MODELING, IDENTIFICATION AND ROBUST CONTROL OF THE UNWINDING-WINDING OF ELASTIC WEB

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Abstract: A web transport system including an unwinder, a load cell and a winder has been modeled from the laws of mechanics. The model parameters are identified using optimization methods. Some peculiar properties of web transport systems such as web tension transmissibility and roller non-circularity effects on web tension are analyzed. The type of control strategy classically used in industrial systems is decentralized with PID controllers. In this paper we present a multivariable H∞ robust control that improves significantly the performances and reduces the coupling between web tension and velocity. *Copyright ©2000 IFAC*

Keywords: manufacturing processes, winding, robust control.

1. INTRODUCTION

The systems transporting paper, metal, polymers or fabric are quite common in the industry. The control of such systems is essentially based on the know-how of the operators. The main studies concerning the control in web handling domain come from Oklahoma University (Reid and Lin, 1993) and Munich University, which suggest web tension observers (Wolfermann, 1997) associated to controls based on fuzzy control and neural networks (Wolfermann, 1995). The multivariable control strategies have recently been proposed in industrial metal transport systems (Geddes and Postlethwaite, 1998) (Grimble, 1999). The industrial central concern is to increase as much as possible the web transport velocity while controlling the tension of the web. Due to the important coupling between web velocity and web tension the scientific community is interested in the practical applications of theoretical works, for instance, on control (Fayaz and Vergé, 1997), identification (Bastogne *et al.*, 1998) or fault detection (Noura *et al.*, 2000).

The studied system has two motors (cf. figure 1) and shows the inherent difficulties of winding systems. The main concern is to prevent the occurrence of web break and fold, and material damage. In case of web break or fold the production line stops, resulting in a waste of time and a lower productivity. In case of material damage, caused by a too high tension or material tension oscillations due to velocity variations, all the web is lost. The web tension should be controlled perfectly even during velocity variations. Consequently the control of the system should reduce the coupling between velocity and tension.

In this paper, the first part presents the modeling made by a synthesis of different existing laws. The second part shows the identification of the model by parameter optimization and the analysis of the measurements obtained on the experimental

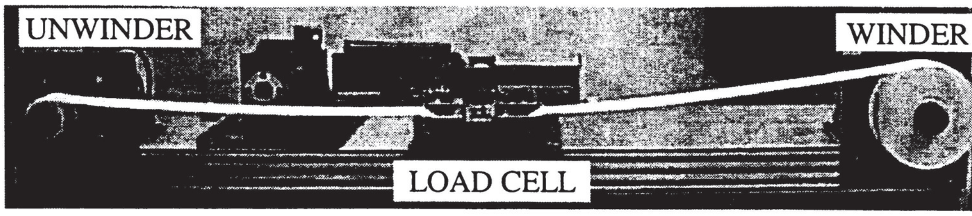

Fig. 1. Experimental setup

setup. The following part shows the synthesis of an H∞ robust controller and the comparison with PID controllers in a point of view of disturbance rejection and reference tracking.

2. MODELING

The model of a web transport system is built from the model of the web tension between two consecutive rolls and the velocity model of each roll.

2.1 *Web tension between two consecutive rolls*

The different modeling in web transport systems are based on three laws:

- Hooke's law: which models the elasticity of the web,
- Coulomb's law: which gives the web tension variation due to the friction and the contact between web and roll,
- Mass conservation law: which expresses the cross coupling between web velocity and web tension .

The web tension between two rolls can be computed based on these three laws.

2.1.1. *Hooke's law* The tension of an elastic web is function of the strain:

$$T = ES\epsilon = ES\frac{L - L_0}{L_0} \tag{1}$$

where ϵ is the web strain, E is the Young modulus, S is the web section, L is the web length under stress and L_0 is the web length without stress.

2.1.2. *Coulomb's law* The study of a web tension on a roll can be considered as a problem of friction between solids (Brandenburg, 1973), (Koç *et al.*, 1999*b*).

On the roll the web tension is constant on the sticking zone which is an arc of length a and varies on a sliding zone which is an arc of length g (cf. figure 2). The web tension between the first contact point of a roll and the first contact point of the following roll is given by:

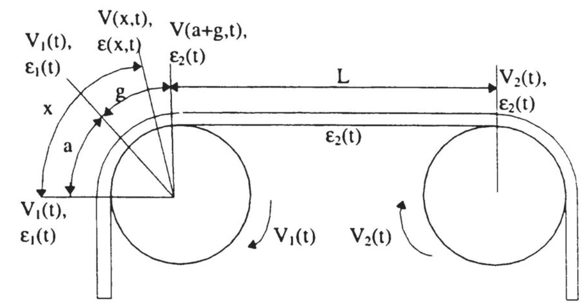

Fig. 2. Web tension on the roll

$$\begin{aligned} \epsilon(x,t) &= \epsilon_1(t) \qquad \text{if } x \le a & (2)\\ &= \epsilon_1(t)e^{\mu(x-a)} \qquad \text{if } a \le x \le a+g & (3)\\ &= \epsilon_2(t) \qquad \text{if } a+g \le x \le L_t & (4)\end{aligned}$$

where μ is the friction coefficient, and $L_t = a + g + L$.
The tension change occurs on the sliding zone. The web velocity is equal to the roll velocity on the sticking zone.

2.1.3. *Mass conservation law* Consider a web of length $l = l_0(1 + \epsilon)$ with a weight density ρ, under an unidirectional stress. The cross section is supposed to be constant. According to the mass conservation law, the mass m of the web remains constant between the state without stress and the state under stress:

$$m = \rho Sl = \rho_0 Sl_0 \Rightarrow \frac{\rho}{\rho_0} = \frac{1}{1+\epsilon} \tag{5}$$

2.1.4. *Tension-velocity relationship* The equation of continuity (Brandenburg, 1973) applied to the web gives:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho V)}{\partial x} = 0 \tag{6}$$

By integrating on the variable x from 0 to L_t (cf. figure 2), taking into account the equation (5), and using the fact that $a + g \ll L$ we obtain (Koç *et al.*, 1999*a*):

$$\frac{d}{dt}\left(\frac{L}{1+\epsilon_2}\right) = \frac{V_1}{1+\epsilon_1} - \frac{V_2}{1+\epsilon_2} \tag{7}$$

This relationship can be simplified by differentiating the left term and using the approximation $\epsilon \ll 1$ and $\frac{1}{1+\epsilon} \approx 1 - \epsilon$ and equation (1):

$$L\frac{dT_2}{dt} = E(V_2 - V_1) + T_1V_1 - T_2(2V_1 - V_2) \quad (8)$$

2.2 *Web velocity on each roll*

Assuming that the web does not completely slide on the roll, the web velocity is equal to the roll linear velocity. The velocity of the k^{th} roll can be obtained through a torque balance:

$$\frac{d}{dt}\left(\frac{J_kV_k}{R_k}\right) = R_k(T_{k+1} - T_k) + K_kU_k \quad (9)$$

where K_kU_k is the motor torque.

For the unwinder and the winder the roll radius R_k and the roll inertia J_k are time dependent and vary largely during the winding process. A solution to reduce the effects of radius and inertia variation is presented by a gain scheduling approach in (Koç *et al.*, 2000) on a system with 3 motors and 2 load cells.

3. IDENTIFICATION

The identification of the complete non linear parametric model is based on the model matching method (Walter and Pronzato, 1997) (see figure 3).

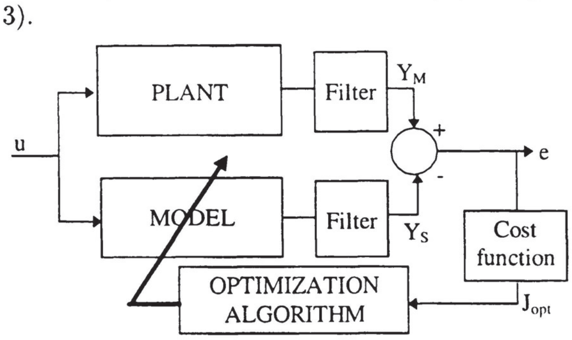

Fig. 3. Model matching method

The model is built and simulated on Matlab-Simulink. The cost function is :

$$J_{opt} = \frac{(T_s - T_m)^T(T_s - T_m)}{\| T_m \|^2} + \frac{(V_s - V_m)^T(V_s - V_m)}{\| V_m \|^2} \quad (10)$$

where T_s, V_s, T_m and V_m are the tensions and velocities simulated and measured respectively. Two optimization algorithms are used: the simplex method (Nelder & Mead) and the Quasi-Newton method, cf. (Walter and Pronzato, 1997). The simplex method gives the smallest cost function and is more robust to the initial values of the parameters. The simulations with the optimized parameters and the measurements are compared on the figure 4.

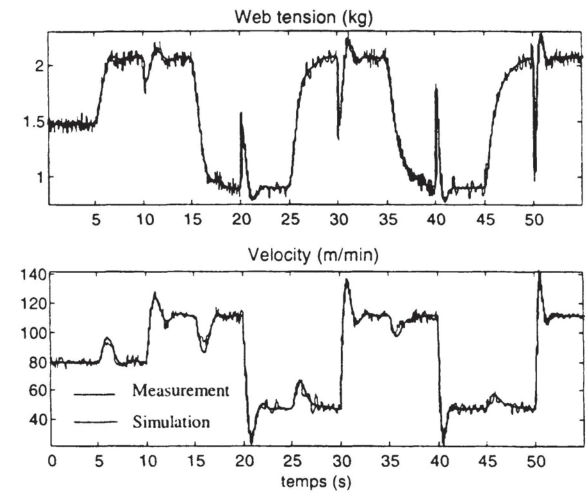

Fig. 4. Identification results

We observe a closed match between the simulations and the measurements. To validate our model we compare the simulation and the measurement obtained at a different operating point with simultaneous changes of tension and velocity (see figure 5).

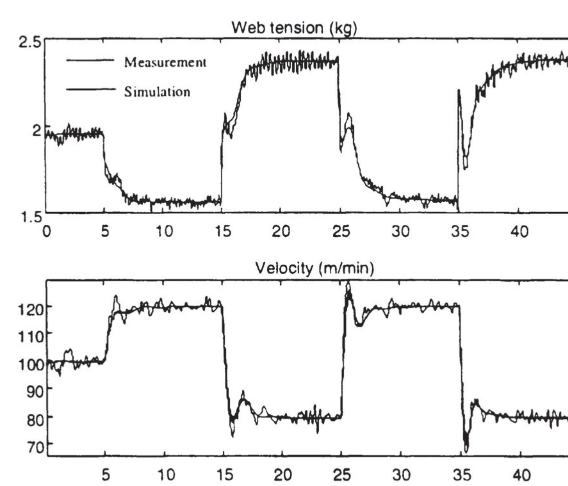

Fig. 5. Graphical validation

We remark that our model is still satisfying even though the process operating conditions are quite pathological.

3.1 *Web tension transmissibility*

The model is based on equation (7) which assumes the transmissibility of the tension in the same direction than the web transport. This equation is in fact valid when the velocity is different from zero. This is shown on the experimental setup by the following test: a perturbation is applied to web tension (with a small blow on the web) on both sides of the load cell, in the case of a web transport in both directions and also at standstill.

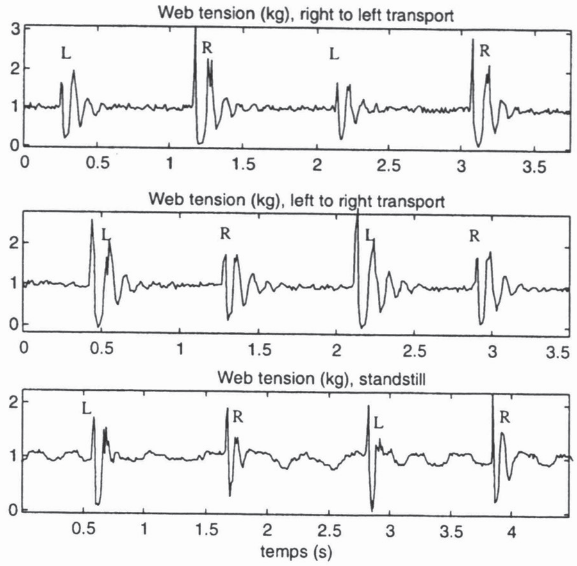

Fig. 6. Web tension transmissibility

We observe that with a perturbation on the left (L in figure 6) of the load cell the tension variation is larger when the web is conveyed from left to right than in the opposite direction. However, when the web is at standstill, the same amplitude of the tension variation is obtained with the perturbation on both sides.

3.2 *Web tension spectrum*

During the experimentation, velocities' oscillations can be observed. This can be explained by the spectrogram of the web tension measured during the winding process. The spectrogram is a 3 dimensional graphic which represents the amplitude of the spectrum, computed with sliding windows, in function of the frequency and the time. The darker is the color, the higher is the the amplitude of the spectrum.

The tension measured by the load cell is the average tension of the tensions in both sides of the load cell. We observe two fundamentals and two harmonics at frequencies that vary slowly during the winding process. These web tension oscillations are due to the perturbations introduced by the non-circularity of the rollers. A roller initially perfectly circular can become non-circular during its storage or transport. The linear velocity is usually maintained constant in web transport lines, and as the radius change the rotational speeds of the unwinder and the winder change during the process operation. The fundamentals seen on

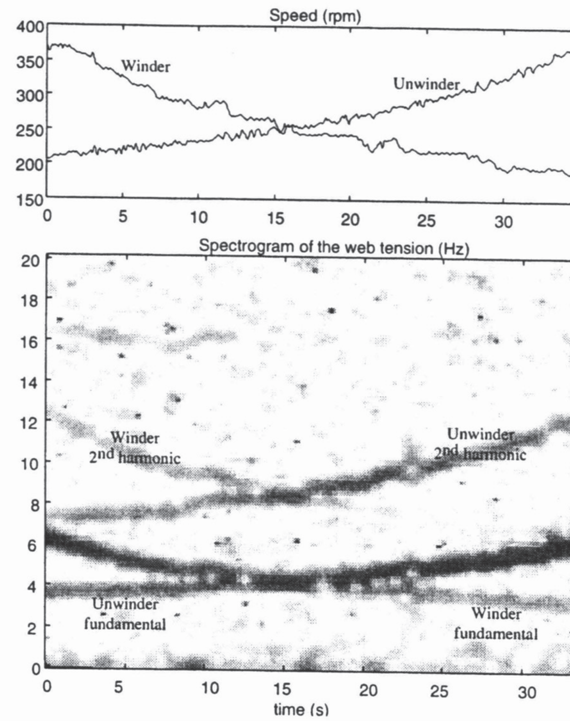

Fig. 7. Web tension spectrogram

figure 7 correspond to the frequency of rotation per second of the unwinding and winding motors.

The mechanical solution, commonly used in the industry, to reduce the web tension oscillations is to add a dancing roll near the unwinder and winder, cf. (Reid and Lin, 1993). A compensation technique using neural networks is also presented in (Wolfermann, 1999) when the rotation frequency is low (inferior to 1 Hz).

4. MULTIVARIABLE CONTROLLER

The figure 8 shows the decentralized control scheme. The web tension is controlled with the winding motor and the web velocity is controlled with the unwinding motor.

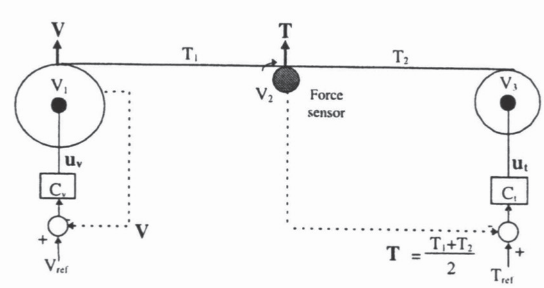

Fig. 8. Experimental setup

The load cell returns the mean of the web tension T_1 and T_2 and the linear velocity V is estimated from the knowledge of the rotational speed and the radius. The motors are synchronous motors and the control signals correspond to torque references computed with a sampling time of 10 ms.

The synthesis of the multivariable controller is done using a linear model obtained by linearization around the nominal point. Due to the elasticity of the web and the coupling existing between velocity and tension the control of winding systems are hardly efficient with classical control strategies. Some methods have been studied to suppress this coupling in a system with two driven rolls, cf. (Jeon *et al.*, 1999). The approach we have developed is a multivariable control with an H∞ robust controller. This approach is compared with PID controllers commonly used in the industry.

The robust H∞ controller is synthesized using the mixed sensitivity H∞ method (Skogestad and Postlethwaite, 1996) (cf. figure 9).

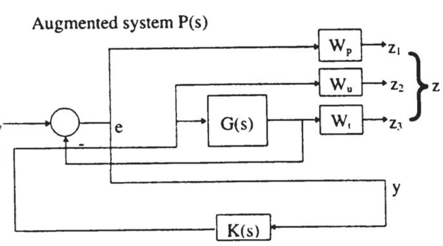

Fig. 9. Mixed sensitivity method

The weight functions W_p, W_u and W_t appear in the closed loop transfer function matrix:

$$T_{zw} := \begin{bmatrix} W_pS \\ W_uKS \\ W_tT \end{bmatrix} \qquad (11)$$

where S is the sensitivity function $S = (I+GK)^{-1}$ and T is the complementary sensitivity function $T = I - S$.

The controller K is calculated using the "γ-iteration" (Doyle - Glover algorithm) (Zhou *et al.*, 1996) (Chiang and Safonov, 1992):

$$\| \gamma T_{zw} \|_\infty := \sup_w \sigma_{max}(\gamma T_{zw}(jw)) < 1 \quad (12)$$

The weight function W_p has a high gain at low frequency to force the sensitivity function to be low at low frequency and therefore reject low frequency perturbations. The goal of the weight function W_u is to avoid large control signals. The weight function W_t has a high gain at high frequency to force the complementary sensitivity function to be low at high frequency and so to be less sensitive to noise cf. (Skogestad and Postlethwaite, 1996) and (Kwakernaak, 1993).

The selected weight functions are:

$$W_p(s) = \begin{bmatrix} \dfrac{0.2s+6}{s+0.02} & 0 \\ 0 & \dfrac{0.2s+6}{s+0.04} \end{bmatrix}$$

$$W_u = I_{3\times 3} \qquad W_t(s) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}$$

The optimum γ found is $\gamma_{opt} = 0.35$ and $\| S \|_\infty = 1dB$. The order of the resulting controller is 9. A comparison between the measurements obtained with PID controllers and an H∞ controller is presented on the figure 10.

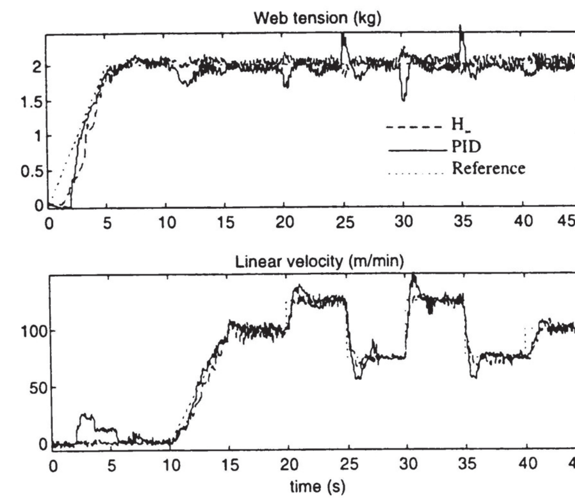

Fig. 10. Comparison of measurements obtained with PID and H∞ controllers

We can observe that the web tension is less sensitive to the velocity variations with the H∞ controller and the velocity is maintained null during the web traction phase.

We can also observe on figure 11 that the H∞ controller is more robust to the working conditions: the tension response is satisfying when a velocity change occurs simultaneously. On the other hand, with the PID controllers, a risk of web break exists.

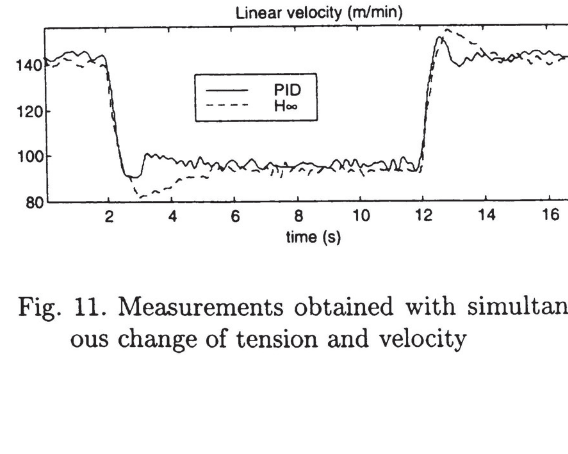

Fig. 11. Measurements obtained with simultaneous change of tension and velocity

5. CONCLUSIONS

The model of a system including an unwinder, a load cell, and a winder has been built from the general laws of physics. This model gives the web tension between the rolls and the velocity of the rolls.

The model is identified by parameter optimization. The transmissibility of the web tension and the non-circularity effects are validated from the measurements acquired on the experimental setup.

To increase production and obtain a high quality wound roll, the web tension must be maintained constant during the change of the velocity. Due to the elasticity of the web an important coupling exits between velocity and tension. The multivariable H∞ control reduces this coupling: the tension variations amplitude is significantly decreased during the velocity changes and the risk of web break is diminished.

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