

Lab04-Church's Thesis

CS363-Computability Theory, Xiaofeng Gao, Spring 2016

* Please upload your assignment to FTP or submit a paper version on the next class

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1. Suggest the natural definition of computability on domain \mathbb{Q} (rational numbers).

Solution:

The computability on domain \mathbb{Q} is that a function $f : \mathbb{Q} \rightarrow \mathbb{Q}$ is computable if there exist a series of computable functions $g_1, g_2, \dots, g_n : \mathbb{Q} \rightarrow \mathbb{Q}$ that f is the composition of the functions $g_1 \dots g_n$.

2. Define $f(n)$ as the n -th digit in the decimal expansion of e . Use Church's Thesis to prove that f is computable. (e is the base of the natural logarithm and can be calculated as the sum of the infinite series: $e = \sum_{n=0}^{\infty} \frac{1}{n!}$)

Proof:

We can obtain an informal algorithm for computing $f(n)$ as follows.

Consider the infinite series: $e = \sum_{n=0}^{\infty} \frac{1}{n!} = \sum_{n=0}^{\infty} h_n$ (say)

Let $s_k = \sum_{n=0}^k h_n$, by elementary theory of infinite series $s_k < e < s_k + 1/10^k$.

Now s_k is rational, so the decimal expansion of s_k can be effectively calculated to any desired number of places using long division.

Thus the effective method of calculation $f(n)$ (given a number n) can be described as:

Find the first $N \geq n + 1$ such that the decimal expansion $s_N = a_0 a_1 \dots a_n a_{n+1} \dots a_N$ does not have all of $a_{n+1} \dots a_N$ equal to 9.

Then put $f(n) = a(n)$.

To see that this gives the required value, suppose that $a_m \neq 9$ with $n < m \leq N$. Then by the above

$$s_N < e < s_N + 1/10^N \leq s_N + 1/10^m$$

Hence $a_0 a_1 \dots a_m \dots < e < a_0 a_1 \dots a_n \dots a_m + 1 \dots$. So the n^{th} decimal place of e is indeed a_n .

Thus by Church's Thesis, f is computable.

3. Suppose there is a two-tape Turing Machine M with alphabet $\Gamma = \{\triangleright, \triangleleft, \square, 1\}$ and state set $Q = \{q_s, \underline{q_1}, q_2, q_h\}$. M has the following specifications. Transform M into a single-tape Turing Machine \tilde{M} , and write down the new alphabet and specifications.

$$\begin{aligned} \langle q_s, \triangleright, \triangleright \rangle &\rightarrow \langle q_1, \triangleright, S, R \rangle \\ \langle q_1, \triangleright, \square \rangle &\rightarrow \langle q_2, 1, R, R \rangle \\ \langle q_2, 1, \square \rangle &\rightarrow \langle q_2, 1, R, R \rangle \\ \langle q_2, \triangleleft, \square \rangle &\rightarrow \langle q_h, \triangleleft, S, S \rangle \end{aligned}$$

Solution:

Alphabet $\Gamma = \{\triangleright, \triangleleft, \square, 1\}$

State set $Q = \{q_s, q_1, q_2, q_h\}$.

Specification:

$$q_s \triangleright Rq_1$$

$$q_1 1 Rq_1$$

$$q_1 \triangleleft 1q_2$$

$$q_2 1 Rq_2$$

$$q_2 \square \triangleleft q_h$$

4. Design a three-tape TM M that computes the function $f(x, y) = x \% y$, where both m and n belong to the natural number set \mathbb{N} . The alphabet is $\{1, \square, \triangleright, \triangleleft\}$, where the input on the first tape is $x + 1$ "1"'s and $y + 1$ "1"'s with a " \square " as the separation. Below is the initial configurations for input (x, y) . The result is the number of "1"'s on the output tape with the pattern of $\triangleright 111 \cdots 111 \triangleleft$. First describe your design and then write the specifications of M in the form like $\langle q_s, \triangleright, \triangleright, \triangleright \rangle \rightarrow \langle q_1, \triangleright, \triangleright, R, R, R \rangle$ and explain the transition functions in detail (especially the meaning of each state).

Initial Configurations

Tape 1:	<table> <tr> <td></td><td>\triangleright</td><td>1</td><td>1</td><td>\dots</td><td>1</td><td>1</td><td>\square</td><td>1</td><td>1</td><td>\dots</td><td>1</td><td>1</td><td>\triangleleft</td><td></td> </tr> </table>		\triangleright	1	1	\dots	1	1	\square	1	1	\dots	1	1	\triangleleft	
	\triangleright	1	1	\dots	1	1	\square	1	1	\dots	1	1	\triangleleft			
	\uparrow $\leftarrow x + 1 \text{ squares} \rightarrow$ $\leftarrow y + 1 \text{ squares} \rightarrow$															
Tape 2:	<table> <tr> <td></td><td>\triangleright</td><td>\square</td><td>\square</td><td>\dots</td><td>\dots</td><td>\dots</td><td></td><td>\square</td><td>\square</td><td>\square</td><td></td> </tr> </table>		\triangleright	\square	\square	\dots	\dots	\dots		\square	\square	\square				
	\triangleright	\square	\square	\dots	\dots	\dots		\square	\square	\square						
	\uparrow															
Tape 3:	<table> <tr> <td></td><td>\triangleright</td><td>\square</td><td>\square</td><td>\dots</td><td>\dots</td><td>\dots</td><td></td><td>\square</td><td>\square</td><td>\square</td><td></td> </tr> </table>		\triangleright	\square	\square	\dots	\dots	\dots		\square	\square	\square				
	\triangleright	\square	\square	\dots	\dots	\dots		\square	\square	\square						
	\uparrow															

Solution:

To realize the function, we first should copy the x and y in the second and third tape; Then, we scan the second and third tape repeatedly from right to left, when we reach the left side of the third tape, we can set a \triangleleft in the second to show the subtraction of $x - y$.

We repeat this procedure, when we reach the left side of the second tape, we can know the result is the number of 1 between \triangleright and the first \triangleleft .

At last, we clear the useless value of the second value and get the result from Tape 2.

Alphabet $\Gamma = \{\triangleright, \triangleleft, \square, 1\}$

State set $Q = \{q_s, q_1, q_2, q_h\}$.

Specification:

$\langle q_s, \triangleright, \triangleright, \triangleright \rangle \rightarrow \langle q_1, \triangleright, \triangleright, R, R, R \rangle$ Start

$\langle q_1, 1, \square, \square \rangle \rightarrow \langle q_2, \square, \square, R, S, S \rangle$ Skip the first "1"

$\langle q_2, 1, \square, \square \rangle \rightarrow \langle q_2, 1, \square, R, R, S \rangle$ Copy x to Tape 2

$\langle q_2, \square, \square, \square \rangle \rightarrow \langle q_3, \triangleleft, \square, R, S, S \rangle$ Copy x done

$\langle q_3, 1, \triangleleft, \square \rangle \rightarrow \langle q_4, \square, \square, R, S, S \rangle$ Skip the first "1"

$\langle q_4, 1, \triangleleft, \square \rangle \rightarrow \langle q_4, \square, 1, R, S, S \rangle$ Copy y to Tape 3

$\langle q_4, \triangleleft, \triangleleft, \square \rangle \rightarrow \langle q_5, \triangleleft, \triangleleft, S, L, L \rangle$ Copy y done

$\langle q_5, \triangleleft, 1, 1 \rangle \rightarrow \langle q_5, 1, 1, S, L, L \rangle$ Substraction

$\langle q_5, \triangleleft, 1, \triangleright \rangle \rightarrow \langle q_6, \triangleleft, \triangleright, S, S, R \rangle$ $x \geq y$

$\langle q_6, \triangleleft, \triangleleft, 1 \rangle \rightarrow \langle q_6, \triangleleft, 1, S, S, R \rangle$ Back to the right side of Tape 3

$\langle q_6, \triangleleft, \triangleleft, \triangleleft \rangle \rightarrow \langle q_5, \triangleleft, \triangleleft, S, L, L \rangle$

Continue subtraction $\langle q_5, \triangleleft, \triangleright, 1 \rangle \rightarrow \langle q_7, \triangleright, 1, S, R, S \rangle$ $x < y$

$\langle q_5, \triangleleft, \triangleright, \triangleright \rangle \rightarrow \langle q_8, \triangleright, \triangleright, S, R, R \rangle$ $x \bmod y = 0$

$\langle q_7, \triangleleft, 1, 1 \rangle \rightarrow \langle q_7, 1, 1, S, R, S \rangle$ Back to the first \triangleleft of Tape 2

$\langle q_7, \triangleleft, \triangleright, 1 \rangle \rightarrow \langle q_9, \triangleleft, 1, S, R, S \rangle$ Back to the first \triangleleft of Tape 2
 $\langle q_8, \triangleleft, 1, 1 \rangle \rightarrow \langle q_9, \triangleleft, 1, S, R, S \rangle$ Answer is 0
 $\langle q_9, \triangleleft, 1, 1 \rangle \rightarrow \langle q_9, \square, 1, S, R, S \rangle$ Clear useless value
 $\langle q_9, \triangleleft, \triangleleft, 1 \rangle \rightarrow \langle q_9, \square, 1, S, R, S \rangle$ Clear useless value
 $\langle q_9, \triangleleft, \square, 1 \rangle \rightarrow \langle q_h, \square, 1, S, S, S \rangle$ Completed!