Lab05-Numbering Programs

CS363-Computability Theory, Xiaofeng Gao, Spring 2015

$*$ Please upload your assignment to FTP or submit a paper version on the next class.

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1. Show that there is a total computable function k such that for each n,
   (a) $k(n)$ is an index of the function $\lfloor \sqrt[n]{x} \rfloor$.
   (b) $W_{k(n)}^{(m)} = \{(y_1, \ldots, y_m) : y_1 + y_2 + \ldots + y_m = n\}$ $(m \geq 1)$.
   (c) $E_{k(n)} = W_n$.

2. (a) Find P_{2057}. Distinguish what are $\phi_{2057}(x)$ and $\phi_{2057}^{(n)}(x_1, \cdots, x_n)$ and their corresponding $W_{2057}(x)$, $E_{2057}(x)$ and $W_{2057}^{(n)}(x)$, $E_{2057}^{(n)}(x)$;
   (b) Let P be the program J(1,2,4), Z(1), S(1). Calculate $\gamma(P)$.

3. Use Cantor's Diagonal Method to complete the following tasks:
   (a) Let $f_0, f_1, \ldots$ be an enumeration of partial functions from $\mathbb{N}$ to $\mathbb{N}$. Construct a function g from $\mathbb{N}$ to $\mathbb{N}$ such that $Dom(g) \neq Dom(f_i)$ for each i.
   (b) f is a partial function from $\mathbb{N}$ to $\mathbb{N}$. Construct a non-computable function g such that $g(x) \simeq f(x)$ for $x \leq m$, here $m \in \mathbb{N}$.

4. (a) Show that the functions s_n^m defined in s-m-n theorem are all primitive recursive.
   (b) Show that for each $m \in \mathbb{N}$ there is a total $(m+1)$-ary computable function s^m such that for all n, $\phi_e^{(m+n)}(\mathbf{x}, \mathbf{y}) \simeq \phi_{s^m(e,\mathbf{x})}^{(n)}(\mathbf{y})$, where $\mathbf{x}, \mathbf{y}$ are m- and n-tuples respectively. (*Hint.* Consider the definition of $s_n^m(e, \mathbf{x})$ given in the proof of s-m-n Theorem. The only way in which n was used was in determining how many of the $r_1, r_2, \ldots$ to transfer to $R_{m+1}, R_{m+2}, \ldots$. Now recall that the effect of P_e depends only on the original contents of $R_1, \ldots, R_{\rho(P_e)}$, where ρ is the function defined as the smallest register number such that none of the registers larger than $\rho(P_e)$ will be affected during the computation of P_e; $\rho(P_e)$ is independent of n.)

5. Alternative Selection of π

   The π function where $\pi(x, y) = 2^x(2y+1) - 1$ can enumerate linearly all pairs of natural numbers $(x, y) \in \mathbb{N} \times \mathbb{N}$. However, it does not generate a completely diagonal-zigzag trace in the first quadrant of the plane. Correspondingly, instead of applying this π function, we can define an alternative bijection π', such that $\pi' : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ and it grows diagonally according to the right figure. Thus we have:

   $\pi'(0,0) = 0$, $\pi'(1,0) = 1$, $\pi'(0,1) = 2$,
   $\pi'(2,0) = 0$, $\pi'(1,1) = 4$, $\pi'(0,2) = 5$, etc.

$x \backslash y$	0	1	2	3	4
0	0	2	5	9	14
1	1	4	8	13	19
2	3	7	12	18	25
3	6	11	17	24	32
4	10	16	23	31	40

   Now please develop a mathematical formula for π', (like the notation of original π), and prove the correctness of your design.

   Hint:
   (a) For the pairs in the diagonal we have the property that $x + y$ is a constant.
   (b) The diagonal given by $x + y = n$ consists of $n + 1$ pairs.
   (c) You can count the elements occurring before the pair (x_0, y_0).