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Multi-Sensor Distributed Fusion Filter for Discrete Stochastic Multi-Delayed Systems with Correlated Noise

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Abstract: This paper is concerned with the distributed fusion estimation problem for discrete-time linear stochastic multi-delayed systems with multiple sensors and correlated noise. Firstly, a new optimal filter in the least mean square sense is presented for discrete stochastic multi-delayed systems with a single sensor, where the white noise filter is used to obtain the optimal state estimate. Then, a distributed optimal scalar-weighted fusion filter is given for discrete-time linear stochastic multi-delayed systems with multiple sensors. A recursive formula for the estimation error cross-covariance matrix between any two local optimal estimates is derived. Compared with the centralized filter, it has a little accuracy loss but better reliability. At last, a simulation example shows the effectiveness of the proposed algorithms.

1. INTRODUCTION

The problem of state estimation for systems with time delays has been investigated widely due to many applications in signal processing, communication and control systems, etc. (Zhang & Xie, 2007). The general approaches to the design of an optimal filter or observer for these systems include the augmented optimal Kalman filter by an augmented state space representation, which has the high implementation cost and large memory, and the non-augmented optimal filter (Mishra & Rajamani, 1975; Priemer & Vacroux, 1969; Raja Rao & Mahalana, 1971; Liang & Christensen, 1975; Liang, 1977), where the optimal filters were directly obtained based on projection theory, avoids the disadvantages of the augmented approach.

For systems with distributed multiple sensors, the centralized filter (Willner, 1976), where all measured sensor data are communicated to a central site for processing, has large computational burden and is not fault tolerant though it can obtain the optimal estimation when there are not faulty sensors. The distributed filter (Hashemipour, 1988), where the local estimates can lead to global optimal or suboptimal estimation according to certain fusion criterion, has better reliability due to its parallel structure that allows a higher input data rate and makes fault detection and isolation easy. Various distributed and parallel versions of the Kalman filter with applications have been investigated. Early, Bar-Shalom (1981) studied the correlation of two sensor subsystems and gave a formula to compute the cross-covariance matrix of two local estimators. Carlson (1990) presented the federated square-root filter where an upper bound of the covariance matrix of the process noise is used and the initial estimation errors between any two local estimators are assumed to be uncorrelated, to avoid the computation of cross-covariance matrix. Kim (1994) gave the maximum likelihood (ML) fusion filter under the assumption of normal distributions. Li et al. (2003) gave the unified fusion rules for centralized, distributed and hybrid fusion architectures based on a unified linear model in weighted least squares (WLS) sense and best linear unbiased estimate (BLUE) sense. Three optimal fusion algorithms weighted by matrices, diagonal matrices or scalars in the linear minimum variance (LMV) sense were presented in (Sun, 2004) where the fusion estimation weighted by matrices is the same as the ML estimation (Kim, 1994) and the distributed estimation in the BLUE sense (Li et al., 2003), but the assumption of normal distributions is not necessary. Further, the weighted fusion algorithms were applied to the distributed fusion filter and smoother for systems with correlated noises (Sun & Deng, 2004; Sun, 2005) and distributed deconvolution estimators (Sun, 2004). In the distributed fusion estimation above, the concerned systems do not involve time delays. The Kalman filter for systems with multiple sensors having different measurement delays were given by a re-organized innovation approach (Zhang et al., 2004; Lu et al., 2005), which involves in series of the computation of multiple filters with the same dimension as the state of the original system. Though it has the same accuracy with the centralized, the reliability can not be guaranteed if there are faulty sensors.

So far, the distributed fusion estimation for multi-sensor systems with multiple time delays in the state has not been investigated although it has wide applications in communication and control. Furthermore, to the best of the author's knowledge, the optimal filter for the single sensor system with correlated noise has not been solved perfectly in (Raja Rao & Mahalana, 1971; Liang & Christensen, 1975; Liang, 1977). Different from them, in the present paper, we give a new optimal filter by using white noise filter (Mendel, 1977) based on innovation analysis approach. Further, the distributed weighted fusion optimal filter is given for discrete-time stochastic multi-delayed systems with multiple sensors based on scalar-weighted optimal fusion algorithm in the LMV sense (Sun, 2004). Compared with the centralized optimal filter, it avoids the high-dimensional computation and large memory, and has better reliability due to the parallel structure. A recursive formula to compute the cross-covariance matrix between any two local estimates is derived.

The rest of this paper is organized as follows. Problem formulation is given in Section 2. The new non-augmented optimal filter with white noise filter is presented for single-sensor system with correlated state and measurement noises in Section 3. The distributed optimal scalar-weighted fusion filter is given in Section 4. In Section 5, a simulation example with three sensors is given. Finally, the conclusions are drawn in Section 6.

2. PROBLEM FORMULATION

Consider the discrete-time linear stochastic multi-delayed system with multiple sensors

$$x(t+1)=\sum_{k=0}^{d}\Phi_k(t)x(t-k)+\Gamma(t)w(t) \tag{1}$$

$$y^{(i)}(t)=H^{(i)}(t)x(t)+v^{(i)}(t)\,,\ i=1,2,\cdots,L \tag{2}$$

where $x(t)\in R^n$ is the state, $y^{(i)}(t)\in R^{m_i}$ is the measurement, $w(t)\in R^r$ and $v^{(i)}(t)\in R^{m_i}$ are white noises, and $\Phi_k(t)$, $\Gamma(t)$ and $H^{(i)}(t)$ are time-varying matrices with compatible dimensions. The superscript (i) denotes the ith sensor and L is the number of sensors. The integer d is the largest time delay in the state.

Assumption 1. $w(t)$ and $v^{(i)}(t)$, $i=1,2,\cdots,L$ are correlated white noises with zero mean and

$$\mathrm{E}\left\{\begin{bmatrix} w(t)\\ v^{(i)}(t)\end{bmatrix}[w^{\mathrm T}(k)\quad v^{(j)\mathrm T}(k)]\right\}=Q_{ij}(t)\delta_{tk}\,,$$

$$Q^{(ij)}(t)=\begin{bmatrix} Q_w(t) & S^{(j)}(t)\\ S^{(i)\mathrm T}(t) & Q_{v^{(ij)}}(t)\end{bmatrix} \tag{3}$$

where the symbol E is the mathematical expectation, the superscript T is the transpose, and δ_{tk} is the Kronecker delta function.

Assumption 2. The initial states $x(-k)$, $k=0,1,\cdots,d$ are independent of $w(t)$ and $v^{(i)}(t)$, $i=1,2,\cdots,L$ and $\mathrm{E}[x(-k)]=\mu_k$, $\mathrm{E}[(x(-k)-\mu_k)(x(-l)-\mu_l)^{\mathrm T}]=P_0(k,l)$, $k=0,1,\cdots,d$; $l=0,1,\cdots,k$.

Our aim is to find the distributed optimal (i.e., linear minimum variance under scalar weighting) fusion filter $\hat{x}^{(o)}(t\,|\,t)$ of state $x(t)$ that is generated by scalar-weighted fusion of local filters $\hat{x}^{(i)}(t\,|\,t)$ from the measurements $(y^{(i)}(0),y^{(i)}(1),\cdots,y^{(i)}(t))$, $i=1,2,\cdots,L$.

3. LOCAL OPTIMAL FILTER

For every single sensor subsystem of system (1)-(2), the state estimation problem can be solved by the augmented approach. But the augmented approach requires expensive computation cost and large memory due to the high system dimension. To avoid these disadvantages, we will derive the optimal filter based on innovation approach. It has the smaller computation cost than the augmented optimal filter, but has identical accuracy. Different from (Raja Rao & Mahalana, 1971; Liang et al., 1975, 1977) where some formulas can not be implemented, the white noise filter is used to obtain the optimal estimator in our approach.

Theorem 1. For the multi-delayed system (1)-(2) with Assumptions 1 and 2, the local optimal Kalman filter based on the i-th sensor subsystem is given by

$$\hat{x}^{(i)}(t+1\,|\,t)=\sum_{k=0}^{d}\Phi_k(t)\hat{x}^{(i)}(t-k\,|\,t)+\Gamma(t)\hat{w}^{(i)}(t\,|\,t) \tag{4}$$

$$\hat{x}^{(i)}(t-k\,|\,t)=\hat{x}^{(i)}(t-k\,|\,t-1)+K^{(i)}(t-k\,|\,t)\varepsilon^{(i)}(t) \tag{5}$$

$$\hat{w}^{(i)}(t\,|\,t)=S^{(i)}(t)Q^{-1}_{\varepsilon^{(i)}}(t)\varepsilon^{(i)}(t) \tag{6}$$

$$\varepsilon^{(i)}(t)=y^{(i)}(t)-H^{(i)}(t)\hat{x}^{(i)}(t\,|\,t-1) \tag{7}$$

$$K^{(i)}(t-k\,|\,t)=P^{(i)}(t-k,t\,|\,t-1)H^{(i)\mathrm T}(t)Q^{-1}_{\varepsilon^{(i)}}(t) \tag{8}$$

$$Q_{\varepsilon^{(i)}}(t)=H^{(i)}(t)P^{(i)}(t,t\,|\,t-1)H^{(i)\mathrm T}(t)+Q_{v^{(i)}}(t) \tag{9}$$

$$P^{(i)}(t+1,t+1\,|\,t)=\sum_{k=0}^{d}\Phi_k(t)P^{(i)}(t-k,t+1\,|\,t)+$$
$$\Gamma(t)\sum_{l=0}^{d}P^{(i)}_{wx}(t,t-l\,|\,t)\Phi_l^{\mathrm T}(t)+\Gamma(t)P^{(i)}_{w}(t\,|\,t)\Gamma^{\mathrm T}(t) \tag{10}$$

$$P^{(i)}(t-k,t+1\,|\,t)=\sum_{l=0}^{d}P^{(i)}(t-k,t-l\,|\,t)\Phi_l^{\mathrm T}(t)+$$
$$P^{(i)}_{xw}(t-k,t\,|\,t)\Gamma^{\mathrm T}(t) \tag{11}$$

$$P^{(i)}(t-k,t-l\,|\,t)=P^{(i)}(t-k,t-l\,|\,t-1)-$$
$$K^{(i)}(t-k\,|\,t)Q_{\varepsilon^{(i)}}(t)K^{(i)^{\mathrm T}}(t-l\,|\,t) \tag{12}$$

$$P^{(i)}_{xw}(t-k,t\,|\,t)=-K^{(i)}(t-k\,|\,t)S^{(i)\mathrm T}(t) \tag{13}$$

$$P^{(i)}_{w}(t\,|\,t)=Q_w(t)-S^{(i)}(t)Q^{-1}_{\varepsilon^{(i)}}(t)S^{(i)\mathrm T}(t) \tag{14}$$

with the initial values $\hat{x}^{(i)}(-k\,|\,-1)=0$ and $P^{(i)}(-k,-l\,|\,-1)=P_0(k,l)$, $k=0,1,\cdots,d$, $l=0,1,\cdots,k$. We define the covariance matrix $P^{(i)}(*,\bullet\,|\,\circ)=\mathrm{E}[\tilde{x}^{(i)}(*\,|\,\circ)\tilde{x}^{(i)\mathrm T}(\bullet\,|\,\circ)]$ of the estimation errors $\tilde{x}^{(i)}(*\,|\,\circ)=x(*)-\hat{x}^{(i)}(*\,|\,\circ)$ and $\tilde{x}^{(i)}(\bullet\,|\,\circ)=x(\bullet)-\hat{x}^{(i)}(\bullet\,|\,\circ)$, with $P^{(i)}(t-k,t-l\,|\,t)=P^{(i)^{\mathrm T}}(t-l,t-k\,|\,t)$. $P^{(i)}_{xw}(t-k,t\,|\,t)=\mathrm{E}[\tilde{x}^{(i)}(t-k\,|\,t)\tilde{w}^{(i)\mathrm T}(t\,|\,t)]$ is the correlated matrix between $\tilde{x}^{(i)}(t-k\,|\,t)=x(t-k)-\hat{x}^{(i)}(t-k\,|\,t)$ and $\tilde{w}^{(i)}(t\,|\,t)=w(t)-\hat{w}^{(i)}(t\,|\,t)$, with $P^{(i)}_{wx}(t,t-k\,|\,t)=P^{(i)\mathrm T}_{xw}(t-k,t\,|\,t)$. $K^{(i)}(t-k\,|\,t)$ is the gain matrix and $\varepsilon^{(i)}(t)$ is the innovation.

Proof. Taking projection on both sides of (1) on the space generated by $(y^{(i)}(0),y^{(i)}(1),\cdots,y^{(i)}(t))$, we have (4). From projection theory, we have (5) and (7), where the gain $K^{(i)}(t-k\,|\,t)$ is defined as follows

$$K^{(i)}(t-k\,|\,t)=\mathrm{E}[x(t-k)\varepsilon^{(i)\mathrm T}(t)]\mathrm{E}^{-1}[\varepsilon^{(i)}(t)\varepsilon^{(i)\mathrm T}(t)] \tag{15}$$

Substituting (2) into (7), we can rewrite (7) as

$$\varepsilon^{(i)}(t)=H^{(i)}(t)\tilde{x}^{(i)}(t\,|\,t-1)+v^{(i)}(t) \tag{16}$$

Substituting (16) into (15) and noting that $x(t-k)=\hat{x}(t-k\,|\,t-1)+\tilde{x}(t-k\,|\,t-1)$ and $\hat{x}(t-k\,|\,t-1)\perp\tilde{x}^{(i)}(t\,|\,t-1)$, where the symbol $\perp$ denotes orthogonality, and the uncorrelation of $x(t-k)$ and $v^{(i)}(t)$, $k\ge 0$, we have (8). (9) can be obtained directly from (16).

From projection theory, we have the white noise filter

$$\hat{w}^{(i)}(t|t)=\hat{w}^{(i)}(t|t-1)+\mathrm{E}[w(t)\varepsilon^{(i)\mathrm{T}}(t)]Q_{\varepsilon^{(i)}}^{-1}(t)\varepsilon^{(i)}(t) \quad (17)$$

Substituting (16) into (17) and noting that $\hat{w}^{(i)}(t|t-1)=0$ and the uncorrelation of $\tilde{x}^{(i)}(t|t-1)$ and $w(t)$, we have (6). Subtracting (4) from (1) yields the prediction error equation as

$$\tilde{x}^{(i)}(t+1|t)=\sum_{k=0}^{d}\Phi_k(t)\tilde{x}^{(i)}(t-k|t)+\Gamma(t)\tilde{w}^{(i)}(t|t) \quad (18)$$

From (18), we have covariance matrix

$$P^{(i)}(t+1,t+1|t)=\mathrm{E}[\tilde{x}^{(i)}(t+1|t)\tilde{x}^{(i)\mathrm{T}}(t+1|t)]$$
$$=\sum_{k=0}^{d}\Phi_k(t)\mathrm{E}[\tilde{x}^{(i)}(t-k|t)\tilde{x}^{(i)\mathrm{T}}(t+1|t)]+\Gamma(t)\mathrm{E}[\tilde{w}^{(i)}(t|t)\tilde{x}^{(i)\mathrm{T}}(t+1|t)] \quad (19)$$

where

$$\mathrm{E}[\tilde{w}^{(i)}(t|t)\tilde{x}^{(i)\mathrm{T}}(t+1|t)]=\sum_{l=0}^{d}\mathrm{E}[\tilde{w}^{(i)}(t|t)\tilde{x}^{(i)\mathrm{T}}(t-l|t)]\Phi_l^{\mathrm{T}}(t)+\mathrm{E}[\tilde{w}^{(i)}(t|t)\tilde{w}^{(i)\mathrm{T}}(t|t)]\Gamma^{\mathrm{T}}(t) \quad (20)$$

Substituting (20) into (19) yields (10). Similarly, (11) can be obtained by using (18).

From (5), we have the estimation error equation as

$$\tilde{x}^{(i)}(t-k|t)=\tilde{x}^{(i)}(t-k|t-1)-K^{(i)}(t-k|t)\varepsilon^{(i)}(t) \quad (21)$$

Then, the estimation error covariance is given by

$$P^{(i)}(t-k,t-l|t)=\mathrm{E}[\tilde{x}^{(i)}(t-k|t)\tilde{x}^{(i)\mathrm{T}}(t-l|t)]$$
$$=P^{(i)}(t-k,t-l|t-1)-\mathrm{E}[\tilde{x}^{(i)}(t-k|t-1)\varepsilon^{(i)\mathrm{T}}(t)]K^{(i)\mathrm{T}}(t-l|t)-K^{(i)}(t-k|t)\mathrm{E}[\varepsilon^{(i)}(t)\tilde{x}^{(i)\mathrm{T}}(t-l|t-1)]+K^{(i)}(t-k|t)Q_{\varepsilon^{(i)}}(t)K^{(i)\mathrm{T}}(t-l|t) \quad (22)$$

From (15), we have

$$\mathrm{E}[\tilde{x}^{(i)}(t-k|t-1)\varepsilon^{(i)\mathrm{T}}(t)]=\mathrm{E}[x(t-k)\varepsilon^{(i)\mathrm{T}}(t)]=K^{(i)}(t-k|t)Q_{\varepsilon^{(i)}}(t) \quad (23)$$

Substituting (23) into (22) yields (12). From (6), we have the white noise filtering error as

$$\tilde{w}^{(i)}(t|t)=w(t)-S^{(i)}(t)Q_{\varepsilon^{(i)}}^{-1}(t)\varepsilon^{(i)}(t) \quad (24)$$

From (21) and (24), we have (13) and (14), where $\tilde{x}^{(i)}(t-k|t)\perp\varepsilon^{(i)}(t)$, the uncorrelation of $\tilde{x}^{(i)}(t-k|t-1)$ and $w(t)$ and $\mathrm{E}[w(t)\varepsilon^{(i)\mathrm{T}}(t)]=S^{(i)}(t)$ are used. □

Corollary. For system (1)-(2), when $\Phi_k(t)=0$, $k=1,2,\cdots,d-1$, i.e., it reduces to the single-delayed d system with correlated noise, we have the corresponding filter.

4. DISTRIBUTED OPTIMAL FUSION FILTER

For system (1)-(2) with multiple sensors, we can use Theorem 1 to obtain the centralized optimal filter by combining all measurements from all sensors into one measurement vector. However, the centralized optimal filter has not the reliability if there are some faulty sensors or sensor data loss. To avoid the shortcoming, we will give the distributed weighted fusion filter by applying the scalar weighting fusion algorithm (Sun, 2004). It has the parallel structure which means its reliability. The cross-covariance matrix of the estimation errors between two local estimates is required, which can be computed by the following Theorem.

Theorem 2. For multi-sensor time-delayed system (1)-(2) with Assumptions 1 and 2, the cross-covariance matrix of the estimation errors between two sensor subsystems is given by

$$P^{(ij)}(t+1,t+1|t)=\sum_{k=0}^{d}\Phi_k(t)P^{(ij)}(t-k,t+1|t)+\Gamma(t)\sum_{l=0}^{d}P_{wx}^{(ij)}(t,t-l|t)\Phi_l^{\mathrm{T}}(t)+\Gamma(t)P_w^{(ij)}(t|t)\Gamma^{\mathrm{T}}(t) \quad (25)$$

$$P^{(ij)}(t-k,t+1|t)=\sum_{l=0}^{d}P^{(ij)}(t-k,t-l|t)\Phi_l^{\mathrm{T}}(t)+P_{xw}^{(ij)}(t-k,t|t)\Gamma^{\mathrm{T}}(t) \quad (26)$$

$$P^{(ij)}(t-k,t-l|t)=P^{(ij)}(t-k,t-l|t-1)+K^{(i)}(t-k|t)Q_{\varepsilon^{(ij)}}(t){K^{(j)}}^{\mathrm{T}}(t-l|t)-K^{(i)}(t-k|t)H^{(i)}(t)P^{(ij)}(t,t-l|t-1)-P^{(ij)}(t-k,t|t-1){H^{(j)}}^{\mathrm{T}}(t){K^{(j)}}^{\mathrm{T}}(t-l|t) \quad (27)$$

$$P_{xw}^{(ij)}(t-k,t|t)=-K^{(i)}(t-k|t)S^{(i)\mathrm{T}}(t)-P^{(ij)}(t-k,t|t-1)H^{(j)}(t)Q_{\varepsilon^{(j)}}^{-1\,\mathrm{T}}(t)S^{(j)\mathrm{T}}(t)+K^{(i)}(t-k|t)Q_{\varepsilon^{(ij)}}(t)Q_{\varepsilon^{(j)}}^{-1\,\mathrm{T}}(t)S^{(j)\mathrm{T}}(t) \quad (28)$$

$$P_w^{(ij)}(t|t)=Q_w(t)-S^{(i)}(t)Q_{\varepsilon^{(i)}}^{-1}(t)S^{(i)\mathrm{T}}(t)-S^{(j)}(t)Q_{\varepsilon^{(j)}}^{-1}(t)S^{(j)\mathrm{T}}(t)+S^{(i)}(t)Q_{\varepsilon^{(i)}}^{-1}(t)Q_{\varepsilon^{(ij)}}(t)Q_{\varepsilon^{(j)}}^{-1}(t)S^{(j)\mathrm{T}}(t) \quad (29)$$

$$Q_{\varepsilon^{(ij)}}(t)=H^{(i)}(t)P^{(ij)}(t,t|t-1)H^{(j)\mathrm{T}}(t)+Q_{v^{(ij)}}(t) \quad (30)$$

with the initial values $P^{(ij)}(-k,-l|-1)=P_0(k,l)$, $k,l=0,1,\cdots,d$. We define $P^{(ij)}(*,\bullet|\circ)=\mathrm{E}[\tilde{x}^{(i)}(*|\circ)\tilde{x}^{(j)\mathrm{T}}(\bullet|\circ)]$ and $P_{xw}^{(ij)}(*,\bullet|\circ)$ $=\mathrm{E}[\tilde{x}^{(i)}(*|\circ)\tilde{w}^{(j)\mathrm{T}}(\bullet|\circ)]$ with $P^{(ij)}(*,\bullet|\circ)=P^{(ji)\mathrm{T}}(\bullet,*|\circ)$ and $P_{xw}^{(ij)}(*,\bullet|\circ)=P_{wx}^{(ji)\mathrm{T}}(\bullet,*|\circ)$.

Proof. Similarly to (19), (25) and (26) can be obtained from $P^{(ij)}(t+1,t+1|t)=\mathrm{E}[\tilde{x}^{(i)}(t+1|t)\tilde{x}^{(j)\mathrm{T}}(t+1|t)]$ and $P^{(ij)}(t-k,t+1|t)=\mathrm{E}[\tilde{x}^{(i)}(t-k|t)\tilde{x}^{(j)\mathrm{T}}(t+1|t)]$ by using (18). From (21), we have

$$P^{(ij)}(t-k,t-l|t)=\mathrm{E}[\tilde{x}^{(i)}(t-k|t)\tilde{x}^{(j)\mathrm{T}}(t-l|t)]=P^{(ij)}(t-k,t-l|t-1)-\mathrm{E}[\tilde{x}^{(i)}(t-k|t-1)\varepsilon^{(j)\mathrm{T}}(t)]K^{(j)\mathrm{T}}(t-l|t)-K^{(i)}(t-k|t)\mathrm{E}[\varepsilon^{(i)}(t)\tilde{x}^{(j)\mathrm{T}}(t-l|t-1)]+K^{(i)}(t-k|t)Q_{\varepsilon^{(ij)}}(t)K^{(j)\mathrm{T}}(t-l|t) \quad (31)$$

From (16), we have

$$\mathrm{E}[\tilde{x}^{(i)}(t-k|t-1)\varepsilon^{(j)\mathrm{T}}(t)]=P^{(ij)}(t-k,t|t-1)H^{(j)\mathrm{T}}(t) \quad (32)$$

Substituting (32) into (31) yields (27). From (21) and (24), we have

$$P_{xw}^{(ij)}(t-k,t|t)=\mathrm{E}[\tilde{x}^{(i)}(t-k|t)\tilde{w}^{(j)\mathrm{T}}(t|t)]=-K^{(i)}(t-k|t)\mathrm{E}[\varepsilon^{(i)}(t)w^{\mathrm{T}}(t)]-\mathrm{E}[\tilde{x}^{(i)}(t-k|t-1)\varepsilon^{(j)\mathrm{T}}(t)]Q_{\varepsilon^{(j)}}^{-1\,\mathrm{T}}(t)S^{(j)\mathrm{T}}(t)+K^{(i)}(t-k|t)\mathrm{E}[\varepsilon^{(i)}(t)\varepsilon^{(j)\mathrm{T}}(t)]Q_{\varepsilon^{(j)}}^{-1\,\mathrm{T}}(t)S^{(j)\mathrm{T}}(t) \quad (33)$$

Substituting (32) into (33) and using Assumption 1 yield (28), where $Q_{\varepsilon^{(ij)}}(t)=\mathrm{E}[\varepsilon^{(i)}(t)\varepsilon^{(j)\mathrm{T}}(t)]$ computed by (30) can be obtained readily from (16). From (24) and Assumption 1, we can obtain (29) readily. □

Based on the local optimal filters in Theorem 1 and cross-covariance matrices in Theorem 2, we have the distributed fusion filter by applying optimal scalar-weighted fusion algorithms in the LMV sense (Sun, 2004) as

$$\hat{x}^{(o)}(t\,|\,t)=\sum_{i=1}^{L}a^{(i)}(t)\hat{x}^{(i)}(t\,|\,t) \tag{34}$$

Note that local optimal filters $\hat{x}^{(i)}(t\,|\,t)$, $i=1,2,\cdots,L$ can be computed by Theorem 1.

Scalar weights $a^{(i)}(t)$, $i=1,2,\cdots,L$ are computed by

$$a(t)=\frac{\Sigma^{-1}(t)e}{e^{\mathrm{T}}\Sigma^{-1}(t)e} \tag{35}$$

where $a(t)=[a^{(1)}(t)\ \cdots\ a^{(L)}(t)]^{\mathrm{T}}$ and $e=[1,1,\cdots,1]^{\mathrm{T}}$ are L-dimension column vectors. $\Sigma(t)=\left(\mathrm{tr}(P^{(ij)}(t,t\,|\,t))\right)$ is an $L\times L$ matrix. The variance of the scalar-weighted optimal fusion filter is computed by

$$P^{(o)}(t,t\,|\,t)=\sum_{i,j=1}^{L}a^{(i)}(t)a^{(j)}(t)P^{(ij)}(t,t\,|\,t) \tag{36}$$

Also we have $P^{(o)}(t,t\,|\,t)\le P^{(i)}(t,t\,|\,t)$, $i=1,2,\cdots,L$.

Remark 1. The distributed weighted fusion optimal predictor and smoother can also be obtained from Theorem 1 and Theorem 2 similarly.

Remark 2. When every local filter has the steady state, the local gain and variance matrices have the steady state values. From Theorem 2, we see that the cross-covariance matrices also have the steady-state values. Then, the distributed steady-state weighted fusion filter can be obtained from (34)-(36). Further, the distributed steady-state weighted fusion filter has the smaller online computational cost than the centralized filter in the fusion centre since the online computation of the scalar weighted fusion of local estimates is only required.

The computation procedure of the distributed weighted fusion filter can be summarized as follows:

Step 1. Compute the local filters $\hat{x}^{(i)}(t\,|\,t)$ and variance matrices $P^{(i)}(t,t\,|\,t)$ by Theorem 1.

Step 2. Compute the cross-covariance matrices $P^{(ij)}(t,t\,|\,t)$, $i\ne j$ by Theorem 2.

Step 3. Compute the distributed weighted fusion filter $\hat{x}^{(o)}(t\,|\,t)$ and variance matrix $P^{(o)}(t,t\,|\,t)$ by (34)-(36).

5. SIMULATION EXAMPLE

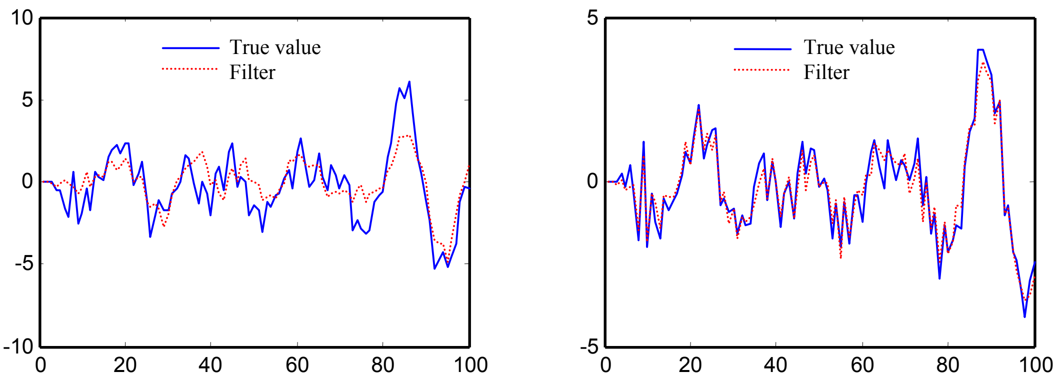
Consider a discrete-time stochastic delayed system with three sensors

$$x(t+1)=\begin{bmatrix}0.8 & 0\\ 0.1 & -0.5\end{bmatrix}x(t)+\begin{bmatrix}-0.4 & 0\\ 0.6 & 0.5\end{bmatrix}x(t-1)+\begin{bmatrix}0.4 & -0.6\\ 0 & 0.5\end{bmatrix}x(t-2)+\begin{bmatrix}1\\ -0.5\end{bmatrix}w(t) \tag{37}$$

$$y^{(i)}(t)=H^{(i)}x(t)+v^{(i)}(t),\ i=1,2,3 \tag{38}$$

where $w(t)$ and $v^{(i)}(t)$, $i=1,2,3$ are correlated noises satisfying the relation $v^{(i)}(t)=\alpha^{(i)}w(t)+\xi^{(i)}(t)$ where $\xi^{(i)}(t)$ with zero mean and variance $Q_{\xi^{(i)}}$ is Gaussian noise independent of $w(t)$ with zero mean and variance Q_w. Our aim is to find the distributed scalar-weighted optimal fusion filter $\hat{x}^{(o)}(t\,|\,t)$.

In simulation, we take *N*=100 sample data, and set $Q_w=1$, $Q_{\xi^{(1)}}=2$, $Q_{\xi^{(2)}}=1$, $Q_{\xi^{(3)}}=0.9$, $H^{(1)}=[0.1,3]$, $H^{(2)}=[0,2]$, $H^{(3)}=[-0.1,1.5]$, $\alpha^{(1)}=0.9$, $\alpha^{(2)}=0.8$, $\alpha^{(3)}=0.7$, the initial values $x(-k)=0$, $P_0(k,l)=0.1I_2$, $k,l=0,1,2$. For every single sensor subsystem, applying Theorem 1 we can obtain local optimal Kalman filters (LF) $\hat{x}^{(i)}(t\,|\,t)$, $i=1,2,3$. From (34)-(36), we can obtain the distributed scalar-weighted fusion filter (DSWFF) $\hat{x}^{(o)}(t\,|\,t)$, which is shown in Fig.1 where solid curves denote the true values and dashed curves denote the estimates. To compare with local filters (LF) and the centralized filter (CF), their estimation error variances are shown in Table.1. From Table.1, we see that DSWFF has higher accuracy than any LF does. Though DSWFF has a lower accuracy than CF, it has better reliability since the parallel structure is used.



(a) Filter of the first state component

(b) Filter of the second state component

Fig.1 Distributed scalar-weighted fusion filter

Table 1. Comparison of accuracy for LF, DSWFF and CF

Filters / Variances	LF1	LF2	LF3	DSWFF	CF
$x_1(t)$	2.0087	2.0343	2.2196	1.9158	1.8045
$x_2(t)$	0.2132	0.2579	0.3603	0.1737	0.1616

6. CONCLUSIONS

This paper has presented a new optimal filtering algorithm for discrete-time stochastic multi-delayed systems with correlated noise based on the innovation analysis approach, where the white noise filter is used. For the system with multiple sensors, we give a distributed fusion filter based on the scalar-weighted optimal fusion algorithm in the linear minimum variance sense. It has better reliability than the centralized filter. A recursive computation for the cross-covariance matrix of estimation errors between any two-sensor subsystems has been derived.

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