Lab07-Recursive and Recursively Enumerable Sets

CS363-Computability Theory, Xiaofeng Gao, Spring 2015

$*$ Please upload your assignment to FTP or submit a paper version on the next class.

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1. Let A, B be subsets of $\mathbb{N}$. Define sets $A \oplus B$ and $A \otimes B$ by

$$A \oplus B = \{2x : x \in A\} \cup \{2x+1 : x \in B\},$$
$$A \otimes B = \{\pi(x,y) : x \in A \text{ and } y \in B\},$$

   where π is the pairing function $\pi(x,y) = 2^x(2y+1) - 1$. Prove that

   (a) $A \oplus B$ is recursive iff A and B are both recursive.

   (b) If $A, B \neq \varnothing$, then $A \otimes B$ is recursive iff A and B are both recursive.

2. Which of the following sets are recursive? Which are r.e.? Which have r.e. complement? Prove your judgements.

   (a) $\{x \mid x \in E_x\}$;

   (b) $\{x \mid x \text{ is a perfect square}\}$;

   (c) $\{x \mid \phi_x \text{ is injective}\}$;

   (d) $\{x \mid P_m(x) \uparrow\}$ (m is fixed).

3. Suppose A is an r.e. set. Prove the following statements.

   (a) Show that the sets $\bigcup\limits_{x \in A} W_x$ and $\bigcup\limits_{x \in A} E_x$ are both r.e.

   (b) Show that $\bigcap\limits_{x \in A} W_x$ is not necessarily r.e. (*Hint*: $\forall t \in \mathbb{N}$ let $K_t = \{x : P_x(x) \downarrow \text{ in t steps}\}$. Show that for any t, K_t is recursive; moreover $K = \bigcup\limits_{t \in \mathbb{N}} K_t$ and $\overline{K} = \bigcup\limits_{t \in \mathbb{N}} \overline{K}_t$.)