PARALLEL ALGORITHMS FOR LARGE SCALE CONTROL AND OPTIMIZATION

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Abstract: In this Keynote Lecture, efficient parallel algorithms are presented on high-performance supercomputers for solution of several fundamental problems: a) complex eigenvalue problem, b) Riccati equation, and c) simultaneous optimization of control and structural systems. The emphasis is on the solution of large-scale problems. Efficiency of the algorithms is evaluated in terms of millions of floating point operations per second (MFLOPS) and the vectorization and parallel processing speedup. The details of the parallel algorithms are presented in several recent journal articles and a book by the authors. *Copyright © 2000 IFAC*

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1. PARALLEL ALGORITHMS FOR SOLUTION OF THE COMPLEX EIGENVALUE PROBLEM

A bottleneck for solution of large-scale control problems is the solution of the complex eigenvalue problem encountered in the solution of the resulting Riccati equation as well as the solution of both open loop and closed loop systems of equations (Adeli and Saleh, 1999).

We investigated the parallel processing suitability of various sequential approaches for finding the eigenvalues and eigenvectors of a real unsymmetric matrix including the LR method (Rutishauser, 1990), the QR method (Stewart, 1973; Golub and Van Loan, 1989), the Double QR method (Stewart, 1973), and the method of matrix iterations (Walsh, 1967; Meirovich, 1980; Dongarra and Sidani, 1993). We found the last method to be the most amenable for development of high-performance parallel algorithms.

Robust and efficient parallel-vector algorithms are presented for solution of the eigenvalue problem of an unsymmetric real matrix using the general approach of matrix iterations and exploiting the architecture of shared memory supercomputers (Adeli, 1992a, b; Adeli and Kamal, 1993) through judicious combination of vectorization, microtasking, and macrotasking (Saleh and Adeli, 1994, 1996). The algorithms are applied to large matrices including one resulting from a 21-story space truss structure and their efficiency is evaluated.

2. PARALLEL ALGORITHMS FOR SOLUTION OF THE RICCATI EQUATION

Riccati equation represents a class of quadratic algebraic equations arising in

a) the study of controlled continuous or discrete dynamic systems,

b) control of turboprop engines, boilers, nuclear reactors, and aircrafts (Morse and Wonham, 1971; Friedland, 1986), and
c) filtering and prediction of noise in applications such as wind gust and updrafts acting on aircrafts and rockets (Johnson, 1971; Rhodes, 1971; Friedland, 1986).

The solution of the Riccati equation is the most time-consuming part of any optimal control problem. It requires an inordinate amount of processing time when applied to large problems. A number of different methods have been proposed to solve the algebraic Riccati equation including the eigenvector approach (Meirovich), the Schur method (Laub, 1979; Meirovich, 1980), the matrix sign function method (Gardiner and Laub, 1986), and the Newton's iterative method (Sandell, 1974). The eigenvector approach lends itself to effective parallel processing if efficient parallel algorithms can be developed for the solution of the complex eigenvalue problem of a general real unsymmetric matrix (Saleh and Adeli, 1996).

Robust and efficient parallel-vector algorithms are presented for solution of the Riccati equations encountered in various control problems on shared-memory multiprocessor machines such as the Cray YMP 8/8128 supercomputer using the eigenvector approach (Saleh and Adeli, 1997). The algorithms are applied to three large examples resulting from a continuous bridge structure, a 21-story space truss structure, and a 12-story space moment-resisting building structure. Efficiency of the algorithms is evaluated in terms of millions of floating point operations per second (MFLOPS) and the speedup. The MFLOPS for the largest example resulting from the 12-story space frame structure is 206.0. The speedup due to parallel processing only for the same example, using seven processors, is 6.3. When vectorization is combined with parallel processing a very significant speedup of 54.4 is obtained using seven processors. It is shown that the algorithms consistently provide stable results for problems of various size while other algorithms show numerical instability for large problems.

3. INTEGRATED CONTROL/STRUCTURAL OPTIMIZATION

Hundreds of journal articles have been published in the area of structural optimization during the past four decades (Adeli, 1994; Adeli and Park, 1998; Adeli and Kumar, 1999; Adeli and Soegiarso, 1999; Sarma and Adeli, 1998, 2000). Similarly, a large number of articles have been published in the area of optimal control (Adeli and Saleh, 1997; Saleh and Adeli, 1998). But, when you combine the two fields the problem becomes complicated and highly numerically intensive. Few journal articles have been published on the solution of the integrated structural/control optimization problem; and the published articles deal with small and academic examples (Khot, 1994). Simultaneous minimization of the weight of the structure and the required level of control forces should be the goal in creating a new generation of adaptive/smart structures (Adeli and Saleh, 1999). Solution of the integrated structural/control optimization problem for large structures requires high-performance computing resources.

Parallel-vector algorithms are presented for simultaneous optimization of control and structural systems (Saleh and Adeli, 1994b). A new parallel-vector algorithm has been developed for computation of the closed eigenvalue and damping factor sensitivities (Adeli and Saleh, 1998). The computational model and parallel vector algorithms have been applied to large steel bridge and multistory space frame structures subjected to various types of dynamic loadings such as impulsive traffic, wind, and earthquake loadings. It is concluded that through adroit use of controllers and the new computational model and algorithms, the weight of the minimum weight structure can be reduced substantially.

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