Lab03-Recursive Function

CS363-Computability Theory, Xiaofeng Gao, Spring 2015

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1. Show that the following functions are computable:

   (a) $lcm(x, y) =$ the least common multiple of x and y,

   (b) $hcf(x, y) =$ the highest common factor of x and y,

   (c) $\phi(x) =$ the number of positive integers less than x which are relatively prime to x. (*Euler's function*) (We say that x, y are *relatively prime* if $hcf(x, y) = 1$.)

2. Let $\pi(x, y) = 2^x(2y + 1) - 1$. Show that π is a computable bijection from $\mathbb{N}^2$ to $\mathbb{N}$, and that the functions π_1, π_2 such that $\pi(\pi_1(z), \pi_2(z)) = z$ for all z are computable.

3. Show the computability of the following functions by minimalisation.

   (a) $f^{-1}(x)$, if $f(x)$ is a total injective computable function.

   (b) $f(a) = \begin{cases} \text{the least non-negative integral root of } p(x) - a \ (a \in \mathbb{N}), \\ \text{undefined if there's no such root,} \end{cases}$

   where $p(x)$ is a polynomial with integer coefficients.

   (c) $f(x, y) = \begin{cases} x/y & \text{if } y \neq 0 \text{ and } y|x, \\ \text{undefined} & \text{otherwise.} \end{cases}$

4. Coding Technology

   Any number $x \in \mathbb{N}$ has a unique expression as

   (1) $x = \sum_{i=0}^{\infty} \alpha_i 2^i$, with $\alpha_i = 0$ or 1, for all i.

   Hence, if $x > 0$, there are unique expressions for x in the forms

   (2) $x = 2^{b_1} + 2^{b_2} + \ldots + 2^{b_l}$, with $0 \leq b_1 < b_2 < \ldots < b_l$ and $l \geq 1$, and

   (3) $x = 2^{a_1} + 2^{a_1+a_2+1} + \ldots + 2^{a_1+a_2+\ldots+a_k+k-1}$. (The expression (3) is a way of regarding x as coding the sequence $(a_1, a_2, \ldots, a_l)$ of numbers)

   Show that each of the functions α, l, b, a defined below is computable.

   (a) $\alpha(i, x) = \alpha_i$ as in the expression (1);

   (b) $l(x) = \begin{cases} l \text{ as in (2),} & \text{if } x > 0, \\ 0 & \text{otherwise;} \end{cases}$

   (c) $b(x) = \begin{cases} b_i \text{ as in (2),} & \text{if } x > 0 \text{ and } 1 \leq i \leq l, \\ 0 & \text{otherwise;} \end{cases}$

   (d) $a(i, x) = \begin{cases} a_i \text{ as in (3),} & \text{if } x > 0 \text{ and } 1 \leq i \leq l, \\ 0 & \text{otherwise;} \end{cases}$