

Lab11-Turing Degree

CS363-Computability Theory, Xiaofeng Gao, Spring 2015

* Please upload your assignment to FTP or submit a paper version on the next class.

* Name: _____ StudentId: _____ Email: _____

1. Find the certificate and certifier for the decision version of the following problems.
 - (a) Subset Sum: Given natural numbers w_1, \dots, w_n , and a target number W , is there a subset of $\{w_1, \dots, w_n\}$ that adds up to precisely W .
 - (b) Metric k-center: Given n cities with specified distances for each pair of cities as d_{ij} , one wants to build k warehouses in different cities and minimize the maximum distance of a city to a warehouse.
 - (c) Set Packing: Given a set U of n elements, a collection S_1, \dots, S_m of subsets of U , find the maximum subsets such that no two of them intersect.
2. (Improving Theorem 9-5.7(d)) Let A, B be any sets.
 - (a) Show that $A \leq_T B$ iff $K^A \leq_m K^B$, and $A \equiv_T B$ iff $K^A \equiv_m K^B$.
 - (b) Show that the previous question can be made effective in the following sense: there is a total computable function f such that if $c_A = \phi_e^B$, then $\phi_{f(e)} : K^A \leq_m K^B$. (*Hint.* Find total computable functions g, h such that (1) if $c_A = \phi_e^B$ then $K^A = W_{g(e)}^B$, (2) $\phi_{h(e)} : W_e^B \leq_m K^B$ for all e .)
3. Turing Sequence

For any set A define a sequence of sets $A^{(n)}$ by $A^{(0)} = A$; $A^{(n+1)} = K^{A^{(n)}}$, and then define $A^{(\omega)} = \{\pi(m, n) : m \in A^{(n)}\}$.

 - (a) Show that $A^{(n)} <_T A^{(\omega)}$ for all n .
 - (b) Show that there is a total computable function h such that $c_{A^{(n)}} = \phi_{h(n)}^{A^{(\omega)}}$ for all n .
 - (c) Suppose that B is a set such that $A^{(n)} \leq_T B$ for all n in the following strong way: there is a total computable function f such that $c_{A^{(n)}} = \phi_{f(n)}^B$, all n . Show that $A^{(\omega)} \leq_T B$.
 - (d) Show that if $A \leq_T B$ then $A^{(n)} \leq_T B^{(n)}$ for all n , and $A^{(\omega)} \leq_T B^{(\omega)}$. (*Hint.* Use the question 2b above, together with (b) and (c).)