CHAPTER

Cables

5

Flexible cables have been used to form structural systems for many centuries. Some of the earliest man-made structures of any size were hanging bridges constructed from jungle vines and creepers, and spanning ravines and rivers. In European literature the earliest description of an iron suspension bridge was published by Verantius in 1607, while ropes have been used in military bridging from at least 1600. In modern times, cables formed by binding a large number of steel wires together are employed in bridge construction where the bridge deck is suspended on hangers from the cables themselves. The cables in turn pass over the tops of towers and are fixed to anchor blocks embedded in the ground; in this manner large, clear spans are achieved. Cables are also used in cable-stayed bridges, as part of roof support systems, for prestressing in concrete beams and for guyed structures such as pylons and television masts.

Structurally, cables are extremely efficient because they make the most effective use of structural material in that their loads are carried solely through tension. Therefore, there is no tendency for buckling to occur either from bending or from compressive axial loads (see Chapter 21). However, many of the structures mentioned above are statically indeterminate to a high degree. In other situations, particularly in guyed towers and cable-stayed bridges, the extension of the cables affects the internal force system and the analysis becomes non-linear. Such considerations are outside the scope of this book so that we shall concentrate on cables in which loads are suspended directly from the cable.

Two categories of cable arise; the first is relatively lightweight and carries a limited number of concentrated loads, while the second is heavier with a more uniform distribution of load. We shall also examine, in the case of suspension bridges, the effects of different forms of cable support at the towers.

5.1 Lightweight cables carrying concentrated loads

In the analysis of this type of cable we shall assume that the self-weight of the cable is negligible, that it can only carry tensile forces and that the extension of the cable does not affect the geometry of the system. We shall illustrate the method by examples.

EXAMPLE 5.1

The cable shown in Fig. 5.1 is pinned to supports at A and B and carries a concentrated load of 10 kN at a point C. Calculate the tension in each part of the cable and the reactions at the supports.

Since the cable is weightless the lengths AC and CB are straight. The tensions T_{CA} and T_{CB} in the parts AC and CB, respectively, may be found by considering the equilibrium of the forces acting at C where, from Fig. 5.1, we see that

$$\alpha = \tan^{-1} 1/3 = 18.4^\circ \quad \beta = \tan^{-1} 1/2 = 26.6^\circ$$

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EXAMPLE 5.1 CONT'D

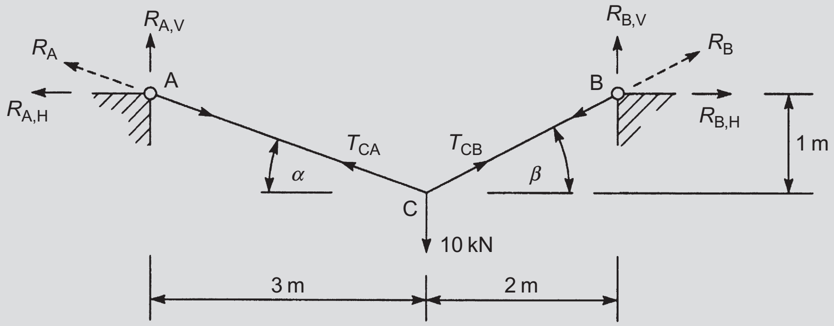

FIGURE 5.1

Lightweight cable carrying a concentrated load.

Resolving forces in a direction *perpendicular* to CB (thereby eliminating T_{CB}) we have, since $\alpha+\beta=45°$

$$T_{CA}\cos 45° - 10\cos 26.6° = 0$$

from which

$$T_{CA} = 12.6\,\text{kN}$$

Now resolving forces horizontally (or alternatively vertically or perpendicular to CA) gives

$$T_{CB}\cos 26.6° - T_{CA}\cos 18.4° = 0$$

so that

$$T_{CB} = 13.4\,\text{kN}$$

Since the bending moment in the cable is everywhere zero we can take moments about B (or A) to find the vertical component of the reaction at A, $R_{A,V}$ (or $R_{B,V}$) directly. Then

$$R_{A,V}\times 5 - 10\times 2 = 0 \tag{i}$$

so that

$$R_{A,V} = 4\,\text{kN}$$

Now resolving forces vertically for the complete cable

$$R_{B,V} + R_{A,V} - 10 = 0 \tag{ii}$$

which gives

$$R_{B,V} = 6\,\text{kN}$$

From the horizontal equilibrium of the cable the horizontal components of the reactions at A and B are equal, i.e. $R_{A,H}=R_{B,H}$. Thus, taking moments about C for the forces to the left of C

$$R_{A,H}\times 1 - R_{A,V}\times 3 = 0 \tag{iii}$$

from which

$$R_{A,H} = 12\,\text{kN}(=R_{B,H})$$

Note that the horizontal component of the reaction at A, $R_{A,H}$, would be included in the moment equation (Eq. (i)) if the support points A and B were on different levels. In this case Eqs (i) and (iii) could be solved simultaneously for $R_{A,V}$ and $R_{A,H}$. Note also that the tensions T_{CA} and T_{CB} could be found from the components of the support reactions since the resultant reaction at each support, R_A at A and R_B at B, must be equal and opposite in direction to the tension in the cable otherwise the cable would be subjected to shear forces, which we have assumed is not possible. Hence

$$T_{CA} = R_A = \sqrt{R_{A,V}^2 + R_{A,H}^2} = \sqrt{4^2 + 12^2} = 12.6\,\text{kN}$$

$$T_{CB} = R_B = \sqrt{R_{B,V}^2 + R_{B,H}^2} = \sqrt{6^2 + 12^2} = 13.4\,\text{kN}$$

as before.

In Ex. 5.1 the geometry of the loaded cable was specified. We shall now consider the case of a cable carrying more than one load. In the cable in Fig. 5.2(a), the loads W_1 and W_2 at the points C and D produce a different deflected shape to the loads W_3 and W_4 at C and D in Fig. 5.2(b). The analysis is then affected by the change in geometry as well as the change in loading, a different situation to that in beam and truss analysis. The cable becomes, in effect, a mechanism and changes shape to maintain its equilibrium; the analysis then becomes non-linear and therefore statically indeterminate. However, if the geometry of the deflected cable is partially specified, say the maximum deflection or sag is given, the system becomes statically determinate.

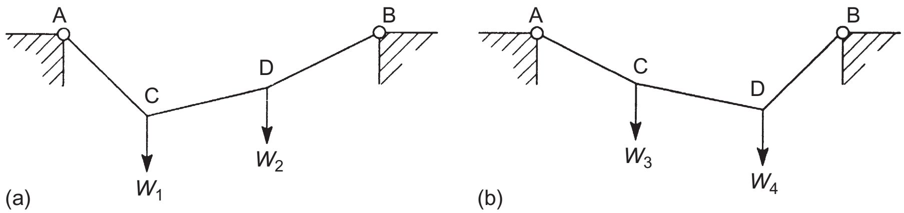

FIGURE 5.2

Effect on cable geometry of load variation.

EXAMPLE 5.2

Calculate the tension in each of the parts AC, CD and DB of the cable shown in Fig. 5.3.

There are different possible approaches to the solution of this problem. For example, we could investigate the equilibrium of the forces acting at the point C and resolve horizontally and vertically. We would then obtain two equations in which the unknowns would be T_{CA}, T_{CD}, α and β. From the geometry of the cable $\alpha = \tan^{-1}(0.5/1.5) = 18.4°$ so that there would be three unknowns remaining. A third equation could be obtained by examining the moment equilibrium of the length AC of the cable about A, where the moment is zero since the cable is flexible. The solution of these three simultaneous equations would be rather tedious so that a simpler approach is preferable.

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EXAMPLE 5.2 CONT'D

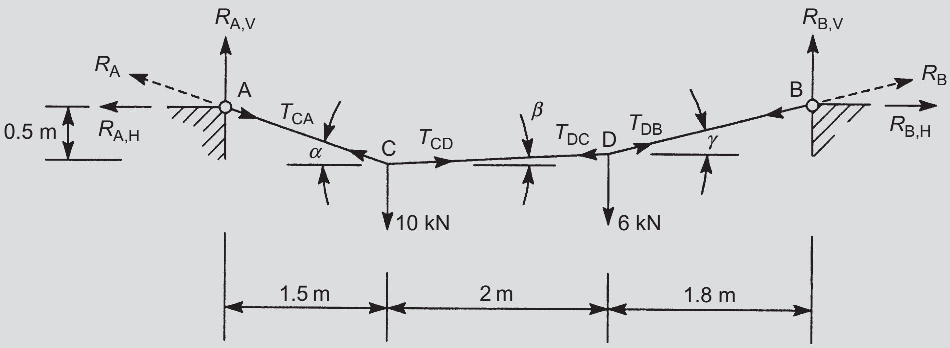

FIGURE 5.3

Cable of Ex. 5.2.

In Ex. 5.1 we saw that the resultant reaction at the supports is equal and opposite to the tension in the cable at the supports. Therefore, by determining $R_{A,V}$ and $R_{A,H}$ we can obtain T_{CA} directly. Hence, taking moments about B we have

$$R_{A,V} \times 5.3 - 10 \times 3.8 - 6 \times 1.8 = 0$$

from which

$$R_{A,V} = 9.2\,\text{kN}$$

Since the cable is perfectly flexible the internal moment at any point is zero. Therefore, taking moments of forces to the left of C about C gives

$$R_{A,H} \times 0.5 - R_{A,V} \times 1.5 = 0$$

so that

$$R_{A,H} = 27.6\,\text{kN}$$

Alternatively we could have obtained $R_{A,H}$ by using the fact that the resultant reaction, R_A, at A is in line with the cable at A, i.e. $R_{A,V}/R_{A,H} = \tan\alpha = \tan 18.4°$, which gives $R_{A,H} = 27.6$ kN as before. Having obtained $R_{A,V}$ and $R_{A,H}$, T_{CA} follows. Thus

$$T_{CA} = R_A = \sqrt{R_{A,H}^2 + R_{A,V}^2} = \sqrt{27.6^2 + 9.2^2}$$

i.e.

$$T_{CA} = 29.1\,\text{kN}$$

From a consideration of the vertical equilibrium of the forces acting at C we have

$$T_{CD}\sin\beta + T_{CA}\sin\alpha - 10 = T_{CD}\sin\beta + 29.1\sin 18.4° - 10 = 0$$

which gives

$$T_{CD}\sin\beta = 0.815 \qquad \text{(i)}$$

From the horizontal equilibrium of the forces at C

$$T_{CD} \cos \beta - T_{CA} \cos \alpha = T_{CD} \cos \beta - 29.1 \cos 18.4^\circ = 0$$

so that

$$T_{CD} \cos \beta = 27.612 \tag{ii}$$

Dividing Eq. (i) by Eq. (ii) yields

$$\tan \beta = 0.0295$$

from which

$$\beta = 1.69^\circ$$

Therefore from either of Eq. (i) or (ii)

$$T_{CD} = 27.6\,\mathrm{kN}$$

We can obtain the tension in DB in a similar manner. Thus, from the vertical equilibrium of the forces at D, we have

$$T_{DB} \sin \gamma - T_{DC} \sin \beta - 6 = T_{DB} \sin \gamma - 27.6 \sin 1.69^\circ - 6 = 0$$

from which

$$T_{DB} \sin \gamma = 6.815 \tag{iii}$$

From the horizontal equilibrium of the forces at D we see that

$$T_{DB} \cos \gamma - T_{CB} \cos \beta = T_{DB} \cos \gamma - 27.6 \cos 1.69^\circ = 0$$

from which

$$T_{DB} \cos \gamma = 27.618 \tag{iv}$$

Dividing Eq. (iii) by Eq. (iv) we obtain

$$\tan \gamma = 0.2468$$

so that

$$\gamma = 13.86^\circ$$

T_{DB} follows from either of Eq. (iii) or (iv) and is

$$T_{DB} = 28.4\,\mathrm{kN}$$

Alternatively we could have calculated T_{DB} by determining $R_{B,H}$ ($=R_{A,H}$) and $R_{B,V}$. Then

$$T_{DB} = R_B = \sqrt{R_{B,H}^2 + R_{B,V}^2}$$

and

$$\gamma = \tan^{-1}\left(\frac{R_{B,V}}{R_{B,H}}\right)$$

Continued

EXAMPLE 5.2 CONT'D

This approach would, in fact, be a little shorter than the one given above. However, in the case where the cable carries more than two loads, the above method must be used at loading points adjacent to the support points.

In Exs 5.1 and 5.2 the support points for the cables are on the same horizontal level thereby simplifying the analysis. We shall now consider a cable system where this is not the case.

EXAMPLE 5.3

Calculate the support reactions and the tension in each part of the cable shown in Fig. 5.4.

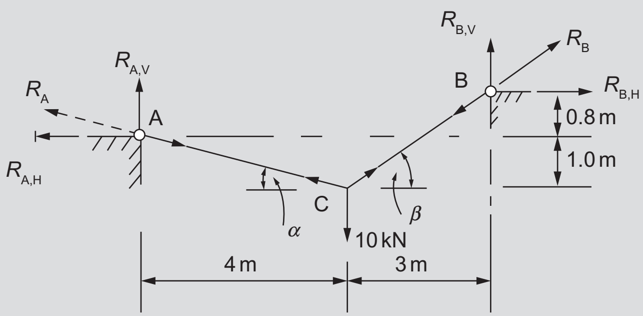

FIGURE 5.4

Cable of Ex.5.3.

The angle α, which the part of the cable AC makes with the horizontal, is given by

$$\alpha = \tan^{-1}\frac{1}{4} = 14.0°$$

Similarly,

$$\beta = \tan^{-1}\frac{1.8}{3} = 31.0°$$

Taking moments about B

$$R_{A,V} \times 7 + R_{A,H} \times 0.8 - 10 \times 3 = 0,$$

which gives

$$R_{A,V} + 0.11R_{A,H} = 4.3 \quad \text{(i)}$$

Taking moments about C

$$R_{A,V} \times 4 - R_{A,H} \times 1 = 0$$

so that

$$R_{A,V} - 0.25R_{A,H} = 0 \quad \text{(ii)}$$

Solving Eqs (i) and (ii) gives

$$R_{A,H} = 11.9\,\text{kN},\ R_{A,V} = 3.0\,\text{kN}.$$

Then

$$T_{A,C} = R_A = \sqrt{R_{A,V}^2 + R_{A,H}^2} = \sqrt{3.0^2 + 11.9^2} = 12.3\,\text{kN}.$$

Resolving vertically

$$R_{B,V} + R_{A,V} = 10,$$

which gives

$$R_{B,V} = 10 - 3 = 7.0\,\text{kN}.$$

Resolving horizontally

$$R_{B,H} - R_{A,H} = 0$$

so that

$$R_{B,H} = 11.9\,\text{kN}$$

Then

$$T_{BC} = R_B = \sqrt{7.0^2 + 11.9^2} = 13.8\,\text{kN}.$$

EXAMPLE 5.4

If the 10-kN load on the cable of Ex. 5.3 is replaced by two 5-kN loads as shown in Fig. 5.5, calculate the sag of the cable at D and the tension in each of its segments.

The angle α is given by

$$\alpha = \tan^{-1}\frac{0.5}{2} = 14.0°$$

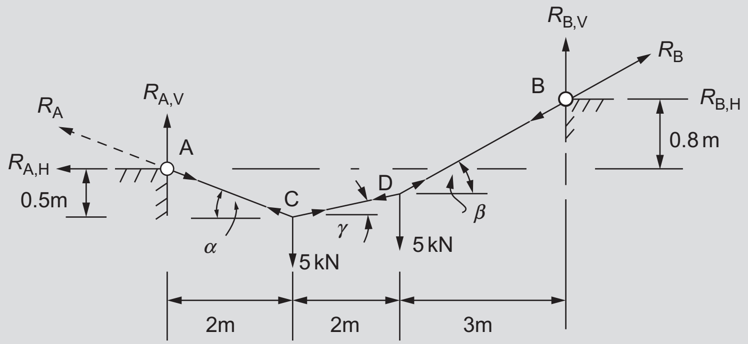

FIGURE 5.5

Cable of Ex.5.4.

Continued

EXAMPLE 5.4 CONT'D

Taking moments about B

$$R_{A,V} \times 7 + R_{A,H} \times 0.8 - 5 \times 5 - 5 \times 3 = 0$$

i.e.

$$R_{A,V} + 0.11\, R_{A,H} = 5.7 \tag{i}$$

Taking moments about C

$$R_{A,V} \times 2 - R_{A,H} \times 0.5 = 0$$

or

$$R_{A,V} - 0.25\, R_{A,H} = 0 \tag{ii}$$

Solving Eqs (i) and (ii)

$$R_{A,H} = 15.8\,\text{kN}, R_{A,V} = 4.0\,\text{kN}.$$

Now resolving forces vertically

$$R_{B,V} + R_{A,V} - 5 - 5 = 0,$$

which gives

$$R_{B,V} = 10 - 4.0 = 6.0\,\text{kN}$$

Resolving forces horizontally

$$R_{B,H} = R_{A,H} = 15.8\,\text{kN}.$$

Then the angle β, which the portion of the cable DB makes with the horizontal, is given by

$$\beta = \tan^{-1}\frac{6.0}{15.8} = 20.8°$$

and the sag of the cable at D relative to B is given by

$$\text{Sag} = 3\tan 20.8° = 1.14\,\text{m}$$

At A

$$T_{AC} = R_A = \sqrt{R_{A,V}^2 + R_{A,H}^2} = \sqrt{4.0^2 + 15.8^2} = 16.3\,\text{kN}.$$

At B

$$T_{BD} = R_B = \sqrt{6.0^2 + 15.8^2} = 16.9\,\text{kN}$$

Also

$$\gamma = \tan^{-1}\left(\frac{1.3 - 1.14}{2}\right) = 4.6°$$

Resolving horizontally at C

$$T_{CD}\cos\gamma - T_{CA}\cos\alpha = 0$$

so that

$$T_{CD} \cos 4.6° - 16.3 \cos 14.0° = 0$$

giving

$$T_{CD} = 15.9\,\text{kN}.$$

5.2 Heavy cables

We shall now consider the more practical case of cables having a significant self-weight.

Governing equation for deflected shape

The cable AB shown in Fig. 5.6(a) carries a distributed load $w(x)$ per unit of its horizontally projected length. An element of the cable, whose horizontal projection is δx, is shown, together with the forces acting on it, in Fig. 5.6(b). Since δx is infinitesimally small, the load intensity may be regarded as constant over the length of the element.

Suppose that T is the tension in the cable at the point x and that $T + \delta T$ is the tension at the point $x + \delta x$; the vertical and horizontal components of T are V and H, respectively. In the absence of any externally applied horizontal loads we see that

$$H = \text{constant}$$

and from the vertical equilibrium of the element we have

$$V + \delta V - w(x)\delta x - V = 0$$

so that, in the limit as $\delta x \rightarrow 0$

$$\frac{\mathrm{d}V}{\mathrm{d}x} = w(x) \tag{5.1}$$

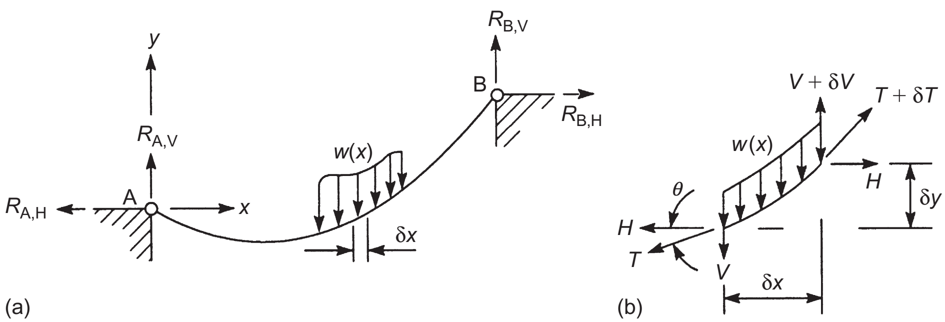

FIGURE 5.6

Cable subjected to a distributed load.

From Fig. 5.6(b)

$$\frac{V}{H} = \tan\theta = +\frac{dy}{dx}$$

where y is the vertical deflection of the cable at any point referred to the x axis.

Hence

$$V = +H\frac{dy}{dx}$$

so that

$$\frac{dV}{dx} = +H\frac{d^2y}{dx^2} \tag{5.2}$$

Substituting for dV/dx from Eq. (5.1) into Eq. (5.2) we obtain the *governing equation* for the deflected shape of the cable. Thus

$$H\frac{d^2y}{dx^2} = +w(x) \tag{5.3}$$

We are now in a position to investigate cables subjected to different load applications.

Cable under its own weight

In this case let us suppose that the weight per actual unit length of the cable is w_s. Then, by referring to Fig. 5.7, we see that the weight per unit of the horizontally projected length of the cable, $w(x)$, is given by

$$w(x)\delta x = w_s\delta s \tag{5.4}$$

Now, in the limit as $\delta s \to 0$, $ds = (dx^2 + dy^2)^{1/2}$

Whence, from Eq. (5.4)

$$w(x) = w_s\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2} \tag{5.5}$$

Substituting for $w(x)$ from Eq. (5.5) in Eq. (5.3) gives

$$H\frac{d^2y}{dx^2} = +w_s\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2} \tag{5.6}$$

Let $dy/dx = p$. Then Eq. (5.6) may be written

$$H\frac{dp}{dx} = +w_s(1 + p^2)^{1/2}$$

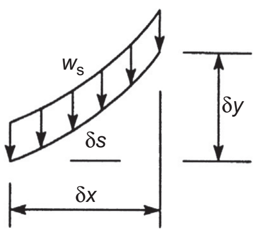

FIGURE 5.7

Elemental length of cable under its own weight.

or, rearranging and integrating

$$\int \frac{\mathrm{d}p}{(1+p^2)^{1/2}} = +\int \frac{w_s}{H}\mathrm{d}x \tag{5.7}$$

The term on the left-hand side of Eq. (5.7) is a standard integral. Thus

$$\sinh^{-1} p = +\frac{w_s}{H}x + C_1$$

in which C_1 is a constant of integration. Then

$$p = \sinh\left(+\frac{w_s}{H}x + C_1\right)$$

Now substituting for p ($=\mathrm{d}y/\mathrm{d}x$) we obtain

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sinh\left(+\frac{w_s}{H}x + C_1\right)$$

which, when integrated, becomes

$$y = +\frac{H}{w_s}\cosh\left(+\frac{w_s}{H}x + C_1\right) + C_2 \tag{5.8}$$

in which C_2 is a second constant of integration.

The deflected shape defined by Eq. (5.8) is known as a *catenary*; the constants C_1 and C_2 may be found using the boundary conditions of a particular problem.

EXAMPLE 5.5

Determine the equation of the deflected shape of the symmetrically supported cable shown in Fig. 5.8, if its self-weight is w_s per unit of its actual length.

The equation of its deflected shape is given by Eq. (5.8), i.e.

$$y = +\frac{H}{w_s}\cosh\left(+\frac{w_s}{H}x + C_1\right) + C_2 \tag{i}$$

Differentiating Eq. (i) with respect to x we have

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sinh\left(+\frac{w_s}{H}x + C_1\right) \tag{ii}$$

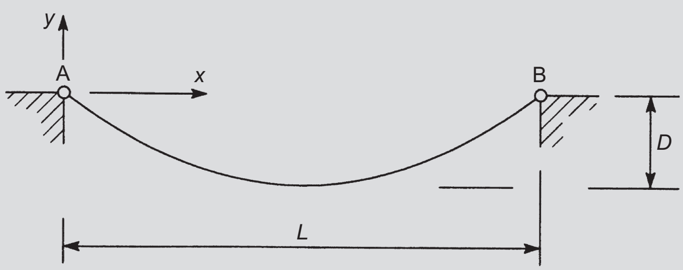

FIGURE 5.8

Deflected shape of a symmetrically supported cable.

Continued

EXAMPLE 5.5 CONT'D

From symmetry, the slope of the cable at mid-span is zero, i.e. $dy/dx=0$ when $x=L/2$. Thus, from Eq. (ii)

$$0=\sinh\left(+\frac{w_s}{H}\frac{L}{2}+C_1\right)$$

from which

$$C_1=-\frac{w_s}{H}\frac{L}{2}$$

Eq. (i) then becomes

$$y=+\frac{H}{w_s}\cosh\left[+\frac{w_s}{H}\left(x-\frac{L}{2}\right)\right]+C_2 \qquad \text{(iii)}$$

The deflection of the cable at its supports is zero, i.e. $y=0$ when $x=0$ and $x=L$. From the first of these conditions

$$0=+\frac{H}{w_s}\cosh\left(-\frac{w_s}{H}\frac{L}{2}\right)+C_2$$

so that

$$C_2=-\frac{H}{w_s}\cosh\left(-\frac{w_s}{H}\frac{L}{2}\right)=-\frac{H}{w_s}\cosh\left(\frac{w_s}{H}\frac{L}{2}\right)\quad (\text{note}: \cosh(-x)\equiv\cosh(x))$$

Eq. (iii) is then written as

$$y=+\frac{H}{w_s}\left\{\cosh\left[+\frac{w_s}{H}\left(x-\frac{L}{2}\right)\right]-\cosh\left(\frac{w_s}{H}\frac{L}{2}\right)\right\} \qquad \text{(iv)}$$

Equation (iv) gives the deflected shape of the cable in terms of its self-weight, its length and the horizontal component, H, of the tension in the cable. In a particular case where, say, w_s, L and H are specified, the sag, D, of the cable is obtained directly from Eq. (iv). Alternatively if, instead of H, the sag D is fixed, H is obtained from Eq. (iv) which then becomes a transcendental equation which may be solved graphically.

Since H is constant the maximum tension in the cable will occur at the point where the vertical component of the tension in the cable is greatest. In the above example this will occur at the support points where the vertical component of the tension in the cable is equal to half its total weight. For a cable having supports at different heights, the maximum tension will occur at the highest support since the length of cable from its lowest point to this support is greater than that on the opposite side of the lowest point. Furthermore, the slope of the cable at the highest support is a maximum (see Fig. 5.6(a)).

Cable subjected to a uniform horizontally distributed load

This loading condition is, as we shall see when we consider suspension bridges, more representative of that in actual suspension structures than the previous case.

For the cable shown in Fig. 5.9, Eq. (5.3) becomes

$$H\frac{d^2y}{dx^2}=+w \qquad (5.9)$$

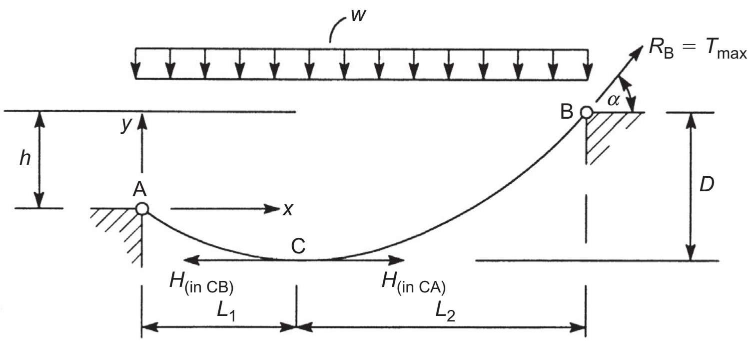

FIGURE 5.9

Cable carrying a uniform horizontally distributed load.

Integrating Eq. (5.9) with respect to x we have

$$H\frac{\mathrm{d}y}{\mathrm{d}x} = +wx + C_1 \tag{5.10}$$

again integrating

$$Hy = +w\frac{x^2}{2} + C_1 x + C_2 \tag{5.11}$$

The boundary conditions are $y=0$ at $x=0$ and $y=h$ at $x=L$. The first of these gives $C_2=0$ while from the second we have

$$H(+h) = +w\frac{L^2}{2} + C_1 L$$

so that

$$C_1 = -\frac{wL}{2} + H\frac{h}{L}$$

Equations (5.10) and (5.11) then become, respectively

$$\frac{\mathrm{d}y}{\mathrm{d}x} = +\frac{w}{H}x - \frac{wL}{2H} + \frac{h}{L} \tag{5.12}$$

and

$$y = +\frac{w}{2H}x^2 - \left(\frac{wL}{2H} - \frac{h}{L}\right)x \tag{5.13}$$

Thus the cable in this case takes up a parabolic shape.

Equations (5.12) and (5.13) are expressed in terms of the horizontal component, H, of the tension in the cable, the applied load and the cable geometry. If, however, the maximum sag, D, of the cable is known, H may be eliminated as follows.

The position of maximum sag coincides with the point of zero slope. Thus from Eq. (5.12)

$$0 = +\frac{w}{H}x - \frac{wL}{2H} + \frac{h}{L}$$

so that

$$x = \frac{L}{2} - \frac{Hh}{wL} = L_1 \quad \text{(see Fig. 5.9)}$$

Then the horizontal distance, L_2, from the lowest point of the cable to the support at B is given by

$$L_2 = L - L_1 = \frac{L}{2} + \frac{Hh}{wL}$$

Now considering the moment equilibrium of the length CB of the cable about B we have, from Fig. 5.9

$$HD - w\frac{L_2^2}{2} = 0$$

so that

$$HD - \frac{w}{2}\left(\frac{L}{2} + \frac{Hh}{wL}\right)^2 = 0 \tag{5.14}$$

Equation (5.14) is a quadratic equation in H and may be solved for a specific case using the formula.

Alternatively, H may be determined by considering the moment equilibrium of the lengths AC and CB about A and C, respectively. Thus, for AC

$$H(D-h) - w\frac{L_1^2}{2} = 0$$

which gives

$$H = \frac{wL_1^2}{2(D-h)} \tag{5.15}$$

For CB

$$HD - \frac{wL_2^2}{2} = 0$$

so that

$$H = \frac{wL_2^2}{2D} \tag{5.16}$$

Equating Eqs (5.15) and (5.16)

$$\frac{wL_1^2}{2(D-h)} = \frac{wL_2^2}{2D}$$

which gives

$$L_1 = \sqrt{\frac{D-h}{D}}L_2$$

But

$$L_1 + L_2 = L$$

therefore

$$L_2 = \left[\sqrt{\frac{D-h}{D}} + 1\right] = L$$

from which

$$L_2 = \frac{L}{\left(\sqrt{\frac{D-h}{D}}+1\right)} \tag{5.17}$$

Then, from Eq. (5.16)

$$H = \frac{wL^2}{2D\left[\sqrt{\frac{D-h}{D}}+1\right]^2} \tag{5.18}$$

As in the case of the catenary the maximum tension will occur, since H=constant, at the point where the vertical component of the tension is greatest. Thus, in the cable of Fig. 5.9, the maximum tension occurs at B where, as $L_2 > L_1$, the vertical component of the tension $(= wL_2)$ is greatest. Hence

$$T_{\max} = \sqrt{(wL_2)^2 + H^2} \tag{5.19}$$

in which L_2 is obtained from Eq. (5.17) and H from one of Eqs (5.14), (5.16) or (5.18). At B the slope of the cable is given by

$$\alpha = \tan^{-1}\left(\frac{wL}{H}\right) \tag{5.20}$$

or, alternatively, from Eq. (5.12)

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_{x=L} = +\frac{w}{H}L - \frac{wL}{2H} + \frac{h}{L} = +\frac{wL}{2H} + \frac{h}{L} \tag{5.21}$$

For a cable in which the supports are on the same horizontal level, i.e. h=0, Eqs (5.12)–(5.14) and (5.19) reduce, respectively, to

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{w}{H}\left(x - \frac{L}{2}\right) \tag{5.22}$$

$$y = \frac{w}{2H}(x^2 - Lx) \tag{5.23}$$

$$H = \frac{wL^2}{8D} \tag{5.24}$$

$$T_{\max} = \frac{wL}{2}\sqrt{1 + \left(\frac{L}{4D}\right)^2} \tag{5.25}$$

We observe from the above that the analysis of a cable under its own weight, that is a catenary, yields a more complex solution than that in which the load is assumed to be uniformly distributed horizontally. However, if the sag in the cable is small relative to its length, this assumption gives results that differ only slightly from the more accurate but more complex catenary approach. Therefore, in practice, the loading is generally assumed to be uniformly distributed horizontally.

EXAMPLE 5.6

Determine the maximum tension and the maximum slope in the cable shown in Fig. 5.10 if it carries a uniform horizontally distributed load of intensity 10 kN/m.

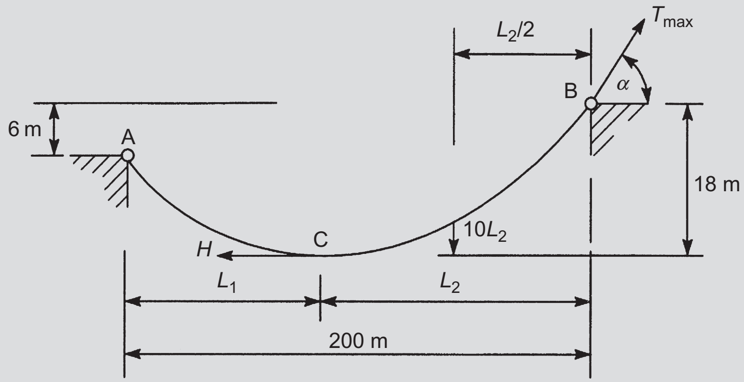

FIGURE 5.10

Suspension cable of Ex. 5.6.

From Eq. (5.17)

$$L_2 = \frac{200}{\left(\sqrt{\frac{18-6}{18}}+1\right)} = 110.1\ \text{m}$$

Then, from Eq. (5.16)

$$H = \frac{10 \times 110.1^2}{2 \times 18} = 3367.2\ \text{kN}$$

The maximum tension follows from Eq. (5.19), i.e.

$$T_{max} = \sqrt{(10 \times 110.1)^2 + 3367.2^2} = 3542.6\ \text{kN}$$

Then, from Eq. (5.20)

$$\alpha_{max} = \tan^{-1}\frac{10 \times 110.1}{3367.2} = 18.1^\circ\ \text{at B}$$

Ex. 5.6 has been solved by direct substitution in Eqs (5.16), (5.17), (5.19) and (5.20). However, working from first principles obviates the necessity of remembering rather unwieldy formulae; the solution of Ex. 5.6 would then proceed as follows:

Taking moments about A for the portion AC of the cable we have

$$H \times 12 - 10L_1(L_1/2) = 0$$

from which

$$H = 5L_1^2/12 \qquad \text{(i)}$$

Now taking moments about B for the portion BC

$$H \times 18 - 10L_2(L_2/2) = 0$$

so that

$$H = 5L_2^2/18 \tag{ii}$$

Equating Eqs (i) and (ii) gives

$$L_1 = (\sqrt{2/3})L_2 = 0.816\,L_2$$

But

$$L_1 + L_2 = 200 \tag{iii}$$

Substituting for L_1 in Eq. (iii) gives

$$L_2 = 110.1\ \text{m}$$

Then, from Eq. (ii)

$$H = 5 \times 110.1^2/18 = 3367.2\ \text{kN}$$

The vertical component of the support reaction at B is $10L_2 = 1101$ kN and the horizontal component is $H = 3367.2$ kN. Then

$$T_{\text{max}} = \sqrt{(1101^2 + 3367.2^2)} = 3542.6\ \text{kN}$$

Finally

$$\alpha_{\text{max}} = \tan^{-1}(1101/3367.2) = 18.1°\ \text{at B}$$

Suspension bridges

A typical arrangement for a suspension bridge is shown diagrammatically in Fig. 5.11. The bridge deck is suspended by hangers from the cables which pass over the tops of the towers and are secured by massive anchor blocks embedded in the ground. The advantage of this form of bridge construction is its ability to provide large clear spans so that sea-going ships, say, can pass unimpeded. Typical examples in the UK are the suspension bridges over the rivers Humber and Severn, the Forth road bridge and the Menai

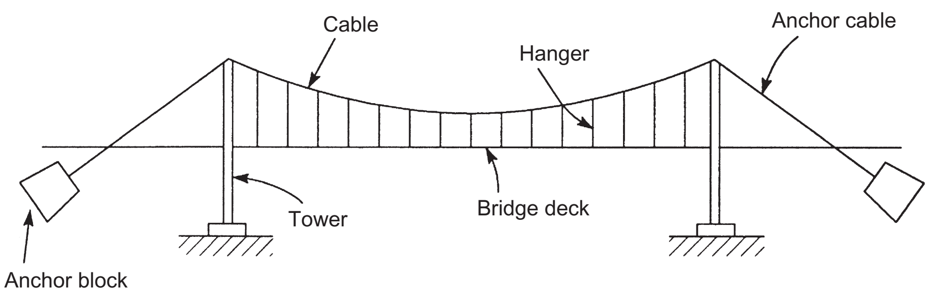

FIGURE 5.11

Diagrammatic representation of a suspension bridge.

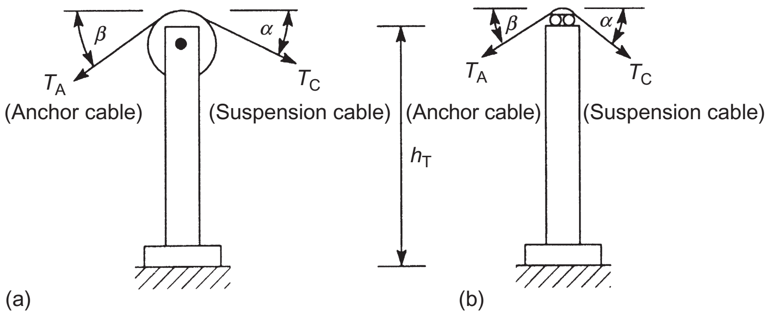

FIGURE 5.12

Idealization of cable supports.

Straits bridge in which the suspension cables comprise chain links rather than tightly bound wires. Suspension bridges are also used for much smaller spans such as pedestrian footbridges and for light vehicular traffic over narrow rivers.

The major portion of the load carried by the cables in a suspension bridge is due to the weight of the deck, its associated stiffening girder and the weight of the vehicles crossing the bridge. By comparison, the self-weight of the cables is negligible. We may assume therefore that the cables carry a uniform horizontally distributed load and therefore take up a parabolic shape; the analysis described in the preceding section then applies.

The cables, as can be seen from Fig. 5.11, are continuous over the tops of the towers. In practice they slide in grooves in saddles located on the tops of the towers. For convenience we shall idealize this method of support into two forms, the actual method lying somewhere between the two. In Fig. 5.12(a) the cable passes over a frictionless pulley, which means that the tension, T_A, in the anchor cable is equal to T_C, the tension at the tower in the suspension cable. Generally the inclination, β, of the anchor cable is fixed and will not be equal to the inclination, α, of the suspension cable at the tower. Therefore, there will be a resultant horizontal force, H_T, on the top of the tower given by

$$H_T = T_C \cos\alpha - T_A \cos\beta$$

or, since $T_A = T_C$

$$H_T = T_C(\cos\alpha - \cos\beta) \tag{5.26}$$

H_T, in turn, produces a bending moment, M_T, in the tower which is a maximum at the tower base. Hence

$$M_{T(max)} = H_T h_T = T_C(\cos\alpha - \cos\beta)h_T \tag{5.27}$$

Also, the vertical compressive load, V_T, on the tower is

$$V_T = T_C(\sin\alpha + \sin\beta) \tag{5.28}$$

In the arrangement shown in Fig. 5.12(b) the cable passes over a saddle which is supported on rollers on the top of the tower. The saddle therefore cannot resist a horizontal force and adjusts its position until

$$T_A \cos\beta = T_C \cos\alpha \tag{5.29}$$

For a given value of β, Eq. (5.29) determines the necessary value of T_A. Clearly, since there is no resultant horizontal force on the top of the tower, the bending moment in the tower is everywhere zero. Finally, the vertical compressive load on the tower is given by

$$V_T = T_C \sin\alpha + T_A \sin\beta \tag{5.30}$$

EXAMPLE 5.7

The cable of a suspension bridge, shown in Fig. 5.13, runs over a frictionless pulley on the top of each of the towers at A and B and is fixed to anchor blocks at D and E. If the cable carries a uniform horizontally distributed load of 120 kN/m, determine the diameter required if the permissible working stress on the gross area of the cable, including voids, is 600 N/mm^2. Also calculate the bending moment and direct load at the base of a tower and the required weight of the anchor blocks.

The tops of the towers are on the same horizontal level, so that the tension in the cable at these points is the same and will be the maximum tension in the cable. The maximum tension is found directly from Eq. (5.25) and is

$$T_{\max} = \frac{120 \times 300}{2}\sqrt{1 + \left(\frac{300}{4 \times 30}\right)^2} = 48466.5\,\text{kN}$$

The maximum direct stress, $\sigma_{\max}$, is given by

$$\sigma_{\max} = \frac{T_{\max}}{\pi d^2/4} \quad (\text{sec Section 7.1})$$

in which d is the cable diameter. Hence

$$600 = \frac{48466.5 \times 10^3}{\pi\, d^2/4}$$

which gives

$$d = 320.7\,\text{mm}$$

The angle of inclination of the suspension cable to the horizontal at the top of the tower is obtained using Eq. (5.20) in which $L_2 = L/2$. Hence

$$\alpha = \tan^{-1}\left(\frac{wL}{2H}\right) = \tan^{-1}\left(\frac{120 \times 300}{2H}\right)$$

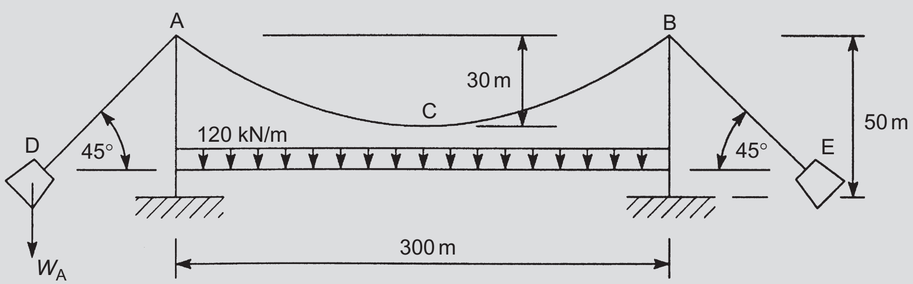

FIGURE 5.13

Suspension bridge of Ex. 5.7.

Continued

EXAMPLE 5.7 CONT'D

where H is given by Eq. (5.24). Thus

$$H = \frac{120 \times 300^2}{8 \times 30} = 45\,000\,\text{kN}$$

so that

$$\alpha = \tan^{-1}\left(\frac{120 \times 300}{2 \times 45\,000}\right) = 21.8°$$

Therefore, from Eq. (5.27), the bending moment at the base of the tower is

$$M_T = 48466.5(\cos 21.8° - \cos 45°) \times 50$$

from which

$$M_T = 536473.4\,\text{kNm}$$

The direct load at the base of the tower is found using Eq. (5.28), i.e.

$$V_T = 48466.5(\sin 21.8° + \sin 45°)$$

which gives

$$V_T = 52269.9\,\text{kN}$$

Finally the weight, W_A, of an anchor block must resist the vertical component of the tension in the anchor cable. Thus

$$W_A = T_A \cos 45° = 48466.5 \cos 45°$$

from which

$$W_A = 34271.0\,\text{kN}.$$

Again, working from first principles and taking moments about A for the portion AC of the cable

$$H \times 30 - 120 \times 150 \times 75 = 0$$

which gives

$$H = 45000\,\text{kN}$$

The horizontal component of the tension in the cable at A is equal to H and the vertical component is equal to $120 \times 150 = 18000$ kN. Then the maximum tension in the cable is

$$T_{max} = \sqrt{(45000^2 + 18000^2)} = 48466.5\,\text{kN}$$

The cable diameter then follows as before and the angle, α, the cable makes with the horizontal at the top of the tower is given by

$$\alpha = \tan^{-1}(18000/45000) = 21.8°$$

Since the cable passes over frictionless pulleys the tension in the anchor cable is equal to the tension in the suspension cable. The resultant horizontal force on the top of a tower is then

$$\text{Resultant horizontal force} = 48466.5(\cos 21.8° - \cos 45°)$$

The bending moment at the base of a tower is then given by

$$M_T = 48466.5(\cos 21.8° - \cos 45°) \times 50 = 536473.4\ \text{kNm}$$

The direct load at the base of a tower is

$$V_T = 48466.5(\sin 21.8° + \sin 45°) = 52269.9\ \text{kN}$$

Finally, the weight of the anchor block is given by

$$W_A = 48466.5 \cos 45° = 34271.0\ \text{kN}$$

PROBLEMS

P.5.1 Calculate the tension in each segment of the cable shown in Fig. P.5.1 and also the vertical distance of the points B and E below the support points A and F.

Ans. $T_{AB} = T_{EF} = 26.9$ kN, $T_{CB} = T_{ED} = 25.5$ kN, $T_{CD} = 25.0$ kN, 1.0 m.

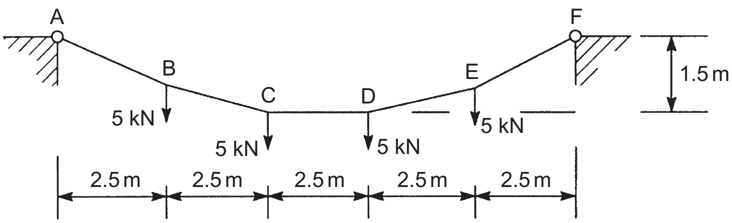

FIGURE P.5.1

P.5.2 Calculate the sag at the point B in the cable shown in Fig. P.5.2 and the tension in each of its segments.

Ans. 0.81 m relative to A. $T_{AB} = 4.9$ kN, $T_{BC} = 4.6$ kN, $T_{DC} = 4.7$ kN.

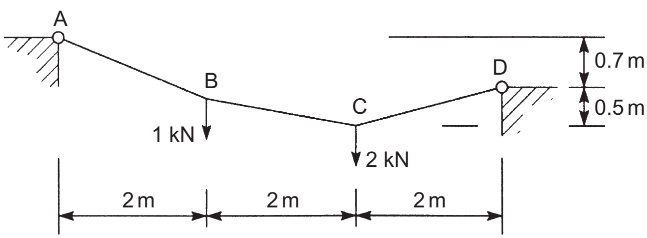

FIGURE P.5.2

P.5.3 Calculate the sag, relative to A, of the points C and D in the cable shown in Fig. P.5.3. Determine also the tension in each of its segments.

Ans. C = 4.2 m, D = 3.1 m, $T_{AB} = 10.98$ kN, $T_{BC} = 9.68$ kN, $T_{CD} = 9.43$ kN, $T_{DE} = 11.0$ kN.

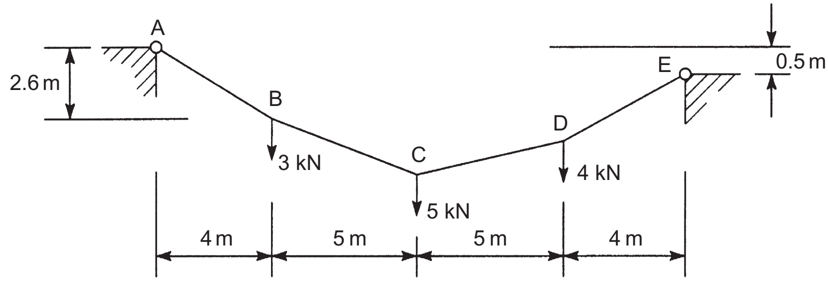

FIGURE P.5.3

P.5.4 A cable that carries a uniform horizontally distributed load of 10 kN/m is suspended between two points that are at the same level and 80 m apart. Determine the minimum sag that may be allowed at mid-span if the maximum tension in the cable is limited to 1000 kN.

Ans. 8.73 m.

P.5.5 A suspension cable is suspended from two points 102 m apart and at the same horizontal level. The self-weight of the cable can be considered to be equivalent to 36 N/m of horizontal length. If the cable carries two concentrated loads each of 10 kN at 34 m and 68 m horizontally from the left-hand support and the maximum sag in the cable is 3 m, determine the maximum tension in the cable and the vertical distance between the concentrated loads and the supports.

Ans. 129.5 kN, 2.96 m.

P.5.6 A cable of a suspension bridge has a span of 80 m, a sag of 8 m and carries a uniform horizontally distributed load of 24 kN/m over the complete span. The cable passes over frictionless pulleys at the top of each tower which are of the same height. If the anchor cables are to be arranged such that there is no bending moment in the towers, calculate the inclination of the anchor cables to the horizontal. Calculate also the maximum tension in the cable and the vertical force on a tower.

Ans. 21.8°, 2584.9 kN, 1919.9 kN.

P.5.7 A suspension cable passes over saddles supported by roller bearings on the top of two towers 120 m apart and differing in height by 2.5 m. The maximum sag in the cable is 10 m and each anchor cable is inclined at 55° to the horizontal. If the cable carries a uniform horizontally distributed load of 25 kN/m and is to be made of steel having an allowable tensile stress of 240 N/mm^2, determine its minimum diameter. Calculate also the vertical load on the tallest tower.

Ans. 218.7 mm, 8990.9 kN.

P.5.8 A suspension cable has a sag of 40 m and is fixed to two towers of the same height and 400 m apart; the effective cross-sectional area of the cable is 0.08 m^2. However, due to corrosion, the effective cross-sectional area of the central half of the cable is reduced by 20%. If the stress in the cable is limited to 500 N/mm^2, calculate the maximum allowable distributed load the cable can support. Calculate also the inclination of the cable to the horizontal at the top of the towers.

Ans. 62.8 kN/m, 21.8°.

P.5.9 A suspension bridge with two main cables has a span of 250 m and a sag of 25 m. It carries a uniform horizontally distributed load of 25 kN/m and the allowable stress in the cables is 800 N/mm^2. If each anchor cable makes an angle of 45° with the towers, calculate:

a. the required cross-sectional area of the cables,
b. the load in an anchor cable and the overturning force on a tower, when
  i. the cables run over a pulley device,
  ii. the cables are attached to a saddle resting on rollers.

Ans. (a) 5259 mm^2, (b) (i) 4207.2 kN, 931.3 kN (ii) 5524.3 kN, 0.

P.5.10 A suspension cable passes over two towers 80 m apart and carries a load of 5 kN/m of span. If the top of the left-hand tower is 4 m below the top of the right-hand tower and the maximum sag in the cable is 16 m, calculate the maximum tension in the cable. Also, if the cable passes over saddles on rollers on the tops of the towers with the anchor cable at 45° to the horizontal, calculate the vertical thrust on the right-hand tower.

Ans. 358.3 kN, 501.5 kN.

P.5.11 A footbridge 2 m wide spans a 25 m wide river. The deck is supported by two cables and the loading on the deck is 7 kN/m^2. Find the greatest and least tension in the cables and the inclination of the cables at the towers if the dip is 3 m. If the cables pass over pulleys at the top of each tower and the anchor cables are inclined at 60° to the horizontal calculate the total thrust and maximum bending moment for a tower height of 7 m. Note that a single tower supports the cables at each end of the footbridge.

Ans. 202 kN, 182 kN, 25°64′, 525 kN, 1137 kNm.