

STRIP THEORY II

Prof. Mark Lowenberg & Prof. Tom Richardson

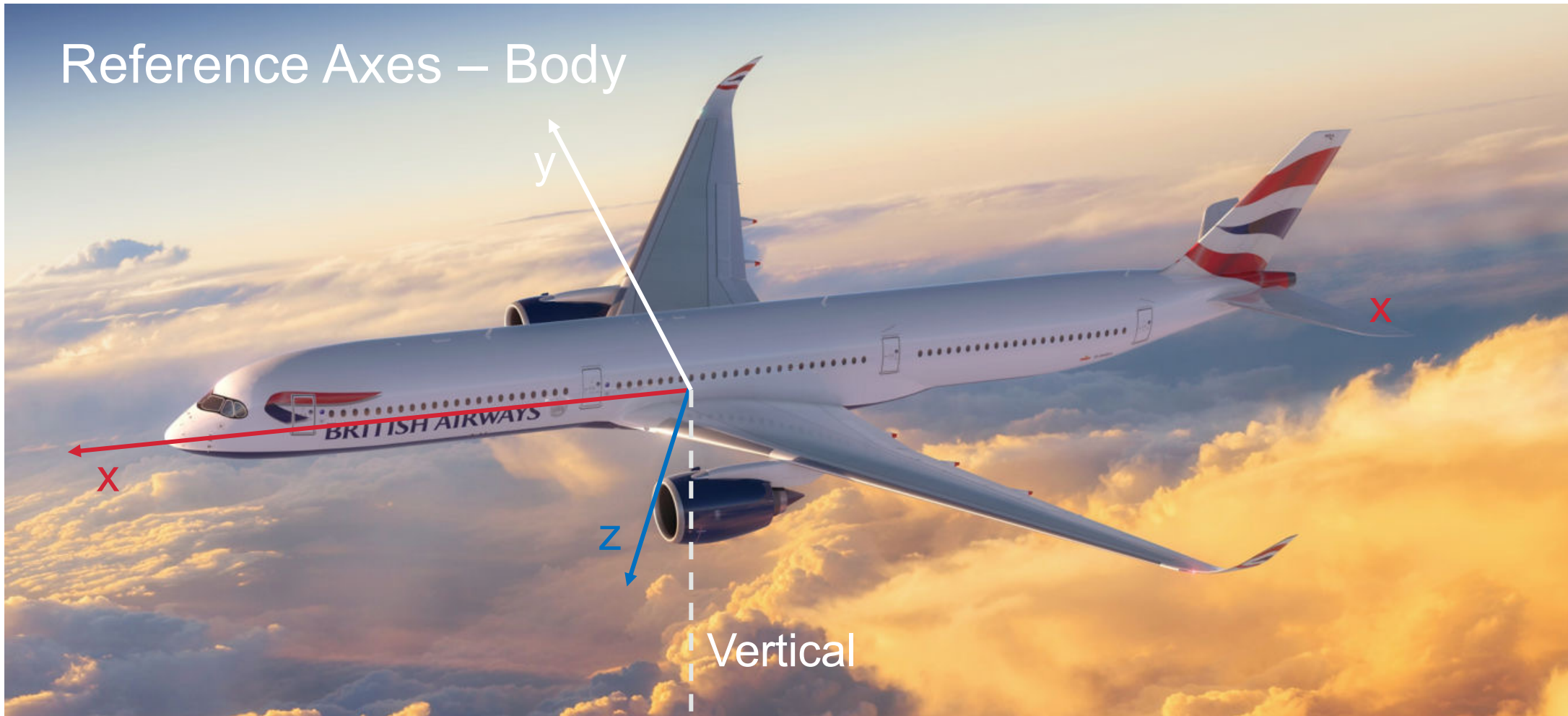
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Bristol and Gloucestershire Gliding Club
Tug Aircraft: 141hp EuroFox 2K



Reference Axes – Body



Airbus A350-1000

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Example Assignment Question from a *Previous Year*

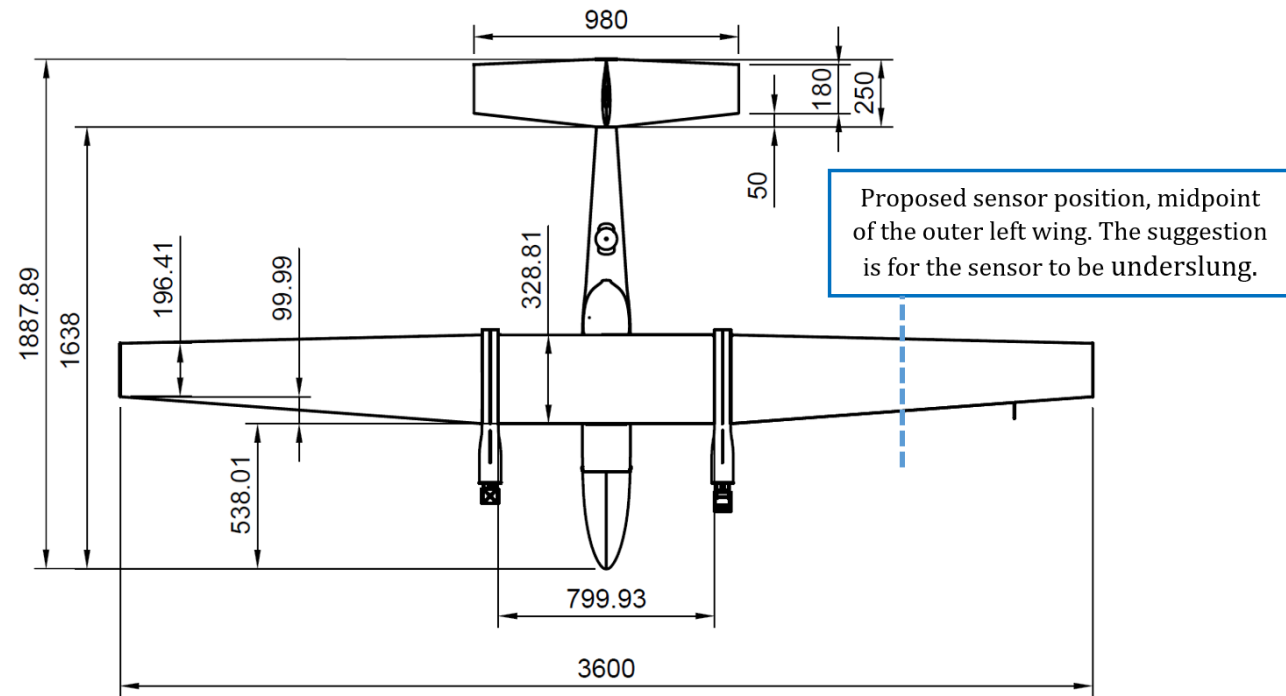


Figure 3: University of Bristol BUDDI plan view (dimensions given in mm)

Figure 3 shows a plan view of the University of Bristol 'BUDDI' aircraft with key dimensions given. One of our academic partners has asked to fly a sensor at the midpoint of the left outer wing section. This sensor has mass m_{sens} and a drag coefficient C_{Dsens} .

Using your knowledge of strip theory, develop the equations required to calculate the control surface deflections to balance out the effect of this sensor in flight. Include assumptions, simplifications, and any relevant parameters you might need. You are not required to solve the equations.

Strip Theory Aerodynamics (last session)

- In practice we can allow for all normal local lift-producing mechanisms, including a trim tab:

$$C_l = a_0 + a_1 \alpha + a_2 \delta + a_3 \beta$$

where the generalized control surface angle δ will specifically be one of ξ , η , ζ .

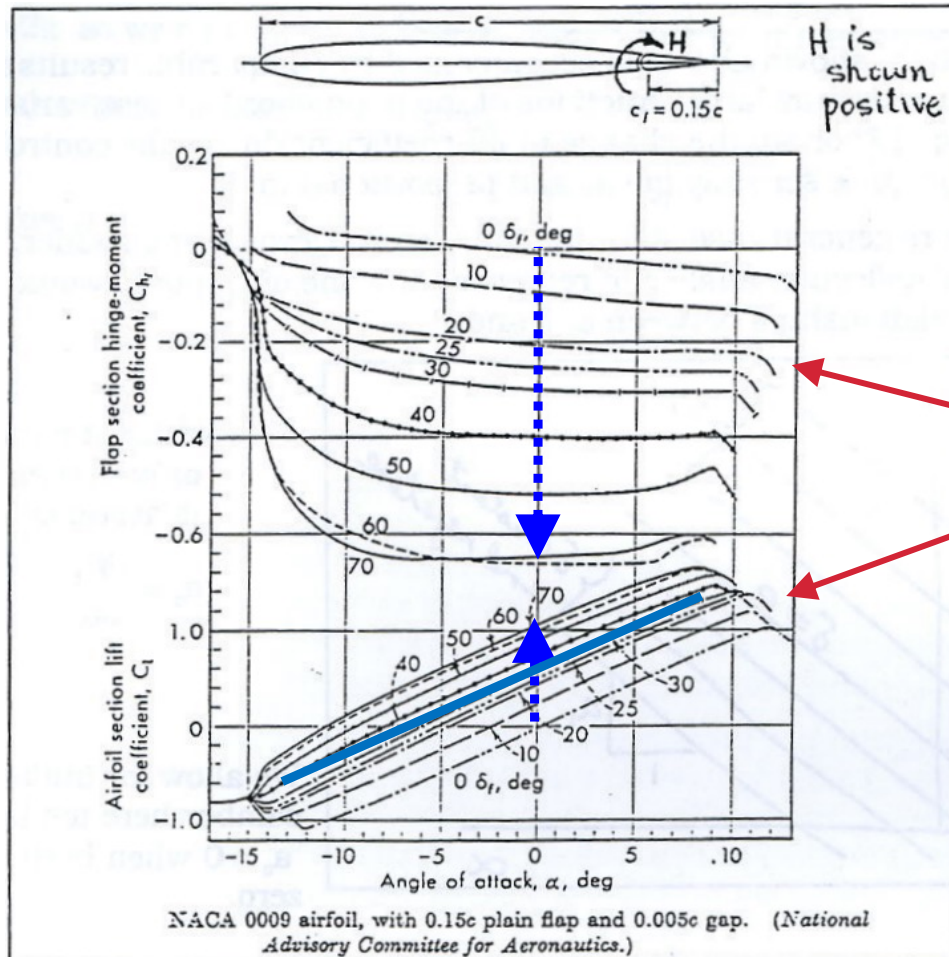
Note: β here is the control tab deflection

$$C_L \equiv \frac{L}{q S} = \frac{L}{\frac{1}{2} \rho u^2 S}$$

Effect of Control Surface Deflection

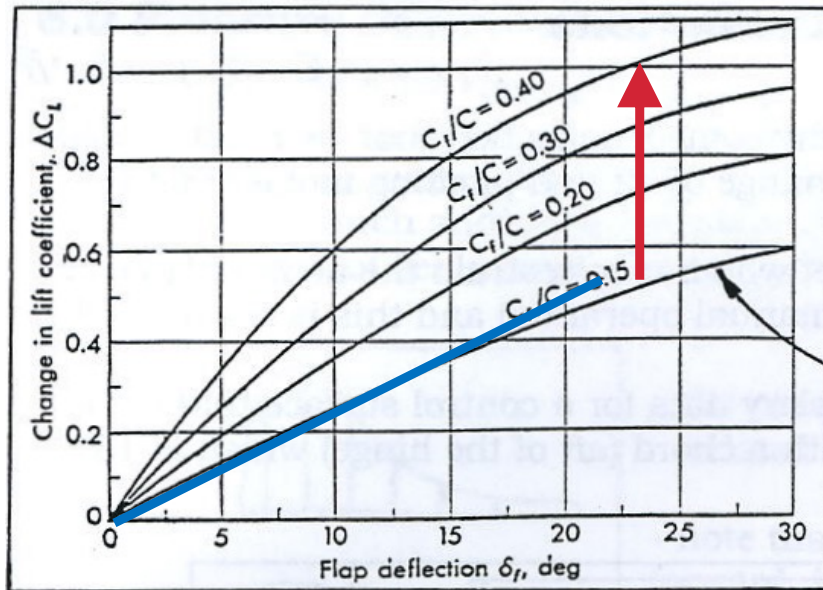
- Control surface deflection can give **non-zero lift**, even when the primary surface is still at **zero incidence**. On the following slide:
- The lower set of curves for the lift coefficient show successive increases in C_L at $\alpha=0$ as the surface deflection δ increases.
- The upper set of curves shows successively greater negative hinge moments as δ increases, again at $\alpha=0$ for example.
- Over the range of incidences $-5^\circ < \alpha < 5^\circ$, both sets of curves appear linear with α . What we need to do in addition is cross-plot from the lower set, keeping $\alpha = \text{constant} = 0^\circ$ and see how the section lift is influence by δ alone.

Effect of a Control Surface Deflection



- Data is given here for a **control surface** that could be a plain flap or perhaps an aileron with a chord (aft of the hinge) which is 15% of the full aerofoil chord.
- Hinge Moment
- Lift Coefficient
- Consider a change in incidence α
- Consider a flap deflection – note **symmetric aerofoil**

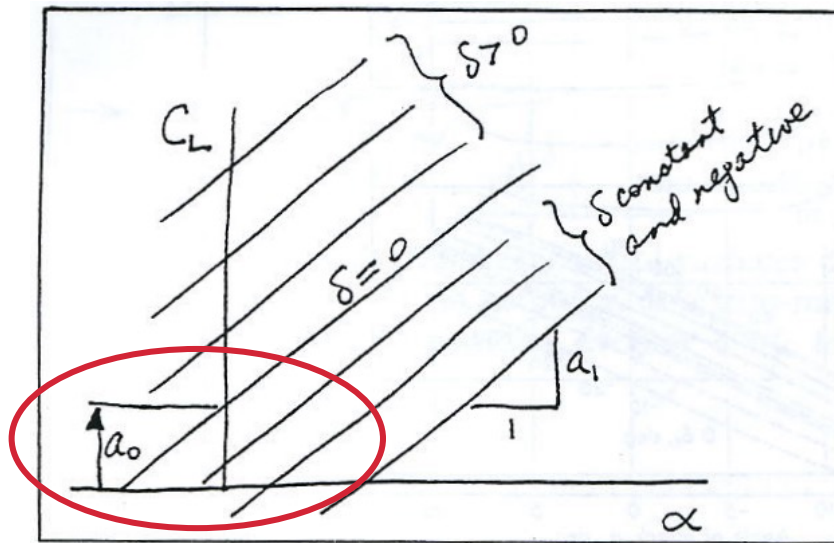
Effect of a Control Surface Deflection



- Results are shown above for control surfaces of **increasing fractions** of the main chord.
- These variations show that for $|\delta| < 15^\circ$ or so, the change in lift coefficient due to *relatively* small control surface deflections appears linear and is **proportional to δ** .

Effect of a Control Surface Deflection

- The more general approximation, allowing for **aileron, elevator and rudder**, and using the general deflection angle δ to represent any one of ξ , η or ζ , would show the following relationships between α , δ and C_L .



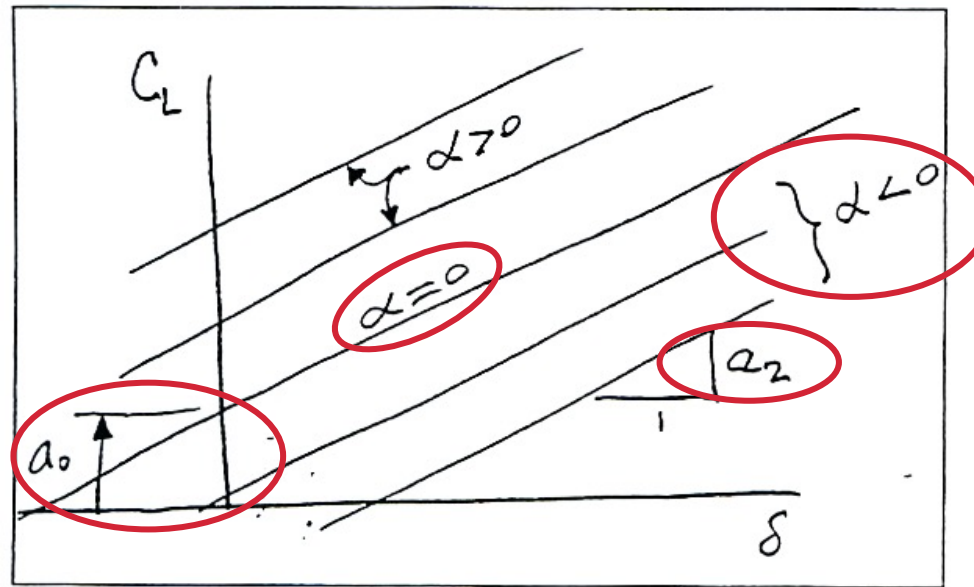
- within a reasonable range of both α and δ the slope is assumed constant at

$$a_1 = \left. \frac{\partial C_L}{\partial \alpha} \right|_{\delta = \text{const.}}$$

- We allow for initial aerofoil camber here too by showing $a_0 \neq 0$ when both α and δ are zero.

Effect of Control Surface Deflection

- Similarly, we expect to find, for several constant values of α :-



- again, with the assumption of a linear response, we can define the slope in this case as:

$$a_2 = \left. \frac{\partial C_L}{\partial \delta} \right|_{\alpha = \text{const.}}$$

Effect of Control Surface Deflection

- Using the two coefficients given on the previous slides we can define the lift coefficient as:

$$C_l = a_0 + a_1\alpha + a_2\delta \quad (3)$$

where the general parameter δ can be replaced by one of the control surface deflection angles ξ , η or ζ .

- Theoretical **thin-aerofoil** theory predicts for a 2D wing with full-span control surface the value:-

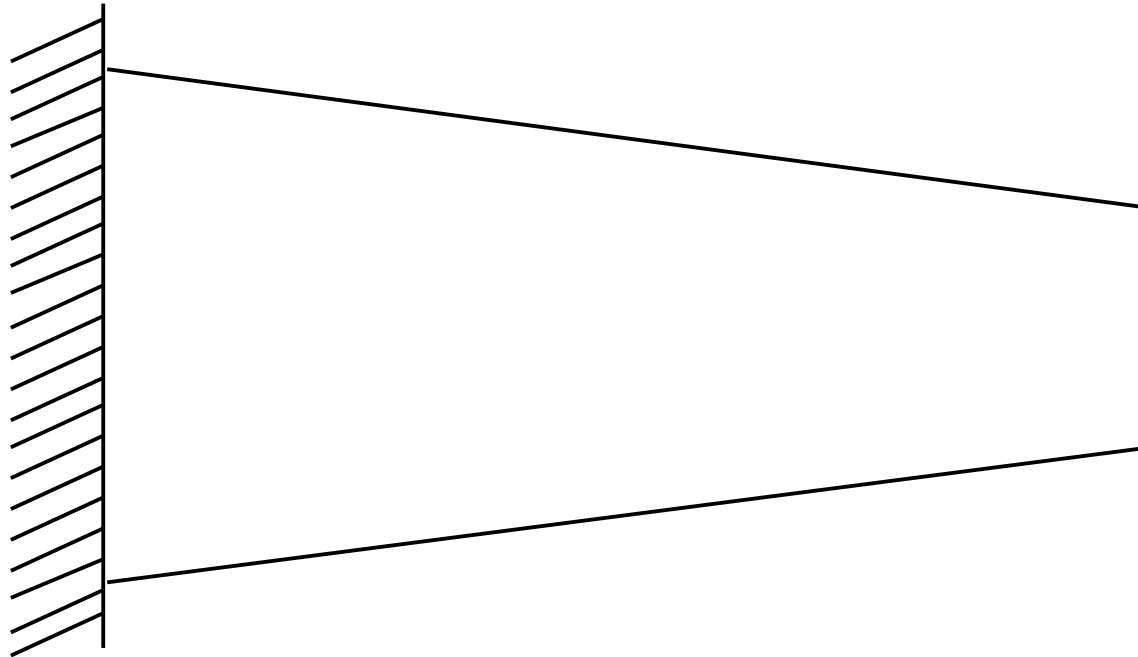
$a_1 = 2\pi$ so we can expect values of order 5 to 6

for α measured in radians or

$a_1 \approx 0.1$ for α measured in degrees.

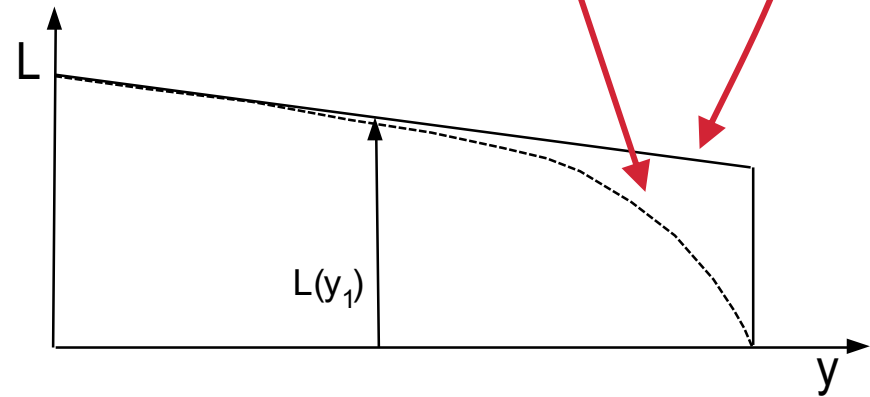
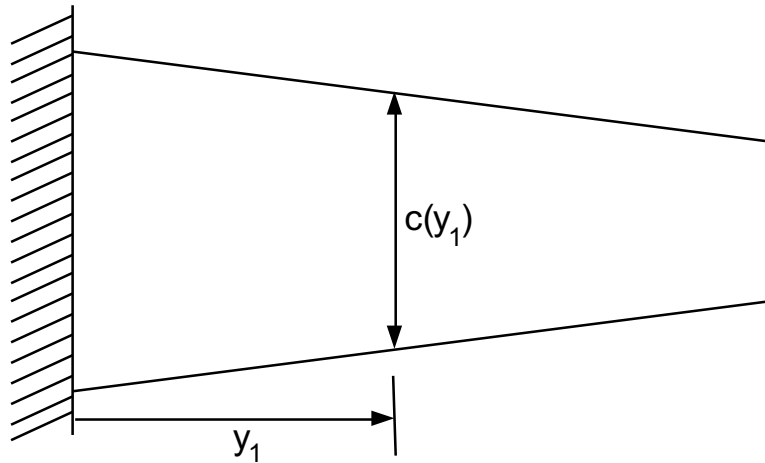
Functions That Vary Across The Span

- Sometimes it is convenient to make one or more of the **coefficients** variable (i.e. $a_i(y)$). Take a simple tapered wing as shown below:



Functions That Vary Across The Span

- If the only **component of incidence** were the flight incidence that is uniform across the span, then the integrand would be all constants except for $c(y)$, and the distribution of lift across the span would reflect the distribution of **chord length**.
- However we know that **true lift distributions** drop to **zero** at the tip, and if we want to be more realistic we can alter the differential to display that reality.



Functions That Vary Across The Span

- One approximation to the spanwise distribution of lift, is to consider the product $a_1\alpha$ to give a **reasonable variation** of the product without trying to portray either factor accurately.
- We might be able to estimate components of α other than that due to **downwash**, whereas the spanwise variation of the **loss in incidence** due to downwash would be more difficult to describe.
- In principle, a_1 is almost constant but we could "shape" it by using a function such as Eqn. 4

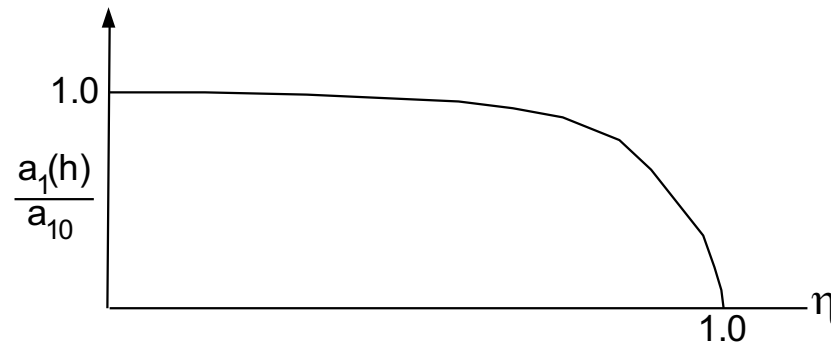
$$a_1(\eta) = a_{10} (1-\eta^2)^{1/2} \quad (4)$$

($\eta = y/s$)

(η is the fractional semi-span)

Note: given if needed

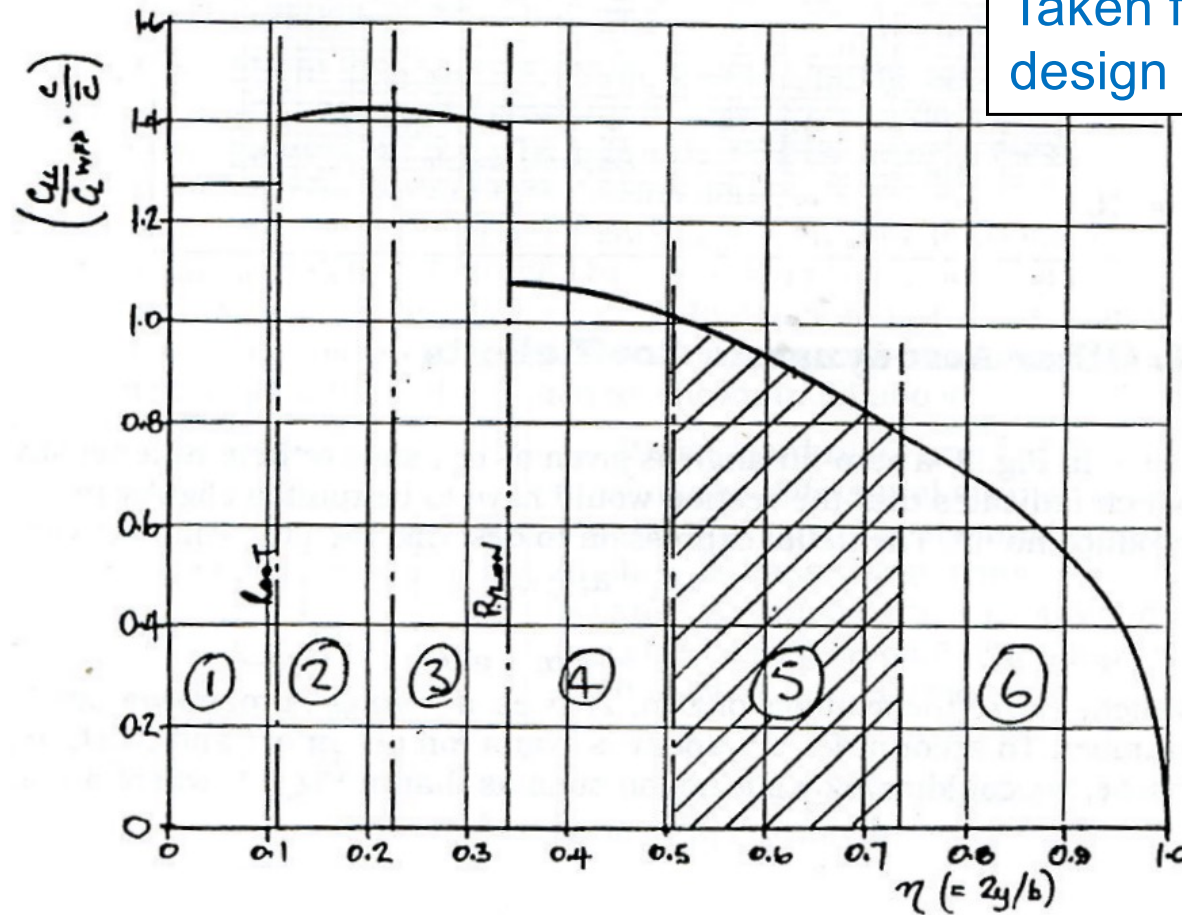
Functions That Vary Across The Span



- The product $a_{10} (1-\eta^2)^{1/2} \alpha(y)$ will drop to zero at the tip even if using a function $\alpha(y)$ that correctly represents camber, constructed twist and flight incidence.
- The elliptical shape given by Eqn.4 is "*about right*". There remains the task of selecting a value for a_{10} the apparent value of the lift-curve slope for the root section.

Functions That Vary Across The Span

Taken from a fourth-year
design manual



A Lift-curve Slope for The Whole Wing

- Where a single value for a_1 is accepted for an aerofoil, the slope $a_1 = \delta C_L / \delta \alpha$ is affected by aspect ratio A and can be represented approximately by:

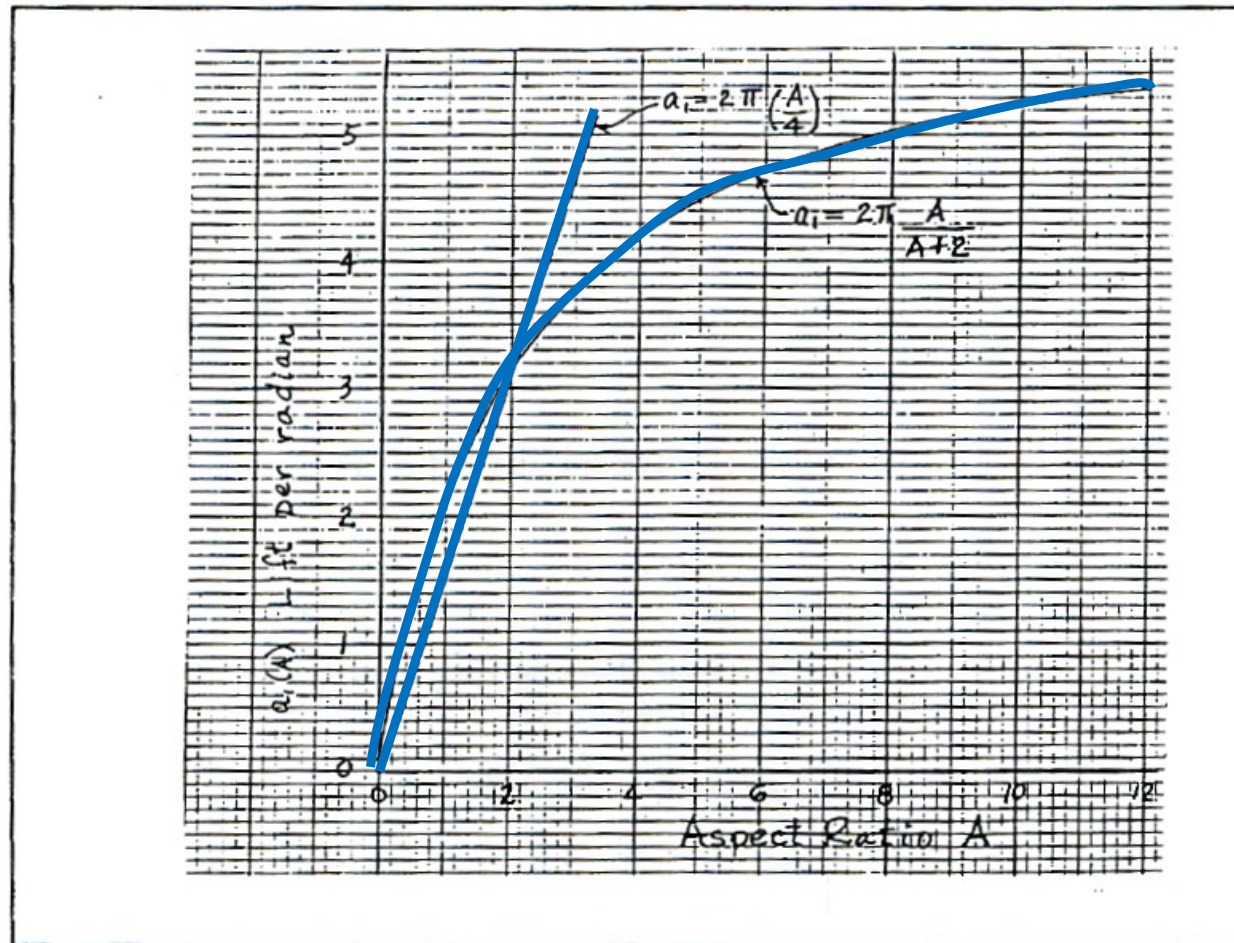
$$a_{10} = 2\pi \times A/(A+2) \quad (5)$$

Note: *even when A is as high as 8, a_1 drops from 2π (the theoretical value for $A \rightarrow \infty$) to only 5.*

- For aspect ratios that are very low ($A \leq 2$), as may be the case for a fin, it is found that the following gives a useful approximation:

$$a_{10} = 2\pi \times A/4 \quad (6)$$

A Lift-curve Slope for The Whole Wing



Other Aerodynamic Coefficients

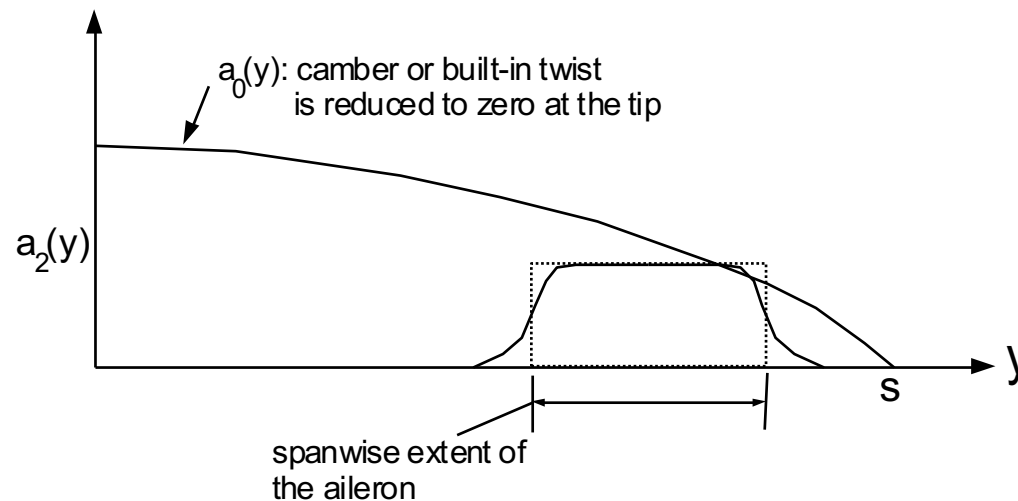
- If the zero-lift angle is taken as α_0 , this is usually a negative value which indicates that the section would have to be rotated slightly nose-down to produce no lift.
- An expression for the lift coefficient could then be given as:

$$\begin{aligned} C_l &= a_1 (\alpha - \alpha_0) \\ &= -a_1 \alpha_0 + a_1 \alpha \end{aligned} \quad (7)$$

which gives $a_0 = -a_1 \alpha_0$, probably a positive number.

Effect of Control Surface Deflection

- To account for the spanwise variation of camber and twist, as described above, we could employ a function such as that shown below, where a realistic shape is shown for a_0 and also a typical shape for a_2 , the coefficient related to the **control surface deflection**.
- "Pure" strip theory would dictate that **lift cannot be changed by such a surface outside its own range** and therefore a_2 should be zero outside the aileron extent, but in reality there will be some 'spill'.



Strip Theory – Rudder Example



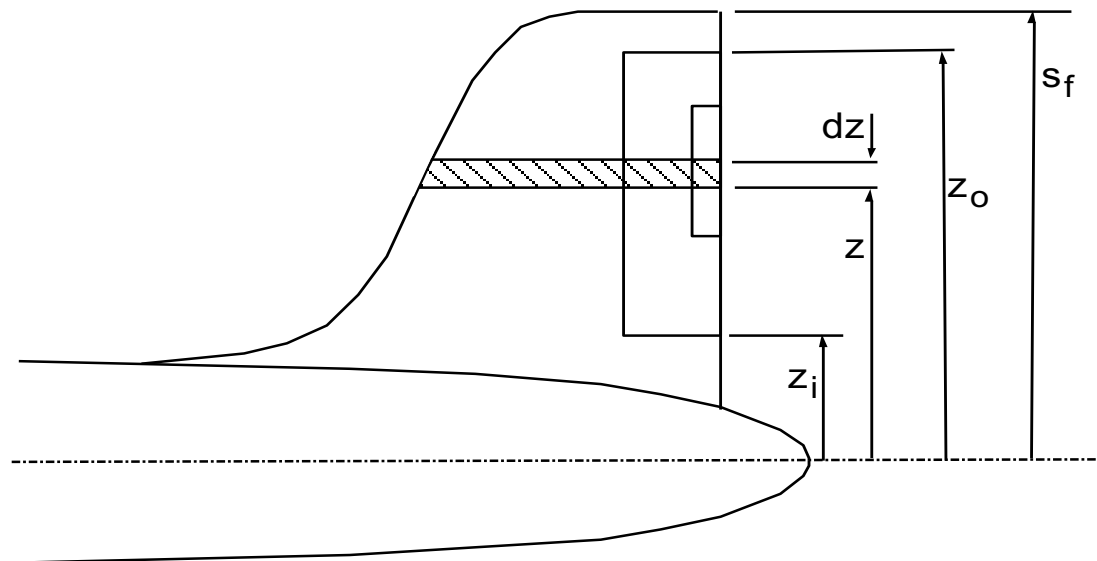
Bell Boeing V-22 Osprey. Wikipedia: Peter Gronemann

Strip Theory – Rudder Example

- For example, take a typical expression for C_l :

$$C_l = a_0 + a_1 \alpha + a_2 \zeta + a_3 \beta \quad (8)$$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
1 2 3 4



Strip Theory – Rudder Example

- For example, take a typical expression for C_l :

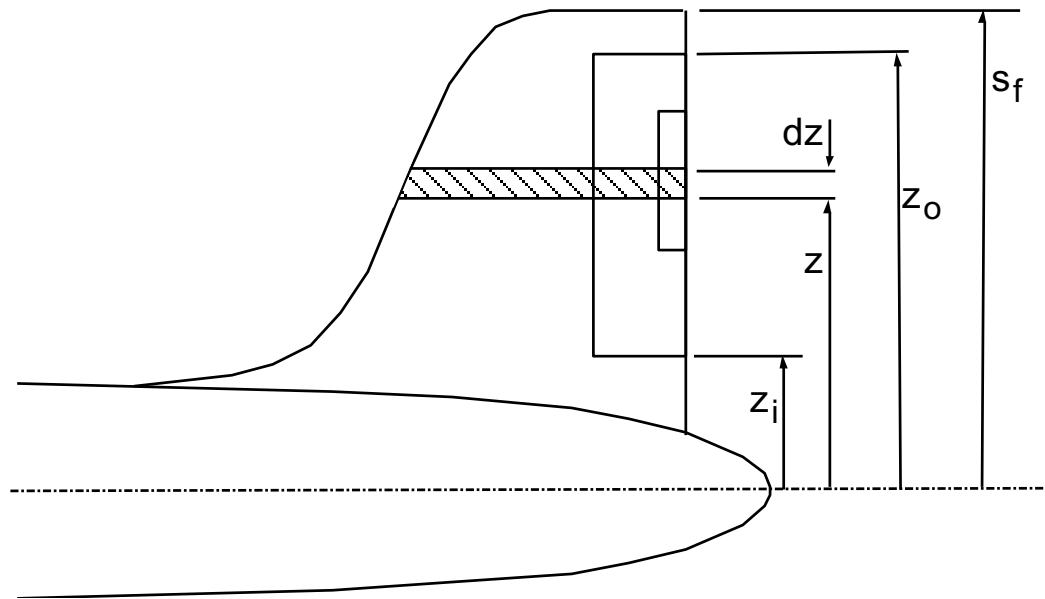
$$C_l = a_0 + a_1 \alpha + a_2 \zeta + a_3 \beta \quad (8)$$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
1 2 3 4

Term 1 We would usually have $a_0=0$ for the symmetric aerofoil which acts as a fin, but a_0 will not generally be zero.

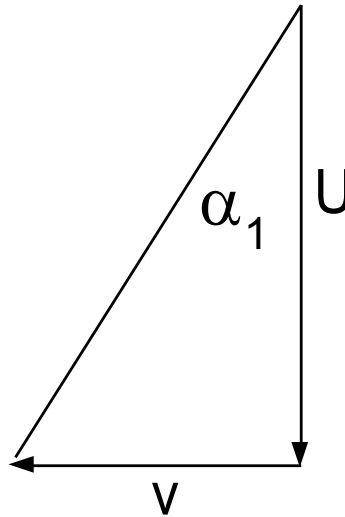
Strip Theory – Rudder Example

Term 2 We need to know what actions are causing an **incidence (lateral)** on the **fin** and this will include any action that leads to a transverse fluid velocity relative to the fin.



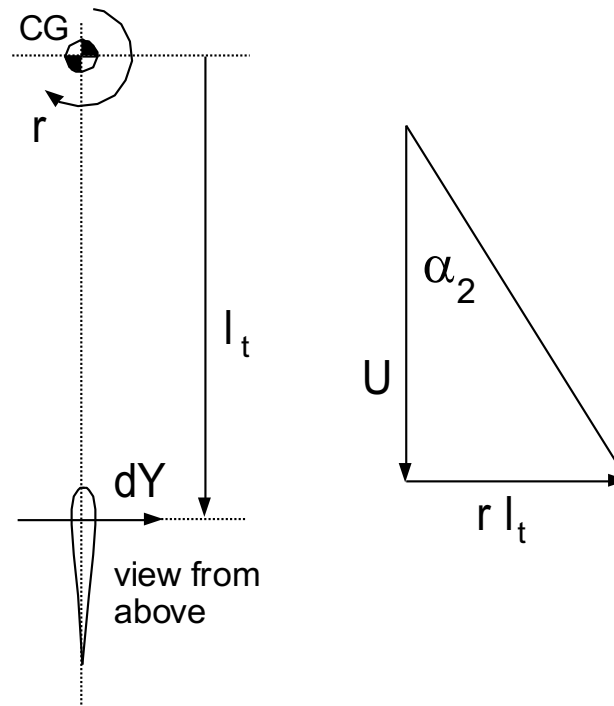
Strip Theory – Rudder Example

- The whole vehicle may be drifting to starboard at velocity v , causing an apparent negative incidence $\alpha_1 = -v/U$ uniformly over the fin.
(*Note:* this sideslip angle v/U is normally referred to as β but we use α_1 here to distinguish it from the tab angle β and to make it consistent with the angles α_2 and α_3)



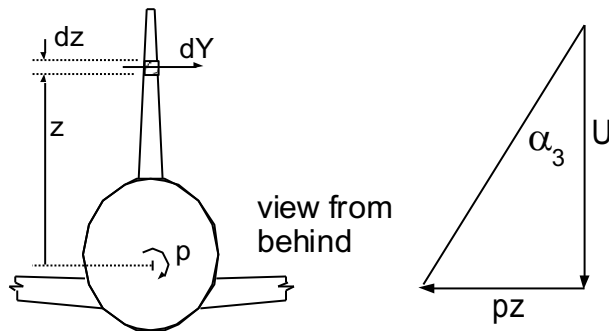
Strip Theory – Rudder Example

- The vehicle may have a yaw rate r that causes lateral motion of the fin of value rl_t (to port), this motion inducing a fluid velocity (from the port) such that incidence $\alpha_2 = rl_t/U$ (positive) is induced over the whole fin.



Strip Theory – Rudder Example

- The vehicle may have a roll rate p that induces a velocity pz of the *strip* (to starboard) and this causes a fluid velocity to be seen from the starboard which leads to $\alpha_3 = -pz/U$. Thus the total incidence at a fin strip due to these three would be:



$$\alpha_{fin} = \frac{r l_t - v - pz}{U}.$$

Strip Theory – Rudder Example

Term 3 Some strips dz pass through the rudder region and some do not, so the range of integration over which the term $a_2\zeta$ is included may be less than the full possible range 0 to s_f .

The expression for $dLift$ within the rudder extent will have the term $a_2\zeta$, whereas the expression for positions z outside this will not.

Term 4 There may even be a tab on the control surface, and a fourth term could be legitimate over an even narrower range of z .

Strip Theory Application

1. Define Geometry and define 'strip' boundaries
2. Develop an expression for **dForce** in general terms:
pressure × area × coefficient
3. If necessary, develop a **dMoment** (with the required moment arm).
4. Determine spanwise functions as needed.
5. Express the full integral with its proper limits. → Integrate
6. Differentiate to get an 'aerodynamic derivative'?

Application of Strip Theory

- Work from the basics
- Do not try to remember the developed expressions
- Explain the assumptions you make and the approximations you use
- Use neat diagrams

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Strip Theory – Rolling Power

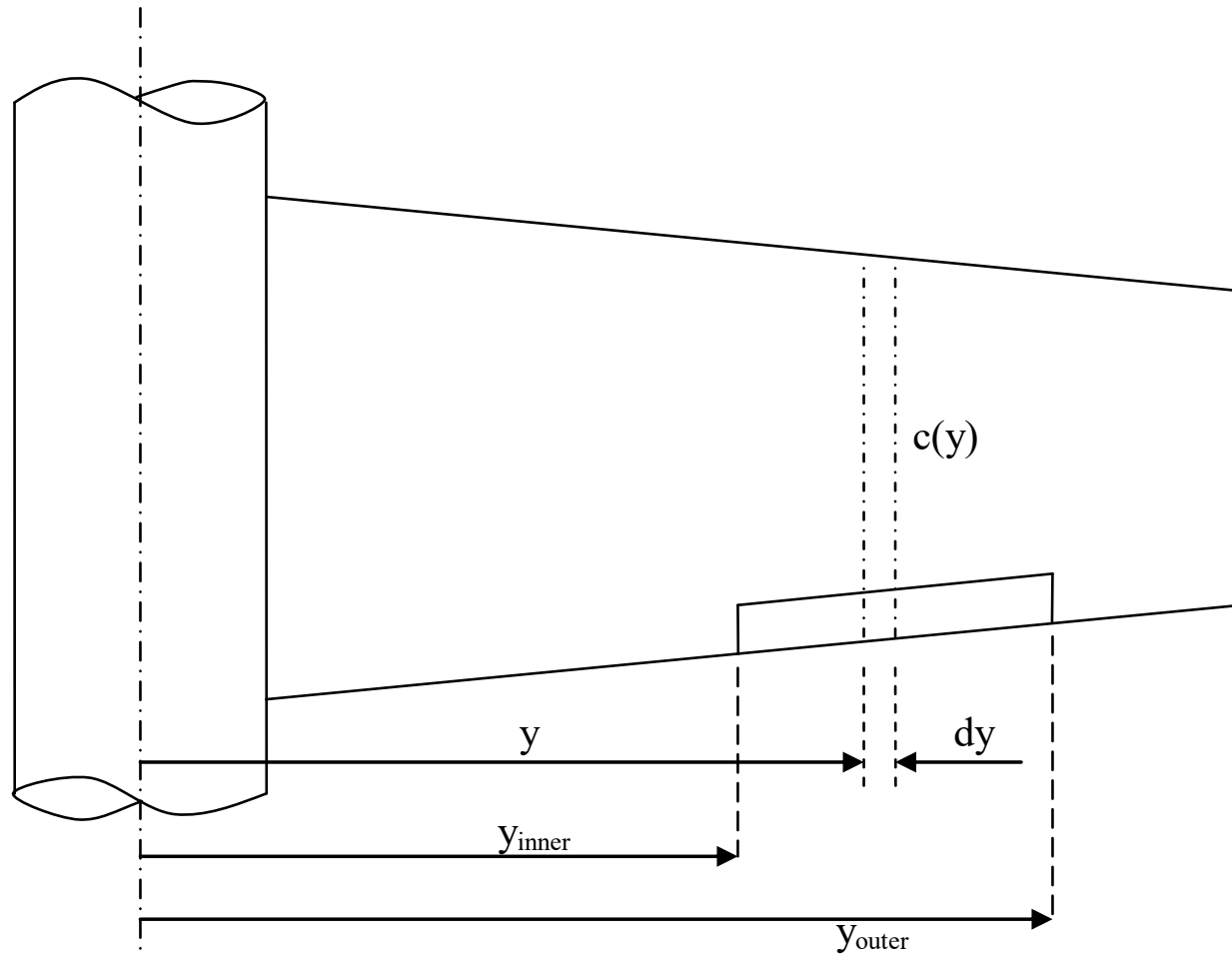


Bell Boeing V-22 Osprey. Wikipedia: Peter Gronemann

Steady Response in Roll

- This section looks at the **steady-state response** to an aileron deflection
- This implies **pure roll**, i.e. no other responses but rotation around the x-axis.
- This becomes unreal after any significant **bank angle** ϕ develops, because then sideslip v would develop also, and this would generate a yaw rate r because of its pressures on the fin; then we would have *three* active variables.
- In this example we ignore all other responses and seek the roll rate p that can be achieved by an aileron deflection of ξ .

Rolling moment due to Ailerons



Rolling moment due to Ailerons

- Local lift coefficient on the strip can have the components:

$$C_l = a_0 + a_1 \alpha + a_2 \xi \quad (9)$$

- And thus the expression for the differential lift on the strip becomes:

$$dLift = \frac{1}{2} \rho U^2 c(y) dy C_l \quad (10)$$

- Differential rolling moment can be expressed as:

$$dL = -y dLift \quad (11)$$

where the negative sign is necessary to display the fact that positive ξ produces a negative rolling moment.

Rolling moment due to Ailerons

- Just before the aircraft begins to roll, the only unbalanced components of lift from C_l which can produce a rolling moment are those from ξ .
- Even the effect of camber (a_0) will be symmetric on the two sides and can therefore be left out of the lift expression.
- As both ξ and y change sign from side to side we can integrate double the moment on one side i.e.:-

$$L(\xi) = -2 \int_{y_i}^{y_o} \frac{1}{2} \rho U^2 y c(y) a_2(y) \xi dy \quad (12)$$

Note: There is a tendency to leap to here!

Rolling moment due to Ailerons

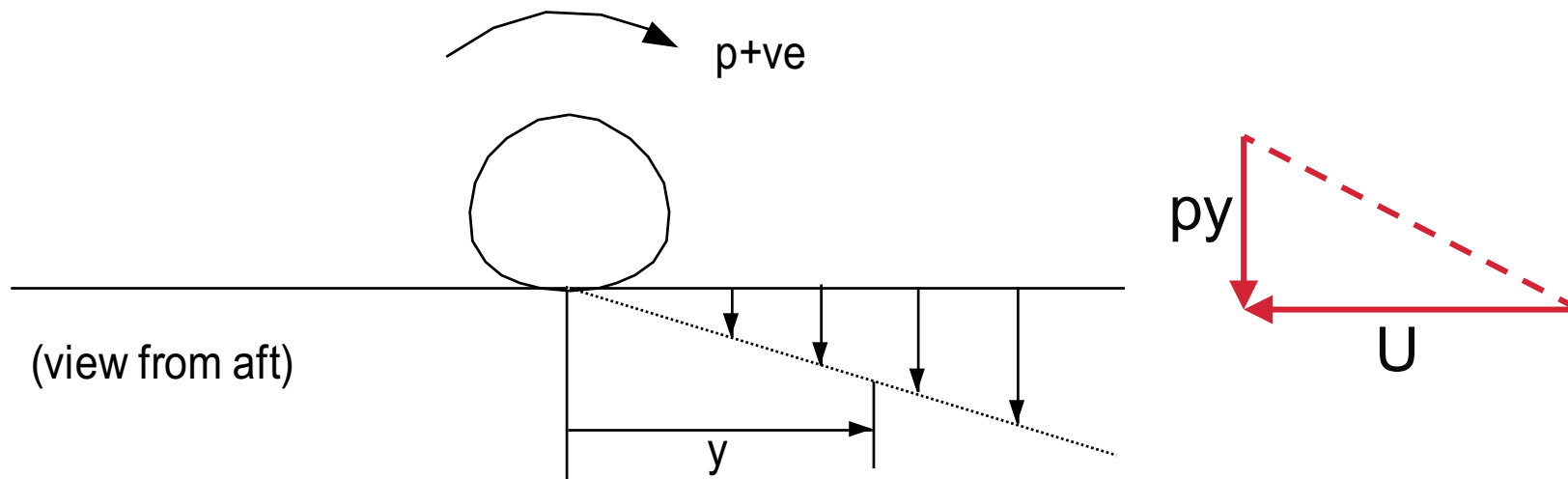
- Now, using pure strip theory we have:-

$$L(\xi) = -\rho U^2 a_2 \xi \int_{y_i}^{y_o} y c(y) dy \quad (13)$$

- This is the total, and yet unbalanced, rolling moment imposed by the ailerons.
- (if there were available a true function that represented the ‘spill’ of aileron-induced pressures near the aileron ends we could use that function $a_2(y)$ under the integral sign instead of using the constant implied here.)

Rolling Moment due to Response in Roll

- In order to develop a general expression we first assume that a steady roll rate p has already developed and then determine the rolling moment due to that rate p , even though it is clear that a positive aileron deflection will actually induce a negative value for p .



Rolling Moment due to Response in Roll

- The downward velocity (due to p) of a section at spanwise y will be py , thus inducing an (upward) incidence py/U and an upward lift (on the right side) as a consequence.
- i.e. the aerodynamic response to the rolling motion is a moment that opposes that motion. Similar to the above, we have:-

$$\begin{aligned} L(p) &= -2 \int_0^s \frac{1}{2} \rho U^2 y c(y) a_1(y) \frac{py}{U} dy \\ &= -\rho U a_1 p \int_0^s y^2 c(y) dy \end{aligned} \tag{14}$$

- And subject to the assumption of *pure* roll, these **two moments** (1) and (2) are the only “external” moments.

Roll Response

- Newton's Second Law, in the rotational sense about the X-axis states that:-

$$I_{xx}\dot{p} = \sum \textit{externally applied moments}$$

- And this is satisfied for $\dot{p} = 0$ (roll acceleration becomes zero when the sum of the applied moments acting on the aircraft from outside is zero) if:-

$$L(\xi) + L(p) = 0 \quad (15)$$

$$-\rho U^2 a_2 \xi \int_{y_i}^{y_o} y c(y) dy - \rho U a_1 p \int_0^s y^2 c(y) dy = 0 \quad (16)$$

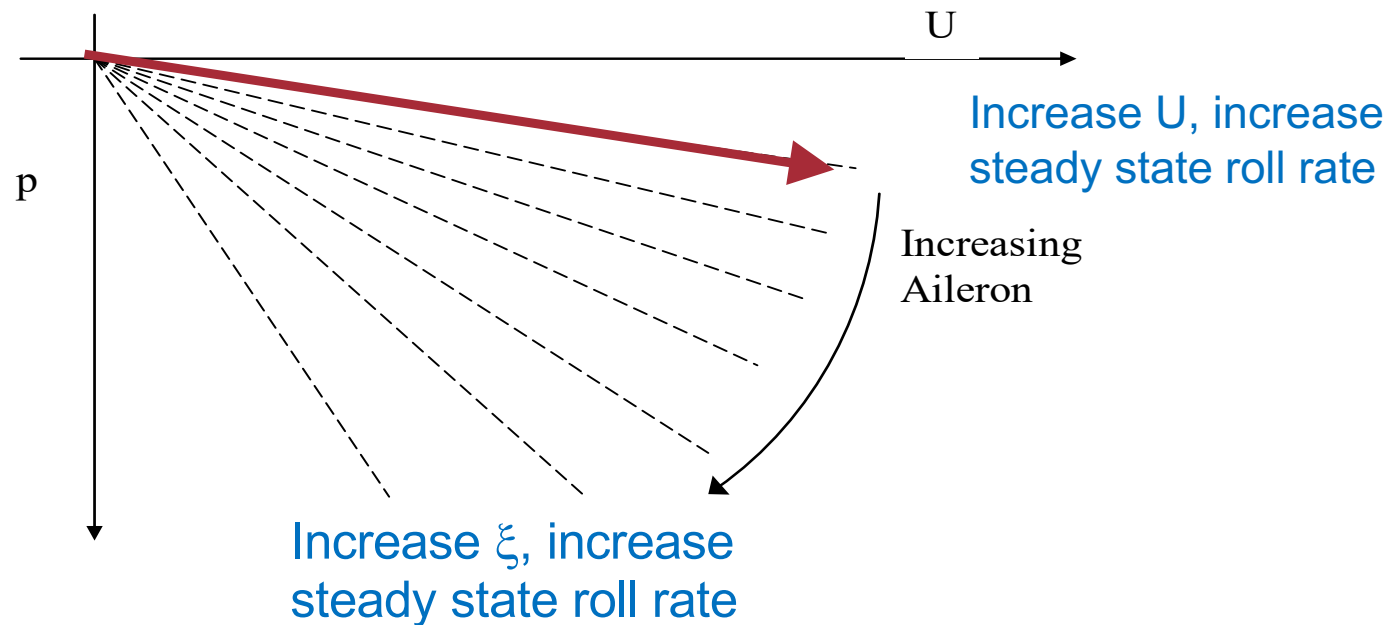
Roll Response

- Then the expression from which we determine the steady roll rate p as a function of ξ is:

$$p = - \frac{a_2 \int_{y_i}^{y_o} y c(y) dy}{a_1 \int_0^s y^2 c(y) dy} \xi U \quad (17)$$

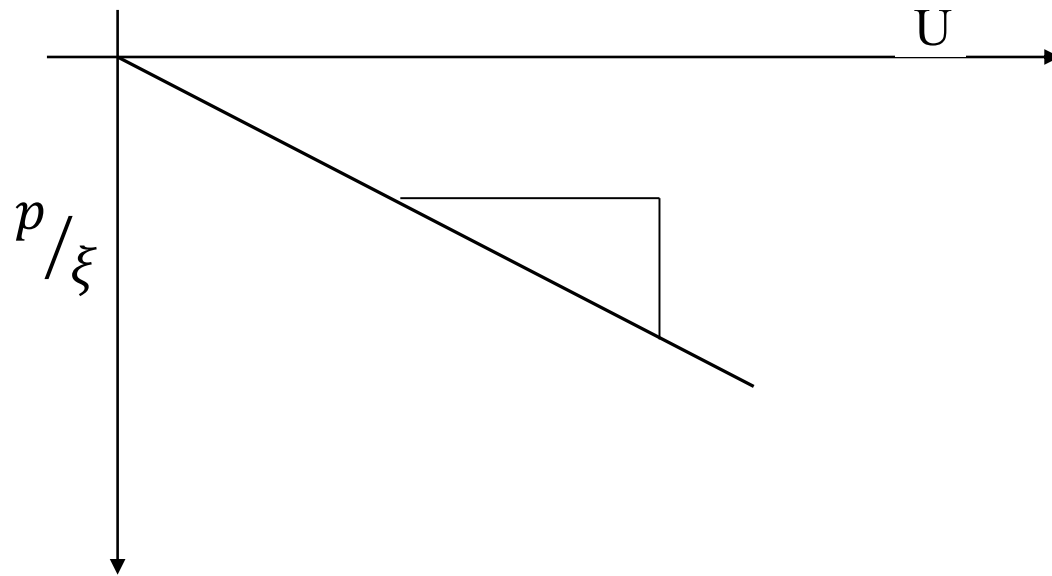
Roll Response

- This can be expressed graphically as



Roll Response

- or we plot "rolling power" as p/ξ



Roll Response

- Note that the slope here is given by:

$$slope = - \frac{a_2 \int_{y_i}^{y_o} y c(y) dy}{a_1 \int_0^s y^2 c(y) dy} \quad (18)$$

- and note that for constant ξ we have p directly proportional to U .

Aerodynamic Derivatives



Bell Boeing V-22 Osprey. Wikipedia: Peter Gronemann

Aerodynamic Derivatives

Note: Mentioned here and used later on in the course

- If an aerodynamic force (or a moment) is known to be a function of several variables, each contributing linearly (note this *assumption*), i.e.,

$$T = T(\alpha, \beta, \gamma \dots)$$

- then for *small changes* in each variable the *total change* in the aerodynamic function can be found using the Chain Rule:

$$T = \frac{\partial T}{\partial \alpha} \delta \alpha + \frac{\partial T}{\partial \beta} \delta \beta + \frac{\partial T}{\partial \gamma} \delta \gamma + \dots \quad (19)$$

Aerodynamic Derivatives

- Since we are often looking for expressions to represent the *departures* from a **steady equilibrium flight condition** we often want the total (but small) change in a force.
- In the case we have been studying above, the rolling moment **L** is a function of **p** and **ξ** so we have:

$$L = L(p, \xi) = \frac{\partial L}{\partial p} p + \frac{\partial L}{\partial \xi} \xi \quad (20)$$

Aerodynamic Derivatives

- The **partial derivatives** by themselves are called "**aerodynamic derivatives**" expressed using the shorthand notation:

$$\frac{\partial T}{\partial u} = T_u$$

- These derivatives display the influence of a variable on a function, e.g. the influence of **u** on **T** and this implies the need to express **T** in terms of **u** before doing the partial differentiation.
- Note:** using (3) and (7), rolling power can be expressed as:

$$\frac{p}{\xi} = -\frac{L_{\xi}}{L_p}$$

Reference Book Example:

- *Flight Dynamics Principles: A Linear Systems Approach to Aircraft Stability and Control (Elsevier Aerospace Engineering) Hardcover – 9 Aug 2007*

$$\dot{L}_\xi = \frac{\partial L}{\partial \xi} \quad \text{Rolling moment due to aileron}$$

This derivative describes the roll control property of the aeroplane and an accurate estimate of its value is important to flight dynamics analysis. With reference to equations (13.182) and Fig. 13.15, the application of simple strip theory enables the rolling moment due to starboard aileron deflection to be written

$$L_{right} = -\frac{1}{2}\rho V^2 \int_{y_1}^{y_2} C_{Ly}|_{right} c_y y \, dy = -\frac{1}{2}\rho V^2 \int_{y_1}^{y_2} (a_0 + a_y \alpha + a_{2A} \xi) c_y y \, dy \quad (13.184)$$

where a_{2A} is the aileron lift curve slope, which is assumed to be constant over the span of the aileron. Similarly, the rolling moment due to port aileron deflection may be written

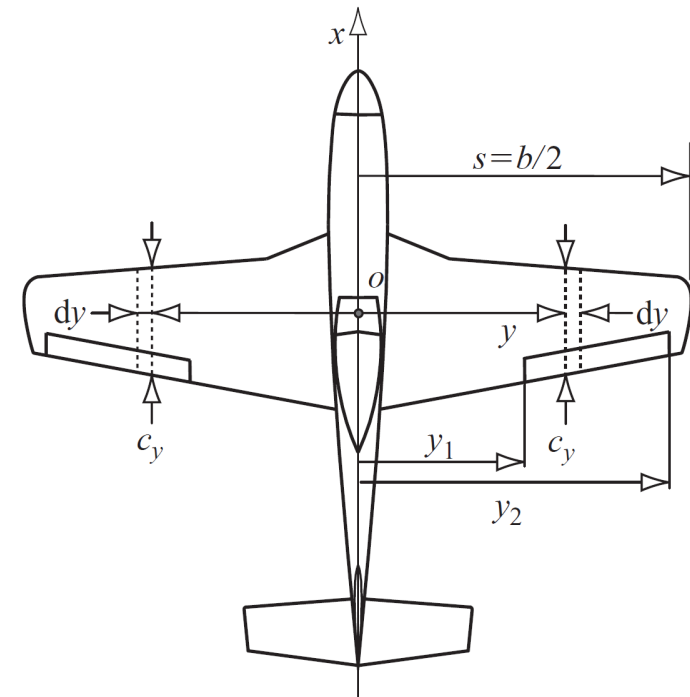
$$L_{left} = \frac{1}{2}\rho V^2 \int_{y_1}^{y_2} C_{Ly}|_{left} c_y y \, dy = \frac{1}{2}\rho V^2 \int_{y_1}^{y_2} (a_0 + a_y \alpha - a_{2A} \xi) c_y y \, dy \quad (13.185)$$

It follows that the total rolling moment may be written

$$\dot{L}_\xi \xi = L_{right} + L_{left} = -\rho V^2 (a_{2A} \xi) \int_{y_1}^{y_2} c_y y \, dy \quad (13.186)$$

Strip Theory

- Section 13.1 (Chapter 13)
- Section 13.4 (Chapter 13)



Next Session

Elevator Angle to Trim

Bristol Box Kite – Bristol Museum & Art Gallery



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STRIP THEORY II

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