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Strongly Stabilizing Controller Synthesis for a Class of MIMO Plants ⋆

H. Özbay * **A. N. Gündeş** **

* *Department of Electrical & Electronics Engineering, Bilkent University, Ankara, 06800 Turkey (e-mail: hitay@bilkent.edu.tr)*
** *Department of Electrical & Computer Engineering, University of California, Davis, CA 95616, USA (e-mail: angundes@ucdavis.edu)*

Abstract: The strong stabilization problem (i.e., stabilization by a stable feedback controller) is studied for unstable MIMO plants with arbitrary number of finitely many poles but no more than two blocking zeros in the extended right half plane. Simple strongly stabilizing controllers, of order not exceeding that of the plant, are obtained for such plants satisfying the parity interlacing property. Connections with earlier design methods are illustrated: for this particular class of plants, it is shown that a sufficient condition appearing in earlier publications is equivalent to the parity interlacing property and hence it is also necessary for the existence of strongly stabilizing controllers. The results are illustrated with numerical examples.

Keywords: Feedback Stabilization; Stable Controllers; Unstable MIMO Plants.

1. INTRODUCTION

In this paper we discuss the strong stabilization problem, i.e., feedback stabilization using a stable controller, for a class of linear time-invariant (LTI), multi-input multi-output (MIMO) systems. It is well known that a given plant is strongly stabilizable if and only if it satisfies the parity interlacing property (p.i.p), that is, the number of poles between every pair of real blocking zeros in the extended right half plane is even, Youla et al. (1974); Vidyasagar (1985). In the single-input multi-output (SISO) case, an interpolation-based procedure is available to derive strongly stabilizing controllers for a given plant, see Doyle et al. (1992); Vidyasagar (1985). Moreover, for such plants, a parameterization of all strongly stabilizing controllers can be obtained using interpolation with infinite dimensional transfer functions, Vidyasagar (1985). On the other hand, extensions of these interpolation techniques to MIMO plants are not currently available. Nevertheless, a vast literature exists on strong stabilization of MIMO plants, see e.g. Campos-Delgado and Zhou (2003); Choi and Chung (2001); Chou et al. (2007); Gumussoy and Ozbay (2005); Halevi (1994); Lee and Soh (2002); Petersen (2006), and their references. In these papers, strongly stabilizing controllers are obtained under certain sufficient conditions. In addition to strong stabilization, most of these papers also consider an H_∞ or H_2 like performance conditions.

In this work, rather than investigating new sufficient conditions for the construction of strongly stabilizing controllers for general MIMO plants, we derive a simple design procedure for a restricted class of plants satisfying the PIP (i.e., *the design procedure works for all strongly stabilizable plants in this class*). The plants we consider satisfy the PIP and have at most two real blocking zeros in the extended right half plane. In this approach, there is no restriction on the number and location of the poles, and on the number of left half plane zeros. Furthermore, the controller order does not exceed that of the plant. We also show that the sufficient condition of Zeren and Ozbay (2000) for general MIMO plants is also necessary for the class of plants considered here.

In Section 2 we define the class of MIMO plants considered for strong stabilization. The design procedure is given in Section 3. Connections with the approach of Zeren and Ozbay (2000) are illustrated in Section 4 with an example. Concluding remarks are made in Section 5.

The notation used in the paper is fairly standard, in particular $\mathbb{R}_{+e} = \{z \in \mathbb{R} \mid z \geq 0\} \cup \{\infty\}$ represents the (extended) non-negative real axis, and define $\mathbb{C}_{+e} := \overline{\mathbb{C}}_{+} \cup \{\infty\}$. The set of real rational functions that are bounded and analytic in $\mathbb{C}_+$ is denoted by $\mathcal{RH}^\infty$. We also say that a matrix valued function is in $\mathcal{RH}^\infty$ if all its entries are in $\mathcal{RH}^\infty$. The norm symbol, $\|\cdot\|$, used in the paper is the usual operator norm defined in $\mathcal{RH}^\infty$.

2. PROBLEM DEFINITION

Consider the standard feedback control system shown in Figure 1, where P is the plant to be controlled and C is the controller to be designed. It is assumed that all entries of the matrices P and C are proper rational functions, and that P and C have no unstable hidden-modes. Let $P = D_p^{-1} N_p$ be a left-coprime-factorization (LCF) of the plant and let $C = N_c D_c^{-1}$ be a right-coprime-factorization (RCF) of the controller, where D_p, N_p, N_c, D_c are appropriate size matrices with all entries in $\mathcal{RH}^\infty$, and D_p^{-1}, D_c^{-1} are proper. A controller $C = N_c D_c^{-1}$ stabilizes the feedback system if and only if the matrix

$$U = D_p D_c + N_p N_c \tag{1}$$

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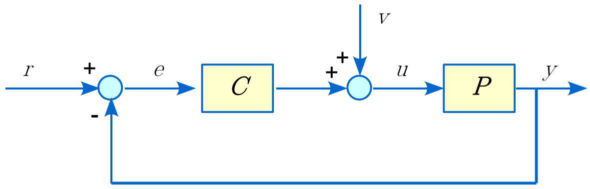

Fig. 1. Feedback Control System

is unimodular, i.e., all entries of U and U^{-1} are in $\mathcal{RH}^\infty$. Moreover, the stabilizing controller is itself stable if and only if this condition is satisfied with a unimodular D_c; in this case we say that C strongly stabilizes P. Let $z_1, \ldots, z_\ell \in \mathbb{R}_{+e}$ be the non-negative real-axis blocking-zeros of P in the extended right-half-plane $\mathbb{C}_{+e}$, i.e., $N_p(z_k) = 0$ for $k = 1, \ldots, \ell$. Then P satisfies the PIP if and only if $\det[D_p(z_k)]$ is sign invariant for $k = 1, \ldots, \ell$ (see e.g. Vidyasagar (1985)).

The plants under consideration for strongly stabilizing controller synthesis here are full row-rank and have no restrictions on the location or number of their poles. The left-half-plane zeros are also unrestricted. These plants may have at most two non-negative real-axis blocking-zeros (including infinity) and no other transmission-zeros in $\mathbb{C}_{+e}$. More specifically, the numerator N_p in any LCF $P = D_p^{-1} N_p$ can be factored as $N_p = N_{pz} N_{pr}$, where N_{pz} contains the blocking-zeros in $\mathbb{C}_{+e}$, and N_{pr} is right invertible by a transfer matrix $N_{pr}^\dagger$, whose entries are in $\mathcal{RH}^\infty$ (since P has no other-transmission zeros in $\mathbb{C}_{+e}$); then N_{pz} can be expressed as

$$N_{pz} = \prod_{i=1}^{\ell_\infty} \frac{1}{(s+a_i)} \prod_{k=1}^{\ell-\ell_\infty} \frac{(s-z_k)}{(s+b_k)} I \ , \tag{2}$$

where $a_i > 0, b_k > 0$ are positive real for $1 \le i \le \ell_\infty$, $1 \le k \le \ell - \ell_\infty$. The controller $C = N_{pr}^\dagger N_{cz} D_c^{-1}$ stabilizes $P = D_p^{-1} N_{pz} N_{pr}$ if and only if

$$U = D_p D_c + N_{pz} N_{cz} \tag{3}$$

is unimodular, and C strongly stabilizes P if in addition D_c is unimodular in (3). The goal then is to find appropriate N_{cz}, D_c whose entries in $\mathcal{RH}^\infty$, with D_c being unimodular, such that U is unimodular. Although design for the general MIMO case is done under certain sufficient conditions as in Barabanov (1996); Campos-Delgado and Zhou (2003); Chou et al. (2003, 2007); Petersen (2006) and Zeren and Ozbay (1999), the design procedure developed here works for every plant satisfying the PIP in the class considered here, with $0 \le \ell_\infty \le \ell \le 2$.

3. STRONGLY STABILIZING CONTROLLER DESIGN

Let the plant be $P = D_p^{-1} N_{pz} N_{pr}$, with N_{pz} as in (2). We assume that $D_p(s)$ is non-unimodular; otherwise P is stable and hence the trivial controller $C = 0$ strongly stabilizes it. Another point to note is that the order of the controllers to be derived in this section depend on the particular coprime factorization which is non-unique. Therefore, in order to obtain the lowest order controller one should try to find the lowest order D_p and N_{pr}.

Case $\ell = 0$: First consider the trivial case, where $\ell = 0$, i.e., $N_{pz} = I$ and hence, $P = D_p^{-1} N_{pr}$ has no blocking-zeros in $\mathbb{C}_{+e}$. Then for any arbitrary unimodular matrix U_c, the stable controller $C = N_{pr}^\dagger (U_c - D_p)$ strongly stabilizes P since (1) is satisfied with $D_c = I$, $N_c = C$, which leads to $U = U_c$.

Case $\ell_\infty = \ell = 1$: In this case (2) becomes

$$N_{pz} = \frac{1}{s+a} I \ , \quad a > 0 \ , \tag{4}$$

and hence, $P = \frac{1}{s+a} D_p^{-1} N_{pr}$ has a blocking-zero at infinity. In the SISO case, these plants have relative-degree one and are called minimum-phase. The PIP is automatically satisfied since there are no finite non-negative zeros. Then the stable controller

$$C = (s+a) N_{pr}^\dagger(s) \ (\, D_p(\infty) - D_p(s)\,) \tag{5}$$

strongly stabilizes P since (1) is satisfied with unimodular $D_c = D_p(\infty)^{-1}$, $N_c = C D_p(\infty)^{-1}$, which leads to $U = I$. Note that $D_p(\infty) - D_p(s)$ is strictly proper, hence the term $(s+a)(D_p(\infty) - D_p(s))$ is proper.

Case $\ell_\infty = 0 < \ell = 1$: In this case (2) becomes

$$N_{pz} = \frac{s-z}{s+b} I \ , \quad z \ge 0 \ , \ b > 0 \ , \tag{6}$$

and hence, $P(s) = \frac{(s-z)}{(s+b)} D_p^{-1} N_{pr}$ has one finite blocking-zero at $z \ge 0$. In the SISO case, these plants have relative-degree zero. The PIP is automatically satisfied because there is only one zero in $\mathbb{C}_{+e}$. Then the stable controller

$$C = \frac{(s+b)}{(s-z)} \ N_{pr}^\dagger(s) \ (\, D_p(z) - D_p(s)\,) \tag{7}$$

strongly stabilizes P since (1) is satisfied with unimodular $D_c = D_p(z)^{-1}$, $N_c = C D_p(z)^{-1}$, which leads to $U = I$.

Case $\ell_\infty = \ell = 2$: In this case (2) becomes

$$N_{pz} = \frac{1}{(s+a_1)(s+a_2)} I \ , \quad a_1 > 0 \, , \ a_2 > 0 \ , \tag{8}$$

and hence, $P = \frac{1}{(s+a_1)(s+a_2)} D_p^{-1} N_{pr}$ has two blocking-zeros at infinity. In the SISO case, these plants have relative-degree two and are called minimum-phase. The PIP is automatically satisfied since there are no finite non-negative zeros. Define

$$D_\infty(s) := (D_p(s) D_p(\infty)^{-1} - I)$$

which is strictly proper and stable. Choose any positive real $\beta > 0$ satisfying

$$\beta > \| \, s \ D_\infty(s) \, \| \ . \tag{9}$$

Let the controller be

$$C = \beta \frac{(s+a_1)(s+a_2)}{(s+\beta)} N_{pr}^\dagger(s) \, (D_p(\infty) - D_p(s)). \tag{10}$$

Since $D_p(\infty) - D_p(s)$ is strictly-proper, the term $\frac{(s+a_1)(s+a_2)}{(s+\beta)} (D_p(\infty) - D_p(s))$ is proper; then all entries of C are in $\mathcal{RH}^\infty$. With $D_c = I$, $N_c = C$, (1) becomes

$$\begin{aligned} U &= D_p(s) + \frac{\beta}{(s+\beta)} (D_p(\infty) - D_p(s)) \\ &= (I + \frac{1}{s+\beta} s D_\infty(s)) D_p(\infty). \end{aligned}$$

For β satisfying (9), by the small-gain theorem, $\| \frac{1}{s+\beta} s D_\infty(s) \| < 1$, which implies U is unimodular. Therefore, the stable controller in (10) strongly stabilizes P.

Case $\ell_\infty = 1 < \ell = 2$: In this case (2) becomes

$$N_{pz} = \frac{(s-z)}{(s+a)(s+b)} I \ , \quad z \geq 0 \ , \ a > 0 \ , \ b > 0 \ , \qquad (11)$$

and hence, $P = \frac{(s-z)}{(s+a)(s+b)} D_p^{-1} N_{pr}$ has a finite blocking-zero at $z \geq 0$ and one at infinity. In the SISO case, these plants have relative-degree one. Define $W := D_p(z)^{-1} D_p(\infty)$, which is obviously nonsingular. For these plants, the PIP is satisfied if and only if $\det W > 0$ since $\det D_p$ is sign invariant at the blocking-zeros of P. The strongly stabilizing controller design proposed here assumes a more conservative condition on W: Let all eigenvalues of $W = D_p(z)^{-1} D_p(\infty)$ have positive real parts (which is equivalent to PIP for single-output plants since $W > 0$). Choose any real $\beta > 0$ satisfying (9) as in the case of 2-zeros at infinity above. With N_{pz} as in (11), let the controller be

$$C = \beta N_{pz}^{-1} N_{pr}^{\dagger}(s)(I - D_p(s) D_p(z)^{-1}) D_p(\infty)(sI + \beta W)^{-1}, \qquad (12)$$

where $\frac{(s+b)}{(s-z)}(I - D_p(s) D_p(z)^{-1})$ is stable. By assumption, $-W$ has eigenvalues in the open left-half-plane and $\beta > 0$ implies $(s+a)(sI + \beta W)^{-1}$ is also stable; then all entries of C are in $\mathcal{RH}^\infty$. With $D_c = I$, $N_c = C$, (1) becomes

$$\begin{aligned} U &= D_p(s) + \beta(I - D_p(s) D_p(z)^{-1}) D_p(\infty)(sI + \beta W)^{-1} \\ &= (s D_p(s) + \beta D_p(\infty))(sI + \beta W)^{-1} \\ &= (I + \frac{1}{s+\beta} \ s D_\infty(s)) \ D_p(\infty)(s+\beta)(sI + \beta W)^{-1} \end{aligned}$$

By (9) the term $(I + \frac{1}{s+\beta} \ s D_\infty(s))$ is unimodular. Since $D_p(\infty)$ is nonsingular, and $(s+\beta)(sI + \beta W)^{-1}$ is unimodular, the controller (12) is strongly stabilizing P.

The strongly stabilizing controller in (12) is designed under the condition that $W = D_p(z)^{-1} D_p(\infty)$ has eigenvalues in the open $\mathbb{C}_+$; while this is a sufficient condition for PIP for MIMO plants, it is equivalent to PIP for single-output systems.

Case $\ell_\infty = 0 < \ell = 2$: We only consider the case where (at least) one of the two finite blocking-zeros is at zero. In this case (2) becomes

$$N_{pz} = \frac{s\,(s-z)}{(s+b_1)(s+b_2)} I \ , \quad z \geq 0 \ , \ b_1 > 0 \ , \ b_2 > 0 \ , \quad (13)$$

and hence, $P = \frac{s(s-z)}{(s+b_1)(s+b_2)} D_p^{-1} N_{pr}$ has a finite blocking-zero at $s = 0$ and another at $z \geq 0$. In the SISO case, these plants have relative-degree zero. Define $V := D_p(0)^{-1} D_p(z)$, which is obviously nonsingular. For these plants, the PIP is satisfied if and only if $\det V > 0$ since $\det D_p$ is sign invariant at the blocking-zeros of P. Similar to the previous case, the strongly stabilizing controller design proposed here assumes a more conservative condition on V: Let all eigenvalues of $V = D_p(0)^{-1} D_p(z)$ have positive real parts (which is equivalent to PIP for single-output plants since $V > 0$). Let us define the stable transfer matrix

$$D_0(s) := \frac{D_p(s) D_p(0)^{-1} - I}{s}$$

and choose any positive real $\rho > 0$ satisfying

$$\rho \, \| \, D_0(s) \, \| < 1 \ . \qquad (14)$$

Let the controller be

$$C = \frac{(s+b_1)(s+b_2)}{(s-z)} N_{pr}^{\dagger}(s) \, [\, D_p(z) - D_p(s) \,] \, (sI + \rho V)^{-1}, \qquad (15)$$

where $\frac{(s+b_1)}{(s-z)} [\, D_p(z_1) - D_p(s) \,]$ is stable. By assumption, $-V$ has eigenvalues in the open left-half-plane and $\rho > 0$ implies $(s+b_2)(sI + \rho V)^{-1}$ is also stable; then all entries of C are in $\mathcal{RH}^\infty$. With $D_c = I$, $N_c = C$, (1) becomes

$$\begin{aligned} U &= D_p(s) + s(D_p(z) - D_p(s)) \ (sI + \rho V)^{-1} \\ &= (\ D_p(s) \rho V + s D_p(z) \ ) \ (sI + \rho V)^{-1} \\ &= ((s + \rho - \rho) I + \rho D_p(s) D_p(0)^{-1}) \ D_p(z)(sI + \rho V)^{-1} \\ &= (I + \frac{s}{s+\rho} \ \rho \ D_0(s)) \ (s+\rho) D_p(z)(sI + \rho V)^{-1} \end{aligned}$$

By (14) the term $(I + \frac{s}{s+\rho} \ \rho \ D_0(s))$ is unimodular. Since $D_p(z)$ is nonsingular and $(s+\rho)(sI + \rho V)^{-1}$ unimodular the controller (15) is strongly stabilizing P.

4. CONNECTIONS WITH THE CONDITION OF ZEREN AND ÖZBAY

In this section we show that a sufficient condition used to design strongly stabilizing controllers in Zeren and Ozbay (2000) is also necessary for the single-output version of the plants considered here.

By (1), the plant $P = D_p^{-1} N_{pz} N_{pr}$ is strongly stabilizable if and only if there exists stable $C = N_{pr}^{\dagger} Q$, $Q \in \mathcal{RH}^\infty$, such that $U := D_p + N_{pz} Q$ is unimodular. Define $R_p := D_p - I$, then $U = I + R_p + N_{pz} Q$. Using the small-gain theorem, a strongly stabilizing controller can be found if there exists $Q \in \mathcal{RH}^\infty$ such that $\| R_p + N_{pz} Q \| < 1$. Since $N_{pz}(z) = 0$ at all blocking-zeros $z \in \mathbb{C}_{+e}$, of P, a feasible $Q \in \mathcal{RH}^\infty$ can be found only if $\| R_p(z) \| < 1$.

Now consider the plants described in Section 3 and let P have a single-output. Therefore $D_p \in \mathcal{RH}^\infty$ and $N_{pz} \in \mathcal{RH}^\infty$ are scalars anr N_{pr} has a single row. The only $\mathbb{C}_{+e}$ zeros of the plant are in $\mathbb{R}_{+e}$, i.e., none are complex. By (2), a coprime factorization is $P = D_p^{-1} N_p = D_p^{-1} N_{pz} N_{pr}$, where N_{pz} contains the $\mathbb{C}_{+e}$ zeros and N_{pr} has a single-column right-inverse in $\mathcal{RH}^\infty$. Without loss of generality, the denominator $D_p \in \mathcal{RH}^\infty$ of $P = D_p^{-1} N_p$ can be expressed as follows: Let $\mathcal{P}_r = \{p_i\}_{i=1}^{k_1} \subset \mathbb{R}_+$, $p_i \geq 0$ denote the non-negative real poles and $\mathcal{P}_i = \{e_i \pm j f_i\}_{i=1}^{k_2} \subset \mathbb{C}_+ \setminus \mathbb{R}_+$, $e_i \geq 0$ denote the complex poles of P in $\mathbb{C}_+$. Then decompose $D_p(s)$ as

$$D_p(s) = D_{pr}(s) D_{pc}(s) \qquad (16)$$

where

$$D_{pr}(s) = \prod_{i=1}^{k_1} \frac{(s - p_i)}{(s + p_i + c_i)}$$

$$D_{pc}(s) = \prod_{i=1}^{k_2} \frac{(\,(s - e_i)^2 + f_i^2\,)}{(\,(s + e_i)^2 + f_i^2 + d_i s\,)}$$

for some $c_i, d_i > 0$. Since $D_p(\infty) = 1$, $R_p = D_p - I$ is strictly-proper.

Suppose that the plant is strongly stabilizable, equivalently the PIP holds (since $D_p(\infty) > 0$, PIP implies

$D_p(z) > 0$). Suppose that the only $\mathbb{C}_{+e}$ zeros of P are in $\mathbb{R}_{+e}$, i.e., P has no complex zeros in $\mathbb{C}_{+e}$. Let $z_{max} \leq \infty \in \mathbb{R}_{+e}$ denote the largest zero of P. Suppose that $p_i \leq z_{max}$ for all $p_i \in \mathcal{P}_r$, which always holds if P is strictly-proper. We now show that under these assumptions, $|R(z)| < 1$ at all $\mathbb{R}_{+e}$ zeros of P: Let $0 \leq z \leq z_{max}$ be any of the $\mathbb{R}_{+e}$ zeros of the plant, i.e., $N_{pz}(z) = 0$. It is obvious that $D_{pc}(z) = \prod_{i=1}^{k_2} \frac{((z-e_i)^2+f_i^2)}{((z+e_i)^2+f_i^2+d_i z)}$ satisfies $0 < D_{pc}(z) \leq 1$. For $D_{pr}(z)$, there are two possibilities: Either this zero $z > p_i$ for all $p_i \in \mathcal{P}_r$, or $z < p_i$ for an even number of the poles $p_i \in \mathcal{P}_r$ since the PIP implies an even number of positive real poles of P in the interval $[z, z_{max}]$ (as well as between any two consecutive positive real zeros). In both cases, it is obvious that $D_{pr}(z) = \prod_{i=1}^{k_1} \frac{(z-p_i)}{(z+p_i+c_i)}$ also satisfies $0 < D_{pr}(z) \leq 1$. Therefore, $0 < D_{pr}(z)D_{pc}(z) \leq 1$ implies $|R_p(z)| = |1 - D_p(z)| < 1$ at all $\mathbb{R}_{+e}$ zeros of P.

In the single-output case, the strongly stabilizing controller can be written as $C = Q = N_{pr}^{\dagger} Q_p$ for some $Q_p \in \mathcal{RH}^{\infty}$. Then using the sufficient condition of Zeren and Ozbay (2000), there exists a strongly stabilizing controller if

$$\inf_{Q_p \in \mathcal{RH}^{\infty}} \|R_p + N_{pz} Q_p\|_{\infty} < 1. \tag{17}$$

For the single-output plant classes (4) and (8), condition (17) is satisfied if and only if $|R_p(\infty)| < 1$: For $N_{pz} = \frac{1}{s+a}$, the optimal solution is

$$Q_p = (s+a)(R_p(\infty) - R_p(s)) = (s+a)(D_p(\infty) - D_p(s));$$

i.e., $Q = N_{pr}^{\dagger} Q_p$ is the same as controller $C = N_{pr}^{\dagger} N_{cz}$ in (5). For $N_{pz} = \frac{1}{(s+a_1)(s+a_2)}$, since $D_p(\infty) = 1$ and β satisfies (9), the optimal solution is

$$Q_p = \frac{\beta N_{pz}^{-1}}{s+\beta}(R_p(\infty) - R_p(s)) = \frac{\beta N_{pz}^{-1}}{s+\beta}(D_p(\infty) - D_p(s));$$

i.e., $Q = N_{pr}^{\dagger} Q_p$ is the same as controller $C = N_{pr}^{\dagger} N_{cz}$ in (10). For the cases (6) and (11) condition (17) is satisfied if and only if $|R_p(z)| < 1$. Necessity is obvious, for the sufficiency see e.g. Section 2.11 of the book by Foias et al (1996). For the case (6), $N_{pz}(s) = \frac{s-z}{s+b}$, the optimal solution is

$$Q_p = N_{pz}^{-1}(R_p(z) - R_p(s)) = N_{pz}^{-1}(D_p(z) - D_p(s)),$$

i.e., $Q = N_{pr}^{\dagger} Q_p$ is the same as controller $C = N_{pr}^{\dagger} N_{cz}$ in (7). In the case (13) the problem (17) is solvable if and only if $|R_p(z)| < 1$ and $|R_p(0)| < 1$. For systems with more than one zero in open $\mathbb{C}_+$ solvability condition for (17) is more complicated than just checking $|R_p(z_i)| < 1$; but if one zero is in $\mathbb{R}_+$ and the other one is on the boundary of $\mathbb{C}_{+e}$, like in (13), then this statement is correct, see Section 2.11.3 of the book by Foias et al (1996).

We showed above that for (4), (8), (6) and (11) strong stabilizability implies $|R_p(z)| < 1$, which in turn implies condition (17). Therefore condition (17) is *necessary* and sufficient for these plants; i.e. there exist strongly stabilizing controllers (equivalently the PIP is satisfied) if and only if there is exists a stable Q_p such that $\|R_p + N_{pz} Q_p\| < 1$. Note that, here we do not require that the plant has no poles on the $j\omega$-axis, which is an assumption in Zeren and Ozbay (2000).

Example. Consider the plant (18), which is taken from Zeren and Ozbay (2000),

$$P = \begin{bmatrix} \frac{(s+1)(s-2)}{(s^2+4s+5)(s-\alpha)} & \frac{(s+2)(s-2)}{(s^2+4s+5)(s-\alpha)} \end{bmatrix} \tag{18}$$

with $\alpha \geq 0$. We see that this plant admits a factorization in the form $P = D_p^{-1} N_{pz} N_{pr}$, where

$$\begin{aligned} D_p(s) &= \frac{s-\alpha}{s+1} \\ N_{pz}(s) &= \frac{s-2}{(s+a)(s+b)} \\ N_{pr}(s) &= \frac{(s+a)(s+b)}{s^2+4s+5} \begin{bmatrix} 1 & \frac{s+2}{s+1} \end{bmatrix}. \end{aligned}$$

This fits into the form (11). Clearly $N_{pz}(2) = 0$ and $N_{pz}(\infty) = 0$, so PIP is satisfied if and only if $\alpha < 2$. The controller proposed in Section 3 for this type of systems is in the form (12), where we can take

$$N_{pr}^{\dagger}(s) = \frac{s^2+4s+5}{(s+a)(s+b)} \begin{bmatrix} 1-c \\ c\frac{(s+1)}{(s+2)} \end{bmatrix}$$

with any $c \in \mathbb{R}$. We have $W = D_p(2)^{-1} D_p(\infty) = 3/(2-\alpha)$, and $W > 0$ if and only if $\alpha < 2$ (equivalent to PIP). Now we need to choose $\beta > 0$ such that $\beta > \|s\ D_{\infty}(s)\|$. For this example, $\|s\ D_{\infty}(s)\| = (\alpha+1)$. Applying the formula (12) we obtain

$$C = \frac{-\beta(1+\alpha)(s^2+4s+5)}{(2-\alpha)(s+1)(s+\frac{3\beta}{2-\alpha})} \begin{bmatrix} 1-c \\ c\frac{(s+1)}{(s+2)} \end{bmatrix}, \tag{19}$$

with any $c \in \mathbb{R}$ and any $\beta > (\alpha+1)$. In this case we have

$$\begin{aligned} U &= \left(\frac{s-\alpha}{s+1}\right) - \frac{\beta(1+\alpha)}{(2-\alpha)} \frac{(s-2)}{(s+1)(s+\frac{3\beta}{(2-\alpha)})} \\ &= \frac{s^2+(\beta-\alpha)s+\beta}{(s+1)(s+\frac{3\beta}{(2-\alpha)})}. \end{aligned}$$

Since $\beta > \alpha$ and $\alpha < 2$, U is unimodular. Note that our condition $\beta > \|s\ D_{\infty}(s)\|$ gave $\beta > (\alpha+1)$. But in fact $\beta > \alpha$ is sufficient. This gap is due to the use of the small gain theorem in our design of Section 3 in guaranteeing that U is unimodular.

For the same system, we can obtain an alternative controller using the method of Zeren and Ozbay (2000). In that approach first note that

$$R_p(s) = D_p(s) - 1 = -\frac{(\alpha+1)}{s+1}$$

and $|R_p(2)| = \frac{\alpha+1}{3} < 1$ if and only if $\alpha < 2$ (equivalent to PIP). Solution of the problem (17) gives

$$Q_p(s) = -\frac{(\alpha+1)(s+b)(s+a)}{3(s+1)(1+\varepsilon s)}$$

for some sufficiently small $\varepsilon > 0$. In order to understand how small ε should be let us now examine U for the controller $C = N_{pr}^{\dagger} Q_p$,

$$\begin{aligned} U(s) &= \frac{s-\alpha}{s+1} - \frac{(\alpha+1)(s-2)}{3(s+1)(1+\varepsilon s)} \\ &= \frac{\varepsilon s^2 + (\frac{2-\alpha}{3} - \varepsilon\alpha)s + \frac{2(\alpha+1)}{3} - \varepsilon\alpha}{(s+1)(1+\varepsilon s)}. \end{aligned}$$

This means that ε satisfying $0 < \varepsilon < \frac{2-\alpha}{3\alpha}$ gives a strongly stabilizing controller

$$C(s) = -\frac{(\alpha+1)(s^2+4s+5)}{3(s+1)(1+\varepsilon s)} \begin{bmatrix} 1-c \\ c\dfrac{(s+1)}{(s+2)} \end{bmatrix}. \qquad (20)$$

Note that the two controllers (19) and (20) are identical if we set $\varepsilon = \frac{2-\alpha}{3\beta}$.

5. CONCLUSIONS

In this paper we considered a class of MIMO plants with at most two blocking zeros in $\mathbb{R}_e$ (no other blocking zeros in $\mathbb{C}_{+e}$), and no restrictions were imposed on the number and locations of the poles. When they satisfy the PIP, these plants are shown to admit strongly stabilizing controllers and we have constructed such controllers explicitly. The order of the strongly stabilizing controllers developed here is equal to the order of the plant.

We have also shown that for the single-output version of the class of plants considered here the sufficient condition of Zeren and Ozbay (2000), given for the construction of strongly stabilizing controllers, is equivalent to PIP.

Strongly stabilizing controller design for MIMO plants with higher number of $\mathbb{C}_+$ blocking zeros is more difficult. For such plants one may have to check the existing sufficient conditions and design methods proposed earlier, see e.g. Campos-Delgado and Zhou (2003); Choi and Chung (2001); Chou et al. (2003, 2007); Gumussoy and Ozbay (2005); Lee and Soh (2002); Petersen (2006); Zeren and Ozbay (1999, 2000), and their references. For example, when we have two complex conjugate zeros placed in such a way that the PIP is about to be violated (as the imaginary part goes to zero) many of the existing finite dimensional (order of the controller is a few multiples of the plant order) controller design techniques fail. Because in this case the minimum order of the strongly stabilizing controllers can be very large (grows as the imaginary part gets smaller), see Smith and Sondergeld (1986). This is the main reason why we have not considered the two finite zero case. Our goal was to derive simple strongly stabilizing controllers for a class of plants using the PIP only. No additional conditions are imposed for this class of plants.

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