DESIGN OF ROBUST PID PARAMETERS FOR DISTRIBUTED PARAMETER PROCESSES

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Abstract: A design method is proposed for a two disk type mixed sensitivity problem for a distributed parameter system whose transfer functions become transcendental functions. The controller is assumed to be PID type. In this paper, a method of systematical design of robust controllers in visualized manner is proposed. The gains of the controller are treated in a parameter plane where three inequality conditions — one regarding stability and two regarding sensitivity and complementary sensitivity functions – are plotted. An intersection of these three areas satisfies the given robust conditions and stability condition. As this method essentially requires only the frequency response data and not necessarily require an exact mathematical model, thus it is versatile and applicable to practical problems. A heat exchanger is taken as an example of a distribution parameter process. By the experiment of the process with disturbances or parameter changes, we confirm the robustness of the controller designed by this method. *Copyright ©2000 IFAC*

Keywords: Robust control, PID control, Parameter, Distributed parameter system, Frequency response

1. INTRODUCTION

Still nowadays PID controllers are used extensively in many process industries. The important role of a PID controller is to keep product quality constant against disturbances and uncertainty in the control object. PID controller has considerable effect, considering that it has very simple structure. It is natural demand to incorporate new technology as the modern control theory gives into it. However, a design method taking account of robustness explicitly is not yet quite satisfactory, because H_∞ control theory is not applicable to a controller with fixed structure. (Ho, Hang & Cao 1995) To solve this difficulty, Saeki 1995 proposed a method plotting families of ellipse curves in parameter plane which indicate regions satisfying sensitivity conditions.

Graphical representation of the admissible PID parameter set has a good perspective and flexibility for design. This paper develops an integrated graphical design method which uses the Saeki's technique as to robustness condition and the parameter plane technique as to stability region and shows that this design method can apply to distributed parameter systems which have transcendental transfer functions and are inappropriate for analytical PID parameter tuning (Isaksson & Graebe, 1999).

In practice, an exact model of the object process can not be expected, in worse cases, the model itself can not be available. For such situation, this method still effective, because the formula 1 essentially uses the values $\{G(j\omega_i),\ i = 1, 2, \cdots\}$ which are obtainable from the frequency response data. Comparison with the Ziegler-Nichols formula (Ziegler & Nichols 1942) and this method shows that this design procedure gives better performance.

To illustrate the effectiveness of this method, PID control of a parallel flow heat exchanger is experimented. The heat exchanger is manipulated by a flow rate of one fluid to regulate the outlet temperature of another fluid. The system is a distributed parameter system and has a pure delay character in its dynamics which means the closed loop system has a narrow stability region and very sensitive to parameter change.

2. DESIGN METHOD

It is assumed that the transfer function of a plant $G(s)$ has been obtained. We consider the control system shown Fig.1 where C(s) represents the transfer function of a PID controller.

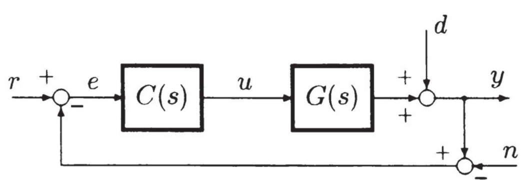

Fig 1. Feedback system

The sensitivity function $S(s)$, and the complementary sensitivity function $T(s)$ of the system are given as follows:

$$S(s) = \frac{1}{1+G(s)C(s)}, \; T(s) = \frac{G(s)C(s)}{1+G(s)C(s)} \quad (1)$$

We consider the following two disk mixed sensitivity problem:

Find controller parameters which satisfy the following conditions:

(1) The closed loop is sufficiently stable.
(2) $|S(j\omega)| < W_S(\omega), \quad \omega \in R$
(3) $|T(j\omega)| < W_T(\omega), \quad \omega \in R$

where W_S, W_T are preassigned weights.
The condition 1 can be represented as a good stability region. The condition 2 is a specification on robust control performance which requires low sensitivity in the low frequency band and the condition 3 is a specification on robust stability which requires low complementary sensitivity in the high frequency band. These last two conditions yield two disk problems and are represented by families of ellipse curves which determine each region. The solution of this problem is an overlap area of these three regions.
PID-controller is assumed to have the following form:

$$C(s) = k_p + k_d \frac{s}{1+s\tau} + k_i \frac{1}{s} \quad (2)$$

We show bellow how to calculate the regions which satisfy three conditions (1),(2),(3) respectively.

Firstly, using parameter plane method, we determine a good stable region K_R.
Secondly using the idea of Saeki (1995) the sensitivity and complementary sensitivity conditions are represented by families of ellipse curves, which define regions K_S, K_T in k_p - k_i parameter plane.

2.1 *Good stable region K_R*

To define the good stable region, we use the parameter plane method which draws a bad stable region by a map of a bad pole's position (for example, the right hand side of s-plane) to the parameter plane.

The closed loop characteristic equation of the system shown in Fig.1 becomes as follows:

$$1 + \left(k_p + \frac{k_i}{s} + k_d \frac{s}{1+\tau s}\right) G(s) = 0 \quad (3)$$

Substituting $s = \sigma + j\omega$, $(\sigma > 0)$ into this equation and separating the real part and the imaginary part make simultaneous equations of k_p and k_i (k_v is fixed) which is solvable when $\omega \neq 0$. The solutions are functions of σ, ω (and k_v) and defines a map s-plane to k_p - k_i parameter plane. When $\omega = 0$, the characteristic equation indicates one parameter family of lines.
The basic idea of the parameter plane method is that if all point in the bad pole position in s-plane are mapped to the parameter plane, each point (k_p, k_i) not belonging to the image never yields bad stable poles $\sigma + j\omega$. Thus the formula is obtained.

Formula 1.
The stable region K_R is the complementary set of union of the regions defined by a set (k_p, k_i)

$$k_p = \frac{\sigma G_I - \omega G_R}{\omega(G_R^2 + G_I^2)} - k_d \frac{2\sigma + \tau(\sigma^2 + \omega^2)}{(\sigma\tau + 1)^2 + \tau^2\omega^2}$$
$$k_i = -\frac{(\sigma^2 + \omega^2) G_I}{\omega(G_R^2 + G_I^2)} + k_d \frac{\sigma^2 + \omega^2}{(\sigma\tau + 1)^2 + \tau^2\omega^2}$$
$$\forall \sigma > 0, \quad \forall w > 0$$

and the family of lines in $k_p - k_i$ plane defined by

$$1 + \left(k_p + \frac{k_i}{\sigma} + k_d \frac{\sigma}{\tau\sigma + 1}\right) g(\sigma) = 0$$
$$\forall \sigma > 0.$$

where

$$G(\sigma + j\omega) = G_R(\sigma, \omega) + G_I(\sigma, \omega) j.$$

2.2 *Small sensitivity egion K_S*

Hereafter the value of G at $s = j\omega$ is written as $G(j\omega) = a(\omega) + jb(\omega)$. Then the condition $|S(j\omega)| < W_S(\omega)$ becomes:

$$(1 + ak'_p + bk'_i)^2 + (bk'_p - ak'_i)^2 > \frac{1}{W_S^2} \quad (4)$$

where

$$k'_p = k_p + \frac{\omega k_d}{1+\tau^2\omega^2}, \quad k'_i = \frac{k_i}{\omega} - \frac{\omega^2 k_d}{1-\tau^2\omega^2}$$

Eq.(4) represents a ellipse in $k_p - k_i$ plane, and gives the following formula.

Formula 2.
The region satisfying $|S(j\omega)| < W_S(\omega)$ is given in the parameter plane $k_p - k_i$ as the outer region of the ellipse.

$$k_p = \frac{1}{W_S\sqrt{a^2+b^2}}\sin\theta - \frac{a}{a^2+b^2} - k_d\frac{\omega^2\tau}{1+\omega^2\tau^2}$$
$$k_i = -\frac{\omega}{W_S\sqrt{a^2+b^2}}\cos\theta - \frac{b\omega}{a^2+b^2} + k_d\frac{\omega^2}{1+\omega^2\tau^2}$$

where $0 \le \theta \le 2\pi$.

2.3 *Small complementary sensitivity egion K_T*

$|T(j\omega)| < W_T(\omega)$ becomes

$$\frac{(ak'_p + bk'_i)^2 + (bk'_p - ak'_i)^2}{(1 + ak'_p + bk'_i)^2 + (bk'_p - ak'_i)^2} < W_T^2 \quad (5)$$

Eq.(5) represents a ellipse in $k_p - k_i$ plane, and gives the following formula.

Formula 3.
The region satisfying $|T(j\omega)| < W_T(\omega)$ is given in the parameter plane $k_p - k_i$
(a) when $0 < W_T < 1$,,
K_T is the inner region of the ellipse below:
(b) when $W_T > 1$
K_T is the outer region of the ellipse below:

$$k_p = \frac{W_T\sin\theta}{\sqrt{a^2+b^2}(1-W_T^2)} + \frac{a}{(a^2-b^2)\left(\frac{1}{W_T^2}-1\right)} - k_d\frac{\omega^2\tau}{1+\omega^2\tau^2}$$
$$k_i = -\frac{W_T\omega\cos\theta}{\sqrt{a^2+b^2}(1-W_T^2)} + \frac{b\omega}{(a^2+b^2)\left(\frac{1}{W_T^2}-1\right)} + k_d\frac{\omega^2}{1+\omega^2\tau^2}$$

where $0 \le \theta \le 2\pi$.

2.4 *Determination of controller parameters*

If there exists an intersection of three regions K_R,K_S and K_T, it means that there exist controller parameters satisfying the three conditions. We can adopt values of the central point of the intersection area.

3. DESIGN OF PID CONTROLLER FOR A HEAT-EXCHANGER

This section presents the synthesis of a PID controller for a parallel flow heat-exchanger shown in Fig.2.

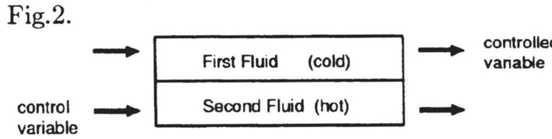

Fig 2. Parallel flow heat exchanger

A parallel flow heat exchanger has a pure delay charactor in its dynamics and is nonminimun phase process, thus it is more difficult to control than the counterpart a counter flow heat exchanger (Kanoh, (1982)).

3.1 *Equation and transfer function*

The parallel flow heat-exchanger's dynamics are described by the following partial differential equations:

$$\begin{cases} \dfrac{\partial\theta_1}{\partial t} + \dfrac{F_1 L}{V_1}\dfrac{\partial\theta_1}{\partial\ell} = \dfrac{kA}{C_{p_1}\rho_1 V_1}(\theta_2 - \theta_1) \\ \dfrac{\partial\theta_2}{\partial t} + \dfrac{F_2 L}{V_2}\dfrac{\partial\theta_2}{\partial\ell} = \dfrac{kA}{C_{p_2}\rho_2 V_2}(\theta_1 - \theta_2) \end{cases} \quad (6)$$

with boundary conditions

$$\theta_1(t,0) = \theta_{1i}, \quad \theta_2(t,0) = \theta_{2i}$$

where $\theta_1(t,\ell)$, $\theta_2(t,\ell)$ represent temperatures of the first and the second fluids at a time t and at a distance ℓ from the entrance and ρ: Density $[kg/m^3]$, A: Heat transfer area $[m^2]$, C: Specific heat $[kJ/kgK]$, F: Flow rate $[m^3/s]$, L: Total passage length $[m]$, k: Overall heat transfer coefficient $[kJ/m^2sK]$, V: Volume of fluid passage $[m^3]$, Control problem is to control the outlet temperature $\theta_1(t,L)$ by manipulation of the flow rate of the second fluid $F_2(t)$. The heat transfer coefficients are dependent of the fluid velocity the relation of which can be linearized in the vicinity of a steady state $\theta_{1s}(\ell)$, $\theta_{2s}(\ell)$, F_{1s}, F_{2s}, k_s. Linearlized equations becomes as follows:

$$\begin{cases} r\dfrac{\partial x_1}{\partial t} + \dfrac{\partial x_1}{\partial \xi} = a_1(x_2 - x_1) \\ \qquad\qquad + b_1 e^{-(a_1+a_2)\xi}u(t) \\ \dfrac{\partial x_2}{\partial t} + \dfrac{\partial x_2}{\partial \xi} = a_2(x_1 - x_2) \\ \qquad\qquad - (a_2 - b_2)e^{-(a_1+a_2)\xi}u(t) \end{cases} \quad (7)$$

where

$$a_1 = \frac{k_s A}{C_{p1}\rho_1 F_{1s}},\quad a_2 = \frac{k_s A}{C_{p2}\rho_2 F_{2s}},\quad b_1 = \frac{m_2 F_{2s} A}{C_{p1}\rho_1 F_{1s}},$$
$$b_2 = \frac{m_2 A}{C_{p2}\rho_2},\quad T_1 = \frac{V_1}{F_{1s}}[s],\quad T_2 = \frac{V_2}{F_{2s}}[s],$$
$$\xi = \ell/L,\quad t = t/T_2,\quad r = T_1/T_2,$$
$$\theta_1 = \theta_{1s}(1+x_1),\ \theta_2 = \theta_{2s}(1+x_2),\ F_2 = F_{2s}(1+u).$$

Using Laplace transformation, we obtain a transfer function as follows:

$$G(s) = \frac{x_1(s,1)}{u(s)}$$
$$= \frac{b_1(e^{p_1} - e^{p_2}) + \beta(s)\left(\dfrac{e^{p_1} - e^{p_3}}{p_1 - p_3} - \dfrac{e^{p_2} - e^{p_3}}{p_2 - p_3}\right)}{p_1 - p_2} \quad (8)$$

where

$$\beta(s) = b_1 s + a_1(a_2 - b_1 - b_2)$$

and $p_3 = -a_1 - a_2$, p_1, p_2 are the roots of the equation:

$$(p + rs + a_1)(p + s + a_2) - a_1 a_2 = 0$$

3.2 *Experimental heat exchanger*

Fig.3 shows the experimental heat exchanger which has $12.25m$ passage length, $1.766 \times 10^{-3} m^3$ passage volumes, and $0.147m^2$ heat exchange area.

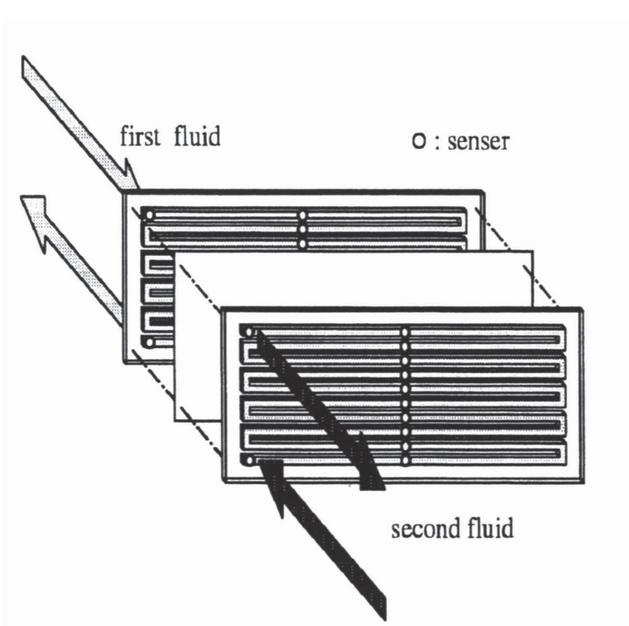

Fig 3. Experimental heat exchanger

3.3 *Frequency response*

From the transfer function (8), the frequency response is calculated and plotted in Fig.4 where the experimental frequency response data also are plotted. There exists a little difference between the two.

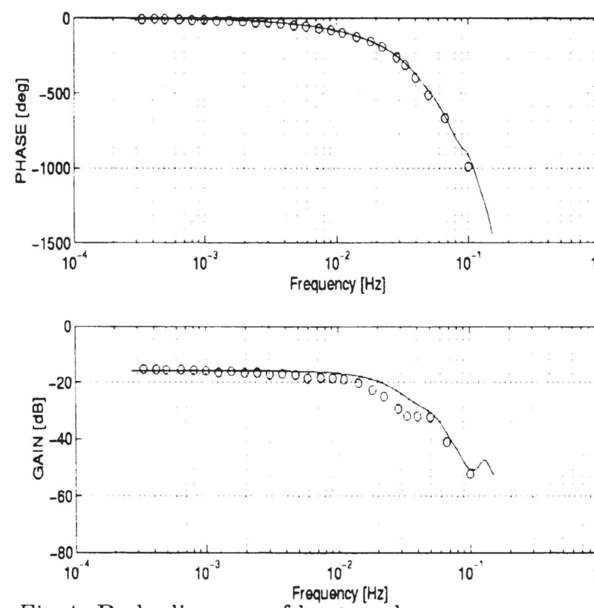

Fig 4. Bode diagram of heat exchanger

3.4 *Stability region*

The stable region calculated by formula 1 is plotted in Fig.5. As shown in Fig.4, the parallel heat exchanger is dominated by the delay, and the derivative action cannot be used.

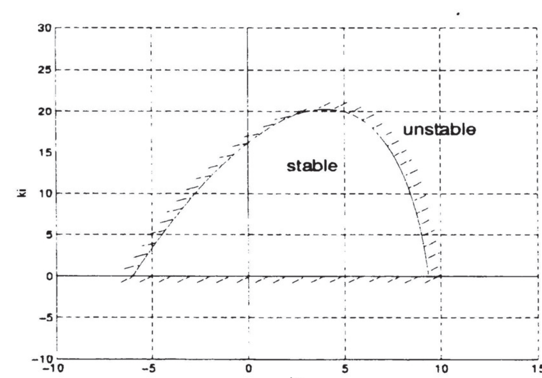

Fig 5. Stable region $K_R(\omega)$

3.5 *Synthesis*

First we adopt the following weight functions:

$$W_S = 0.08\left|\frac{200s+1}{10s+7}\right|, \quad W_T = 1.3\left|\frac{0.1s+1}{2.0s+1}\right|$$

Then we have three plots of the regions K_R,K_S and K_T as shown in Fig.5,Fig.6 and Fig.7.

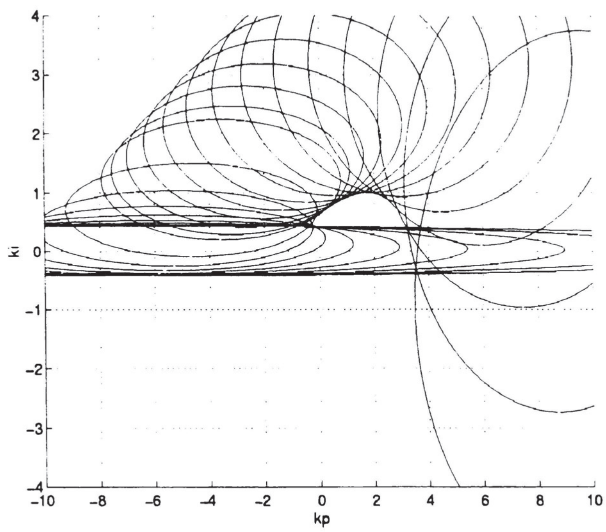

Fig 6. Permissible sets $K_S(\omega)$

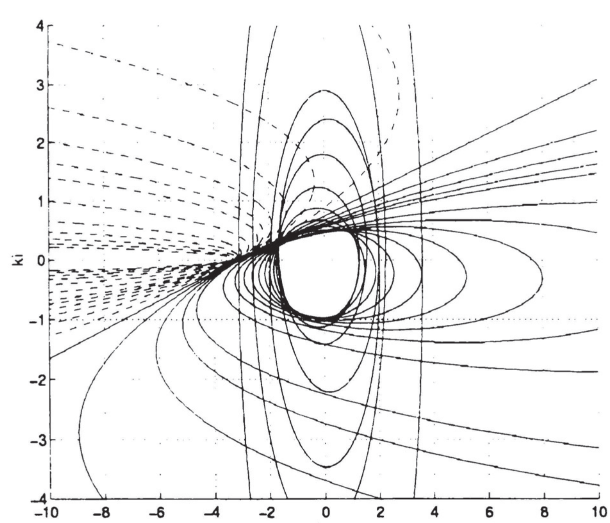

Fig 7. Permissible sets $K_T(\omega)$

Fig.8 is the gain diagram for $[k_p, k_i]$=[0.7,0.43] at which all three condition are satisfied. The solid lines represent S, and T, the broken lines represent W_S, W_T. Both S and T are above the graphs of W_S, W_T over all frequencies.

3.6 *Design on experimental data*

This method does not require any exact mathematical model or analytical transfer function, it only require frequency response data, $\{\omega_i,\ Gain_i,\ Phase_i,\ \ i = 1, 2, \cdots\}$. From the data, we can make $G(j\omega)$ from the following relation:

$$G(j\omega_i) = Gain_i \cdot e^{jPhase_i}$$

Fig.9 is plotted from the experimental data.

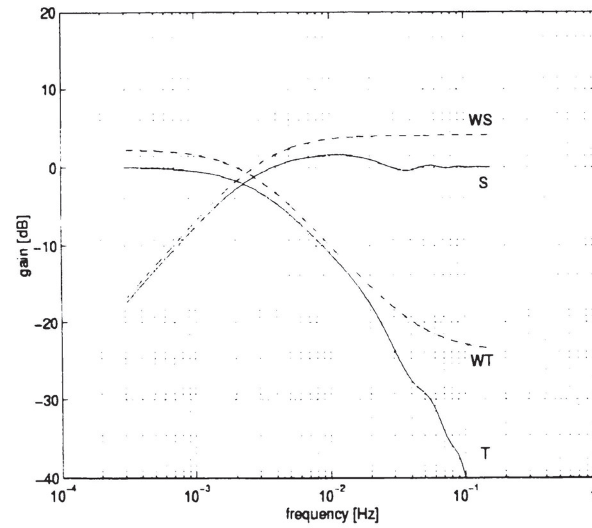

Fig 8. Gain diagram of S, T, W_S and W_T (Analysis)

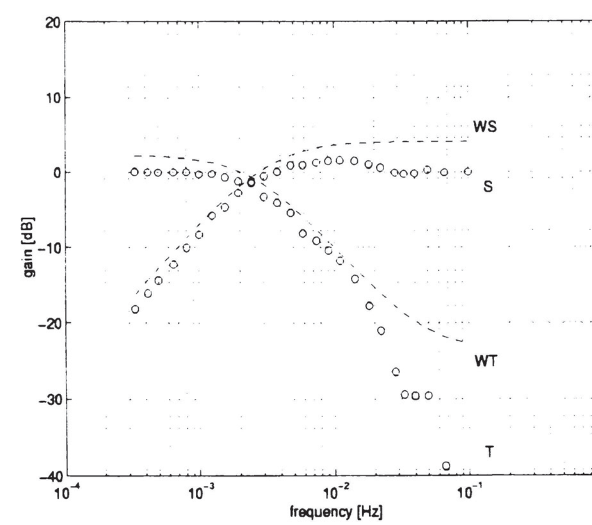

Fig 9. Gain diagram of S, T, W_S and W_T (Experiment)

Comparison of Fig.8 and Fig.9 makes sure that if we have much frequency response data, we can tune the robust PID parameters.

4. CONTROL EXPERIMENT

Control experiments of set response are made for the parameters obtained from this method using analytical model (ANAL), using experimental data (DATA) and from other control laws for comparison; Ziegler Nichols (NZ) method and trial and error (TE) method. The oscillating frequency in the NZ method is determined from the stability limit of Fig.5, and the parameters in TE method are chosen as the best one from tens of step responses. We determine the following parameter values:

Table 1

	name	k_p	k_i	line
ANAL	K_{FR}	1.3,	0.51	solid
DATA	K_{Trf}	0.7,	0.43	dashed
ZN	K_{ZN}	4.3,	0.97	dotted
TE	K_{Third}	2.0,	0.23	broken

Fig.11 shows step responses without parameter perturbation. K_{ZN} is oscillatory and out of the adjustable range, while K_{FR}, and K_{Trf} show good response. Expectedly K_{Trf} gives better response than K_{FR} and K_{Third} gives the best response a little over that of this method.

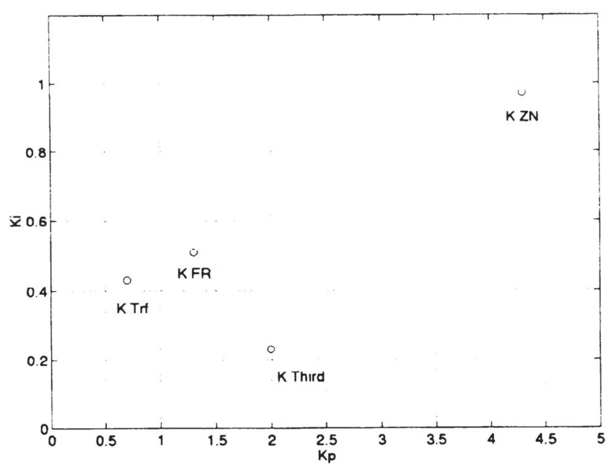

Fig 10. Gain parameters of the controller

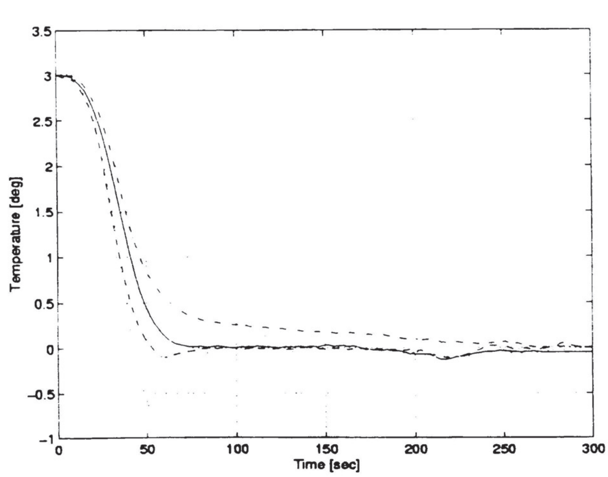

Fig 11. Response without parameter changes

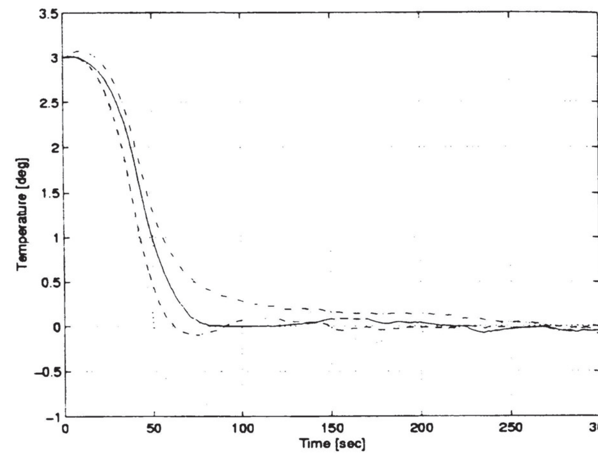

Fig 12. Response with parameter changes

When there is parameter perturbation, as shown in Fig.12 K_{FR} keeps unchanged but K_{Third} becomes little oscillatory.

5. CONCLUSION

This paper proposes a method of tuning robust PID controller parameters for practical systems including distributed parameter systems. We improve the method proposed by Saeki (1995) and combine it with the parameter plane method for the stable region. This method does not require any exact mathematical model or analytical transfer function, it only require frequency response data for many cases, which is advantage of this method. Experiments for a parallel flow heat exchanger with parameter perturbation shows that the controller parameters obtained by this method have more robustness than other PID parameters.

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