Dynamics and Control of Linear Systems
Vibrations & Aeroelasticity

Academic year 2024-2025 coursework specification

Vibrations and aeroelasticity applications in aerospace

Students *must* submit their own individual and independent work. All submitted reports will be checked for plagiarism.

Coursework description

This part of the *coursework* constitutes 50% of the final mark in the unit. The coursework centres on vibrations and aeroelasticity applications in an aerospace context. The coursework is divided into 5 tasks. **Each task contributes 20% to the overall mark for the *coursework***. Further marking information is provided in section *Marking* (page 3).

Each task must be answered on a single page of your *individual* reports. The final report covering this theme will consist of five pages dedicated to the technical tasks (see separate comments regarding appendices). This assessment closes a 10cp worth component of *Dynamics and Control of Linear Systems*. With this credit weight, the component amounts to approximately 100 hours of your work overall. Within this context and assuming nominal engagement with the unit, the coursework is designed to be completed during five days of individual work. Those students who wish to start with tasks 1 and 2 can read *Appendix* first to appreciate the context and to familiarise with the provided *Matlab* tool.

Contents

AfCh … Amplitude-frequency characteristics

DOF … degree of freedom

EoM … Equation of Motion

FRF … Frequency Response Function

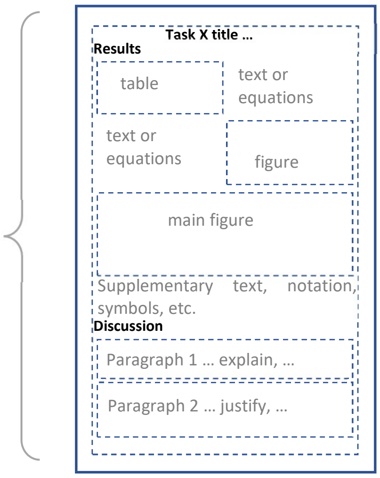
ODE … ordinary differential equation

abc … typed Matlab text

Coursework specification and guidance

Each task is specified in terms of its technical objective and the expected delivery format. All tasks should have page layout consistent with the following example. Each task specification offers further requirements and recommendations. Results and Discussion sections are compulsory.

Example A4 page layout



Writing recommendations and page formatting:

- A4 page size, all page margins minimum 2 cm and maximum 2.5 cm wide.
- In the text, font size 10pt or 11pt, single line spacing.
- Use figure captions, include and use SI units, label or annotate where appropriate, use legends.
- Describe all figures and tables using a brief description.
- In figures, use variety of line styles, data markers, choose suitable font size, **ensure clear visibility**.
- Use formal language, avoid the use of the first person in discussions or elsewhere.
- Preferably avoid the use of photographed hand-drawn sketches.
- In discussions, focus on covering the required discussion topics.
- Preferably avoid trivial, obvious, repetitive, or irrelevant discussion points.
- "*briefly*", "*brief statement*", "*short paragraph*", etc. means two, three or maximum four lines of text, more text is seen as the inability to offer a succinct observation or discussion point.

- All required references can be separated from the main discussion. A separate section or footnotes can be used to include the references.

Marking:

- Each task contributes 20% to the total 100%. In each task: 50% of the mark is derived from "*Results*" and 30% from "*Discussion*" in terms of their degree of completion and their technical quality; 20% of the mark is derived from the organisation (e.g., layout, flow, clarity), technical delivery (e.g., figures, tables, equations) and English style.
- **Every report is checked for plagiarism.**
- *Do not use text of this coursework document or graphics in your reports.*

Academic integrity and AI statement:

- See "*Using AI in Assessment*" (https://www.bristol.ac.uk/bilt/sharing-practice/guides/guidance-on-ai/using-ai-in-assessment/). This is a **Category 2** assessment with regard to the *report production*.
- AI/generative tools can be used to explore the concepts and improve learning productivity.
- The use of grammar tools to change phrases, sentences, paragraphs, structure, flow, etc. as well as the use of translation tools in a similar context exceeds the scope of **category 2** assessment.
- Each task must be completed based on your own intellectual ability without collaboration with any human or machine third party.
- All resources provided in this DCLS theme (Vibrations and Aeroelasticity) can be used to develop the tasks whilst adhering to the standards of academic integrity (e.g., referencing, quoting, etc.).

Task 1: Single degree-of-freedom analysis (20% of 100%)

Specification: Use the provided *Matlab App* to find the physical parameters of the aerofoil (mass moment of inertia of the aerofoil about its hinge, torsional damping, torsional stiffness). Present the following compulsory *Results* and *Discussion* points:

- Results:
    - Use suitably chosen simulations to find the aerofoil physical parameters. In each case, briefly (one or two sentences) state the method or approach used (use referencing where relevant), include the key formula (no need to explain the individual terms) before substitution and after substitution of the numerical values. Clearly identify the results and units!
    - To evidence your way of parameter identification, include one support figure which shows an example of the simulation(s) used to determine the selected physical parameter.
    - The main figure must show: (**1**) the 1DOF FRF obtained using a sequence of a chosen number of harmonic excitations (*see* Hints) in the Matlab App, overlayed with (**2**) the FRF calculated analytically. Also include (**3**) the 1DOF undamped natural frequency.
- Discussion:
    - *First*, given the adopted parameter identification methods, briefly discuss which of the three found parameters is subjected to the most significant identification error (or uncertainty) and why it is so. *Second*, use the Internet to find an academic paper or industrial report which discusses similar tasks in the real-world aerospace context. Explain your motivation behind the choice.
    - Each discussed point must be limited to a maximum three-line paragraph.

Guidance, suggestions, hints:

- The default system in the Matlab App, after its start, is 2DOF and needs to be modified to *approximate* the 1DOF aerofoil system. This can be done by reducing the significance of the TVA part in the system ("a grain of sand glued to the surface of the wing").
- Use suitable free or forced conditions to obtain such responses that can be effectively used to calculate the required parameters.
- For each FRF (or AfCh, see later) use an appropriate number of frequency points to show the specifics of the shape of the function. Typically, between 10 and 20 points suffices for such use.
- *Note*: based on how it is defined in the lecture notes, AfCh can be seen as a function that presents the specific steady-state response to *any* form of harmonic excitation. FRF is a special function which presents the steady-state harmonic amplitude of an output DOF across a frequency range normalised by the input load amplitude. Based on this, an FRF can be seen as the AfCh produced for the case with a single load with the magnitude equal to one.

Task 2: Tuning the Tuned Vibration Absorber (20% of 100%)

Specification: Study and tune the TVA such that it absorbs the resonance of your own 1DOF system. Present the following compulsory *Results* and *Discussion* points:

- Results:
    - Perform tuning of the TVA to absorb the resonance in your system. Document your working by giving a brief outline of the process (2 or 3 sentences), include the tuning formula before and after the substitution of the numerical values. Where required justify briefly (1 sentence) your choices. The TVA mass must not exceed 5% of the *equivalent mass* of the aerofoil (*see* Hints).
    - The main figure must contain the information identified using the simulations: (**1**) previously found 1DOF AfCh reused from **Task 1**; (**2**) 2DOF aerofoil AfCh for the case where the TVA is undamped; (**3**) 2DOF aerofoil AfCh for the case where the TVA damping is 10%.
- Discussion:
    - *First*, given the choices adopted in your TVA tuning, describe the impact of the damping on the features of your presented AfCh characteristics. *Second*, use the Internet to find an academic paper or industrial report which discusses a similar task in the real-world general engineering context. Explain your motivation behind the choice.
    - Each discussed point must be limited to a maximum three-line paragraph.

Guidance, suggestions, hints:

- Use your own 2DOF system configuration in the *Matlab App*, in combination with the suitable range of harmonic inputs, to calculate the amplitude frequency characteristics (AfCh functions) such that the shape specifics of the AfCh are revealed well.
- To determine the *equivalent mass* of the aerofoil, use the concept of "*radius of gyration*" from Mechanics formulated between the aerofoil hinge and the TVA attachment point.
    - *Note*: mass moment of inertia = equivalent mass × (radius of gyration)^2, where "radius of gyration" is known because we know where the "equivalent mass" is located for the purposes of this comparison.
- The "%-value TVA damping" represents the damping ratio of the TVA spring-damper-mass system when in isolation from the primary system (aerofoil) and acting as a 1DOF system.

Task 3: Equations of motion for 2 DOF system and eigenvalue problem (20% of 100%)

Specification: A mechanical part of the flight control system is described in *Figure 1*. The system is modelled below as a 2DOF lumped parameter system. Study the natural frequencies of the system.

Assumptions: The lever is massless and rigid. The link is massless and rigid. Masses 1 and 2 move horizontally only without friction. The lever pivots about point O. L_1 and L_2 are defined as follows: $L_1 = a \times L$, $L_2 = (1-a) \times L$, where a is in the range between 0 and 1. Choose mass 2 not to exceed 5 % of mass 1.

Freely define *your own reasonably chosen* parameters such that the natural frequencies do not exceed 50 Hz when a=0.5. If not the same, the stiffness values should not differ by more than one order of magnitude. Assume small angles where necessary.

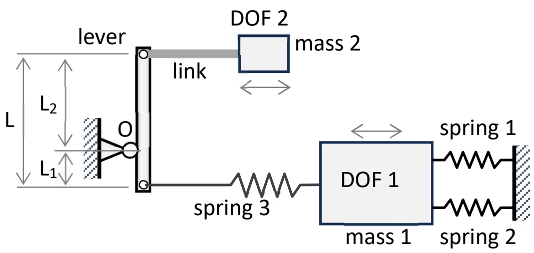


Figure 1 Mechanical part of the flight control system.

Solve the following tasks:

- Results:
    - Derive the EoM using **Lagrange's approach**. Identify the relevant coordinates and clearly annotate the system parameters. Write down the full Lagrangian and briefly explain its composition. Derive the mass and stiffness matrices of the system.
    - Calculate the natural frequencies of the system for at least 15 different values of parameter a within its specified range. Briefly (two or three sentences) describe the process and methods used to complete this task.
    - The main figure must contain the calculated changes of the natural frequencies with the varying parameter shown on the horizontal axis. Clearly annotate the figure, axes and any potential special cases.
- Discussion*:*
    - *First*, given the arrangement of the system, discuss briefly the observed trends in the natural frequencies. When doing so, particularly focus on the impact of the lever in the system. *Second*, given the observations and insights found in the first discussion point, indicate which vibrations and aeroelasticity applications could benefit from this system arrangement. When doing so, refer to the existing industrial case across the broader aerospace discipline. Evidence your claim by including a reference to a news article, journal paper, industrial report or Internet web page.
    - Each discussed point must be limited to a maximum three-line paragraph.

Guidance, suggestions, hints:

- *When made available*, consider the next task to double-check the correctness of your EoM derivations.

Task 4: Damping in vibrating systems (20% of 100%)

Specification: An extended model of the flight control system introduced in Task 3 is shown in *Figure 2* below. An additional viscous damper is placed between mass 2 and the ground. A harmonic excitation force is applied to mass 1. Study the influence of the damping on the resonant vibrations of the system.

Assumptions: All assumptions discussed in Task 3 apply in this task too.

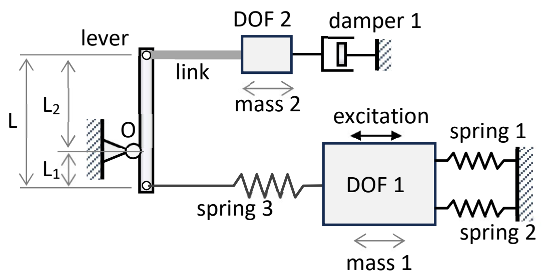


Figure 2 Extended composition of the mechanical part of the flight control system.

Solve the following tasks:

- Results:
    - Derive the EoM using **Newton's approach**. Sketch the free body diagrams (include mass 1, 2 and lever) where all relevant coordinates and loads are clearly identified. Write down the corresponding EoM in their final form. Identify the mass, damping and stiffness matrices.
    - Calculate and plot the AfCh of mass 1 under the influence of the harmonic force when a=0.5 and across the frequency range which includes the undamped natural frequencies of the system. Choose the viscous damping parameter such that the resonant peaks are visible.
    - Repeat the calculation and plot the resulting AfCh in the same figure for the cases where a=0.01, 0.05, 0.1, and 0.7.
- Discussion:
    - *First*, briefly discuss the role of the damper in the system as manifested by the analysis of a single selected AfCh. Then, discuss briefly the role of the damper in the context of varying parameter a. The paragraph which includes this discussion should not exceed four lines of text. *Second*, briefly discuss the practical significance of the observations made in the first paragraph and identify an aspect of aerospace systems where these insights could be used to improve the design or performance. This second paragraph must not exceed three lines of text and can be supplemented by a relevant external reference.

Guidance, suggestions, hints:

Consider the previous task to double-check the correctness of your EoM derivations.

Task 5: Aeroelastic analysis of a 2 DOF system (20% of 100%)

Specification: Consider a 2 DOF pitching-heaving aeroelastic system shown in *Figure 3*. Use the course resources (slides, examples, guides) to develop this task and define *your own (i.e., modified)* problem parameters such that the system experiences a binary flutter instability within the studied air speed range.

Assumptions: Assume small angles where necessary. The aerodynamic centre can be assumed to be localised one quarter of the chord's length from the leading edge. The positions of the aerodynamic centre, centre of mass and elastic axis (pivot point) are not identical.

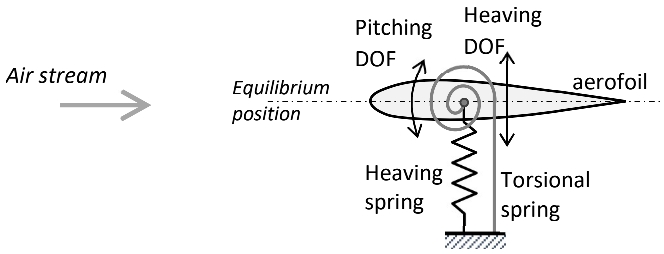


Figure 3 Pitching-heaving aeroelastic system.

Solve the following tasks:

- <u>Results</u>:
  - Present the table with the system parameters you have chosen. Then, demonstrate the required aeroelastic behaviour by including a figure with the upper half of the complex plane and the aeroelastic eigenvalue loci when varying air speed. In the figure, clearly identify the initial, final and flutter air speed cases.
  - Using the eigenvalue loci or other related information, determine the flutter speed U_F and flutter frequency ω_F for your own problem. Briefly describe the process used to obtain the result and estimate the associated flutter speed uncertainty or error.
  - Use numerical integration to compute and plot the free vibration response of the system for these operational conditions: U=0, $0.6\times U_F$, $0.95\times U_F$ and $1.1\times U_F$.
  - Finally, assume unit vertical harmonic loading in the aerodynamic centre. Using this loading and the aeroelastic model, calculate the complex-valued steady state response of the system for the excitation frequency $0.99\times\omega_F$ and the air speed U_F.
- <u>Discussion</u>:
  - *First*, in one short paragraph, with reference to the presented results, discuss the significance and implications of structural damping in the system. *Second*, use the Internet to find two academic papers or industrial reports (one before 1972 and one after 2021), which discuss a similar task in the real-world aerospace context. Explain your motivation behind the choices.
  - Each discussed point must be limited to a maximum three-line paragraph.
- *Appendix* (maximum one additional page expected, the font size and the column count can be adjusted accordingly): include a copy of the *working Matlab* codes which you developed and used to produce the results and graphs in this task.

Guidance, suggestions, hints:
Use quasi-steady aerodynamic theory to determine the aerodynamic loading.

Appendix

Background and context

"*Fighter aircraft have been designed to fly and maneuver at high angles of attack and at high loading conditions. At these high angles of attack, the flow separates from the sharp leading edges of the wing and leading-edge extension (LEX) forming a strong vortical flow that maintains the stability of the aircraft. However, the leading-edge vortices break down upstream of the vertical tails. The breakdown flow impinges upon the vertical tail surfaces causing severe structural fatigue and has lead to their premature fatigue failure, costing millions of dollars every year for inspections and repairs.*" [Sheta&Huttsell, 2003]

Moses, RW, Active vertical tail buffeting alleviation on a twin-tail fighter configuration in a wind tunnel, CEAS International Forum on Aeroelasticity and Structural Dynamics. 1997.

Sheta, EF, Huttsell, LJ., Characteristics of F/A-18 vertical tail buffeting. Journal of fluids and structures, 2003 Mar 1;17(3):461-77.

Within the context of this coursework, you are a dynamics specialist asked to analyse a problem of all-moving vertical stabiliser buffeting on a sixth-generation fighter aircraft. A range of tasks needs to be completed to characterise the system and propose a tuned vibration absorber to suppress the resonant buffeting.

The technical tasks involve (see also the formal task description):

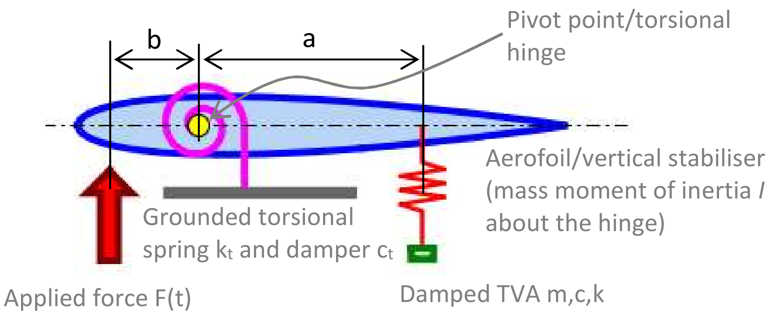
1. *Single DOF analysis* (using Matlab App).
2. *Tuning the Tuned Vibration Absorber* (using Matlab App).
3. EoM for 2 DOF system and eigenvalue problem (full model derivation).
4. *Damping in vibrating systems* (full model derivation).
5. Aeroelastic analysis of a 2 DOF system (full model simulation)

To complete the first two tasks, the provided Matlab system model (called "*App*") will be used as a form of a "*digital twin*" which represents the overall system with the unknown vertical stabiliser properties (later also called "*wing*" or "*aerofoil*"). The other tasks are conceptually linked to the first two tasks but need to be solved independently of the *Matlab App*.

The tool and the modelled system

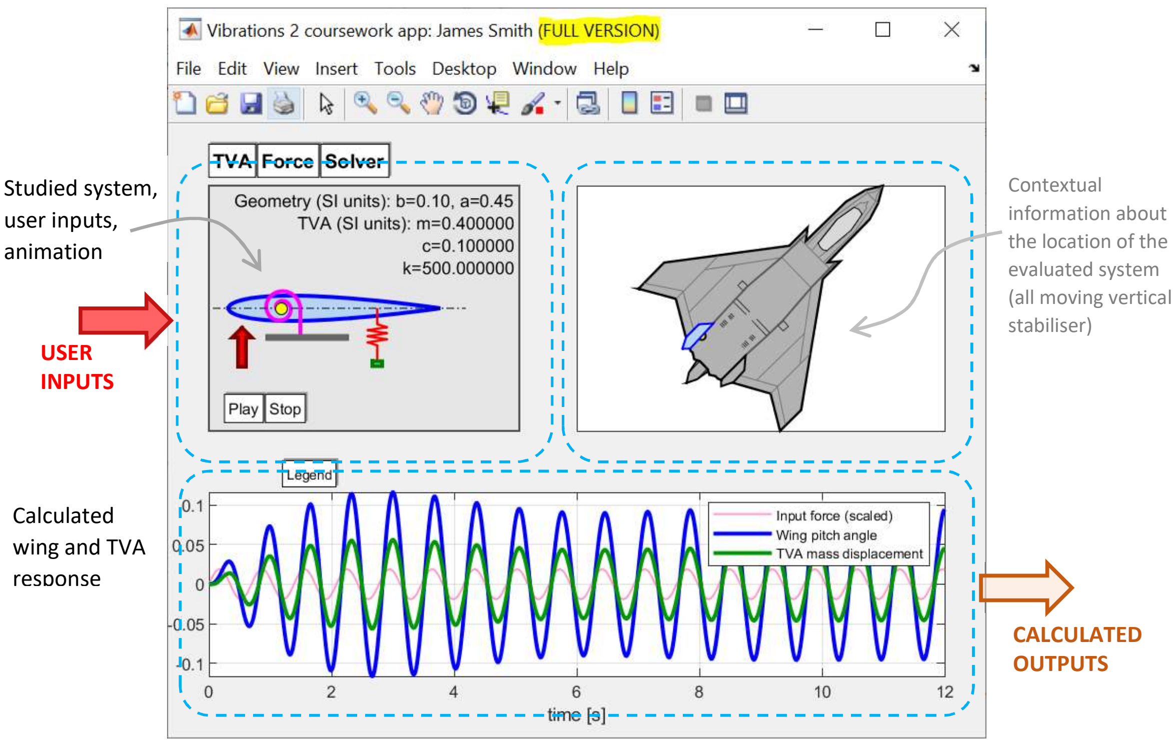
To support the first two tasks of this coursework, a simple *Matlab App* is provided in P-file format to avoid the necessity or possibility of the source code modification. The app represents a model of the problematic vertical stabiliser with the unknown physical properties which will be determined in **Task 1**.

The system's visual representation is shown in the following figure together with its constituent components, model elements and relevant geometry information (note the offset dimensions).



This is a *two degree-of-freedom* problem where the first DOF is realised by the aerofoil pitching and the second DOF is linked with the vertical motion of the TVA mass.

The application window is shown in the following figure where the main user blocks are identified and described.

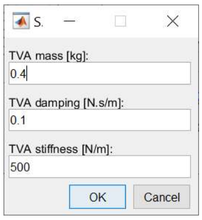


IMPORTANT: You must use the FULL version of the *Matlab App* ~~which can be downloaded from "***Assessment Information***" folder located in the "***Assessment, submission and feedback***" section.~~ which is provided for this coursework. In the FULL version, all students have their own individual aerofoil parameters to work with.

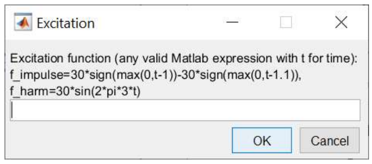
User functionality

After each user-specified parameter update, the system's *vibration response* is automatically updated and shown in the bottom panel. The *user inputs* are applied in the top left panel. The top right panel serves for simple contextual information about the overall system layout.

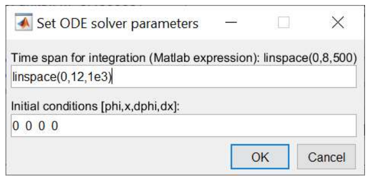
The user input windows can be activated by a mouse click on: "**TVA**" or the spring-mass visualisation of the damped TVA (see figure below) to change the mass, damping and stiffness of the TVA; "**Force**" or the red arrow visualisation of the input force to change the time-dependent excitation function (*see* the figure below); "**Solver**" to adjust the ODE time integration range, discretisation and the initial conditions.



Specify TVA parameters



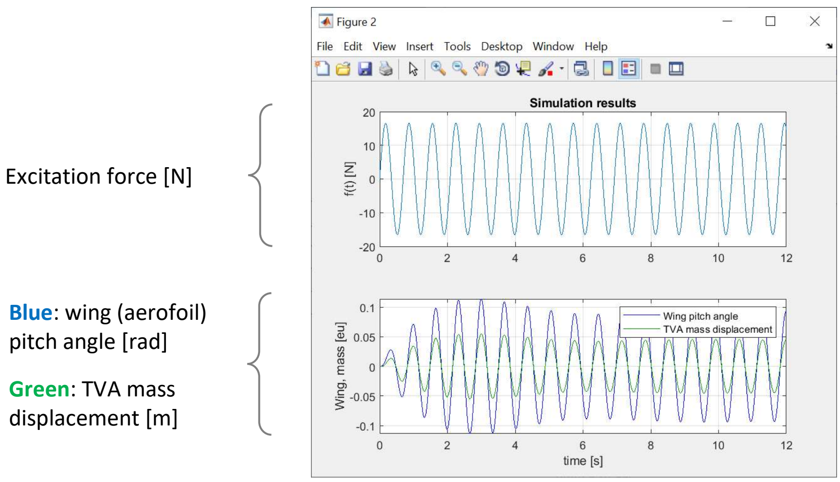
Specify excitation function



Specify ODE integration options

The *time-dependent excitation function* can be any valid *Matlab* expression with the specific parameter values and a time symbol "t". Examples of suitable functions are: "1" (a constant unit step at t=0 sec), "30*sin(2*pi*3*t)" (a harmonic force at 3 Hz), "0" (a zero-force leading to free vibrations for non-zero initial conditions), "0.1*t" (a slow ramp function). More complicated function specifications are possible. The *four initial conditions* represent: (**1**) initial aerofoil angular displacement in [rad]; (**2**) vertical TVA displacement in [m]; (**3**) aerofoil angular velocity in [rad/s]; (**4**) vertical TVA velocity in [m/s].

For precise analysis purposes, the calculated vibration responses can be plotted separately after a mouse click on any of these lines. A separate window will be generated which will show the applied force in [N] (the top subplot). The bottom subplot contains the aerofoil angular displacement (shown in blue) in [rad], and the vertical TVA mass coordinate (shown in green) in [m].



A mouse click on one of the calculated vibration response curves also updates the content of the variable "`var_tvalab`" which is present in the base workspace (i.e., accessible through the command line) and can be used for further analysis in or outside of Matlab. The columns in this matrix represent [time,dof1,dof2,force] and the rows represent the time instants for which the responses were calculated.

An animation of the obtained responses can be started (stopped) by clicking on "Play" ("Stop") text fields. A click on the "Legend" text field switches on/off a legend in the calculated vibration response panel.

App start and initial configuration

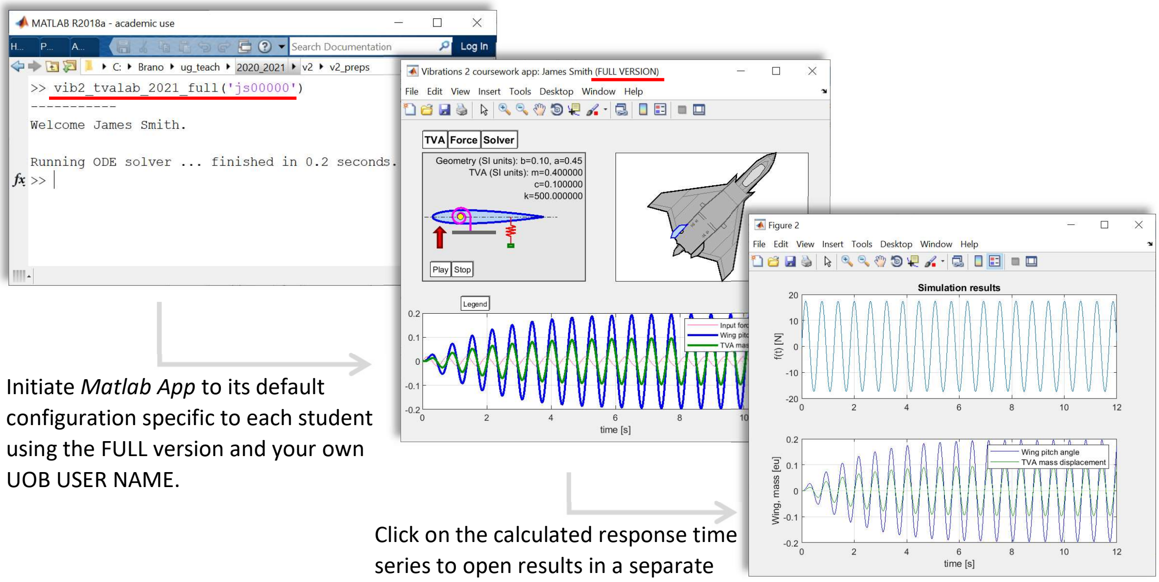
Each student must download their own <u>FULL version</u> from the "*Assessment Information*" folder located in the "*Assessment, submission and feedback*" section of the unit to be able to work with their own problem specification. In other words, every student has their own specific aerofoil parameter values to determine in **Task 1**.

The FULL version of *Matlab App* must be started using a single input argument which represents the student's "UOB USER NAME" typed in lower case on the *Matlab* command line. The P-file must be located in the directory visible to Matlab, e.g., current working directory.

Example:

```
vib2_tvalab_2025_full('js00000')  % use your own UOB USER NAME
```

After this, the application starts and is ready to be used for further analysis:



Initiate *Matlab App* to its default configuration specific to each student using the FULL version and your own UOB USER NAME.

Click on the calculated response time series to open results in a separate window for further analysis.

The default initial state of the app is represented by the student's specific aerofoil parameters, the identical and adjustable TVA parameters, and semi-randomised harmonic excitation force. The force and TVA parameters can be changed. The aerofoil parameters are fixed and specific to each student.

Final remarks: It is recommended to run the app on your own machines, assuming you have *Matlab* installation. However, in case of problems of any kind on your local machines, you can work with this app on your own UOB remote desktops. This option was assessed and was found to be a feasible option for the use of this tool. This *Matlab App* does not assume or require the use of any special toolbox, only the core *Matlab* functionality is used.

End of the document.