Aerospace Vehicle Design And System Integration 3

CADE30007

(AVDASI 3 - Aircraft Propulsion, Performance and Sustainable Operations)

# Aircraft Performance

## Lecture 3

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# Overview

A brief Introduction to Helicopter performance

Helicopter performance in axial flight

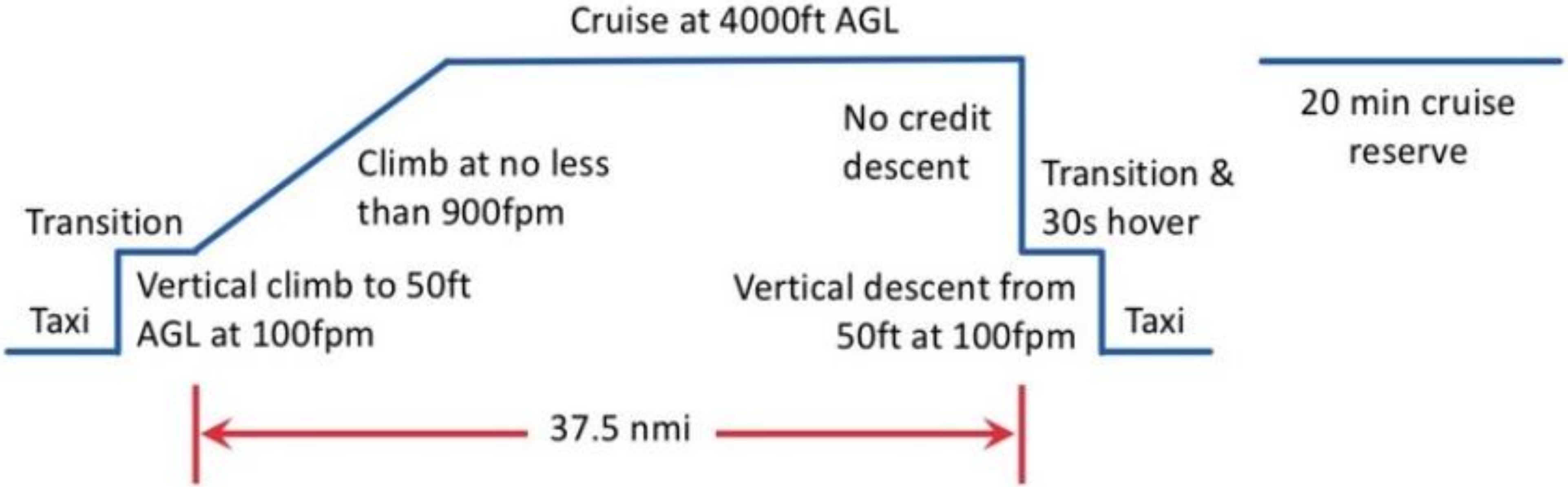
Helicopter performance in forward flight

# Introduction to Helicopter Performance

# Helicopter Design Requirements

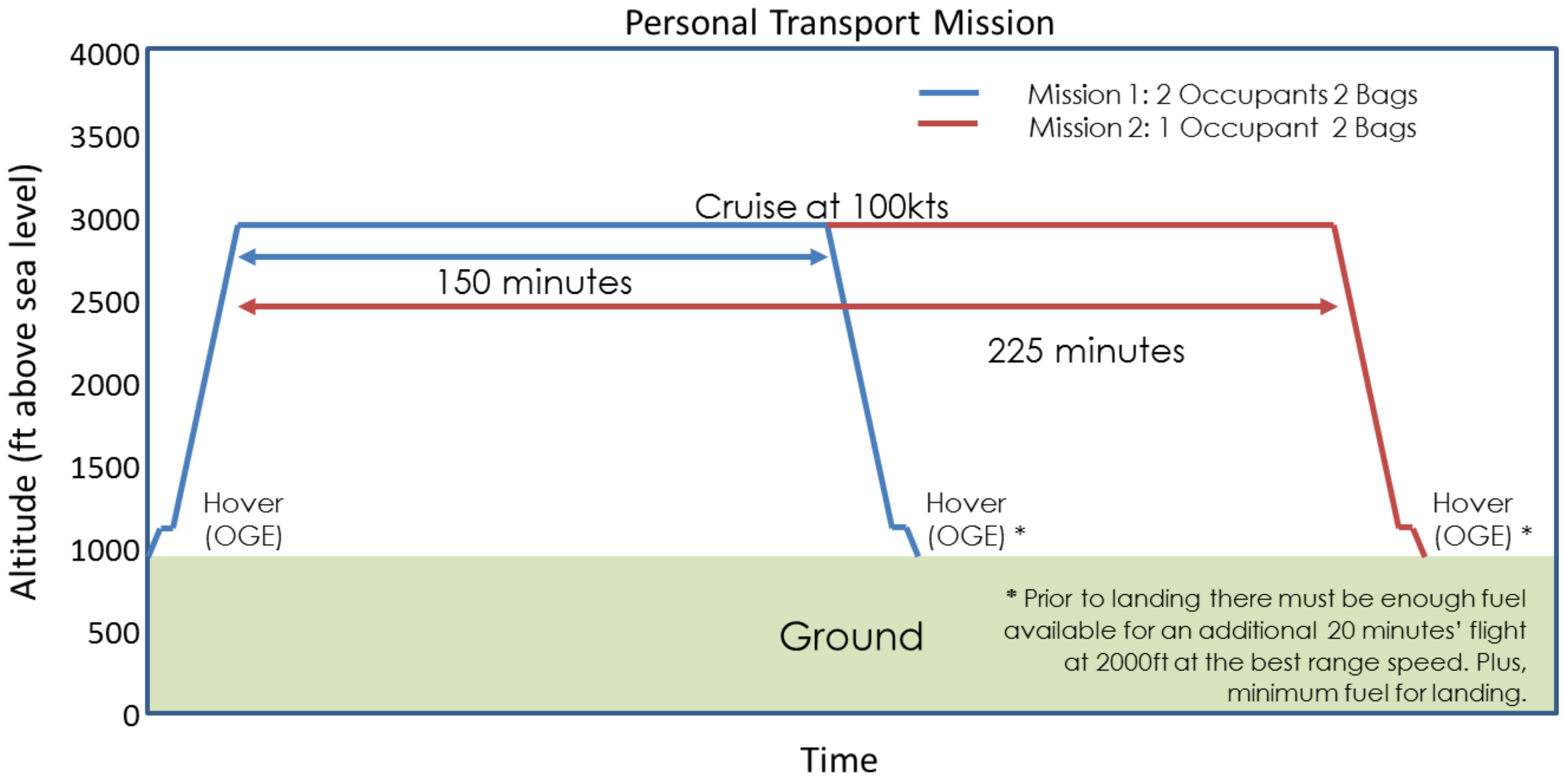
- **General requirements**, e.g. payload, speed, range, altitude, cost, etc.
- **Key mission requirements**, e.g. transportation, firefighting , training, surveillance, military, …
- **Compliance with the applicable airworthiness standards** (CS27, CS29 …)
- **Inoperable Engine requirements** (urban area, traffic controlled area, …)

# Helicopter mission profiles




AGL: Above Ground Level

# Typical Helicopter Mission



# Rotorcraft Configurations

Helicopters

Autogyros

Compound Helicopters

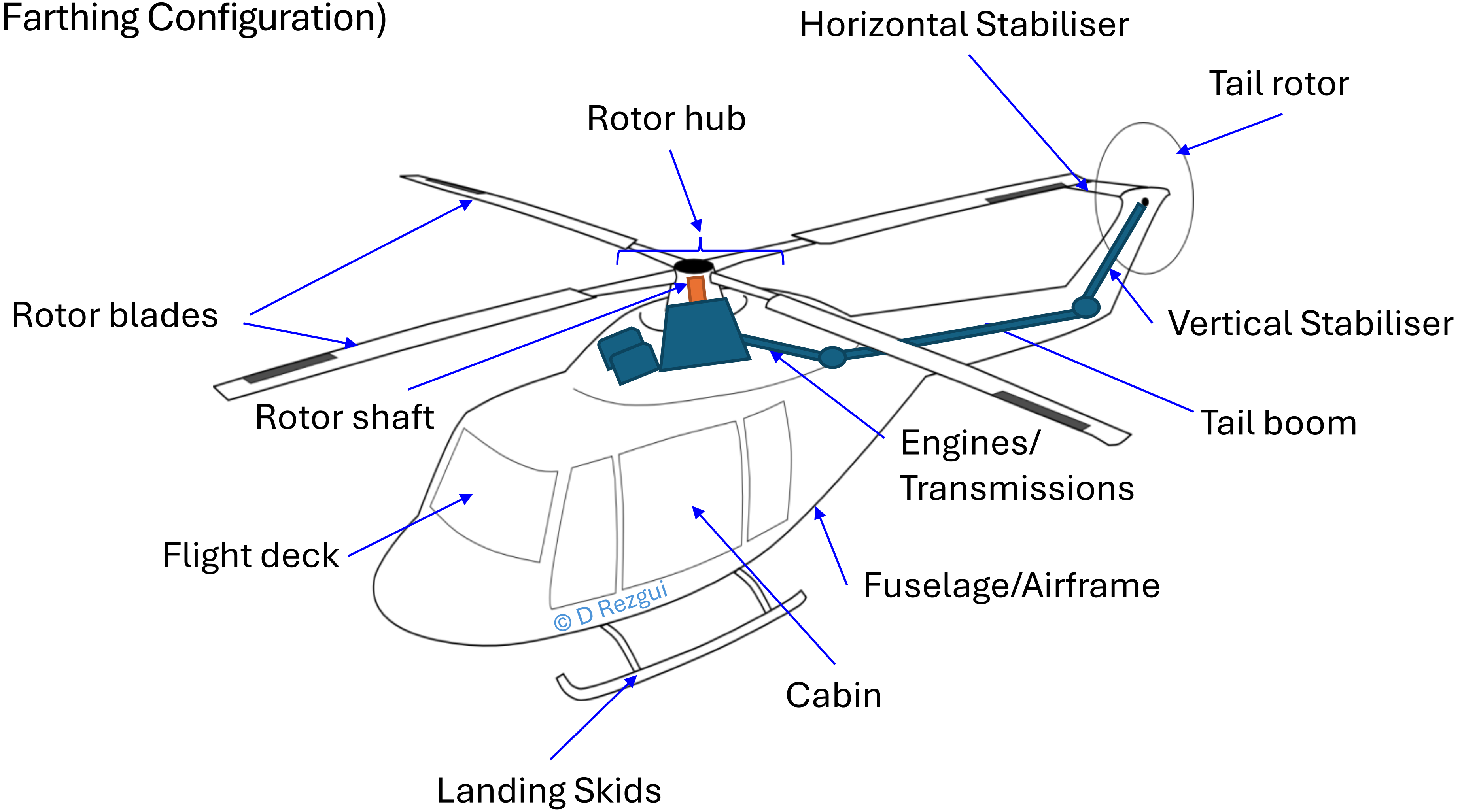
Multi-rotors

# When we think of a helicopter……

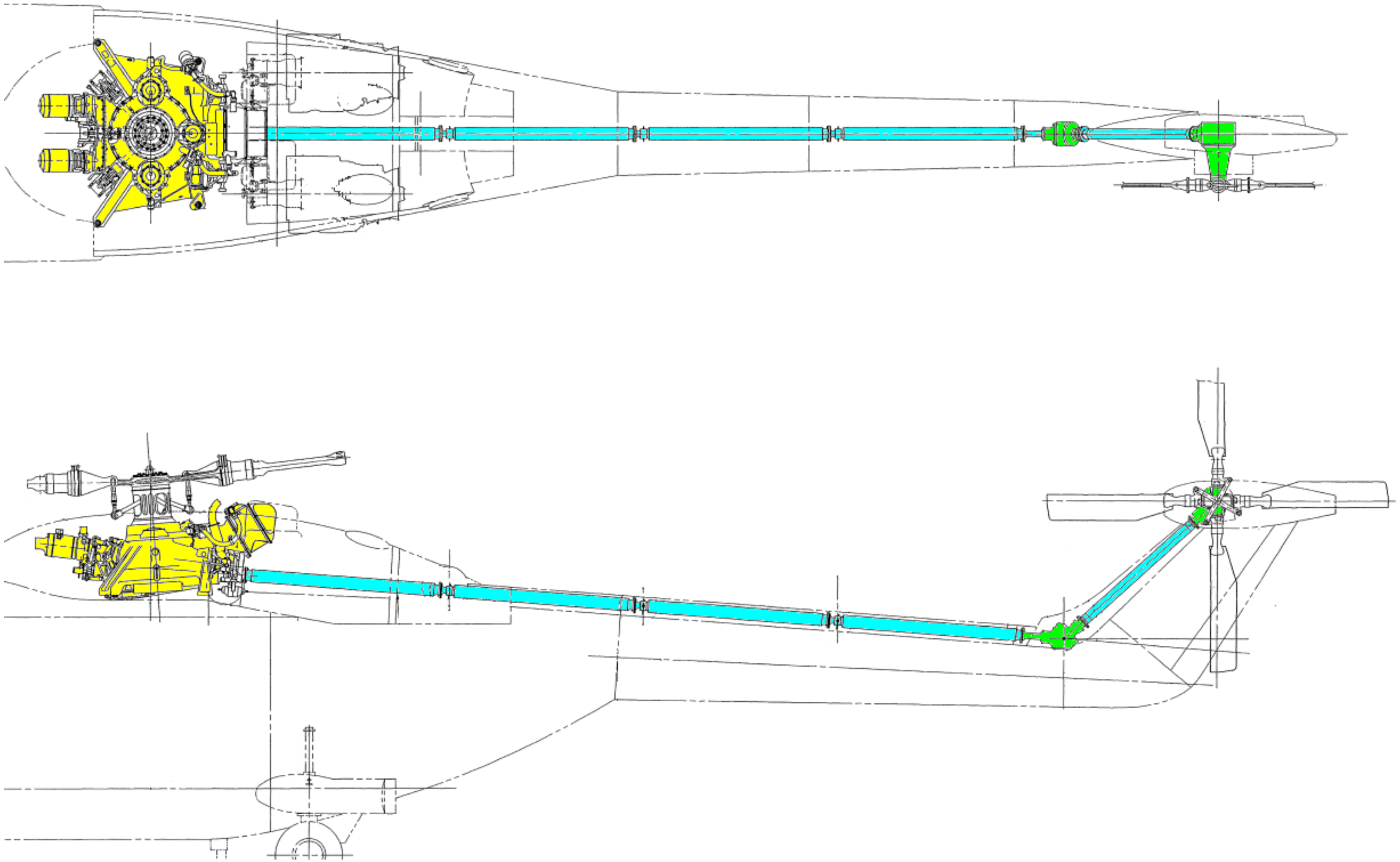
This is the kind of image that often comes to mind.
Hence the penny-farthing configuration is used in this course.

# Main Helicopter Parts

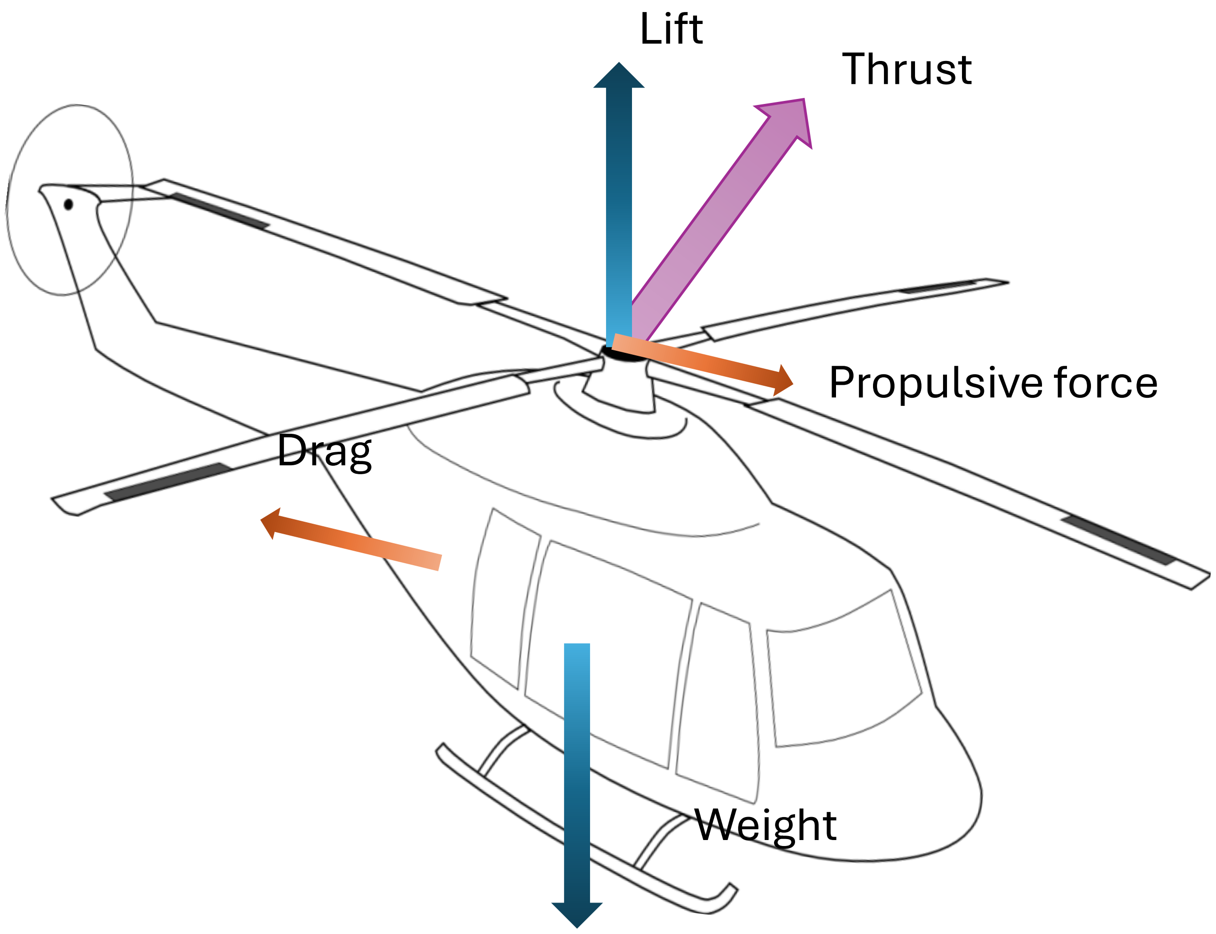
(Penny Farthing Configuration)

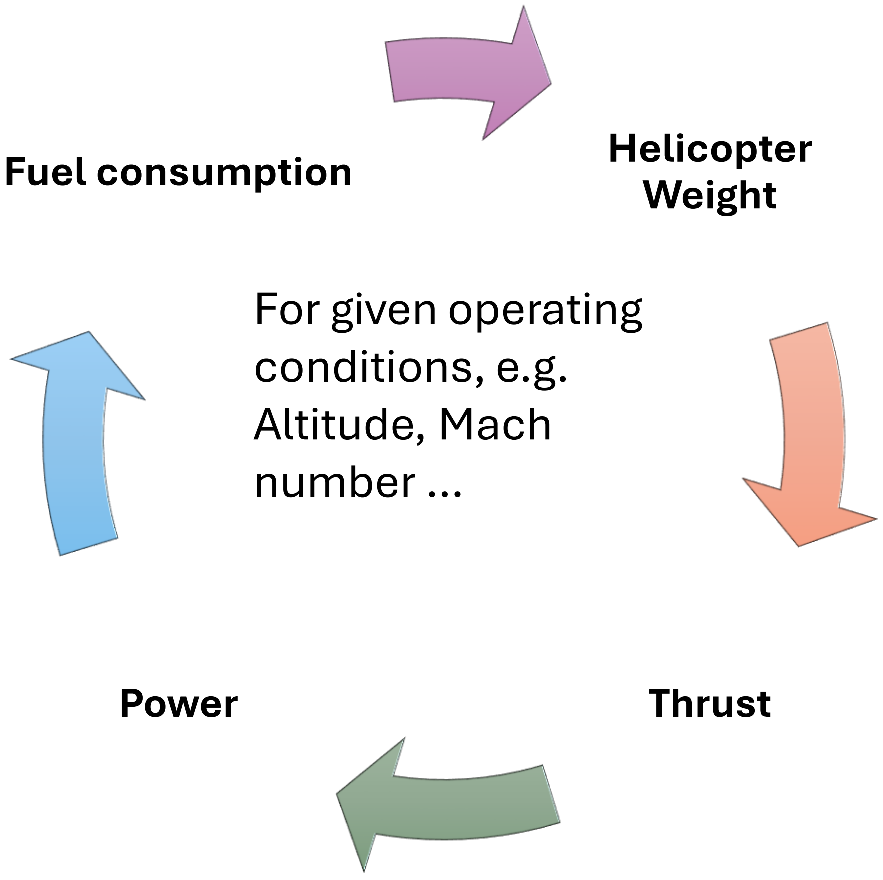


# A Typical Helicopter Drive Train



# A Simplistic View of Performance Model

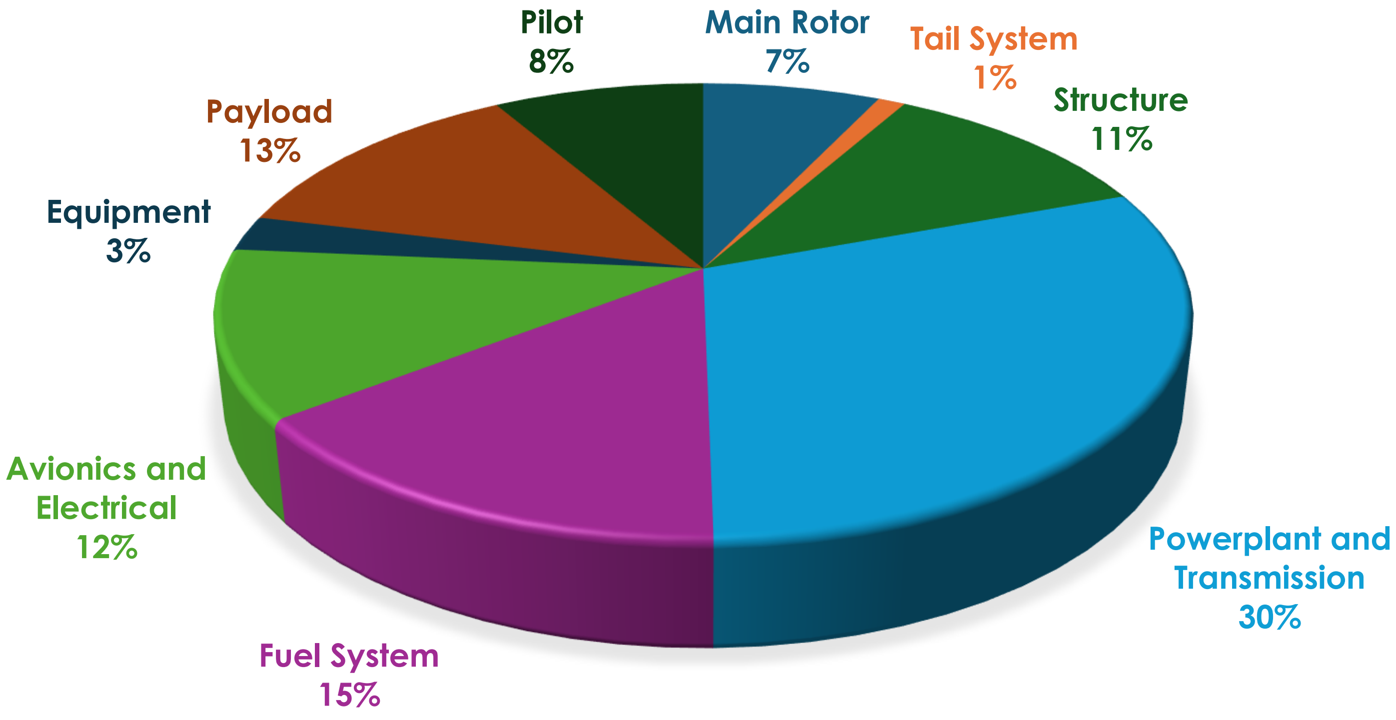


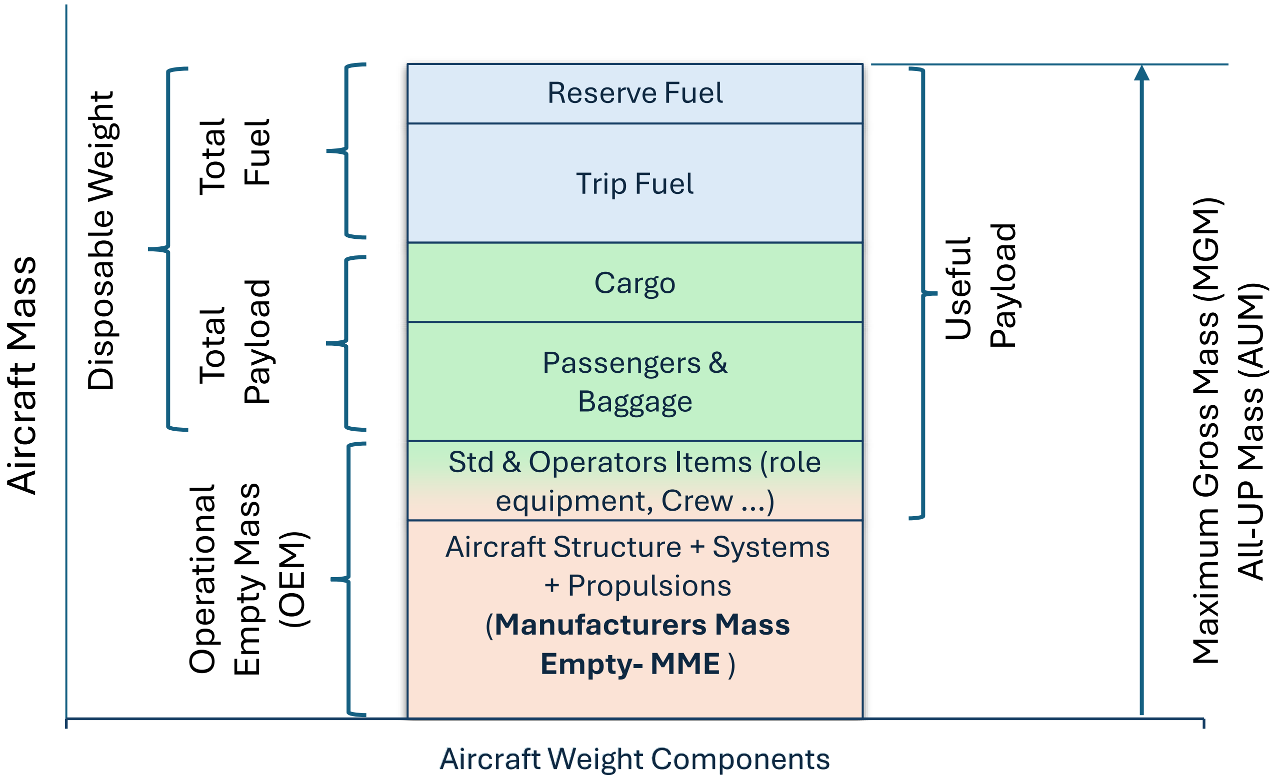
# Performance Calculations

- **Power / Energy consumption is dependent on**
  - Rotor geometry and helicopter size
  - Mission capabilities, hover, maximum airspeed, service ceiling
  - The required aircraft maximum gross mass
    - MGM = empty mass + basic equipment + crew + role equipment + fuel + payload
    - A target empty mass fraction appropriate to the role can be assumed
- **Calculate aircraft power required to meet performance requirements**
  - Rotor radius, tip speed, chord
  - Compare power requirements to power available (engine & transmission)
  - Mission fuel can be refined using typical engine specific fuel consumption values

# Typical Weight Breakdown



# Helicopter Mass (Weight) Breakdown




**Aircraft mass breakdown in one trip:**

$Mass_{final} = Mass_{initial}$ - Trip Fuel

# Definitions

| | |
|---|---|
| Disposable weight fraction (DWF) | Disposable weight (DW) / All Up Weight (AUW) |
| Empty weight fraction (EWF) | Empty weight (EW) / All Up Weight (AUW) |
| Useful weight fraction (UWF) | Useful weight (UW) / All Up Weight (AUW) |
| Fuel load fraction (FLF) | Fuel load (FL) / Payload (PL) |

# Power Breakdown

- In general, the total power required for a helicopter is:

$$\boldsymbol{P_{tot} = P_i + P_0 + P_p + P_{TR} + P_t + P_{aux}}$$

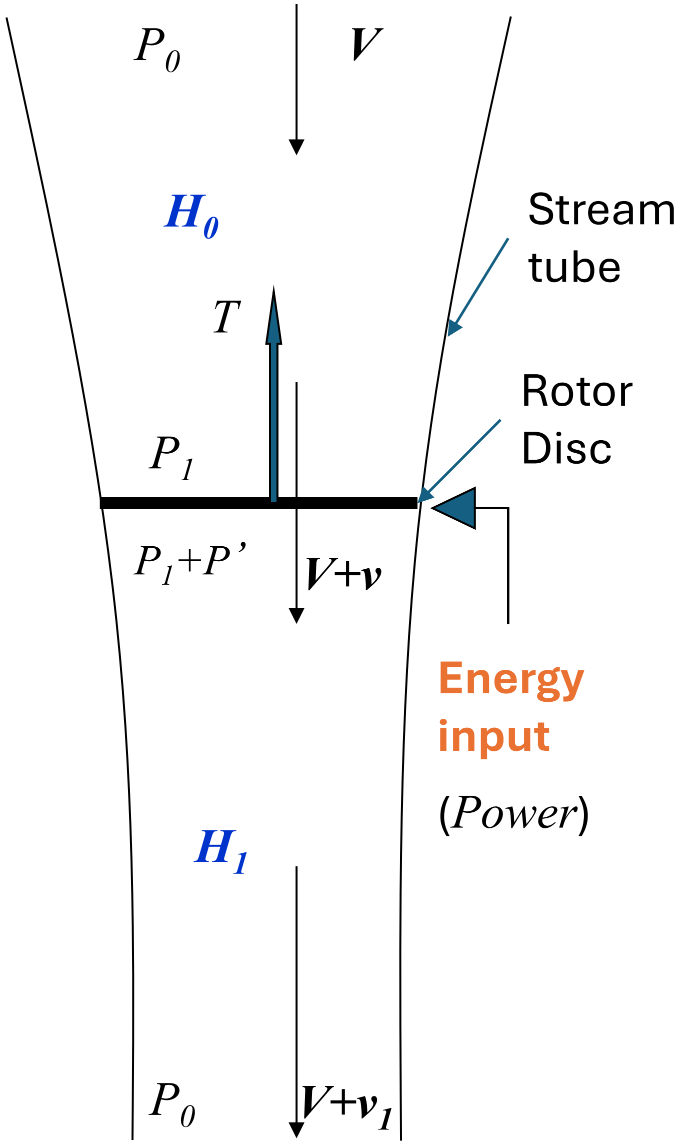
| | |
|---|---|
| $P_i$ | is the **induced power** (of the main rotor) needed to produce useful thrust |
| $P_0$ | is the **profile power** to overcome the blade drag (for the main rotor) |
| $P_p$ | is the **parasitic power** to overcome the drag of the airframe. This is typically zero in hover. |
| $P_{TR}$ | is the total **tail rotor power**, which also has induced and profile components |
| $P_t$ | is the power due to **transmission losses** |
| $P_{aux}$ | is the power needed to power the **auxiliary systems** of the helicopter |

# Helicopter performance in axial flight

# **Reminder:** Actuator Disc (Momentum) Theory in Axial Flight

Thrust: $T = 2\rho A(V + v)v$

Power: $P = T(V + v)$



# Momentum Theory in Hover

For a lifting rotor in **hover**, when the onset velocity $V = 0$

The induced velocity in hover:

$$v_h = \sqrt{\frac{T}{2\rho A}}$$

The induced power in hover:

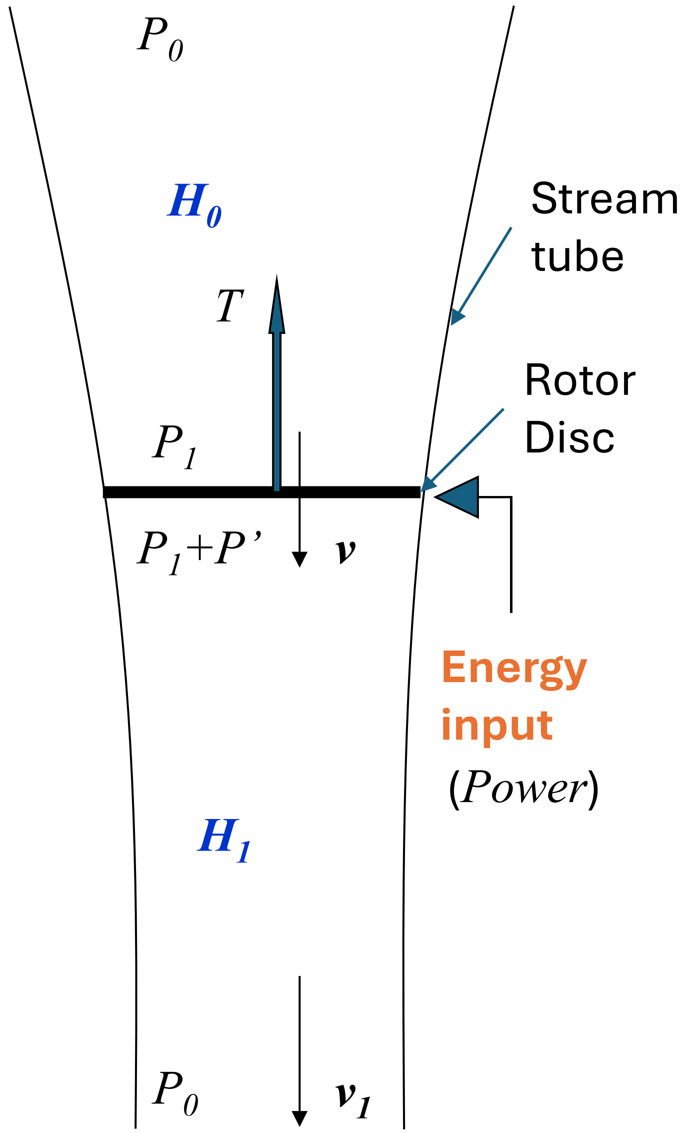
$P_h = T v_h$ Hence

$$P_h = \frac{T^{3/2}}{\sqrt{2\rho A}}$$

The main rotor thrust can be given as:

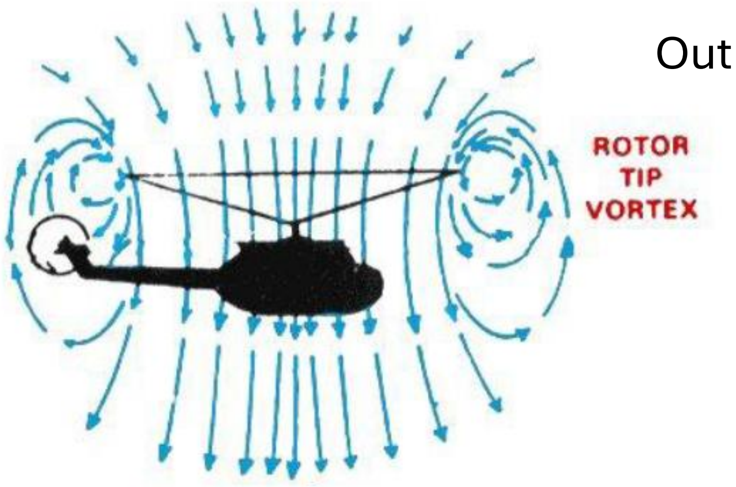
$$\boldsymbol{T} = \boldsymbol{AUW}(\mathbf{1} + \boldsymbol{DLF})$$

where DLF is a download factor due to airframe drag

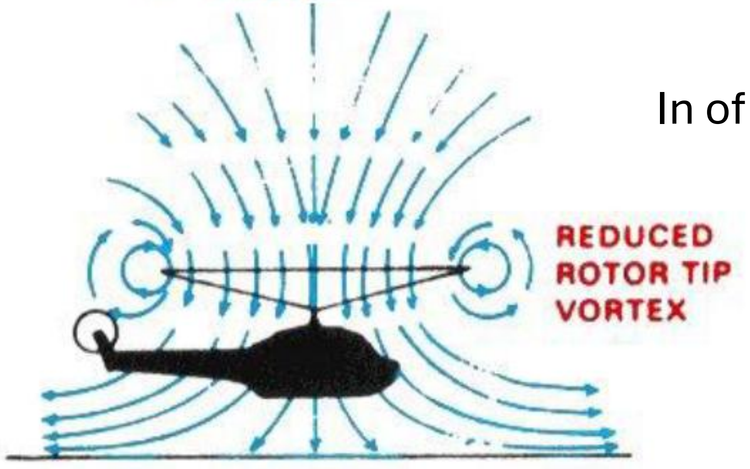


# Operation in Ground Effects

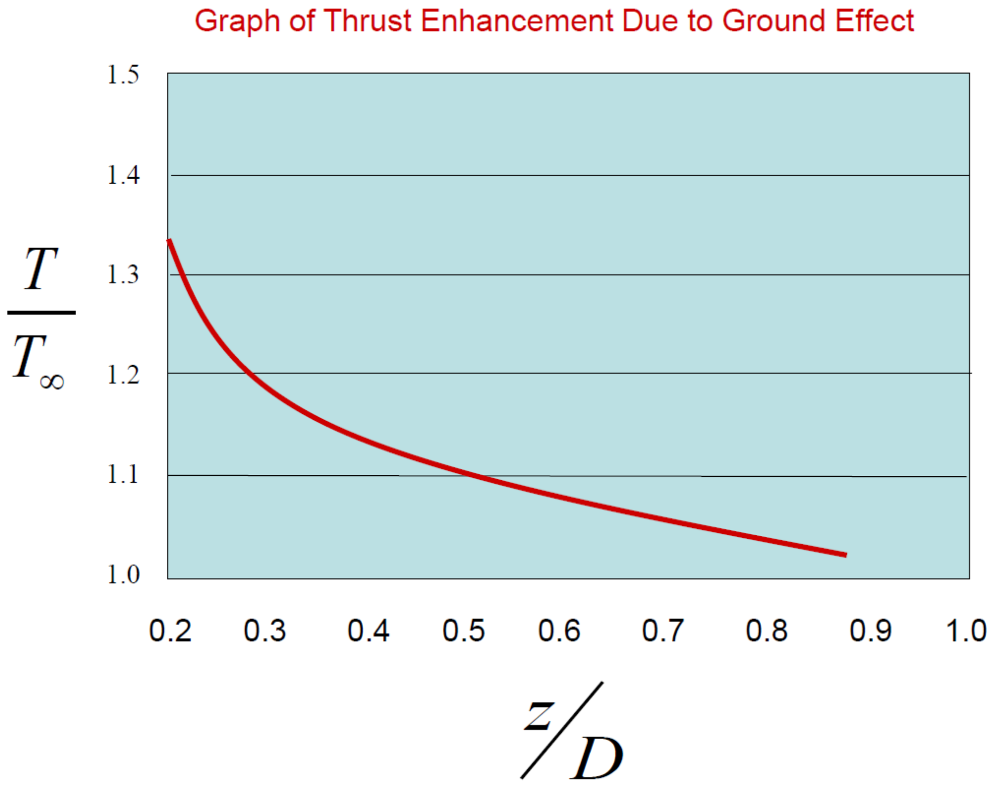
***T***: Thrust with ground effect
***T*****∞**: Thrust without ground effect
***z***: height of rotor from ground
***D***: rotor diameter




Out of Ground Effects (OGE)




In of Ground Effects (IGE)




Or in terms of power for a constant thrust:

$$\frac{P_{IGE}}{P_{OGE}} = k_g(z, D) \leq 1$$

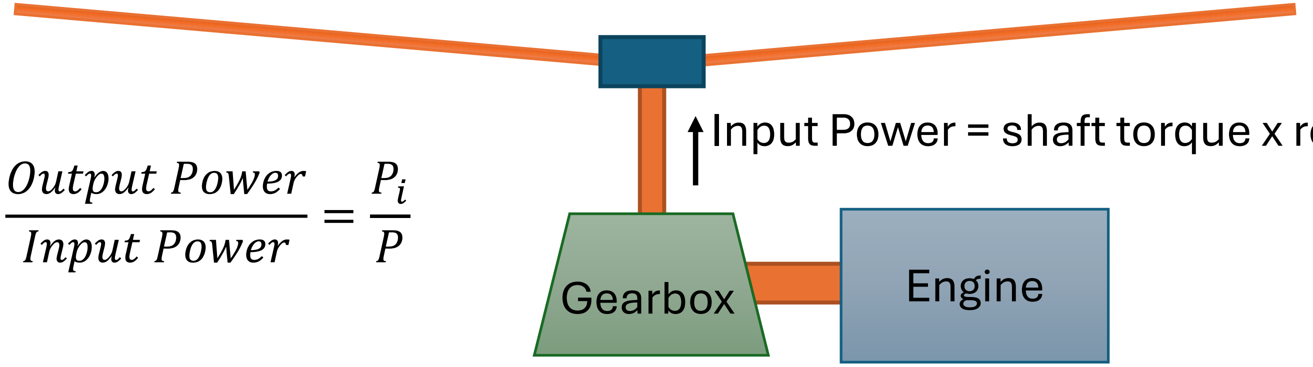
# Rotor Efficiency

Input Power → System with efficiency $\eta$ → Output Power

$$\eta = \frac{Output\ Power}{Input\ Power}$$

For a Rotor:

Output Power (to produce Thrust)




Input Power = shaft torque x rotor speed

$$\eta_{rotor} = \frac{Output\ Power}{Input\ Power} = \frac{P_i}{P}$$

# Rotor Efficiency in the Hover

"**Figure of Merit**" is defined as:

$$\eta_r = FoM = \frac{P_i}{P} = \frac{Tv}{P}$$

$P_i$ is the induced power
$P$ is the rotor shaft power

$$FoM = \frac{T}{P}\sqrt{\frac{T}{2\rho A}} = \frac{T}{P}\frac{1}{\sqrt{2}}\sqrt{\frac{T}{\rho\pi R^2}}$$

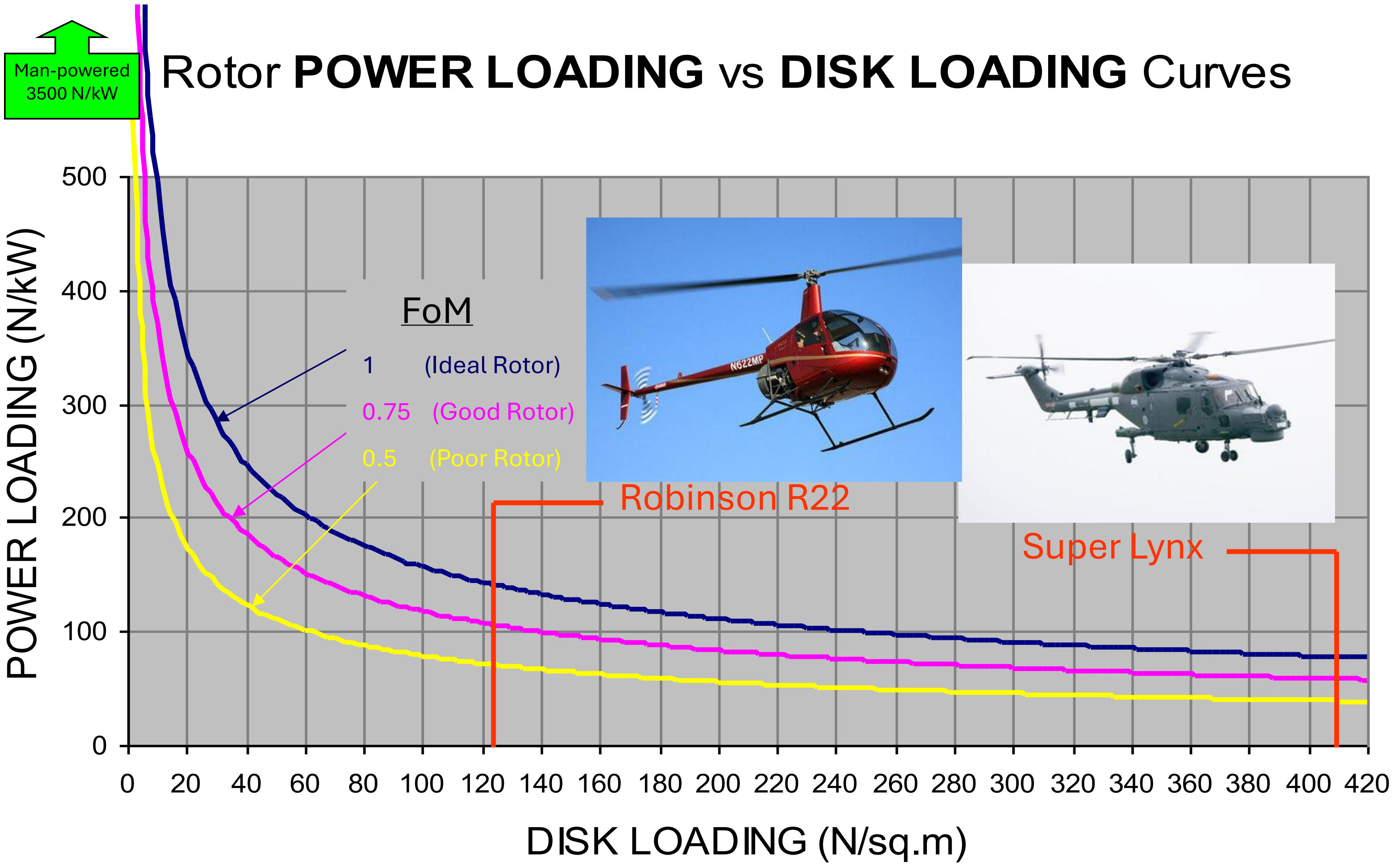
$$FoM = PL\sqrt{\frac{DL}{2\rho}}$$

Where $\frac{T}{P} = \mathrm{PL}$ (known as Power Loading) and $\frac{T}{A} = \mathrm{DL}$ (known as Disk Loading)

Then $$\mathrm{PL} = 1.565 FoM \frac{1}{\sqrt{\mathrm{DL}}}$$

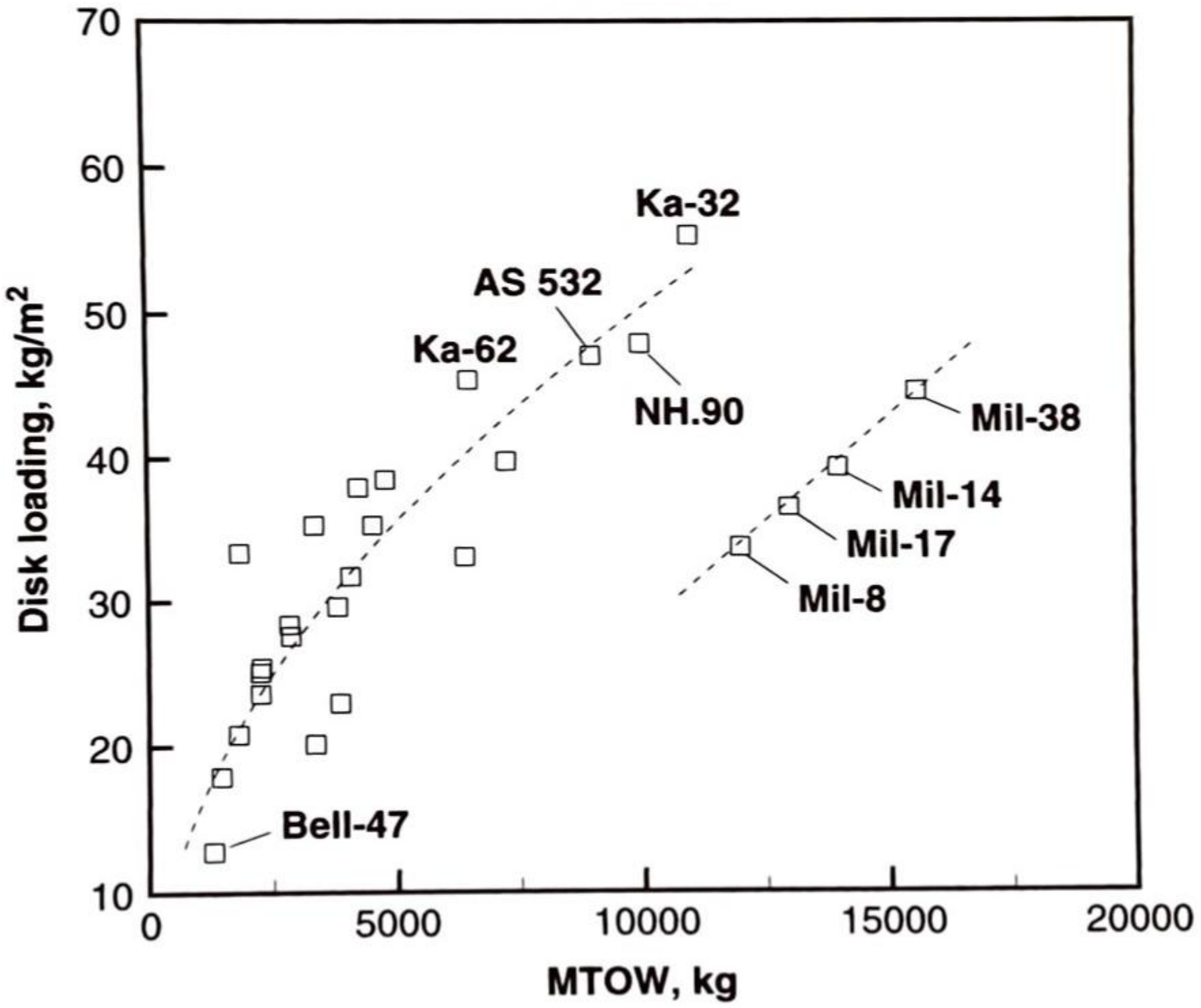
<u>This is Dimensional !</u> (based upon $\rho$ =1.225 kg/m$^3$)

# Power Loading vs. Disc Loading



# Disk Loading (typical values)

- Some highly specialised or stretched larger helicopters have disc loadings greater than **12 *lb/ft²* (~58 kg/m²)**
- A new design for a large transport type of helicopter would favour a lower disc loading to provide margins for in-service weight growth.
- At the small end of the range of helicopter sizes typical disc loading values can be found well below **10 *lb/ft²*** *(typical values* **3-10 *lb/ft²***, **~15-50 kg/m² ).**
- Very light helicopters can have a smaller disc loading.




(Figure adopted from Ref [1])

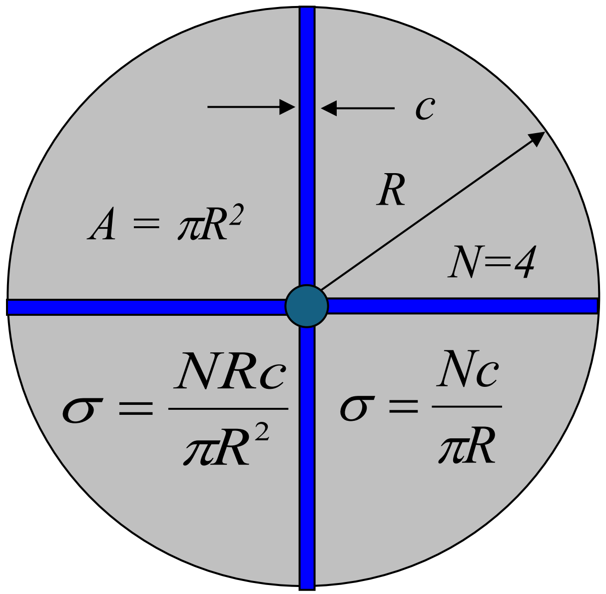
# Rotor Solidity

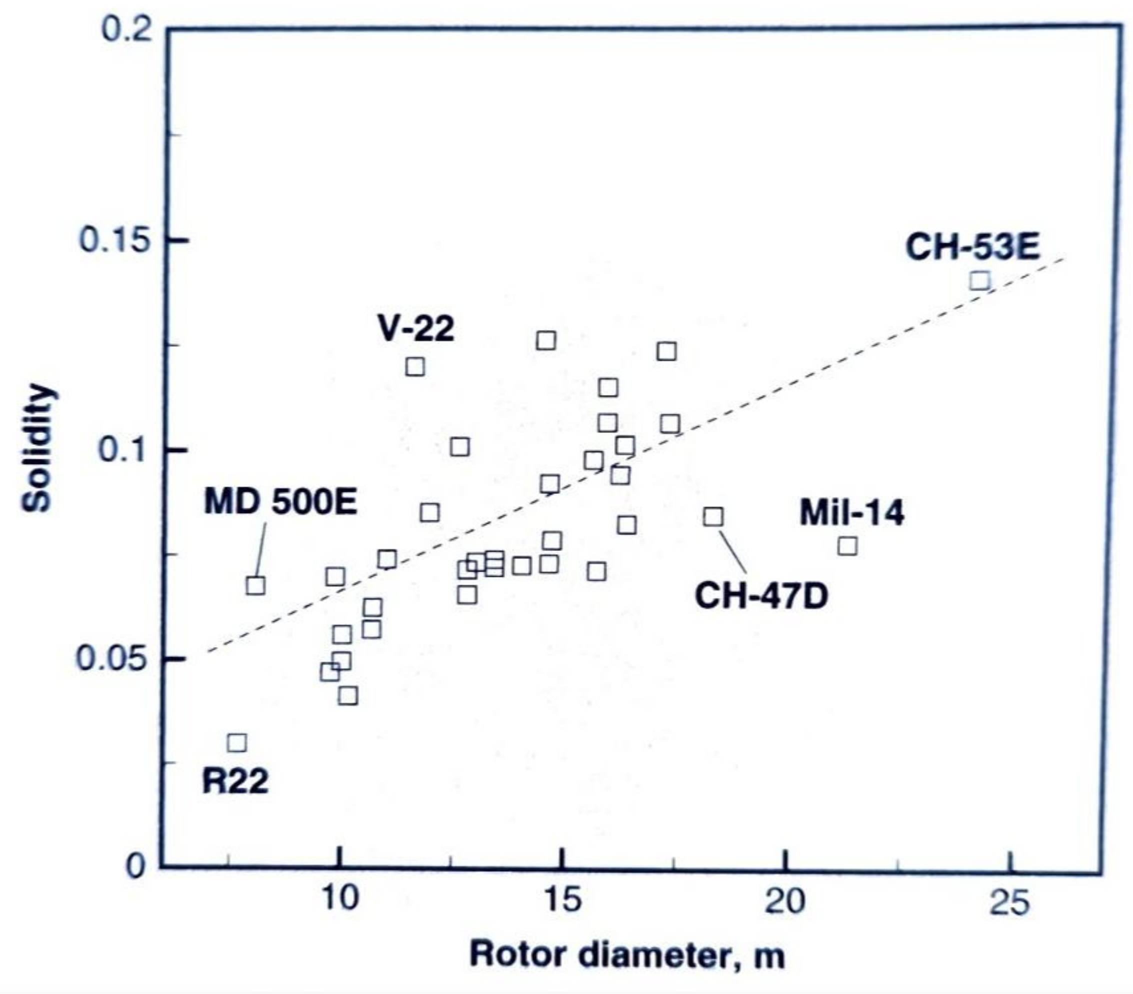
$\boldsymbol{\sigma}$ is the solidity defined as

$$\boldsymbol{\sigma = \frac{A_b}{A} = \frac{Blade\ Area}{Disc\ Area} = \frac{Nc}{\pi R}}$$

*N* is the number of blades

*c* is the blade chord




(Figure adopted from Ref [1])

# Rotor Performance Coefficients

Thrust Coefficient $\boldsymbol{C_T = \frac{T}{\rho A(\Omega R)^2}} = \frac{T}{\rho A V_t^2}$ where $V_t = \Omega R$ is the blade tip speed

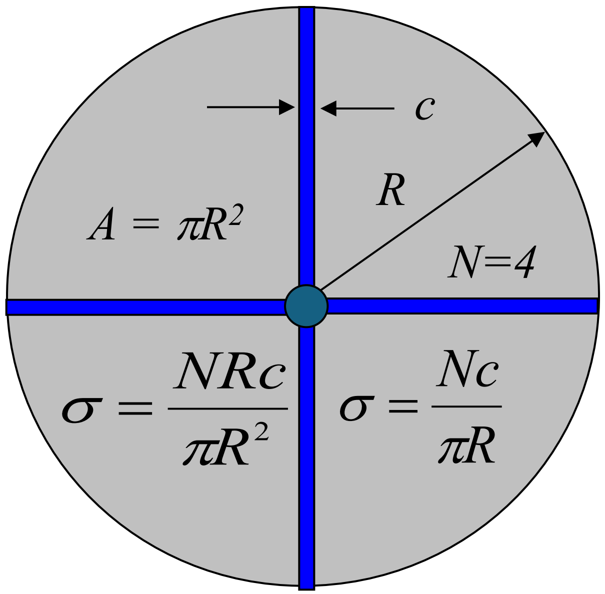
**Blade Loading:**

$$\boldsymbol{C_T/\sigma = \frac{T}{\rho A\sigma(\Omega R)^2} = \frac{T}{\rho NcR(\Omega R)^2} = \frac{T}{\rho NcRV_t^2}}$$

Where $\boldsymbol{\sigma}$ is the solidity

*N* is the number of blades and *c* is the blade chord

$$\boldsymbol{\overline{CL} = 6\ C_T/\sigma}$$



# Rotor Performance Coefficients

**Torque Coefficient** $\boldsymbol{C_Q = \frac{Q}{\rho A\, R(\Omega R)^2}}$ and **Power Coefficient** $\boldsymbol{C_P = \frac{P}{\rho A(\Omega R)^3}}$

Therefore, $C_p = C_Q$ since $P = Q\Omega$

**Advance ratio** : $\mu = \frac{V_F \cos(\alpha)}{\Omega R}$, where $V_F$ is the helicopter forward speed and $\alpha$ is the angle of attack of the rotor

**Inflow ratio**: $\lambda = \frac{V_V + v}{\Omega R}$, where $V_V$ is the helicopter Vertical speed. In Hover $\lambda = \frac{v}{\Omega R}$

# Figure of Merit

- $FoM = \frac{Tv}{P} = \frac{C_T}{C_P}\lambda = \frac{1}{\sqrt{2}}\frac{C_T^{3/2}}{C_P}$

For Ideally Twisted Blade

Induced power coefficient

$$C_P = \boxed{\frac{C_T^{3/2}}{\sqrt{2}}} + \boxed{\frac{\sigma\delta}{8}}$$

Profile power coefficient

where $\delta = \overline{C_D}$ is an average drag coefficient for the blade.

- $\overline{C_D}$ is a function of blade pitch

Therefore, $FoM = \frac{\frac{C_T^{3/2}}{\sqrt{2}}}{\frac{C_T^{3/2}}{\sqrt{2}} + \frac{\sigma\delta}{8}}$

# FoM for general cases

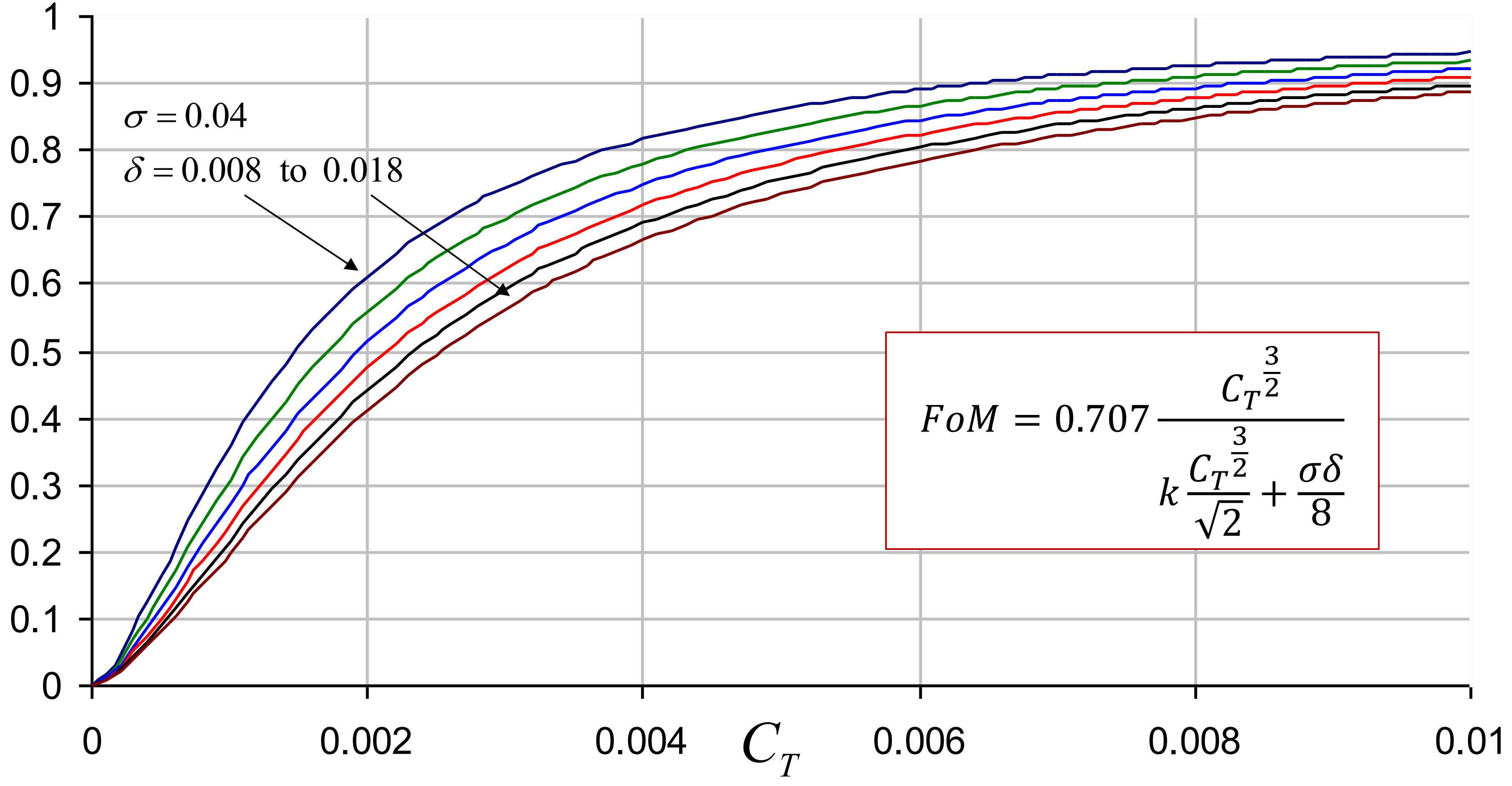
$$C_P = k\frac{C_T^{3/2}}{\sqrt{2}} + \frac{\sigma\delta}{8}$$

$$FoM = \frac{\frac{C_T^{3/2}}{\sqrt{2}}}{k\,\frac{C_T^{3/2}}{\sqrt{2}} + \frac{\sigma\delta}{8}}$$

Where $k$ is the induced power correction factor ($k = 1.1\ to\ 1.15$)

# Maximising the Figure of Merit

Figure of Merit against Coefficient of Thrust for a range of delta's (aerofoil profile drag coefficients) based on a fixed value of rotor solidity ($\sigma = 0.04$).



# Endurance

The fuel flow for a power plant delivering a power *P*

$$m_f = SFC\ P$$

The weight loss due to fuel consumption

$$\frac{dW}{dt} = -\frac{dW_f}{dt} = -g\ SFC\ P = -g\ SFC\ \rho A(\Omega R)^3 C_P$$

We also have

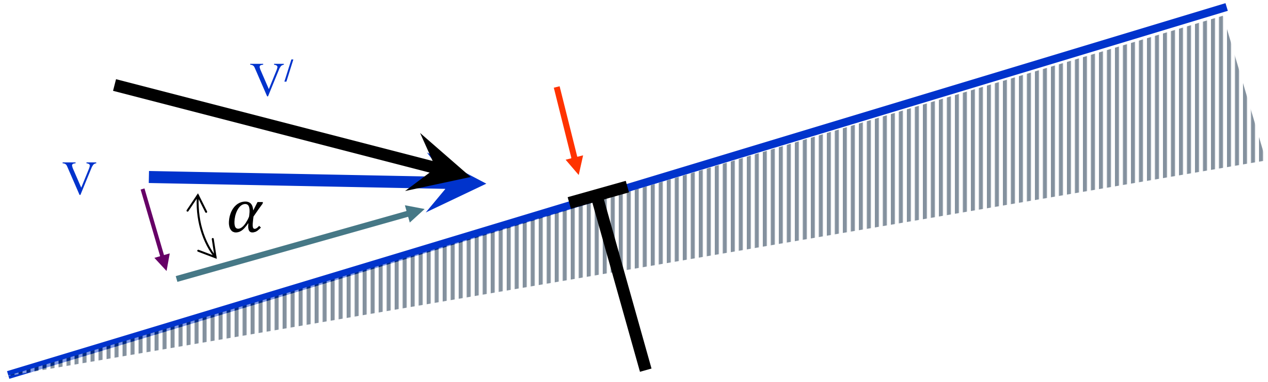
$$\frac{dW}{dt} = \frac{dT}{dt} = \ \rho A(\Omega R)^2 C_T$$

Therefore, the endurance can be given as

$$E = \frac{1}{g\ SFC}\frac{1}{\Omega R}\int_{end}^{Initial} \frac{dC_T}{C_P}$$

# Helicopter Performance in Forward Flight

# Induced Velocity in Translational Flight




In hover, the thrust is $\boldsymbol{T = (\rho A v) 2v}$ and this gives $\boldsymbol{v = \sqrt{\dfrac{T}{2\rho A}}}$

For translational flight, the unit mass flow $\boldsymbol{(\rho A v)}$ has increased as it now includes the translational component of flow through the rotor. So unit mass flow is $\rho A V'$

Where

$$V' = \sqrt{(V\cos\alpha)^2 + (V\sin\alpha + v)^2} = \sqrt{V^2 + 2Vv\sin\alpha + v^2}$$

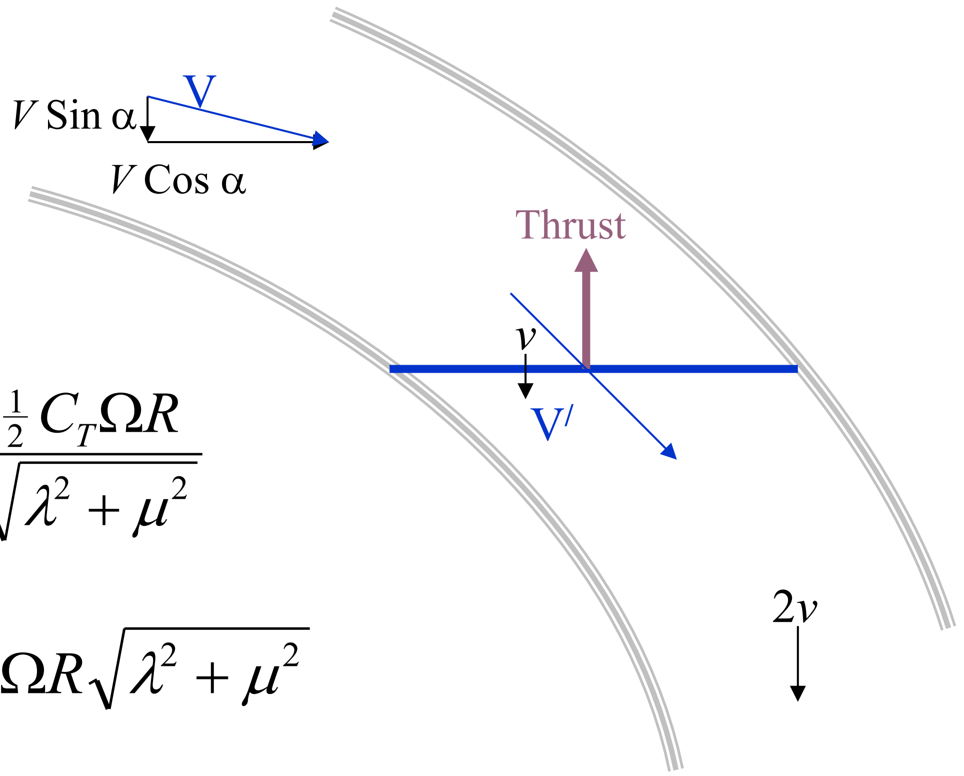
# Induced Velocity in Translational Flight

Thus $V'$ is the vector sum of the translational and induced velocities so in the original thrust equation:

$$\boldsymbol{T = (\rho A V')2v}$$

where

$$V' = \sqrt{(V \sin\alpha + v)^2 + (V\cos\alpha)^2}$$




So $$v = \frac{T}{2\rho A V'}$$

$$v = \frac{\frac{1}{2} C_T \Omega R}{\sqrt{\lambda^2 + \mu^2}}$$

Knowing that $\lambda = \dfrac{V\sin\alpha + v}{\Omega R}$, $\mu = \dfrac{V\cos\alpha}{\Omega R}$ then $V' = \Omega R\sqrt{\lambda^2 + \mu^2}$

Thus $$v = \frac{C_T \rho A (\Omega R)^2}{2\rho A \Omega R\sqrt{\lambda^2 + \mu^2}}$$ or $$v = \frac{C_T(\Omega R)}{2\sqrt{\lambda^2+\mu^2}} = \frac{\lambda_h^2}{\sqrt{\lambda^2+\mu^2}}$$

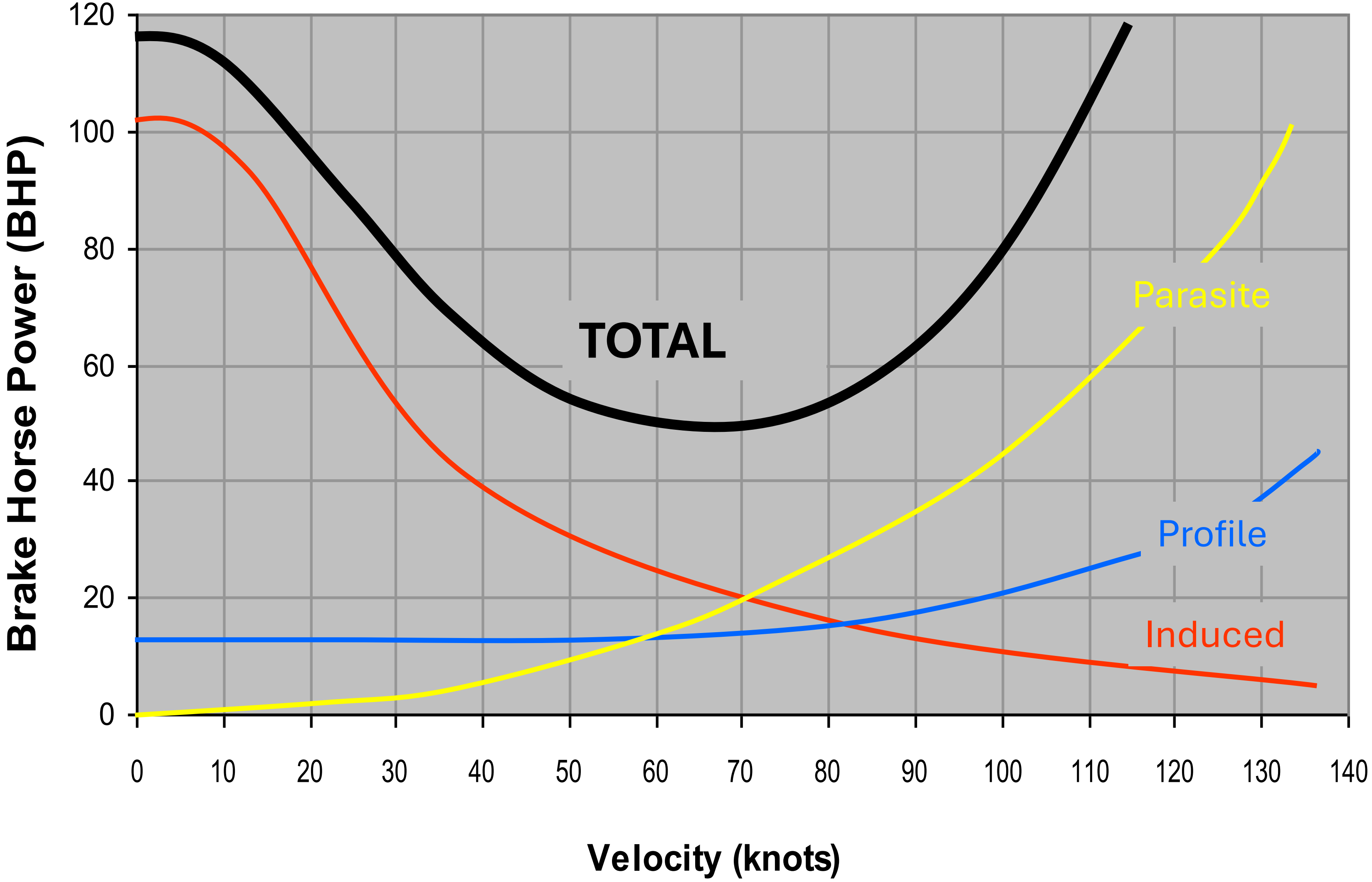
# Power of Main Rotor in Forward Flight

- The main rotor power can be given as:
- $\boldsymbol{P_{MR} = P_i + P_0 + P_p}$
- $P_{MR} = kT_{MR}v + \frac{1}{8}\rho\, V_t^3 NcR\,\overline{C_D}\left\{1 + 4.7\left(\frac{V}{V_t}\right)^2\right\} + \frac{1}{2}\rho V^2 A_f C_{D_f} V$ (From Ref [2])
- $P_{MR} = \underbrace{kT_{MR}v}_{\text{Induced power}} + \underbrace{\frac{1}{8}\rho A\, V_t^3 \sigma\overline{C_D}\left\{1 + 4.7\left(\frac{V}{V_t}\right)^2\right\}}_{\text{Profile power}} + \underbrace{\frac{1}{2}\rho V^2 A_f C_{D_f} V}_{\text{Parasitic power}}$

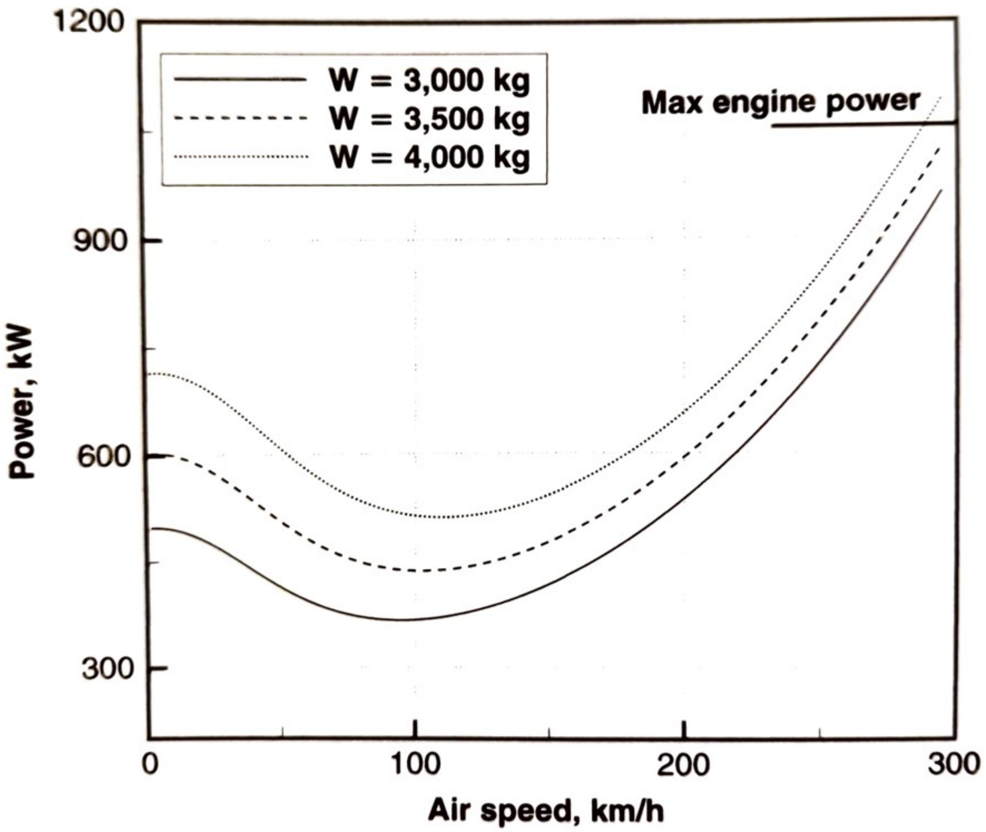
$A_f$ is the reference area for fuselage aerodynamic forces

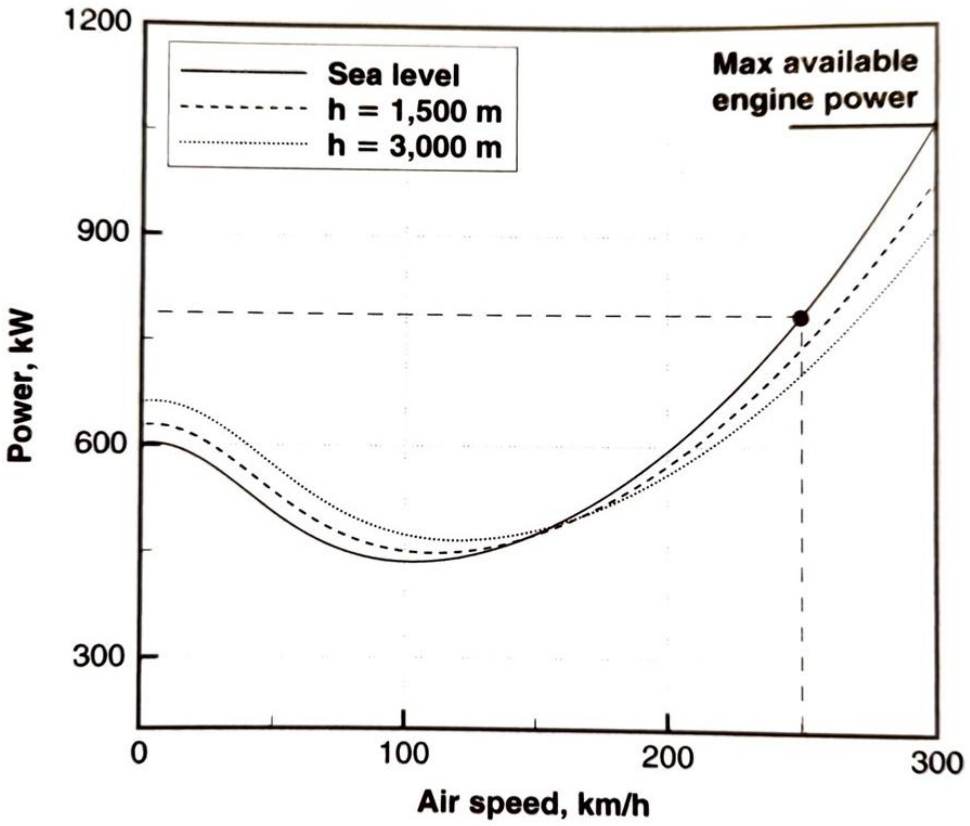
$C_{D\_f}$ is the fuselage drag coefficient based on $A_f$

# Breakdown of Helicopter Power



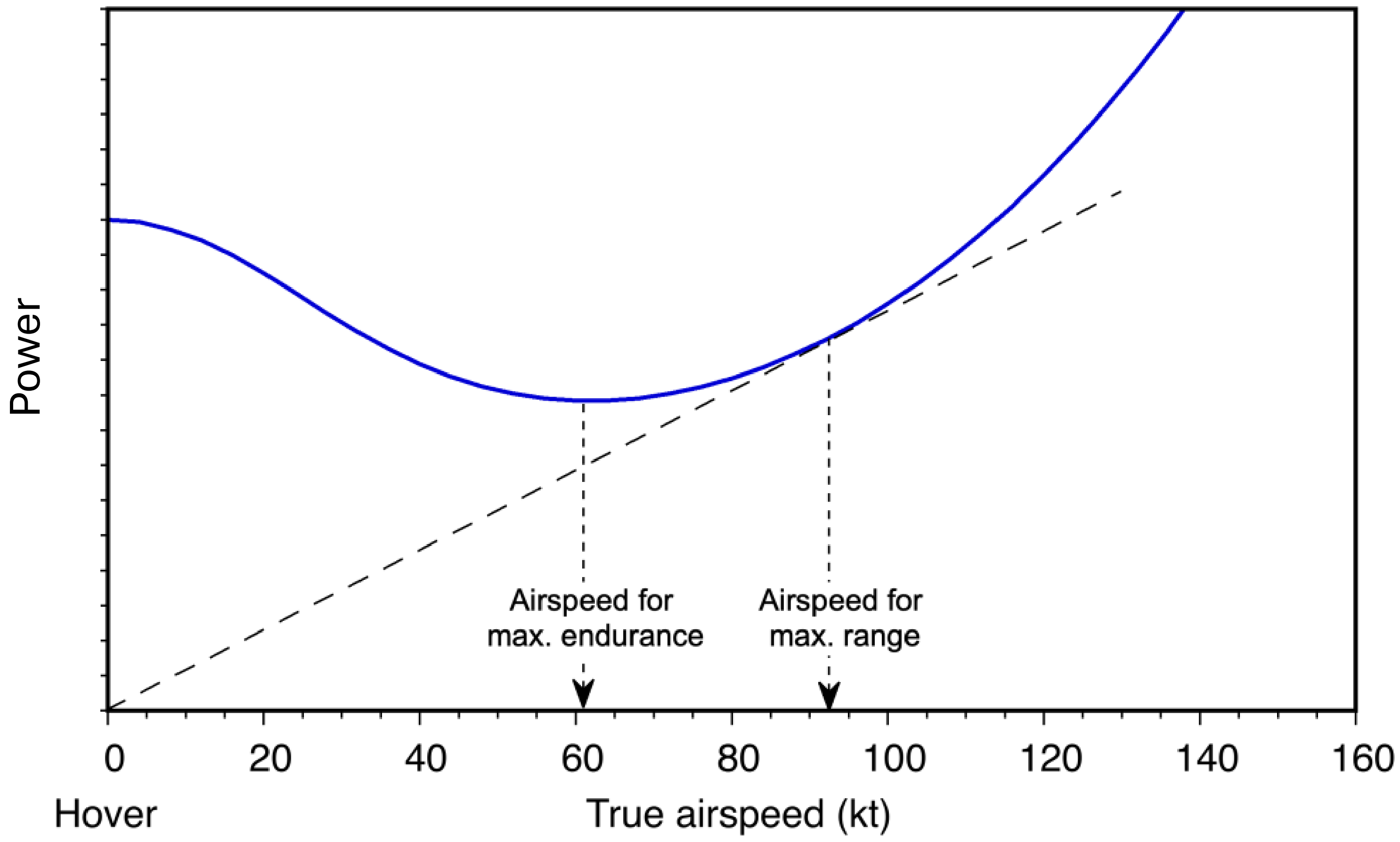
# Effects of Weight and Altitude on Power



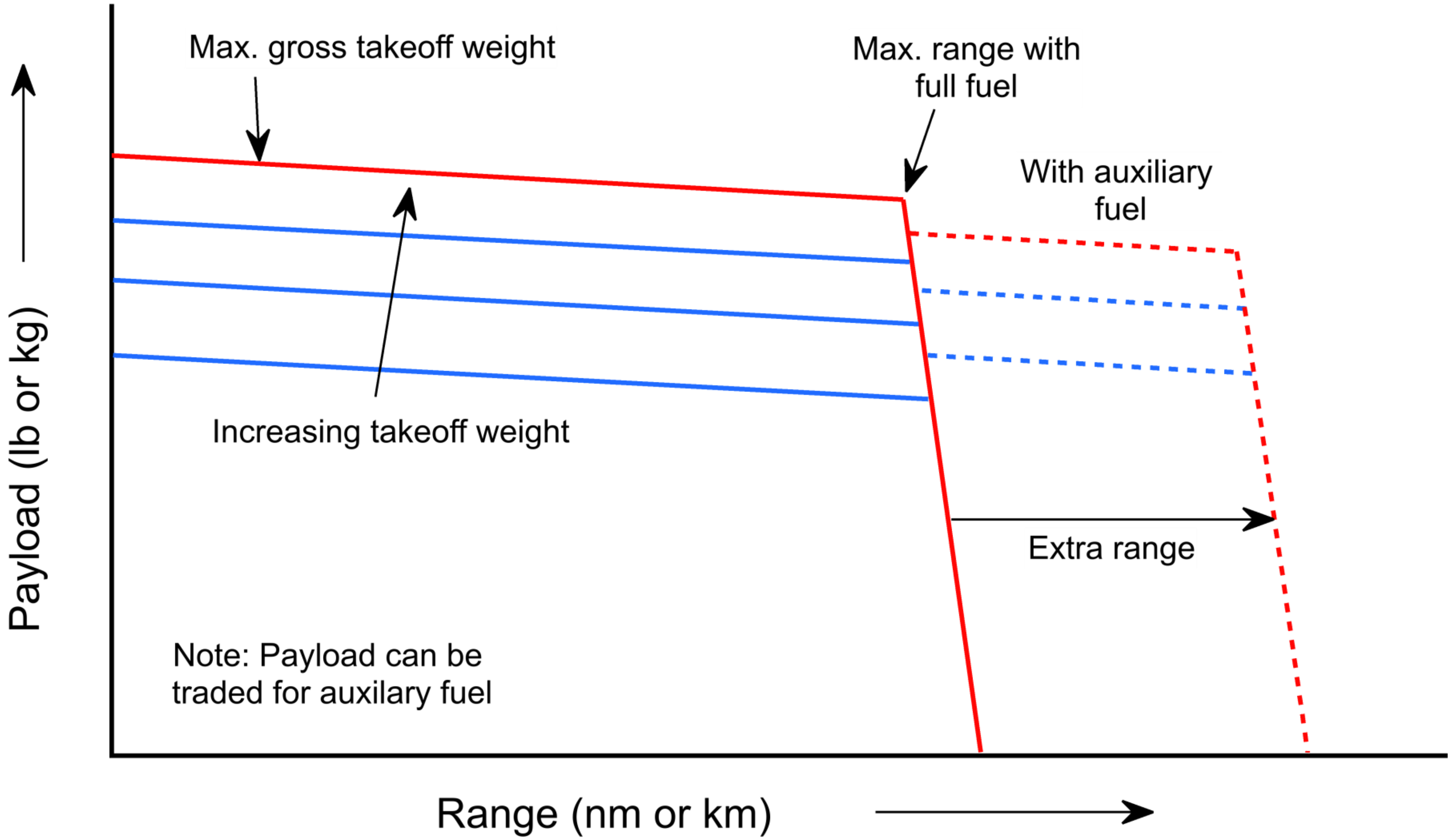




(Figures adopted from Ref [1])

# Maximum Range and Maximum Endurance



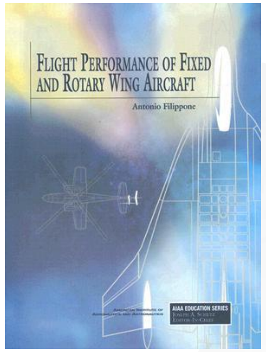
# Payload Range Diagram



# Questions

# References and recommended textbooks


[1] Antonio Filippone, **Flight Performance of Fixed and Rotary Wing Aircraft**, American Institute of Aeronautics and Astronautics, Inc (AIAA) and Butterworth-Heinmann (Elsevier), 2006.



[2] Wieslaw Zenon Stepniewski and  C. N. Keys, **Rotary-wing Aerodynamics**, Courier Corporation, 1984,