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STRUCTURAL PROPERTIES OF INVERSE MODELS REPRESENTED BY BOND GRAPH

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Abstract: Inverse models are widely used for control or design purposes. In both cases, the maximum order of differentiation of each output appearing in the model inversion constitutes relevant information since it characterizes the regularity of the trajectory to be followed. Although this property can be determined from algebraic manipulation of the model, this paper shows how it can be obtained directly from the bond graph model using graphical procedures. *Copyright © 2008 IFAC*

Keywords: structural properties, essential order, inverse model, decoupling, Bond Graph

1. INTRODUCTION

Since the introduction of the bond graph principles, the interest of bond graph was shown as a basis of complex and multidisciplinary system modelling. Researches were also carried out to develop system analysis using the graphic support of bond graph and the concept of causality. Several authors gave an interesting light on the equation organisation in models (Van Dixhorm and Evans, 1974) and introduced several procedures based on causal assignment to determine system dynamic properties and to help at the control law synthesis. The majority of these procedures are based on the concepts of power lines and causal paths. A power line between two components is a series of power bonds and junction structure elements connecting these two components (Wu and Youcef-Toumi, 1995). An input-output (I/O) causal path (u_j, y_i) is a path from a source u_j to a detector y_i (Sueur and Dauphin-Tanguy, 1989; Sueur and Dauphin-Tanguy, 1991). While causal paths are defined for causal bond graphs, power lines are an acausal concept. The length of a causal path between an output y_i and an input u_j is equal to the number of dynamical elements in integral causality met in this path in the bond graph in preferential integral causality (BGI) (Rahmani, et al., 1992). The concept of "disjoint" I/O causal paths was defined by Ngwompo, et al. (1997) and it was used to solve inversion problems. In fact, two input-output causal paths are said to be disjoint if they have no element in common. The concept of "independent" I/O power lines was defined by Ngwompo, et al. (2005) to improve inversion procedures. In fact, two I/O power lines are independent if they do not share a common variable.

Recently, the bicausality concept introduced formally by Gawthrop (Gawthrop, 1995 and 1997), has initiated a new philosophy in regard to a bond graph model. This concept established new rules for causal assignment, and made a new range of problem accessible. This principle has been successfully applied in design or sizing problems by Ngwompo (Ngwompo, et al., 1999 and 2001) and in control synthesis. Causality has to be regarded as a graphical principle to check the physical validity and the mathematical complexity of a given problem. The concept of bicausality permits also to search what kind of information is required to solve a problem and which mathematical properties are needed to keep a physical meaning for the solution (Bideaux, et al., 2006b). A major contribution of the bicausality concept is that it settles reliable means to study inverse problems (Gawthrop, 2000; Ngwompo, et al., 2005; Bideaux, et al., 2006a). This last control problem is addressed in this paper for square linear time-invariant systems. A brief description of algebraic results on this topic is done. Starting from the concept of infinite zero order, a bond graph procedure is established and illustrated by two examples.

2. INVERSIBILITY AND FEEDBACK DECOUPLING

2.1. Infinite structure analysis

We consider the linear time-invariant system described by:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{1}$$

with $x \in R^n$, $u \in R^p$, and $y \in R^p$. We assume that the system (1) is full column rank, with a transfer matrix $T(s)$ strictly proper and defined by:

$$T(s) = C(sI - A)^{-1}B \,. \tag{2}$$

The rank of $T(s)$ is equal to p and the Smith-McMillan form at infinity of $T(s)$ is given as follows:

$$T(s) = B_1(s)\Lambda(s)B_2(s)\,, \tag{3}$$

where $B_1(s)$ and $B_2(s)$ are biproper and Λ =diag($s^{-n_1},\ldots,s^{-n_p}$). The transfer matrix $T(s)$ has only infinite zeros whose orders are $n_1,\ldots,n_p$ and therefore the transfer matrix of the inverse system $T^{-1}(s)$ has only infinite poles.

Let n_i' denotes the order of the infinite zero of (A,B,c_i) where c_i is the i^{th} row of C. This order is generally named relative degree of the i^{th} output and defined by:

$$n_i' = \inf\{k \in N \,/\, c_i A^{k-1} B \neq 0,\ k=1,\ldots, n\}. \tag{4}$$

Commault, et al (1986) defined the essential orders n_{ie} which were used thereafter for solving row by row decoupling problems. Their work was based on the concept of essential row which was introduced by Cremer (1971). A row w_i of a given matrix W is said to be essential if it is not linearly dependent of other rows of W. The essential orders are determined from the Toeplitz matrices Γ_μ, defined by:

$$\Gamma_\mu = \begin{bmatrix} CB & 0 & 0 & \cdots & 0 \\ CAB & CB & 0 & & 0 \\ \vdots & & & \ddots & \vdots \\ \vdots & & & & 0 \\ CA^{\mu-1}B & \cdots & \cdots & CAB & CB \end{bmatrix} \quad \mu \geq 1$$

Commault, et al. (1986) illustrated that for a right-invertible system (A, B, C) and having $n_{\sup}$ its supremal order of infinite zeros, the essential order of the i^{th} output is defined by:

$$n_{ie} = \inf\{k \in N \,/\, [c_i A^{k-1} B | \ldots | c_i B | 0 \ldots 0] \text{ is essential in } \Gamma_{n_{\sup}}\} \tag{5}$$

Thus, we can deduce that:
(i) $n_{ie} \geq n_i' \quad \forall$ i=1,…,p
(ii) $n_{ie} \leq n_{\sup} \quad \forall$ i=1,…,p

For any right-invertible system (A, B, C), Commault, et al. (1986) define the essential orders as follows:

$$n_{ie} = \sum_{j=1}^{p} n_j - \sum_{k=1}^{p-1} n_k' \tag{6}$$

$$\forall \, i \in \{1,\ldots p\} \qquad k \in \{1,\ldots p\}/\{i\}$$

Sain and Massey (1969) defined the condition of invertibility for square system ($p \times p$) by using the Toeplitz matrices. This square system is invertible if and only if:

$$rank(\Gamma_n) - rank(\Gamma_{n-1}) = p. \tag{7}$$

The inversibility is a necessary condition for the decoupling. The square system ($p \times p$) defined by the triplet (A, B, C) is decouplable by a static feedback of the form $u = Kx + Lv$ with L a non-singular matrix if and only if one of these three equivalent conditions is satisfied:

(i) The decoupling matrix B^* is non-singular (Falb and Wolovich, 1967):

$$B^* = \begin{pmatrix} c_1 A^{n_1'-1} B \\ \vdots \\ c_i A^{n_i'-1} B \\ \vdots \\ c_p A^{n_p'-1} B \end{pmatrix}. \tag{8}$$

(ii)
$$\sum_{i=1}^{m} n_i = \sum_{i=1}^{m} n_i' \,. \tag{9}$$

This condition using the infinite structure was defined by Dion and Commault (1993).

(iii)
$$n_{ie} = n_i' \qquad \forall \, i \in \{1,..,p\} \tag{10}$$

This condition using essential orders was defined by Commault, et al. (1986).

2.2. Interpretation of essential orders

Gilbert (1969) showed that there is a class of invertible systems not decouplable by static feedback but which require a dynamic extension to achieve decoupling by feedback, which corresponds to a dynamic feedback. In this case, and more particularly for non square systems which are only right invertible, the essential order n_{ie} of the i^{th} output, that is superior to its relative degree n'_i, corresponds to the highest derivation order of the i[th] output appearing in the inverse model. Let us prove this assertion of the essential orders. In fact it is enough to demonstrate that the essential order n_{ie} corresponds to the order of the pole at infinity of the i[th] column of $T^{-1}(s)$. The main ideas of the proof come from (Commault et al., 1986). We start from the Hermite form of (1) to write $T(s)$, which is assumed to be right invertible but non necessarily square (with dimension $p\times m$), as

$$T(s)=\left[R(s)\mid 0\right]\,B(s), \tag{11}$$

where $B(s)$ is a $(m\times m)$ biproper matrix and $R(s)$ is a $(p\times p)$ invertible matrix.

Theorem 1: The essential order n_{ie} corresponds to the order t_i of the pole at infinity of the i[th] column of the transfer matrix of the inverse system $T^{-1}(s)$.

Proof: The proof will be given for i =1 (for the 1[st] column). For other columns, the same proof holds by rows permutations. Consider the factorization (11) of $T(s)$. As $T(s)$ is right-invertible, it follows

$$T^{-1}(s)=B^{-1}(s)\left[\frac{R^{-1}(s)}{0}\right]. \tag{12}$$

Write $R(s)$ as $R(s)=\left[\frac{r(s)}{\overline{R}(s)}\right]$, where $r(s)$ the first row of $R(s)$. Let us factorize $\overline{R}(s)$ as follows:

$$\overline{R}(s)=\overline{B}_1(s)\overline{\Lambda}(s)\overline{B}_2(s). \tag{13}$$

With $\overline{B}_1(s)$ and $\overline{B}_2(s)$ biproper and

$$\overline{\Lambda}(s)=\left[\begin{array}{ccc|c} s^{-n'_2} & & & 0\\ & \ddots & & \vdots\\ & & s^{-n'_p} & 0\end{array}\right].$$

Then,

$$R(s)=\left[\frac{r(s)\overline{B}_2^{-1}(s)}{\overline{B}_1(s)\overline{\Lambda}(s)}\right]\overline{B}_2(s). \tag{14}$$

Thereafter, $R(s)$ can be written as

$$R(s)=\overline{\overline{B}}_1(s)\overline{\overline{R}}(s)\overline{B}_2(s), \tag{15}$$

where $\overline{\overline{B}}_1(s)=\left[\begin{array}{c|c}1 & 0\\ \hline 0 & \overline{B}_1(s)\end{array}\right]$ is a $(p\times p)$ biproper matrix

$$\overline{\overline{R}}(s)=\left[\frac{r_1(s)\overline{B}_2^{-1}(s)}{\overline{\Lambda}(s)}\right]=\left[\begin{array}{cccc}\beta_1 & \cdots & \cdots & \beta_P\\ \hline s^{-n'_2} & & & 0\\ & \ddots & & \vdots\\ & & s^{-n'_p} & 0\end{array}\right]$$ is a $(p\times p)$ matrix and $\{\beta_1,\ldots,\beta_p\}$ are rational functions

Denote $\overline{T}(s)=\overline{\overline{B}}_1(s)\overline{\overline{R}}(s)$, so that $R(s)=\overline{T}(s)\overline{B}_2(s)$. Then, $R^{-1}(s)$ and $\overline{\overline{R}}^{-1}(s)$ have the same infinite structure. This infinite structure corresponds to those of $\left[\frac{R^{-1}(s)}{0}\right]$ and $\left[\frac{\overline{\overline{R}}^{-1}(s)}{0}\right]$. We have

$$\overline{\overline{R}}^{-1}(s)=\frac{1}{\det(\overline{\overline{R}}(s))}\,adj(\overline{\overline{R}}(s)). \tag{16}$$

The first column (i=1) of $adj(\overline{\overline{R}}(s))$ is

$$\left[\begin{array}{cccc}0 & \ldots & 0 & \prod_{j=2}^{p}s^{-n'_i}\end{array}\right]^T.$$

According to (15), we have

$$\overline{\overline{R}}(s)=\overline{\overline{B}}_1^{-1}(s)R(s)\overline{B}_2^{-1}(s). \tag{17}$$

The Smith-MacMillan form at infinity of $R(s)$ is given by:

$$R(s)=B_3(s)\Lambda(s)B_4(s), \tag{18}$$

with $B_3(s)$ and $B_4(s)$ biproper and

$\Lambda(s)=\begin{bmatrix}s^{-n_1} & & \\ & \ddots & \\ & & s^{-n_p}\end{bmatrix}$.Thus,

$$\det(\overline{\overline{R}}(s))=\beta(s)\prod_{j=1}^{p}s^{-n_j} \tag{19}$$

with $\beta(s)$ a biproper function. The first column of $\overline{\overline{R}}^{-1}(s)$ is finally $\left[\begin{array}{cccc}0 & \ldots & 0 & \dfrac{\prod_{j=2}^{p}s^{-n'_j}}{b(s)\prod_{j=1}^{p}s^{-n_j}}\end{array}\right]^T$.

Thus, the order of the pole at infinity of the first column of $T^{-1}(s)$ is $t_1=\sum_{j=1}^{p}n_j-\sum_{j=2}^{p}n'_j$, which corresponds to the essential order of the first output (i=1).

3. BOND GRAPH APPROACH

The relative degree n'_i is equal, on a bond graph model, to the length l_i of the shortest causal path between the i[th] output (D_e or D_f) and all the inputs (S_e or S_f) (Rahmani, et al., 1996). The order n'_i represents the minimal and necessary number of derivations of this output to make appear explicitly at least one of the entries, see (Bertrand, et al., 1997; Dauphin-Tanguy, et al., 2000).

The number of the infinite zeros of the bond graph model is equal to the number of *disjoint* I/O causal paths and that their orders n_k are computed as in equation (20), where L_k is the sum of the lengths of the k shortest I/O *disjoint* causal paths. (Dauphin-Tanguy, et al., 1999 and 2000)

$$\begin{aligned} n_1 &= L_1 \\ n_k &= L_k - L_{k-1} \end{aligned} . \tag{20}$$

Proposition 1: We define, on the bond graph model, the essential order of the i[th] output described by equation (6) as follows:

$$n_{ie} = L_k - \sum_{j=1}^{p-1} l_j \tag{21}$$

$\forall$ i $\in$ {1,…p} $\forall$ j$\in$ {1,…,p}/{ i}

In a bond graph model, a sufficient condition for the existence of the inverse model of the square system $(p \times p)$ is that there is a single choice of independent I/O power lines. If there are several sets of I/O power lines then it is necessary to show that the determinant of the transfer matrix is not zero (Ngwompo, et al., 2005). An invertible system represented by a bond graph model is decouplable by a static feedback if the following equivalent conditions are verified (Rahmani, et al., 1996):

(i) $$L_m = \sum_{i=1}^{m} l_i \, . \tag{22}$$

(ii) $$\{n_i\} = \{n'_i\}. \tag{23}$$

Proposition 2: the bond graph model is decouplable by a static feedback if the following condition is satisfied

$$l_i = L_k - \sum_{j=1}^{p-1} l_j \tag{24}$$

$\forall$ i $\in$ {1,…p} $\forall$ j$\in$ {1,…,p}/{ i}

This condition represents a bond graph interpretation of condition (10).

4. EXAMPLES

4.1 Example 1

Let us consider the electrical network (Figure 1) and the associated integral bond graph model (Figure 2).

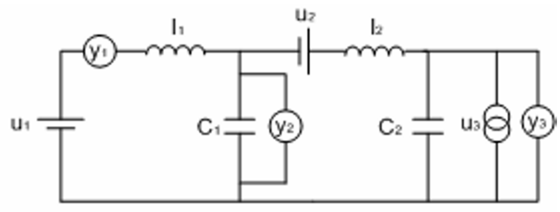

Fig.1. Electrical network

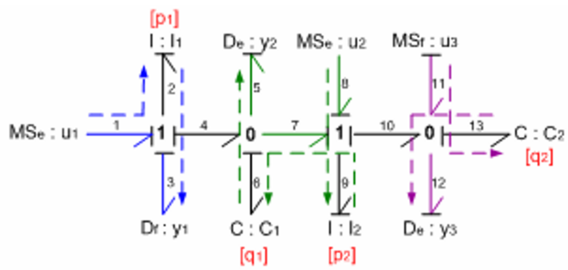

Fig.2. Bond graph model in preferential integral causality (BGI)

The state, input and output vectors are given by: $x = [p_1 \; p_2 \; q_1 \; q_2]^T$; $u = [u_1 \; u_2 \; u_3]^T$ and $y = [y_1 \; y_2 \; y_3]^T$. This system has three independent I/O power lines: (u_1, y_1), (u_2, y_2) and (u_3, y_3), then the system is invertible. The disjoint I/O causal paths are (u_1, y_1): e_1-e_2-f_2-f_3, (u_2, y_2): e_8-e_9-f_9-f_7-f_6-e_6-e_5 and (u_3, y_3): f_{11}-f_{13}-e_{13}-e_{12}, then using the length of these causal paths, $L_3 = 4$ = 1+2+1.

We have $l_1 = n'_1 = 1$, $l_2 = n'_2 = 2$ and $l_3 = n'_3 = 1$, then $L_3 = l_1 + l_2 + l_3$ and the system is decouplable by static feedback. The orders of the infinite zeros are: $n_1 = L_1 = 1$, $n_2 = L_2 - L_1 = 2$ and $n_3 = L_3 - L_2 = 1$. The essential orders are given by : $n_{1e} = L_3 - n'_2 - n'_3 = 1$; $n_{2e} = L_3 - n'_1 - n'_3 = 2$ and $n_{3e} = L_3 - n'_1 - n'_2 = 1$. In this case, the essential orders are equal to the relative degrees.
To find the inverse model, we define the bicausal bond graph model (BBG) on Figure 3 in replacing each source and each detector by respectively a double detector and a double source and in propagating the bicausality along the O/I power lines. The bicausality propagation is done according to SCAPI procedure (Ngwompo, et al., 2001).

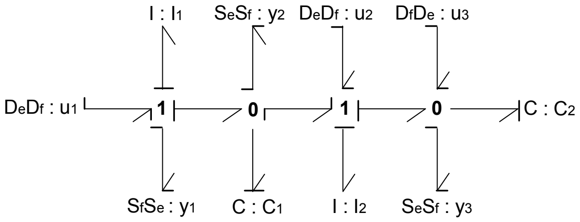

Fig.3. Bicausal bond graph model

The inverse model is directly obtained from the BBG:

$$\begin{cases} u_1 = I_1 \dot{y}_1 + y_2 \\ u_2 = -C_1 I_2 \ddot{y}_2 + I_2 \dot{y}_1 - y_2 + y_3 \\ u_3 = C_2 \dot{y}_3 + C_1 \dot{y}_2 - y_1 \end{cases} \tag{25}$$

The highest derivation order of each output corresponds to the relative degree.

4.2 *Example 2*

Let us consider a second example (Figure 4). The BGI model is represented on Figure 5.

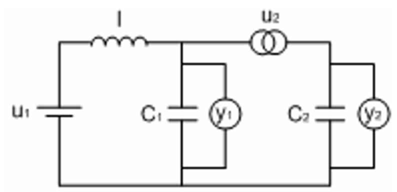

Fig.4. Example 2

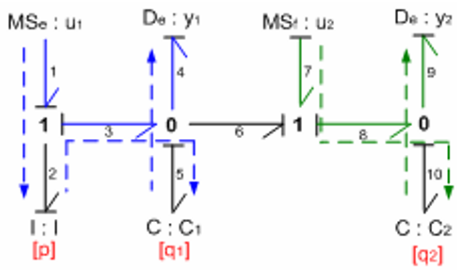

Fig.5. BGI model

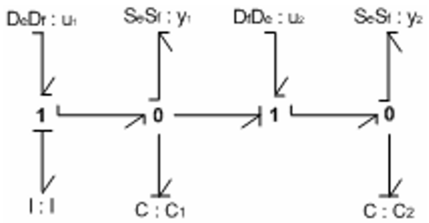

Fig.6. BBG model

The state, input and output vectors are given by: $x = [p\ q_1\ q_2]^T$; $u = [u_1\ u_2]^T$ and $y = [y_1\ y_2]^T$. This system has a single set of independent I/O power lines: (u_1, y_1) and (u_2, y_2) then the system is invertible. The disjoint I/O causal paths are (u_1, y_1): e_1-e_2-f_2-f_3-f_5-e_5-e_4 (u_2, y_2): f_7-f_8-f_{10}-e_{10}-e_9, then using the length of these causal paths, $L_2 = 3 = 2+1$.

$l_1 = n'_1 = 1$ and $l_2 = n'_2 = 1$ then $L_3 \neq l_1 + l_2$ and the system is not decouplable by static feedback. The orders of the infinite zeros are: $n_1 = L_1 = 2$ and $n_2 = L_2 - L_1 = 1$. The essential orders are: $n_{1e} = L_2 - n'_2 = 2$ and $n_{2e} = L_2 - n'_1 = 2$.

The inverse model is directly computed from BBG model obtained as previously (Figure 6).

$$\begin{cases} u_1 = IC_1 \ddot{y}_1 + IC_2 \ddot{y}_2 + y_1 \\ u_2 = C_2 \dot{y}_2 \end{cases} \tag{26}$$

Let us denote by d_i i =1,2 the numbers of derivations of outputs in the inverse model, then $d_1 = 2$ and $d_2 = 2$. We remark that $d_1 \geq n'_1$ and $d_2 \geq n'_2$, so the highest derivation order of each output in the inverse model is different from the relative degree. This number corresponds to the essential order n_{ie} .

5. CONCLUSION

It has been shown that the order of essentiality of the system outputs can be graphically obtained from the bond graph model in a simple way. This approach seems to be an important complement to previous algebraic procedures developed in the literature. This enables to detect as soon as possible (directly after the bond graph modelling step), relevant information in regard with several aspects of the system design: sizing and control. First it gives the required regularity of each trajectory characterizing the design specifications related to an output in the system. Second, it enables to determine directly if the decoupling with a static feedback is possible, or if not, the dimension of the required dynamic extension. Finally, if the outputs are flat, it characterizes an important property of the inverse model.

As it is usually the case with the bond graph approach, an obvious utilization of this property can be done at the physical model level. For example, at the design step, it will be easy to modify the system (and consequently the model) in order to define an adequate structure according to the design or control requirements. In fact, the determination of essential orders on the bond graph model can help to synthesize a choice of dynamic extension.

The determination of essential orders on nonlinear bond graph model is possible for two reasons. Firstly, the essential order was defined for nonlinear systems (Glumineau and Moog, 1989). Secondly, the bond graph procedures for determining structural properties in general and essential orders in particular remain unchanged for nonlinear system. Nevertheless, a demonstration can be more convincing.

REFERENCES

Bertrand, J.M., C. Sueur and G. Dauphin-Tanguy (1997). On the finite and infinite structures of bond graph models. *Proc. IEEE Int. Conf. Syst. Man. Cybern.*, **vol.3**, pp.2472-2477.

Bideaux, E., J. Laffite, A. Derkaoui, W. Marquis-Favre, S. Scavarda and F. Guillemard (2006a). Design of a Hybrid Vehicle Powertrain using an Inverse Methodology. *Journal Européen des Systèmes Automatisés*, **vol.40**, pp.279-290.

Bideaux, E., W. Marquis-Favre and S. Scavarda (2006b). Equilibrium set investigation using bicausality, Mathematical and Computer Modelling of Dynamical Systems. *Taylor & Francis. I. Troch.*, **vol.12**, pp.127-140.

Commault, C., J. Descusse , J.M. Dion, J.F. Lafay and M. Malabre (1986). New decoupling invariants: the essential orders. *Int. J. of Control*, **vol.44**, **n°3**, pp.689-700.

Cremer, M. (1971). A precompensator of minimal order for decoupling a linear multivariable system. *Int. J. of Control*, **vol.14**, **n°6**, pp.1089-1103.

Dauphin-Tanguy, G., A. Rahmani and C. Sueur (1999). Bond graph aided design of controlled systems. *Simul. Pract. Theory*, **vol.7, n°5-6**, pp.493-512.

Dauphin-Tanguy, G. et al. (2000). *Les bond graphs*, 383p, Hermes Science, Paris.

Dion, J.M. and C. Commault (1993). Feedback decoupling of structured systems. *IEEE Trans. Autom. Control*, **vol.38**, **n°7**, pp1132-1135.

Falb, P.L. and W.A. Wolovich (1967). Decoupling in the Design and Synthesis of Multivariable Control Systems. *IEEE Trans. Autom. Control*, **vol.AC-12**, **n°6**, pp.651-659.

Gawthrop, P.J. (1995). Bicausal bond graphs, *Proc. of ICGBM'95*, Las Vegas, pp.83-88.

Gawthrop, P.J. (1997). Control system configuration: Inversion and bicausal bond graphs. *Proc. of ICGBM'97*, Phoenix, pp.97-102.

Gawthrop, P.J. (2000). Physical Interpretation of Inverse Dynamics Using Bicausal Bond Graphs. *J. Franklin Inst.*, **vol.337**, pp.743-769.

Gilbert, E.G. (1969). Decoupling of multivariable systems by state feedback. *S.I.A.M. J. of Control*, **vol.7**, **n°1**, pp50-63.

Glumineau, A. and C. H. Moog (1989). Essential orders and the non-linear decoupling problem, *Int. J. of Control*, **vol.50, n°5**, pp.1825-1834.

Ngwompo, R.F, S. Scavarda and D. Thomasset (1997). Structural inversibility and minimal of multivariable linear systems: A bond graph approach. *Simul. Counc. Proc. Ser.*, **vol.29**, **n°1**, pp.109.

Ngwompo, R.F, S. Scavarda and D. Thomasset (1999). Inversion of linear time invariant SISO systems modeled by bond graph. *J. Franklin Inst.*, **vol.333B**, **n°2**, pp.157-174.

Ngwompo, R.F., S. Scavarda and D. Thomasset (2001). Physical model-based inversion in control systems design using bond graph representation, Part 1: theory. *Proc. Inst. Mech. Eng. Part I J. Syst. Control Eng.*, **vol.215**, **n°2**, pp.95-103.

Ngwompo, R.F., E. Bideaux and S. Scavarda (2005). On the role of power lines and causal paths in bond graph – based model inversion. *ICBGM'05*, **vol.37(1)**, pp.5-10.

Rahmani, A., C. Sueur and G. Dauphin-Tanguy (1992). Formal determination of controllability/observability matrices for multivariable systems modelled by bond graph. *Proceeding of IMACS/SICE Int. Symposium of Robotics, Machatronics and Manufacturing System'92*, pp573-580.

Rahmani, A., C. Sueur and G. Dauphin-Tanguy (1996). On the infinite structure of systems modelling by bond graph: feedback decoupling. *Proc. IEEE Int. Conf. Syst. Man. Cybern.*, **vol.3**, pp.1617-1622.

Sain, M.K. and J.L. Massey (1969). Invertibility of linear time-invariant dynamical systems, *IEEE Trans. Autom. Control*, **vol.AC-14**, **n°2**, pp.141-149.

Sueur, C. and G. Dauphin-Tanguy (1991). Bond graph approach for structural analysis of MIMO linear systems., *J. Franklin Inst.*, **vol.328**, **n°1**, pp.55-70.

Sueur, C. and G. Dauphin-Tanguy (1989). Structural controllability/observability of linear systems represented by bond graphs. *J. Franklin Inst.*, **vol.326**, **n°6**, pp.869-883.

Van Dixhoorn, J.J and F.J. Evans (1974). *Physical structure in systems theory: network approaches to engineering and economics*; 305p, Academic-Press, London – New York – San Fransisco.

Wu, S.-T. and K. Youcef-Toumi (1995). On the relative degrees and zero dynamics from physical system modelling. *J. Dyn. Syst. Meas. Control Trans. ASME*, **vol.117**, **n°2**, pp.205-217.