

## ONLINE ESTIMATION OF SHIP STEERING DYNAMICS AND ITS APPLICATION IN DESIGNING AN OPTIMAL AUTOPILOT

Minh-Duc Le\*

\* Graduate School of Engineering, Hiroshima University. Higashi Hiroshima-shi  
Kagamiyama 1-4-1, Hiroshima 739-8527, Japan  
Tel. (Mobile): (81) 070-5671-2114. Email: melinh@ipc.hiroshima-u.ac.jp

**Abstract:** Recursive Least Square (RLS) Algorithm applied to a Multivariate Auto-Regressive (MAR) process is used to estimate ship steering dynamics online. The estimation method is then linked to the Linear Quadratic (LQ) Algorithm to design an optimal autopilot for steering ships.

The estimation method was applied to several ships and model ships and in all the cases the estimated parameters converged well. The design algorithm was used to construct a tracking system for course keeping and course changing manoeuvres. Simulation results for the ships show the robustness of the estimation method and prove that the autopilot has very good performance. Copyright © 2000 IFAC

**Keywords:** estimation and identification, autoregressive models, least-squares algorithm, quadratic control, ship control, design systems

### 1. INTRODUCTION

Ship steering dynamics is of interest when evaluating ship manoeuvrability as well as when designing ship autopilots and steering systems. Designing a computer-based autopilot for ships is always a challenging task in marine control engineering. Ships operating in seawater are often strongly influenced by unpredictable environmental disturbances such as wind, wave and current. Therefore to navigate safely and economically, the ship must have a robust autopilot system with good steering characteristics. To design such a robust computer-based autopilot system that can be adapted well to the changes of the environment a suitable mathematical model representing ship steering dynamics should be constructed. And, one of the challenging problems involved in designing the computer-based autopilot is to find a suitable estimation method for a chosen model.

Methods for determining ship steering dynamics with high accuracy have been of the focus of many studies over a long period of time. Astrom and Kallstrom (1976) did the pioneer work in identification of ship steering dynamics. Abkowitz (1980) presented results of the identification of ship hydrodynamic characteristics based on Extended Kalman Filter, the results have long been seen as excellent. More recently Le, et al. (2000) has proposed a new and effective method for estimation of ship linear hydrodynamic coefficients. Since the main parameters in ship steering dynamics are the linear coefficients in ship hydrodynamic characteristics, methods for the estimation of ship hydrodynamic coefficients also contribute to the development of methods for estimation of ship steering dynamics.

In modern control theory, identification algorithms are often combined with appropriate control laws to construct automatic control systems. In marine control, much research has been carried out in this

direction. Several authors have applied stochastic approach to analysis and control of ship motion (Astrom (1976) Ohtsu *et al.* (1979), Kallstrom (1979), Holzinger (1990) and Fossen (1994)). More recently, Wellstead, et al. (1991) combined a self-tuning control algorithm with the RLS algorithm to design control systems. Since then, the self-tuning control algorithm has been developed into a route-tracking controller in the PID form for ships (Mizuno *et al.*, 1989) and autopilots for ships (Nguyen, *et al.*, 1998, 2000; Nguyen, and Le, 2000). Besides this, the RLS algorithm was also combined with an optimal control law to design an adaptive dynamic positioning system for vessels (Iida, 1990).

This paper presents a new method for online estimation of ship steering dynamics and its application to design an optimal autopilot system for ships. Ship steering dynamics is expressed by a MAR model with unknown parameters to be estimated. The RLS algorithm is used as an online estimator to estimate the parameters of the assumed model. The parameters estimated by the RLS algorithm are then used as the input to calculate control gain of the LQ optimal control law for designing the autopilot. The estimation method was applied to estimate steering dynamics of several ships and model ships. The design method was verified by computer simulation of several ships and model ship during course keeping and course changing manoeuvres.

In this paper, Section 2 describes the method for estimation of ship steering dynamics and the results of its application to several ships and model ships. Section 3 presents the LQ control algorithm and its applications to design the optimal autopilot using steering estimated parameters as described in Section 2. The design procedure is emphasized and a comprehensive set of application results of the designing method is included in this section. Finally, Section 4 highlights some conclusions based on estimation results and computer simulation results applying the optimal autopilot, and some directions for future research.

## 2. ESTIMATION OF SHIP STEERING DYNAMICS

This section describes in detail the application of the RLS algorithm to a MAR process for online estimation of ship steering dynamics. Several results of the estimation method applied to various ships and model ships are also presented.

### 2.1 RLS algorithm applied to estimate parameters of a MAR model

Ship dynamics can be described by the following state-space equation (Astrom, and Kallstrom, 1976):

$$\dot{x} = Ex + Fu + De \quad (1)$$

where  $x(t)$ ,  $u(t)$ ,  $e(t)$  are state vector, control vector and disturbance vector, respectively;  $E$ ,  $F$ ,  $D$  are the corresponding matrices, to be estimated. In this study both  $x(t)$ ,  $e(t)$  are assumed to be measured or at least obtained by state estimation; in fact they were calculated from measurement of ship position.

The above equation can be conveniently expressed by a MAR process of order  $p$ , as shown in (2):

$$v_i = \omega + \sum_{l=1}^p A_l v_{i-l} + \epsilon_i, \quad (i = p, \dots, N) \quad (2)$$

Here  $v_i \in R^m$  is a time series of state vectors, observed as equally spaced instant  $i$ ;  $\omega \in R^m$  is a parameter vector of intercept terms and is included to allow for a nonzero mean of time series  $v_i$ ;  $A_l \in R^{m,m}$  are matrices of unknown (constant) coefficients expressed the process;  $\epsilon_i \in R^m$  are random vectors of noises, with the present of  $\omega$ ,  $\epsilon_i$  can be assumed to have zero mean and covariance matrix  $C$ . Relations between matrices in formulas (1) and (2) can be easily derived using Multivariate Auto-regressive theory or differential formula.

Denote the augmented state vector by (3) (with  $n = m+1$ ) and the augmented coefficient matrix by (4):

$$v_i = [1^T \ v_{i-1}^T \ \dots \ v_{i-p}^T]^T \in R^n, \quad (i = p, \dots, N) \quad (3)$$

$$A = [\omega \ A_1 \ \dots \ A_p] \in R^{m,n} \quad (4)$$

, the MAR( $p$ ) model (2) can be rewritten as:

$$v_i = A v_i + \epsilon_i, \quad (i = p, \dots, N) \quad (5)$$

The parameter matrix  $A$  can then be estimated using RLS Algorithm with the availability of measurement  $v_i$  and  $v_i$ . Introducing the matrices:

$$U = \sum_{i=p}^N v_i v_i^T; \quad W = \sum_{i=p}^N v_i v_i^T \quad (6)$$

, then the best linear unbiased estimate for the matrix  $A$  is derived as:

$$\hat{A} = W U^{-1} \quad (7)$$

, and an estimate for the covariance matrix  $C$  is given by:

$$\hat{C} = \frac{1}{N-n} \sum_{i=p}^N \hat{\epsilon}_i \hat{\epsilon}_i^T \quad \text{with } \hat{\epsilon}_i = v_i - \hat{A} v_i \quad (8)$$

In practice, to progressively reduce the emphasis placed on past information a forgetting factor (FF)  $\lambda$  with value between 0.95 to 0.998 (Wellstead et al., 1991, Hang et al., 1993) was also used.

The RLS algorithm applied to estimate ship steering dynamics is summarized as follows.

At time interval  $i$ :

- Step 1: Form vectors  $v_i$  and  $v_i$  using new data according to formulas (2) and (3).
- Step 2: Add new values to  $U$  and  $W$  according to (6), noting that the FF can be used.
- Step 3: Calculate the estimates for matrices  $A$  and  $C$  using formulas (7) and (8).
- Step 4: Wait for the next step to elapse and loop back to Step 1 until the end of the estimation process.

## 2.2 Results of estimation of ship steering dynamics

Usually the input and control variables of the state-space equation (1) expressing ship steering dynamic are chosen as follows:  $x = [v \ r \ \psi]^T$  where  $v$ ,  $r$ ,  $\psi$  respectively are ship sway, angular velocities and heading angle, and  $u = \delta$  with  $\delta$  is the rudder deflection. Since the order of the differential equation (1) is 1, it would be suitable to choose the parameters in equation (2) as:  $p = 1$ ,  $m = 4$ ,  $n = 5$ . In this case, expressions of vectors  $v_i$  and  $v_i$  are derived as follows:

$$v_i = [v_i \ r_i \ \psi_i \ \delta_i]^T \in R^4, (i = p, \dots, N) \quad (9)$$

$$v_i = [1 \ v_i \ r_i \ \psi_i \ \delta_i]^T \in R^5, (i = p, \dots, N) \quad (10)$$

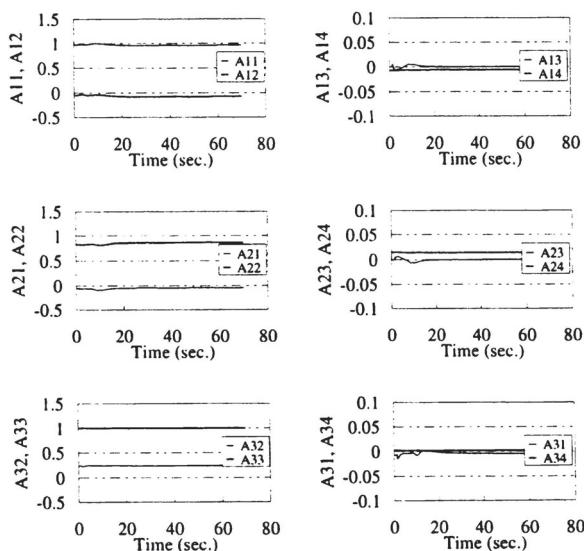


Fig.1. Time series of estimated parameters for the model ship SR221B (sampling time = 0.25 sec.)

Parameters of ship steering dynamics then can be estimated using the four steps given in 2.1

Figure 1 shows the time series of the estimated parameters for SR221B, a model ship. Data for the estimation was collected from a 10deg. Zigzag test in a towing tank. Estimation results for a training ship (the Shioji Maru) is shown in Figure 2 and data was collected from a 20deg. Zigzag trial. In these figures, only parameters concerning the input vector  $x$  are given, as they are the parameters of the steering dynamics. From both figures, it is clear that the estimated parameters of ship steering dynamics converge very well and this proves the effectiveness of the estimation method.

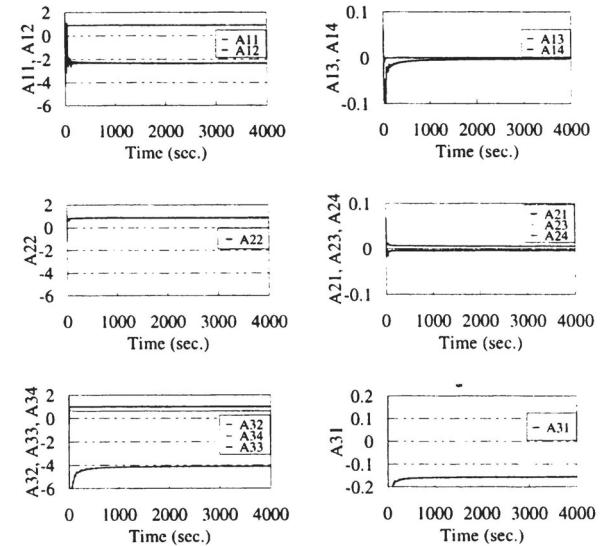


Fig.2. Time series of estimated parameters for the ship Shioji Maru (sampling time = 0.50 sec.)

## 3. OPTIMAL AUTOPILOT DESIGN

In this section the LQ algorithm is briefly presented, then the combination of the estimation method described in the previous section and LQ algorithm for designing an optimal autopilot is discussed in more detail. The design procedure is emphasized and a comprehensive set of application results of the designing method is included.

### 3.1 The LQ optimal control algorithm

Suppose that the output  $y(t)$  of the control system (1) is expressed by the following equation:

$$y = Gx + Hu \quad (11)$$

where  $G$  and  $H$  are corresponding matrices. To design an optimal controller for tracking a time varying reference (desired) trajectory  $y_d(t)$ , let define  $\tilde{y} = y - y_d$  the trajectory error vector, and

$$J = \sum_{t=0}^{N-p} [\tilde{y}^T Q \tilde{y} + u^T P u] \quad (12)$$

the performance index, where  $Q \geq 0$  and  $P > 0$  are weighting matrices. Solution of the LQ Tracker Problem is a control law that minimizes the performance index (12) and can be expressed by following equation (Fossen, 1994):

$$u = G_1 x + G_2 y + G_3 e \quad (13)$$

here gain matrices  $G_1$ ,  $G_2$ ,  $G_3$  are calculated from:

$$\begin{cases} G_1 = -P^{-1} F^T R \\ G_2 = -P^{-1} F^T (E + FG_1)^{-T} G^T Q \\ G_3 = P^{-1} F^T (E + FG_1)^{-T} RD \end{cases} \quad (14)$$

,with  $R$  the solution of the discrete-time Riccati equation:

$$RE + E^T R - RFP^{-1}F^T R + \tilde{Q} = 0 \quad (15)$$

$$\tilde{Q} = G^T Q G \quad (16)$$

### 3.2 Procedure of optimal autopilot design

From equation (13), it is clear that if the parameters of the state-space equation (1) have been estimated, the optimal solution for the control vector  $u(t)$  can be calculated. Therefore, a combination of the estimation method presented in Section 2 and the LQ algorithm could give an approach to designing an optimal autopilot.

To design a control system, the input and output variables should be chosen. Choosing of the input variable was described in Section 2. The output variables (described by vector  $y$  in equation (11)) are usually chosen based on the desirable control output. For ship steering systems  $v$ ,  $r$ ,  $\psi$  are often chosen as the output variables (in simpler cases, only  $\psi$  may be chosen, but in more general tracking controllers some other variables such as ship position, ship surge velocity and so on can also be added). Then matrix  $G$  is given as:

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (17)$$

The procedure for designing the optimal autopilot can be summarized by following steps:

At time interval  $i$ :

(a) Step 1: Form vectors  $v_i$  and  $v_i$  using new data according to formulas (2) and (3), and output

vector  $y$ .

(b) Step 2: Estimate parameters of MAR process (5) using formulas (6), (7) and (8), then calculate the ship steering dynamics (matrices  $E$ ,  $F$ ,  $D$  in the state-space equation (1)).

Matrix  $G$  is given by (17) and matrix  $H$  can be easily calculated from matrix  $A$ , which has already been estimated in (5).

(c) Step 3: Use the results of Step 2 to find solution  $R$  of the discrete-time Riccati equation (15) and then calculate gain matrices  $G_1$ ,  $G_2$ ,  $G_3$  according to formula (14).

(d) Step 4: Substitute new values of  $G_1$ ,  $G_2$ ,  $G_3$  and the current values of input, output and disturbance vector  $x_i$ ,  $u_i$ ,  $e_i$  into equation (13) for the new optimal value of the control vector  $u$ .

(e) Step 5: Wait for the next step to elapse and loop back to Step 1 until the end of the control process.

Among the practical aspects of designing and implementing such an autopilot, the main task is to choose design parameters such as proper weightings ( $Q$  and  $P$ ) in the performance index function, sampling time and initial parameters during implementing computer simulations and full-scale experiments. The values of weighting matrices ( $Q$  and  $P$ ) are usually chosen based on the aims of the designing optimal autopilots. For example, when considering the energy saving problem one may choose a large value for  $P$  (compared to value of  $Q$ ) while if the accuracy of the control process is emphasized, the large value should be chosen for  $Q$ . Sampling time could be decided based on the allowable rate of rudder deflection and also on a proper sampling rate of measurement equipments. Parameters of ship steering dynamics can be estimated with or without knowing the initial values. Strip theory can be applied to estimate the initial values for the parameters. Fossen (1994) gives formulas for this purpose, but he also cautions that care should be taken when using those formulas (derived from strip theory) since some rough approximations have been made. However, the values are highly useful as a priori information for a recursive parameter estimator. For using the initial values of the parameters, formulas for continuous least-squares estimator were derived and are given in the Appendix.

### 3.3 Simulation results of applying the design method

The RLS algorithm was successfully applied to estimate ship steering dynamics as presented in section two and can be linked to the LQ optimal control algorithm for designing an optimal autopilot as analysed above. The procedure of designing an optimal autopilot given in 3.2 was applied to some ships and model ships. Simulations of course keeping

and course changing manoeuvres using the optimal autopilot were performed for those ships and model ships. Figure 3 gives an example of computer simulation results of the above manoeuvres for the SR221B model ship. The simulation was performed for an 80-0-40 manoeuvre that means the ship starting from zero-degree course was ordered to change to 80-degree course, then to zero-degree course and finally to 40-degree course, after each of the changing course manoeuvres, the ship was ordered to keep the course stable for a period of about 200 seconds. Rudder deflection was limited in the range of -35 to +35 degrees as usually required for most ships. The ship changed to the new course properly and there were only two slight oscillations before stability has achieved. The overshoot that did occur, however, was rather small. The course keeping manoeuvres were performed very well.

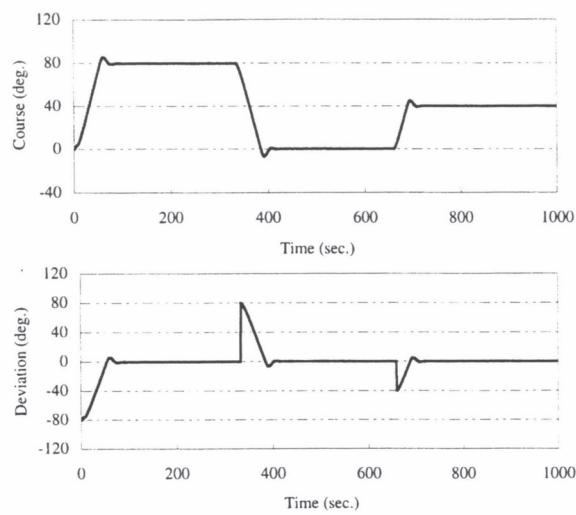


Fig.3. Time series of ship responses (course and deviation) for model ship SR221B during keeping and changing course maneuvers

### 3.4 Evaluation of the design algorithm

For a conventional autopilot design approach (such as the PD control law, the SISO MAR model) ship heading is often oscillated several times before achieving stability on the new course. As mentioned above, using the current design algorithm the course-changing manoeuvres were performed rather well: there were very few slight oscillations during each manoeuvre. That means the autopilot design based on the current algorithm has performed better than an autopilot design based on a conventional approach would have.

In addition, this design algorithm has also proved its robustness as showed in Figure 4. During each of the course changing processes, the parameters of the steering dynamics were changed. At the beginning of the course changing action especially the largest

variations in the parameters occurred, however they quickly rebounded (to stable values). This shows that the autopilots could adapt well to environment changes.

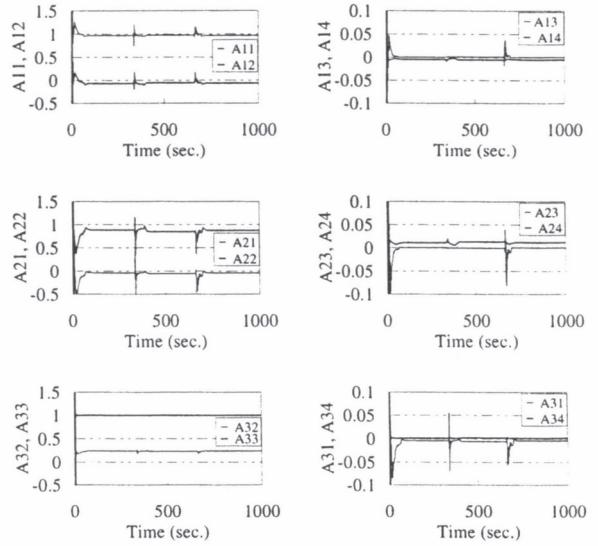


Fig.4. Time series of estimated parameters for the model ship SR221B during course keeping and course changing maneuvers (using autopilot)

## 4. CONCLUSIONS AND FUTURE RESEARCH

Recursive Least Square (RLS) Algorithm applied to a Multivariate Auto-Regressive (MAR) process was successfully used to estimate ship steering dynamics online. The estimation method was then combined with the Linear Quadratic (LQ) Algorithm to design an optimal autopilot for steering ships. The estimated parameters of steering dynamics for the ships and ship models converged well and computer simulation results showed that using the described design approach, the optimal autopilot had excellent performance both with keeping and changing the courses as desired. Since the method for estimation of ship steering dynamics can be applied both to scale model tests and full scale ship trials, it provides a possibility to analyse effects due to scaling.

To verify the optimal autopilot design approach full-scale trials should also be carried out for investigation of the steering characteristics in practice and its ability to adapt to the environmental. Moreover, it is expected the optimal autopilot will further be developed into an optimal route-tracking controller for ships.

## ACKNOWLEDGEMENTS

The sincerest acknowledgement is expressed to Mr. Duc-Hung Nguyen of the Tokyo University of Mercantile Marine for his valuable discussions on

topics in this paper and for providing the data of the 20deg. Zigzag trial of the training ship Shioji Maru. Hearty thanks are expressed to people of Houryuji (Hiroshima, Japan), especially to their leaders, Mrs. Houmyou Saitou and Mr. Shodo Seta, for their best mutual and financial support in this study.

## APPENDIX – DERIVATION OF FORMULAS FOR CONTINUOUS LEAST-SQUARES ESTIMATOR

Continuous Least-squares estimate of a MAR process (5) can be obtained by minimizing the integral square error with respect to parameter matrix  $\hat{A}$ :

$$\min I = \int_0^t \|v(\tau) - \hat{A}(\tau)v(\tau)\|^2 d\tau \quad (18)$$

Differentiating  $I$  with respect to  $\hat{A}$  gives:

$$0 = \frac{\partial I}{\partial A} = -2 \int_0^t [v(\tau) - \hat{A}(\tau)v(\tau)]v^T(\tau)d\tau \quad (19)$$

Defining the estimator gain matrix  $K$  as:

$$K(t) = \int_0^t v(\tau)v^T(\tau)d\tau \quad (20)$$

Differentiating of (19) with respect to time yields:

$$\dot{\hat{A}} \left[ \int_0^t v(\tau)v^T(\tau)d\tau \right] = [v(t) - \hat{A}v(t)]v^T(t) \quad (21)$$

Finally, the parameter update law is derived using notations (8) and (20):

$$\dot{\hat{A}} = \hat{\epsilon}(t)v^T(t)K(t) \quad (22)$$

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