Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Estimating Cutting Forces in Micromilling by Input Estimation from Closed-loop Data

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Abstract: Estimation of cutting forces in micromilling from the signals of a spindle with Active Magnetic Bearings (AMB) is treated as an input estimation problem. For the closed-loop AMB system, a minimum mean square error input estimator with an adjustable delay is derived. This filter is based on the Wiener filter, where the unknown input is treated as white noise filtered by known dynamics and the controller is assumed to be known. It is shown that controller knowledge can be replaced by a perfect measurement of the control signal. Simulation results demonstrate the applicability of the presented approach.

Keywords: Unknown input estimation, active magnetic bearings, manufacturing systems.

1. INTRODUCTION

The topic of this paper concerns an application in the area of micro-manufacturing. In particular micromilling is considered, which entails the scaling of conventional milling to the microdomain. Tools with diameters of less than $0.5mm$ are used to manufacture components with arbitrary 3D features in a range of materials. Application of such components can be found in medical areas as well as in automotive and electronics industries. More than in conventional milling, it is important in micromilling to monitor the cutting forces during the milling process in order to maintain a stable cutting process. These forces can be measured directly with force transducers, however commercially available systems are limited by their bandwidth and the additional space needed in the machine. In the area of Machine Tool Design and Manufacturing, techniques have been proposed to indirectly measure the cutting forces by adding position sensors or accelerometers in the spindle housing or elsewhere in the machine, measuring the frequency response from the tooltip to the sensor, and using a Kalman filter approach to obtain cutting force estimates (see e.g. Chae & Park (2007) and the references therein). When the milling is performed by a spindle with Active Magnetic Bearings (AMBs), the active nature of the bearings can be employed to observe the cutting forces from the signals of the bearings. In AMB spindles, the rotor is levitated by generating electromagnetic forces at the front and rear side of the rotor, as well as in the axial direction (see figure 1). A stable system is obtained by using position measurements in a closed-loop to control the currents of the electromagnets.

The problem of observing the cutting forces from measurements of the currents and displacements in an AMB spindle, can be considered an input estimation problem. The cutting forces constitute an unknown input to a partially closed loop dynamical system (see figure 2). Measurements of the control input y_1 and the outputs y_2 are available, viz. the currents through the coils and the displacement of the rotor respectively, which are used to estimate the unknown input u_2, representing the cutting forces. When it is assumed that the rotational speed of the milling spindle is known and fixed, this system is adequately modeled by a time-invariant system. Hence in this paper we confine ourselves to the treatment of LTI systems and we assume an appropriate model of the spindle setup is available,

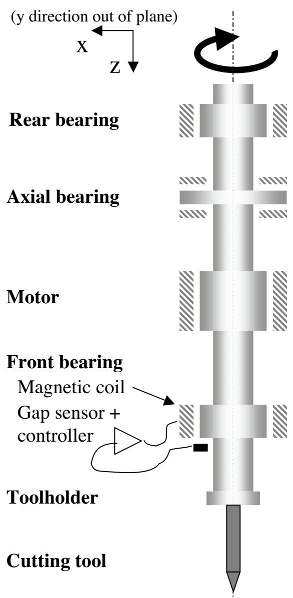

Fig. 1. Schematic of an AMB spindle

either obtained by modeling from first principles or by system identification.

A priori it is known that cutting forces consist of a slowly varying offset related to the cutting depth, with on top of that a signal that is periodic in nature due to the rotation of the milling tool. The frequency is dependent on the rotational speed as well as the number of cutting teeth. In order to obtain cutting force information meaningful for micro-milling process research, it is important to obtain cutting force estimates with high bandwidth, small error, and small phase distortion up to the bandwidth frequency to preserve the waveform. Having the cutting force estimates available instantly is of less importance and a small processing delay can be tolerated.

The problems of input estimation and state reconstruction for systems with unknown inputs has received considerable attention over the past few decades (see Sunaram, & Hadjicostis (2007) and the references therein). These two problems are closely related, in fact as was shown by (Hou & Müller, 1992), when a state estimator exists for systems with unknown inputs, also estimates of the unknown input can be obtained. Existence conditions and a procedure for input estimator design was formulated by Hou & Patton (1998). Many of these results consider systems in a deterministic setting. Results on input observers for systems with noisy measurements are also available. One of the first results in which no a priori structure on the inputs is assumed, was published by Glover (1969). Darouach et al. studied the problem of unbiased minimum variance estimation for discrete-time time-varying stochastic systems with unknown inputs, both in the absence and presence of feedthrough of the unknown input (Darouach & Zasadzinski, 1997; Darouach, Zasadzinski & Boutayeb, 2003). Kitanidis (1987) solved the state estimation problem for the case part of the inputs are unknown. Continuing on the results of Kitanidis, Gillijns & De Moor introduced recursive unbiased minimum variance recursive filters (Gillijns, & De Moor, 2007a; Gillijns & De Moor, 2007b).

In the case probablistic information on the input is available, a common approach is to augment the system matrix (Hostetter & Meditch, 1973; Park et al., 2000). Since in our application we have a priori information on the spectral content of the input signal, such approach has been selected. The problem that we will address in this paper is to minimize the Mean Square Error (MSE) in the input estimates, while the bias of the estimator is not constrained to zero. As a delay in the estimation result is acceptable, the advantage of minimizing the MSE criterion is that the smallest estimation error can be obtained even if that implies that there is some delay. Compared to the existing techniques for cutting force estimation in Manufacturing (like Chae & Park (2007)), the approach in this paper has a number of favorable features. First, it uses the information that is already available in the AMB spindle and hence no additional sensors are needed. A solution is presented to obtain input estimates from an unstable plant in a partially closed-loop of which the control law can be unknown (which is the case for the setup in our laboratory). Secondly, often the random-walk model is used to represent the unknown cutting force, which incorporates little a priori knowledge. Here the spectral information available on the cutting forces is used to obtain estimates with a smaller error. Last, the estimator has an adjustable delay, giving the possibility to make a trade-off between a small estimation error and a small time-lag.

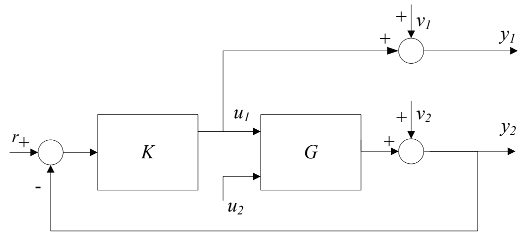

Fig. 2. Block diagram of closed-loop input estimation problem

This paper is organized as follows. After defining the problem in section 2, we first discuss some generic assumptions in section 3. In section 4 the solution to the problem will be derived based on the Wiener filter. Simulation results with the presented approach are discussed in section 5.

2. PROBLEM FORMULATION

Consider the closed-loop system as depicted in figure 2. Plant G is a linear time-invariant discrete-time system given by

$$\begin{aligned} x(t+1) &= Ax(t) + B_1 u_1(t) + B_2 u_2(t) \\ y_2(t) &= Cx(t) + v_2(t) \end{aligned} \tag{1}$$

where x is the state vector with $dim(x) = n$, u_1 is the control input with $dim(u_1) = m_1$, u_2 is the unknown input vector with $dim(u_2) = m_2$, y_2 the output vector with $dim(y_2) = p_2$. Matrices A, B_1, B_2 and C are known matrices with appropriate dimensions, where A has no unit-circle eigenvalues, (A, C) is detectable and (A, B_1) and (A, B_2) stabilizable. Without loss of generality it will be assumed that $rank(B_2) = m_2$ and $rank(C) = p_2$. The system is stabilized by controller $K(z)$, of which no exact knowledge is available. Vector y_1 with $dim(y_1) = m_1$ provides a measurement of vector u_1. Measurement noise sequence v_2 with $dim(v_2) = p_2$ is assumed to be a zero mean white noise process with known covariance matrix $R_{v_2} > 0$. Measurement noise v_1 with $dim(v_1) = p_1$ and spectral density Φ_{v_1} is uncorrelated with v_2. The problem is to construct a causal linear time-invariant filter on measurements $\bar{y} = col(y_1, y_2)$ to get estimate $\hat{u}_2$ such that for some fixed time-lag $N \geq 0$,

$$\mathbb{E}|\hat{u}_2(t) - u_2(t-N)|^2 \tag{2}$$

is minimized.

3. PRELIMINARIES

In the course of this paper the following assumptions are made. It will be assumed that all signals are exponentially bounded and that all spectra are rational. Furthermore, it is assumed that triplet (A, B_2, C) has no invariant zeros on or outside the unit disc, i.e. for every $\lambda \in \mathbb{C}$, $|\lambda| \geq 1$

$$rank \begin{bmatrix} A - \lambda I & B_2 \\ C & 0 \end{bmatrix} = n + m_2. \tag{3}$$

When this is not satisfied, there exist a initial state and non-decaying exponential input which cannot be distinguished from the zero input. Hence this condition is necessary to ensure that for $t \to \infty$ the mapping between the input and the output is injective. Note that this condition also implies that $p_2 \geq m_2$, which is satisfied in the application at hand.

The superscript asterix will be used to denote the adjoint operator: $G^*(z) = G^T(z^{-1})$.

4. INPUT ESTIMATION BY WIENER FILTERING

4.1 Introduction

In this section we will derive the solution of the problem described in section 2. This we will do as follows. First in section 4.2 it is shown that under reasonable assumptions, the input sequence can be represented as white noise filtered by known dynamics. Subsequently, in section 4.3, the input estimation problem is solved for the case full information on controller K is available. This solution is then used in 4.4 to derive the input estimator in case no information on K is available.

4.2 Spectral model of the unknown input sequence

Given the knowledge that is available on the cutting forces, a stochastic signal model of the input sequence is the most natural. Hence, concerning input u_2 the following assumptions are adopted:

A1 u_2 is a realization of a stationary stochastic process that is uncorrelated with $v_{1,2}$;
A2 u_2 has known rational spectrum $\Phi_u(z)$;
A3 $\|\Phi_u(e^{j\omega})\|_2 \to 0$ for $\omega \to \pi$.

These assumptions allow to model a large class of input signals. Under assumptions A1-A3, there exists a stable, minimal LTI system G_u with state space realization given by

$$\begin{aligned} \xi(t+1) &= A_u \xi(t) + B_u w(t) \\ u_2(t) &= C_u \xi(t) \end{aligned} \tag{4}$$

with $w(t)$ a white noise process with covariance $R_u > 0$, such that $\Phi_u(z) = G_u(z) R_u G_u^*(z)$. Here B_u has full column rank, and C_u has full row rank. Note that assumption A3 implies that the feedthrough in G_u can indeed be ignored. Defining $G_1(z) = C(zI - A)^{-1}B_1$ and $G_2(z) = C(zI - A)^{-1}B_2$, we obtain the cascaded system $G_c(z) = [\, G_1(z) \;\; G_2(z)G_u(z) \,]$ that admits the state-space representation

$$\begin{aligned} x(t+1) &= A_c x(t) + B_{c,1} u_1(t) + B_{c,2} w(t) \\ y_2(t) &= C_c x(t) + v_2(t) \end{aligned} \tag{5}$$

with

$$A_c = \begin{bmatrix} A_u & 0 \\ B_2 C_u & A \end{bmatrix}, \quad B_{c,1} = \begin{bmatrix} 0 \\ B_1 \end{bmatrix}, \quad B_{c,2} = \begin{bmatrix} B_u \\ 0 \end{bmatrix}$$
$$C_c = [\, 0 \;\; C \,].$$

4.3 Known controller

We will start analyzing the input estimation problem by solving it for the case complete information on the controller is available.

Proposition 4.1. Consider the closed-loop system described in section 2 and let unknown input u_2 satisfy assumption A1-A3. In addition assume complete information on controller K is available. Let F be the causal estimator for u_2 according to $\hat{u}_2 = F\bar{y}$, such that (2) is minimized. Then F is of the form $[\, 0 \;\; F_2 \,]$, i.e. y_1 is not used to construct estimate $\hat{u}_2$. F_2 is given by

$$\begin{aligned} F_2(z) = F'(L - B_{c,1}K(z)) \\ + F''(T^{-1}(z) + G_L(z)B_{c,1}K(z)) \end{aligned} \tag{6}$$

where

$$\begin{aligned} F' &= z^{-N} [\, C_u \;\; 0 \,] (zI - \bar{A}_c)^{-1} \\ F'' &= z^{m-N-1} [\, C_u \;\; 0 \,] \sum_{m=1}^{N} P(\bar{A}_c^T)^{m-1} C_c^T R^{-1} \end{aligned} \tag{7}$$

and

$$\begin{aligned} T^{-1}(z) &= I - G_L(z)L \\ G_L(z) &= C_c(zI - \bar{A}_c)^{-1} \qquad \bar{A}_c = A_c - LC_c \\ L &= A_c P C_c^T R^{-1} \qquad R = R_{v_2} + C_c P C_c^T \end{aligned} \tag{8}$$

and P is the unique positive definite solution of the discrete algebraic Ricatti equation (DARE)

$$P = A_c P A_c^T + B_{c,2} R_u B_{c,2}^T - LRL^T. \tag{9}$$

Proof For brevity we will only give a sketch of the proof. From estimation theory it follows that under the given conditions, the causal filter F that minimizes (2) is the Wiener filter. In order to arrive at the equations for F, denote the cross spectrum of the unknown input u_2 and $\bar{y}$ as $\Phi_{u_2\bar{y}}$, and the spectral density of $\bar{y}$ as $\Phi_{\bar{y}}$. Let $M(z)$ be a minimum phase function and $\bar{R}$ a positive definite matrix, such that $\Phi_{\bar{y}}(z) = M(z)\bar{R}M^*(z)$ is the canonical spectral factorization of $\Phi_{\bar{y}}$. Then F is given by (Kailath et al., 2000)

$$F = \{z^{-N}\Phi_{u_2\bar{y}} M^{-*}\}_+ \bar{R}^{-1} M^{-1} \tag{10}$$

where $\{\cdot\}_+$ represents the causal part of the expression between the curly brackets. We will derive an expression for the spectral factorization of $\Phi_{\bar{y}}$, and for $\Phi_{u_2\bar{y}}$, which are then used to derive the causal filter F.

Spectral factorization of $\Phi_{\bar{y}}$ The objective here is to derive expressions for M and $\bar{R}$ such that $\Phi_{\bar{y}} = M\bar{R}M^*$. Observe from figure 2 that $\bar{y} = \bar{G}u_2 + \bar{H}\bar{v}$, with

$$\bar{G} = \begin{bmatrix} -KS \\ S \end{bmatrix} G_2, \quad \bar{H} = \begin{bmatrix} I & -KS \\ 0 & S \end{bmatrix}, \quad \bar{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

and $S = (1 + G_1 K)^{-1}$. With this, it follows that the spectral density of $\bar{y}$ is given by

$$\Phi_{\bar{y}} = \bar{G}\Phi_{u_2}\bar{G}^* + \bar{H} \begin{bmatrix} \Phi_{v_1} & 0 \\ 0 & R_{v_2} \end{bmatrix} \bar{H}^*$$

Note that

$$\bar{G}\Phi_{u_2}\bar{G}^* = \bar{H} \begin{bmatrix} 0 & 0 \\ 0 & G_2\Phi_u G_2^* \end{bmatrix} \bar{H}^*$$

so that

$$\Phi_{\bar{y}} = \bar{H} \begin{bmatrix} \Phi_{v_1} & 0 \\ 0 & \Psi \end{bmatrix} \bar{H}^*,$$

where we define the Popov function $\Psi = G_2\Phi_u G_2^* + R_{v_2} = G_{c,2} R_u G_{c,2}^* + R_{v_2}$. Factorization of Φ_{v_1} and Ψ yields the desired factorization of $\Phi_{\bar{y}}$. As by assumption $\Phi_{v_1}(e^{j\omega}) > 0$ for all $\omega \in [-\pi, \pi]$, there exist a minimum phase function $V(z)$ and a positive definite matrix R_{v_1} such that $VR_{v_1}V^*$ is the canonical spectral factorization of Φ_{v_1}. Observe that $G_{c,2}$ is not necessarily stable. However, the properties of G and G_u in combination with (3) ensure that there exist a minimum-phase function $T(z)$ and a positive definite matrix R, such that $\Psi = TRT^*$, where T and R satisfy (8) and (9) (Kailath et al., 2000). With this we obtain

$$\Phi_{\bar{y}} = M \begin{bmatrix} R_{v_1} & 0 \\ 0 & R \end{bmatrix} M^* \tag{11}$$

with $M = \begin{bmatrix} V & -KST \\ 0 & ST \end{bmatrix}$ minimum phase. Indeed, $M^{-1} = \begin{bmatrix} V^{-1} & V^{-1}K \\ 0 & N \end{bmatrix}$ is stable due to the stability of V^{-1}, K and $N = (ST)^{-1}$.

Cross spectrum $\Phi_{u_2\bar{y}}$ The cross spectrum between unknown input u_2 and $\bar{y}$ is given by

$$\Phi_{u_2\bar{y}} = \Phi_{u_2}\bar{G}^* = G_u R_u G_u^* \bar{G}^*. \tag{12}$$

Derivation of the causal filter Combining (10), (11) and (12) yields

$$F = \{z^{-N} G_u R_u G_u^* \bar{G}^* M^{-*}\}_+ \begin{bmatrix} R_{v_1}^{-1} & 0 \\ 0 & R^{-1} \end{bmatrix} M^{-1}. \tag{13}$$

With some algebra, it can be verified that $G_u^* \bar{G}^* M^{-*} = \left[\, 0 \;\; B_{c,2}^T G_L^* \,\right]$. With this we obtain

$$F = \left[\, 0 \;\; \{z^{-N} G_u R_u B_{c,2}^T G_L^*\}_+ R^{-1} N \,\right] \tag{14}$$

which proves the first part of the proposition. We continue by deriving the causal part of $z^{-N} G_u R_u B_{c,2}^T G_L^* =: z^{-N} W(z)$. In order to do this, we will find a strictly causal function $W_1(z)$ and an anti-causal function $W_2(z)$ such that $W(z) = W_1(z) + W_2(z)$. Partition $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$ according to the partitioning of A_c and let $Z = [\, P_{11} \;\; P_{12} \,]$. From (9) it follows that $B_u R_u B_{c,2}^T = Z - A_u Z \bar{A}_c^T$. With this, $W(z)$ can be written as

$$W(z) = \left[\, C_u (zI - A_u)^{-1} \;\; I \,\right] \begin{bmatrix} Z - A_u Z \bar{A}_c^T & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} (z^{-1}I - \bar{A}_c^T)^{-1} C_c^T \\ I \end{bmatrix}. \tag{15}$$

Note that any function of the form

$$S_{12}(z) = \left[\, C_1 (zI - A_1)^{-1} \;\; I \,\right] \begin{bmatrix} K & L \\ M & N \end{bmatrix} \begin{bmatrix} (z^{-1}I - A_2^T)^{-1} C_2^T \\ I \end{bmatrix}$$

is invariant under transformation

$$\begin{bmatrix} K & L \\ M & N \end{bmatrix} \rightarrow \begin{bmatrix} K - Q + A_1 Z A_2^T & L + A_2 Q C_2^T \\ M + C_1 Q A_2^T & N + C_1 Q C_2^T \end{bmatrix}$$

for any matrix Q with appropriate dimensions. Applying this transformation to (15) with $Q = Z$, it follows that

$$W_1(z) = C_u (zI - A_u)^{-1} A_u Z C_c^T \tag{16}$$

$$W_2(z) = C_u Z \bar{A}_c^T (z^{-1}I - \bar{A}_c^T)^{-1} C_c^T + C_u Z C_c^T. \tag{17}$$

It follows immediately that $\{z^{-N} W(z)\}_+ = z^{-N} W_1(z) + \{z^{-N} W_2\}_+$. Note that $W_2(z)$ can be expanded as

$$W_2(z) = [\, C_u \;\; 0 \,] \sum_{m=1}^{\infty} z^{m-1} P (\bar{A}_c^T)^{m-1} C_c^T \tag{18}$$

from which it follows that the causal part of $z^{-N} W_2(z)$ is given by

$$\{z^{-N} W_2(z)\}_+ = [\, C_u \;\; 0 \,] \sum_{m=1}^{N} z^{m-N-1} P (\bar{A}_c^T)^{m-1} C_c^T$$

Hence, $F = \left[\, 0 \;\; (z^{-N} W_1(z) + \{z^{-N} W_2\}_+) R^{-1} N \,\right]$, which in combination with the identities

$$W_1(z) R^{-1} N(z) = [\, C_u \;\; 0 \,] (zI - \bar{A}_c)^{-1} (L - B_{c,1} K(z))$$

$$N(z) = T^{-1}(z) + G_L(z) B_{c,1} K(z)$$

yields the result of (6). □

For increasing delay N, the estimation error obtained with F decreases, allowing to trade off lower estimation error against longer delay. Also note that for $N = 0$ a filter structure similar to the Kalman filter is obtained.

4.4 Controller unknown

As expected, the filter obtained in section 4.3 depends on the controller $K(z)$. If explicit information on $K(z)$ is not available, we can use the input y_1 to construct an estimator that does not depend on $K(z)$.

Proposition 4.2. Consider the closed-loop system described in section 2, where no information on the controller K is available. Let u_2 satisfy A1-A3. If the noise on y_1 is negligible, the estimate $\hat{u}_2$ obtained with causal input estimator $\hat{u}_2 = \breve{F}\bar{y}$ where $\breve{F}$ is given by

$$\breve{F}(z) = F' \left[\, B_{c,1} \;\; L \,\right] + F''([\, 0 \;\; I \,] - G_L(z) [\, B_{c,1} \;\; L \,]) \tag{19}$$

with F', F'', G_L, and L as defined in (7) and (8), minimizes (2).

Indeed $\breve{F}$ does not depend on K, which implies that for any control law, under the given conditions, optimal input estimates are obtained. This we can prove as follows.

Proof We have demonstrated the optimality of the input estimator (6) for the case K is known. If v_1 negligible, we may substitute $-K(z)y_2$ by y_1 to obtain

$$y = F'(L y_2 + B_{c,1} y_1) + F''(T^{-1}(z) y_2 - G_L(z) B_{c,1} y_1). \tag{20}$$

This can be rewritten to (19). □

5. SIMULATION RESULTS

The input estimation approach has been tested in simulation using Matlab/Simulink. A simulation model has been developed based on the properties of the AMB spindle setup in our laboratory. Signals from the axial AMB are not considered in this case. System G results from a finite element modeling which after balanced truncation and discretization with $T = 25\mu s$ yielded a model with $n = 32$, $m_1 = 4$ (current inputs), $m_2 = 2$ (cutting force inputs in x and y direction) and $p_2 = 4$ (displacement outputs). Output noise covariance matrix is diagonal, i.e. $R_v = \sigma_v^2 I$, with $\sigma_v = 10^{-7}m$. In the simulation a PID controller is applied to stabilize each of the four magnetic bearings. The input estimators demonstrated here however do not use the information on the controller and are designed as described in section 4.4. For the cutting forces, a waveform is chosen in accordance with a model describing the cutting forces when milling with a micro-endmill with two teeth (Dow, Miller & Garrard, 2000).

We will discuss the simulation results for two cases. First, for a simple model for the input force, we will compare the results obtained with an estimator with and without delay. At higher rotational speeds estimation results will improve if the spectral information on the cutting forces is used when designing estimators. This will be investigated in the second test.

5.1 Test 1: Input observers with $N = 0$ and with $N > 0$

In this test for the input a simple model is chosen, i.e. a random walk-like model ($A_u = 0.999 \cdot I$, $B_u = C_u = I$). Figure 3 shows plots of the simulated response of the AMB system to the chosen input signal (rotational frequency is $10,000$ rpm). In this figure only the signals from the front radial x bearing are displayed. An estimator has been implemented with no delay ($N = 0$) and one with a delay of $N = 80$ time steps. The result of both estimators is depicted in figure 4 (x-direction only), where the result of the delayed estimator has been shifted by 80 time steps in order to compare it with the input signal. We observe a number of things. A first difference is that the result of the estimator for $N = 0$ is more noisy than that of $N = 80$. This can be explained by figure 5(a) and (b). Figure 5(a) depicts amplitude response of $F_2(z)G_2(z)|_{1,1}$, viz. the transfer function from the first element of unknown input u_2 to the first element of its estimate $\hat{u}_2$. Figure 5(b) shows the response of the input estimator to the measurement noise in y_2, i.e. it shows spectral density $F_2(e^{j\omega})R_{v_2}F_2^*(e^{j\omega})|_{1,1}$. These figures show that the delayed estimator has an equal bandwidth as the estimator with $N = 0$, but has steeper roll-off at higher frequences. Moreover, the high frequency measurement noise is better suppressed. A more striking difference is that the estimator for $N = 0$ has a delay of around 21 time steps, whereas the estimator for $N = 80$ has no extra delay. To investigate this, we need to inspect the phase response of both filters. It is well known that a filter representing a pure delay has a linear phase characteristic. In filter design it is therefore common to evaluate the group delay, which is the first order derivative of the phase response function. The more constant the group delay of a filter is in the pass band, the more it will act as a delay and the least it will distort the waveform of the input signals in the pass band. In figure 6 we have depicted the group delay of $F_2(z)G_2(z)|_{1,1}$ for both estimators. This figure clearly shows that both estimators have a constant group delay for low frequencies. Here the delayed estimator is also outperforming the estimator for $N = 0$, as its group delay is constant up till the bandwidth of the estimator whereas the group delay $N = 0$ for increasing frequencies slightly drifts off before reaching the bandwidth of the estimator.

An interesting conclusion from this simulation is that the delay of 21 time steps obtained for $N = 0$ is the minimum delay that can be obtained under criterion (2). Indeed, choosing an N smaller than 21 will not result in a filter with a smaller total delay (N+the additional delay in the filter). Moreover, if the noise on y_2 is increased, a filter with a lower bandwidth and a higher group delay is obtained. This is logical since if the noise on the output measurements is increased, it will take longer before the system's response is strong enough to be discerned from the noise.

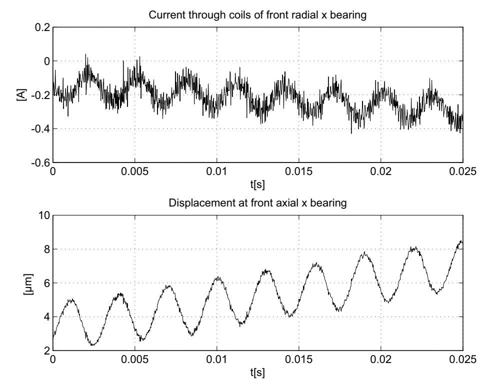

Fig. 3. Displacement and current signals of the front AMB bearing in x-direction (i.e. first elements of y_1 and y_2)

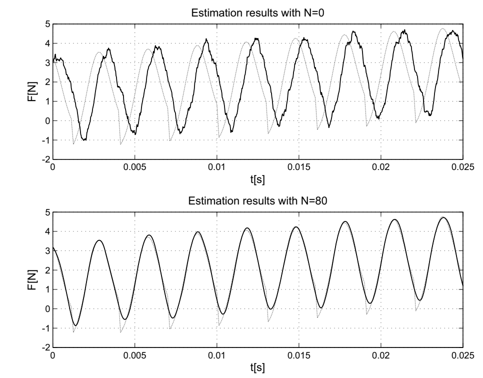

Fig. 4. Estimation results for the cutting force in x-direction (first element of $\hat{u}_2$) for $N = 0$ (upper figure) and $N = 80$ (lower figure)

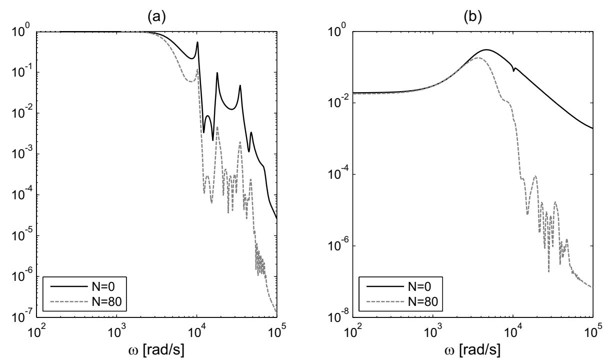

Fig. 5. (a) Amplitude response from $u_{2,1}$ to $\hat{u}_{2,1}$ for $N = 0$ (solid) and $N = 80$ (dotted), (b) Spectral density of noise in $\hat{u}_{2,1}$ for $N = 0$ and $N = 80$

5.2 Test 2: Improved stochastic model for the input

The simulation described in the previous section is performed at a relatively low rotational speed. At higher rotational speeds, i.e. at $50,000$ rpm, estimation errors made by the estimator with $N = 0$ are quite large, whereas the delayed estimator can still produce good estimates though. Obviously increasing the rotational frequency of the AMB spindle results in cutting forces with higher frequencies. As with any mechanical system, the response of the AMB system descreases as the exciting frequencies

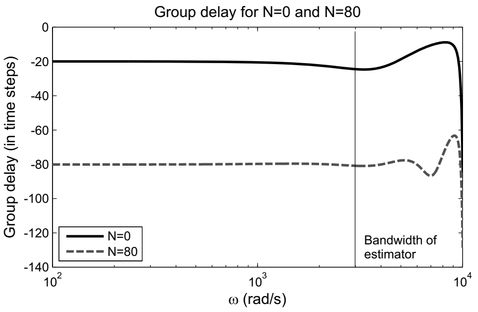

Fig. 6. Group delay of $F_2(z)G_2(z)|_{1,1}$ for N=0 and N=80

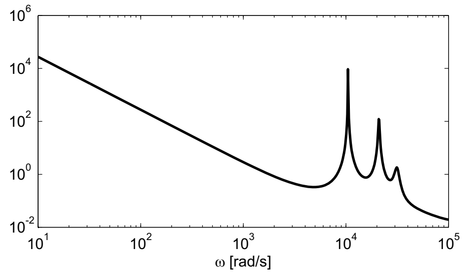

Fig. 7. Spectral model for the cutting forces in simulation test 5.2

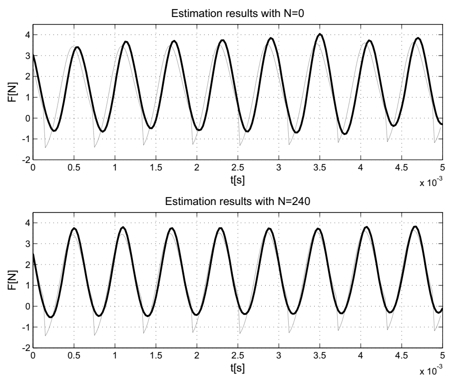

Fig. 8. Estimation results obtained for the cutting force in x-direction (first element of $\hat{u}_2$) with the improved input force model ($N = 0$ and $N = 240$)

increase, resulting in a decreased signal to noise ratio. Hence, we can improve the estimation result if we use the a priori information on the spectral content of the cutting force signal. To verify this, a spectral model for the input has been chosen that has high power in the low frequencies regions and incorporates peaks at the first three harmonics of the cutting force signal (i.e. at 1.7kHz, 3.3kHz and 5kHz). The resulting power spectral density is depicted in figure 7. In figure 8 the results obtained with an estimator with $N = 0$ and $N = 240$ are compared. Again estimation results obtained by the delayed estimator have a smaller error at the cost of time delay.

6. CONCLUSION

For the application of micromilling with an AMB spindle, a minimum mean square error input estimator has been developed to estimate the cutting forces. As this estimator uses data already available in the closed-loop AMB system, no additional sensors are needed. If exact measurements of the control currents of the magnetic bearings are available, no knowledge on the AMB controller is needed. The estimator has an adjustable delay allowing to trade off the estimation error against the lag in the estimation results. Given the dynamics of the plant and the measurement noise characteristics, there exists a minimum delay that can be attained. Estimation results can be improved by using the a priori information on the spectral content of the cutting forces.

ACKNOWLEDGEMENTS

This research is supported by MicroNed and the Delft Center for Mechatronics and Microsystems.

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