

# REVIEW & STATE SPACE INTRO

steve.bullock@bristol.ac.uk

bristol.ac.uk

<https://unsplash.com/@cookayne>



# **Feedback (from you)**

- 10/158 responses
- More guidance
- I'm still unsure what I should be doing
- More coverage of basic control concepts
- More linking back to previous learning
- I appreciate the mixture of theory and MATLAB
- Please be clear of when coursework is set
  - the lecturer has said it will be set on a certain day but it has not



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Mentimeter

(Start of lecture) Current feeling/comments...

fast bold  
inspiration creative  
leader focus  
transpiration

Menti

Control L3



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Current feeling...



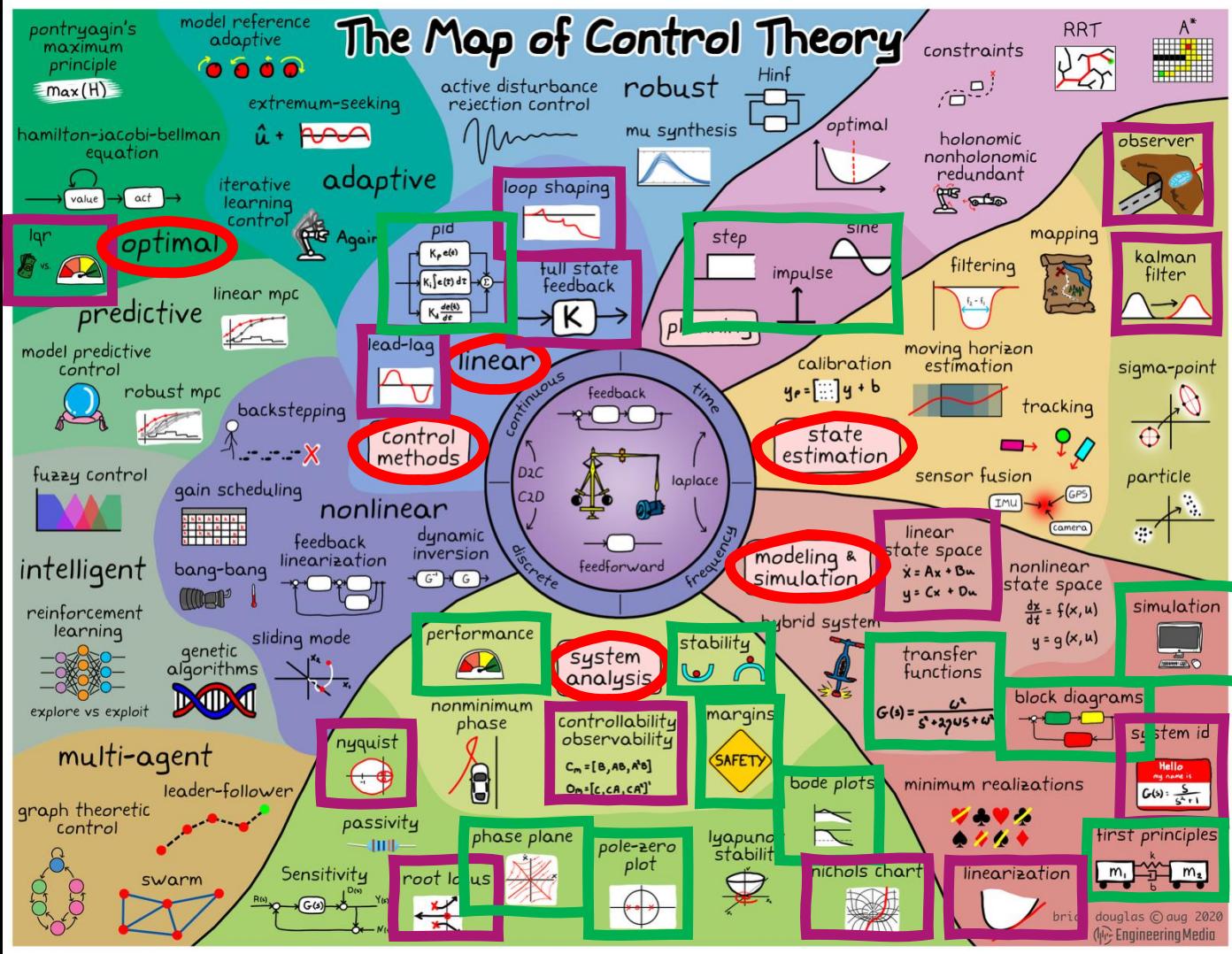
(After concept outline) Current feeling...

(After recap) Current feeling/comments...

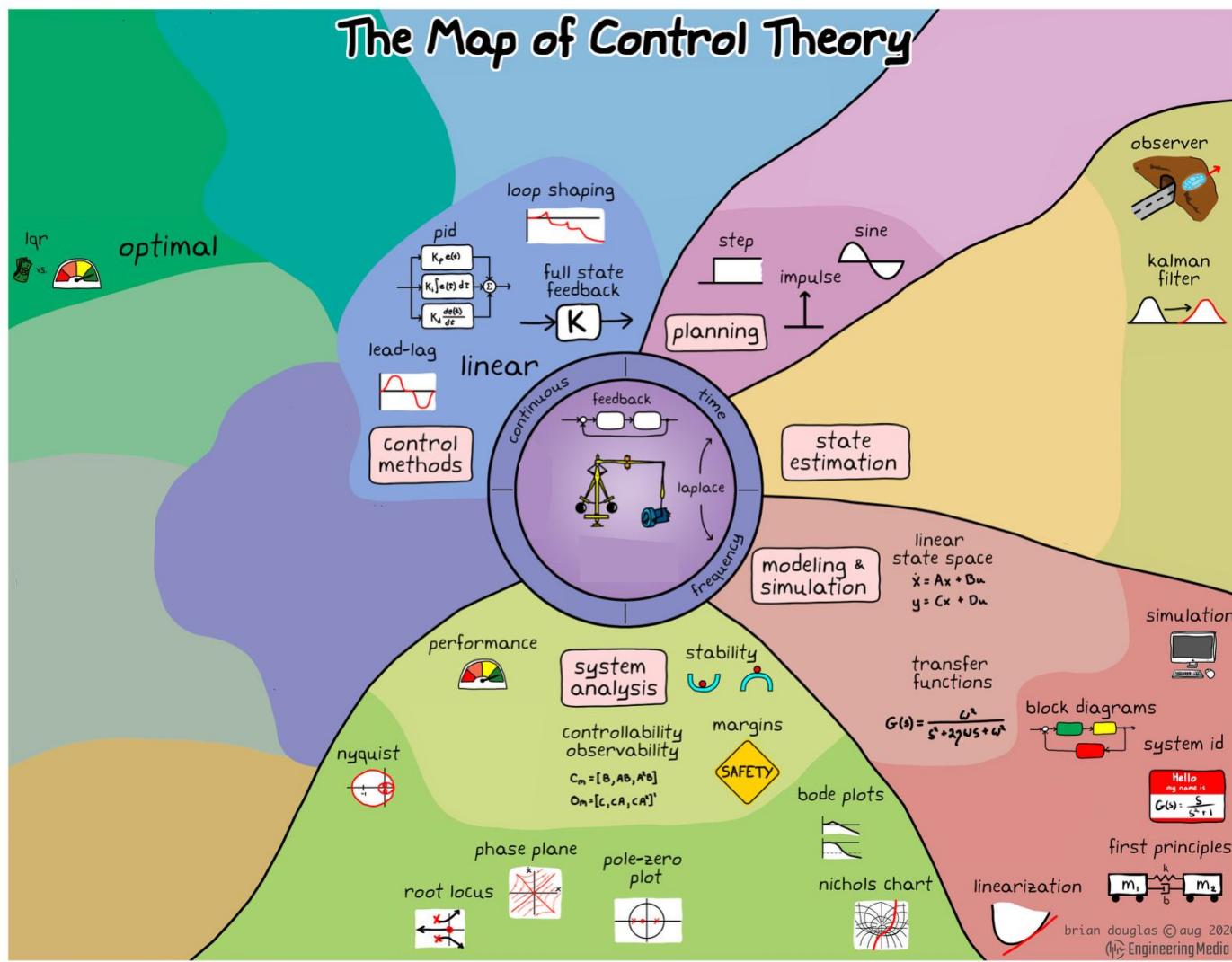
# **Today**

- Review syllabus
  - What did I assume you knew?
  - What did you actually know?
  - What do you want a refresher on?
  - What else do you want clarity on?
- Review delivery approach
  - What do you think so far? (Be honest).
  - How shall we proceed?
- Review and recap as necessary
- Introduce State Space models and control

# The Map of Control Theory



# The Map of Control Theory



# Recommended text: access and mapping

- Dorf mapping doc on Bb
  - But last 4 versions (and more?) are largely identical
- Doc includes access and etiquette guidance

CHAPTER 1	Introduction to Control Systems	CHAPTER 1	Introduction to Control Systems	CHAPTER 1	Introduction to Control Systems	CHAPTER 1	Introduction to Control Systems
Preface	15	Preface	15	Preface	xi	Preface	xiii
About the Authors	27	About the Authors	27	About the Authors	xxv	About the Authors	xxv
CHAPTER 2 Mathematical Models of Systems							
2.1	Introduction	2.1	Introduction	2.1	Introduction	2.1	Introduction
2.2	Differential Equations of Physical Systems	2.2	Differential Equations of Physical Systems	2.2	Differential Equations of Physical Systems	2.2	Differential Equations of Physical Systems
2.3	Linear Approximation of Physical Systems	2.3	Linear Approximation of Physical Systems	2.3	Linear Approximation of Physical Systems	2.3	Linear Approximation of Physical Systems
2.4	The Laplace Transform	2.4	The Laplace Transform	2.4	The Laplace Transform	2.4	The Laplace Transform
2.5	The Transfer Function of a Linear System	2.5	The Transfer Function of a Linear System	2.5	The Transfer Function of a Linear System	2.5	The Transfer Function of a Linear System
2.6	Block Diagram Models	2.6	Block Diagram Models	2.6	Block Diagram Models	2.6	Block Diagram Models
2.7	Signal-Flow Graph Models	2.7	Signal-Flow Graph Models	2.7	Signal-Flow Graph Models	2.7	Signal-Flow Graph Models
2.8	State Variable Models	2.8	State Variable Models	2.8	State Variable Models	2.8	State Variable Models
2.9	The Simulation of Systems Using Computer Design Examples	2.9	The Simulation of Systems Using Computer Design Examples	2.9	The Simulation of Systems Using Computer Design Examples	2.9	The Simulation of Systems Using Computer Design Examples
2.10	Sequential Design Example: Disk Drive Read System	2.10	Sequential Design Example: Disk Drive Read System	2.10	Sequential Design Example: Disk Drive Read System	2.10	Sequential Design Example: Disk Drive Read System
2.11	Summary	2.11	Summary	2.11	Summary	2.11	Summary
2.12	Skills Check	2.12	Skills Check	2.12	Skills Check	2.12	Skills Check
2.13	Exercises	2.13	Exercises	2.13	Exercises	2.13	Exercises
2.14	Problems	2.14	Problems	2.14	Problems	2.14	Problems
2.15	Design Problems	2.15	Design Problems	2.15	Design Problems	2.15	Design Problems
2.16	Terms and Concepts	2.16	Terms and Concepts	2.16	Terms and Concepts	2.16	Terms and Concepts
CHAPTER 3 State Variable Models							
3.1	Introduction	3.1	Introduction	3.1	Introduction	3.1	Introduction
3.2	The State Variables of a Dynamic System	3.2	The State Variables of a Dynamic System	3.2	The State Variables of a Dynamic System	3.2	The State Variables of a Dynamic System
3.3	The State Differential Equation	3.3	The State Differential Equation	3.3	The State Differential Equation	3.3	The State Differential Equation
CHAPTER 4 Feedback Control System Characteristics							
4.1	Introduction	4.1	Introduction	4.1	Introduction	4.1	Introduction
4.2	Error Signal Analysis	4.2	Error Signal Analysis	4.2	Error Signal Analysis	4.2	Error Signal Analysis
4.3	Sensitivity of Control Systems to Parameter Variation	4.3	Sensitivity of Control Systems to Parameter Variation	4.3	Sensitivity of Control Systems to Parameter Variation	4.3	Sensitivity of Control Systems to Parameter Variation
4.4	Error Signals in a Feedback Control System	4.4	Error Signals in a Feedback Control System	4.4	Error Signals in a Feedback Control System	4.4	Error Signals in a Feedback Control System
4.5	Control of the Transient Response	4.5	Control of the Transient Response	4.5	Control of the Transient Response	4.5	Control of the Transient Response
4.6	Steady-State Error	4.6	Steady-State Error	4.6	Steady-State Error	4.6	Steady-State Error

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*Of the greater teachers—  
when they are gone,  
their students will say:  
we did it ourselves.*

*Dedicated to  
Lynda Ferrera Bishop  
and  
Joy MacDonald Dorf  
In grateful appreciation*

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*Dedicated to the memory of  
Professor Richard C. Dorf*



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# This hour - recap

- 1. Mathematical modelling
  - ODEs
  - LTI ODEs
  - Linearisation
  - Frequency domain methods
- 2. Feedback control
  - PID control
  - Block diagrams
- In Bb/Control/Recaps/link
  - Recap slides
  - Problem sheets
  - MATLAB code

**See slides and code in MATLAB drive**

# **More practice:**

- Modelling (ODEs → TFs): Dorf ch2 (any version!)
- PID control: Brian Douglas – [Understanding PID control](#)



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# STATE SPACE MODELS

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# **Overview**

**See slides and code in MATLAB drive**

# **State Space – Steve's version**

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{D}(t)\mathbf{u}(t)$$

where:

$\mathbf{x}(\cdot)$  is called the "state vector",  $\mathbf{x}(t) \in \mathbb{R}^n$ ;

$\mathbf{y}(\cdot)$  is called the "output vector",  $\mathbf{y}(t) \in \mathbb{R}^q$ ;

$\mathbf{u}(\cdot)$  is called the "input (or control) vector",  $\mathbf{u}(t) \in \mathbb{R}^p$ ;

$\mathbf{A}(\cdot)$  is the "state (or system) matrix",  $\dim[\mathbf{A}(\cdot)] = n \times n$ ,

$\mathbf{B}(\cdot)$  is the "input matrix",  $\dim[\mathbf{B}(\cdot)] = n \times p$ ,

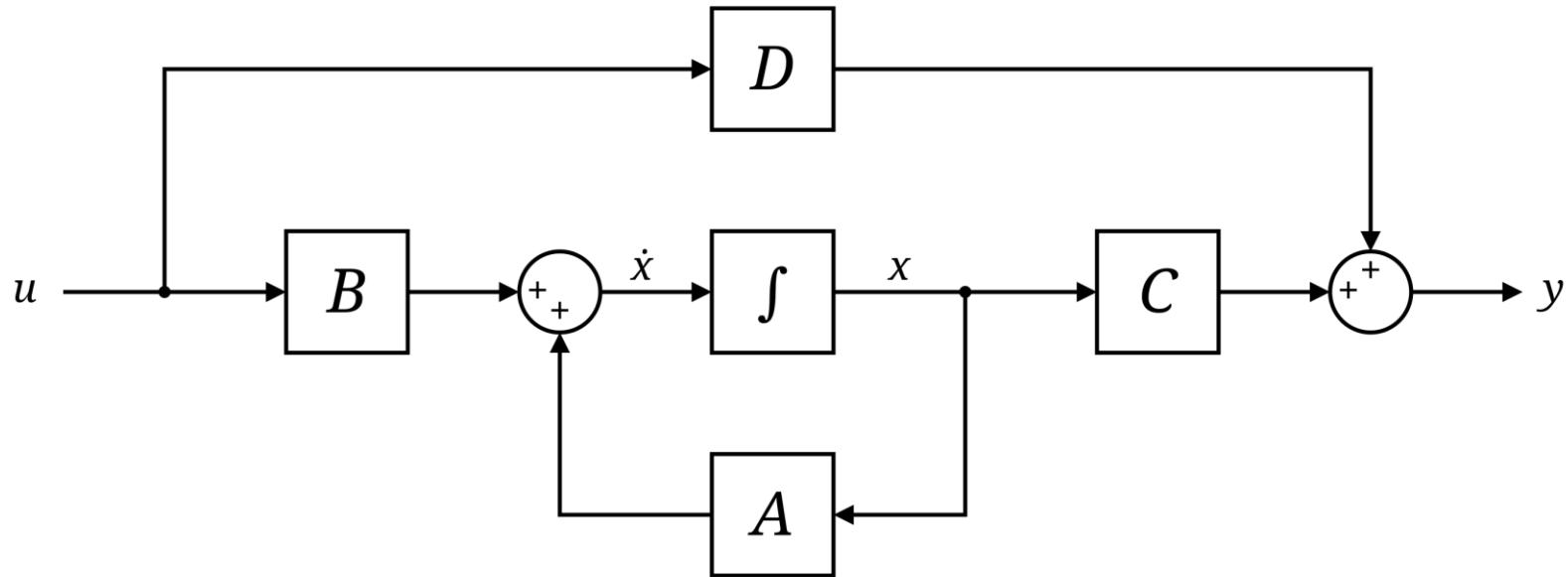
$\mathbf{C}(\cdot)$  is the "output matrix",  $\dim[\mathbf{C}(\cdot)] = q \times n$ ,

$\mathbf{D}(\cdot)$  is the "feedthrough (or feedforward) matrix" (in cases where the system model does not have a direct feedthrough,  $\mathbf{D}(\cdot)$  is the zero matrix),  $\dim[\mathbf{D}(\cdot)] = q \times p$ ,

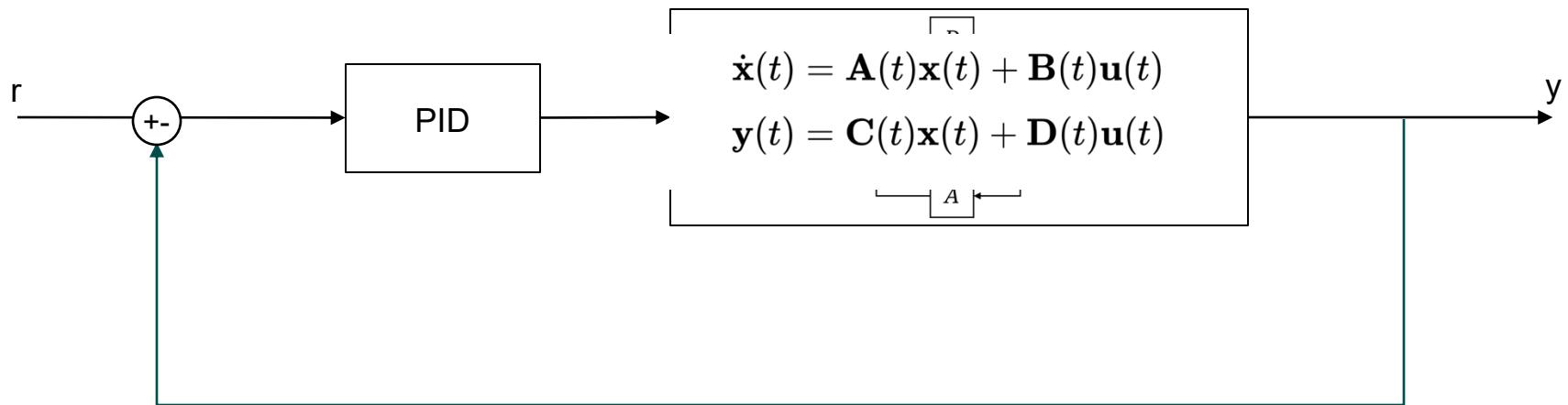
$$\dot{\mathbf{x}}(t) := \frac{d}{dt}\mathbf{x}(t).$$

Continuous time-invariant

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)\end{aligned}$$



Remember! This is the dynamics of whatever system you put into it.  
Could be a ‘plant’ on its own, i.e.:



# What does it look like?

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{D}(t)\mathbf{u}(t)$$

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.313 & 56.7 & 0 \\ -0.0139 & -0.426 & 0 \\ 0 & 56.7 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 0.232 \\ 0.0203 \\ 0 \end{bmatrix} [\delta]$$

$$y = [ \ 0 \ 0 \ 1 \ ] \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix}$$

([CTMS pitch model](#))

# **What does it look like?**

- Quanser model on MATLAB Drive

# Looking ahead

