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Advanced structures and materials

Lecture 5: Measuring fracture toughness

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21/10/2025

Course Content

Lecture 1: Modes of failure	Lecture 2: Case studies	Lecture 3: Introduction to fracture mechanics – Part 1	Lecture 4: Introduction to fracture mechanics – Part 2	Lecture 5: Measuring fracture toughness
• Why study failure? • Concept of strain energy and toughness • Ductile, brittle failure • Fractography • Factors affecting ductile to brittle transition	• Historical Examples • Design philosophies	• Introduction to fracture mechanics • Theoretical stress approach to fracture • Stress intensity factor	• Griffith's energy balance approach • Irwin's energy balance approach	• Fracture process zone and geometrical considerations • Measuring toughness • Anisotropic materials

The story so far…

- The importance of fracture mechanics in understanding material failure has been introduced
- Inglis:
  - Stress singularity close to sharp crack
- Griffith
  - concept of an energy balance, the relationship between stress, crack length atomic bonds strength
- Irwin
  - The plastic deformation ahead of crack contributes to toughness
  - Stress intensity factor and loading modes
    - I. opening
    - II. shear sliding
    - III. tearing

$$\sigma_{tip} \approx 2\sigma\sqrt{\frac{a}{\rho_r}}$$

$$\sigma = \sqrt{\frac{E\gamma}{2\pi a}}$$

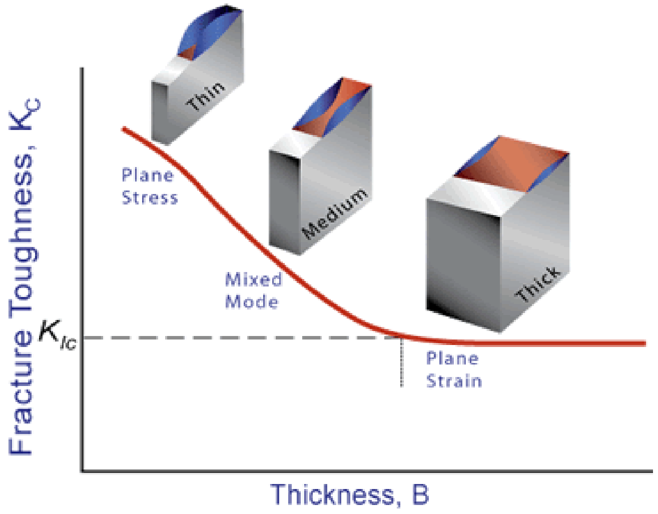
$$G = \frac{(\sigma\sqrt{\pi a})^2}{E}$$

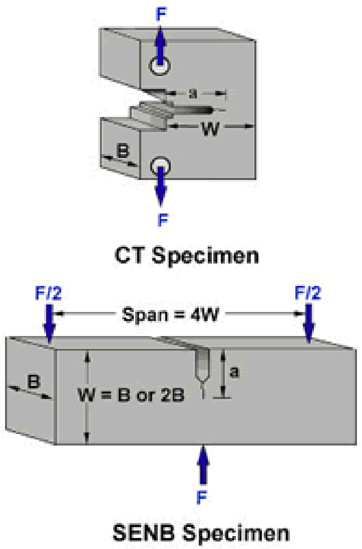
$$G_c = \frac{(\sigma_s\sqrt{\pi a})^2}{E}$$

$$G_C = \frac{K_C^2}{E}$$

Measurement of Fracture Toughness

- Objective: Understand the influence of geometry and loading method on fracture toughness
  - Describe the importance of Plane Strain
  - Explain the role of material anisotropy in fracture toughness

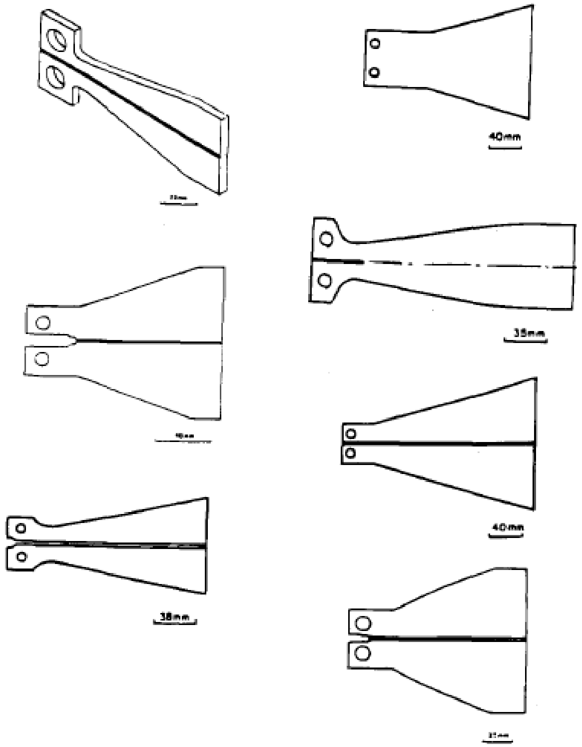




- It is important to understand the role of material thickness and application of loading direction in establishing fracture toughness

- This can be accomplished via understanding of
  - Plane Strain and Plane Stress
  - Crack tip state of tension
  - Isotropic vs Anisotropic Toughness

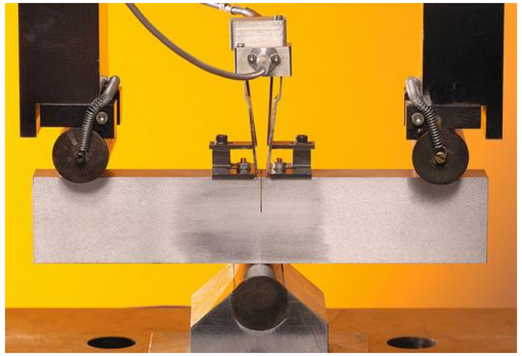
Test configurations

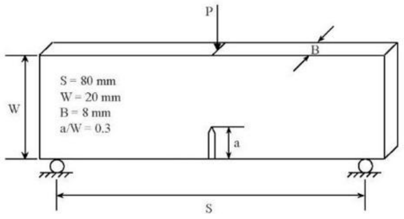


Tapered Double Cantilever Bend (TDCB)

Compact Tension (CT)

Single edged notched bend (SENB)



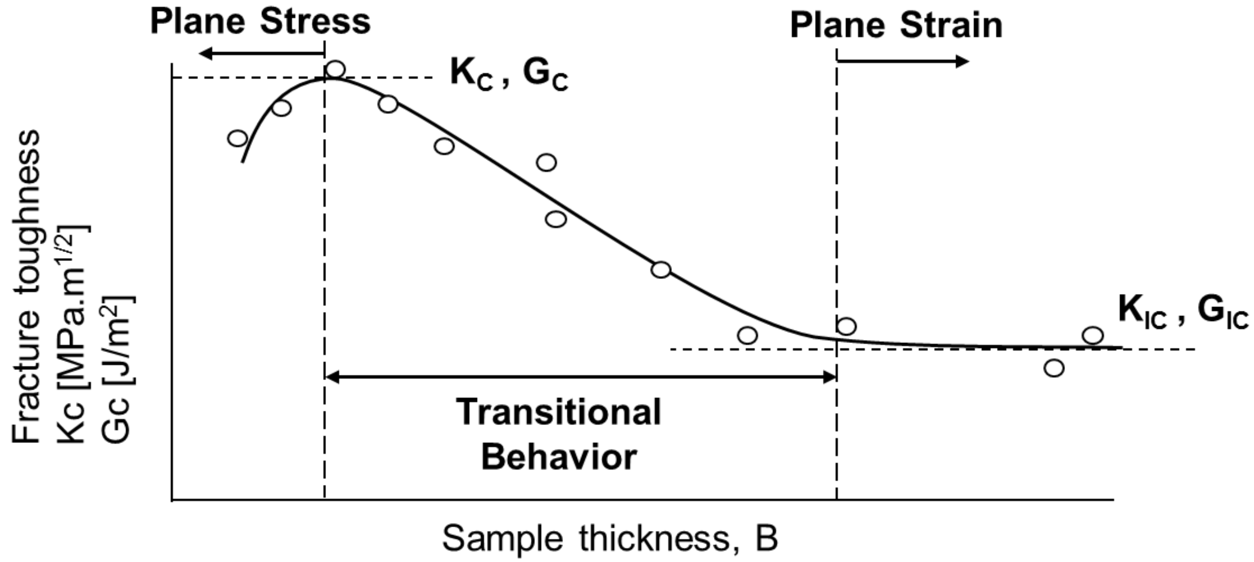


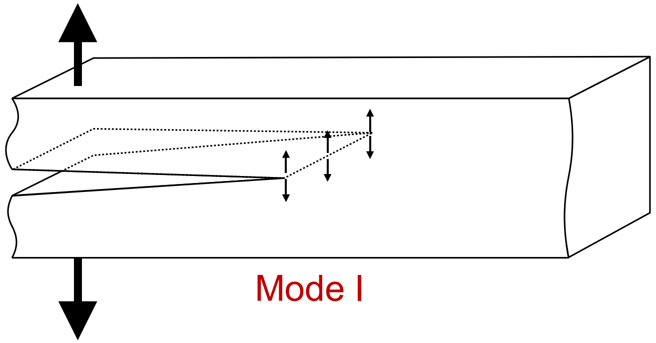
Double Cantilever Bend (DCB)

Measuring the mode I fracture toughness

- The critical stress intensity factor required to extend the crack under an opening mode is called the **mode I fracture toughness** K_{Ic}.

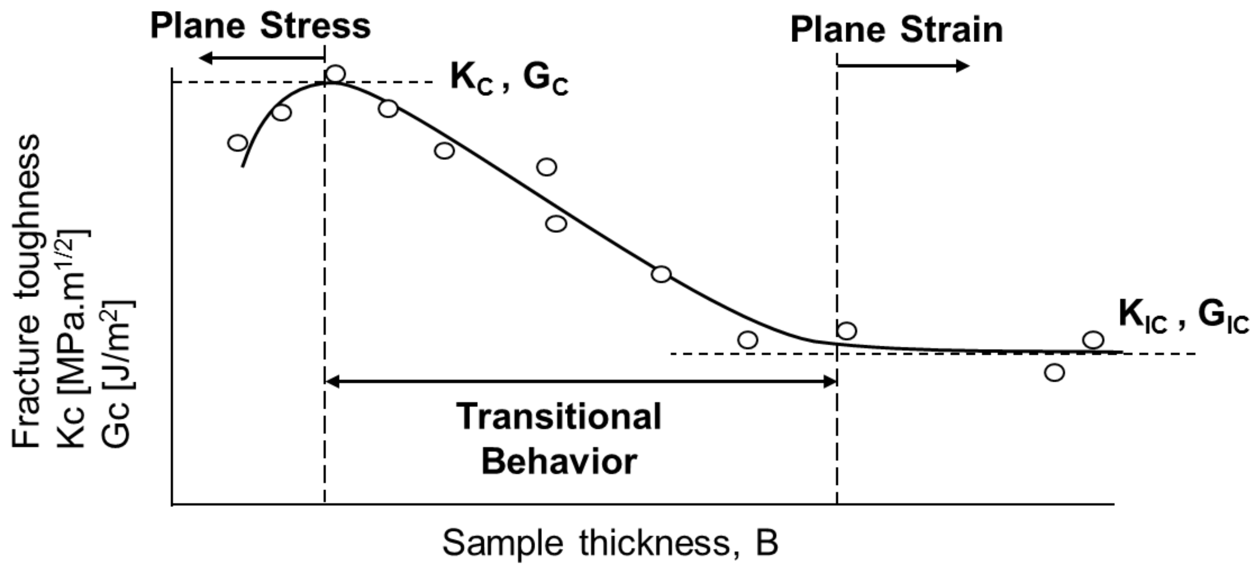
- But measured fracture toughness can depend on sample sizes, so we need to improve control of the test dimensions.





Role of specimen thickness

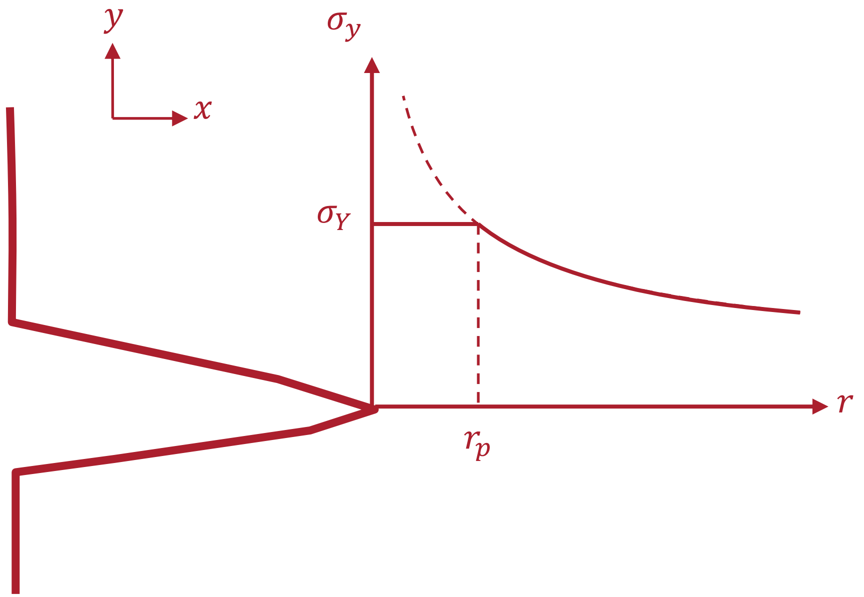
- Specimens having standard proportions but different in absolute size produce different values for K_C.
- The **stress states** depend on the specimen thickness (B) until the thickness exceeds some critical dimension.
- Once the thickness exceeds the critical dimension, the value of K_C becomes relatively constant, which is regarded as the true material property called the **plane-strain fracture toughness** K_{IC}.

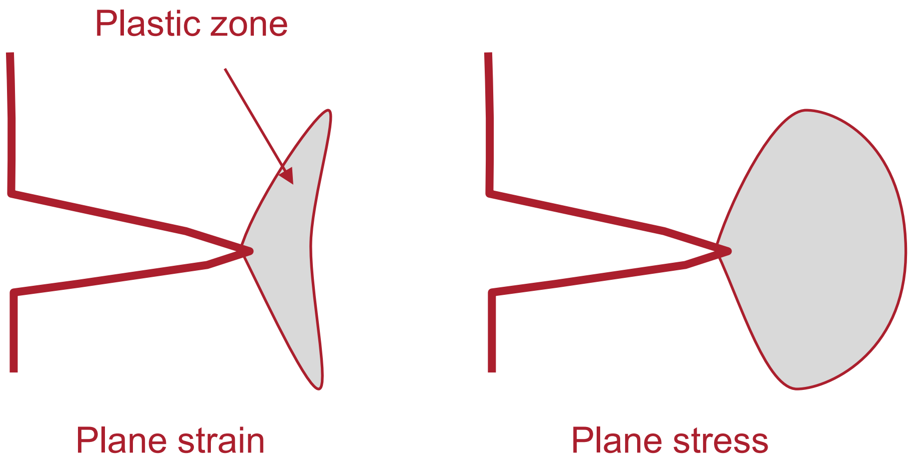


Why is the result so thickness dependent? – Constraint !

Plastic zone

- When a crack is loaded the stress increases exponentially, recall $1/\sqrt{r}$
- However a material will yield before the stress reaches this singularity
- This yield region is known as the **Plastic zone**, or **Fracture process zone**
- It is clear that this region depends on the nature of the stress ahead of the crack
- Plane Stress $r_p \approx$ 3x Plane Strain r_p



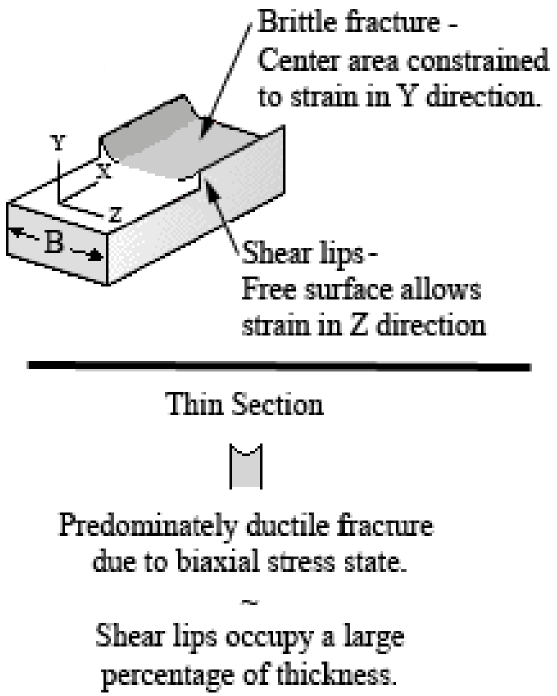


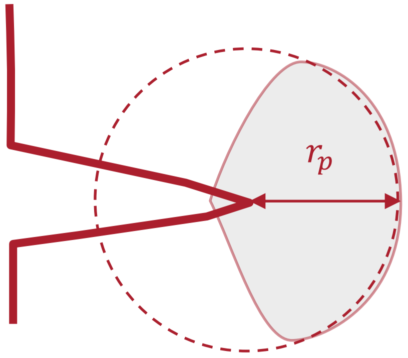
Key slide!

Plastic zone – Plane Stress

- For thin sheets there is no stress in thickness direction, i.e. $\sigma_{zz} = 0$
- This stress state is known as Plane Stress
- However there's still a ε_{zz} which results in a bi-axial stress state such that the material fractures in a characteristic ductile manner, with a 45° shear lip being formed at each free surface.
- The Plastic zone radius can be approximated:

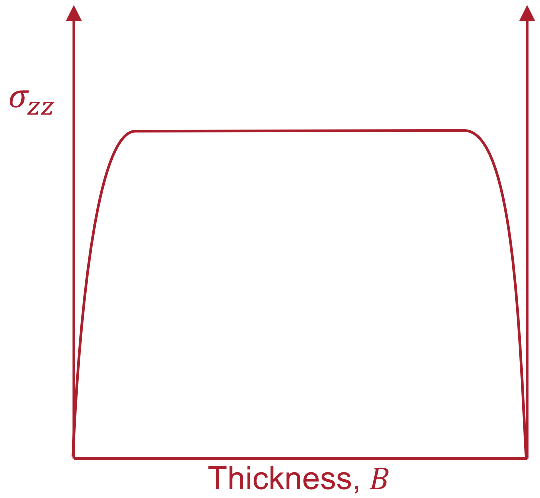
$$r_p = \frac{1}{2\pi}\left(\frac{K_C}{\sigma_y}\right)^2$$





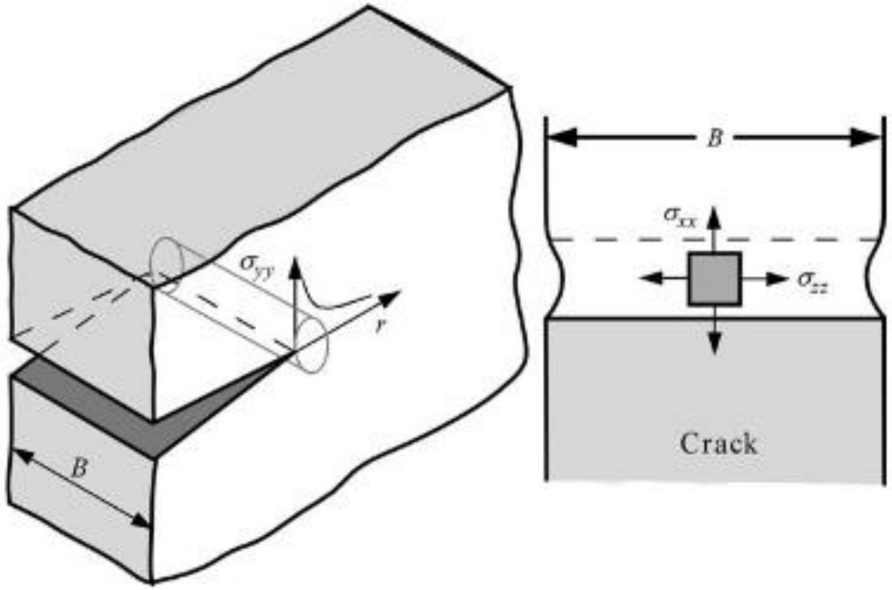
Plane stress

Plastic zone – Plane Strain



- For a thick material the stress in the thickness direction becomes significant, $\sigma_{zz} \gg 0$ i.e. there's a tri-axial stress state
- With the thickness contraction effectively being $\varepsilon_{zz} = 0$
- This stress state is known as Plane Strain
- The Plastic zone radius can be approximated:

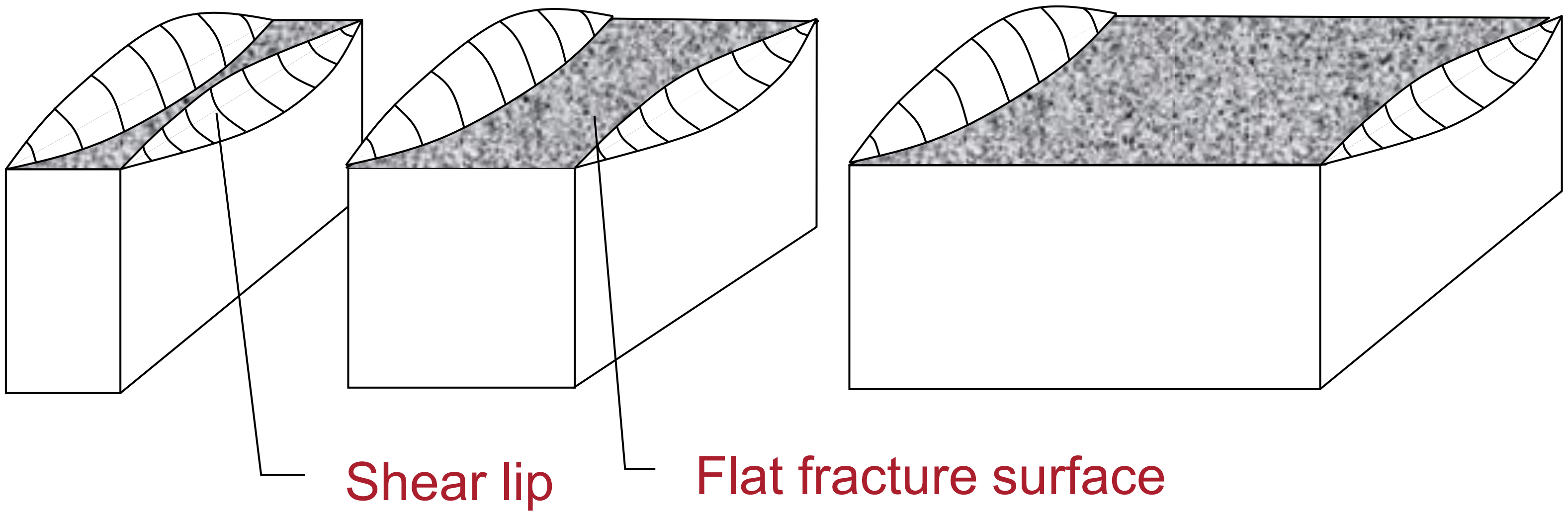
$$r_p = \frac{1}{6\pi}\left(\frac{K_{IC}}{\sigma_y}\right)^2$$



T.L. Anderson, Fracture Mechanics

Shear lips

- As the crack approaches the surface it tends to run out into a ‘shear lip’.
- For small samples, the shear lips are the dominant failure mechanism.
- At the thickest sample size the contribution of the shear lips is trivial and we have a valid measurement of the plane strain fracture toughness.



Example

- High Strength Steel

$$\sigma_{\text{YS}} = 1640\ MPa$$

$$K_{\text{IC}} = 50 MPa\sqrt{m}$$

- Plane strain r_p

$$r_p = \frac{1}{6\pi}\left(\frac{50}{1640}\right)^2 = 0.049mm$$

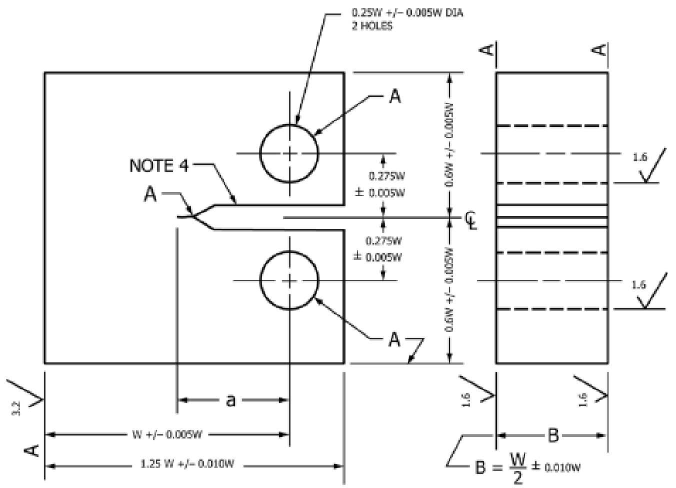
  - This is about twice the grain diameter, i.e. very localised plasticity

	Yield Strength		K_{Ic}	
Material	***MPa***	***ksi***	$MPa/\sqrt{m}$	$ksi/\sqrt{in}$
	Metals			
Aluminum Alloy[a] (7075-T651)	495	72	24	22
Aluminum Alloy[a] (2024-T3)	345	50	44	40
Titanium Alloy[a] (Ti-6Al-4V)	910	132	55	50
Alloy Steel[a] (4340 tempered @260HS)	1640	238	50.0	45.8
Alloy Steel[a] (4340 tempered @425HS)	1420	206	87.4	80.0
	Ceramics			
Concrete	—	—	0.2–1.4	0.18–1.27
Soda-Lime Glass	—	—	0.7–0.8	0.64–0.73
Aluminum Oxide	—	—	2.7–5.0	2.5–4.6
	Polymers			
Polystyrene (PS)	—	—	0.7–1.1	0.64–1.0
Poly(methyl methacrylate) (PMMA)	53.8–73.1	7.8–10.6	0.7–1.6	0.64–1.5
Polycarbonate (PC)	62.1	9.0	2.2	2.0

[a] **Source:** Reprinted with permission, *Advanced Materials and Processes,* ASM International, © 1990.

Mode I fracture toughness ASTM standard

- Compact Tension Specimen
- ASTM Standard
  - ASTM-E399-12 *“Standard Test Method for Linear-Elastic Plane-Strain Fracture Toughness K_{IC} of Metallic Materials”*



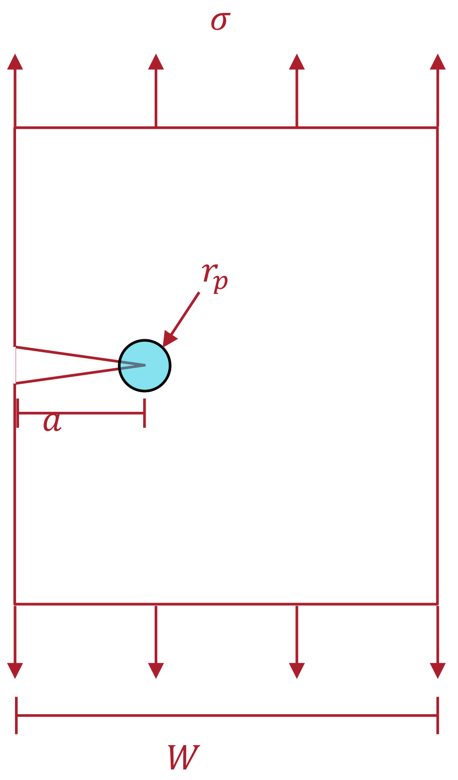
- Must ensure plane strain conditions - thick specimens

$$K_{IC} = \frac{P_C}{B\sqrt{W}} f\left(\frac{a}{W}\right)$$

$$f\left(\frac{a}{W}\right) = \frac{\left(2+\frac{a}{W}\right)\left[0.886+4.64\frac{a}{W} - 13.32\left(\frac{a}{W}\right)^2 + 14.72\left(\frac{a}{W}\right)^3 - 5.6\left(\frac{a}{W}\right)^4\right]}{\left(1-\frac{a}{W}\right)^{3/2}}$$

LEFM validity

- For LEFM equations to be valid the approximate plastic zone radius ahead of the crack must be $\boldsymbol{r_p < a/50}$
- Similarly $\boldsymbol{r_p < (W - a)/50}$
- An additional requirement for Plane strain crack is the thickness, *B* relative to the plastic zone radius must satisfy $\boldsymbol{r_p < B/50}$



$$\sigma_s = \frac{K_{IC}}{\beta\sqrt{\pi a}}$$

Example

Material	K_{IC} [MPa m$^{1/2}$]	σ_y [MPa]	Plane strain r_p	B_{min}	LEFM Valid
High strength Aluminium alloy	25	500	0.013mm	6mm	Yes
High strength Steel	60	1500	0.1mm	5mm	Yes
PMMA	1.5	50	0.05mm	2.5mm	Yes
Medium strength structural steel	80	450	1.7mm	85mm	Yes*

*LEFM valid for specimen sizes with minimum 85mm thickness, minimum crack of 85mm and minimum width of 170mm

To characterise K_{IC} large and thick specimen for a laboratory coupon standard is needed

Mode I fracture toughness ASTM standard

- When you load the material typical load curves look as shown

- What value do you take for the critical load P_c ?

- You must make correct engineering decision on this quantity as this will directly affect the fracture toughness

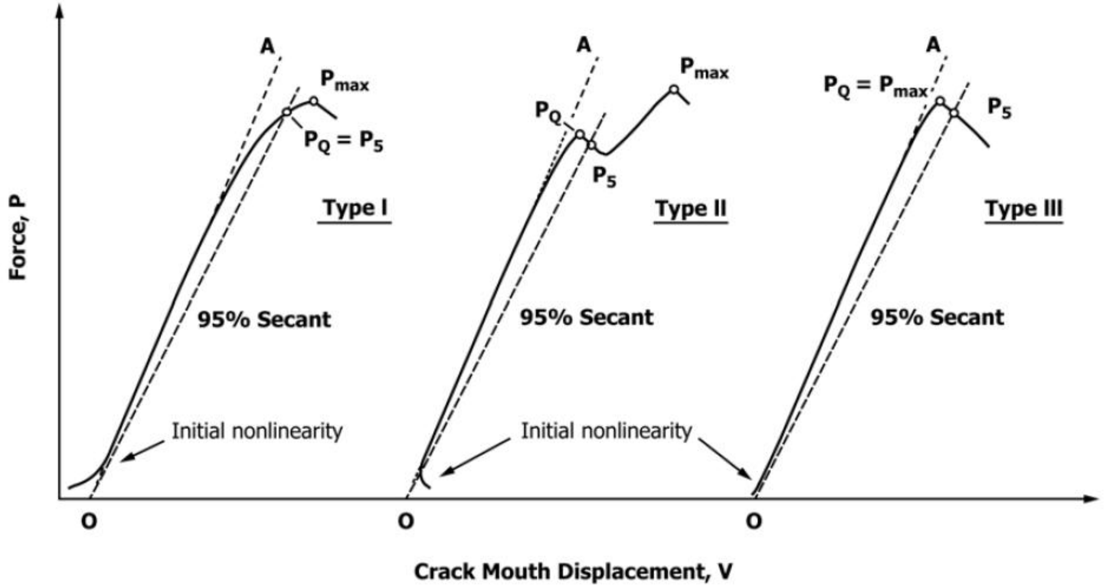
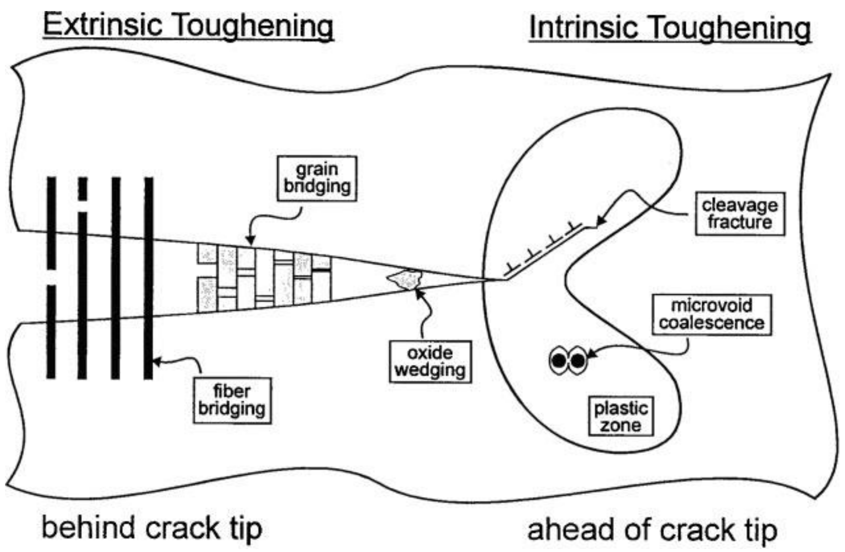


FIG. 7 Principal Types of Force-Displacement (CMOD) Records

Factors that influence fracture toughness

- Extrinsic and Intrinsic toughness mechanisms
  - Schematic illustration of the mutual competition between intrinsic mechanisms of damage which act ahead of the crack tip to promote crack advance and extrinsic mechanisms of crack-tip shielding that act mainly behind the crack tip to impede crack advance.
- Crack tip radius
  - Notching techniques
  - Does it create small enough crack tip?
  - Is it uniform, no external damage to specimen?



To recap

- Cracks and defects exist in all engineering structures
- Cracks can be represented as flat free surfaces in regions where tri-axial stresses dominate – which makes Linear Elastic Fracture Mechanics assumption correct
- This means the stress field can be characterised and the K_{IC} value at the onset of crack extension can be correctly accepted as a material property (plane-strain fracture toughness)
- Factors have been calculated for a variety of crack types, the most commonly used being the interior crack in a plate of infinite width and an edge crack in a plate of semi-infinite width.

Mode I fracture toughness energy approach

- Energy conservation

$$\frac{d}{dA}(U + W - F) = 0$$

$$G = \frac{dW}{Bda} = \frac{d}{Bda}(F - U) \qquad U = \frac{1}{2}P\delta \qquad F = Pd\delta$$

- Under **fixed displacement**:

$$F = 0$$

- Energy dissipation

$$G = -\frac{dU}{Bda} = -\frac{\delta}{2B}\frac{dP}{da}$$

- Under **fixed load**:

$$G = \frac{dF}{Bda} - \frac{dU}{Bda}$$

- Energy absorption

$$G = \frac{P}{B}\frac{d\delta}{da} - \frac{P}{2B}\frac{d\delta}{da} = \frac{P}{2B}\frac{d\delta}{da}$$

Mode I fracture toughness energy approach

- Compliance is the inverse of stiffness
  - If something is compliant - It will deform more

$$C = \frac{\delta}{P} \Rightarrow CP = \delta$$

- Under **fixed displacement**:

$$\frac{d(CP)}{da} = \frac{d\delta}{da} \qquad P\frac{dC}{da} + C\frac{dP}{da} = 0 \qquad \frac{dP}{da} = -\frac{P}{C}\frac{dC}{da}$$

- Energy dissipation

$$G = -\frac{\delta}{2B}\frac{dP}{da} = -\frac{CP}{2B}\frac{dP}{da} \Rightarrow \boxed{G = \frac{P^2}{2B}\frac{dC}{da}}$$

- Under **fixed load:**

$$\frac{d(CP)}{da} = \frac{d\delta}{da} \qquad P\frac{dC}{da} = \frac{d\delta}{da}$$

- Energy absorption

$$G = \frac{P}{2B}\frac{d\delta}{da} \Rightarrow \boxed{G = \frac{P^2}{2B}\frac{dC}{da}}$$

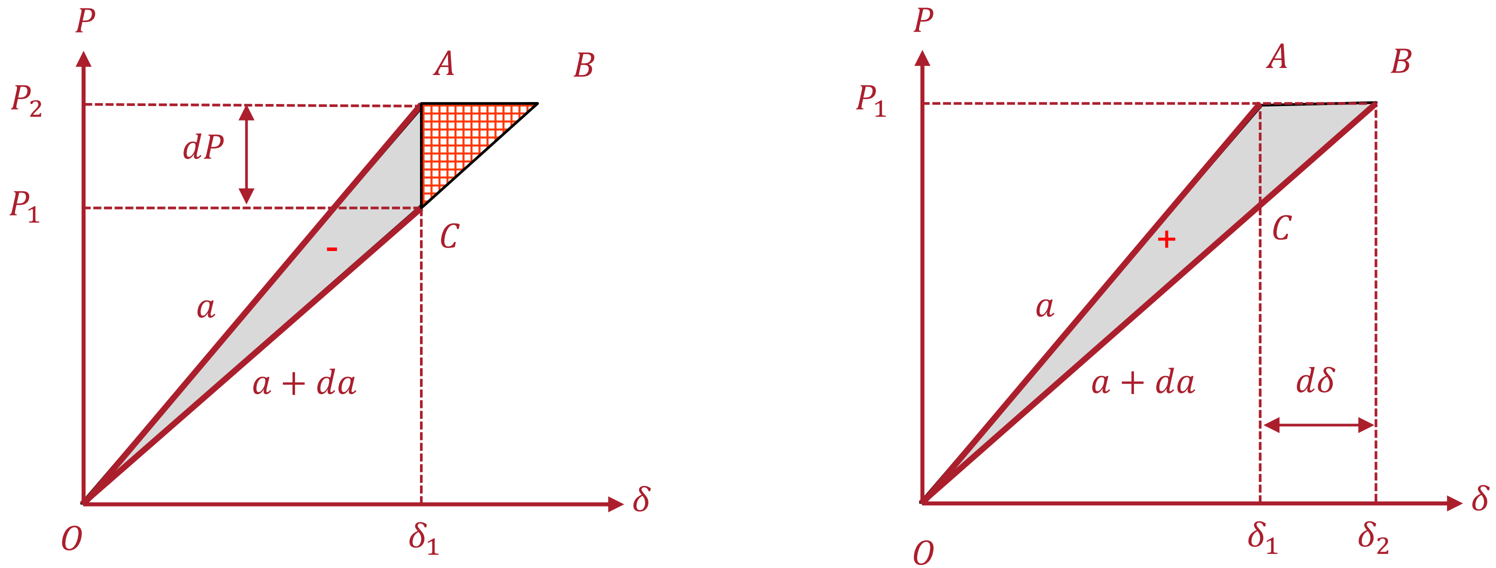
Strain energy released rate G under both **fixed displacement** and **fixed load**, can now be written in the same form.

Strain energy, compliance and crack growth

Key slide!

$$G = \frac{P^2}{2B}\frac{dC}{da}$$

- This form of the equation is very useful as now we can relate the fracture energy to the compliance of the system



Under **fixed displacement** Under **fixed load**

Another way to explain - *G* is the area between the loading curve

Elastic Plastic Fracture Mechanics (EPFM)

- There will come a time where it is required to characterize a ductile or highly tough material.
- To do this, a procedure has been developed called the Crack Tip Opening Displacement (CTOD)
- Crack tip plasticity makes the crack behave as if it where longer, $a+r_p$
- This method calculates the displacement at the physical crack tip

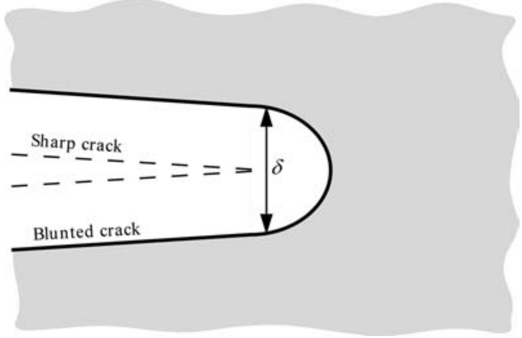


FIGURE 3.1 Crack-tip-opening displacement (CTOD). An initially sharp crack blunts with plastic deformation, resulting in a finite displacement (δ) at the crack tip.

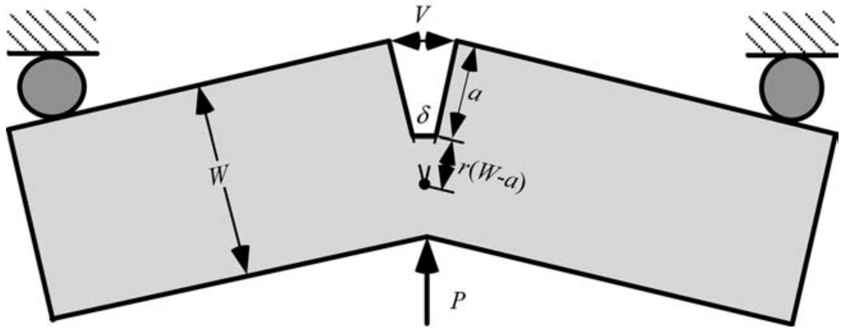


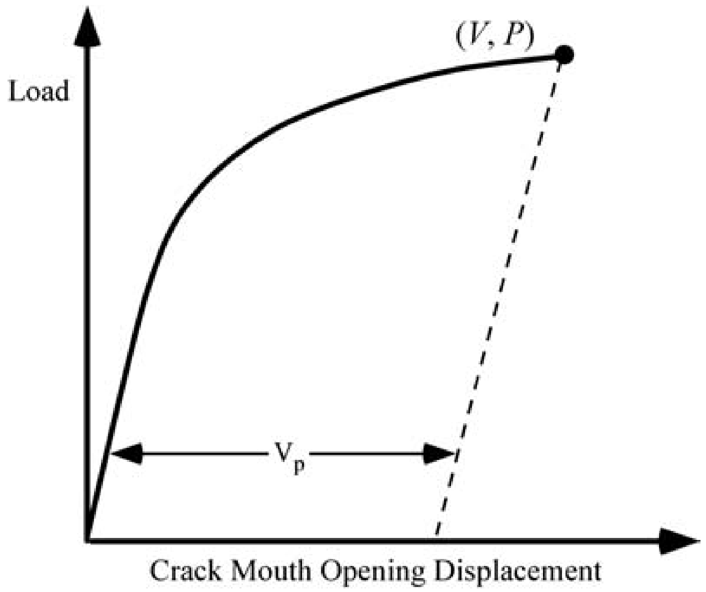
FIGURE 3.5 The hinge model for estimating CTOD from three-point bend specimens.

T.L. Anderson, Fracture Mechanics

Elastic Plastic Fracture Mechanics (EPFM)

- Crack Tip Opening Displacement (CTOD) Procedure
- Separating the elastic and plastic components of the CTOD we get:

$$\delta = \delta_{el} + \delta_p = \frac{K_I^2}{m\sigma_{YS}E'} + \frac{r_p(W-a)V_p}{r_p(W-a)+a}$$



Where *m* is a dimensionless constant ~1 for plane strain and ~2 for plane stress

r_p is a rotational factor 0.44 for typical materials and test specimen geometries

E' is effective modulus:

$$E' = E \quad \text{for plane stress} \qquad E' = \frac{E}{1-\nu^2} \quad \text{for plane strain}$$

- Elastic strain energy dissipation or absorption determines the fracture energy J_C.

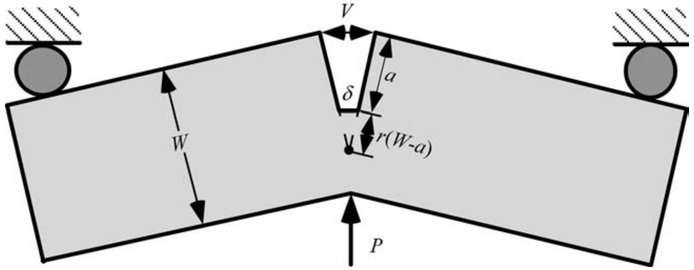


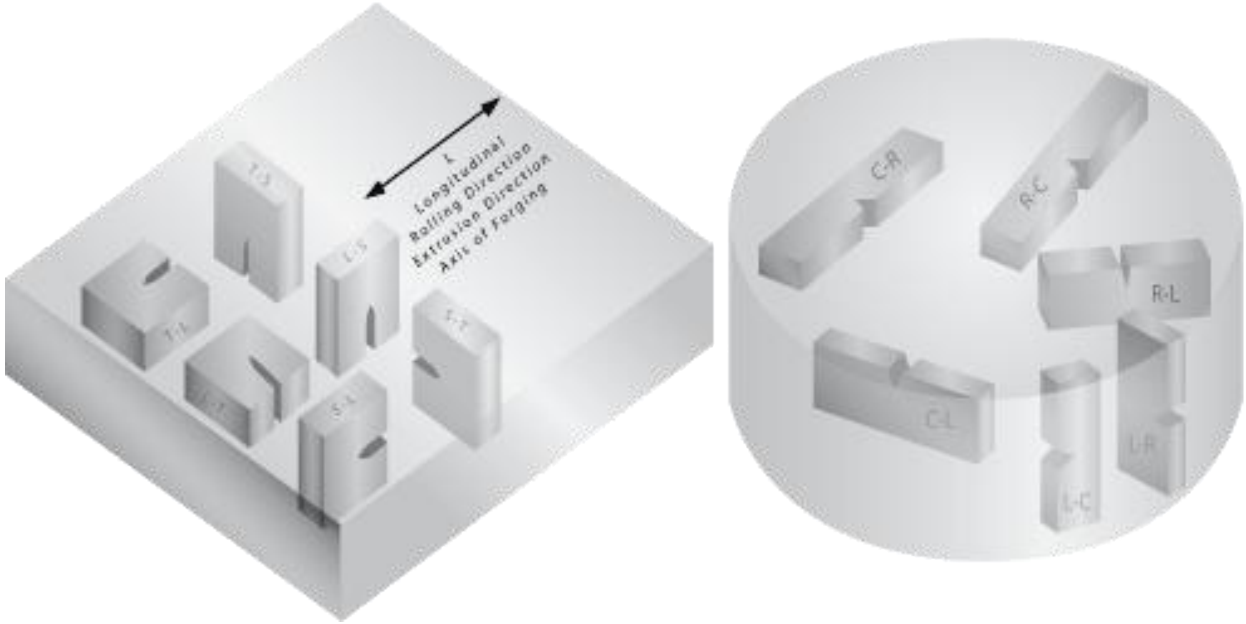
FIGURE 3.5 The hinge model for estimating CTOD from three-point bend specimens.

T.L. Anderson, Fracture Mechanics

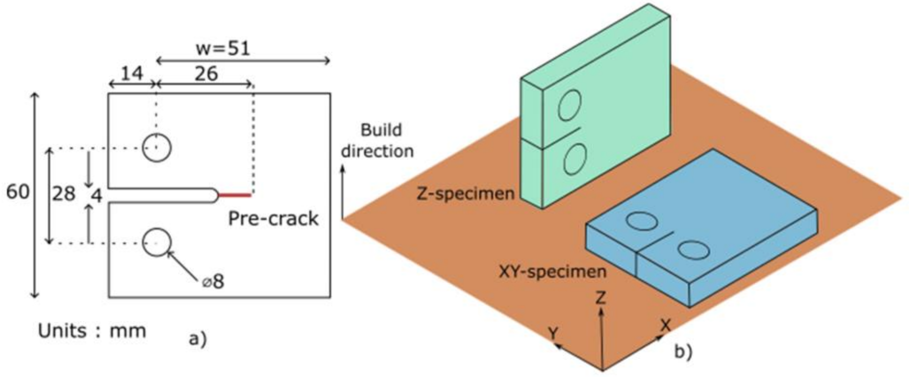
Isotropic Materials vs Anisotropic Materials

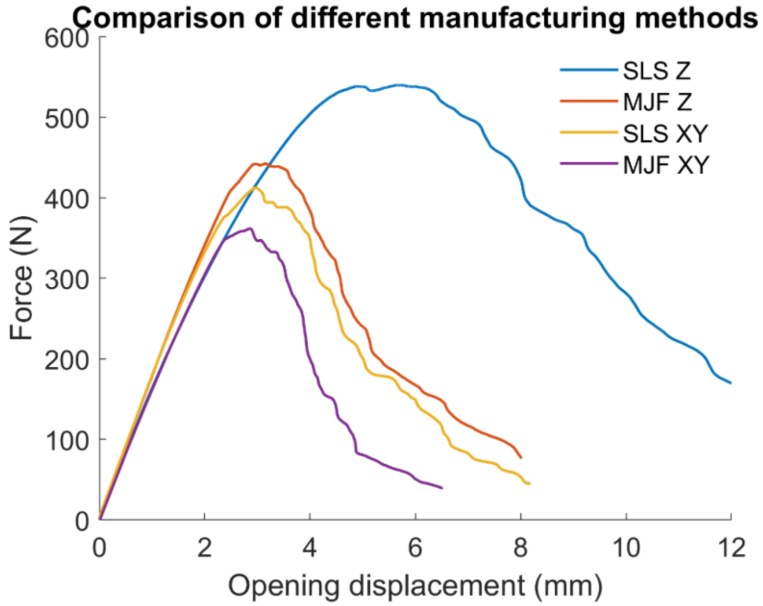
Isotropic material

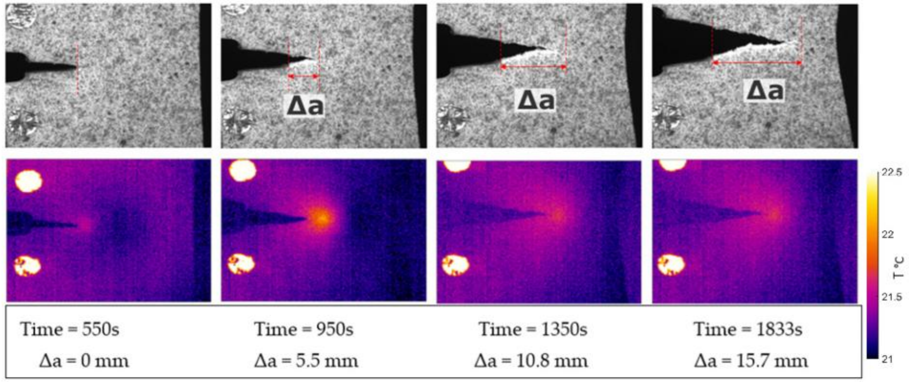
- Remember an Isotropic material (steel, aluminium) has a stress-strain relationship that is independent of orientation of the coordinate system at that point,
  - Same elastic properties (E, ν) in all directions

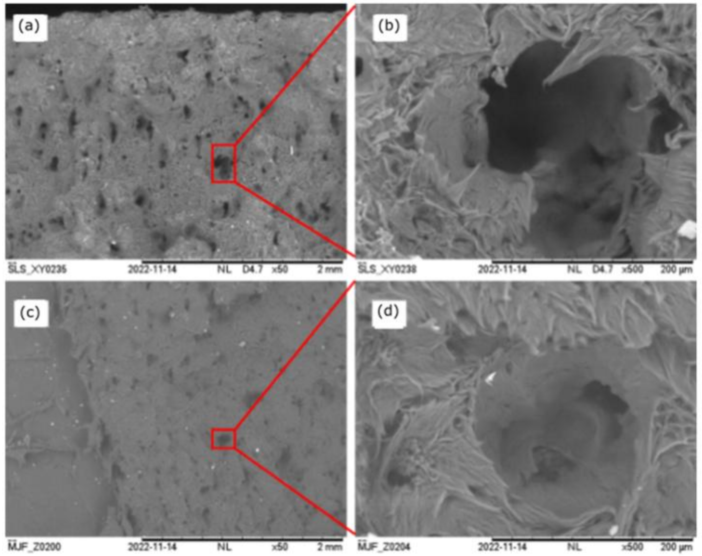


Build orientation effects

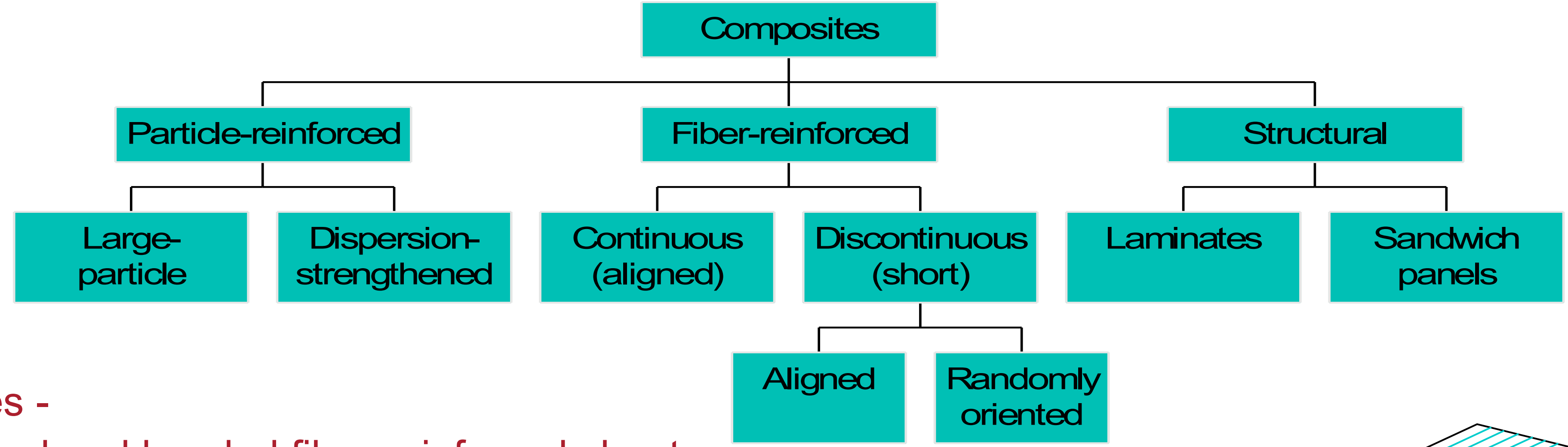




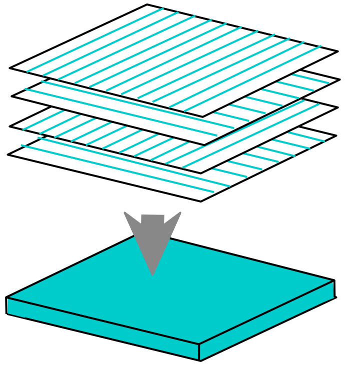




Anisotropic material

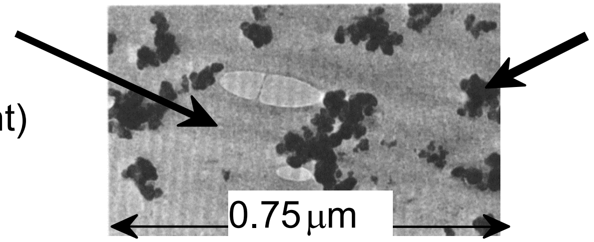


- Laminates -
  - stacked and bonded fiber-reinforced sheets
  - stacking sequence: e.g., 0º/90º
  - benefit: balanced in-plane stiffness



- Automobile tire rubber

matrix: rubber (compliant)



particles: carbon black (stiff)

Why composites?

- The main advantages of composites

  - High stiffness

  - High strength

  - Light weight

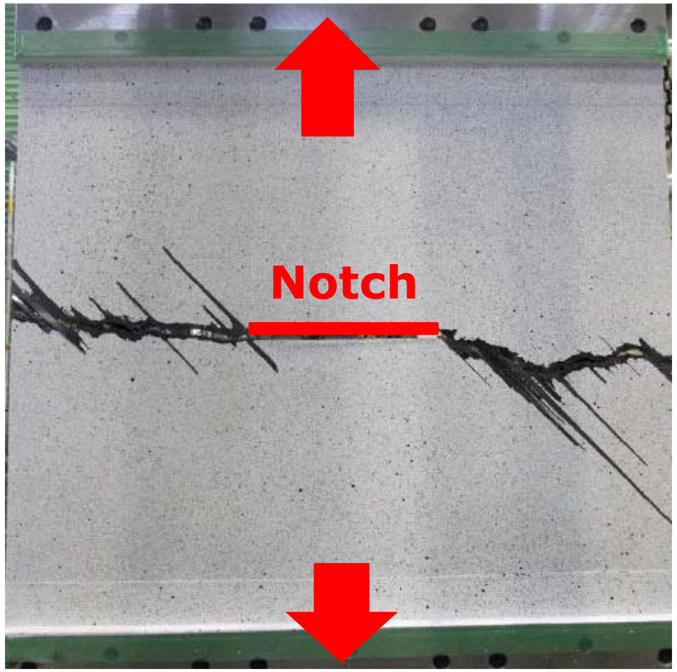
- Understanding failure of composite structures

  - Strength scaling

  - Notch sensitivity

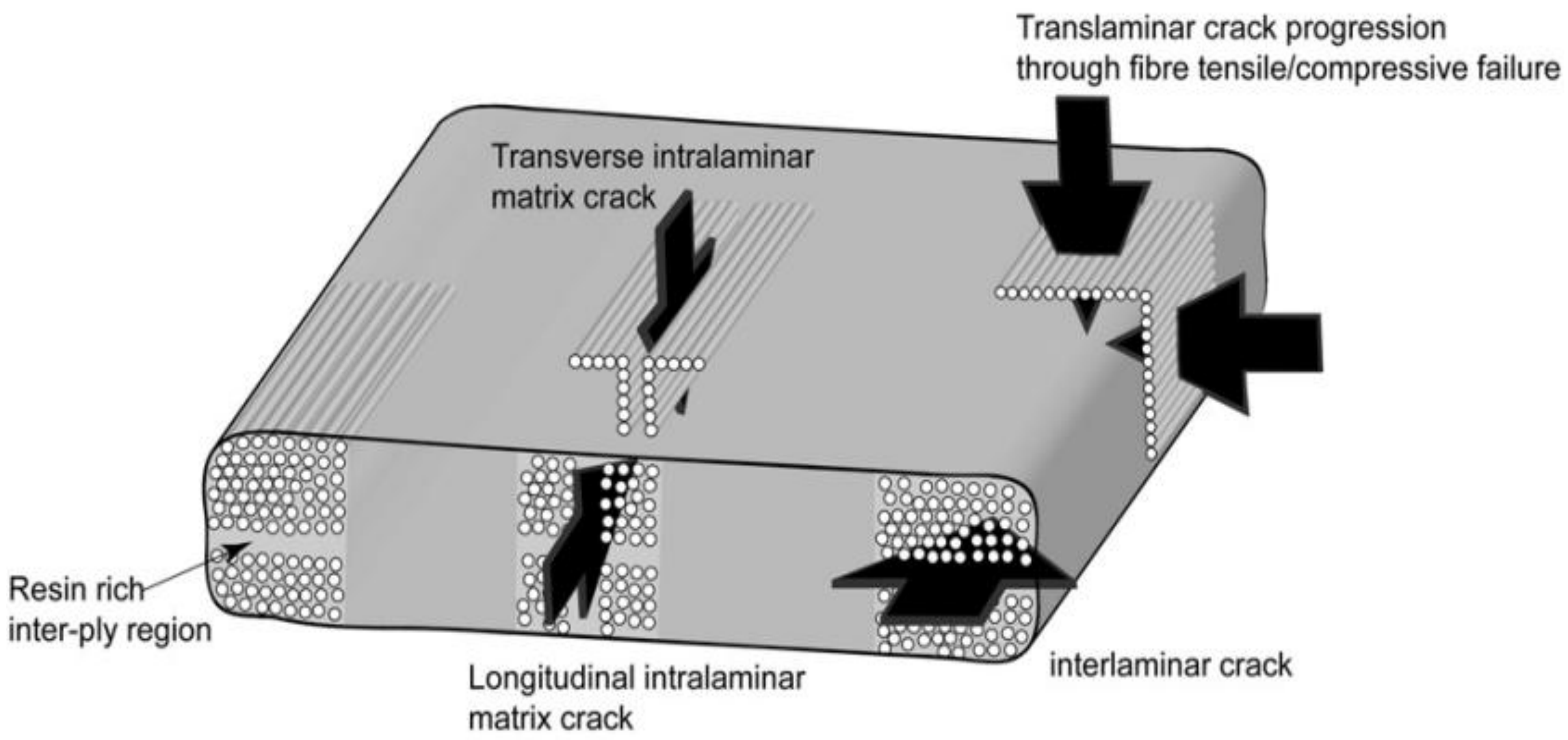
  - Trans-laminar fracture

Ferrari F40



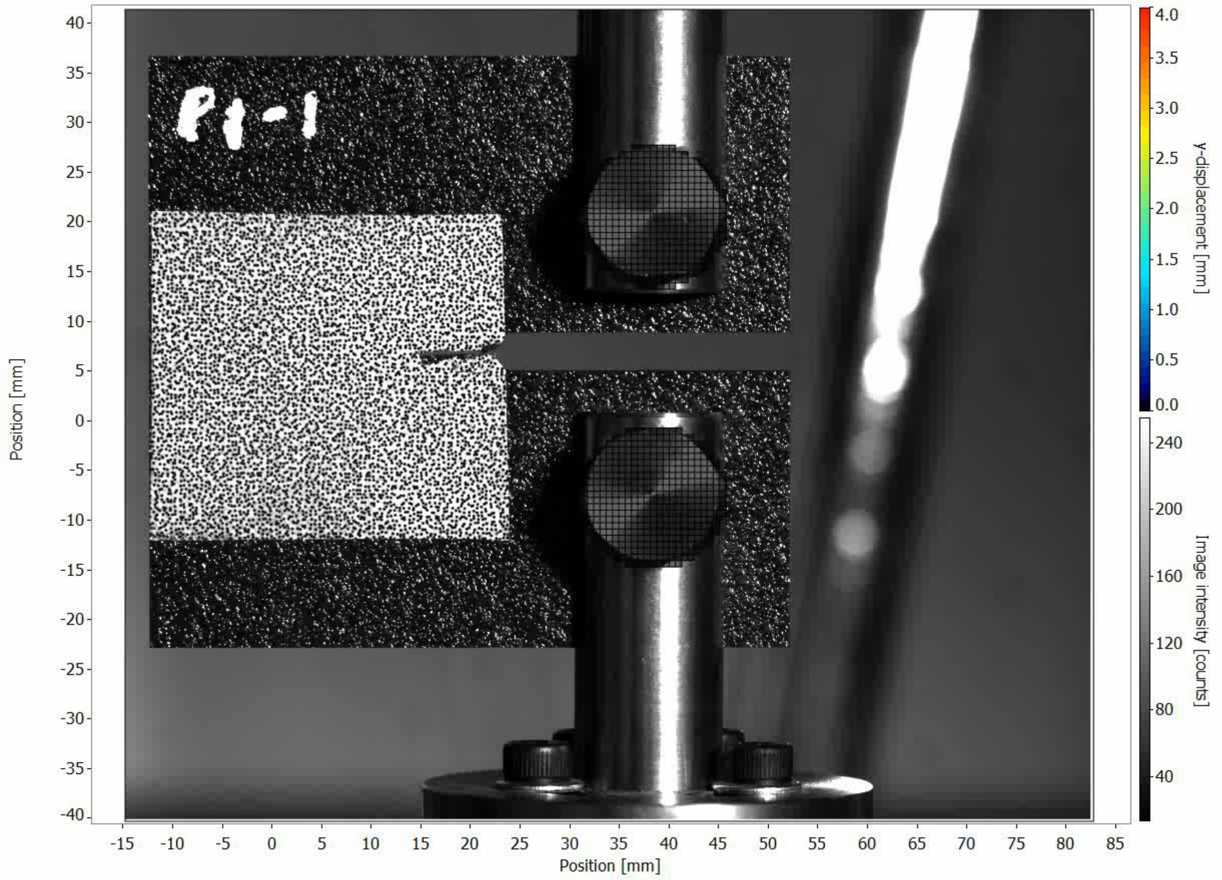
Failure of composite laminates

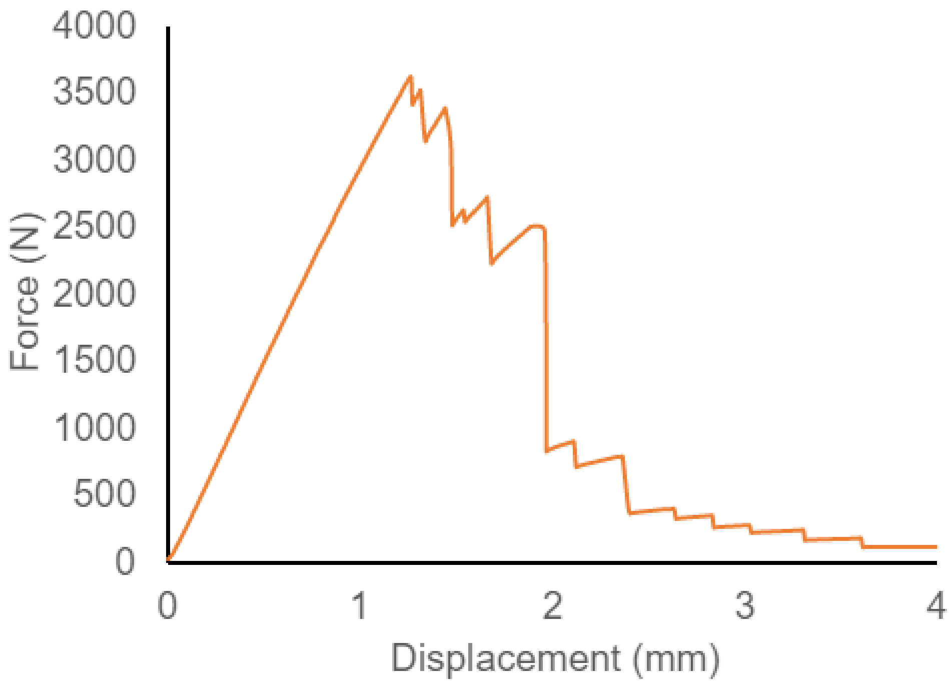
- A general lack of knowledge about how damage initiates and progresses

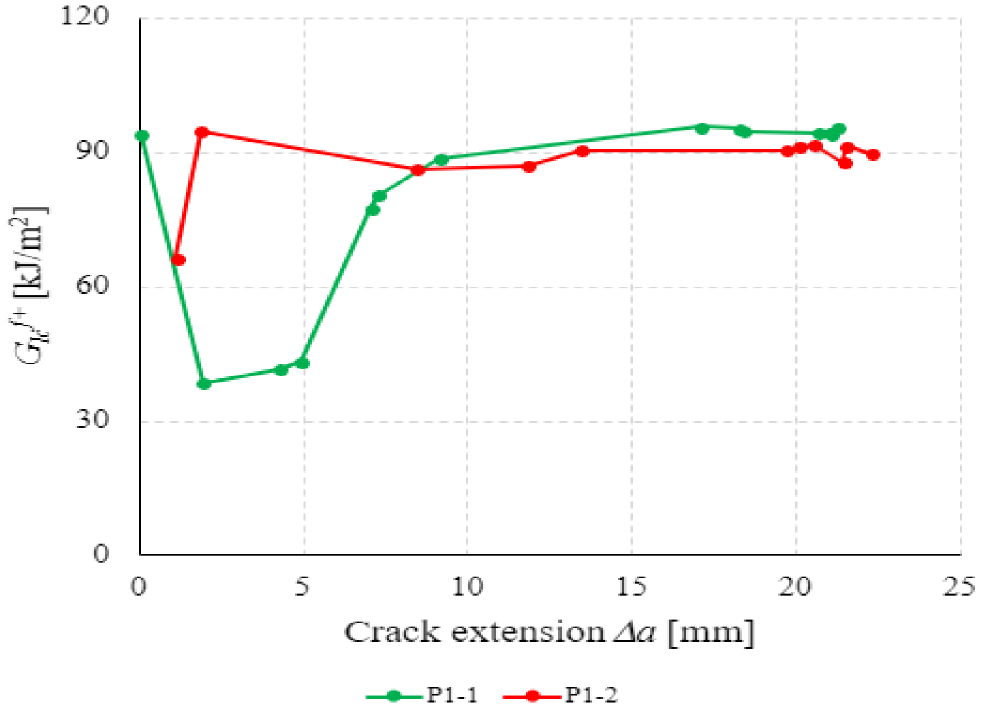


Overview of ply-level failure

Test example - Composites

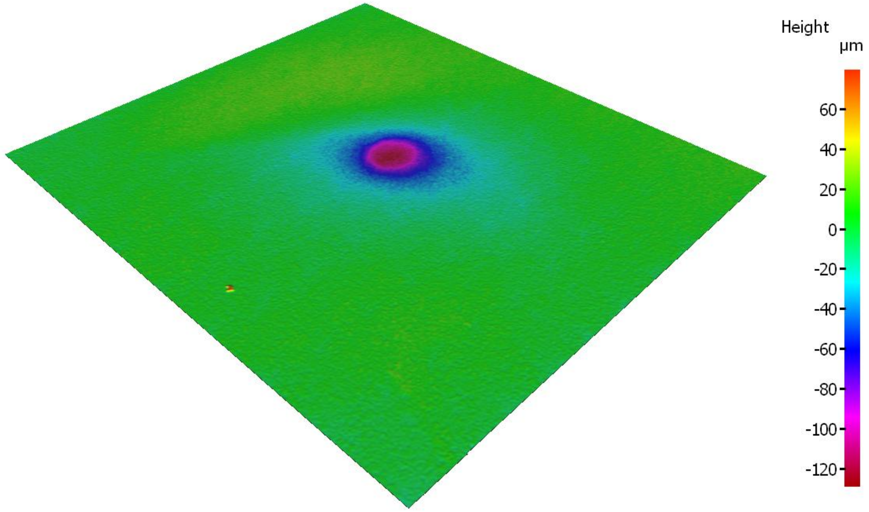


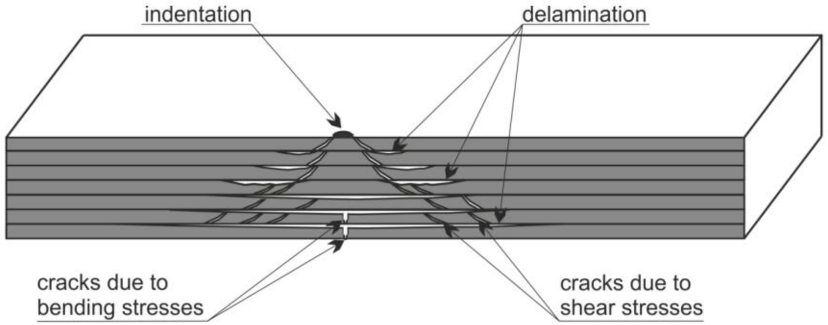


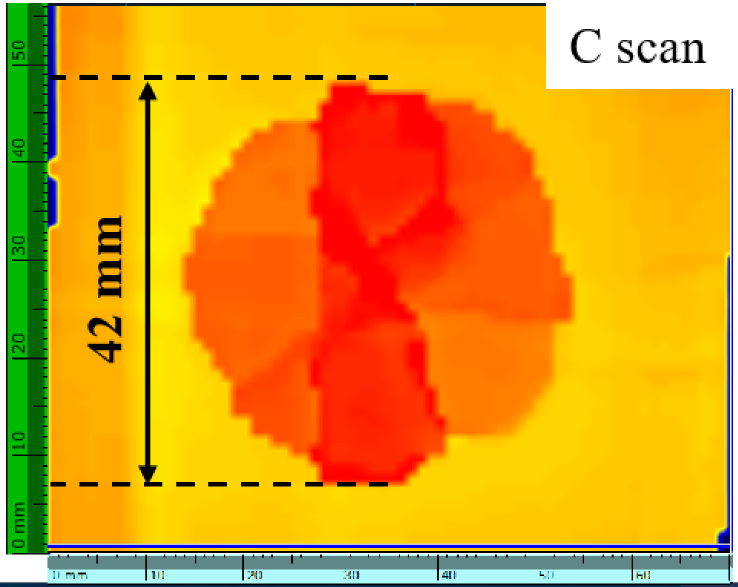


Bonus content: Structural Health Monitoring (SHM)

bird strike



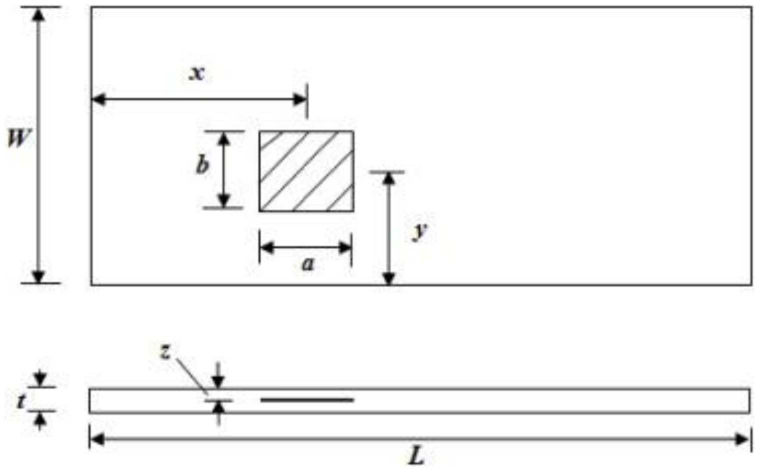


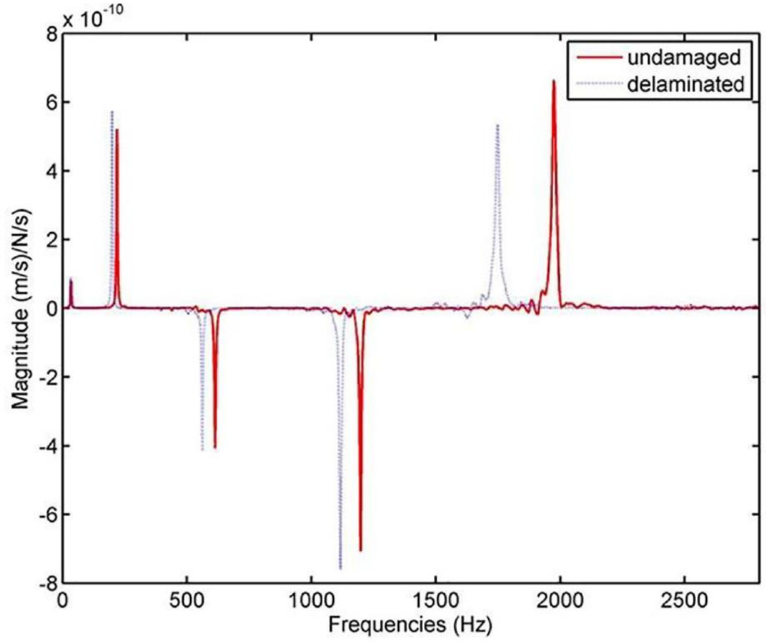


- Impact loading during service life causes damage
- Delamination, matrix cracking and fibre failure

Forward problem

- Known location, size and interface of damage
- Evaluate change in natural frequencies

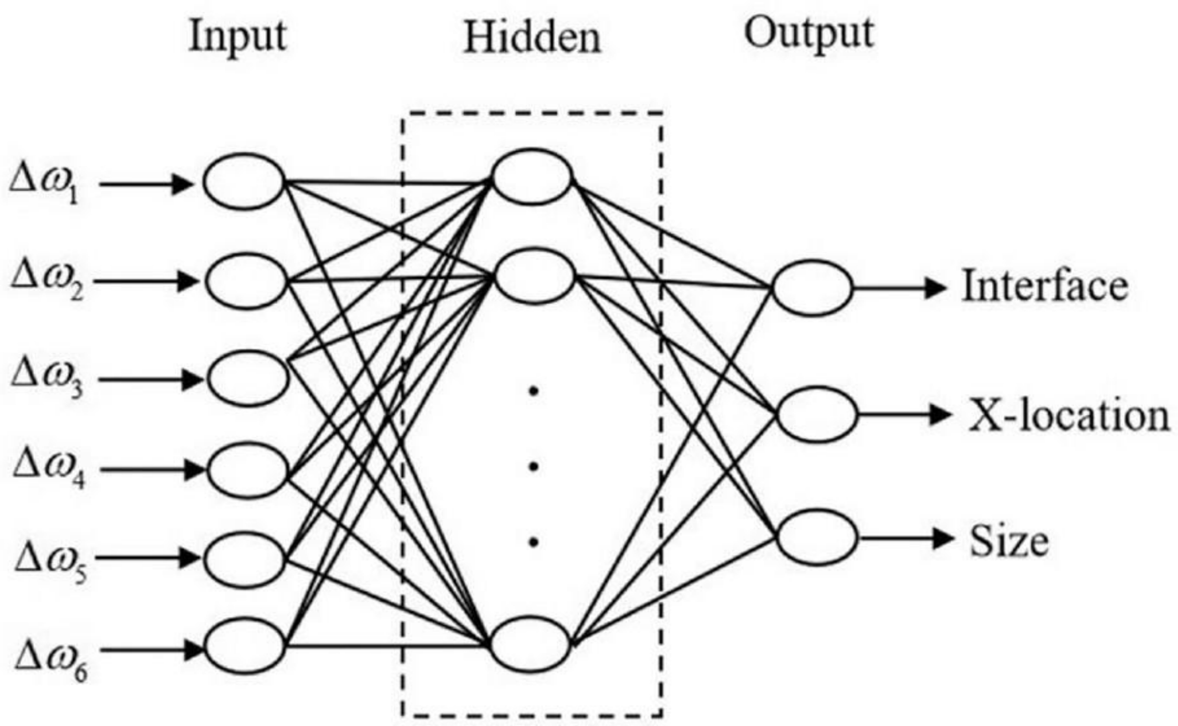




Zhang Z et al.. Vibration-based assessment of delaminations in FRP composite plates. Composites Part B: Engineering. 2018 Jul 1;144:254-66.

Inverse problem

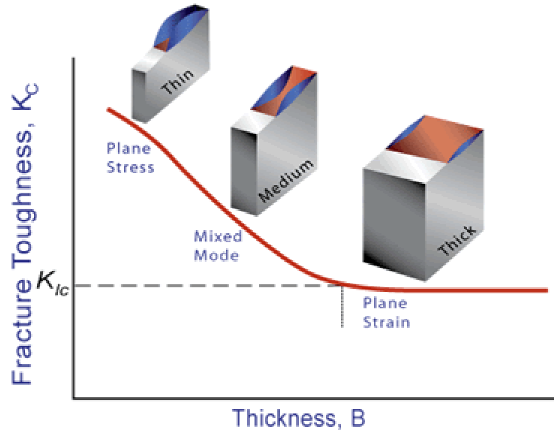
- Predict location, size and interface of damage from frequency shifts



He M et al. A comparison of machine learning algorithms for assessment of delamination in fiber-reinforced polymer composite beams. Structural Health Monitoring. 2021

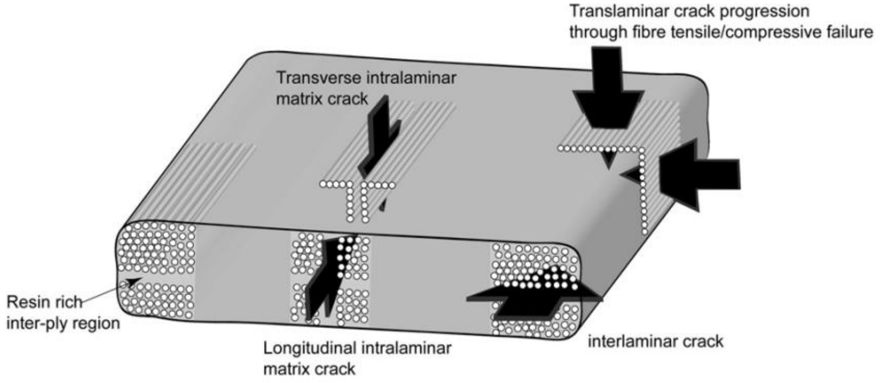
Concluding remarks

- It is important to understand the role of material thickness and application of loading direction in establishing fracture toughness

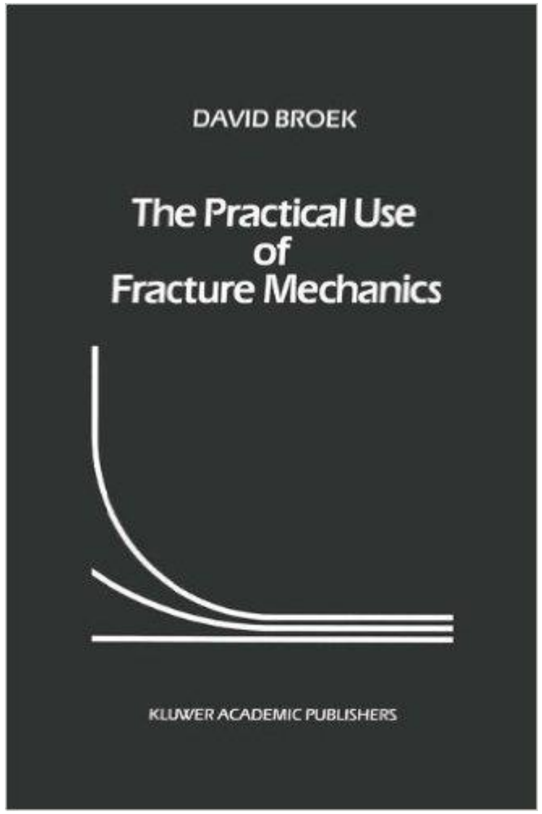


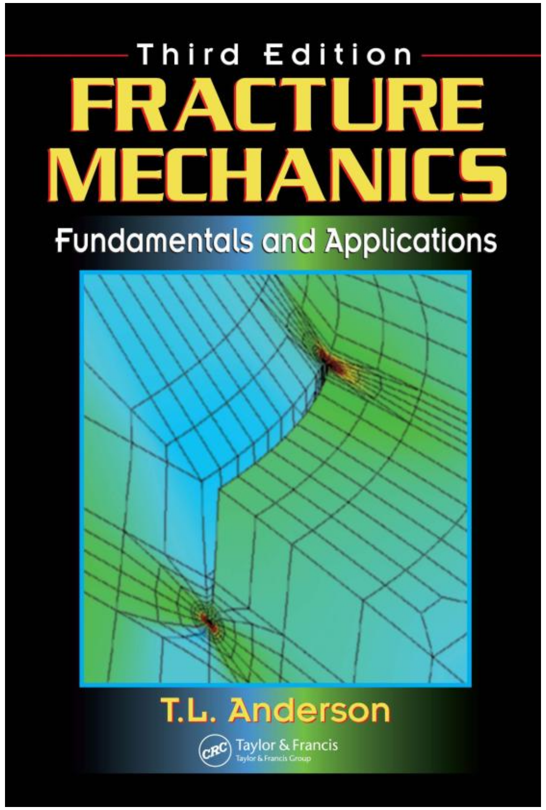
- This has been demonstrated via the introduction of the following concepts:

  - Plane Strain and Plane Stress
  - LEFM vs. EPFM
  - Isotropic vs. Anisotropic Toughness



Recommended Books





- The Practical Use of Fracture Mechanics
  - D. Broek
- Fracture Mechanics Fundamentals and Applications – T.L. Anderson (3rd Edition)