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Nonlinear sliding-mode control of a multi-motor web-winding system without tension sensor

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Abstract: A sliding-mode (SM) feedback linearisation control system is designed for a multi-motor web-winding system. First, an ideal feedback linearisation control system is adopted in order to decouple the tensions and velocity of the web-winding system; then to enhance the performance of the control system in the presence of uncertainties, an SM feedback linearisation control system is applied, which consists of an SM velocity controller and two SM tension controllers; a decentralised version of the proposed controller is developed. Two tension observers are suggested to eliminate the need of load cells in a web-winding system. Finally, the effectiveness and capability of the proposed control strategy is verified by computer simulation.

1 Introduction

A web is any material which is manufactured and processed in a continuous, flexible form. Web-winding systems are widely used in various fields such as textile, paper polymer or metal factories [1].

Modelling and control of web-handling systems have been studied in several literatures [1–4]. So far, many industrial web transport systems have used decentralised PI-type controllers. However, increasing requirement on control performance and better handling of elastic web material have led to a search for more complex robust control strategies [3, 4]. One of the objectives in such systems is to increase web velocity as much as possible, while controlling web tension over the entire production line. The main problem is independent control of the web velocity and tensions, to prevent web breaks, folding and damage [3].

The previous research works on web-handling systems control are mainly based on linear control methods; however, the web-winding system is a typical nonlinear multi-variable coupled system; hence the control methods, with linear approximated models, usually cannot meet the control requirements. Linear models of web-winding systems are only valid around particular speed and tension values.

In this paper, a robust nonlinear controller is designed for a three-motor web-winding system, which can be extended to web-winding systems with more motors. Improved performance is normally achieved by using a nonlinear model rather than a linear one and then using robust control methods like H_∞ [3], μ synthesis [4] etc. for a safe control of the web throughout the whole industrial process.

First, we present a feedback linearisation control for a web-winding system to obtain a decoupled relationship between inputs and outputs. Then to ensure the stability of the system and to guarantee the performance, despite the model uncertainty, a sliding-mode (SM) controller system is proposed.

To reduce the burden of computation in the controller and information communication among subsystems of a web-winding system, a decentralised version of the suggested controller is used for each subsystem. The designed decentralised control approach only requires limited neighbourhood knowledge.

Two tension observers are introduced. Using these observers, the load cells, used to detect tensions, are omitted. Based on the stability of linear time-varying systems, it is proved that the estimated tensions converge to real values.

The contributions of this paper can be summarised as follows: (1) a robust nonlinear controller is proposed for web-winding systems; (2) a decentralised version of the robust nonlinear controller is proposed; (3) two tension observers are introduced to omit load cells in measuring tensions.

2 Description and modelling of the web-winding system

A scheme of a three-motor setup is presented in Fig. 1. The inputs to system are the torque control signals (u_0, u_1, u_2) of the motors; its measurements are the unwinder and winder web tensions t_1 and t_2 and the web velocities v_0, v_1 and v_2.

It is common in the web-handling industry to divide a process line into several tension zones by denoting the span between two successive driven rollers as a tension zone. Since the free roller dynamics has a negligible effect on operation, the assumption that the free rollers do not contribute to web dynamics is reasonable and is often of value in practice, and they are extensively used in the industry [1]. This assumption will be used in developing the dynamic model in this section.

The line in Fig. 1 consists of the unwind/rewind rolls and one intermediate driven roller. There are two load cells to measure tensions t_1 and t_2 and three encoders to measure v_0, v_1 and v_2. Further, it will also be assumed that the web is elastic and there is no web slip on the rollers.

Fig. 1 shows three sections in the web line; the unwind section, master speed roller and rewind section. The name master speed roller is given to a driven roller that sets the reference web transport speed for the entire web line and is, generally, the first driven roller upstream of the unwind roll in almost all web process lines. The unwind/rewind rolls

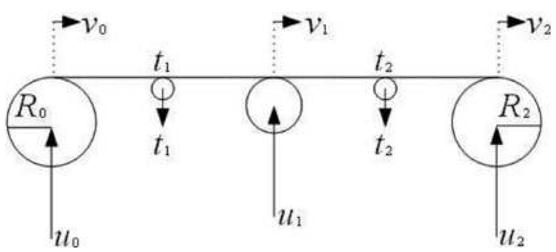

Figure 1 *A web line with three motors*

release/accumulate material to/from the master speed roller. Thus, their radii and inertias are time-varying.

The velocity is imposed by the master traction motor, whereas the web tension is controlled by the unwinding and winding motors.

The dynamics of the whole system is presented in the following [1]

$$\dot{v}_0 = \frac{R_0^2}{J_0}t_1 - \frac{b_{f0}}{J_0}v_0 - \frac{R_0 n_0}{J_0}u_0 - c_{f0}v_0^2 \tag{1}$$

$$\dot{t}_1 = \frac{SE}{L_1}(v_1 - v_0) + \frac{1}{L_1}t_0 v_0 - \frac{1}{L_1}t_1 v_1 \tag{2}$$

$$\dot{v}_1 = \frac{R_1^2}{J_1}(t_2 - t_1) - \frac{b_{f1}}{J_1}v_1 + \frac{R_1 n_1}{J_1}u_1 \tag{3}$$

$$\dot{t}_2 = \frac{SE}{L_2}(v_2 - v_1) + \frac{1}{L_2}t_1 v_1 - \frac{1}{L_2}t_2 v_2 \tag{4}$$

$$\dot{v}_2 = -\frac{R_2^2}{J_2}t_2 - \frac{b_{f2}}{J_2}v_2 + \frac{R_2 n_2}{J_2}u_2 + c_{f2}v_2^2 \tag{5}$$

$$\dot{R}_i \simeq (i-1)\frac{t_w}{2\pi}\frac{v_i}{R_i}, \quad i = 0, 2 \tag{6}$$

$$J_i = n_i^2 J_{mi} + J_{ci} + \frac{\pi}{2}b_w \rho_w (R_i^4 - R_{ci}^4), \quad i = 0, 2 \tag{7}$$

$$c_{fi} = \frac{t_w}{2\pi J_i}\left(\frac{J_i}{R_i^2} - 2\pi\rho_w b_w R_i^2\right), \quad i = 0, 2 \tag{8}$$

where t_w is the web thickness, b_w the web width and ρ_w the density of the web material; L_1 and L_2 are the lengths of the web span between the master speed roller and unwind/rewind rollers, respectively; S is the area of cross-section of the web; E is the modulus of elasticity of the web material; t_0 is the constant tension of the unwinder roll; for $i = 0, 1, 2$, n_i represents the gearing ratios between the ith motor shaft and the corresponding roll-shaft; b_{fi} is the friction coefficient of the ith roll; R_i and J_i represent the radius and the total inertia of the ith roll, respectively; J_{m0} and J_{m2} are the inertias of all the rotating elements on the motor side, which includes the inertia of the motor armature, driving pulley (or gear), driving shaft etc.; J_{c0} and J_{c2} are the inertias of the driven shaft and the core mounted on it; R_{c0} and R_{c2} are the radii of the empty core mounted on the unwind/rewind roll-shafts.

Notice that (6) are approximates because the thickness affects the rate of change of the radius of the roll only after each revolution of the roll; the continuous approximation is valid since the thickness is generally very small.

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3 Ideal feedback linearisation control

In the web-winding dynamics described by (1)–(5), u_0, u_1 and u_2 are the control inputs, and t_1, v_1 and t_2 are the system outputs. Thus, the web-winding system is a coupled dynamical system. As there is no direct relation between the outputs and inputs, it is difficult to design the control inputs u_0, u_1 and u_2 so that the system outputs t_1, v_1 and t_2 can track the desired trajectories accurately. Hence, the nonlinear state feedback theory is used to eliminate this coupling relationship to simplify the design of the control system. The web-winding system model has to be converted to a suitable form such that the feedback linearisation can be applied. From (1)–(5) and differentiating (2) and (4), the relationship between inputs and outputs of the dynamic model can be represented as follows

$$\begin{bmatrix} \ddot{t}_1 \\ \dot{v}_1 \\ \ddot{t}_2 \end{bmatrix} = \mathbf{b} + \mathbf{A} \begin{bmatrix} u_0 \\ u_1 \\ u_2 \end{bmatrix} \tag{9}$$

where vector $\mathbf{b}$ and matrix $\mathbf{A}$ are functions of states and system parameters as follows

$$\mathbf{A} = \begin{bmatrix} \frac{R_0 n_0}{L_1 J_0}(SE - t_0) & \frac{R_1 n_1}{L_1 J_1}(SE - t_1) & 0 \\ 0 & \frac{R_1 n_1}{J_1} & 0 \\ 0 & \frac{R_1 n_1}{L_2 J_1}(t_1 - SE) & \frac{R_2 n_2}{L_2 J_2}(SE - t_2) \end{bmatrix} \tag{10}$$

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{11}$$

$$\begin{aligned} b_1 = {} & \frac{SE}{L_1}\left[\frac{R_1^2}{J_1}(t_2 - t_1) - \frac{b_{f1}}{J_1}v_1 - \frac{R_0^2}{J_0}t_1 + \frac{b_{f0}}{J_0}v_0 + c_{f0}v_0^2\right] \\ & + \frac{1}{L_1}t_0\left[\frac{R_0^2}{J_0}t_1 - \frac{b_{f0}}{J_0}v_0 - c_{f0}v_0^2\right] \\ & - \frac{1}{L_1}v_1\left[\frac{SE}{L_1}(v_1 - v_0) + \frac{1}{L_1}t_0 v_0 - \frac{1}{L_1}t_1 v_1\right] \\ & - \frac{1}{L_1}t_1\left[\frac{R_1^2}{J_1}(t_2 - t_1) - \frac{b_{f1}}{J_1}v_1\right] \end{aligned}$$

$$b_2 = \frac{R_1^2}{J_1}(t_2 - t_1) - \frac{b_{f1}}{J_1}v_1$$

$$\begin{aligned} b_3 = {} & \frac{SE}{L_2}\left[-\frac{R_2^2}{J_2}t_2 - \frac{b_{f2}}{J_2}v_2 - \frac{R_1^2}{J_1}(t_2 - t_1) + \frac{b_{f1}}{J_1}v_1 + c_{f2}v_2^2\right] \\ & + \frac{1}{L_2}v_1\left[\frac{SE}{L_1}(v_1 - v_0) + \frac{1}{L_1}t_0 v_0 - \frac{1}{L_1}t_1 v_1\right] + \frac{1}{L_2} \\ & \times t_1\left[\frac{R_1^2}{J_1}(t_2 - t_1) - \frac{b_{f1}}{J_1}v_1\right] - \frac{1}{L_2}v_2\left[\frac{SE}{L_2}(v_2 - v_1) \right. \\ & \left. + \frac{1}{L_2}t_1 v_1 - \frac{1}{L_2}t_2 v_2\right] - \frac{1}{L_2}t_2\left[-\frac{R_2^2}{J_2}t_2 - \frac{b_{f2}}{J_2}v_2 + c_{f2}v_2^2\right] \end{aligned}$$

If the system parameters are available and all system states are measurable, an ideal feedback linearisation control system can be designed as follows

$$\begin{bmatrix} u_0 \\ u_1 \\ u_2 \end{bmatrix} = \mathbf{A}^{-1}\left(-\mathbf{b} + \begin{bmatrix} I_{t_1} \\ I_{v_1} \\ I_{t_2} \end{bmatrix}\right) \tag{12}$$

where I_{t_1}, I_{v_1} and I_{t_2} are the new control inputs and can be designed as

$$I_{t_1} = \ddot{t}_1^* - k_1 \dot{e}_{t_1} - k_2 e_{t_1} \tag{13}$$

$$I_{v_1} = \dot{v}_1^* - k_3 e_{v_1} \tag{14}$$

$$I_{t_2} = \ddot{t}_2^* - k_4 \dot{e}_{t_2} - k_5 e_{t_2} \tag{15}$$

in which for $i = 1, \ldots, 5$ k_i is a positive constant, the quantities with * are reference commands and $e_{t_1} = t_1 - t_1^*$, $e_{v_1} = v_1 - v_1^*$ and $e_{t_2} = t_2 - t_2^*$ are tracking errors.

Substituting (13)–(15) into (12) and using (9), one can obtain

$$\ddot{e}_{t_1} + k_1 \dot{e}_{t_1} + k_2 e_{t_1} = 0 \tag{16}$$

$$\dot{e}_{v_1} + k_3 e_{v_1} = 0 \tag{17}$$

$$\ddot{e}_{t_2} + k_4 \dot{e}_{t_2} + k_5 e_{t_2} = 0 \tag{18}$$

By properly choosing the values of k_i, $i = 1, \ldots, 5$, the desired system dynamics such as rise time, overshoot and settling time can be easily designed.

It is worthwhile to mention here that the total relative degree obtained from (9) is equal to the order of the original system, namely 5; hence there is no internal dynamics. With the control law in the form of (12), we thus obtain an input-state linearisation of the original nonlinear system.

4 SM feedback linearisation control system

In this section, an SM control system is proposed in order to generate the torque control signals that are capable of forcing the web-winding velocity and tensions to track their

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associated reference signals despite the uncertainties. This SM control system comprises an SM velocity controller and two SM tension controllers as follows

$$\begin{bmatrix} u_0 \\ u_1 \\ u_2 \end{bmatrix} = \bar{\mathbf{A}}^{-1}\left(-\bar{\mathbf{b}} + \begin{bmatrix} I^a_{t_1} \\ I^a_{v_1} \\ I^a_{t_2} \end{bmatrix}\right) \tag{19}$$

where $I^a_{t_1}$, $I^a_{v_1}$ and $I^a_{t_2}$ are the new actual control inputs; $\bar{\mathbf{A}}$ and $\bar{\mathbf{b}}$ can be obtained using the nominal parameter values without external disturbance. Consider the system parameters' variations and external disturbances, the relationship between the inputs and outputs of the dynamic model can be rewritten as

$$\begin{bmatrix} \ddot{t}_1 \\ \dot{v}_1 \\ \ddot{t}_2 \end{bmatrix} = \bar{\mathbf{b}} + \bar{\mathbf{A}}\begin{bmatrix} u_0 \\ u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} \psi_{t_1} \\ \psi_{v_1} \\ \psi_{t_2} \end{bmatrix} = \begin{bmatrix} I^a_{t_1} + \psi_{t_1} \\ I^a_{v_1} + \psi_{v_1} \\ I^a_{t_2} + \psi_{t_2} \end{bmatrix} \tag{20}$$

where the lumped uncertainty vector $[\psi_{t_1} \quad \psi_{v_1} \quad \psi_{t_2}]^{\mathrm{T}} = \boldsymbol{\Delta}\mathbf{b} + \boldsymbol{\Delta}\mathbf{A}[u_0 \quad u_1 \quad u_2]^{\mathrm{T}}$.

4.1 SM velocity controller

To control the velocity with the main motor in the web system, a first-order switching surface is chosen as follow [5–7]

$$S_{v_1} = e_{v_1}(t) + k_3\int_0^t e_{v_1}(\tau)\,\mathrm{d}\tau \tag{21}$$

Differentiating S_{v_1} with respect to time and using (20), the following can be obtained

$$\dot{S}_{v_1} = \psi_{v_1} + I^a_{v_1}(t) - \dot{v}_1^* + k_3 e_{v_1}(t) \tag{22}$$

Now, an SM velocity controller is proposed in the following

$$I^a_{v_1} = \dot{v}_1^* - k_3 e_{v_1}(t) - \phi_{v_1}S_{v_1}(t) - \rho_{v_1}\operatorname{sgn}(S_{v_1}(t)) \tag{23}$$

where $\rho_{v_1} > |\psi_{v_1}| + \eta_{v_1}$, ρ_{v_1}, η_{v_1} and ϕ_{v_1} are positive constants and $\operatorname{sgn}(\cdot)$ is the signum function. The term $-\phi_{v_1}S_{v_1}(t)$ in (23) affects the rate at which the sliding surface is attained [7]. It can be established that with this control law, the inequality

$$S_{v_1}\dot{S}_{v_1} \le -\phi_{v_1}S^2_{v_1} - \eta_{v_1}|S_{v_1}| \le 0 \tag{24}$$

is satisfied, which implies

$$\lim_{S_{v_1}\to 0^+} \dot{S}_{v_1} < 0 \quad \text{and} \quad \lim_{S_{v_1}\to 0^-} \dot{S}_{v_1} > 0 \tag{25}$$

Consequently, the sliding surface is attractive. Moreover, it can be shown that from (24) the sliding surface is reached in the following finite time

$$t_s = \frac{1}{\phi_{v_1}}\ln\left(1 + \frac{\phi_{v_1}|S_{v_1}(0)|}{\eta_{v_1}}\right) \tag{26}$$

It follows that the switching function satisfied $S_{v_1}(t) = 0$ for all $t > t$, which in turn implies $\dot{S}_{v_1}(t) = 0$; $\dot{e}_{v_1} + k_3 e_{v_1} = 0$ and the desired dynamics in (17) can be achieved.

It is worthwhile to mention that the reaching time of this modified SM controller is smaller than that of a conventional integral SM controller without the term $-\phi_{v_1}S_{v_1}(t)$ in (23).

4.2 SM tension controllers

To control the tensions with the winder/rewinder motors in the web system, two second-order switching surfaces are chosen as

$$S_{t_1} = \dot{e}_{t_1} + k_1 e_{t_1} + k_2\int_0^t e_{t_1}(\tau)\,\mathrm{d}\tau \tag{27}$$

$$S_{t_2} = \dot{e}_{t_2} + k_4 e_{t_2} + k_5\int_0^t e_{t_2}(\tau)\,\mathrm{d}\tau \tag{28}$$

Differentiating S_{t_1} with respect to time and using (20), the following can be obtained

$$\dot{S}_{t_1} = \psi_{t_1} + I^a_{t_1}(t) - \ddot{t}_1^* + k_1\dot{e}_{t_1} + k_2 e_{t_1} \tag{29}$$

Now, an SM tension controller is proposed in the following

$$I^a_{t_1} = \ddot{t}_1^* - k_1\dot{e}_{t_1} - k_2 e_{t_1} - \phi_{t_1}S_{t_1}(t) - \rho_{t_1}\operatorname{sgn}(S_{t_1}(t)) \tag{30}$$

where $\rho_{t_1} > |\psi_{t_1}| + \eta_{t_1}$; ρ_{t_1}, η_{t_1} and ϕ_{t_1} are positive constants.

It can be established that with this control law the inequality

$$S_{t_1}\dot{S}_{t_1} \le -\phi_{t_1}S^2_{t_1} - \eta_{t_1}|S_{t_1}| \tag{31}$$

is satisfied.

When the sliding motion takes place, $S_{v_1}(t) = 0$ and $\dot{S}_{v_1}(t) = 0$, which yields the desired dynamics in (16).

A similar controller can be obtained for t_2.

5 Decentralised control

The controller developed in the previous section is a centralised controller. In web-winding systems, we confront the control of complex MIMO systems with many states

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which that requires many calculations as seen in the centralised control design of this paper. In this section, we want to place some constrains on the flow of states or information. In particular, we will consider the control of decentralised systems where there are constraints on information exchange between subsystems. Decentralised control systems often arise from either the physical inability of subsystem information exchange or the lack of computing capabilities required for a single central controller. Furthermore, at times, it may be more convenient to design a controller in a decentralised framework since each subsystem is often much more simple than the composite MIMO system. Within a decentralised framework, the overall system is broken into some subsystems each with its own inputs and outputs. A decentralised control law is then defined using local subsystem signals. Fig. 2 shows such a control configuration for a three-motor web-winding system.

The subsystems have interconnections; these interconnections can be viewed as disturbances. Here we use an SM control for each subsystem to have robustness in the presence of uncertainties and to reject these disturbances.

$$\begin{aligned}\dot{t}_1 &= b_1 + a_1 u_0 + \psi_1 = I_1^a + \psi_1\\ \dot{v}_1 &= b_2 + a_2 u_1 + \psi_2 = I_2^a + \psi_2\\ \dot{t}_2 &= b_3 + a_3 u_2 + \psi_3 = I_3^a + \psi_3\end{aligned} \tag{32}$$

where

$$a_1 = \frac{R_0 n_0}{L_1 J_0}(SE - t_0), \quad a_2 = \frac{R_1 n_1}{J_1}, \quad a_3 = \frac{R_2 n_2}{L_2 J_2}(SE - t_2)$$

$$b_1 = \frac{SE}{L_1}\left[\frac{R_1^2}{J_1}(-t_1) - \frac{R_0^2}{J_0}t_1 + \frac{b_{f0}}{J_0}v_0 + c_{f0}v_0^2\right] + \frac{1}{L_1}t_0\left[\frac{R_0^2}{J_0}t_1 - \frac{b_{f0}}{J_0}v_0 - c_{f0}v_0^2\right] - \frac{1}{L_1}t_1\left[\frac{R_1^2}{J_1}(-t_1)\right]$$

$$b_2 = -\frac{b_{f1}}{J_1}v_1$$

$$b_3 = \frac{SE}{L_2}\left[-\frac{R_2^2}{J_2}t_2 - \frac{b_{f2}}{J_2}v_2 - \frac{R_1^2}{J_1}(t_2) + c_{f2}v_2^2\right] - \frac{1}{L_2}v_2\left[\frac{SE}{L_2}(v_2) - \frac{1}{L_2}t_2 v_2\right] - \frac{1}{L_2}t_2\left[-\frac{R_2^2}{J_2}t_2 - \frac{b_{f2}}{J_2}v_2 + c_{f2}v_2^2\right]$$

I_i^a are the new actual control inputs; a_i and b_i can be obtained using the nominal parameter values. In fact, a_i and b_i are just functions of the ith subsystem states. The interconnections and uncertainties are considered in ψ_i functions. It is also worthwhile to note that in this decentralised scheme, a_i and b_i have simpler functions with respect to the centralised scheme previously presented.

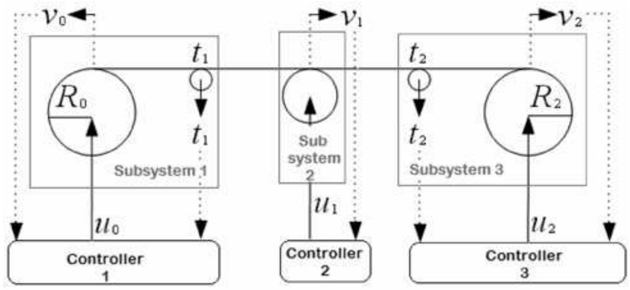

Figure 2 *Schematic of decentralised control of web-winding system using subsystems information*

Here we assume ψ_i functions are bounded or at least the uncertainty parts of ψ_i functions are bounded and their interconnection parts or approximations are known (which yield a partial decentralised control). Using the same switching surfaces as for the centralised controller, similar SM controllers can be designed for each subsystem.

6 Web-line tensions observer

In this section, two observers are introduced to estimate of tensions.

Assume the speeds are measurable with some encoders and using (2) and (4), the following tension observer is proposed

$$\dot{\hat{t}}_1 = \frac{SE}{L_1}(v_1 - v_0) + \frac{1}{L_1}t_0 v_0 - \frac{1}{L_1}\hat{t}_1 v_1 \tag{33}$$

$$\dot{\hat{t}}_2 = \frac{SE}{L_2}(v_2 - v_1) + \frac{1}{L_2}\hat{t}_1 v_1 - \frac{1}{L_2}\hat{t}_2 v_2 \tag{34}$$

The error dynamics of this observer is given as follows

$$\begin{bmatrix}\dot{\tilde{t}}_1\\ \dot{\tilde{t}}_2\end{bmatrix} = \mathbf{M}(t)\begin{bmatrix}\tilde{t}_1\\ \tilde{t}_2\end{bmatrix}, \quad \mathbf{M}(t) = \begin{bmatrix}-\frac{v_1(t)}{L_1} & 0\\ \frac{v_1(t)}{L_2} & -\frac{v_2(t)}{L_2}\end{bmatrix} \tag{35}$$

here for $i = 1, 2$, $\tilde{t}_i = t_i - \hat{t}_i$.

Equation (35) represents a time-varying linear system. One might conjecture that (35) will be stable if at any time the eigenvalues of $\mathbf{M}(t)$ all have negative real parts; but it is not enough for stability. A simple result, however, is that the time-varying system (35) is asymptotically stable if the eigenvalues of $\mathbf{M}(t) + \mathbf{M}^{\mathrm{T}}(t)$ (all of which are real) remain strictly in the left-half complex plane [8].

We have

$$\mathbf{M}(t) + \mathbf{M}^{\mathrm{T}}(t) = \begin{bmatrix}-\frac{2\,v_1(t)}{L_1} & \frac{v_1(t)}{L_2}\\ \frac{v_1(t)}{L_2} & -\frac{2\,v_2(t)}{L_2}\end{bmatrix} \tag{36}$$

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The eigenvalues of (36) are given as follows

$$\lambda_{1,2} = -\left(\frac{v_1}{L_1} + \frac{v_2}{L_2}\right) \pm \sqrt{\left(\frac{v_1}{L_1} + \frac{v_2}{L_2}\right)^2 - \frac{4v_1v_2}{L_1L_2} + \frac{v_1^2}{L_2^2}} \leq -\lambda \quad \text{for } v_1 < \frac{4L_2}{L_1}v_2 \tag{37}$$

where λ is a positive constant; thus the system (35) is asymptotically stable and the estimation errors converge to zero. To quantify this observer convergence rate, consider the following Lyapunov function

$$V = \tilde{t}^{\mathrm{T}}\tilde{t} \tag{38}$$

where $\tilde{t}^{\mathrm{T}} = [\tilde{t}_1\ \tilde{t}_2]$.

Since

$$\dot{V} = \tilde{t}^{\mathrm{T}}(\mathbf{M}(t) + \mathbf{M}^{\mathrm{T}}(t))\tilde{t} \leq -\lambda\tilde{t}^{\mathrm{T}}\tilde{t} = -\lambda V \tag{39}$$

so that

$$\tilde{t}^{\mathrm{T}}\tilde{t} = V < V(0)\mathrm{e}^{-\lambda^t} \tag{40}$$

As can be seen from (40), the convergence rate is lower than λ, which is dependent on speed. To solve this problem, add the terms $m_1(t_1 - \hat{t}_1)$ and $m_2(t_2 - \hat{t}_2)$ to the right-hand sides of (33) and (34), respectively

$$\dot{\hat{t}}_1 = \frac{SE}{L_1}(v_1 - v_0) + \frac{1}{L_1}t_0v_0 - \frac{1}{L_1}\hat{t}_1v_1 + m_1(t_1 - \hat{t}_1) \tag{41}$$

$$\dot{\hat{t}}_2 = \frac{SE}{L_2}(v_2 - v_1) + \frac{1}{L_2}\hat{t}_1v_1 - \frac{1}{L_2}\hat{t}_2v_2 + m_2(t_2 - \hat{t}_2) \tag{42}$$

where m_1 and m_2 are positive constants.

The next step is to substitute t_1 and t_2 from (1) and (5) in (41) and (42), respectively, to obtain

$$\dot{\hat{t}}_1 = \frac{SE}{L_1}(v_1 - v_0) + \frac{1}{L_1}t_0v_0 - \frac{1}{L_1}\hat{t}_1v_1 + \frac{m_1J_0}{R_0^2}\dot{v}_0 + \frac{m_1b_{\mathrm{f0}}}{R_0^2}v_0 + \frac{m_1J_0c_{\mathrm{f0}}}{R_0^2}v_0^2 + \frac{m_1n_0}{R_0}u_0 - m_1\hat{t}_1 \tag{43}$$

$$\dot{\hat{t}}_2 = \frac{SE}{L_2}(v_2 - v_1) + \frac{1}{L_2}\hat{t}_1v_1 - \frac{1}{L_2}\hat{t}_2v_2 - \frac{m_2J_2}{R_2^2}\dot{v}_2 - \frac{m_2b_{\mathrm{f2}}}{R_2^2}v_2 + \frac{m_2J_2c_{\mathrm{f2}}}{R_2^2}v_2^2 + \frac{m_2n_2}{R_2}u_2 - m_2\hat{t}_2 \tag{44}$$

From these equations, it is possible to obtain $\hat{t}_1$ and $\hat{t}_2$; but the existence of speed derivatives in (43) and (44) deteriorates the estimated tensions, which can be eliminated. Define the following new variables

$$\hat{T}_1 = \hat{t}_1 - \frac{m_1J_0}{R_0^2}v_0 \tag{45}$$

$$\hat{T}_2 = \hat{t}_2 + \frac{m_2J_2}{R_2^2}v_2 \tag{46}$$

Substituting $\hat{t}_1$ and $\hat{t}_2$ from (45) and (46) into (43) and (44) yields

$$\dot{\hat{T}}_1 = \frac{SE}{L_1}(v_1 - v_0) + \frac{1}{L_1}t_0v_0 - \left(\frac{1}{L_1}v_1 + m_1\right) \times \left(\hat{T}_1 + \frac{m_1J_0}{R_0^2}v_0\right) + \frac{m_1b_{\mathrm{f0}}}{R_0^2}v_0 + \frac{m_1J_0c_{\mathrm{f0}}}{R_0^2}v_0^2 + \frac{m_1n_0}{R_0}u_0 - m_1\frac{\mathrm{d}(J_0/R_0^2)}{\mathrm{d}t}v_0 \tag{47}$$

$$\dot{\hat{T}}_2 = \frac{SE}{L_2}(v_2 - v_1) + \frac{1}{L_2}v_1\left(\hat{T}_1 + \frac{m_1J_0}{R_0^2}v_0\right) - \left(\frac{1}{L_2}v_2 + m_2\right) \times \left(\hat{T}_2 - \frac{m_2J_2}{R_2^2}v_2\right) - \frac{m_2b_{\mathrm{f2}}}{R_2^2}v_2 + \frac{m_2J_2c_{\mathrm{f2}}}{R_2^2}v_2^2 + \frac{m_2n_2}{R_2}u_2 + m_2\frac{\mathrm{d}(J_2/R_2^2)}{\mathrm{d}t}v_2 \tag{48}$$

The last terms on the right-hand sides of (47) and (48) may be eliminated or calculated using (6) and (7) as follows

$$\frac{\mathrm{d}(J_i/R_i^2)}{\mathrm{d}t} = \left(\frac{\partial J_i}{\partial R_i}\dot{R}_i\right)\frac{1}{R_i^2} - J_i\left(\frac{-2}{R_i^3}\dot{R}_i\right) = \left(2\pi b_{\mathrm{w}}\rho_{\mathrm{w}} + J_i\frac{2}{R_i^4}\right) \times (i-1)\frac{t_{\mathrm{w}}}{2\pi}v_i, \quad i = 0, 2 \tag{49}$$

Using (47) and (48) one can obtain $\hat{T}_1$ and $\hat{T}_2$; then from (45) and (46) $\hat{t}_1$ and $\hat{t}_2$ are obtained. Considering (41) and (42), the error dynamics of this observer is given as follows

$$\begin{bmatrix}\dot{\tilde{t}}_1 \\ \dot{\tilde{t}}_2\end{bmatrix} = \mathbf{N}(t)\begin{bmatrix}\tilde{t}_1 \\ \tilde{t}_2\end{bmatrix}, \quad \mathbf{N}(t) = \begin{bmatrix} -\frac{v_1(t)}{L_1} - m_1 & 0 \\ \frac{v_1(t)}{L_2} & -\frac{v_2(t)}{L_2} - m_2 \end{bmatrix} \tag{50}$$

Table 1 Parameters of the web-winding system

$b_{\mathrm{w}} = 0.1$ m	$b_{\mathrm{f0}} = b_{\mathrm{f1}} = b_{\mathrm{f2}} = 0.001$ Nms
$t_{\mathrm{w}} = 0.275$ mm	$J_{\mathrm{m0}} = J_{\mathrm{m2}} = 0.005$ kg m^2
$E = 0.16^*10^9$ N/m^2	$J_{\mathrm{c0}} = J_{\mathrm{c2}} = 0.01$ Kg m^2
$L_1 = L_2 = 0.95$ m	$n_0 = n_1 = n_2 = 1$

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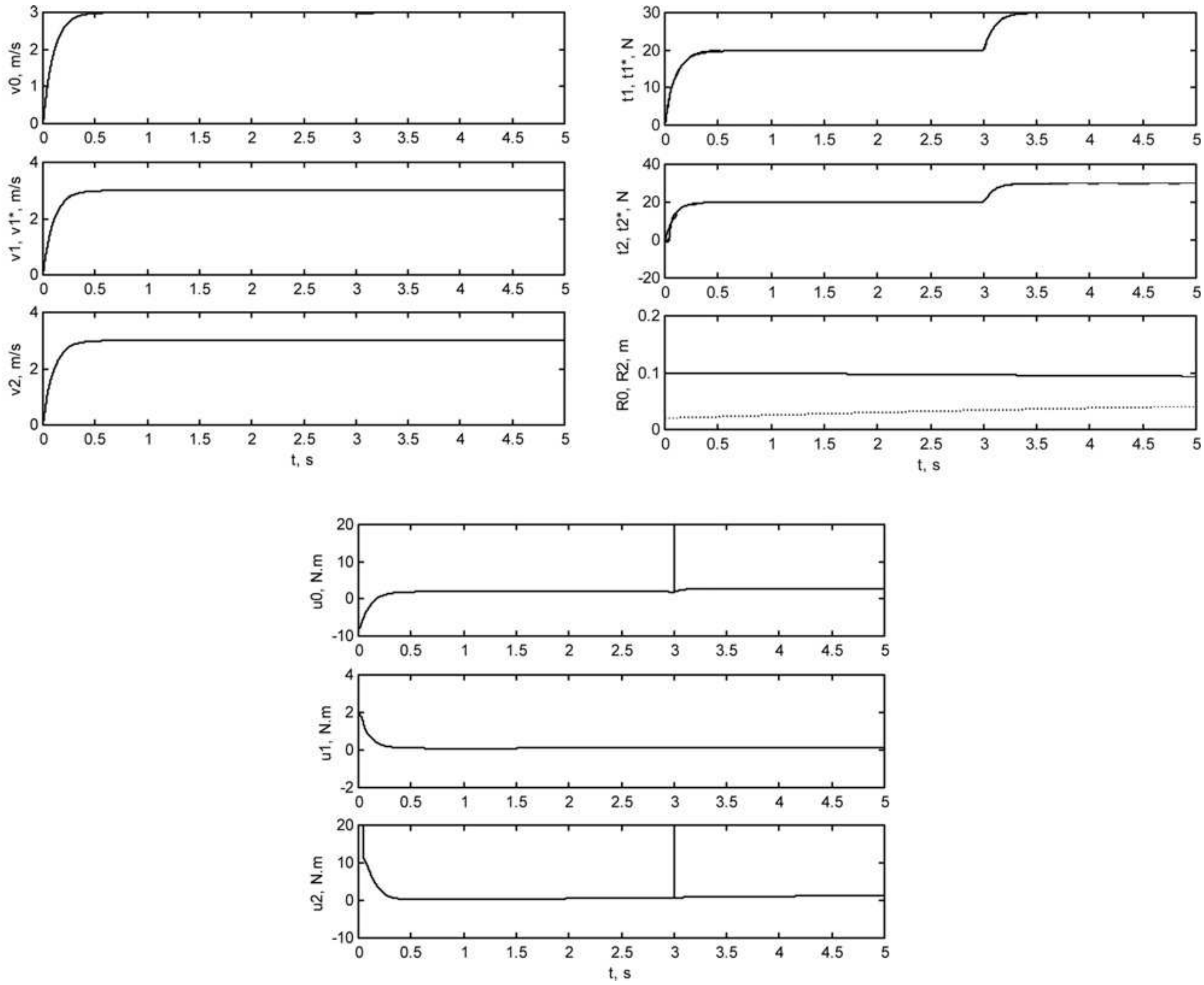

Figure 3 *Simulation results using centralised controller without tension observer: ——— states, R_0 and controls; - - - - - - - R_2; – – – – – - references*

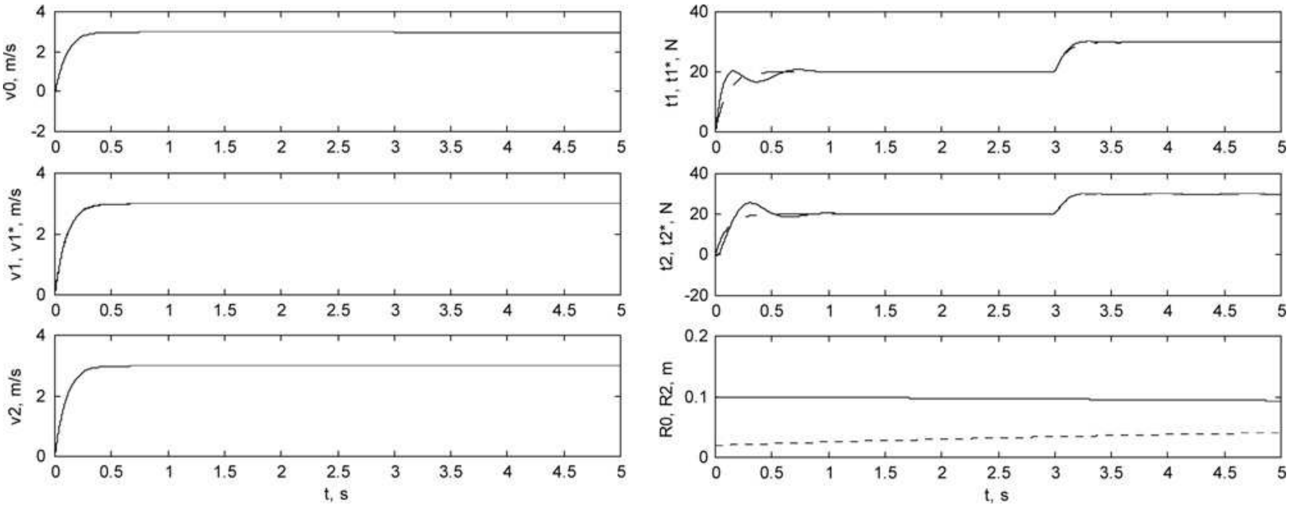

Figure 4 *Simulation results using decentralised controller without tension observer: ——— states and R_0 - - - - - - - R_2; and – – – – – - references*

doi: 10.1049/iet-cta.2008.0118

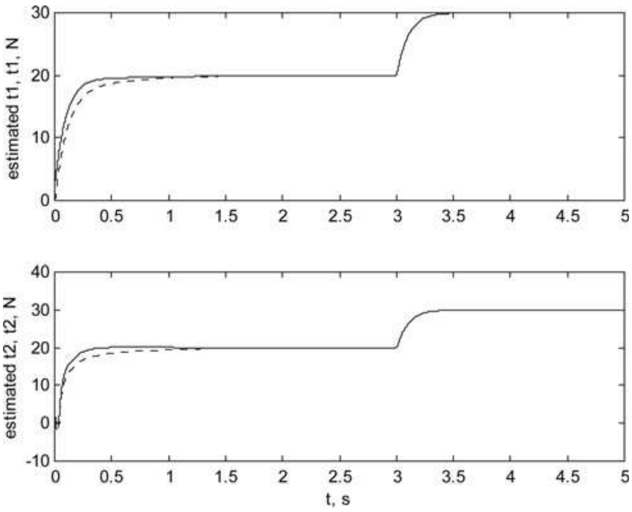

Figure 5 *Simulation results using centralised controller and tension observer: ———— estimated tensions; - - - - - - - tensions*

The eigenvalues of $\mathbf{N}(t)+\mathbf{N}^{\mathrm{T}}(t)$ are given as follows

$$\lambda_{1,2}=-\left(\frac{v_1}{L_1}+\frac{v_2}{L_2}+m_1+m_2\right)\pm\sqrt{\left(\frac{v_1}{L_1}+\frac{v_2}{L_2}+m_1+m_2\right)^2-\frac{4v_1v_2}{L_1L_2}+\frac{v_1^2}{L_2^2}-\frac{4v_1}{L_1}m_2-\frac{4v_2}{L_2}m_1-4m_1m_2} \tag{51}$$

By appropriate choosing of m_1 and m_2, these eigenvalues become negative enough to have a fast error dynamics, but it is worthwhile to note that choosing $m_1=m_2=0$, the parameters needed for this observer are L_1, L_2, S and E, which are available in most cases with good accuracy.

7 Simulation results

The web-winding system parameters used to obtain simulation results are given in Table I [2].

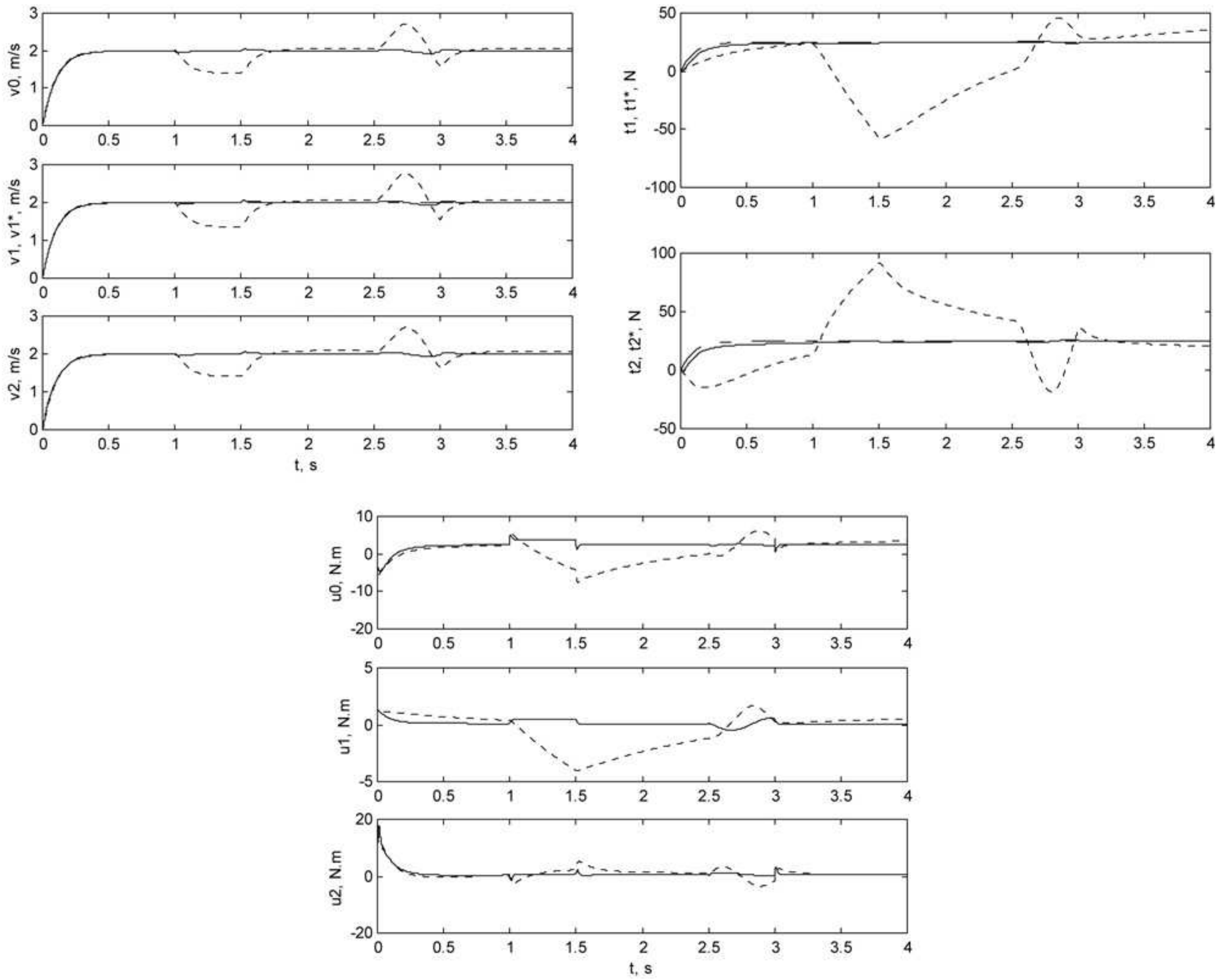

Figure 6 *Simulation results using centralised controller and tension observer with and without SM controller: ———— states and controls with SM controller; - - - - - - - states and controls without SM controller; — - — - — - references*

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To avoid chattering in SM controllers, the signum functions are replaced with saturation functions. In addition, limiters are used to restrict control signals.

Simulation results in Figs. 3–5 are obtained for the system detuned conditions, assuming initial errors of +40% in J_{m0}, J_{m2}, J_{c0}, J_{c2} and −30% in b_{f0}, b_{f1}, b_{f2}, b_w, ρ_w.

Fig. 3 shows simulation results using centralised controller without tensions observer. Fig. 4 shows simulation results using the decentralised controller without the tension observer. In both centralised and decentralised controller schemes, using the proposed robust nonlinear controller, even in the presence of uncertainties, the velocities and tensions track the reference values in a short time.

Fig. 5 shows real and estimated tensions using the centralised controller and tension observer (33) and (34) or equivalently $m_1 = m_2 = 0$ in (47) and (48). The initial conditions for the tension observer are assumed to be $\hat{t}_1(0) = 3$, $\hat{t}_2(0) = 2$. The estimated values converge to the corresponding actual values with a fast dynamics.

To investigate the robustness of the proposed SM controller in the presence of disturbances and more deviations in parameters and to compare the performance of the proposed SM feedback linearisation control system with the ideal feedback linearisation control system, Fig. 6 shows simulation results using the centralised controller and tension observer with $m_1 = m_2 = 0$ and the initial conditions $\hat{t}_1(0) = 4$, $\hat{t}_2(0) = 4$. To obtain these results, a disturbance tension (15 N) is applied in the unwinder zone between $t = 1$ s and $t = 1.5$ s; a disturbance tension ($20\sin(10t)$) is applied in the rewinder zone between $t = 2.5$ s and $t = 3$ s. Moreover, assume initial errors of −50% in $J_{m0}, J_{m2}, J_{c0}, J_{c2}$ and +70% in b_{f0}, b_{f1}, b_{f2}, b_w, ρ_w. Fig. 6 illustrates that, using the SM control system, even in these circumstances, the references are followed properly.

8 Conclusion

This paper has discussed a robust nonlinear controller for web-winding systems. The robust nonlinear tracking controller has been designed, capable of controlling the velocity of the main motor and the tensions of the web-winding system separately. The proposed controllers are able to track the desired velocity and tensions perfectly in the presence of uncertainties. In addition, the proposed controller is not made for a particular operating point and is general. A decentralised version of the proposed controller is introduced, which comprises three separate controllers. The decentralised control is developed considering constraints on information exchanges between subsystems. Two tensions observers are also introduced, and it is proved that the estimated tensions converge to their real values. The effectiveness and validity of the proposed control method are shown by simulation results.

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10 References

[1] PAGILLA P.R., SIRASKAR N.B., DWIVEDULA R.V.: 'Decentralized control of web processing lines', *IEEE Trans. Control Syst. Technol.*, 2007, **15**, (1), pp. 106–117

[2] KOC H., KNITTEL D., DE MATHELIN M., ABBA G.: 'Modeling and robust control of winding systems for elastic webs', *IEEE Trans. Control Syst. Technol.*, 2002, **10**, (2), pp. 197–208

[3] BENLATRECHE A., OSTERTAG E., KNITTEL D.: 'H_∞-feedback decentralized control by BMI optimization for large scale web handling systems'. Proc. 2006 American Control Conf., Minneapolis, Minnesota, USA, 14–16 June 2006, pp. 619–625

[4] LAROCHE E., KNITTEL D., KOC H., DE MATHELIN M.: 'Web handling system robustness analysis via μ- analysis'. Proc. 2001 IEEE Int. Conf. Control Applications, Mexico city, Mexico, September 2001, pp. 948–953

[5] UTKIN V., GULDNER J., SHI J.: 'Sliding mode control in electromechanical systems' (Taylor & Francis Ltd, London, 1999)

[6] UTKIN V., SHI J.: 'Integral sliding mode in systems operating under uncertainty conditions'. Proc. 35th Conf. Decision and Control, Kobe, Japan, December 1996, pp. 4591–4596

[7] EDWARDS C., SPURGEON S.K.: 'Sliding mode control: theory and applications' (Taylor & Francis Ltd, London, 1998)

[8] SLOTINE J.J.E., WEIPING L.: 'Applied nonlinear control' (Prentice-Hall, Englewood Cliffs, NJ, 1991)