Table 1: Various Sets

Set	Definition	Theorem	Example	Counter Example
Recursive Set	$c_A(x) = \begin{cases} 1, & \text{if } x \in A, \\ 0, & \text{if } x \notin A. \end{cases}$ is computable.	① Recursive Function Theorems ② Closure: A,B are r. $\Rightarrow \overline{A}$, $A \cup B$, $A \cap B$ are r. ③ Rice Theorem: $\varnothing \subsetneq \mathscr{B} \subsetneq \mathscr{C}_1 \Rightarrow$ '$\phi_x \in \mathscr{B}$' is undecidable. ④ Any Theorems for Decidable Predicates.	$\mathbb{N}$, $\mathbb{Z}$, $\mathbb{E}$, $\mathbb{O}$, $\mathbb{P}$ Any finite set	$\{x \mid \phi_x \text{ is total}\}$ $\{x \mid x \in W_x\}$ $\{x \mid \phi_x = \mathbf{0}\}$
Recursively Enumerable Set (r.e. set)	$\chi_A(x) = \begin{cases} 1, & \text{if } x \in A, \\ \uparrow, & \text{if } x \notin A. \end{cases}$ is computable.	① Index $\leftrightarrow$ ② Listing $\begin{cases} \text{③ Equivlence} \\ \exists \text{ infinite } r. \subseteq r.e. \\ r. \Leftrightarrow \exists f \in \mathscr{C}_1 \upuparrows, Ran(f) \end{cases}$ ④ Normal Form $\begin{cases} \text{⑤ Uniformization} \\ \text{⑥ Graph} \\ \text{⑦ Quantifier Constraction} \end{cases}$ ⑧ Complementation (A is $r.$ $\Leftrightarrow A, \overline{A}$ are r.e.) ⑨ Closure (A,B are r.e. $\Rightarrow A \cap B$, $A \cup B$ are r.e.) ⑩ Rice-Shapiro: $\mathscr{A} \subseteq \mathscr{C}_1$, $\{x \mid \phi_x \in \mathscr{A}\}$ is r.e., then $\forall f \in \mathscr{C}_1$, $f \in \mathscr{A} \Leftrightarrow \exists$ finite $\theta \subseteq f$ with $\theta \in \mathscr{A}$	all recursive set non-recursive r.e. set $\{x \mid x \in W_x\}$ $\{x \mid \phi_x(x) = 0\}$ $\{x \mid W_x \neq \varnothing\}$ $\{x \mid x\ 7's \text{ in } \pi\}$	$\{x \mid x \notin W_x\}$ $\{x \mid \phi_x \text{ is total}\}$ $\{x \mid \phi_x \text{ is not total}\}$
Productive Set	A is productive if $\exists$ total $g \in \mathscr{C}_1$ s.t. $\forall\ W_x \subseteq A$, $g(x) \in A \setminus W_x$	① Reduction Theorem A is productive and $A \leq_m B \Rightarrow B$ is productive ② Quasi-Rice Theorem $\mathscr{B} \subsetneq \mathscr{C}_1$, $f_\varnothing \in \mathscr{B} \Rightarrow \{x \mid \phi_x \in \mathscr{B}\}$ is productive ③ Quasi-Listing Theorem Productive set has r.e. subset	$\{x \mid \phi_x(x) \neq 0\}$ $\{x \mid c \notin W_x\}$ $\{x \mid c \notin E_x\}$ $\{x \mid \phi_x \text{ is not total}\}$	① r.e. set ② doesn't have r.e. subset
Creative Set	$\begin{cases} A \text{ is r.e.;} \\ \overline{A} \text{ is productive.} \end{cases}$	① Quasi-Rice Theorem $\mathscr{A} \subseteq \mathscr{C}_1$, $A = \{x \mid \phi_x \in \mathscr{A}\}$. If A is r.e., $A \neq \varnothing$, $\mathbb{N}$, then A is creative	$\{x \mid \phi_x(x) = 0\}$ $\{x \mid c \in W_x\}$ $\{x \mid c \in E_x\}$	simple set
Simple Set	$\begin{cases} A \text{ is r.e.;} \\ \overline{A} \text{ is infinite;} \\ \overline{A} \text{ contains no infinite} \\ \text{r.e. subset.} \end{cases}$	① Characteristic Theorem (A simple set is neither recursive nor creative) ② Existence Theorem (There is a simple set)	If A, B are simple: $A \oplus B$ is simple $A \otimes B$ is not simple $\overline{\overline{A} \otimes \overline{B}}$ is simple	Any recursive set Any creative set