Lab06-Universal Programs

CS363-Computability Theory, Xiaofeng Gao, Spring 2015

$*$ Please upload your assignment to FTP or submit a paper version on the next class.

$*$ Name:________ StudentId: ________ Email: ________

1. Show that there is a decidable predicate $Q(x, y, z)$ such that
   (a) $y \in E_x$ if and only if $\exists z.Q(x, y, z)$
   (b) if $y \in E_x$ and $Q(x, y, z)$, then $\phi_x((z)_1) = y$.
2. Show that there is a total computable function $k(x)$ such that for any x, $\phi_{k(x)} = c_{\neg M(x)}$, where M is a decidable predicate and $\phi_x = c_M$.
3. Show that there is a total computable function $s(x, y)$ such that for all x, y, $E_{s(x,y)} = E_x \cup E_y$.
4. Prove the equivalent of example 5 in Chapter 5-3.1 for the operations of substitution and minimalisation, namely:
   (a) Fix $m, n \geq 1$; there is a total computable function $s(e, e_1, \ldots, e_m)$ such that (in the notation of theorem 2.2) $\phi^{(n)}_{s(e,e_1,\ldots,e_m)} = Sub(\phi^{(m)}_e; \phi^{(n)}_{e_1}, \phi^{(n)}_{e_2}, \ldots, \phi^{(n)}_{e_m})$.
   (b) Fix $n \geq 1$; there is a total computable function $k(e)$ such that for all e, $\phi^{(n)}_{k(e)}(\mathbf{x}) \simeq \mu y(\phi^{(n+1)}_e(\mathbf{x}, y) = 0)$. (We could extend the notation of theorem 2.2 in the obvious way and write $\phi^{(n)}_{k(e)} = Min(\phi^{(n+1)}_e)$.)