Symbol List for Computability Theory

CS-363, Xiaofeng Gao's Section

Description:

This symbol list includes almost every commonly used symbols and notations in Computability Theory. It also provides some comparisons so that you can easily catch the points. Please refer our textbook page 241-245 (Notation Section) for the detail definition indices of each symbol.
(Note: some of the symbols are introduced in class and can only be checked from our slides. Please also notice that the symbols used in this class may not be restricted to the tables listed below.)

1. Sets and Collections

Table 1: Sets and Collections

Set		Collection			
$\mathbb{N}$	natural numbers	$\mathscr{C}, \mathscr{C}_n$	computable functions	$\mathscr{I}$	set of URM instructions
$\mathbb{Z}$	integers	$\mathscr{R}_0$	total recursive functions	$\mathscr{P}$	set of URM programs
$\mathbb{E}$	even numbers	$\mathscr{R}$	(partial) recursive functions	$\mathscr{S}$	statements of language L
$\mathbb{O}$	odd numbers	$\mathscr{PR}$	primitive recursive functions	$\mathscr{T}, \mathscr{F}$	true false statements of L
$\mathbb{R}$	rational numbers	$\mathscr{TC}$	Turing computable functions	$\mathscr{P}r$	provable statements

2. URM Related Computations

Table 2: URM Instructions and Functions

URM Instruction		URM Operation	
$Z(n)$	zero instruction	U_i^n	projection function
$S(n)$	successor instruction	PQ	concatenation of programs
$T(m,n)$	transfer instruction	$\rho(P)$	denotes registers affected by P
$J(m,n,q)$	jump instruction	$P[l_1, \cdots, l_n \to l]$	computing $R_{l_1}, \cdots, R_{l_n}$ to R_l
$O(n)$	oracle instruction	$r_n \coloneqq x$	r_n becomes x

Table 3: URM vs URMO

URM		URMO	
P_n	nth URM program $P = \gamma^{-1}(n)$	P_n^χ, Q_n^χ	nth URMO program P with oracle χ
$\mathscr{C}, \mathscr{C}_n$	computable functions	$\mathscr{C}^\chi$	χ-computable functions
$\mathscr{R}, \mathscr{R}_0$	(partial) recursive functions	$\mathscr{R}^\chi$	χ-partial recursive functions
$\phi_a^{(n)}, \phi_a$	functions computed by P_a	$\phi_m^{\chi,n}$, ϕ_m^χ	functions computed by Q_m^χ
$W_a^{(n)}, W_a$	domain of $\phi_a^{(n)}$, ϕ_a	W_m^χ	domain of ϕ_m^χ
$E_a^{(n)}, E_a$	range of $\phi_a^{(n)}$, ϕ_a	E_m^χ	range of ϕ_m^χ
ψ_U^n, ψ_U	universal functions	$\psi_U^{\chi,n}$, ψ_U^χ	universal function for χ-computability
K	$\{x : x \in W_x\}$	K^χ	$\{x : x \in W_x^\chi\}$
$f_P^{(n)}$	n-ary function computed by P	$f_P^{\chi,n}$	n-ary function computed by P^χ

Note: P^A, $\mathscr{C}^A$, ϕ_m^A, W_m^A, E_m^A, K^A are relativized notions for A-computability

3. Reducibility and Degree

Table 4: m-degree vs T-degree

m-degree		T-degree	
$A \leq_m B$	A is many-one reducible to B	$A \leq_T B$	A is Turing reducible to B
$\equiv_m$	many-one equivalent	$\equiv_T$	Turing equivalent
$d_m(A)$	the m-degree of A	$d_T(A)$	the Turing degree of A
$\mathbf{a} \leq_m \mathbf{b}$	partial order on m-degrees	$\mathbf{a} \leq \mathbf{b}$	partial order on T-degree
$\mathbf{0}_m$	m-degree of recursive sets	$\mathbf{0}$	T-degree of recursive sets
$\mathbf{0}'_m$	m-degree of K	$\mathbf{0}'$	T-degree of K
$\mathbf{o}$, $\mathbf{n}$	m-degree of $\varnothing$, $\mathbb{N}$	A', $\mathbf{a}'$	jump of A, $\mathbf{a}$
$\mathbf{a} \underline{\cup} \mathbf{b}$	least upper bound of degrees $\mathbf{a}$, $\mathbf{b}$	$\mathbf{a} \mid \mathbf{b}$	$\mathbf{a}$, $\mathbf{b}$ are incomparable degrees

4. Other Notations

Table 5: Special Functions and Sets

Special Function		Special Set
γ	program coding function	$K = \{x \mid x \in W_x\}$
θ_n	$(n+1)$th statement of $\mathscr{S}$	$\overline{K} = \{x \mid x \notin W_x\}$
c_M	characteristic function of M	$Fin = \{x \mid W_x \text{ is finite}\}$
χ_M	partial characteristic function	$Inf = \{x \mid W_x \text{ is infinite}\}$
p_x	xth prime number	$Cof = \{x \mid W_x \text{ is cofinite}\}$
$(x)_y$	power of p_y occurring in x	$Tot = \{x \mid \phi_x \text{ is total}\}$
$\pi(x, y)$	a pairing function	$\overline{Tot} = \{x \mid \phi_x \text{ is not total}\}$
$x \dot{-} y$	cut-off subtraction	$Con = \{x \mid \phi_x \text{ is total and constant}\}$
$sg(x)$, $\overline{sg}(x)$	signum functions	$Rec = \{x \mid W_x \text{ is recursive}\}$
$rm(x, y)$	remainder function	$Ext = \{x \mid \phi_x \text{ is extensible to some } f \in \mathscr{R}_0\}$
$qt(x, y)$	quotient function	$Accp = \{x \mid c \in W_x\}$
$\mu z < y(\cdots)$	least z less than y	$Print = \{x \mid c \in E_x\}$
$\mu y(f(\mathbf{x}, y){=}0)$	minimalisation operator	$Pr^* = \{n \mid \mathsf{n} \in \mathsf{K} \text{ is provable}\}$
$\mathsf{0}, \mathsf{1}, \mathsf{x}, \mathsf{y}, \mathsf{R}$	symbols in a logical languages	$Ref^* = \{n \mid \mathsf{n} \notin \mathsf{K} \text{ is provable}\}$
$\mathsf{n} \in \mathsf{K}$	formal counterpart of $n \in K$	$K_0 = \{x : \phi_x(x) = 0\}$, $K_1 = \{x : \phi_x(x) = 1\}$
$\mathsf{S}_n(e, \mathbf{x}, y, t)$	'$P_e(\mathbf{x}) \downarrow y$ in $\leq t$ steps'	$A \oplus B = \{2x \mid x \in A\} \cup \{2x + 1 \mid x \in B\}$
$\mathsf{H}_n(e, \mathbf{x}, t)$	'$P_e(\mathbf{x}) \downarrow$ in $\leq t$ steps'	$A \otimes B = \{\pi(x, y) \mid x \in A \text{ and } y \in B\}$