# Appendix 1: AeroTrim: A Symmetric Trim Calculator for Subsonic Flight Conditions

***A Mathcad Program Written by M. V. Cook, Version Date 1 September 2012***
***Given the operating condition and some basic geometric and aerodynamic data for a conventional aircraft, this programme calculates an estimate of the symmetric trim state of the aircraft for a chosen airspeed range. The programme is limited to subsonic flight in the troposphere; however, it may be developed easily for application to a wider range of operating conditions and aircraft configurations. Data given are best estimates for the Cranfield Jetstream laboratory aircraft.***

***1. Aircraft Flight Condition***
***(Insert values to define aircraft operating condition)***

***Altitude (ft)*** $ht := 6562$ ***Convert to m*** $ht := 0.3048 \cdot ht$

***Aircraft mass (kg)*** $m := 6300$

***cg position (%c)*** $h := 0.29$

***Flight path angle (deg)*** $\gamma_e := 0$ ***Convert to rad*** $\gamma_e := \dfrac{\gamma_e}{57.3}$

***Gravity constant (m/s²)*** $g := 9.81$

***2. Air Density Calculation***
***Valid for troposphere only—up to 36,000 ft***

***Gas constant (Nm/kgK)*** $R := 287.05$

***Lapse rate (K/m)*** $lr := -0.0065$

***Temperature (K)*** $Temp := 288.16 + lr \cdot ht$

***Air density (kg/m³)*** $\rho := 1.225 \cdot \left(\dfrac{Temp}{288.16}\right)^{-\left[\left(\frac{g}{lr \cdot R}\right)+1\right]}$

***Density ratio*** $\sigma := \dfrac{\rho}{1.225}$

***Check results***
$Temp = 275.15$
$\rho = 1.006$
$\sigma = 0.822$

***3. Set up Velocity Range for Computations***
***Note that true airspeed is assumed unless otherwise stated***

***Counter*** $i := 0..10$

***(set counter to number of velocity test points required)***

***True airspeed range (knots)*** $V_{knots_i} := 100 + 15 \cdot i$

***(set initial velocity and velocity increment as required)***

***True airspeed (m/s)*** $V_i := V_{knots_i} \cdot 0.515$

***Equivalent airspeed (knots)*** $V_{eas_i} := V_{knots_i} \cdot \sqrt{\sigma}$

***4. Aircraft Geometry—Constant***

***(Insert values defined by aircraft geometry)***

***Wing Geometry***

| | | | |
|---|---|---|---|
| ***Wing area ($m^2$)*** | $S := 25.08$ | | |
| ***Wing span (m)*** | $b := 15.85$ | | |
| ***Wing mean chord (m)*** | $c_w := 1.716$ | | |
| ***Sweep 1/4$c_w$ (deg)*** | $\Lambda := 0$ | | |
| ***z coordinate of 1/4$c_w$ point above (−ve) or below (+ve) ox body axis (m)*** | $z_w := 0.45$ | | |
| ***Wing rigging angle (deg)*** | $\alpha_{wr} := 1.0$ | ***Convert to rad*** | $\alpha_{wr} := \frac{\alpha_{wr}}{57.3}$ |

***Tailplane Geometry***

| | | | |
|---|---|---|---|
| ***Tailplane area ($m^2$)*** | $S_T := 7.79$ | | |
| ***Tailplane span (m)*** | $b_T := 6.6$ | | |
| ***Tail arm, 1/4$c_w$ to 1/4$c_t$ (m)*** | $l_t := 6.184$ | | |
| ***z coordinate of 1/4$c_w$ point above (−ve) or below (+ve) ox body axis (m)*** | $z_T := -1.435$ | | |
| ***Tail setting angle (deg)*** | $\eta_T := 1.5$ | ***Convert to rad*** | $\eta_T := \frac{\eta_T}{57.3}$ |
| ***Fuselage diameter or width (m)*** | $F_d := 1.981$ | | |

***Engine Installation***

| | | | |
|---|---|---|---|
| ***Thrust line z coordinate above (−ve) or below (+ve) ox body axis (m)*** | $z_\tau := 0.312$ | | |
| ***Engine thrust line angle (deg) relative to ox body axis (+nose up)*** | $\kappa := 0$ | ***Convert to rad*** | $\kappa := \frac{\kappa}{57.3}$ |

***5. Wing-Body Aerodynamics***

***(Insert values defined by the installed wing aerodynamic design)***

| | | | |
|---|---|---|---|
| ***Wing-body $C_L$-$\alpha$ (per rad)*** | $a := 5.19$ | | |
| ***Maximum lift coefficient*** | $C_{Lmax} := 1.37$ | | |
| ***Zero lift pitching moment*** | $C_{m0} := -0.0711$ | | |
| ***Zero lift drag coefficient*** | $C_{D0} := 0.03$ | | |
| ***Zero lift angle of attack (deg)*** | $\alpha_{w0} := -2$ | ***Convert to rad*** | $\alpha_{w0} := \frac{\alpha_{w0}}{57.3}$ |
| ***Wing-body aero centre*** | $h_0 := -0.08$ | | |

## *6. Tailplane Aerodynamics*

***(Insert values defined by the tailplane aerodynamic design)***

***Tail plane $C_L$-α (per rad)*** $a_1 := 3.2$

***Elevator $C_L$-η (per rad)*** $a_2 := 2.414$

***Zero lift downwash angle (deg)*** $\varepsilon_0 := 2.0$ ***Convert to rad*** $\varepsilon_0 := \dfrac{\varepsilon_0}{57.3}$

## *7. Wing and Tailplane Calculations*

***Aspect ratio*** $Ar := \dfrac{b^2}{S}$

***Wing semi-span (m)*** $s := \dfrac{b}{2}$

***Tail arm, cg to 1/4$c_t$ (m)*** $l_T := l_t - c_w \cdot (h - 0.25)$

***Tail volume*** $V_T := \dfrac{S_T \cdot l_T}{S \cdot c_w}$

***Check results***
$Ar = 10.017$
$s = 7.925$
$l_T = 6.115$
$V_T = 1.107$

## *8. Downwash at Tail*

***Ref:-Stribling, C.B. "Basic Aerodynamics", Butterworth Ltd., 1984.***

***Tail position relative to wing (% of span)*** $x := \dfrac{l_t}{b}$ $z := \dfrac{z_w - z_T}{b}$

$$d_{\varepsilon\alpha} := \frac{a}{\pi^2 \cdot Ar} \cdot \sum_{fi=5}^{85} \frac{0.5 \cdot \cos\left(\frac{fi \cdot \pi}{180}\right)^2}{\sqrt{x^2 + \left(0.5 \cdot \cos\left(\frac{fi \cdot \pi}{180}\right)\right)^2 + z^2}} \cdot \left[ \frac{\left[x + \sqrt{x^2 + \left(0.5 \cdot \cos\left(\frac{fi \cdot \pi}{180}\right)\right)^2 + z^2}\right]}{\left(0.5 \cdot \cos\left(\frac{fi \cdot \pi}{180}\right)\right)^2 + z^2} + \frac{x}{(x^2 + z^2)} \right] \cdot \frac{\pi}{180}$$

***Check result*** $d_{\varepsilon\alpha} = 0.279$

## *9. Induced Drag Factor*

***Ref:- Shevell, R.S. "Fundamentals of Flight", 2nd edition, Prentice Hall Inc., 1983.***
***Drag polar is defined, $C_D = C_{D0} + KC_L^2$, where K is the induced drag factor.***

***Fuselage drag factor*** $S_d := 0.9998 + 0.0421 \cdot \left(\dfrac{F_d}{b}\right) - 2.6286 \cdot \left(\dfrac{F_d}{b}\right)^2 + 2.000 \cdot \left(\dfrac{F_d}{b}\right)^3$

***Empirical constant*** $k_D := -3.333 \cdot 10^{-4} \cdot \Lambda^2 + 6.667 \cdot 10^{-5} \cdot \Lambda + 0.38$

***Oswald efficiency factor*** $e := \dfrac{1}{\pi \cdot Ar \cdot k_D \cdot C_{D0} + \dfrac{1}{(0.99 \cdot s_d)}}$

***Induced drag factor*** $K := \dfrac{1}{\pi \cdot Ar \cdot e}$

***Check result***
$s_d = 0.968$
$k_D = 0.38$
$e = 0.713$
$K = 0.045$

## *10. Basic Performance Parameters*

***Minimum drag speed (knots)*** $V_{md} := \left[\left(\sqrt{\dfrac{2 \cdot m \cdot g}{\rho \cdot S}}\right) \cdot \left(\dfrac{K}{C_{D0}}\right)^{0.25}\right] \cdot \dfrac{1}{0.515}$

***Equivalent minimum drag speed (knots)*** $V_{mdeas} := V_{md} \cdot \sqrt{\sigma}$

***Stall speed (knots)*** $V_{stall} := \sqrt{\dfrac{2 \cdot m \cdot g}{\rho \cdot S \cdot C_{Lmax}}} \cdot \dfrac{1}{0.515}$

***Equivalent stall speed (knots)*** $V_{stalleas} := V_{stall} \cdot \sqrt{\sigma}$

***Neutral point—controls fixed*** $h_n := h_0 + V_T \cdot \dfrac{a_1}{a} \cdot (1 - d_{\varepsilon\alpha})$

***Static margin—controls fixed*** $K_n := h_n - h$

## *11. Trim Calculation*

***The trim computation finds the trim condition for each speed defined in the speed range table and for the flight condition defined in Section 1.***

***Initial seed values for solve block***

$C_L := 0.7 \quad C_{LW} := 0.5 \quad C_D := 0.02 \quad C_\tau := 0.4 \quad \alpha_e := 0.1 \quad C_{LT} := 0.1$

***Trim solve block***

Given

***Total normal force (oz body axis)*** $2 \cdot \dfrac{m \cdot g}{\rho \cdot (V)^2 \cdot S} \cdot \cos(\alpha_e + \gamma_e) = C_L \cdot \cos(\alpha_e) + C_D \cdot \sin(\alpha_e) + C_\tau \cdot \sin(\kappa)$

***Total axial force (ox body axis)*** $2 \cdot \dfrac{m \cdot g}{\rho \cdot (V)^2 \cdot S} \cdot \sin(\alpha_e + \gamma_e) = C_\tau \cdot \cos(\kappa) - C_D \cdot \cos(\alpha_e) + C_L \cdot \sin(\alpha_e)$

***Total drag coefficient*** $C_D = C_{D0} + K \cdot {C_L}^2$

***Wing-body lift coefficient*** $C_{LW} = a \cdot (\alpha_e + \alpha_{wr} - \alpha_{w0})$

***Pitching moment (about cg)*** $0 = [C_{m0} + (h - h_0) \cdot C_{Lw}] - V_T \cdot C_{LT} + C_\tau \cdot \dfrac{z_\tau}{c_w}$

***Tail lift coefficient*** $C_{LT} = (C_L + C_{LW}) \cdot \dfrac{S}{S_T}$

$\mathrm{Trim}(V) := \mathrm{Find}(\alpha_e, C_\tau, C_D, C_{LT}, C_{LW}, C_L)$

***End of trim solve block***

## *12. Trim Variables Calculation*

$\alpha_{e_i} := \mathrm{Trim}(V_i)_0$ $C_{\tau_i} := \mathrm{Trim}(V_i)_1$ $C_{D_i} := \mathrm{Trim}(V_i)_2$

$C_{LT_i} := \mathrm{Trim}(V_i)_3$ $C_{Lw_i} := \mathrm{Trim}(V_i)_4$ $C_{L_i} := \mathrm{Trim}(V_i)_5$

***Wing incidence*** $\alpha_{w_i} := \alpha_{e_i} + \alpha_{w_r}$

***Trim elevator angle*** $\eta_{e_i} := \dfrac{C_{LT_i}}{a_2} - \dfrac{a_1}{a_2} \cdot \left[\alpha_{w_i} \cdot (1 - d_{\varepsilon\alpha}) + \eta_T - \alpha_{wr} - \varepsilon_0\right]$

***Pitch attitude*** $\theta_{e_i} := \gamma_e + \alpha_{w_i} - \alpha_{w_r}$

***Tail angle of attack*** $\alpha_{T_i} := \alpha_{w_i} \cdot (1 - d_{\varepsilon\alpha}) + \eta_T - \varepsilon_0 - \alpha_{wr}$

***Lift to drag ratio*** $L_{D_i} := \dfrac{C_{Lw_i}}{C_{D_i}}$

## *13. Conversions of Angles to Degrees*

$\alpha_{w_i} := \alpha_{w_i} \cdot 57.3$ $\alpha_{e_i} := \alpha_{e_i} \cdot 57.3$ $\theta_{e_i} := \theta_{e_i} \cdot 57.3$

$\alpha_{T_i} := \alpha_{T_i} \cdot 57.3$ $\eta_{e_i} := \eta_{e_i} \cdot 57.3$ $\gamma_e := \gamma_e \cdot 57.3$

## *14. Total Trim Forces Acting on Aircraft*

***Total lift force (N)*** $L_i := 0.5 \cdot \rho \cdot (V_i)^2 \cdot S \cdot C_{L_i}$

***Total drag force(N)*** $D_i := 0.5 \cdot \rho \cdot (V_i)^2 \cdot S \cdot C_{D_i}$

***Total thrust (N)*** $T_i := 0.5 \cdot \rho \cdot (V_i)^2 \cdot S \cdot C_{\tau_i}$

# SUMMARY RESULTS OF TRIM CALCULATION

## *15. Definition of Flight Condition*

| | | | |
|---|---|---|---|
| ***Aircraft weight (N)*** | $m \cdot g = 6.18 \cdot 10^4$ | ***Minimum drag speed (knots)*** | $V_{md} = 155.307$ |
| ***Altitude (ft)*** | $ht \cdot 3.281 = 6.562 \cdot 10^3$ | ***Equivalent minimum drag speed (knots)*** | $V_{mdeas} = 140.769$ |
| ***Flight path angle (deg)*** | $\gamma_e = 0$ | ***Stall speed (knots)*** | $V_{stall} = 116.092$ |
| ***cg position (%$c_w$)*** | $h = 0.29$ | ***Equivalent stall speed (knots)*** | $V_{stalleas} = 105.225$ |
| ***Neutral point —controls fixed*** | $h_n = 0.412$ | ***Static margin-controls fixed*** | $K_n = 0.122$ |

## *16. Trim Conditions as a Function of Aircraft Velocity*

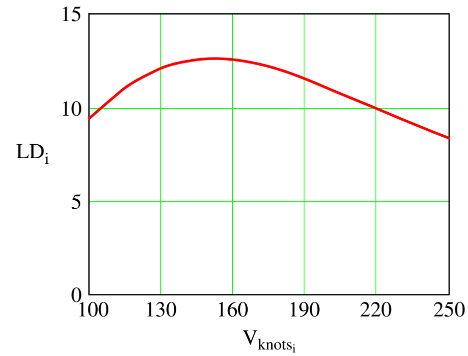
***Angles in degrees, velocity in m/s, forces in N unless indicated otherwise.***

| $V_{knots_i}$ | $V_i$ | $C_{L_i}$ | $C_{D_i}$ | $C_{Lw_i}$ | $C_{LT_i}$ | $LD_i$ | $C_{\tau_i}$ | $\alpha_{w_i}$ | $\alpha_{e_i}$ | $\theta_{e_i}$ | $\alpha_{T_i}$ | $\eta_{e_i}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 100 | 51.5 | 1.799 | 0.174 | 1.64 | 0.514 | 9.409 | 0.181 | 16.105 | 15.105 | 15.105 | 10.107 | −1.208 |
| 115 | 59.225 | 1.374 | 0.114 | 1.258 | 0.375 | 11.017 | 0.116 | 11.885 | 10.885 | 10.885 | 7.066 | −0.46 |
| 130 | 66.95 | 1.081 | 0.082 | 0.994 | 0.282 | 12.106 | 0.083 | 8.97 | 7.97 | 7.97 | 4.965 | 0.1 |
| 145 | 74.675 | 0.872 | 0.064 | 0.805 | 0.215 | 12.603 | 0.064 | 6.885 | 5.885 | 5.885 | 3.462 | 0.521 |
| 160 | 82.4 | 0.717 | 0.053 | 0.665 | 0.167 | 12.573 | 0.053 | 5.346 | 4.346 | 4.346 | 2.353 | 0.842 |
| 175 | 90.125 | 0.6 | 0.046 | 0.56 | 0.13 | 12.154 | 0.046 | 4.181 | 3.181 | 3.181 | 1.513 | 1.091 |
| 190 | 97.85 | 0.51 | 0.042 | 0.478 | 0.102 | 11.496 | 0.042 | 3.277 | 2.277 | 2.277 | 0.862 | 1.287 |
| 205 | 105.575 | 0.438 | 0.039 | 0.413 | 0.08 | 10.72 | 0.039 | 2.564 | 1.564 | 1.564 | 0.348 | 1.444 |
| 220 | 113.3 | 0.381 | 0.036 | 0.361 | 0.063 | 9.912 | 0.036 | 1.99 | 0.99 | 0.99 | −0.066 | 1.572 |
| 235 | 121.025 | 0.334 | 0.035 | 0.319 | 0.048 | 9.123 | 0.035 | 1.523 | 0.523 | 0.523 | −0.403 | 1.677 |
| 250 | 128.75 | 0.295 | 0.034 | 0.284 | 0.036 | 8.383 | 0.034 | 1.136 | 0.136 | 0.136 | −0.681 | 1.764 |

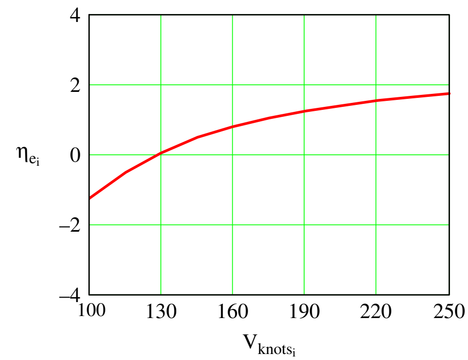
| $V_{knots_i}$ | $V_i$ | $L_i$ | $D_i$ | $T_i$ |
|---|---|---|---|---|
| 100 | 51.5 | $6.023 \cdot 10^4$ | $5.834 \cdot 10^3$ | $6.042 \cdot 10^3$ |
| 115 | 59.225 | $6.083 \cdot 10^4$ | $5.053 \cdot 10^3$ | $5.146 \cdot 10^3$ |
| 130 | 66.95 | $6.115 \cdot 10^4$ | $4.643 \cdot 10^3$ | $4.688 \cdot 10^3$ |
| 145 | 74.675 | $6.134 \cdot 10^4$ | $4.494 \cdot 10^3$ | $4.518 \cdot 10^3$ |
| 160 | 82.4 | $6.146 \cdot 10^4$ | $4.535 \cdot 10^3$ | $4.548 \cdot 10^3$ |
| 175 | 90.125 | $6.154 \cdot 10^4$ | $4.722 \cdot 10^3$ | $4.729 \cdot 10^3$ |
| 190 | 97.85 | $6.16 \cdot 10^4$ | $5.025 \cdot 10^3$ | $5.029 \cdot 10^3$ |
| 205 | 105.575 | $6.165 \cdot 10^4$ | $5.424 \cdot 10^3$ | $5.426 \cdot 10^3$ |
| 220 | 113.3 | $6.17 \cdot 10^4$ | $5.907 \cdot 10^3$ | $5.908 \cdot 10^3$ |
| 235 | 121.025 | $6.174 \cdot 10^4$ | $5.465 \cdot 10^3$ | $6.465 \cdot 10^3$ |
| 250 | 128.75 | $6.179 \cdot 10^4$ | $5.089 \cdot 10^3$ | $7.089 \cdot 10^3$ |

*17. Some Useful Trim Plots*

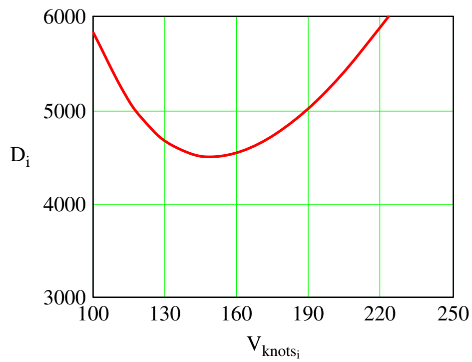
***Lift to drag ratio with true airspeed.***



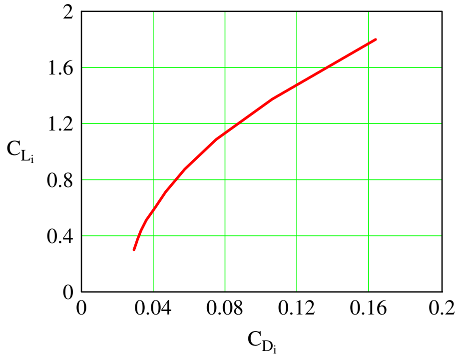

***Elevator angle (deg) to trim with true airspeed.***




***Total drag (N) variation with true airspeed.***



***Drag polar.***




***- End of program -***