Computer Security and Cryptography

CS381

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Organization

- Week 1 to week 16 (2016-02-24 to 2016-06-08)
- 东上院502
- Monday 3-4节; week 9-16
- Wednesday 3-4节; week 1-16
- lecture 10 + exercise 40 + random tests 40 + other 10
- Ask questions **in** class – counted as points
- Turn ON your mobile phone (after lecture)
- Slides and papers:
  - http://202.120.38.185/CS381
    - **computer-security**
  - http://202.120.38.185/references
- TA: '薛伟佳' icelikejia@qq.com，'黄格仕' <huang.ge.shi@foxmail.com>
- Send homework to: laix@sjtu.edu.cn and to TAs

Rule: do not disturb others!

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Contents

- Introduction -- What is security?
- Cryptography
  - Classical ciphers
  - Today's ciphers
  - Public-key cryptography
  - Hash functions/MAC
  - Authentication protocols
- Applications
  - Digital certificates
  - Secure email
  - Internet security, e-banking

Network security
- SSL
- IPSEC
- Firewall
- VPN

Computer security
- Access control
- Malware
- DDos
- Intrusion

Examples
- Bitcoin
- Hardware
- Wireless

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References

- W. Stallings, *Cryptography and network security - principles and practice*， Prentice Hall.
- W. Stallings, 密码学与网络安全：原理与实践（第4版），刘玉珍等译，电子工业出版社，2006
- Lidong Chen, Guang Gong, *Communication and System Security*, CRC Press, 2012.
- A.J. Menezes, P.C. van Oorschot and S.A. Vanstone, *Handbook of Applied Cryptography*. CRC Press, 1997, ISBN: 0-8493-8523-7, http://www.cacr.math.uwaterloo.ca/hac/index.html
- B. Schneier, *Applied cryptography*. John Wiley & Sons, 1995, 2nd edition.
- 裴定一,徐祥, 信息安全数学基础, ISBN 978-7-115-15662-4, 人民邮电出版社,2007.

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contents

- Public-key cryptosystems:
  - RSA - factorization
  - DH , ElGamal -discrete logarithm
  - ECC
- Math
  - Fermat's and Euler's Theorems & ø(n)
  - Group, Fields
  - Primality Testing
  - Chinese Remainder Theorem
  - Discrete Logarithms

IT-security and Cryptography

- **Issues in Information security**
  - **Scientific like**
    - **Confidentiality**
    - **Authentication**
    - **Access control**
    - **Integrity**
    - **Non-repudiation**
  - **More engineering**
    - **Virus protection**
    - **Intrusion prevention**
    - **Copyright protection**
    - **Content filtering**

Cryptography

Cryptology
(from the Greek for 'hidden word')

Cryptography –密码编码学
Code making

Cryptanalysis-密码分析
Code breaking 破译

Confidentiality
Secrecy

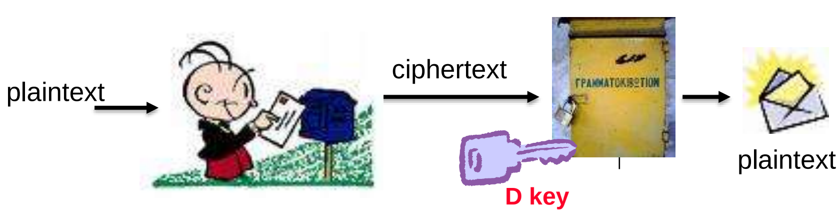
Authenticity
Data   entity

Integrity
Random number

Confidentiality and authenticity are independent attributes

☆ Confidentiality

– Confidentiality : information is not disclosed to unauthorized individuals, entities, or processes. [ISO]
– Mechanism to achieve confidentiality--Encryption:

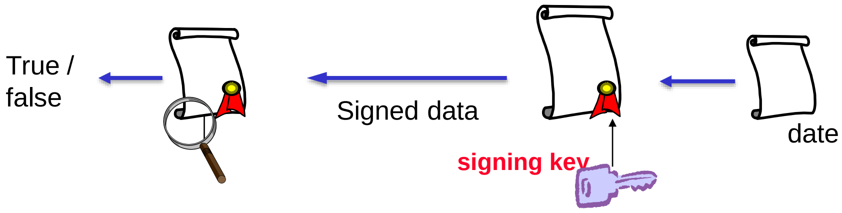


Only the user knowing the decryption key can recover plaintext

–"who can *read* the data"

Authenticity

- Authenticity: assurance of the claimed identity of an entity. [ISO]
- Example: ID-card, password, digital signature



Only the user knowing the secret-key can generate valid signature

"who *wrote* the data"

remark

- Understanding cryptography from the point of view of “read/write” is essential and useful.
- When an application or a functionality involves secret-key, it is helpful to decide whether it is a read or write problem, then pick up the correct approach: encryption or authentication.
- Example: copy-right protection, e-banking access, on-line transaction, e-voting, etc.

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ciphersystems

cipher
- Asymmetric (public-key)
- Symmetric (secret-key)
  - Block cipher
  - Stream cipher

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cryptosystems

- symmetric cipher, secret-key cryptosystem: encryption key and decryption key are essentially the same, it is easy to derive one from the other.
  - Example: DES, RC2, IDEA, AES
- asymmetric cipher, public-key cryptosystem: encryption key and decryption key are different, it is difficult to derive one (private decryption key) from the other (public encryption key).
  - Example: RSA, ElGamal, ECC

- Symmetric --- sharing some secret
- Asymmetric --- sharing some trusted information

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Two cryptosystems

Symmetric-key

- Advantages
  - high data throughput
  - Short size
  - primitives to construct various cryptographic mechanisms
- Disadvantages
  - the key must remain secret at both ends.
  - $O(n^2)$ keys to be managed for n users.

Public-key

- Advantages
  - Only the private key must be kept secret
  - Achieve non-repudiation (digital signature)
  - O(n) keys to be managed
- Disadvantages
  - low data throughput
  - much larger key sizes

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The usage

- Public-key cryptography
  - signatures (particularly, non-repudiation) and key management
- Symmetric-key cryptography
  - encryption and some data integrity applications
- Private keys must be larger (e.g., 1024 or 2048 bits for RSA) than secret keys (e.g., 64 or 128 bits)
  - most attack on symmetric-key systems is an exhaustive key search
  - public-key systems are subject to “short-cut” attacks (e.g., factoring)
- **Hybrid system**: Use public-key to encrypt a session-key, then use the symmetric session key to encrypt document.

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One-way functions

- Oneway function f: X ->Y, given x, easy to compute f(x); but for given y in f(X), it is hard to find x, s.t., f(x)=y.
  - Prob[f(A(f(x))=f(x))] < 1/p(n) (TM definition, existence unknown)
  - Example: hash function, discrete logarithm;
- Keyed function f(X,Z)=Y, for known key z, it is easy to compute f(.,z)
  - Block cipher (fix c, f(c,.) is a oneway function)
- Keyed oneway function: f(X,Z)=Y, for known key z, it is easy to compute f(.,z) but for given y, it is hard to x,z, s.t., f(x,z)=y.
  - MAC function: keyed hash h(z,X), block cipher CBC
- **Trapdoor oneway function** $f_T(x)$: easy to compute and hard to invert, but with additional knowledge T, it is easy to invert.
  - Public-key cipher; RSA: $y=x^e \bmod N$, T: N=p*q

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Number Theory - Divisibility

- Divisibility

  For any two integers a,b， $a+b$, $a-b$, $a*b$ are all integers, but a/b may not be an integer.

  $a=b*q+r$， where $b>r\geq 0$.

  q is the quotient, and r is the remainder.
- If $r=0$, we call b divides a, denoted by $b|a$； otherwise we call b does not divide a, denoted by $b\nmid a$。

For $a,b,c\in Z$，

- If $a|b$, then $a|(bc)$；
- If $a|b$ and $a|c$, then $a|(b+c)$ and $a|(b-c)$；
- for $i,a,b\in Z$， if $a=bq+r$, $i|a$ and $i|b$, then $i|r$.

2016/4/5

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Prime Numbers

- prime numbers only have divisors of 1 and self
  - they cannot be written as a product of other numbers
  - note: 1 is prime, but is generally not of interest
- eg. 2,3,5,7 are prime, 4,6,8,9,10 are not
- prime numbers are central to number theory
- list of prime number less than 200 is:

  2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97
  101 103 107 109 113 127 131 137 139 149 151 157 163 167 173 179 181
  191 193 197 199

Prime Factorisation

- to **factor** a number n is to write it as a product of other numbers: $n = a \times b \times c$
- factoring a number is relatively hard compared to multiplying the factors together to generate the number
- the **prime factorisation** of a number n is when its written as a product of primes
  - eg. $91 = 7 \times 13$; $3600 = 2^4 \times 3^2 \times 5^2$
- Any number can be written as a product of prime powers

$$a = \prod_{p \in P} p^{a_p}$$

Relatively Prime Numbers

- two numbers a, b are **relatively prime** if they have **no common divisors** apart from 1
  - eg. 8 & 15 are relatively prime since factors of 8 are 1,2,4,8 and of 15 are 1,3,5,15 and 1 is the only common factor
- conversely one can determine the greatest common divisor by comparing their prime factorizations and using least powers
  - eg. $300=2^1\times 3^1\times 5^2$ $18=2^1\times 3^2$ hence $GCD(18,300)=2^1\times 3^1\times 5^0=6$

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GCD and LCM

- *d* is the greatest common divisor of *a* and *b* if
  - $d|a$ and $d|b$;
  - If $f|a$ and $f|b$, then $f|d$;

  denoted by $d=\gcd(a,b)$, or (a,b).
- If $d|ab$, and $gcd(d,a)=1$, then $d|b$.

- *m* is the least common multiple of *a* and *b* if
  - $a|m$ and $b|m$;
  - If $a|n$ and $b|n$, then $m|n$;

  Denoted by $m=lcm(a,b)$, or $[a,b]$.

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The Euclid Algorithm

- gcd(a,b)=d
  - Fact 1： gcd(a,b)= gcd(b, $a-b$)；
  - Fact 2: if $a=qb+r$, then gcd(a,b)= gcd(b,r)；
  - Fact 3: there exists s,t， such that gcd(a,b)=$sa+tb$
- With the Euclid algorithm to determine d= gcd(a,b);
- With the extended Euclid algorithm to determine $d=sa+tb$;

The Euclid Algorithm

- The Euclid Algorithm to determine gcd(a,b)
  - $a=k_1b+r_1$ $\quad 0<r_1<b$
  - $b=k_2 r_1 +r_2$ $\quad 0<r_2< r_1$
  - $r_1 = k_3 r_2 +r_3$ $\quad 0<r_3< r_2$
  - ……
  - $r_{n-2} = k_n r_{n-1} +r_n$ $\quad 0<r_n< r_{n-1}$
  - $r_{n-1} = k_{n+1} r_n +r_{n+1}$ $\quad r_{n+1}=0$
- gcd(a,b)=gcd(b, r_1)=gcd(r_1 ,r_2)=…= r_n

☆ The extended Euclid algorithm

determine gcd(a,b)=$sa+tb$

- Input $b \geq a > 0$;
- Initialize $s_0=b$; $s_1=a$; $u_0=0$; $u_1=1$; $v_0=1$; $v_1=0$; $n=1$
- While $s_n>0$ do
  put $n=n+1$;
  write $s_{n-2}=q_n s_{n-1}+s_n,\ 0 \leq s_n < s_{n-1}$
  put $u_n=q_n u_{n-1}+u_{n-2}$;
  put $v_n=q_n v_{n-1}+v_{n-2}$;
- Put $s=(-1)^n u_{n-1}$; $t=(-1)^{n-1} v_{n-1}$;

Congruence

- If a and b are integers, we say that a is congruent to b modulo m if $m|(a-b)$.

  We write $a \equiv b \bmod n$
- $a \equiv a' \pmod m \Leftrightarrow m \mid (a-a')$
- $ka \equiv kb \pmod m$ not$\Rightarrow$ $a \equiv b \pmod m$
- If $ka \equiv kb \pmod m$ and $\gcd(k,m)=d$, then
  $a \equiv b \pmod{m/d}$

Euler Totient Function

Euler Totient Function

$$\phi(m) = \#\{j,\ \gcd(j, m)=1,\ 0 \le j \le m-1\}$$

Exa. $\phi(15)=\#\{1,2,4,7,8,11,13,14\}=8$

- for p prime, $\varphi(p)= p-1$, $\varphi(p^k) = p^k - p^{k-1}$
- gcd(a,b)=1, $\varphi(ab)=\varphi(a)\varphi(b)$

•Euler's Theorem: if $\gcd(a,m)=1$

then $a^{\phi(m)} \equiv 1 (\mathrm{mod}\ m)$

•Fermat's Theorem : for a prime *p*,

- if $\gcd(p,a)=1$, then $a^{p-1} \equiv 1 (\mathrm{mod}\ p)$
- $a^p \equiv a\ (\mathrm{mod}\ p)$

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RSA Public Key Cryptosystem

- The Inventors
  - R - Ron Rivest
  - S - Adi Shamir
  - A - Leonard Adleman
  - (2002 Turing Award)
- The Trap-Door One-Way Function
  - The exponentiation function $y = f(x) = x^e \bmod n$ can be computed with reasonable effort.
  - Its inverse $x = f^{-1}(y)$ is difficult to compute.
- The Hard Problem Securing the Trap Door
  - based on the hard problem of factoring a large number into its prime factors.

RSA Key Setup

- each user generates a public/private key pair:
  - selecting two large primes at random p, q
  - computing their system modulus n=p.q
    - note $\phi(n)=(p-1)(q-1)$
  - selecting at random the encryption key e
    - where $1<e<\phi(n)$, $\gcd(e,\phi(n))=1$
  - solve following equation to find decryption key d
    - $e.d\equiv 1 \bmod \phi(N)$ and $0\le d\le n$
- publish their public encryption key: PK={e,n}
- keep secret private decryption key: SK={d,p,q}

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RSA public-key encryption

- Encrypt with (e, n)
  - ciphertext：$0<M<n$, ciphertext $C\equiv M^{e}\ (\mathrm{mod}\ n)$.
- Decrpt with (d, n)
  - ciphertext：C ciphertext：$M\equiv C^{d}\ (\mathrm{mod}\ n)$

Alice $PK_A=(n_A, e_A)$
$SK_A=(p_A, q_A, d_A)$

Bob $PK_B=(n_B, e_B)$
$SK_B=(p_B, q_B, d_B)$

Get PK_B,
Compute C

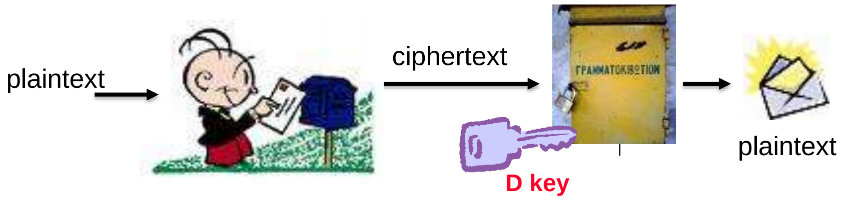
$C=E_{PKB}[M]=(M)^{eB} \bmod n_B$ →

$C^d=(M^e)^d=M^{k\phi(n)+1}=M^{k\phi(n)}M=M$

$M=E_{SKB}[C]=(C)^{dB} \bmod n_B$

Confidentiality

- Confidentiality : information is not disclosed to unauthorized individuals, entities, or processes. [ISO]
- Mechanism to achieve confidentiality--Encryption:

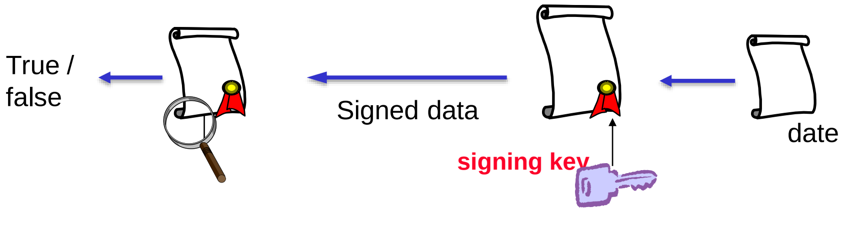


Only the user knowing the decryption key can recover plaintext

–"who can *read* the data"

Authenticity

- Authenticity: assurance of the claimed identity of an entity. [ISO]
- Example: ID-card, password, digital signature



Only the user knowing the secret-key can generate valid signature

"who *wrote* the data"

RSA digital signature

- Parameters PK={e,n} , SK={d,p,q} as before.
- The signature of the message M is S
  - $S \equiv M^d \pmod{n}$ (signing)
- receiver recover the message
  - $M \equiv S^e \pmod{n}$ (verification)

Alice		Bob
$S \equiv M^{dA} \pmod{n_A}$	$\xrightarrow{S}$	$M \equiv S^{eA} \pmod{n_A}$

Bob verify that only Alice can generate S
--M must be redundant (has clear structure)

RSA digital signature

Alice $PK_A = (n_A, e_A)$
$SK_A = (p_A, q_A, d_A)$

Bob $PK_B = (n_B, e_B)$
$SK_B = (p_B, q_B, d_B)$

Compute $H(M)$
Compute the signature
$S = H(M)^{dA} \bmod nA$

$\xrightarrow{(M,S)}$

Get PK_A,
(1) From M, **compute** $H(M)$
(2) From S, **recover**
$H(M) = E_{PKA}[S] = (S)^{eA} \bmod n_A$
(3) Check if $H(M) = H(M)$

In real use, a hash function is used to
•prevent S(xy)=S(x)S(y)
•provide redundancy

RSA digital signature

- M, a public hash function H with domain of $\{0,1,\ldots,n-1\}$。
- Signature

  Compute the hash value of M, and get $H(M) \in \{0,1,\ldots,n-1\}$

  The input of hash function is of arbitrary length.

  Sign $H(M)$ with the private key d, and get

  $S \equiv H(M)^d \pmod n$

  Send (M, S) to the receiver
- Verification

  After getting (M,S), recover $V \equiv S^e \pmod n$, and verify $V=H(M)$

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The trap-door

- For an integer $n=pq$, given M and e, modular exponentiation $C \equiv M^e \pmod n$ is a simple operation;
- Given $C \equiv M^e \pmod n$, to find $M \equiv C^{1/e} \pmod n$ is a difficult problem;
- When the prime factorization of n is known (trapdoor), to find $M \equiv C^{1/e} \pmod n$ is easy.

Knowing d $\Leftrightarrow$ knowing the factorization

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Cost of factorization

- For currently known algorithms, to complexity of factoring large number n is about

  $\exp(b^{1/3} \log^{2/3}(b))$ $b=\log(n)$

- Record:
  - RSA: 768-bit modulo (2010) , RSA 640-bit (2005)
  - Special Numbers: $2^{1039}-1$ (2007) , $6^{353}-1$ (2006)
- Question: Integer factorization ⇔ Breaking RSA (?)
- Size of n: now 1024-bit (5year?); recommended: 2048-bit

RSA module Length (EMV)

Length	Current Expiry Date	
1024 bits	31 Dec 2009	
1152 bits	31 Dec 2017	
1408 bits	31 Dec 2023	
1984 bits	31 Dec 2023	

2014 recommendation

Parameters of RSA

- length of n is at least 1024 bits
- p and q are large.
- $|p-q|$ is large
- p,q should be random/strong prime numbers.
  $p=2p'+1$, $q=2q'+1$, where p' q' are both primes
- $d>n^{1/4}$
- Public-key e: can be small for efficiency
  - ISO9796 allows 3, (problems?)
  - EDI $2^{16}+1=65537$

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Exercise 7 – RSA

1. PayTV systems require that only the paid customers can watch the program, which of the 5 security services can be used to achieve this goal?
2. Determine the complexity (number of arithmetic operations) of
   - computing gcd(a,b);
   - computing RSA encryption $C=M^e \bmod n$
3. Limitation of raw RSA signature：Only when M has redundancy structure, can the signature be securely verified. Why?
4. For RSA, it requires $|p-q|$ should not be small. Task: design an attack if $|p-q|$ is smaller than 10000.
5. Show that in RSA, knowing $\phi(n)$ is equivalent to knowing the factorization of n
   - Deadline: before next lecture
   - Format: Subject: CS381-某某某-EX.#

Summary

- Public-key cryptosystems:
  - RSA - factorization
  - DH , ElGamal -discrete logarithm
  - ECC
- Math
  - Fermat's and Euler's Theorems & ø(n)
  - Group, Fields
  - Primality Testing
  - Chinese Remainder Theorem
  - Discrete Logarithms