OPTIMAL PATH IN HYBRID DYNAMIC SYSTEMS

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Abstract: This work treats the analysis and control of hybrid systems using hybrid controllers. The goal is to find the optimal solution according to a performance measure and to verify the solution in order to assure correct run or to avoid some misbehaviour. Abstraction is applied over the events generated by continuous signals in order to modify the set of controllable and observable events, and apply Ramadge and Wonham theory of Discrete Event Systems. Optimisation procedure applied over reachability ways obtains the optimal solution and gives the continuous signals to be applied to the system in order to obtain the best performance measure. *Copyright© 2000 IFAC*

Keywords : Hybrid, Reachability, Optimization.

1. INTRODUCTION

Hybrid systems are dynamic systems with continuous signals and discrete events, which can be interpreted as states with continuous dynamic behaviour approximated by ordinary differential equations. This work treats a particular class of hybrid dynamic systems, i.e. systems represented by linear stable continuous dynamics in each state.

In the hybrid systems control, the goal is to find an admissible and controllable path between the initial point in the starting state and the goals end conditions. A controllable system, is a system, which enable to drive through the optimal path towards the goal, although possible uncontrollable events may be encountered in the path.

We present a methodology to solve the problem to find the optimal path with respect to a performance measure, which uses different software tools in order to obtain: a continuous abstraction, a controllable path and the optimal control for a class of hybrid dynamic systems.

Ramadge & Wonham's theory (Wonham 87) along with the TCT software gives the Supremal Controllable Sublanguage of the legal language generated for a Discrete Event System (DES). Continuous abstraction transforms predicates over continuous variables into controllable events, and modifies the set of uncontrollable and unobservable discrete events.

The verification of the hybrid system assures reachability for the solution, together with safety and liveness for the system. This is useful to synthesise safety controllers (Puri and Varaiya 95).

Verification of the Supremal solutions determines the right paths that control the system, reaching all the predicates in the transition states and the goal's end conditions.

An optimisation method determines the optimal way and the continuous control to minimise the applied performance measure.

2. REACHABLE SETS

Let us consider the following SISO linear system (1)

$$\begin{aligned} \dot{X} &= AX + BU \\ Y &= CX \end{aligned} \qquad (1)$$

With $X \in R^2$ and A stable, and U whitin limit bounds $|U| < U_{max}$

For a stable system, the state evolves to the stationary solution when stabilisation time has been reached. The union of the maximum limits on the different trajectories gives the reach set of the system for U(t) in an interval range.

The reach set of the system with C=[0 k] is initial condition independent:

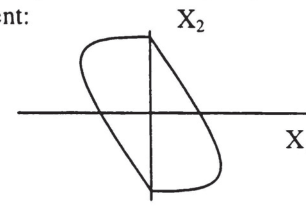

Figure 1 : Reachability set of a state

X_1 and X_2 limits are the maximum response to the Bang-Bang control. Inside of the region are all the reachable states with finite time for the system (1).

Reachability analysis of the system must consider two possible situations, either reaching the bound limits in the transition conditions or reaching the goal.

In the first case, the intermediate state reachability analysis determines if the system crosses the bound limits of the linear predicate (2), which leads the system to the next state. Linear predicates are predicates composed by linear inequalities connected by a first order logic operator (✳), and with constants $k_i \in \mathbf{R}$.

$$k_1 X_0 \geq k_1 * k_2 X_2 \geq k_3 \qquad (2)$$

$$\begin{aligned} X_i &= (k_4, k_5) \\ X_f &= (k_6, k_7) \end{aligned} \qquad (3)$$

Theorem 1: If part of the reachable set (figure 1) is superposed with this region (2), then there exists a finite signal control U ∈ [U_{min} , U_{max}] which activates the condition transition (2).

In the case when the constraint is in the End State, reachability is to be defined as the ability to reach the goal X_f (3) on the hybrid system.

Theorem 2: If the end condition is inside the reachable set (figure1), then there exist a finite signal control U ∈ [U_{min} , U_{max}] which translates the system (1) from some initial condition in the end state to the goal X_f (3) or End point.

Using the combination of this two properties this verification procedure assures the reachability of the End point X_f(3) for a hybrid system, starting at another state with initial condition X_i(3). We call this a Reachable Way (RchW). Safety property is assured if there are no solution (RchW) to any non-permitted state.

3 CONTINUOUS ABSTRACTION

To apply the RW theory (Wonham 87) of discrete event systems on a hybrid system, we must abstract the continuous signals and project the events generated for these signals, to obtain observable and controllable sets of events.

Continuous signals produce an event in the system when it crosses the limit (2). If this event is deterministic it can be abstracted. The problem is to determine the controllability of this event, in order to assign it to the corresponding set, $\{\Sigma_c, \Sigma_{uc}, \Sigma_o, \Sigma_{uo}\}$, composed of controllable, uncontrollable, observable and unobservable sets of events.

The set Σ_{uc} of uncontrollable events has to be projected to another set, by eliminating the uncontrollable events that are indirectly controllable, or continuous reachable, by the continuous control U (t).

$$\Sigma_{uc} = P(\Sigma'_{uc}) \qquad (4)$$

These events are those for which the region generated by the linear predicate (2) crosses the reachable set.

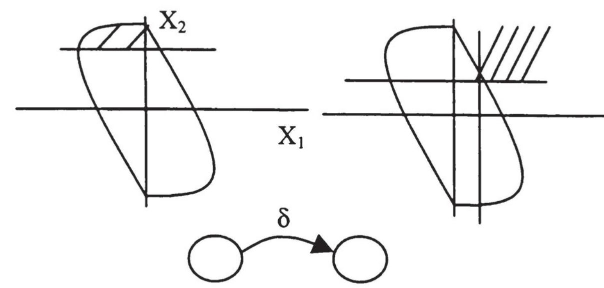

Figure 2: predicates δ : $X_2 > K_1$ and $X_1 > K_1 \wedge X_2 > K_2$

States with more tan one uncontrollable transition can change the behavior of the graph, when the events can be converted into controllable events, this transitions become deterministic.

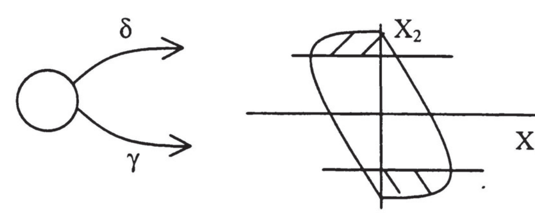

Figure 3: predicates $X_2 > K_1$ and $X_2 < K_2$

Σ_{uo} is the projection over the unobservable events, generated by the discrete or continuous signals, eliminating the set of observable events by the continuous signals analysis (Lemmon and Antsaklis 94).

$$\Sigma_{uo} = P(\Sigma'_{uo}) \qquad (5)$$

Unobservable events are detected when the state is out of the reach set of the model, indicating the use of the other reach set, this means that other model and other state are to be considered. They are not observable in continuous sense because it is not possible to determine the state change instant by identification methods. Otherwise it is observable in discrete sense if the state crosses the reach bounds.

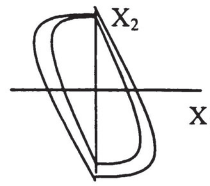

Figure 4: Observability of the state

The transition is true unobservable if the event is not presented and is unidentifiable by continuous identification, and doesn't go out of the reach set of the current model.
At the same time the uncontrollable events that have been eliminated are added to the set of observable events. A similar procedure will be applied to the unobservable events.

$$\Sigma_c \cap \Sigma_{uc} = 0 \ , \ \Sigma_o \cap \Sigma_{uo} = 0 \qquad (6)$$

These sets are disjoint (6)

The RW theory (Wonham 87) defines a Controllable Sublanguage, with respect to a language that is composed of erased unobservable events:

$$\overline{K}\Sigma_u \cap L \subseteq \overline{K} \qquad (7)$$

This means that uncontrollable events present in some prefix of the Controllable Sublanguage $\overline{K}$, permitted for the language L of the system, is part of the Controllable Sublanguage. And the Supremal Controllable Sublanguage is the union of the different Controllable Sublanguages.

4 FAULTS

A fault is said to be detectable in discrete sense, if there exist a transition in the system model that leads to detection in a finite number of steps, and is detectable in continuous sense if the identification method determines a failure state.

Fault is isolable in discrete sense if there exist a transition, which gives different behavior or next states for different faults. And the fault is isolable in continuous sense if the identification method determines a unique failure state (Larson 99).

Unobservable events may be failure events or other events that cause changes in the state system not recorded by sensors.

The procedure for fault detection is the following:

1) The continuous signal doesn't progress towards the transition
2) Wait the for a discrete event to indicate the failure in the system.
3) Analysis of the continuous signals: model identification.
4) Representation of the state in the reachable sets.

These four steps allow to detect a wide range of faults. Non-identifiable faults in steps 1-3, can be detected if the representation of the state (1) is out of the reachable set of the current state.

5 OPTIMISATION

To find the optimal solution of RchW for the evolution of the system from X_i , to X_f (3) going trough different states and minimising the performance measure, optimisation methods (Kirk 70) are applied over the set of state sequences of RchW.

For the minimum time problem the performance measure is:

$$J = t_f - t_0 = \int_{t_0}^{t_f} dt \qquad (8)$$

Where t_f is the first instant of time when $x(t)$ intersects the target set in the end state. Bellman's principle of optimality is applicable in problems that don't present interaction between lateral states.

$$V^*(x) = \min_{p \in \Pi} \left\{ g(x,p) + V^*(x'(x,p)) \right\} \qquad (9)$$

Where Π is the set of states and g(x,p) is the cost of local state, and V^* (x') the optimal cost of the rest.

In the transitions defined by Y limits, the $\dot{Y}$ in the limit and the signal control U(t) are calculated by dynamic programming (DP) in order to minimise the global time cost. The problem is composed of three parts, the first is the cost of the first model to approach the system to the limit, the second part is the first model to cross the limit, and the third part is the second model to go through the new limit. The signal applied U(t) in each part is the Bang-Bang (Kirk 70) control in order to optimise the performance measure. In this sense the global optimisation is obtained as the local optimisation of the collateral states.

The interaction between states prohibits the use of local state optimisation. The solution is obtained recursively taken into account the collateral pairs of states until they converges (Esteva 98).

6 EXAMPLE:

A simple illustrative example show the possibilities of this method. Results are not very interesting because of the strong simplification of the problem.

Let us consider a car with the following speed model, in Km/h:

$$A=\begin{bmatrix} -.7-G^{\wedge 3} & -1 \\ 1 & 0 \end{bmatrix} \qquad B=\begin{bmatrix}1 & 0\end{bmatrix}$$
$$C=\begin{bmatrix}0 & 35*G\end{bmatrix}$$

With U restricted to [0,1] and G the gear number.

Determine the RchW and the optimal control, to drive the vehicle as fast as possible from stop (initial state) to end conditions which are defined by a speed of 90Km/h and acceleration of .15m/s , which is the maximum speed to safety take the curve.

1) The first step is to define the graph evolution:

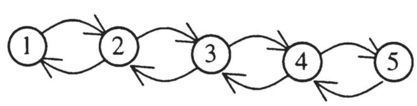

Figure 5.1 : Evolution state graph.

2) Second step is to determine the transition conditions of reachability.

The state transitions are triggered in order to maximize the acceleration, they are active when the next state presents a higher acceleration.

1 to 2 condition transition: when speed ≥28km/h and acceleration ≥ 1m/s
2 to 3 condition transition: when speed ≥57km/h and acceleration ≥ .4m/s
3 to 4 condition transition: when speed ≥82km/h and acceleration ≥ .24 m/s
4 to 5 condition transition: when speed ≥115km/h and acceleration ≥ .1 m/s

The state transitions reachability conditions are the following:

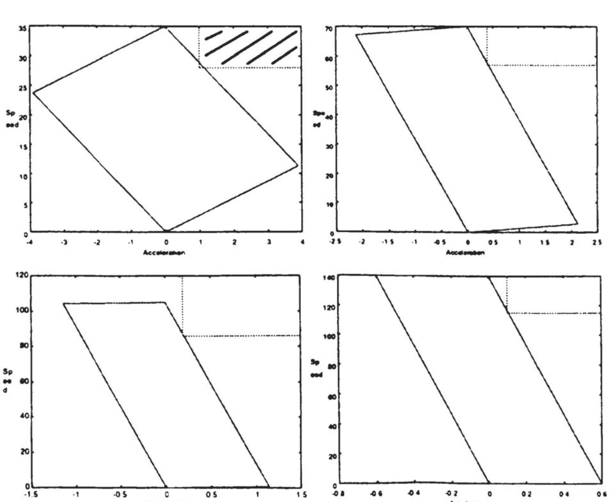

Figure 5.1: Transition conditions reachability for 1[st], 2[nd] , 3[rd] and 4[th] gear

The graph shows that transitions, 1 to 2, 2 to 3, and 3 to 4, are reachable.

The reachabilty set in finite time is given into these limits:

1: 3.9 m/s at 11.25 km/h, -3.9 m/s at 23.5 km/h, and maximum 35 km/s.
2: 2.12 m/s at 2.8 km/h, -2.12 m/s at 67.2 km/h, and maximum 70 km/h .
3: 1.14 m/s at .66 km/h and -1.14 m/s at 104 km/h and maximum 105 km/h.
4: .6 m/s at .18 km/h and -.6 m/s at 139 km/h and maximum 140 km/h.
5: .38 m/s at .12 km/h and -.38 m/s at 174 km/h and maximum 175 km/h.

The end condition reachability , speed : 90 km/h and acceleration: .15m/s , as seen in the figure:

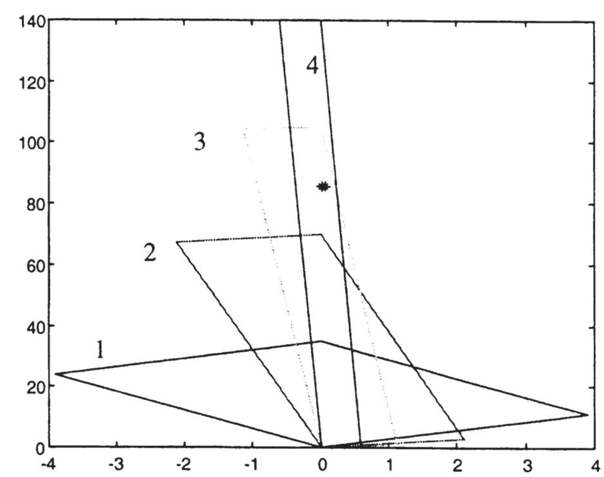

Figure 5.2 : End conditions reachability.

In the case of multiple solutions, the end point reachability is obtained by evaluating the different solutions in order to determine the optimum. When the models have similar rising times, the optimal state will be the one having its reach set limits most distant to the end point.

3) Apply the abstraction over continuous signals, to obtain the set of observable and controllable events.

The events generated by the continuous signals are reachable, this implies to belong to at the set of controllable events:

$\sum'_c$ = {transition: 1-2, 2-3, 3-4, 4-5}

4) Step to obtain controllable languages.

TCT (Thistle 94) yields the Supremal Controllable Sublanguages to drive the system from state 1 to state X (2,3,4,or 5):
Once the reachability criteria of the states are met, the path trough those states will determine the RchW.

Two possible RchW : 1 -> 2 -> 3 and 1 -> 2 -> 3 -> 4.
The 5th gear doesn't appear in the solution because the transition 4->5 is not reachable before the End point.
The two possibilities must be analysed to determine the fastest possibility. In this case, without delays in the transitions of the states, the fastest one is 1 -> 2 -> 3 -> 4, because the 4th gear is faster that the 3th one when its use is possible.

5) Optimization method calculates the control signal U(t), which concludes that the minimum time is 40 seg.

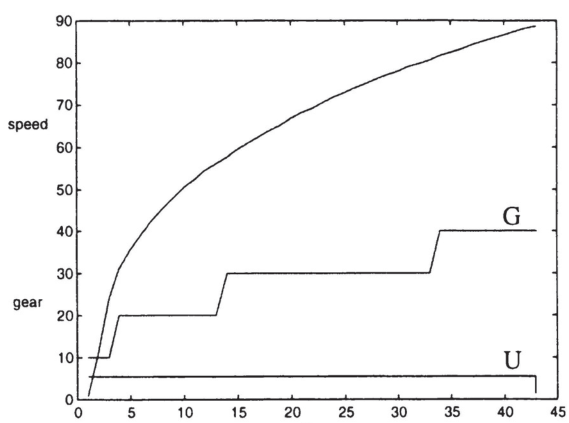

Figure 5.3: Continuous signal U(t), and discrete G

6) Fault detection.
In cases when the fault cannot be detected by inspection of the continuous signals, for example when the wind perturbation doesn't allow identification of the exact model. Faults can be detected by inspection of the reachable sets for the automatic gear-box.

When the state is out of the current reachable set. This method determines that the actual state (gear) is bigger than the expected state when the state crosses the limits of the reach set for the side of the velocity, and otherwise in the side of acceleration, the actual state is a lower gear.

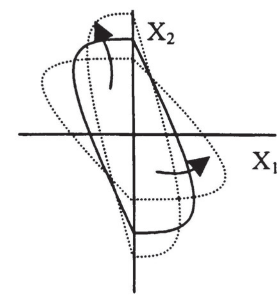

Figure 5.4: Observable faults

7 CONCLUSION

This paper shows a methodology to work with hybrid systems in order to control and minimise performance measures. Integrators can be used in this methodology for robot path planning, so dynamic scenarios may assure safe paths, that is, without collisions.

To automate this procedure is difficult because a particular analysis must be carried out for each problem, and besides, different optimisation methods can be applied. Now work is being done in this sense to generate high level code, which allows the modelisation (Esteva 99) and verification of the hybrid systems.

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