EIGENSTRUCTURE ASSIGNMENT DESIGN FOR PROPORTIONAL-INTEGRAL OBSERVERS — THE DISCRETE-TIME CASE

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Abstract: A complete parametric design approach for proportional-integral observers of multivariable discrete-time linear systems is proposed based on eigenstructure assignment technique. Complete parameterizations for all the observer gains as well as the eigenvector matrix of the observer system matrix are established in terms of three sets of design parameters which satisfy three basic and simple constraints. The proposed approach provides all the degrees of freedom and has great potential in applications. An illustrative example shows the effect of the proposed approach. *Copyright ©2000 IFAC*

Key words: Linear systems; proportional-integral observers; eigenstructure assignment, Sylvester matrix equations; parametric solutions.

1. INTRODUCTION

It is well-known that integral actions are useful in control systems design to achieve steady state accuracy (Anderson and Moore 1989). By introducing an integral term in observer design, Wojciechowsky (1978) first proposed the proportional-integral (PI) observers for SISO-linear time invariant systems. The idea was later generalized to the multivariable linear system case by Kaczorek (1979) and Shafai and Carroll (1985) to improve robustness against parameter variations and step disturbances. Up till now, this type of observers have attracted the attention of quite a few researchers (Beale and Shafai 1989, Niemann *et al.* 1995, Shafai *et al.* 1996, Saif (1993), Söffker *et al.* 1995 and Niemann 1998).

A PI observer offers certain degrees of freedom. By utilizing these degrees of freedom, properties such as LTR/LQR and robustness against parameter uncertainties and nonlinearities, can be achieved. However, by now there has not been such a method for design of PI observers which provides fully the degrees of design freedom. Beale and Shafai (1989) adopted the observable canonical form of the open-loop system in designing PI observers. Just as the pole assignment approach adopting the control canonical form does not provides degrees of design freedom, their method actually has lost all the part of the degrees of freedom associated with the design of the proportional gain. With the type of LQG designs (e.g., Shafai, *et al.* 1996), certainly all the degrees of freedom have been used up in minimizing the cost function.

The purpose of this paper is to propose a complete parametric design approach for PI observers, which provides all the degrees of the design freedom. Following the same lines of our former work in eigenstructure assignment (Duan 1992, 1993a, 1994, 1995, 1998, and 1999), complete parameterizations are given for all the gain matrices in terms of three parts of design parameters. The condition of existence of a PI observer is turned into three simple constraints on these design parameters. Moreover, the parametric expression for the corresponding eigenvector matrix of the observer system matrix is also obtained. The proposed approach uses the right coprime factorization of the given open-loop system, to which many methods have been available. The approach provides all the degrees of freedom which can be utilized to achieve various desired system specifications and performances, and thus has great potentials in applications.

2. PROBLEM FORMULATION

Consider the following discrete-time linear system:

$$\begin{cases} x(t+1) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{2.1}$$

where $x \in R^n$, $u \in R^r$ and $y \in R^m$ are respectively the state vector, the control vector and the output vector, and A, B, C are known real matrices of appropriate dimensions satisfying the following assumptions:

A1: The matrix C is of full-row rank.

A2: The matrix pair $[\,A \quad C\,]$ is observable.

For the system (2.1), a full-order PI observer is in the following form (Anderson and Moore 1989, Niemann and Stoustrup 1992a and Söffker *et al.* 1995):

$$\begin{cases} \widehat{x}(t+1) = (A - LC)\widehat{x}(t) + Ly(t) \\ \qquad\qquad\quad + Bu(t) + F\omega(t) \\ \omega(t+1) = \omega(t) + K(y(t) - C\widehat{x}(t)) \end{cases} \tag{2.2}$$

where $\widehat{x} \in R^n$ is the estimated state vector, $w \in R^p$ is a vector representing the integral of the weighted output estimation error, and $L \in R^{n\times m}$, $F \in R^{n\times p}$ and $K \in R^{p\times m}$ are the observer gains. Specifically, the matrices L and F are called the proportional and integral gain, respectively, and the matrix K is called the output estimation error weighting gain.

Definition 2.1: System (2.2) is said to be a PI observer for system (2.1). If for arbitrary initial conditions $x(0)$ and $\widehat{x}(0)$ and any input $u(t)$, the following relations hold:

$$\lim_{t\to\infty} \omega(t) = 0 \quad \text{and} \quad \lim_{t\to\infty} e(t) = 0 \tag{2.3}$$

where

$$e(t) = \widehat{x}(t) - x(t) \tag{2.4}$$

From (2.1) and (2.2), it can be obtained that

$$\begin{bmatrix} e(t+1) \\ \omega(t+1) \end{bmatrix} = A_0 \begin{bmatrix} e(t) \\ \omega(t) \end{bmatrix} \tag{2.5}$$

where

$$A_0 = \begin{bmatrix} A - LC & F \\ -KC & I_p \end{bmatrix} \tag{2.6}$$

Thus the system (2.2) is a PI observer for system (2.1) if and only if the matrix A_0 is Schur stable, i.e., it has all eigenvalues with modulus less than 1. Further in view that nondefective matrices have lower eigenvalue sensitivities than defective ones (Kautsky *et al.* 1985 and Duan 1993b), our problem of PI observer design for system (2.1) can be stated as follows.

Problem PIO: Given matrices $A \in R^{n\times n}$, $B \in R^{n\times r}$ and $C \in R^{m\times n}$, find complete parameterizations for the matrices L, F and K such that the matrix A_0 in (2.6) is nondefective and Schur stable.

3. THE GENERAL SOLUTION

This section provides the general solution to the Problem PIO proposed in Section 2.

3.1 Some Basic Relations

Since the matrix A_0 is required to be nondefective, it has a diagonal Jordan matrix

$$\Lambda = \operatorname{diag}[\,s_1 \quad s_2 \quad \dots \quad s_{n+p}\,] \tag{3.1}$$

where s_i, $i = 1, 2, \dots, n+p$ are not necessarily distinct, but satisfy the following constraint to ensure that A_0 is real and Schur stable:

Constraint $C1$: $\{s_i,\ i = 1, 2, \dots, n+p\}$ is self-conjugate and $|\,s_i\,| < 1$, $i = 1, 2, \dots, n+p$

Further, denote the left eigenvector matrix of A_0 by

$$U = \begin{bmatrix} T \\ V \end{bmatrix}, \quad T,\ V \in R^{n\times(n+p)} \tag{3.2}$$

Then, the following must hold:

$$\det \begin{bmatrix} T \\ V \end{bmatrix} \neq 0 \tag{3.3}$$

and by definition,

$$[\,T^T \quad V^T\,] \begin{bmatrix} A - LC & F \\ -KC & I_p \end{bmatrix} = \Lambda [\,T^T \quad V^T\,] \tag{3.4}$$

Equation (3.4) can clearly be decomposed equivalently into

$$T^T(A - LC) - V^TKC = \Lambda T^T \quad (3.5)$$

and

$$T^TF = (\Lambda - I)V^T \quad (3.6)$$

By introducing

$$-Z^T = T^TL + V^TK \quad (3.7)$$

equation (3.5) can be further written as

$$T^TA + Z^TC = \Lambda T^T \quad (3.8)$$

3.2 Solution of Matrices T and Z

The matrices T and Z are determined by equation (3.8), which is a generalized Sylvester matrix equation in the dual form studied by Duan 1992, 1993 and 1996.

Due to Assumptions A1 and A2, the following right coprime factorization holds:

$$(sI - A^T)C^T = N(s)D^{-1}(s) \quad (3.9)$$

where $N(s)$ and $D(s)$ are a pair of right coprime polynomial matrices with real coefficient and are of dimensions $n \times m$ and $m \times m$, respectively.

Lemma 3.1: Let assumptions $A1$ and $A2$ hold, then all the matrices T and Z satisfying (3.8) can be parameterized as follows:

$$T = [N(s_1)g_1 \quad \ldots \quad N(s_{n+p})g_{n+p}] \quad (3.10)$$

$$Z = [D(s_1)g_1 \quad \ldots \quad D(s_{n+p})g_{n+p}] \quad (3.11)$$

with $g_i \in R^m$, $i = 1, 2, \ldots, n+p$, being a group of arbitrary parameter vectors.

Proof: Taking the transpose of equation (3.8), gives

$$A^TT + C^TZ = T\Lambda \quad (3.12)$$

By applying the Theorem 2 in Duan (1993 a) to the above equation, the expressions (3.10) and (3.11) can be obtained.

□

3.3 Solution of Matrices V and F

In view of constraint $C1$, the matrix V can be directly obtained from (3.6) as

$$V^T = (\Lambda - I)^{-1}T^TF \quad (3.13)$$

Further using (3.1) and (3.10), gives

$$V = [v_1 \quad v_2 \quad \ldots \quad v_{n+p}] \quad (3.14a)$$

with

$$v_i = (s_i - 1)^{-1}F^TN(s_i)g_i \quad (3.14b)$$

where the matrix F has been taken as a parameter matrix in the expression for V.

It follows from (3.2), (3.10) and (3.14) that the general parametric form for the eigenvector matrix is in the following form

$$U = \begin{bmatrix} N(s_1)g_1 & N(s_2)g_2 & \ldots & N(s_{n+p})g_{n+p} \\ v_1 & v_2 & \ldots & v_{n+p} \end{bmatrix} \quad (3.15)$$

with v_i, $i = 1, 2, \ldots, n+p$ given by (3.14b). Therefore, condition (3.3) can be converted into the following constraint on this parameter matrix F as well as the other design parameters s_i, f_i, $i = 1, 2, \ldots, n+p$:

Constraint $C2$: $\det[U(F, s_i, f_i)] \neq 0$

It is worth pointing out that the integral gain matrix F does not have a parametric expression. This gain matrix has been taken as a design parameter which can be an arbitrary matrix of dimension $n \times p$ determined from Constraint C2.

3.4 Solution of Matrices L and K

Having obtained the matrices T, V and Z with Constraint $C2$ satisfied, the matrices L and K can be derived directly from (3.7) as follows.

$$\begin{bmatrix} L \\ K \end{bmatrix} = -[T^T \quad V^T]^{-1}Z^T \quad (3.16)$$

where T, Z and V are parameterized by (3.10), (3.11) and (3.14). However, in order that the matrices L and K obtained from (3.16) are real, it is necessary and sufficient to add the following constraint.

Constraint C3: $f_i = \bar{f}_l$ if $s_i = \bar{s}_l$

To summarize, the following conclusion about solution to Problem PIO proposed in Section 2 is given as follows.

Theorem 3.1 : Let Assumptions A1 and A2 be met. Then all the solutions to Problem PIO are given by (3.16) through (3.10), (3.11) and (3.14) with the parameters F and s_i, g_i, $i = 1, 2, \ldots, n+p$ satisfying Constraints C1-C3. Furthermore, the left eigenvector matrix of the matrix A_0 is parameterized by (3.15).

4. FURTHER REMARKS

This section gives some further remarks about the general solution proposed in Section 3.

4.1 Existence of Solutions to Constraints C1-C3

Theorem 3.1 indicates that Problem PIO has solutions if and only if there exist parameters F and s_i, g_i, $i = 1, 2, ..., n+p$ satisfying Constraints C1-C3. Now the questions is, are there always exist parameters F and s_i, g_i, $i = 1, 2, ..., n+p$ satisfying Constraint C1-C3? The following result provides a positive answer to this question (proof omitted).

Theorem 4.1 : Let Assumptions A1 and A2 hold. Then the following three statements are equivalent:
(1) The following condition holds:

$$\text{rank}\left(\begin{bmatrix} A-I & F \\ C & 0 \end{bmatrix}\right) = n+p \qquad (4.1)$$

(2) The following matrix pair is observable:

$$\left(\begin{bmatrix} A & F \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} C & 0 \end{bmatrix}\right) \qquad (4.2)$$

(3) The eigenvalues of the matrix A_0 are arbitrarily assignable by selecting matrices L and K.

Proof: Note that the observability of the matrix pair $[A \;\; C]$ is equivalent to the following

$$\text{rank}\left(\begin{bmatrix} sI-A \\ C \end{bmatrix}\right) = n, \quad \forall s \in \mathbb{C} \quad (4.3)$$

$(1) \Leftrightarrow (2):$ Statement (2) holds if and only if

$$\text{rank}\left(\begin{bmatrix} sI-A & -F \\ 0 & (s-1)I_p \\ C & 0 \end{bmatrix}\right) = n+p, \forall s \in \mathbb{C}$$

When $s \neq 1$, the above relation holds automatically because of (4.3). When $s = 1$, the above condition reduces to (4.1). Therefore, the two statements (1) and (2) are equivalent.

$(2) \Leftrightarrow (3):$ Since

$$A_0 = \begin{bmatrix} A & F \\ 0 & I_p \end{bmatrix} + \begin{bmatrix} L \\ K \end{bmatrix} \begin{bmatrix} C & 0 \end{bmatrix}$$

it follows from the well known pole assignment theory that the eigenvalues of the matrix A_0 can be arbitrarily assigned by selecting matrices L and K if and only if the matrix pair in (4.2) is observable.

□

In view of Theorem 4.1, the following constraint on the parameter matrix F is introduced:

Constraint $C4$: $\text{rank}\left(\begin{bmatrix} A-I & F \\ C & 0 \end{bmatrix}\right) = n+p$

It follows from Theorem 4.1 that Constraints C1-C3 always have solutions when the above Constraint C4 is satisfied.

4.2 The Inverse Matrix in (3.16).

If by the stage of solving L and K the matrices T, Z and V have already been fixed (constant), it is better to solve the matrices L and K from the following equation

$$\begin{bmatrix} T^T & V^T \end{bmatrix} \begin{bmatrix} L \\ K \end{bmatrix} = -Z^T \qquad (4.4)$$

using some numerically reliable algorithms for linear equations (e.g. the QR algorithm). However, there indeed exist applications which require general expressions of the gains (e.g. the minimum gain problem which minimizes the norm of the gain). In the following, it is shown that the order of the inverse matrix can be reduced by using the well-known Matrix Inversion Lemma, and two separate general expressions for the matrices L and K are derived.

Note that (3.3) implies

$$\text{rank}\,(T) = n \qquad (4.5)$$

thus there exists a matrix

$$P = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \in R^{(n+p)\times(n+p)}(P) \neq 0 \qquad (4.6)$$

such that

$$\det(P_1 T^T) \neq 0 \qquad (4.7)$$

Pre-multiplying the matrix P both sides of (4.4), gives

$$\begin{bmatrix} P_1 T^T & P_1 V^T \\ P_2 T^T & P_2 V^T \end{bmatrix} \begin{bmatrix} L \\ K \end{bmatrix} = -\begin{bmatrix} P_1 Z^T \\ P_2 Z^T \end{bmatrix} \qquad (4.8)$$

In view of (4.7) and the Matrix Inversion Lemma, Constraint $C2$ can be simplified as

$$\det\left(P_2 V^T - P_2 T^T (P_1 T^T)^{-1} P_1 V^T\right) \neq 0$$

Again using the Matrix Inversion Lemma, the matrices L and K are readily obtained as follows.

$$L = -(M_{11} P_1 + M_{12} P_2) Z^T \qquad (4.9)$$

and

$$K = -(M_{21} P_1 + M_{22} P_2) Z^T \qquad (4.10)$$

where

$$\begin{cases} M_{22} = \left(P_2 V^T - P_2 T^T (P_1 T^T)^{-1} P_1 V^T\right)^{-1} \\ M_{21} = -M_{22} P_2 T^T (P_1 T^T)^{-1} \\ M_{12} = -(P_1 T^T)^{-1} P_1 V^T M_{22} \\ M_{11} = (P_1 T^T)^{-1} (I - P_1 V^T M_{21}) \end{cases} \qquad (4.11)$$

4.3 *The degrees of freedom*

It follows from Theorem 3.1 that the design parameters involved in a PI observer design using the proposed parametric approach are F and s_i, g_i, $i = 1, 2, \ldots, n+p$. In view that an eigenvector of a matrix is not unique, one nonzero element in each vector g_i can be fixed to 1. Thus all these design parameters give a degree of freedom of $mp + (m-1)(n+p) + (n+p) = m(n+2p)$. These degrees of freedom should be well utilized to achieve some desired system performance, such as, disturbance decoupling, LTR/LQR, etc.

5. AN ILLUSTRATIVE EXAMPLE

Consider a linear system in the form of (2.1) with the following parameters (Duan 1993):

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5.1}$$

It is easy to verify that the Assumptions A1 and A2 are met. Further, it can be obtained that a pair of solutions to the right coprime factorization (3.9) are

$$N(s) = \begin{bmatrix} s-1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \; D(s) = \begin{bmatrix} s^2 - s - 1 & 1 \\ 0 & s \end{bmatrix} \tag{5.2}$$

In the following, a PI observer in the form of (2.2) with $p = 1$ will be designed for the above system (5.1). For simplicity, the eigenvalues s_i, $i = 1 \sim 4$ are restricted to be real and less than 1, thus Constraint C1 holds. In this case, the vectors g_i, $i = 1 \sim 4$ can also be restricted to be real, then Constraint C3 also holds. Denote

$$g_i = \begin{bmatrix} g_{i1} \\ g_{i2} \end{bmatrix}, \; i = 1 \sim 4$$

and

$$F^T = [\, f_1 \;\; f_2 \;\; f_3 \,]$$

we then have, following (3.10), (3.11) and (3.14), the columns of the matrices T, Z and V as

$$t_i = \begin{bmatrix} (s_i - 1) g_{i1} \\ g_{i1} \\ g_{i2} \end{bmatrix}, \; i = 1 \sim 4 \tag{5.3}$$

$$z_i = \begin{bmatrix} (s_i^2 - s_i - 1) g_{i1} + g_{i2} \\ s_i g_{i2} \end{bmatrix}, \; i = 1 \sim 4 \tag{5.4}$$

and

$$v_i = \frac{1}{s_i - 1}[(s_i - 1) f_1 g_{i1} + f_2 g_{i1} + f_3 g_{i2}] \tag{5.5}$$
$$i = 1 \sim 4$$

Therefore, Constraint C2 is

$$\det\left(\begin{bmatrix} t_1 & t_2 & t_3 & t_4 \\ v_1 & v_2 & v_3 & v_4 \end{bmatrix}\right) \neq 0 \tag{5.6}$$

where t_i and v_i, $i = 1 \sim 4$, are given by (5.3) and (5.5). Further, Constraint C4 for this system can be obtained as

$$f_2 \neq 0 \text{ or } f_3 \neq 0 \tag{5.7}$$

Thus, with f_i, $i = 1, 2, 3$ chosen satisfying (5.7), there always exist parameters s_i, g_{ij}, $i = 1 \sim 4$, $j = 1, 2$ satisfying (5.6). As a matter of fact, the contrary condition to (5.6), i,.e,

$$\det\left(\begin{bmatrix} t_1 & t_2 & t_3 & t_4 \\ v_1 & v_2 & v_3 & v_4 \end{bmatrix}\right) = 0 \tag{5.8}$$

can be viewed as a supper-plane in a $8 + 4 + 3 = 15$ dimensional space, thus the possibility of an arbitrary choice of these 15 parameters f_i, $i = 1, 2, 3$ and s_i, g_{ij}, $i = 1 \sim 4$, $j = 1, 2$ forming a point on this supper-plane is very small. From this point, an arbitrary choice of these parameters may very possibly satisfy constraint (5.6). In real applications, these degrees of freedom should be sought to meet certain system specifications. In the following, two groups of these parameters are specified, and the corresponding solutions are obtained.

When the parameters are chosen as

$$f_1 = f_2 = 0, \; f_3 = 1$$

$$s_i = -0.1 \times i, \; g_{ij} = i - j; \; i = 1 \sim 4, \; j = 1, 2$$

we have

$$U = \begin{bmatrix} 0 & -1.2 & -2.6 & -4.2 \\ 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ -0.9091 & 0 & -0.7692 & -1.4286 \end{bmatrix}$$

and

$$L = \begin{bmatrix} 2.9 & -1 \\ 4.24 & -1.2 \\ 0.82 & 0.1 \end{bmatrix}, \; K^T = \begin{bmatrix} 2.002 \\ 0 \end{bmatrix}, \; F = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

When the parameters are chosen as

$$s_i = -0.1 \times i, \; i = 1 \sim 4; \; f_j = j, \; j = 1 \sim 3$$

$$g_{11} = g_{31} = g_{22} = g_{42} = 0$$
$$g_{21} = g_{41} = g_{12} = g_{32} = 1$$

we have

$$U = \begin{bmatrix} 0 & -1.2 & 0 & -1.4 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ -2.7273 & -0.6667 & -2.3077 & -0.4286 \end{bmatrix}$$

and

$$L = \begin{bmatrix} 1.6 & 0.5674603 \\ 2.68 & 0.9987302 \\ -1 & 1.4 \end{bmatrix}$$

$$K^T = \begin{bmatrix} 0 \\ 0.4766667 \end{bmatrix}, \quad F = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

6. CONCLUSION

This paper proposes a simple complete parametric approach for PI observers of multivariable discrete-time linear systems. It is revealed that all the degrees of freedom involved in the design are composed of the following:

1) the eigenvalues of the observer system matrix;
2) a group of parameter vectors of dimension equal to the number of system outputs, and
3) the integral gain matrix.

Complete parametric expressions for all the observer gains as well as the eigenvector matrix are given in terms of these design parameters.

The design parameters can be utilized to achieve some desired system specifications and performances. This aspect of work will be considered in separate papers.

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