IFAC

PWARX Traffic Network Hybrid
Controller based on 0-1 Classification

YoungWoo Kim *** Tatsuya Kato ** Shigeru Okuma ***
Tatsuya Narikiyo ****

* Nagoya University, Nagoya, Japan
(e-mail: ywkim@esi.nagoya-u.ac.jp, Tel:+81-52-789-5583).
** Aisin Seiki Co., Aichi prefecture, Japan
(e-mail: kato@okuma.nuee.nagoya-u.ac.jp).
*** Nagoya University, Nagoya, Japan,
(e-mail: okuma@okuma.nuee.nagoya-u.ac.jp).
**** Toyota Technological Institute, Nagoya, Japan,
(e-mail: n-tatsuo@toyota-ti.ac.jp).

Abstract: In this paper, we propose a new design method for the traffic network hybrid feedback controller. In the proposed method, the PWARX classifier describes the nonlinear feedback control law of the traffic control system that the output of the previously developed controller is reproduced applying the 0-1 classifications of the PWARX systems. The proposed method is a hierarchical classification procedure, where the cluster splitting process is follows by the piecewise fitting process to compute the cluster guard and dynamics, and the cluster updating process to find new center points of the clusters. The usefulness of the proposed method is verified through some numerical experiments.

1. INTRODUCTION

With the increasing number of automobile and complication of traffic network, the traffic flow control becomes one of significant economic and social issues in urban life. Desirable characteristics of the traffic network control can be stated as follows: (1) precise modeling of traffic flow, (2) well-defined formulation with algebraic manipularity, and (3) applicability to large-scale traffic network. To meet these characteristics many approaches have been proposed in the Traffic Flow Control (TFC) community.

In the microscopic approach (Kato [2000], Nagel et al. [1992]), the behavior of each vehicle is represented based on the relationships with those of neighboring vehicles. However, this approach is not suitable for traffic signal control of the inner-city districts because it requires enormous computational efforts to obtain all vehicles' behaviors in real time. Furthermore, the simulation for all the combinations of the traffic signals in a given network will most likely be impossible. On the other hand, a fluid approximation model (Lighthill et al. [1955], Payne [1971], Prigogine et al. [1971], Cremer et al. [1986]) was proposed to represent traffic flow in the macroscopic approach. Although this model expresses well the (continuous) behavior of the traffic flow on the freeway, it is unlikely that this model can be directly applied to the urban traffic network, since urban traffic network involves discontinuities of the traffic flow density, controlled by stop signals at the intersections.

The traffic network control system is typical hybrid systems with nonlinear dynamics. In our previous study (Kato et al. [2005], Kato et al. [2006]), the traffic flow model was formulated based on the Mixed Integer Non-Linear Programming (MINLP) problem, where the mixed integer nonlinear traffic control problem is recast to the convex programming problem.

In this paper, we propose a new design method for the traffic network hybrid feedback controller. The method reported in Kato et al. [2006] is an elaborate contrivance to avoid redundant introduction of binary variables. Since the output of the traffic network controller is the 0-1 binary signals, the control output is reproduced in this paper applying 0-1 classifications of the PWARX systems. In the proposed method, the PWARX classifier describes the nonlinear feedback control law of the traffic control system. This implies we don't need the time-consuming searching process of the solver such as the Branch-and-Bound algorithm to solve the mixed integer nonlinear programming (MINLP) problem, and furthermore the exactly same solutions or very similar solutions are obtained in a very short time.

The classification problem we address in this paper is a special problem where the output y is a 0-1 binary variable, and very good classification performance is desirable even with very large number of the introduced clusters. If we plot the observational data in a same cluster in the x-y space, it will show always zero inclination, since we have the binary output, i.e., all the components of θ, a and b except for f will be zeros. This implies we need consideration for the binary output. A new performance criterion is presented in this paper to consider not only previously covariance of θ, but also the covariance of y. The proposed method is a hierarchical classification procedure, where the cluster splitting process is introduced to the cluster with worst classification performance at every iteration which includes 0-1 mixed values of y. The cluster splitting process is follows by the piecewise fitting

10.3182/20080706-5-KR-1001.2856

Fig. 1. HPN model of the intersection

process to compute the cluster guard and dynamics, and the cluster updating process to find new center points of the clusters. The usefulness of the proposed method is verified through some numerical experiments.

2. TRAFFIC FLOW MODELING

2.1 Traffic flow dynamics

The Traffic Flow Control System (TFCS) is the collective entity of traffic network, traffic flow and traffic signals. In the previous study (Kato et al. [2005]), the traffic flow model was proposed, where the traffic flow between the successive section i and j is described as follows.

$$
q(k_i(\tau), k_j(\tau))
$$
$$
= \begin{cases}
-\left(\dfrac{k_i(\tau) + k_j(\tau)}{2}\right) v_f \left(1 - \dfrac{k_i(\tau) + k_j(\tau)}{2k_{jam}}\right) \\
\quad if \ k_i(\tau) \geq k_j(\tau) \\
-v_{f_i}\left(1 - \dfrac{k_i(\tau)}{k_{jam}}\right) k_i(\tau) \\
\quad if \ k_i(\tau) < k_j(\tau) \ and \ c_i(\tau) > 0 \\
-v_{f_j}\left(1 - \dfrac{k_j(\tau)}{k_{jam}}\right) k_j(\tau) \\
\quad if \ k_i(\tau) < k_j(\tau) \ and \ c_i(\tau) \leq 0
\end{cases} \tag{1}
$$

, where $q_i(\tau)$ is the traffic flow i.e., the number of vehicles passing through the boundary per unit time of two successive traffic sections at time τ, $k_i(\tau)$ is the traffic density i.e., the number of vehicles on the ith l_i meters long section, and $v_i(\tau)$ is the traffic flow speed i.e., the average speed of the traffic flow $q_i(\tau)$. Note that $v_i(\tau)$ is obtained by

$$
v_i(\tau) = v_{f_i} \cdot \left(1 - \frac{k_i(\tau)}{k_{jam}}\right) \tag{2}
$$

$$
q_i(\tau) = -\frac{(k_i(\tau) + k_j(\tau))}{2} \frac{v_i(k_i(\tau)) + v_j(k_j(\tau))}{2} \tag{3}
$$

supposing that the density k_* and average velocity v_* of the flow in i and jth sections are almost identical. With eq.(3), the flow dynamics can be uniquely defined. Here, k_{jam} is the density in which the vehicles on the roadway are spaced at the minimum intervals (traffic-jammed), and v_{f_i} is the free velocity, that is, the velocity of the vehicle when no other car exists in the same section.

2.2 Traffic network model at an intersection

In this subsection, we develop the traffic network model at an intersection. Figure 1 shows the HPN model of the jth intersection, where the notation for other than southward entrance lane is omitted. In Fig.1, $l_{j,S}$ and $l_{j,N}$ are the length of the districts $p_{c,j_{IS}}$ and $p_{c,j_{ON}}$, and the numbers of the vehicles at $p_{c,j_{IS}}$ and $p_{c,j_{ON}}$ are $k_{j_{IS}} \cdot l_{j,IS}$ and $k_{j_{ON}} \cdot l_{j,ON}$, respectively. The vehicles in $p_{c,j_{IS}}$ are assumed to have the probability $\zeta_{j,SW}$, $\zeta_{j,SN}$, and $\zeta_{j,SE}$ to proceed into the district corresponding to $p_{c,j_{OW}}$, $p_{c,j_{ON}}$, and $p_{c,j_{OE}}$ as follows,

Fig. 2. Outline of the proposed controller

$$
k_{j_{SW}}(\tau) = k_{j_{IS}}(\tau)\zeta_{j,SW}, \tag{4}
$$
$$
k_{j_{SN}}(\tau) = k_{j_{IS}}(\tau)\zeta_{j,SN}, \tag{5}
$$
$$
k_{j_{SE}}(\tau) = k_{j_{IS}}(\tau)\zeta_{j,SE}. \tag{6}
$$

Note that these probabilities are determined by the traffic network structure, and satisfy $0 \leq \zeta_{j,SW}(\tau) \leq 1$, $0 \leq \zeta_{j,SN}(\tau) \leq 1$, $0 \leq \zeta_{j,SE}(\tau) \leq 1$, and $\zeta_{j,SW}(\tau) + \zeta_{j,SN}(\tau) + \zeta_{j,SE}(\tau) = 1$. Therefore, the traffic flows of the three directions are represented by

$$
q\left(k_{j_{SN}}(\tau), k_{j_{ON}}(\tau)\right), \tag{7}
$$
$$
q\left(k_{j_{SW}}(\tau), k_{j_{OW}}(\tau)\right), \tag{8}
$$
$$
q\left(k_{j_{SE}}(\tau), k_{j_{OE}}(\tau)\right). \tag{9}
$$

Since the probability ζ includes the affection of yellow light, yellow light is not explicitly represented in Fig. 1.

3. TRAFFIC NETWORK CONTROL SYSTEM

3.1 MLDS-like representation

The traffic flow q is the function of k_i and k_j, and contain nonlinearity because of the multiplications of the two variables as in Eq.(1). Since q is also the function of traffic light, the traffic flow adjoining the intersection i can be represented by introducing the (continuous) auxiliary variable z as follows

$$
z_{i,\tilde{i}}(\kappa) = q\left(\frac{\boldsymbol{x}_i(\kappa)}{l_i}, \frac{\boldsymbol{x}_{\tilde{i}}(\kappa)}{l_{\tilde{i}}}\right) u_j(\kappa) \tag{10}
$$

, where the traffic light at the intersection i is $u_i(\kappa)$. Here, $GREEN(RED)$ light of the north-south orientation and $RED(GREEN)$ light of the east-west orientation are represented by 0 (1). With z of Eq.(10), the state equation and the constraint inequality are formulated as follows

$$
\boldsymbol{x}(\kappa+1) = \boldsymbol{A}\boldsymbol{x}(\kappa) + \boldsymbol{B}\boldsymbol{z}(\kappa) \tag{11}
$$
$$
\boldsymbol{E_{2\kappa}}\boldsymbol{z}(\kappa) \leq \boldsymbol{E_{1\kappa}}\boldsymbol{u}(\kappa) + \boldsymbol{E_{4\kappa}}\boldsymbol{x}(\kappa) + \boldsymbol{E_{5\kappa}} \tag{12}
$$

, where $\boldsymbol{x} = [x_1, x_2, ..., x_{n_x}]^T$, $u_j \in \{0, 1\}$, $\tilde{i} \in t_j^{\bullet}$ and $\boldsymbol{A}$ is the matrix with the suitable dimension. The equation (12) represents the logical relationship (proposition) of Eq.(1), the maximum capacity constraints of each place, the firing speed of each transition respectively and so on.

Although the mixed logic dynamical system is represented in a compact form as Eq.(11) and Eq.(12), this cannot be directly applied to the model predictive control scheme, since z has the multiplication of the three decision variables (z amounts to $\alpha\, u \cdot x^2$).

3.2 MINLP problem

The Model Predictive Control (MPC)is one of the well-known paradigms for optimizing the systems with constraints and uncertainties. In the MPC policy, the control input at each sampling instant is decided based on the prediction of the behavior for the next several sampling periods called the prediction horizon. In order to formulate

the optimization procedure, first, Eq.(11) is modified to evaluate the state and input variables in the prediction horizon as follows:

$$\boldsymbol{x}(\kappa + \lambda|\kappa) = \boldsymbol{A}^{\lambda}x(\kappa)$$
$$+ \sum_{\eta=0}^{\lambda-1}\{\boldsymbol{A}^{\boldsymbol{\eta}}(\boldsymbol{B}\boldsymbol{z}(\kappa + \lambda - 1 - \eta|\kappa)$$
$$\cdot\boldsymbol{u}(\kappa + \lambda - 1 - \eta|\kappa))\}, \qquad (13)$$

where $\boldsymbol{x}(\kappa + \lambda|\kappa)$ denotes the predicted state vector at sampling index $\kappa + \lambda$, which is obtained by applying the input sequence, $u(\kappa), \cdots, u(\kappa+\lambda)$ to Eq.(11) starting from the state $\boldsymbol{x}(\kappa|\kappa) = \boldsymbol{x}(\kappa)$.

The following performance criterion is introduced to maximize the traffic flow.

$$J(\kappa) = \sum_{\eta=0}^{H-1}\sum_{i=0}^{n_z-1} z_i(\kappa + \eta), \qquad (14)$$

Then the MINLP problem for the traffic network control can be formulated as follows.

$$\begin{aligned}
minimize \quad &-J(\kappa) \\
s.t. \quad \boldsymbol{x} \quad &\in \Re^{\,r}, \\
\boldsymbol{u} \quad &\in \{0,1\}^{l}, \\
\forall \eta \in [1, H], \quad &\boldsymbol{x}\,(\kappa+\eta+1) = \boldsymbol{A}\boldsymbol{x}\,(\kappa+\eta)+\boldsymbol{B}\boldsymbol{z}(\kappa+\eta) \\
\forall \eta \in [1, H], \quad &\boldsymbol{E}_{2\kappa+\eta}\boldsymbol{z}(\kappa + \eta) \le \boldsymbol{E}_{1\kappa+\eta}\boldsymbol{u}(\kappa + \eta) \\
+ &\boldsymbol{E}_{4\kappa+\eta}\boldsymbol{x}(\kappa + \eta) + \boldsymbol{E}_{5\kappa+\eta}
\end{aligned}$$

,where $r = Hn_x, l = Hn_z$.

4. 0-1 CLASSIFICATION BASED ON PWARX SYSTEM

The Fig.2 describes the block diagram of the proposed controller design method, where the MINLP controller is constructed to control the traffic flow in each traffic intersection in a decentralized manner. The traffic inflow from the outside and outflow to the outside are closely affected by the traffic flow in the adjoining traffic intersections. In order to construct the classification map, we need the data of the input and output variables of the MINLP controller obtained by applying to various traffic situation of the network. For this purpose, we adopted a Cellular Automaton based simulator in this paper.

4.1 Classification problem of hybrid dynamics

The PWARX (Piece-Wise Auto Regressive eXogeneous) system is a well-formulated classification technique for the hybrid and nonlinear dynamics. The PWARX system contains the state vector $\boldsymbol{x}$ which consists of the past inputs and past outputs of the system as

$$\boldsymbol{x}(\kappa) = [y'(\kappa - 1), y'(\kappa - 2), \cdots, y'(\kappa - n_a), \qquad (15)$$
$$u'(\kappa - 1), u'(\kappa - 2), \cdots, u'(\kappa - n_b)]$$

and this vector is certainly involved in one of the polyhedral convex regions defined by

$$\chi_i = \{\boldsymbol{x}|\boldsymbol{V}_i\boldsymbol{x}(\kappa) \le \boldsymbol{W}_i\}. \qquad (16)$$

The entire behavior of the state vector is represented in a piece-wise manner. The dynamics of each region is defined as follows

$$f_i(\boldsymbol{x}(\kappa)) = \boldsymbol{\theta}_i\boldsymbol{\rho}(\kappa) \qquad (17)$$

, where $\boldsymbol{\rho}(\kappa)$ is $[\boldsymbol{x}(\kappa), 1]'$, and $\boldsymbol{\theta}$ is the coefficient vector as follows.

$$\boldsymbol{\theta}_i = [a_{i,1}, \cdots, a_{i,n_a}, b'_{i,1}, \cdots, b'_{i,n_b}, f_i]' \qquad (18)$$

The classification problem we address in this paper is a special problem where the output y is a 0-1 binary variable and very good classification performance is desirable even with very large number of the introduced clusters. If we plot the observational data in a pure (not mixed) cluster in the $\boldsymbol{x}$-$y(k)$ space, it will show always zero inclination, since we have the binary output, i.e., all the components of θ, a and b expect for f will be zeros. Therefore the value of $f(\boldsymbol{x}(\kappa))$ must be only 0 or 1.

For this type of clustering problem, the conventional PWARX system cannot well reproduce the restricted 0-1 output variable. Since they simultaneously obtain the clusters and the (linear) dynamics of the clusters applying the least squared method to each of the fixed number of clustering region, the overall accuracy of their reproduced model is not so high. Furthermore they are very sensitive to the initialization concerning the number of the clusters, the position of the initial cluster, and so on.

4.2 0-1 classification based on modified PWARX system

The desirable outputs in a pure cluster of the problem addressed in this paper are continued by the same values, 0 or 1 in the $\boldsymbol{x}$-y space. All the values except for the offset variable f among the parameters of θ will be zeros, i.e., the dynamics in θ-$\boldsymbol{x}$ space will be almost same. Therefore in the conventional PWARX system, the regions with same dynamics are often considered to be included in the same cluster.

The proposed method described below is a hierarchical PWARX system for 0-1 classification as follows.

Step 1   (*Initialization Process*) Set the cluster number, s, the number of the splitting clusters, s_r, the cluster centers, μ_i ($i \in [1, s]$), the initial data group number N, the renew data group number N' and the threshold values $\epsilon > 0$ and $\gamma > 0$. Using K-means, obtain small N data groups so that neighboring data may be belonging to the same groups.

Step 2   (*Piecewise Fitting Process*) Obtain the cluster D_i of ξ points which minimizes the following performance criterion.

$$J_{\chi} = \sum_{i=1}^{s}\sum_{\xi_j \in D_i} ||\xi_j - \mu_i||^2_{R_j^{-1}} \qquad (19)$$

Obtain the guard $\boldsymbol{V}_i$ and $\boldsymbol{W}_i$ by solving the quadratic problem for all i and i' which satisfy $1 \le i \le s$ and $1 \le i' \le s$ ($i \ne i'$) as follows.

$$\text{find } \boldsymbol{V}_{i,i'} \text{ and } \boldsymbol{W}_{i,i'} \qquad (20)$$
$$\text{minimize } \boldsymbol{V}_{i,i'}\boldsymbol{V}_{i,i'}^T \qquad (21)$$
$$\text{subject to } \zeta_l(\boldsymbol{V}_{i,i'}^T\boldsymbol{x_l} + \boldsymbol{W}_{i,i'}) \ge 1 \qquad (22)$$

119

, where l is the data number and ζ is defined as follows.

$$\zeta_l = \begin{cases} 1 & if \quad \xi(\boldsymbol{x_l}) \in D_i \\ -1 & if \quad \xi(\boldsymbol{x_l}) \in D_{i'} \end{cases} \quad (23)$$

Here $\xi(\boldsymbol{x})$ is the function which obtains the corresponding value of ξ from $\boldsymbol{x}$, i.e., ξ is a translation of $\boldsymbol{x}$ in the θ-$\boldsymbol{x}$ space. Then $\boldsymbol{V}_i$ and $\boldsymbol{W}_i$ are obtained as follows.
$\boldsymbol{V}_i = [\boldsymbol{V}_{i,1}^T, \cdots, \boldsymbol{V}_{i,i-1}^T, \boldsymbol{V}_{i,i+1}^T, \cdots, \boldsymbol{V}_{i,s}^T]^T$ and
$\boldsymbol{W}_i = [\boldsymbol{W}_{i,1}, \cdots, \boldsymbol{W}_{i,i-1}, \boldsymbol{W}_{i,i+1}, \cdots, \boldsymbol{W}_{i,s}]^T$.

Step 3   (*Cluster Updating Process*) Update the centers μ according to the following formula.

$$\tilde{\mu}_i = \frac{\sum_{j:\xi_j \in D_i} \xi_j w_j}{\sum_{j:\xi_j \in D_i} w_j} \quad (24)$$

If $max||\tilde{\mu}_i - \mu_i||) < \epsilon$, go to Step 4, otherwise set

$$\mu = \tilde{\mu}_i \quad (25)$$

and go to Step 2.

Step 4   (*Cluster Splitting Process*) Obtain J_i for all $i \in [1, s]$ which is defined by

$$J_i = \sigma^2(y(\kappa)). \quad (26)$$

Step 4-1   For all $i \in [1, s]$, do the following. If $J_i \le \gamma$, do the following

$$\chi = \chi - \chi_i \quad (27)$$

$$\chi_i = \{\boldsymbol{x} | \boldsymbol{V}_i \boldsymbol{x} \le \boldsymbol{W}_i\} \quad (28)$$

, otherwise set new centers of the s_r clusters, μ_r in D_i randomly, and do the following.

$$s = s + s_r \quad (29)$$

Here, $\sigma^2(y(\kappa))$ is the covariance of $y(\kappa)$ in the cluster D_i.

Step 4-2   Set i_m as follows.

$$i_m = \arg \min_{i \in [1,s]} \sigma^2(y(\kappa)) \quad (30)$$

Step 4-3   If $J_{i_m} \le \gamma$, terminate with success, otherwise, obtain N' data group of the corresponding resion of D_{i_m} and go to Step 2.

In Step 2, R_j is defined as

$$R_j = \begin{bmatrix} V_j & 0 \\ 0 & Q_j \end{bmatrix} \quad (31)$$

, where

$$V_j = \frac{S_j}{c - (n_a + n_b) + 1}(\Phi_j' \Phi_j)^{-1} \quad (32)$$

$$Q_j = \sum_{(x,y) \in C_j} (x - m_j)(x - m_j)' \quad (33)$$

$$\Phi_j = \begin{bmatrix} x_1 & x_2 & \cdots & x_c \\ 1 & 1 & \cdots & 1 \end{bmatrix} \quad (34)$$

$$S_j = y_{c_j}'(I - \Phi_j(\Phi_j'\Phi_j)^{-1}\Phi_j')y_{c_j} \quad (35)$$

$$m_j = \frac{1}{c} \sum_{(x,y) \in C_j} \boldsymbol{x}, j = 1, \cdots, N \quad (36)$$

Fig. 3. Density of traffic flow

$$\xi_j = [(\theta_j)', m_j'] \quad (37)$$

$$w_j = \frac{1}{\sqrt{(2\pi)^{(2n_a+2n_b+1)}\det(R_i)}}. \quad (38)$$

V_j is the empirical covariance matrix which measures the relevance criterion, Q_j is the scatter matrix which measures the sparsity of the data in the cluster j, S_j is the sum of the squared residuals, C_j is the cluster in the $\boldsymbol{x}$ space, x_j is the vector of the regressor belonging to C_j, y_{c_j} is the output vector included in C_j.

5. GENERATION OF TRAFFIC FLOW DATA

5.1 Cellular Automaton based traffic network simulator

When we construct the input-output PWA map of the traffic network controller, the traffic flow data with sufficient amounts of a variety traffic situation must be provided. However it is generally impossible to take the experimental data, directly applying the developed traffic controller to the real traffic system. Therefore in this paper, we used Cellular Automaton model, which is well known to reproduce real traffic flow dynamics.

5.2 Simulation Environment

For this simulation, we used following traffic network: we try to develop the traffic controller for intersection and entire traffic network consists of 4 intersections connected with each other as Fig.4. The length of each block is 1000 [m] long and the length of each section is 500 [m] long, all with 2 lanes bi-directionally. We empirically took the traffic flow data using the CA based simulator, where the sampling interval of CA is 1 sec and the sampling interval in the traffic network data saving is 10 sec. This is for the traffic network controller construction.

In order to consider a variety of traffic situation, we used two types of the traffic flow dynamics as follows: a sinusoidal wave is for considering the steady state traffic flow with a variety size, and a square wave is for considering the non-steady state traffic flow such as the effect from the stalled traffic at the adjoining section(s) or change of the adjoining traffic signal. Furthermore, in order to consider the combination of the traffic flow from the adjoining 4 sections, the periodes of the waves are set to be different as 2000, 4000, 8000, and 16000 sec for the sinusoidal wave and 100, 200, 400, and 800 sec for the square wave, respectively. The figure 3 are the traffic flow data from 4 directions in this paper, where k_E, k_W, k_S and k_N imply the traffic inflows from east, west south and north sides of the intersection and 5000 patterns of traffic situations during 50000 seconds are simulated.

6. CLASSIFICATION RESULTS

6.1 Comparison with conventional PWARX system

In this subchapter, the conventional PWARX system is compared with our proposed method. Table 1 and 2 compare the cluster number and data number in the clusters

	Total	Red	Blue	Mixed
Proposed	225	116	109	0
Conventional	100	30	32	38
	200	67	96	37
	300	127	135	38
	400	185	183	32
	500	228	255	17

Table 1. Comparison of cluster number (H=1)

	Cluster Number	Data Number of Red Clusters	Blue	Mixed
Proposed	225	1171	1327	0
Conventional	100	614	853	1031
	200	926	1113	459
	300	984	1180	334
	400	1014	1250	234
	500	1095	1263	140

Table 2. Comparison of data number in the cluster (H=1)

	Total	Red	Blue	Mixed
Proposed	228	116	112	0
Conventional	100	32	29	39
	200	78	83	39
	300	136	136	28
	400	186	184	30
	500	240	246	14

Table 3. Comparison of cluster number (H=3)

	Cluster Number	Data Number of Red Clusters	Blue	Mixed
Proposed	228	1175	1323	0
Conventional	100	609	715	1174
	200	830	1089	579
	300	1040	1245	213
	400	1047	1227	224
	500	1148	1266	84

Table 4. Comparison of data number in the cluster (H=3)

Fig. 4. Traffic network

of the results obtained by applying the proposed method and conventional method. In the Table 1 and 2, the conventional method is applied with the initial cluster number of 100, 200, 300, 400 and 500 respectively. Although most of data are well classified using the conventional PWARX system with introduction of large number of clusters, 2.8 and 1.6 percents of the total data were not correctly classified. In contrast the proposed method was perfectly classified introducing relatively small cluster number.

7. TRAFFIC NETWORK CONTROL SIMULATION RESULTS

7.1 Case study example for traffic network control

In this subsection, the effectiveness of our proposed method for the large-scale traffic network control with the arterial roads shown in Fig.6. The proposed 0-1 PWARX classification based controller is adopted to each intersection. In Fig.4, the center intersection is surrounded by black line called control block (CB), where the traffic

	Passed Vehicles
No Control	30089
Proposed controller with $H=1$	39363
Proposed controller with $H=2$	39246
Proposed controller with $H=3$	51059
Proposed controller with $H=4$	50697
Proposed controller with $H=5$	40939

Table 5. Comparison of Control Performance

flow information such as the traffic density is measured for the 0-1 classification of the center intersection traffic network controller. Fig.4 illustrates that sixteen control blocks constitute the entire traffic network where the sensory information at each boundary of CBs is shared with the control of the adjoining blocks. Note that two arterial roads are running north-south (second road from the left) and east-west (second road from the top), respectively.

We assume that from the outside of the network, the traffic flows of vehicles move into the network with random speeds, whereas the traffic flows inside the network, move from the network with the speed of the maximum velocity (no congestion arises and affects the traffic flow inside the network). We used (14) as a performance criterion. All results are obtained from simulations over 30 minutes, where the sampling interval T_s is 10 [sec].

Table 5 shows the obtained solutions by applying the proposed method in the case that there are 2 arterial roads. The simulation result shows that the proposed method has good performance. Note that our method has always better or equal solutions, compared with the cases of 'No Control'. Here 'No Control' implies the traffic signals are changed every 30 minutes. The reason why the controller with H is 4 or 5 is worse than the case with $H = 3$ is that the traffic network is dynamically changing system as shown in Fig. 4. we used the random probabilities of left and right turning at every intersection. Therefore too long prediction horizon may deteriorate the control performance. If traffic jam occurs in a specific section, the controllers in the adjoining section or intersection will take action to alleviate the congestion that a new action will be taken during long horizon H. In other words the controller with $H=3$ is good enough for this problem.

7.2 Comparison of computational efforts

Lastly the computational amounts are investigated in this subchapter. Table 6 compares the computational efforts of the results obtained by applying the proposed 0-1 classification based method and the MINLP controller reported in Kato et al. [2006]. It is known that the solution method of the hybrid dynamical system is extremely burdensome, requiring exponential time of the binary variable number. In contrast, the proposed method requires only 0.07 seconds regardless of the binary variable number. In this simulation, the plant input u_P (controller output $u(\kappa)$) is obtained by applying the MINLP controller with MPC horizon $H=3$ as saw in the result of Table 5. The same solutions are obtained using only 5.7×10^{-2} percent time the MINLP controller requires.

	Computation Time[sec]
Proposed	0.0746
Proposed controller with $H=1$	0.8095
Proposed controller with $H=2$	20.5678
Proposed controller with $H=3$	123.2846
Proposed controller with $H=4$	286.0026
Proposed controller with $H=5$	912.6755

Table 6. Comparison of Computational Efforts

8. CONCLUDING REMARKS

In this paper we have proposed a new design method for the traffic network hybrid feedback controller. Since the output of the traffic network controller is the 0-1 binary signals, the output of the developed controller has been reproduced applying the 0-1 classifications of the PWARX systems. The developed PWARX classifier describes the nonlinear feedback control law of the traffic control system. As we checked in the chapter 7, very good solutions are obtained in a very short time, compared with the one obtained with the conventional MINLP controller.

REFERENCES

T. Kato, Y. Kim and S. Okuma Model Predictive Control of Traffic Flow Based on Hybrid System Modeling *ICCAS*, pages 368–373, 2005.

T. Kato, Y. Kim, S. Okuma and T. Narikiyo Large-Scale Traffic Network Control Based on Convex Programming Coupled with B & B Strategy *IEEJ Trans. EIS* volume 126, no 6, pages 752–760, 2006.

S.Darabha and K.R.Rahagopal Intelligent cruise control systems and traffic flow stability *Transportation Research Part C* volume 7, pages 329–352, 1999.

Y. Kato Traffic Flow Simulation by Cellular Automaton Method *JSIE* volume 15, no 2 pages 242–250, 2000.

K. Nagel and M. Schreckenberg Cellular Automaton Model for Freeway Traffic *Journal de Physique I France* pages 2–2221, 1992.

H.J. Payne Models of freeway traffic and control in Mathematical Models of Public Systems *Simulation Council Proceedings Series* volume 1, no 1, pages 51–61, LaJolla, California, 1971.

M.J. Lighthill and G.B. Whitham On kinematic waves II. A theory of traffic flow on long crowded roads *Proc. R. Soc. London Ser. A* volume 229, pages 281, 1955.

I. Prigogine and R. Herman Kinetic Theory of Vehicular Traffic *Elsevier, New York* , 1971.

M. Cremer and J. Ludwig A fast simulation model for traffic flow on the basis of Boolean operations *Mathematics and Computers in Simulation* volume 28, pages 297–303, 1986.