Lab08-Various Sets

CS363-Computability Theory, Xiaofeng Gao, Spring 2015

∗ Please upload your assignment to FTP or submit a paper version on the next class.
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1. Use the Rice-Shapiro theorem to show that the following problems are not partially decidable:
   (a) W_x is finite;
   (b) W_x is infinite;
   (c) W_x is cofinite;
   (d) ϕ_x is extensible to a total recursive function.
2. Prove the following statements.
   (a) If B is r.e. and $A \cap B$ is productive, then A is productive.
   (b) If C is creative and A is an r.e. set such that $A \cap C = \varnothing$, then $C \cup A$ is creative.
3. If $A \oplus B = \{2x : x \in A\} \cup \{2x+1 : x \in B\}$, $A \otimes B = \{\pi(x,y) : x \in A \text{ and } y \in B\}$, prove the following statements.
   (a) Suppose B is r.e. If A is creative, then so are $A \oplus B$ and $A \otimes B$ (provided $B \neq \varnothing$).
   (b) If B is recursive, then the implications in (a) reverse.
4. Suppose that f is a total computable function, A is a recursive set and B is an r.e. set. Show that $f^{-1}(A)$ is recursive and that $f(A)$, $f(B)$ and $f^{-1}(B)$ are r.e., but not necessarily recursive. What extra information about these sets can be obtained if f is a bijection?
5. Let $\mathscr{B}$ be a set of unary computable functions, and suppose that $g \in \mathscr{B}$ is such that for all finite $\theta \subseteq g$, $\theta \notin \mathscr{B}$. Prove that the set $\{x : \phi_x \in \mathscr{B}\}$ is productive. (*Hint.* Follow the first part of the proof of the Rice-Shapiro theorem.)