ELEVATOR ANGLE TO TRIM

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Tug Aircraft: 141hp EuroFox 2K

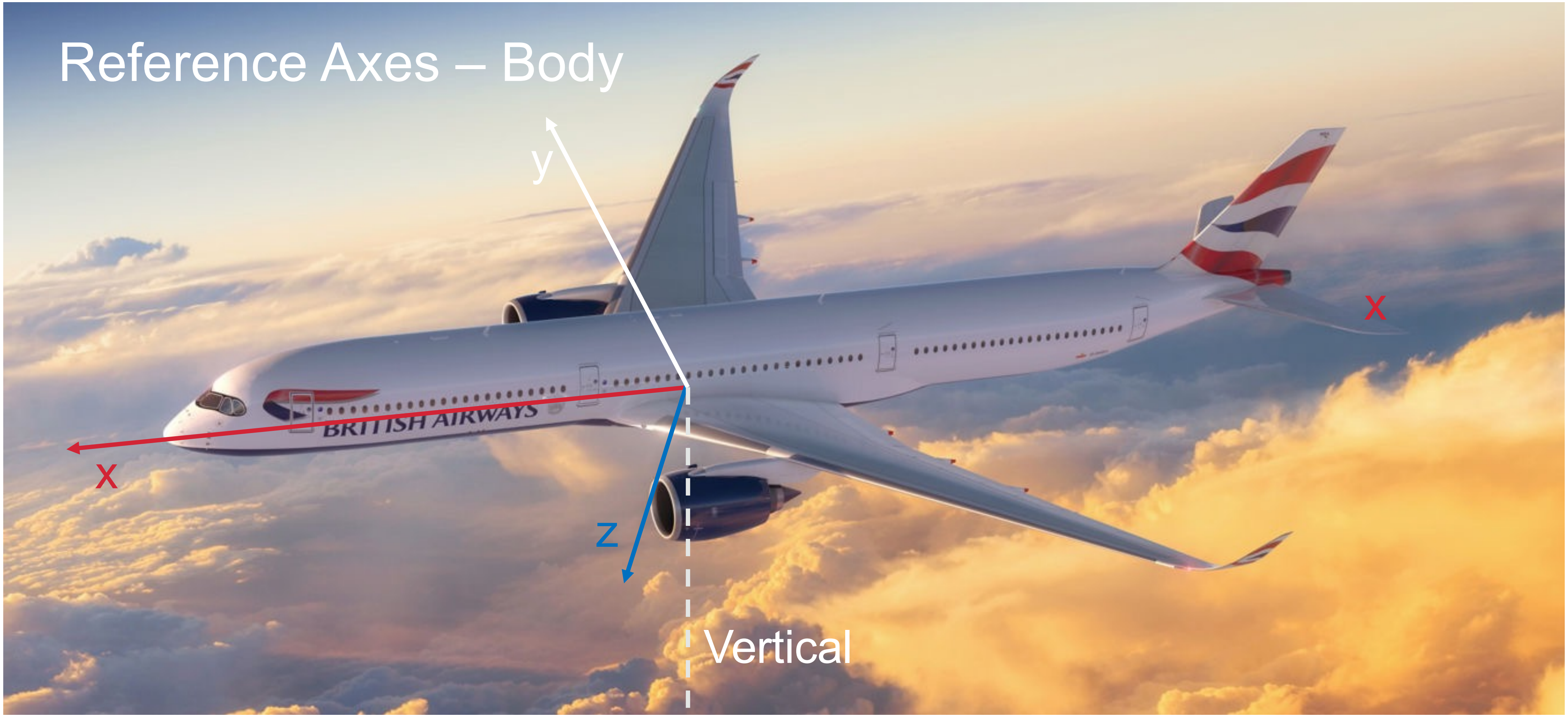
Flight Dynamics Principles – Book References

- Note: some of the nomenclature and derivations differ from those we use in lectures.
- These references are for the 2007 edition.

  Flight Dynamics Principles: A Linear Systems Approach to Aircraft Stability and Control (Elsevier Aerospace Engineering) Hardcover – 9 Aug 2007

- Chapter 3: Static equilibrium and trim
  - *Section 3.1 – Trim equilibrium*
  - *Section 3.2 – The pitching moment equation*
  - *Section 3.3 – Longitudinal static stability*
  - Section 3.4 – Lateral static stability
  - Section 3.5 – Directional static stability
  - Section 3.6 – Calculation of aircraft trim condition

- Chapter 4: The equations of motion
  - *Section 4.1 – The equations of motion of a rigid symmetric aircraft*
  - Section 4.2 – The linearised equations of motion
  - Section 4.3 – The decoupled equations of motion
  - Section 4.4 – Alternative forms of the equations of motion



Airbus A350-1000

https://www.britishairways.com

Tail Configurations

Three types of tail configuration can be used to achieve trimmed conditions across the flight envelope:

a. Fixed tail, plus elevator

- in this case trim is achieved by adjusting the elevator to a datum position, probably different from $\eta = 0$, with only the remaining elevator deflection being available for manoeuvring;
- we shall discuss later how to find the right fixed position for the horizontal tail (permanently built-in) – the *tail setting angle.*

Tail Configurations

b. Trimming tail, plus elevator

- the *whole* horizontal tail can be rotated in pitch to achieve trim with $\eta = 0$; the elevator is then used only for additional (more rapidly changing) control (manoeuvring).

c. All-flying tail, no elevator

- the datum position is selected for trim; control is then effected by additional movements of the whole tail away from this datum value.

Downwash and Wake Effects at the Tail

Equation for the Elevator Angle to Trim?

Local Airspeed

- The velocity over the tail will not normally be the same as the free stream. Note: T-tails are less affected.
- In general there will be a reduction in airspeed due to wing and fuselage drag.
- In contrast, there will be an increase in airspeed from the engine/propeller slipstream (short take-off video).
- We can account for these effects using an 'efficiency factor' η_{T}. This can be modelled as:

$$U_T^2 = \eta_T U^2$$

but η_{T} is often simply taken to be 1.0

(assume to be 1.0 in the exam/assessment unless told otherwise).

Local Airspeed

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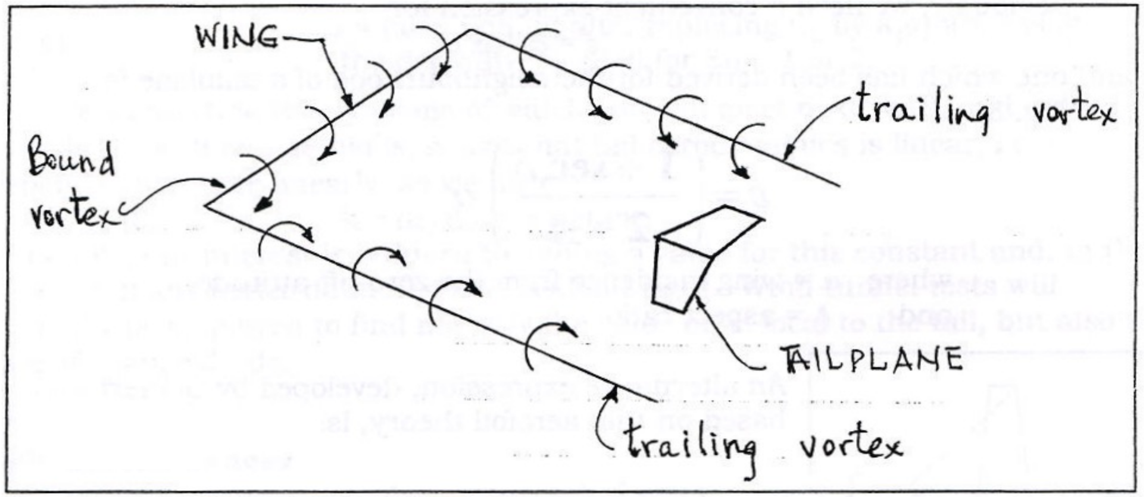
Downwash and Wake Effects at the Tail

- Downwash created by vortices shed downstream from the main wing can alter the flow over the horizontal tail.

- Consider the flow field at the horizontal tailplane, where the wing wake is well-developed – we would expect that downwash would be significant.



A wing will produce a downwash field for the tailplane from a (wing) wake that stretches from upstream to downstream of the tail.

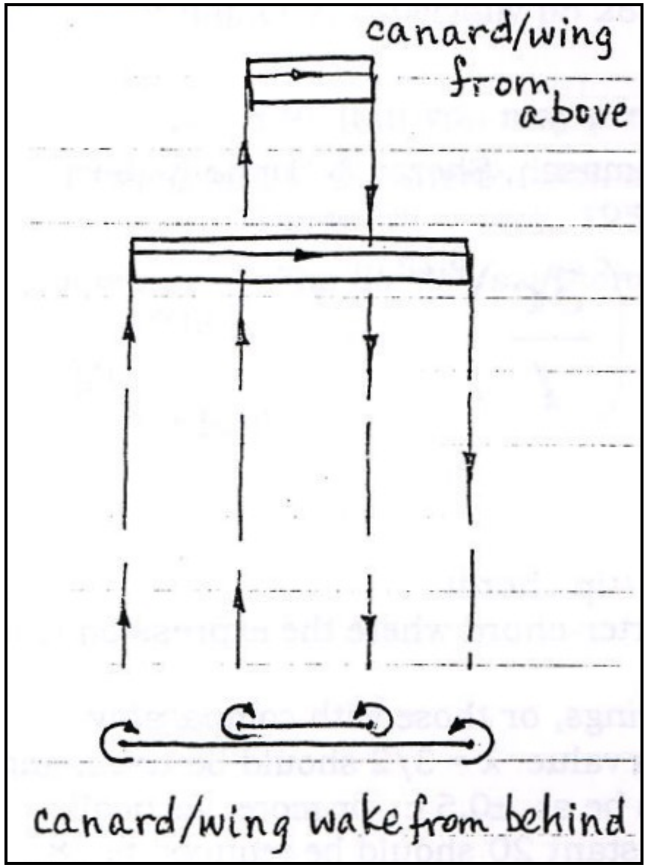
Downwash and Wake Effects at the Tail

- The angles of attack seen by the two lifting surfaces are not going to be equal and the distribution of α across the two surfaces from tip to tip is not going to be equal. (*More on this in subsequent units* – this year we will consider a representative point.)
- Generally, the forward surface (while it is developing lift) will have a downward influence on the flow downstream so that the rearward surface 'sees' a 'lesser α'.
- The lifting wing induces a downwash field in its wake.
- Downwash also changes quite noticeably with wing angle of attack.

> *The change of angle is referred to as the downwash angle ε (epsilon).*

The Downwash Field

- A wing will normally affect the whole of a tailplane with the wider wake causing downwash over the full span of the tail.
- The effect of a canard surface will vary depending on whether the point considered is within or without the span of the canard.
- A canard will produce additional downwash within the span of the canard
- *but* upwash (leading to increased α) outboard of its span.
- *We will consider the conventional configuration.*



The Downwash Field

https://fyfluiddynamics.com/

Simple Downwash Models for the Tail

- Downwash at the tail is directly proportional to the lift being developed at the wing.
- In order to express the effective tail angle of attack α_T we need to express ε (loss of 'incidence') at the tail in terms of α_{wing}.
- An approach is to employ a term $\alpha(1-k)$, where:

$$k = \partial\varepsilon/\partial\alpha \qquad (1)$$

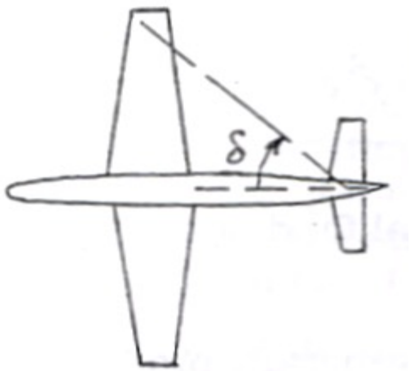
- Therefore $$\alpha_T = \alpha - \frac{\partial\varepsilon}{\partial\alpha}\alpha$$

  - where α is actually α_{wing}, from the zero-lift attitude;
  - both of these terms become zero when there is no wing lift.

Simple Downwash Models for the Tail

- We need an expression for: $\varepsilon = \frac{\partial \varepsilon}{\partial \alpha}\alpha$ (2)

- One model which represents the downwash at the tailplane is:



$$\varepsilon = \left(\frac{1 + \sec\delta}{2 + A}\right)\alpha \qquad (3)$$

(No need to memorise)

where α = wing incidence from the zero-lift attitude

and A = aspect ratio.

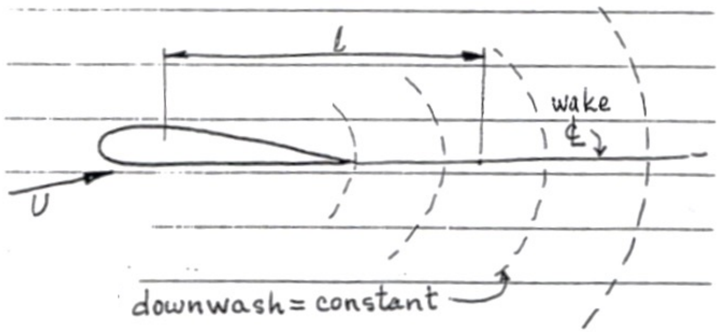
Simple Downwash Models for the Tail

- An alternative expression, developed by Glauert and based on thin aerofoil theory, is:

$$\varepsilon = \left(\frac{4}{\pi}\right)^2 \left(\frac{1+\sec\delta}{2+A}\right)\alpha \qquad (4)$$

- An alternative empirical expression is:

$$\varepsilon = 20C_L \frac{\lambda^{0.3}}{A^{0.725}} \left(\frac{3c}{l}\right)^{0.25} \qquad (5)$$



where: ε is in degrees

c is the mean wing chord

λ is the wing taper ratio (root chord/tip chord)

l is the distance aft of the wing quarter-chord

Simple Downwash Models for the Tail

- Different approximations can be made for different tail positions and wing shapes.

- A linear approximation can be used for ε dependency on α, so we have:

$$k = \frac{\partial \varepsilon}{\partial \alpha} = \text{constant}$$

- In the absence of better data, this constant can be approximated with $k = 0.5$.

Elevator-Angle-to-Trim

Elevator-Angle-to-Trim

The three equations valid in the plane of symmetry are:

- transverse: $$L_w + L_T = nW \quad (6)$$
- axial: $$F = D \quad (7)$$
- pitch: for the moment we shall choose the a.c. of the wing to take the moments about, leading to:

$$M_{ac} = M_0 + nWx\bar{c} - L_T l_T \quad (8)$$

$$= 0 \quad \text{for equilibibrium}$$

Elevator-Angle-to-Trim

Introduction

To achieve longitudinal balance we need to satisfy equations (6) & (8), and trim the aircraft to meet the following requirements:

a) fly fast enough to produce the required lift,

b) achieve an angle of attack, related to the flight speed, to satisfy the conditions in Eqn. (6); since the wing lift is dominant we tend to think of this as a requirement for wing α though it is not that simple, i.e. need to consider the distribution of lift related to (c).

Elevator-Angle-to-Trim

(c) to achieve the pitch balance to satisfy Eqn. (8) by adjusting the tail lift;

- Noting that adjustment of tail lift requires an equal and opposite change in wing lift to keep the balance given by Eqn. (6).
- We will use the elevator to change the tail lift and achieve the required balance.

Hence, we need to be able to calculate η (required elevator deflection) for a given speed.

And we will assume a fixed and ‘sufficient’ speed of flight.

Elevator-Angle-to-Trim

$U \rightarrow$ Fixed

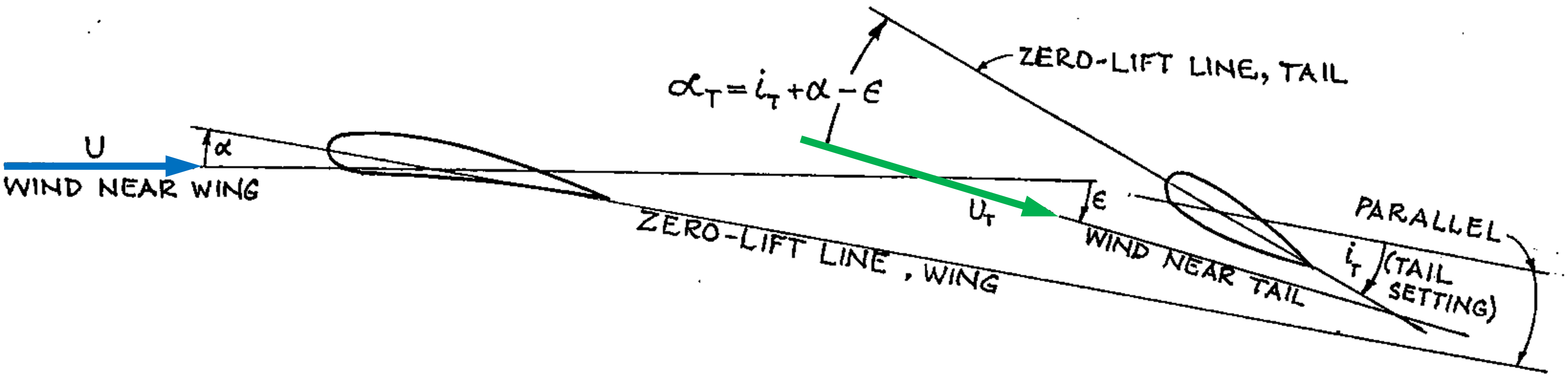
$\alpha \rightarrow$ Variable

$\eta \rightarrow$ Variable

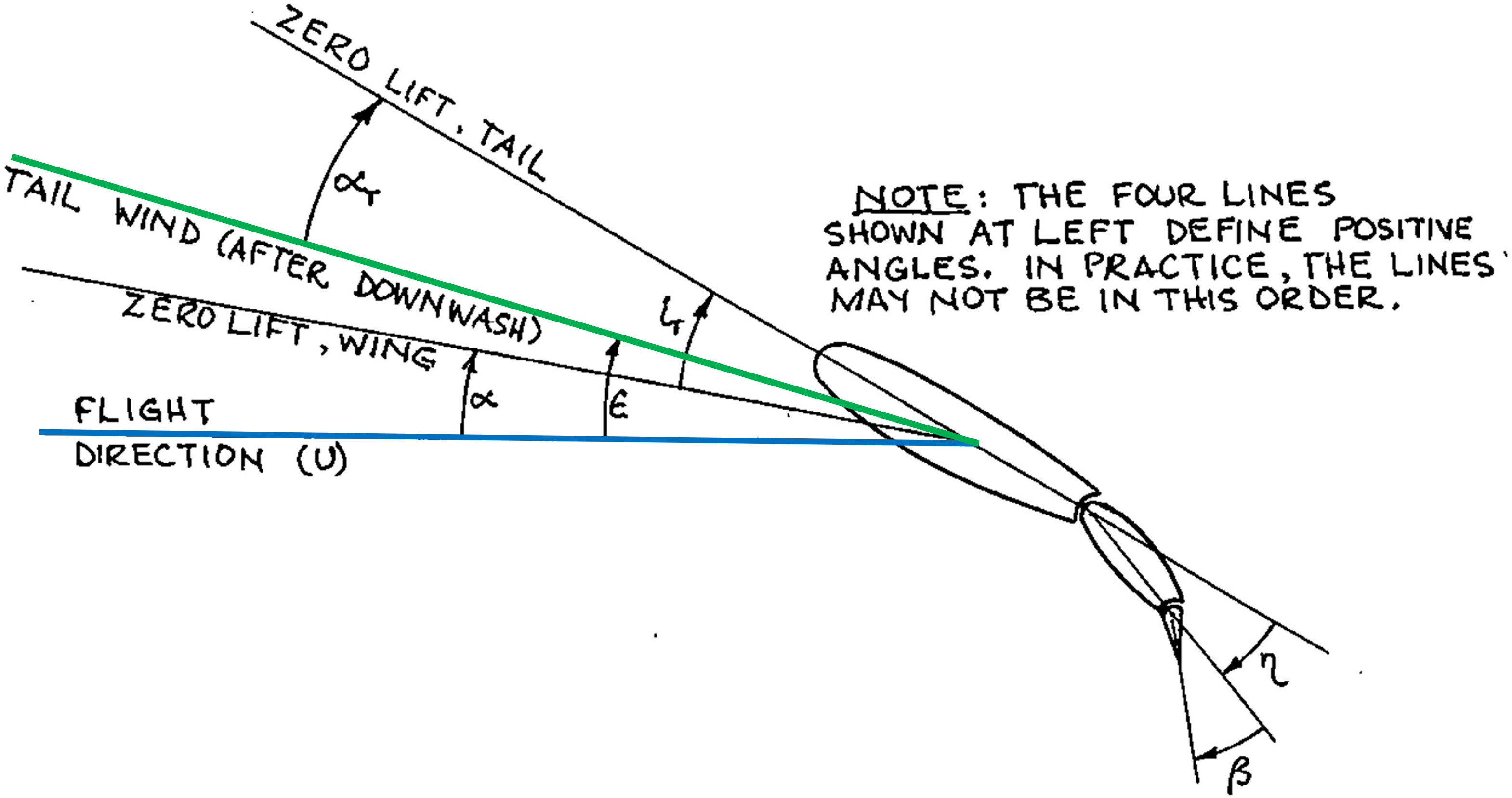
$i_T \rightarrow$ Variable (tail setting angle)

An Expression for the Elevator Angle η

- Assume a conventional configuration.
- Note that the tailplane is attached to the fuselage with a different basic incidence, greater than the wing incidence by the setting angle i_T.



An Expression for the Elevator Angle η



An Expression for the Elevator Angle η

- Thus, as illustrated on the previous slide, effective angle of attack at the tail is:

$$\alpha_T = i_T + \alpha - \varepsilon \tag{9}$$

w subscript implicit *i.e. wing angle of attack*

- The downwash from the wing, which alters the angle of attack at the tail, is proportional to C_{L_W} and this in turn is proportional to α so we can put

$$\varepsilon = \frac{\partial \varepsilon}{\partial \alpha} \alpha \tag{10}$$

- which implies exactly what was stated in the previous section on downwash/wake, namely that

$\varepsilon = 0$ for $\alpha = 0$ (no wing lift).

An Expression for the Elevator Angle η

Eqn. (9) above then becomes

$$\alpha_T = i_T + \alpha\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right) \tag{11}$$

$$= i_T + \alpha(1 - k)$$

where $$k = \frac{\partial\varepsilon}{\partial\alpha}$$

Approach in design could be, e.g., to find/choose i_T; then fix i_T and find η for a new trim condition.

An Expression for the Elevator Angle η

- Then, to involve the elevator explicitly, we write the tail lift coefficient as:

$$C_{L_T} = a_{1_T}\alpha_T + a_{2_T}\eta$$
$$C_{L_T} = a_{1_T}(i_T + \alpha[1-k]) + a_{2_T}\eta \qquad (12)$$

- which implies a symmetric section for the horizontal tailplane, having $a_{0_T} = 0$.

- We can now use Eqn. (12) from Session 9, the moment equation, to solve for balance but be careful about the choice of reference point – a.c. versus c.g.

Choice of Lift Coefficient C_L

- Eqns. (11) & (12) from the Session 9 slides show that we should really use the total C_L, with the consequence that:

$$C_{M_{ac}} = C_{M_0} + xC_L - \bar{V}C_{L_T} \qquad (13)$$

$$= C_{M_0} + x\left[C_{L_W} + \frac{S_T}{S}C_{L_T}\right] - \bar{V}C_{L_T}$$

$$= C_{M_0} + xC_{L_W} + \left(x\frac{S_T}{S} - \bar{V}\right)C_{L_T}$$

- and this, rather inconveniently(!), makes both the C_{L_W} term and the C_{L_T} term dependent on x.

Choice of Lift Coefficient C_L

Consider the factor $\left(x\frac{S_T}{S} - \bar{V}\right)$:

Examples of the values of the two terms in this expression for a few civil aircraft are:

	Twin Otter	Cessna 402B	Airbus	B-747
$\bar{V}$	1.155	0.958	1.06	1.0
$x\,S_T\,/\,S$	0.013 to 0.02	0.029 to 0.05	0.025 to 0.04	about ±0.025

The implication is that relative to $\bar{V}$ (the upper row of the table), $x\frac{S_T}{S}$ (lower row in the table) can be ignored without great loss of accuracy.

Moment Balance Using η

- One of the virtues in simplifying Eqn. (13) to a near copy of the first form shown above, namely:

$$C_{M_{ac}} = C_{M_0} + xC_{L_W} - \bar{V}C_{L_T} \qquad (13a)$$

- is that we might want to take partial derivatives of the kind

$$\frac{\partial}{\partial \alpha} \quad \text{or} \quad \frac{\partial}{\partial C_L}$$

stability?

- Note: $C_{L_W} = a_1 \alpha$

subscript w implied

Elevator-Angle-to-Trim

- Now we can combine Eqn. (12) (lift acting on the tail) & (13) (moment-balance):

$$C_{M_0} + xC_{L_W} - \bar{V}a_{1_T}(i_T + \alpha[1-k]) - \bar{V}a_{2_T}\eta = 0 \qquad (15)$$

- Note that we have terms in C_{L_W} and in α that could be combined. We replace α with C_{L_W}/a_1 and gather terms in C_{L_W} to obtain:

$$C_{M_0} - \bar{V}a_{1_T}i_T - C_{L_W}\left[\bar{V}\frac{a_{1_T}}{a_1}(1-k) - x\right] - \bar{V}a_{2_T}\eta = 0 \qquad (16)$$

Elevator-Angle-to-Trim

and thus the elevator angle required to obtain this trim is:

$$\boxed{\eta_{trim} = \frac{1}{\bar{V} a_{2_T}} \left\{ C_{M_0} - \bar{V} a_{1_T} i_T - C_{L_W} \left[\bar{V} \frac{a_{1_T}}{a_1} (1 - k) - x \right] \right\}} \qquad (17)$$

which displays, among other things, that the elevator-angle-to-trim is a linear function of C_{L_W}.

Choosing i_T

- Generally, for a fixed tailplane, i_T will be chosen to obtain trim in cruise without having to use the elevator, i.e. η will be nearly zero.

- Obviously, changes in speed and c.g. position alter the need for η so, at best, the value of i_T that requires $\eta=0$ can be correct at only one (instantaneous) condition.

- For an adjustable horizontal tail, the whole cruise can be flown at $\eta=0$ while i_T is adjusted continually.

Named Laima, after the ancient Latvian deity of good fortune, this little plane set a record by flying across the Atlantic without a pilot – 71 years after Lindbergh's historic solo flight.

On August 21, 1998, Laima became the first unmanned aircraft to cross the North Atlantic. The crossing was completed within 15 minutes of schedule after a flight of 2,044 miles in a time of 26 hours and 45 minutes.

Insitu Aerosonde Laima:
https://www.museumofflight.org/aircraft/insitu-areosonde-laima

Launch weight 13.1kg

https://www.youtube.com/watch?v=KSVjJjOK-4Y

Next Session

Static Stability

Gossamer Albatross arrives at France, 12 June 1978. (AeroEnvironment, Inc.)

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