classfellow

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3CS12

Parameter Estimation

Q Let $(x1, x2 \ldots)$ be random sample of size n taken from a Normal Population with parameter mean $= \theta 1$ and variance $= \theta 2$. The max. Likelihood Estimation of these two parameter.

→ PDF of Normal distribution :-

$$f(x) = \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{1}{2}\left(\frac{x-\theta_1}{\sqrt{\theta_2}}\right)^2}$$

$$\theta_2 = \sigma^2$$
$$\theta_1 = \mu$$

ATQ, $x_1, x_2 \ldots$ are random value from the distribution which makes likelihood as

$$\alpha = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{1}{2}\frac{(x_i-\theta_1)^2}{\theta_2}}$$

Taking log on both side

$$\log \alpha = \log\left(\sqrt{2\pi\theta_2}^{\,-n} \prod_{i=1}^{n} e^{-\frac{1}{2}\frac{(x_i-\theta_1)^2}{\theta_2}}\right)$$

$$\log(\alpha) = -\frac{n}{2}\log(2\pi\theta_2) + \left(\frac{-1}{2\theta_2}\right)\sum_{i=1}^{n}(x_i-\theta_1)^2$$

Differentate wrt θ_1

$$\frac{1}{\alpha}\frac{\partial \alpha}{\partial \theta_1} = \frac{1}{\alpha\theta_2}\sum_{i=1}^{n} \alpha (x_i - \theta_1)$$

equating $\frac{\partial L}{\partial \theta_1} = 0$

$$\alpha \cdot \frac{1}{2\theta_2}\sum_{i=1}^{n} \alpha (x_i - \theta_1) = 0$$

either $\alpha = 0$

or $\frac{1}{\alpha\theta_2}\sum_{i=1}^{n} 2 (x_i - \theta_1) = 0$

$$n\theta_1 = \Sigma x_i$$

$$\theta_1 = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\boxed{\theta_1 = \frac{1}{n}\sum_{i=1}^{n} x_i}$$ θ_1 = sample mean

Now differentate ~~to~~ wrt θ_2

$$\frac{\partial \alpha}{\partial \theta_2}\left(\frac{1}{\alpha}\right) = \frac{-n}{2}\frac{\alpha\pi}{2\pi\theta_2} + \sum_{i=1}^{n}(x_i - \theta_1)^2 \frac{1}{(2\theta_2^2)}$$

Putting $\frac{\partial \alpha}{\partial \theta_2} = 0$

$$\sum_{i=1}^{n} (x_i - \theta_1)^2 = n\theta_2$$

$$\theta_2 = \frac{\sum_{i=1}^{n} (x_i - \theta_1)^2}{n}$$

$$\boxed{\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2}$$

θ_2 = Sample Variance