Lab04-Church's Thesis

CS363-Computability Theory, Xiaofeng Gao, Spring 2015

∗ Please upload your assignment to FTP or submit a paper version on the next class.

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1. Suggest the natural definition of computability on domain $\mathbb{Q}$ (rational numbers).

2. Prove that the following functions are URM-computable by Church's Thesis.

   (a) Ackermann function $\psi(x, y)$.

   (b) $g(n) = n^{th}$ digit in the decimal expansion of e (e is the basis for natural logarithms).

3. $\mathbb{A}$ and $\mathbb{B}$ are two domains other than $\mathbb{N}$. Assume there exist intuitively computable functions $\alpha : \mathbb{A} \to \mathbb{N}$ and $\alpha^{-1} : \mathbb{N} \to \mathbb{A}$ as encoding and decoding functions from $\mathbb{A}$ to $\mathbb{N}$ respectively (we have $\beta : \mathbb{B} \to \mathbb{N}$ and $\beta^{-1} : \mathbb{N} \to \mathbb{B}$ similarly). Then answer the following questions.

   (a) To prove that $f : \mathbb{A} \to \mathbb{B}$ is computable, we need to construct an $f^* : \mathbb{N} \to \mathbb{N}$ and analyze its computability. How to calculate f^*?

   (b) Reversely, for a computable function $g : \mathbb{N} \to \mathbb{N}$, we can obtain an intuitively computable function $g' : \mathbb{A} \to \mathbb{B}$ from g. What is g' in notation of g, α and β?

   (c) Find a new coding function γ (with the help of α and β) to deal with computability on domain $\mathbb{A} \times \mathbb{B}$. (Hint: Consider π function given by $\pi(x, y) = 2^x(2y + 1) - 1$.)

4. Design a three-tape TM M that computes the function $f(m, n) = m \times n$, where both m and n belong to the natural number set $\mathbb{N}$. The alphabet is $\{1, \Box, \triangleright, \triangleleft\}$, where the input on the first tape is $m + 1$ "1"'s and $n + 1$ "1"'s with a "$\Box$" as the separation. Below is the initial configurations for input (m, n). The result is the number of "1"'s on the output tape with the pattern of $\triangleright 111 \cdots 111 \triangleleft$. First describe your design and then write the specifications of M in the form like $\langle q_S, \triangleright, \triangleright, \triangleright \rangle \to \langle q_1, \triangleright, \triangleright, R, R, R \rangle$ and explain the transition functions in detail (especially the meaning of each state).

Initial Configurations

Tape 1:	$\triangleright$	1	1	$\cdots$	1	1	$\Box$	1	1	$\cdots$	1	1	$\triangleleft$
	$\uparrow$	$\leftarrow m + 1$ squares $\rightarrow$						$\leftarrow n + 1$ squares $\rightarrow$					
Tape 2:	$\triangleright$	$\Box$	$\Box$	$\cdots$ $\cdots$ $\cdots$							$\Box$	$\Box$	$\Box$
	$\uparrow$												
Tape 3:	$\triangleright$	$\Box$	$\Box$	$\cdots$ $\cdots$ $\cdots$							$\Box$	$\Box$	$\Box$
	$\uparrow$												