Genetic optimisation of Smith predictor controller under pre-prescribed constraints.

Genetic Algorithms (GAs) were first introduced by John Holland (1975), as a general purpose optimisation tool, and were firstly proposed for control system tuning by Porter & Jones (1992).

The basic feature of this GA is it's ability to minimise any cost function under a set of pre-prescribed constraints. In this case it is desirable to minimise the ISE and at the same time ensure a certain GM & DM constraints are met. The aim of the optimisation is to minimise the ISE. However, this process is subject to both GM and DM constraints. The effect of including GM & DM constraints is to reduce the controller gains. As the GM and DM constraints are independent, one of the constraints is always met before the other, and for this reason, they can be assessed independently. Thus in the case of GM constraints, three constraint values were considered, namely Gm>2, GM>3, GM>4, for a range of the plant parameter ratio (Td/Tp). The ISE values are plotted in Figure 8,9,10. In addition the single parameter Smith predictor controller ISE values are also plotted on the same graphs for the same GM constraints. In the case of DM constraints, three values were also considered, namely DM=100%, 200% and 400% again for a range of plant parameters ratio (Td/Tp). The ISE values are plotted in figure 11, 12, 13. In addition the single parameter Smith predictor controller ISE values are also plotted on the same graphs for the same DM constraints.

It is important to note that if both constraints are applied simultaneously, the resulting ISE can be inferred from the worst case ISE of the constraints considered independently. Thus it can be concluded that for any simultaneous constraints up to GM > 4 and DM > 400%, the loss in performance of the SP SPC compared to the two parameter PI-SPC is less than 4%. These results confirm the hypothesis that setting Ti = Tp in the PI-SPC results in near optimal performance. Finally, in order to assess the quality of the SP-SPC, a PI controller and a SP-SPC were optimally tuned so as to minimise their respective ISE subject to the robustness constraints of GM>2 and DM>100%. This was done over the same range of T_d/T_p , these results were also obtained using the genetic algorithm These results are plotted in figure 7.a, and demonstrate that the SP-SPC is able out perform the standard PI controller by approximately 15% over the whole range of T_d/T_p. Moreover, it can be argued that the SP-SPC controller is easier to tune than the PI controller. This is because the tuning of the SP-SPC is carried out in the form of an identification process. In this case the identification process involves the three plant parameters K, T_d and T_p. These three parameters can be identified from open loop step response data, or computed on line by running the model in parallel to the process, and adjusting the parameters until a good fit between the two sets of data takes place. The selection of the tuning parameter kp is done using the two graphs in Figures 6 and 7, once the desired GM & DM have been chosen. This process is easier than tuning a PI controller and has the benefit of assuring robustness constraints.

5. Conclusion

Robustness of the controller has shown to be better defined by gain margin and delay margin rather than the conventional phase margin as the latter can lead to incorrect results. Moreover, the use of Pade' approximations in the design process is also shown to give rise to misleading results. The standard process control model was then adopted for the design of the SP-SPC and a set of controller tuning curves were produced which facilitate the design of SP-SPC with pre-prescribed gain margins and delay margins. In this paper the robust design of SP-SPC has been investigated for GM & DM constraints. The results show that for simultaneous constraints of GM>4 & DM>400%, the performance obtained by a PI-SPC can be emulated by a SP-SPC with only small performance losses. Finally, the results of the SP-SPC were compared to optimally tuned PI controllers when subject to the same robustness constraints.

References

Cohen,G.H., and Coon, G.A., 1953, Trans. Am. Soc. Mech. Engrs, Julay, 827.

Smith,O.J.M, "Closer Control of Loops with Dead Time." Chem. Eng. Progress,53(5),217-219(1957).

Donoghue, J, F., 1977, I.E.E.E. Trans. indust. Electron. Control Instrum.,21, 109.

Palmor, Z. J., and Shinnar, R., 1981, Design of advanced process controllers. AICHE Journal, 27, 793_805.

Morari,M and Zafiriou,E.,1989,Robust Process Controller design for linear systems having multiple time delays. AICE Journal,25(6),1043-1057.

Landau. I D, Robust digital control of systems with time delay. Int. J Control, 1995, vol.62, 325_347.

Landau, ID,Lozano R and M'Saad M "Adaptive control". Springer-Verlag(1999).

Laughlin, D. L., Rivera, D. E., and Morari, M. 1987, Smith predictor design for robust performance. International Journal of Control, 46, 477_504.

Wang,Zi-Qin and Sigurd Skogestad. Int. J. Control, 1993, vol. 57, No. 6, 1405_1420.

Anderson,B.DO., and Moore,J.,1969,Linear Optimal Control(Englewood liffs,NJ:Prentice_Hall)

Holland J H (1975), "Adaptation in Natural and Artificial Systems",1st MIT Press edition, ISBN.

Porter B and Jones A H, (1992), "Genetic Tuning of PID Controllers", Electronic Letters, Vol.28(9)pp. 843-844.

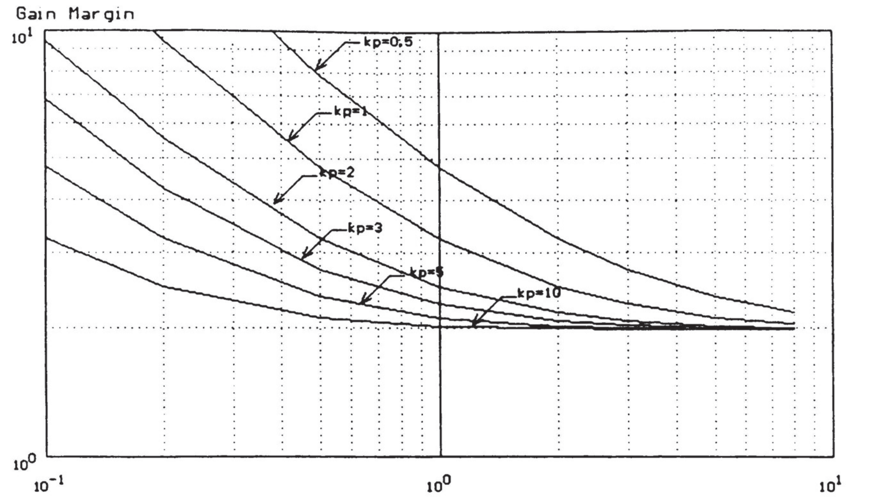

Fig 6. Robustness curves for selecting kp given desired gain margin and Td/Tp

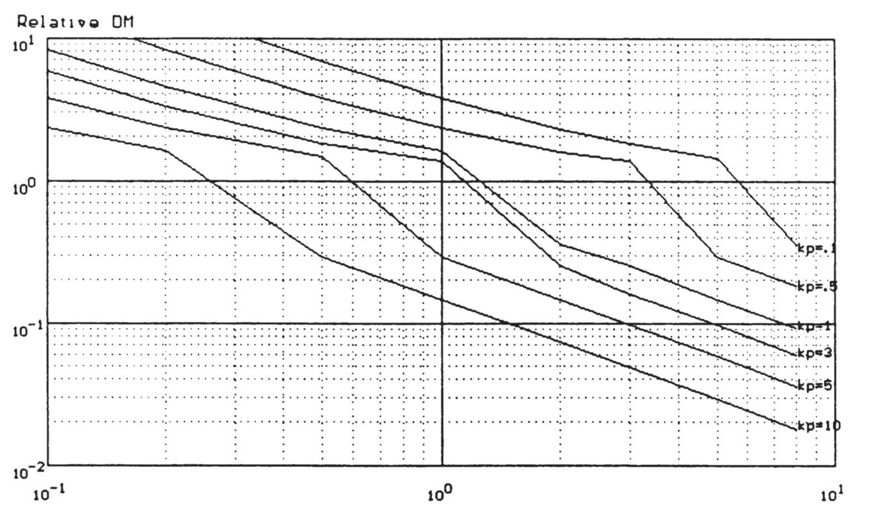

Fig 7. Robustness curves for selecting kp given desired delay margin and Td/Tp

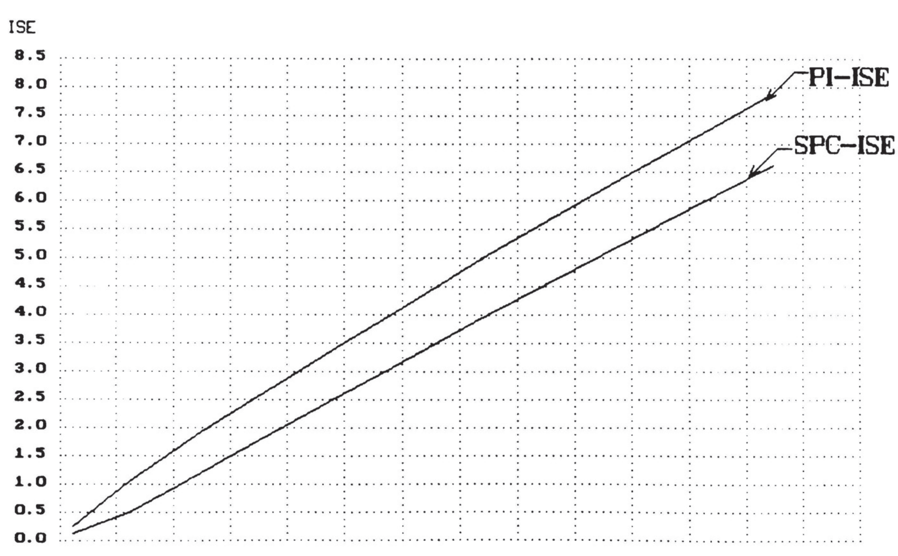

Fig 7a. Performance of SP-SPC & PI controller as the ratio Td/Tp is varied