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Proportional and Proportional-Integral Controllers for a Nonlinear Hydraulic Network ⋆

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Abstract: We consider the problem of regulating to a reference value pressures across components in a nonlinear hydraulic network of a reduced-size yet meaningful district heating system with two end-users. Exploiting the analogy between electrical and hydraulic networks, we derive a nonlinear model for the system. Then we design and analyze a proportional and a proportional-integral controller which guarantee semi-global practical and, respectively, asymptotic regulation of the pressures.

1. INTRODUCTION

Plug and Play Process Control is a Research Program which investigates control problems for complex systems with a modular structure motivated by a number of case studies. Because of the modular structure, a fundamental aspect of the project is to understand how to detect the addition of new components to the systems, and to reconfigure correspondingly the controllers. In this paper we focus on one of these case studies, a district heating system (Kallesøe (2007)), for which we derive the model, formulate the control problem, and propose a solution designing proportional and proportional-integral controllers. Other papers (see Knudsen et al. (2007)) apply system identification techniques to understand how the model modifies when components are added to the system.

Presently district heating systems are designed to meet the needs of a given number of end users. Increased demand due to possible expansion of the district may require a re-design of the entire heating system. A possible alternative is to employ a distributed design approach in which the increased demand is met by adding *locally* a new set of pipelines and pumps. This requires to turn from a centralized control design to a distributed control design, in which each pump is controlled to meet the demands of a single end user notwithstanding the presence of the remaining users connected through the network (see Figure 1).

In this document, we examine a simple yet meaningful example of a district heating system with two users. Exploiting basic circuit theory tools we first derive a nonlinear model of the control system under study. Then, depending on the model adopted to describe the demand of the user, we propose two simple controllers. If we assume that the demand is modeled by an erratic time-varying signal, then we prove that a *proportional* controller guarantees semi-global practical regulation, meaning that the controller is able to guarantee the behavior of the system to be arbitrarily close to the demand of the users, although it may not ever converge to the exact demand. The analysis is Lyapunov-based and exploits variations of the Lyapunov functions introduced in Teel and Praly (1995). On the other hand, if we assume that the reference signal modeling the demand of the users is a (piece-wise) constant signal of the time, then we design a *proportional-integral controller* which asymptotically meets the demand. In this case, the analysis is carried out relying on the theory of nonlinear output regulation (Isidori et al. (2003), Serrani et al. (2000)).

There has been a renewed interest in the control of hydraulic systems. In particular, we mention a couple of contributions about irrigation channels where the interested reader can find more references. In Cantoni *et al.* (2007), the focus is on a cascade of irrigation channels. This allows the authors to regard each pipe as independent and see the terms due to the other pipes as a disturbance. The tools are essentially linear. *Nonlinear* control techniques have been employed e.g. in Besançon *et al.* (2001), to control an irrigation channel. Models and techniques, however, appear to be different from those in the present paper.

In the next section a district heating system is introduced. The dynamic model of the hydraulic network of a reduced-size system is presented in Section 3. The proportional controller and the proportional-integral controller are studied in Section 4 and 5, respectively. Numerical results are illustrated in Section 6.

2. DISTRICT HEATING SYSTEMS

In the pipeline network of Figure 1, four different components are present: *heat exchangers*, *pipelines*, *valves*, and *pumps*. As the time constants of the heat dynamics are very much slower than the hydraulic dynamics, in the following only the hydraulic phenomena of the network are treated. Therefore, it is only necessary to take into

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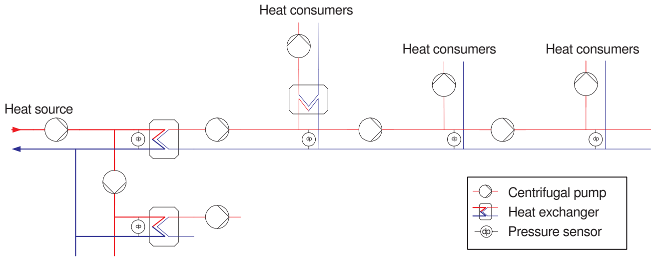

Fig. 1. Structure of a distributed district heating system.

account pumps, pipes and valves.

Pump model. Pumps, together with their connection points to the hydraulic network are depicted in Fig. 3. Models for such pumps are derived in Kallesøe (2005). In this paper, we regard the pump as a device which is able to deliver a desired pressure difference $h_j - h_i$, where h_i, h_j are the pressures at the nodes relative to a common pressure value. The pressure difference $h_j - h_i$ is viewed as a *control input* (see Section 3).

Pipe model. The pipe model is derived under the assumption that the flow is uniformly distributed along a cross section of the pipe, and that the flow is turbulent. The model for the kth pipe is the following:

$$J_k \frac{dq_k}{dt} = (h_i - h_j) - K_{pk}|q_k|q_k \tag{1}$$

where $h_i - h_j$ is the pressure across the pipe, q_k is the flow through the pipe, and J_k and K_{pk} are constant parameters of the pipe.

Valve model. The valves are normally viewed as pipe fittings. They can be modeled by a quadratic relationship between the pressure across the valve and the flow through the valve (Roberson and Crowe (1999)). That is,

$$h_i - h_j = K_{vk}|q_k|q_k \ , \tag{2}$$

where $h_i - h_j$ is the pressure across the valve, q_k is the flow through the valve, and K_{vk} is a variable denoting the change of hydraulic resistance of the valve.

3. MODEL DERIVATION

The district heating system under consideration is composed of three pumps and three heat exchangers, and supplies two apartment buildings, see Fig. 2. Based on network theory, we derive below a model for the system. We exploit the well-known analogy between electrical and hydraulic circuits, and replace voltages and currents with, respectively, pressures and flows. The first step is to identify *fundamental* circuits in which the flows are *independent* variables. Loosely speaking, independent flow variables are variables such that any other flow variable can be derived from the former ones. These can be identified by simple topological arguments. We briefly remind how to proceed and refer the interested reader to e.g. Desoer and Khu (1969) for details. The first step is to associate a *graph* $\mathcal{G}$ to the circuit of Figure 3, obtained from the circuit by taking its nodes as nodes of the graph and replacing each component with an edge. A *tree* $\mathcal{T}$ of the graph $\mathcal{G}$ is any connected sub-graph with no cycles. A *co-tree* of $\mathcal{T}$ is a subgraph of $\mathcal{G}$ containing exactly those edges of $\mathcal{G}$ which do not belong to $\mathcal{T}$. A fundamental circuit, then, is obtained by adding a single edge of the co-tree to the tree. The flow through the added edge of the co-tree is an independent flow variable. In the present case, the fundamental circuits are 2, easily recognizable even by inspection. The first one is composed by the sequence of nodes $n_1, n_2, \ldots, n_{10}$ and the components among them. The independent variable, denoted by q_1, is the flow through the pipeline c_4. The second circuit is identified by the nodes $n_1, n_2, n_{11}, n_{12}, n_{13}, n_8, n_9, n_{10}$, with independent variable q_2 equal to the flow through pipeline c_{11}.

We can now proceed directly to the derivation of the model.

For the first circuit, we write the pressure balance along the circuit starting e.g. from node n_1. We obtain:

$$\begin{aligned} -\Delta h_{c_2} - \Delta h_{c_3} - \Delta h_{c_4} + \Delta h_{c_5} - \Delta h_{c_6} - \Delta h_{c_7} \\ -\Delta h_{c_8} - \Delta h_{c_9} - \Delta h_{c_{10}} + \Delta h_{c_1} = 0 \ , \end{aligned} \tag{3}$$

where Δh_{c_i} denotes the pressure across the component c_i:

- Δh_{c_5}, Δh_{c_1} are the pressure values delivered by the pumps present in the circuit, and as such are interpreted as *control* inputs;
- Δh_{c_6}, $\Delta h_{c_{10}}$ are the pressures across the valves, which have constitutive laws given by (see (2)):
$$\Delta h_{c_6} = K_{v6} q_1^2 \ , \quad \Delta h_{c_{10}} = K_{v10}(q_1 + q_2)^2 \ ;$$
- Δh_{c_i}, for $i \in \mathcal{P}_1 = \{2, 3, 4, 7, 8, 9\}$ represent the pressure drops across the pipes. Observe that for components c_i with $i \in \mathcal{P}_{12} = \{2, 9\}$ the pressure drop depends on the sum $q_1 + q_2$. Hence, the relations between flow and pressure drops are (see (1)):

$$\begin{aligned} J_i \dot{q}_1 &= -K_{pi} q_1^2 + \Delta h_{c_i} \ , i \in \mathcal{P}_1 \setminus \mathcal{P}_{12} \\ J_i(\dot{q}_1 + \dot{q}_2) &= -K_{pi}(q_1 + q_2)^2 + \Delta h_{c_i} \ , i \in \mathcal{P}_{12} \ . \end{aligned} \tag{4}$$

Summing up equations (4), we have

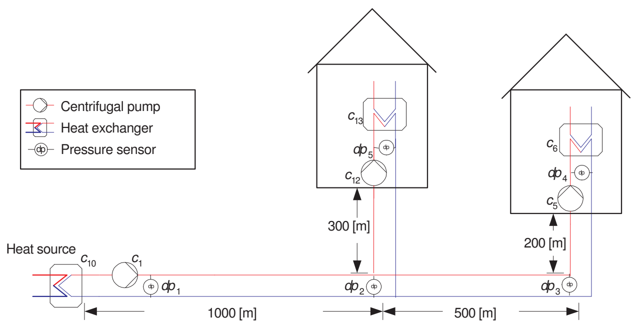

Fig. 2. A sketch of a small District Heating System

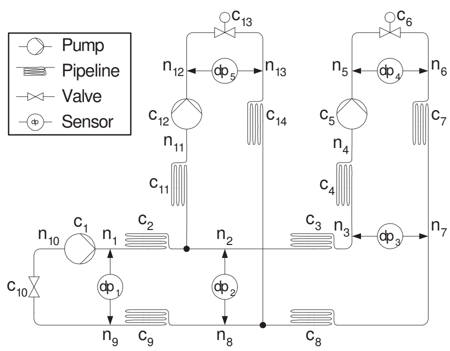

Fig. 3. The hydraulic network diagram.

$$
\begin{aligned}
&(\sum_{i\in\mathcal{P}_1} J_i)\dot{q}_1 + (\sum_{i\in\mathcal{P}_{12}} J_i)\dot{q}_2 = \\
&\quad -(\sum_{i\in\mathcal{P}_1} K_{pi})q_1^2 - (\sum_{i\in\mathcal{P}_{12}} K_{pi})(q_2^2+2q_1q_2) + \sum_{i\in\mathcal{P}_1}\Delta h_{c_i} \ .
\end{aligned}
$$

From (3), we then obtain

$$
\begin{aligned}
&-(\sum_{i\in\mathcal{P}_1} J_i)\dot{q}_1 - (\sum_{i\in\mathcal{P}_{12}} J_i)\dot{q}_2 - (\sum_{i\in\mathcal{P}_1} K_{pi})q_1^2 - \\
&\quad (\sum_{i\in\mathcal{P}_{12}} K_{pi})(q_2^2+2q_1q_2) + (\Delta h_{c_5}+\Delta h_{c_1}) - \\
&\quad (K_{v6}+K_{v10})q_1^2 - K_{v10}(q_2^2+2q_1q_2) = 0 \ .
\end{aligned}
$$

Similarly for the second circuit we have ($\mathcal{P}_2 = \{2, 9, 11, 14\}$)

$$
\begin{aligned}
&-(\sum_{i\in\mathcal{P}_2} J_i)\dot{q}_2 - (\sum_{i\in\mathcal{P}_{12}} J_i)\dot{q}_1 - (\sum_{i\in\mathcal{P}_2} K_{pi})q_2^2 - \\
&\quad (\sum_{i\in\mathcal{P}_{12}} K_{pi})(q_1^2+2q_1q_2) + (\Delta h_{c_{12}}+\Delta h_{c_1}) - \\
&\quad (K_{v13}+K_{v10})q_2^2 - K_{v10}(q_1^2+2q_1q_2) = 0 \ .
\end{aligned}
$$

We can simplify the equations with few calculations and setting $P_k = -\sum_{i\in\mathcal{P}_{12}} J_i / \sum_{i\in\mathcal{P}_k} J_i$, $k = 1, 2$, to obtain,

$$
\begin{aligned}
\mathcal{J}_1\dot{q}_1 &= -\mathcal{K}_{11}q_1^2 + u_1 - 2\mathcal{K}_{12}q_1q_2 - \mathcal{K}_{13}q_2^2 \\
\mathcal{J}_2\dot{q}_2 &= -\mathcal{K}_{21}q_2^2 + u_2 - 2\mathcal{K}_{22}q_1q_2 - \mathcal{K}_{23}q_1^2
\end{aligned}
\tag{5}
$$

with the coefficients $\mathcal{K}$s, $\mathcal{J}$s, and the control inputs u_i

$$
\begin{aligned}
&\mathcal{J}_1 = \sum_{i\in\mathcal{P}_1} J_i + P_2\sum_{i\in\mathcal{P}_{12}} J_i \ , \ \mathcal{J}_2 = \sum_{i\in\mathcal{P}_2} J_i + P_1\sum_{i\in\mathcal{P}_{12}} J_i \ , \\
&\mathcal{K}_{11} = \sum_{i\in\mathcal{P}_1} K_{pi} + K_{v6} + K_{v10} + P_2(\sum_{i\in\mathcal{P}_{12}} K_{pi} + K_{v10}) \ , \\
&\mathcal{K}_{12} = 2(\sum_{i\in\mathcal{P}_{12}} K_{pi} + K_{v10})(1+P_2) \ , \\
&\mathcal{K}_{13} = \sum_{i\in\mathcal{P}_{12}} K_{pi} + K_{v10} + P_2(\sum_{i\in\mathcal{P}_2} K_{pi} + K_{v13} + K_{v10}) \ , \\
&\mathcal{K}_{21} = \sum_{i\in\mathcal{P}_2} K_{pi} + K_{v13} + K_{v10} + P_1(\sum_{i\in\mathcal{P}_{12}} K_{pi} + K_{v10}) \ , \\
&\mathcal{K}_{22} = 2(\sum_{i\in\mathcal{P}_{12}} K_{pi} + K_{v10})(1+P_1) \ , \\
&\mathcal{K}_{23} = \sum_{i\in\mathcal{P}_{12}} K_{pi} + K_{v10} + P_1(\sum_{i\in\mathcal{P}_1} K_{pi} + K_{v6} + K_{v10}) \ , \\
&u_1 = \Delta h_{c_5} + \Delta h_{c_1} + P_2(\Delta h_{c_{12}} + \Delta h_{c_1}) \ , \\
&u_2 = \Delta h_{c_{12}} + \Delta h_{c_1} + P_1(\Delta h_{c_5} + \Delta h_{c_1}) \ .
\end{aligned}
$$

The measured and controlled outputs coincide. We have:

$$
y_1 = K_{v6}q_1^2 \ , \ y_2 = K_{v13}q_2^2 \ . \tag{6}
$$

The control goal is to regulate to a constant r the outputs y_1, y_2, despite of the (not measured) variations of the values of K_{v6}, K_{v13}, due to the (time-varying) demands of the end-users. On the other hand, parameters K_{pi}, J_i and K_{v10} are usually known, although this is not required by the two controllers introduced in the next section.

4. PROPORTIONAL CONTROLLER FOR PRACTICAL REGULATION

The system to consider – upon renaming the coefficients and the control inputs – is of the form

$$
\begin{aligned}
\dot{q}_1 &= -\beta_{11}q_1^2 - \beta_{12}q_1q_2 - \beta_{13}q_2^2 + u_1 \\
\dot{q}_2 &= -\beta_{21}q_2^2 - \beta_{22}q_1q_2 - \beta_{23}q_1^2 + u_2 \\
y_1 &= \alpha_1 q_1^2 \\
y_2 &= \alpha_2 q_2^2 \ .
\end{aligned}
\tag{7}
$$

Since all the coefficients above depend (*smoothly*) on K_{v6} and K_{v13} which are uncertain, the coefficients are uncertain as well. Before the analysis, we introduce the error variables $e_i = x_i - r$ (supposed, without loss of generality, to be available for feedback) and consider a preliminary change of coordinates, namely $x_i = \alpha_i q_i^2$, with α_i constant (but see the Remark following the proof of Lemma 1 below), $i = 1, 2$, which yields the following model

$$
\begin{aligned}
\dot{x}_1 &= \tilde{f}_1(x_1, x_2, \alpha_1, \alpha_2) + 2\alpha_1^{1/2}x_1^{1/2}u_1 \\
\dot{x}_2 &= \tilde{f}_2(x_1, x_2, \alpha_1, \alpha_2) + 2\alpha_2^{1/2}x_2^{1/2}u_2 \\
e_1 &= x_1 - r \\
e_2 &= x_2 - r \ ,
\end{aligned}
\tag{8}
$$

where $\tilde{f}_1$ and $\tilde{f}_2$ are functions defined as

$$
\begin{aligned}
\tilde{f}_1(x_1, x_2, \alpha_1, \alpha_2) &= -2\alpha_1^{-1/2}\beta_{11}x_1^{3/2} - \\
&\quad 2\alpha_2^{-1/2}\beta_{12}x_1x_2^{1/2} - 2\alpha_1^{1/2}\alpha_2^{-1}\beta_{13}x_1^{1/2}x_2 \\
\tilde{f}_2(x_1, x_2, \alpha_1, \alpha_2) &= -2\alpha_2^{-1/2}\beta_{21}x_2^{3/2} - \\
&\quad 2\alpha_1^{-1/2}\beta_{22}x_2x_1^{1/2} - 2\alpha_2^{1/2}\alpha_1^{-1}\beta_{23}x_2^{1/2}x_1 \ .
\end{aligned}
\tag{9}
$$

Observe that both $\tilde{f}_1$ and $\tilde{f}_2$ are *smooth* functions of their arguments, provided that these arguments range over sets which do not include the zero. In practice, r is a known constant value. Nevertheless, for the sake of generality we shall suppose that the value of r can range over a compact set $\mathcal{R} \subset I\!R_+$, namely $\mathcal{R} = \{r : 0 < r_m \le r \le r_M\}$. In what follows, the following terminology will be in use: a trajectory is *attracted* by a set Q if it is defined for all $t \ge 0$, and it belongs to Q for all $t \ge T$, with $T > 0$ a finite time. Our control goal is the following:

Practical Pressure Regulation Problem. Given system (8), parameters $0 < x_m \le x_M$, a compact interval of reference values $\mathcal{R}$, a compact set of initial conditions

$$
\mathcal{X} = \{x \in I\!R_+^2 \ : \ x_m \le x_i \le x_M \ , \ i = 1, 2\} \ , \tag{10}
$$

and an arbitrarily small positive number ϵ, find controllers of the form $u_i = N_i e_i$, $i = 1, 2$, such that, for any $r \in \mathcal{R}$, every trajectory $x(t)$ of the closed-loop system with initial condition in $\mathcal{X}$ is attracted by the set $\{e \in I\!R^2 \ : \ |e_i| \le \epsilon \ , \ i = 1, 2\}$.

The following result, in which restrictions on the set of initial conditions $\mathcal{X}$ are imposed, is instrumental to solve the problem above:

Lemma 1. For any choice of parameters $0 < \gamma < 1$, $x_M > 0$, any compact set $\mathcal{R} \subset I\!R_+$, any compact set

$$
\mathcal{X} = \{x \in I\!R_+^2 \ : \ r_M - (1-\gamma)r_m \le x_i \le x_M \ , \ i = 1, 2\} \ ,
$$

and for any arbitrarily small positive number ϵ, there exist gains $N_i^* < 0$ such that for all $N_i \le N_i^*$, the control laws $u_i = N_i e_i$ guarantee that, for any $r \in \mathcal{R}$, every trajectory $x(t)$ of the closed-loop system with initial condition in $\mathcal{X}$ is attracted by the set $\{e \in I\!R^2 \ : \ |e_i| \le \epsilon \ , \ i = 1, 2\}$.

Proof. In the error variables, system (8) writes as

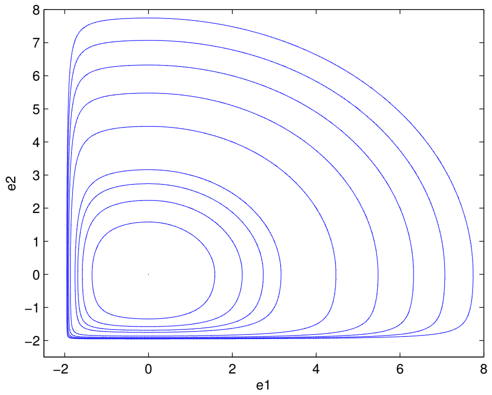

Fig. 4. Level sets for the Lyapunov function $V(e)$, with $n = 2$, $d = 64$, $r_m = 2$, $\gamma = 0.5$.

$$\begin{aligned} \dot{e}_1 &= \tilde{f}_1(e_1 + r, e_2 + r, \alpha_1, \alpha_2) + 2\alpha_1^{1/2}(e_1 + r)^{1/2}u_1 \\ \dot{e}_2 &= \tilde{f}_2(e_1 + r, e_2 + r, \alpha_1, \alpha_2) + 2\alpha_2^{1/2}(e_2 + r)^{1/2}u_2 \,. \end{aligned} \tag{11}$$

Observe that the coefficients (*high frequency gains*) which multiply the controls u_i depend on the state variables e_i. To guarantee that the control action never vanishes, we first show that the high frequency gains are bounded away from zero showing that the variables e_i, $i = 1, 2$, evolve in a set such that $e_i(t) > -r_m$.
Consider first the set of initial conditions of (11). Observe that $e_i(0) \geq -(1-\gamma)r_m =: -e_m$. On the other hand, because of the arbitrariness of x_M, there exists (an arbitrarily large) $d > 1$ such that $e_i^2(0) \leq d^2$, if $e_i(0) \geq 0$. We introduce now a Lyapunov function for which a level set exists (cf. Fig. 4) such that: (a) it includes the set of initial conditions above, and (b) it lies in a portion of the state space such that $e_i > -r_m$. To take into account the *asymmetry* of the set of initial conditions, we propose the following Lyapunov function, inspired by similar functions in Teel and Praly (1995):

$$V(e_1, e_2) = \begin{cases} \dfrac{r_m^2\gamma^2 e_1^2}{r_m^2 - e_1^2}\,, & e_1 \leq 0 \\ e_1^2\,, & e_1 \geq 0 \end{cases} + \begin{cases} \dfrac{r_m^2\gamma^2 e_2^2}{r_m^2 - e_2^2}\,, & e_2 \leq 0 \\ e_2^2\,, & e_2 \geq 0\,. \end{cases}$$

The Lyapunov function is continuously differentiable and satisfies properties (a) and (b) above. In fact, it is easily verified (Teel and Praly (1995)) that the set of initial conditions is included in the level set $S_1 = \{e : V(e) \leq 2(\max\{e_m, d\})^2\}$. On the other hand, if the state (e_1, e_2) belongs to the level set S_1, then

$$e_i^2 \leq \frac{2(\max\{e_m, d\})^2}{2(\max\{e_m, d\})^2 + r_m^2\gamma^2} r_m^2\,, \qquad \text{if } e_i \leq 0\,,$$

and $e_i^2 \leq 2(\max\{e_m, d\})^2$, if $e_i \geq 0$, so that $e_i^2 < r_m^2$ if $e_i \leq 0$, and $e_i^2 < 2(\max\{e_m, d\})^2 + 1$ if $e_i \geq 0$. In particular, if $e_i \leq 0$, we can suppose the existence of $0 < \hat{\gamma} < 1$ such that $e_i \geq -(1-\hat{\gamma})r_m$, and this in turn implies $e_i + r \geq \hat{\gamma} r_m$, that is the 2 high frequency gains in (11) are bounded away from zero. In other words, $e_i + r \geq \hat{\gamma} r_m$ as far as the state evolves within the level set S_1. Since, the set of initial conditions is contained in S_1, we are now left with proving that $\dot{V} < 0$ on (a subset of) this level set. We have:

$$\begin{aligned} \dot{V}(e_1, e_2) = \sum_{i=1,2} \frac{\partial V}{\partial e_i} [\tilde{f}_i(e_1 + r, e_2 + r, \alpha_1, \alpha_2) + \\ 2\alpha_i^{1/2}(e_i + r)^{1/2}u_i]\,, \end{aligned}$$

$$\frac{\partial V}{\partial e_i} = \begin{cases} \dfrac{r_m^4\gamma^2}{(r_m^2 - e_i^2)^2} 2e_i & e_i \leq 0 \\ 2e_i & e_i \geq 0\,. \end{cases}$$

Note that, as far as $V(e) \leq 2(\max\{e_m, d\})^2$,

$$\gamma^2 \leq \frac{r_m^4\gamma^2}{(r_m^2 - e^2)^2} \leq \frac{((2(\max\{e_m, d\})^2 + r_m^2\gamma^2)^2}{r_m^4\gamma^2}\,. \tag{12}$$

The first term in the sum is bounded from above by a constant M_i, as the $\tilde{f}_i$ are smooth functions. Let now $u_i = N_i e_i$, $N_i < 0$. We have:

$$\dot{V}(e_1, e_2) \leq \sum_{i=1,2} \left[M_i + \gamma^2 4\alpha_i^{1/2}(\hat{\gamma} r_m)^{1/2} N_i e_i^2 \right]\,.$$

Consider now a level set S_2 included in the cube $\{e \in I\!R^2 : |e_i| \leq \epsilon\,,\ i = 1, 2\}$, and a cube included in S_2. Let $2\epsilon'$ be the length of the side of the latter cube. By definition of ϵ', the error vector e which does not belong to S_2 is such that $|e_j| \geq \epsilon'$ for at least an index j. Then, outside S_2,

$$\dot{V}(e_1, e_2) \leq \sum_{i=1,2} M_i + \gamma^2 4\alpha_j^{1/2}(\hat{\gamma} r_m)^{1/2} N_j \epsilon'^2\,.$$

Then it is easy to see that setting

$$N_j^* = -\frac{\sum_{i=1,2} M_i + 1}{\gamma^2 4\alpha_j^{1/2}(\hat{\gamma} r_m)^{1/2}\epsilon'^2}\,, \qquad j = 1, 2\,, \tag{13}$$

for all $N_j \leq N_j^*$, $\dot{V}(e) < 0$ for all e in the set $S_1 \setminus S_2$, and for the α_js ranging over compact sets. Hence, the state converges to the level set S_2, where $|e_i| \leq \epsilon$, $i = 1, 2$, in finite time and stays there from that time on. ■

Remark. One can tackle also the case in which the α_is (i.e. the users' demands) are *time-varying* with a bounded derivative $\dot{\alpha}_i$. In fact, a time varying α_i implies that in the equations (11) the term $\dot{\alpha}_i(e_i + r)/\alpha_i$ adds up, whose effect can be incorporated in the bound M_i, larger than before. Hence, the effect of time-varying parameters α_i can be counteracted by an appropriate redesign of the gains N_is.

A straightforward consequence of Lemma 1 is the main result of this section and shows that the restriction on $\mathcal{X}$ can be removed.

Proposition 1. For any choice of the parameters $0 < x_m \leq x_M$, any compact set $\mathcal{R} \subset I\!R_+$, and for any arbitrarily small positive number ϵ, there exist gains $N_i^* < 0$ such that for all $N_i \leq N_i^*$, the control laws $u_i = N_i e_i$ solve the Practical Pressure Regulation Problem.

Proof. Consider the special case in which $\mathcal{R} = \{r\}$. Apply Lemma 1 with $r_m = r_M = r$ and $\gamma = x_m/r$. (We can assume without loss of generality that $x_m < r$, so that $0 < \gamma < 1$.) In particular, the set $\mathcal{X}$ in Lemma 1 takes the form (10). Then Lemma 1 proves the existence of gains N_i^* which solve the Practical Pressure Regulation Problem in

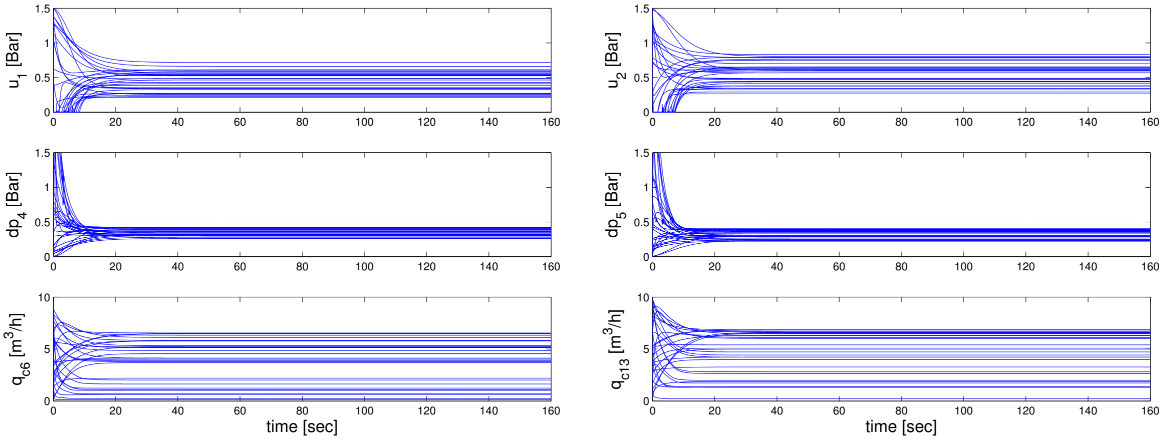

Fig. 5. The control inputs u_1 and u_2, the controlled variable dp_4 and dp_5, and the flow through valve c_6 and c_{13} obtained with a proportional controller.

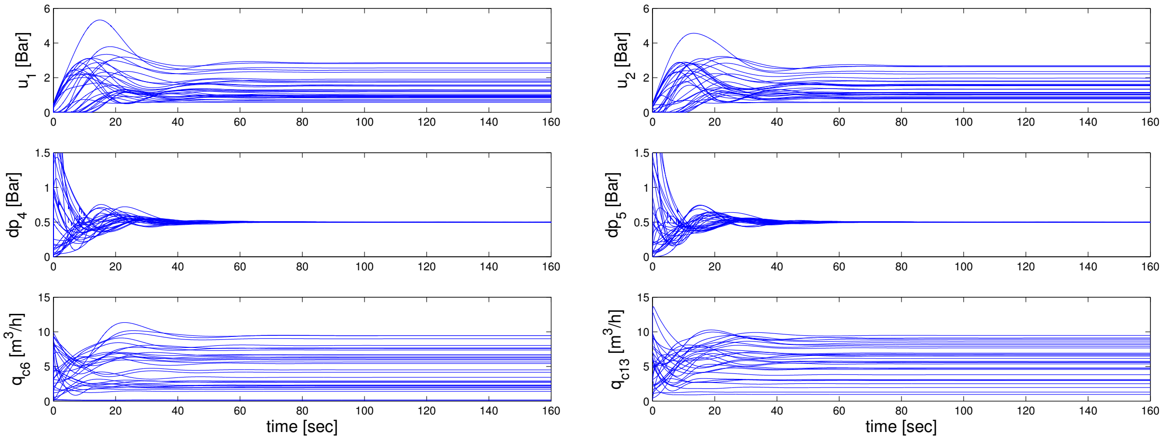

Fig. 6. The control inputs u_1 and u_2, the controlled variables dp_4 and dp_5, and the flow through valve c_6 and c_{13} obtained with a proportional-integral controller.

the case $\mathcal{R} = \{r\}$. Hence, to prove the proposition, we only need to prove that the same continues to hold for r which ranges over the compact set $\mathcal{R} = \{r : 0 < r_m \leq r \leq r_M\}$. To this purpose, it is enough to prove that the gains N_i^* depend smoothly on r, and then the gains which give the thesis are those obtained by taking the minimum of $N_i^*(r)$ for $r \in \mathcal{R} = \{r : 0 < r_m \leq r \leq r_M\}$. Details are omitted for the sake of conciseness.

5. ASYMPTOTIC REGULATION

We have already shown that a simple error-feedback proportional controller guarantees that the controlled output converges in finite time to a set of points which are arbitrarily close to the desired set-point value. We know from basic arguments of control theory that, if we consider the system *linearized* around an operating point, a simple proportional-integral controller would guarantee asymptotic regulation to zero of the tracking error. Then one may wonder if the same kind of controller would work for the *nonlinear* system. The objective of this section is to show that this is actually the case. The departing point is again system (8). We also consider in this case the (trivial) dynamics which generates the reference value r and the unknown parameters α_1, α_2, which here are assumed to be constant (or piece-wise constant):

$$\dot{w} := [\dot{r} \quad \dot{\alpha}_1 \quad \dot{\alpha}_2]^T = [0 \quad 0 \quad 0]^T = s(w)\,. \tag{14}$$

Loosely speaking, asymptotically regulating the pressure drop to the desired set-point value r means finding u which asymptotically steers to zero the output e of the system (8),(14). Differently from the previous section, we consider here a *dynamic* error-feedback controller which takes the general form:

$$\begin{aligned} \dot{\xi} &= \phi(\xi, e) \\ u &= \theta(\xi, e)\,. \end{aligned} \tag{15}$$

The output regulation problem we tackle can then be cast as follows (Isidori et al. (2003)).

Asymptotic Pressure Regulation Problem. Given system (8),(14), parameters $0 < x_m \leq x_M$, a compact set $\mathcal{W} \subset I\!R^3_+$, and a compact set of initial conditions

$$\mathcal{X} = \{x \in I\!R^2_+ \,:\, x_m \leq x_i \leq x_M \;,\; i = 1,2\}\,,$$

find a controller (15) and a set Ξ such that for the closed-loop system (8)-(15):

(a) the trajectory $(x(t), \xi(t), w(t))$ is bounded,
(b) $\lim_{t\to+\infty} e(t) = 0$,

for every initial condition in the set $\mathcal{X} \times \Xi \times \mathcal{W}$.

We design the controller (15) following the theory of nonlinear output regulation as exposed for instance in Chapter 1 of Isidori et al. (2003). We refer the interested reader to e.g. Serrani et al. (2001), Isidori et al. (2003) for a study of the subject in its full generality. In the present case, the design leads to a controller of the form

$$\begin{aligned} \dot{\xi} &= Ke \\ u &= \xi + Ne \,. \end{aligned} \tag{16}$$

with K and N *diagonal* matrices. The set Ξ of initial conditions of (16) is any compact set in $\mathbb{R}^2$. Then the following can be proven:

Lemma 2. Given system (8), (14), for any choice of the parameters $0 < \gamma < 1$, and $x_M > 0$, any compact set $\mathcal{W} \subset I\!R^3_+$, any compact set

$$\mathcal{X} = \{x \in I\!R^2_+ \,:\, r_M - (1-\gamma) r_m \leq x_i \leq x_M \,,\; i = 1, 2\} \,,$$

there exists a pair of diagonal (2×2) matrices (K, N) such that the controller (16) solves the Asymptotic Pressure Regulation Problem.

In the lemma we put a restriction on the set of initial conditions $\mathcal{X}$. Nevertheless, arguments analogous to those in Proposition 1 allow us to show that the controller (16) solves the Asymptotic Pressure Regulation Problem with no restriction on the set $\mathcal{X}$. Details are omitted for the sake of conciseness.

6. NUMERICAL RESULTS

The performance of the controllers are illustrated by applying them to the system of Fig. 3 and then computing the solution. The parameters of the three valves c_{10}, c_6 and c_{13} are, respectively, $K_{v10} = 1.25 \cdot 10^{-3}$, $K_{v6} \in [5.0 \cdot 10^{-3}; \infty]$ and $K_{v13} \in [5.0 \cdot 10^{-3}; \infty]$. The values of K_{v6} and K_{v13} range over sets (which we shall assume to be arbitrarily large but *compact*), as the valve position is a function of the load of the heat exchanger (set by the end-user) modeled by the valve.

First the proportional controllers for controlling the pressure drops dp_4, i.e. y_1, and dp_5, i.e. y_2, are tested. The stability of these controllers is tested by performing 30 simulations with different operating conditions. The reference value of the controllers is in all 30 simulations set to 0.5 $[bar]$, and the controller parameter is the same for both controllers, i.e. $N_1 = N_2 = -3$. The valve resistances K_{v6} and K_{v13} are randomly picked within a range in such a way that the flows through the valves are distributed between 0.08 and 10 $[m^3/h]$, when the pressures are at the desired reference value. As seen from (7), the system contains two independent flows. The initial values of these are also changed for each simulation. In particular, the initial values of q_1 and q_2 are uniformly distributed between 0.01 and 10 $[m^3/h]$.

The results obtained are shown in Fig. 5. From the simulation results it is seen that the system is stable. It is also seen that there is a steady state error compared to the reference value 0.5 $[bar]$, as it is expected with a proportional controller. This of course influences the flows, which have their maximum values around 7 $[m^3/h]$ instead of the expected 10 $[m^3/h]$. The steady state errors can be forced to *approach zero* by increasing the gains of the proportional controllers.

Secondly, the proportional-integral controllers are tested. This is done under the same conditions as with the proportional controllers. Again 30 simulations are performed. The values of K_{v6} and K_{v13}, and the initial values of the flows are chosen in the same way as in the previous tests. The controller parameters are the same for both controllers and are $N_1 = K_1 = N_2 = K_2 = -1$.

The results obtained with the proportional-integral controller regulating the pressure drops dp_4 and dp_5 are shown in Fig. 6. It is seen that the system is stable and that the reference value is reached for both pressures under the different valve positions and initial conditions.

7. CONCLUSIONS

We have presented elementary controllers for semi-globally regulating pressure drops in the hydraulic network of a district heating system case study proposed within the Plug and Play Process Control Project. Simulation results show good agreement with the theory. Next steps will be focused on various aspects, such as taking into account constraints, also to improve overall performance of the heating system; re-designing controllers with reduced energy consumption; detection schemes to asses the addition of new components with possible reconfiguration; extension of the results to district heating systems with a more complex structure.

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