Lab04-Church's Thesis

CS363-Computability Theory, Xiaofeng Gao, Spring 2016

$*$ Please upload your assignment to FTP or submit a paper version on the next class

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1. Suggest the natural definition of computability on domain $\mathbb{Q}$ (rational numbers).
   Solution:
   The computability on domain Q is that a function $f : \mathbb{Q} \to \mathbb{Q}$ is computable if there exist a series of computable functions $g_1, g_2 ... g_n$:$\mathbb{Q}$→$\mathbb{Q}$ that f is the composition of the functions $g_1 ... g_n$.

2. Define $f(n)$ as the n-th digit in the decimal expansion of e. Use Church's Thesis to prove that f is computable. (e is the the base of the natural logarithm and can be calculated as the sum of the infinite series: $e = \sum\limits_{n=0}^{\infty} \dfrac{1}{n!}$)
   Proof:
   We can obtain an informal algorithm for computing $f(n)$ as follows.
   Consider the infinite series: $e = \sum_{n=0}^{\infty} \frac{1}{n!} = \sum_{n=0}^{\infty} h_n(say)$
   Let $s_k = \sum_{n=0}^{k} h_n$, by elementary theory of infinite series $s_k < e < s_k + 1/10^k$.
   Now s_k is rational, so the decimal expansion ofs_k can be effecively calculated to any desired number of places using long division.
   Thus the effective method of calculation $f(n)$(given a number n) can be described as:
   Find the first $N \geq n + 1$ such that the decimal expansion $s_N = a_0 a_1 ... a_n a_{n+1} ... a_N$ does not have all of $a_{n+1} ... a_N$ equal to 9.
   Then put $f(n) = a(n)$.
   To see that this gives the required value, suppose that $a_m \neq 9$ with $n < m \leq N$. Then by the above
   $$s_N < e < s_N + 1/10^N \leq s_N + 1/10^m$$
   Hence $a_0 a_1 ... a_m ... < e < a_0 a_1 ... a_n ... a_m + 1 ...$. So the n^{th} decimal place of e is indeed a_n.
   Thus by Church's Thesis, f is computable.

3. Suppose there is a two-tape Turing Machine M with alphabet $\Gamma = \{\rhd, \lhd, \Box, 1\}$ and state set $Q = \{q_s, q_1, q_2, q_h\}$. M has the following specifications. Transform M into a single-tape Turing Machine $\widetilde{M}$, and write down the new alphabet and specifications.

$$\begin{array}{rcl}
\langle q_s, \rhd, \rhd \rangle & \to & \langle q_1, \rhd, S, R \rangle \\
\langle q_1, \rhd, \Box \rangle & \to & \langle q_2, 1, R, R \rangle \\
\langle q_2, 1, \Box \rangle & \to & \langle q_2, 1, R, R \rangle \\
\langle q_2, \lhd, \Box \rangle & \to & \langle q_h, \lhd, S, S \rangle
\end{array}$$

   Solution:
   Alphabet $\Gamma = \{\rhd, \lhd, \Box, 1\}$
   State set $Q = \{q_s, q_1, q_2, q_h\}$.
   Specification:

$$q_s \triangleright Rq_1$$
$$q_1 1 Rq_1$$
$$q_1 \triangleleft 1 q_2$$
$$q_2 1 Rq_2$$
$$q_2 \Box \triangleleft q_h$$

4. Design a three-tape TM M that computes the function $f(x, y) = x\%y$, where both m and n belong to the natural number set $\mathbb{N}$. The alphabet is $\{1, \Box, \triangleright, \triangleleft\}$, where the input on the first tape is $x + 1$ "1"'s and $y + 1$ "1"'s with a "$\Box$" as the separation. Below is the initial configurations for input (x, y). The result is the number of "1"'s on the output tape with the pattern of $\triangleright 111 \cdots 111 \triangleleft$. First describe your design and then write the specifications of M in the form like $\langle q_S, \triangleright, \triangleright, \triangleright\rangle \to \langle q_1, \triangleright, \triangleright, R, R, R\rangle$ and explain the transition functions in detail (especially the meaning of each state).

Initial Configurations

Tape 1:	$\triangleright$	1	1	$\cdots$	1	1	$\Box$	1	1	$\cdots$	1	1	$\triangleleft$	
	$\uparrow$	$\leftarrow x+1$ squares $\rightarrow$						$\leftarrow y+1$ squares $\rightarrow$						
Tape 2:	$\triangleright$	$\Box$	$\Box$	$\cdots$ $\cdots$ $\cdots$							$\Box$	$\Box$	$\Box$	
	$\uparrow$													
Tape 3:	$\triangleright$	$\Box$	$\Box$	$\cdots$ $\cdots$ $\cdots$							$\Box$	$\Box$	$\Box$	
	$\uparrow$													

Solution:
To realize the function, we fisrt should copy the x and y in the second and third tape;
Then, we scan the second and third tape repeatedly from right to left, when we reach the left side of the third tape, we can set a $\triangleleft$ in the second to show the subtraction of $x - y$.
We repeat this procedure, when we reach the left side of the second tape, we can know the result is the number of 1 between $\triangleright$ and the first $\triangleleft$.
At last, we clear the useless value of the second value and get the result from Tape 2.

Alphabet $\Gamma = \{\triangleright, \triangleleft, \Box, 1\}$
State set $Q = \{q_s, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9, q_h\}$.
Specification:
$\langle q_s, \triangleright, \triangleright, \triangleright\rangle \to \langle q_1, \triangleright, \triangleright, R, R, R\rangle$ Start
$\langle q_1, 1, \Box, \Box\rangle \to \langle q_2, \Box, \Box, R, S, S\rangle$ Skip the first "1"
$\langle q_2, 1, \Box, \Box\rangle \to \langle q_2, 1, \Box, R, R, S\rangle$ Copy x to Tape 2
$\langle q_2, \Box, \Box, \Box\rangle \to \langle q_3, \triangleleft, \Box, R, S, S\rangle$ Copy x done
$\langle q_3, 1, \triangleleft, \Box\rangle \to \langle q_4, \Box, \Box, R, S, S\rangle$ Skip the first "1"
$\langle q_4, 1, \triangleleft, \Box\rangle \to \langle q_4, \Box, 1, R, S, S\rangle$ Copy y to Tape 3
$\langle q_4, \triangleleft, \triangleleft, \Box\rangle \to \langle q_5, \triangleleft, \triangleleft, S, L, L\rangle$ Copy y done
$\langle q_5, \triangleleft, 1, 1\rangle \to \langle q_5, 1, 1, S, L, L\rangle$ Substraction
$\langle q_5, \triangleleft, 1, \triangleright\rangle \to \langle q_6, \triangleleft, \triangleright, S, S, R\rangle$ $x > y$
$\langle q_6, \triangleleft, \triangleleft, 1\rangle \to \langle q_6, \triangleleft, 1, S, S, R\rangle$ Back to the right side of Tape 3
$\langle q_6, \triangleleft, \triangleleft, \triangleleft\rangle \to \langle q_5, \triangleleft, \triangleleft, S, L, L\rangle$ Continue substraction
$\langle q_5, \triangleleft, \triangleright, 1\rangle \to \langle q_7, \triangleright, 1, S, R, S\rangle$ $x < y$
$\langle q_5, \triangleleft, \triangleright, \triangleright\rangle \to \langle q_8, \triangleright, \triangleright, S, R, R\rangle$ x mod y = 0
$\langle q_7, \triangleleft, 1, 1\rangle \to \langle q_7, 1, 1, S, R, S\rangle$ Back to the first $\triangleleft$ of Tape 2

$\langle q_7, \triangleleft, \triangleright, 1\rangle \to \langle q_9, \triangleleft, 1, S, R, S\rangle$ Back to the first $\triangleleft$ of Tape 2
$\langle q_8, \triangleleft, 1, 1\rangle \to \langle q_9, \triangleleft, 1, S, R, S\rangle$ Answer is 0
$\langle q_9, \triangleleft, 1, 1\rangle \to \langle q_9, \square, 1, S, R, S\rangle$ Clear useless value
$\langle q_9, \triangleleft, \triangleleft, 1\rangle \to \langle q_9, \square, 1, S, R, S\rangle$ Clear useless value
$\langle q_9, \triangleleft, \square, 1\rangle \to \langle q_h, \square, 1, S, S, S\rangle$ Completed!