COMPUTER-AIDED ANALYSIS AND DESIGN OF ROBUST CONTROL SYSTEMS USING ROOT LOCUS TECHNIQUES

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Abstract: The problem of robust control systems analysis and synthesis is currently one of the central problems of the control theory. There exist series of program packages for investigation and design of uncertain control systems based on the corresponding analytical methods, which r application allows either to verify the given system stability or to calculate the parameters deviations from the nominal values ensuring the system hurwitz or generalized stability. This paper presents the software package based on application of root sensitivity functions, root loci fields, root loci portraits of uncertain control systems. Its application allows not only to verify the uncertain (interval) control system stability, but, unlike the existing ones, also to provide with recommendations for calculation of the variable parameters limit values ensuring the system robust stability in the cases when the nominal system is unstable. *Copyright © 2000 IFAC*

Keywords: computer aided control systems design, analysis, root locus, stability, sensitivity, robustness, program package.

1. INTRODUCTION

Emerging CAD packages for control systems analysis and design was caused by development of the new methods for dynamic systems analysis and synthesis at the beginning of the 50th. Unlike the existing classical methods, they were sophisticated in utilization, featuring high formalization level and required a lot of calculations necessary for their realization.

The first control systems CAD packages creation was preceded by developing separate program complexes solving particular tasks of the Control Theory. By the end of the 60th the series of calculating methods and algorithms (Ash et al., 1968, Filby, 1972 and Pan et al., 1978) were developed for computer-aided analysis of control systems using the root locus method. The first substantial developments in the area of control systems CAD emerged at the beginning of the 70th. The major program packages for dynamic systems computer aided analysis and synthesis that were developed in the Western Europe and USA in the 70-80th are described by Dzhamshydy, 1989. The review of calculating algorithms development for solving basic problems of linear MIMO control systems described in the state space is represented by Petkov et al., 1985.

The program complex GRLT (Rimsky et al., 1978) has been developed that is the specially designed set of programs for analysis and design of dynamic systems of various classes on the basis of the General Root Locus Theory (Rimsky et al., 1978). Methods and algorithms for computer aided solving the basic tasks of the Control Theory are considered (Belova et al., 1979). They cover linear control systems stability and quality and nonlinear control systems absolute stability investigations. Calculating methods (Voronov et al., 1984) for computer aided analysis and optimal synthesis of complicated nonlinear systems of high degrees. Algorithms and programs for control systems computer aided analysis and synthesis have been worked out (Bolnokin et al., 1986). Unlike preceding ones, they cover also stochastic systems; the stochastic methods of analysis were used.

In 1990 the book of V.V. Solodovnikov has been published. It presents theoretical problems, technique for control systems computer aided CAD. The author pinpoints that even though in general control systems are complicated nonlinear multivariable systems with distributed parameters, in practice the linear theory is widely applied for there calculation, as there are no the generalized methods for analysis and synthesis of the systems of such a kind.

The above survey shows that the existing control systems CAD packages don't comprise the means for generating root sensitivity functions, root locus fields and instruments of there utilization for dynamic systems analysis and synthesis. They also don't include the tools for investigating uncertain control systems robust properties using root locus techniques.

These paper suggests program complex including applications for uncertain linear control systems CAD on the basis of root locus methods. They are based on the analytical root locus equations. The polynomially described control systems are considered. Applied are root sensitivity functions, root locus fields, root loci and root portraits of interval control systems.

2. PROGRAM COMPLEX FOR CONTROL SYSTEMS ANALYSIS AND SYNTHESIS USING ROOT LOCUS FIELDS AND ROOT SENSITIVITY FUNCTIONS

The program complex described here has been worked out as further development of the basic program complex *GRLT* (General Root Locus Theory) that is described in details in the book by Rimsky et al., 1978 and is aimed at dynamic systems investigation. It is developed for utilization within control systems CAD packages and consists of four subsystems: *RLSENS, STAB, RLFLD* and *ANALRL* that form the integrated part of the sub-complex *LINEAR SYSTEMS ANALYSIS AND DESIGN* of the basic complex *GRLT* aimed at analysis and design of the linear control systems.

Subsystem is a group of programs that are united by the main program into the structure being ready for execution on the computer and used for solving the required task of analysis and synthesis. The main (supervising) program being the calculating model of the task solving algorithm organizes calls of the subsystem subroutines. Informational exchange between the subroutines is organized by the coordinated I/O data lists. Every subroutine call has its definite place in the structure of the main program.

Due to the modular structure of the subsystems the same program modules (subroutines) can be used for generating various subsystems (sub-complexes), creating new subsystems. Subsystems include both functional basic subroutines (modules of the first level) and elementary basic subroutines (modules of the second level). Among them there are modules forming the kernel of the basic complex *GRLT*, e.g. elementary basic subroutines for multiplying and adding polynomials, functional basic subroutines for generating basic modules of root loci equations, etc.

Consider subsystems of the program complex described.

Subsystem *RLSENS* serves for calculating functions of root sensitivity to system parameters variations (Rimsky et al., 1998). It ensures calculation of the module and the argument of the root sensitivity vector at any root locus point.

Subsystem *STAB* serves for calculation of the dynamic system stability margin in conditions of its characteristic equation coefficients variation (Nesenchuk, 1990). Aperiodic and oscillation stability margin of a system is calculated.

Subsystem *RLFLD* is used for generating plane stationary scalar root locus fields of various types represented by there level lines and synthesis of the systems satisfying given quality requirements on the bass of the method ensuring system robust quality in conditions of parameter uncertainty (Rimsky and Nesenchuk, 1996)

Subsystem *ANALRL* provides with analysis of the root locus curves on display. Root loci and root portraits of Kharitonov's polynomials are generated for the given interval control system and their analysis is carried on. The system variable parameter and dynamic characteristics are calculated for the given state of the system. In case stability test is negative, recommendations are offered for determination of the parameters variation intervals ensuring system asymptotic stability on the basis of the corresponding method that has been developed (Nesenchuk, 1999).

Consider the above described program package application to the interval control system parameter design.

3. APPLICATION TO THE INDUSTRIAL ROBOT MANIPULATOR CONTROL SYSTEM PARAMETER DESIGN

Most industrial robots are used for transportation of various items (parts), e.g. the robots used for installing parts and machine tools in the cutting machines adjustments, for moving parts and units, etc. Robots loads change with variation of the weights of the items they carry, i.e. the load inertia moment j_l varies, and, as it is included into the formulas for calculating almost all the characteristic equation coefficients, it causes their variation.

Currently during the design procedure robots parameters values in such cases are obtained by the technique of tests and mistakes (iteration techniques). Presented here is the example demonstrating the computer program functioning that ensures parameter synthesis of the interval control systems described by polynomials based on the corresponding analytical method (Nesenchuk, 1999) using root loci and root portraits of Kharitonov's polynomials. Conduct parameter synthesis of the manipulator chain (shoulder) control system in conditions of its parameters uncertainty using the above mentioned analytical calculation method (program *ANALRL*).

The simplified version of the structure of the manipulator chain subordinate control is shown in fig. 1.

In fig. 1:
φ_g and φ - angles of rotation, given and of the motor shaft correspondingly;
K_s and K_p - gains of regulators by speed and position correspondingly;
K_1 and K_2 - photo-electric sensor coefficients;
U_g - the input voltage;
T - time constant of PI regulator;
ω - angular speed;
the transfer function

$$W'_p(p) = W_p(p)p$$

where $W_p(p)$ is the plant transfer function.

Write the manipulator chain control system characteristic equation

$$p^4 + \frac{R_A}{L_A}p^3 + \frac{C_eC_M}{j_mL_A}p^2 + \frac{C_MK_1K_s}{j_mL_AT}p + \frac{C_MK_2K_pK_s}{j_mL_AT} = 0$$

or as

$$a_0p^4 + a_1p^3 + a_2p^2 + a_3p + a_4 = 0 \quad (1)$$

where

$$a_0 = 1;\ a_1 = R_A/L_A;\ a_2 = \frac{C_eC_M}{(j_m + j_l)L_A};$$

$$a_3 = \frac{C_MK_1K_s}{(j_m + j_l)L_AT};\ a_4 = \frac{C_MK_2K_pK_s}{(j_m + j_l)L_AT}.$$

R_A - motor anchor resistance;
L_A - anchor inductance;
j_l - load inertia moment;
j_m - anchor inertia moment;
C_e - electric-mechanical ratio of the motor;
C_M - constructive constant of the motor.

Suppose the robot chain has the following nominal parameters:

R_A =0,63 Ω;
L_A =0.0014 henry;
$j_l = 2.04 \cdot 10^{-5} kg \cdot m^2$;
$j_m = 40.8 \cdot 10^{-5} kg \cdot m^2$;
$C_e = 0.16 \frac{V \cdot s}{rad}$;
$C_M = C_e$;
T=0.23c;
K_1 =66.7, K_2=250;
K_s = 0.078, K_p = 2.5.

After substitution of the nominal values into (1) write the chain characteristic equation

$$p^4 + 0.5 \cdot 10^3 p^3 + 0.427 \cdot 10^5 p^2 + 0.6 \cdot 10^7 p + 0.56 \cdot 10^8 = 0 \quad (2)$$

Let coefficients of the characteristic equation (2) vary within the following limits:

$$a_0 = 1;\ 0.4 \cdot 10^3 \le a_1 \le 0.5 \cdot 10^3;$$
$$0.373 \cdot 10^3 \le a_2 \le 0.427 \cdot 10^3;$$
$$0.52 \cdot 10^7 \le a_3 \le 0.6 \cdot 10^7;$$
$$0.488 \cdot 10^9 \le a_4 \le 0.56 \cdot 10^9.$$

The limit values of the intervals of coefficients variation are entered to the input of the program *ANALRL* (described above) for computer aided investigation of control systems with variable parameters. During the package functioning the Kharitonov's polynomials of the system characteristic equation are formed, there root loci are generated.

According to the corresponding task algorithm (Nesenchuk, 1999) and the obtained values after the stability test it has been stated that the given coefficients intervals don't comply with the asymptotic stability conditions. Therefore, following the algorithm, asymptotic stability has been ensured by setting up one of the system characteristic equation coefficients, in our case it is a_4. The main result of the subsystem functioning is the following stability condition of the interval system:

$$-\infty \le a_4 \le 0.280 \cdot 10^9,$$

that has been formed as an output of the program. It means that at $\bar{a}_4 = 0.280 \cdot 10^9$ and $\underline{a}_4 = 0$ the system asymptotic stability is ensured. The other coefficients intervals remain as given. Knowing the values of equation (1) coefficients variation intervals, ensuring the system asymptotic stability, it is easy to choose the corresponding values of the system

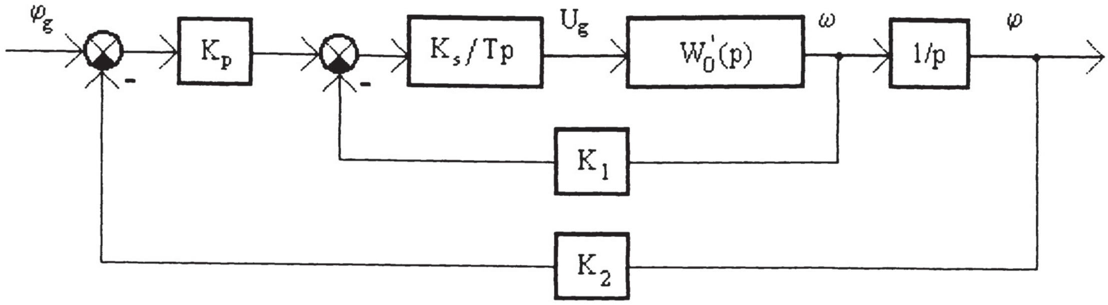

Fig. 1. Structure of the control system for the manipulator shoulder position control contour

parameters ensuring stable performance of the industrial robot. These results are based on the corresponding analytical method of interval control systems parameter design which is described in details in (Nesenchuk, 1999).

The advantage of this application is that it ensures interval control system asymptotic stability just by setting up the interval of only one coefficient of the system characteristic equation.

The innovation of the approach is that the developed method for the interval dynamic systems parameter synthesis allows not only to verify system stability or calculate maximal parameters deviations from their nominal values (when the system is stable), but, unlike the existing approaches, also to obtain the intervals of parameters variation when the initial verified system is unstable, i.e. to ensure the system robustness within the certain parameter variation intervals. The approach is descriptive and simple. Its application allows to exclude the labor-consuming iteration procedure of parameter design and substitute it by the simple calculation procedure based on the analytical method described above.

The Program Package operates on computers IBM PC in the media of Windows version 4.0 and higher, program languages Pascal and C.

ACKNOWLEDGEMENT

This work was supported by the International Science and Technology Center (Project B-104-98).

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