University of BRISTOL

LONGITUDINAL BALANCE

Prof. Mark Lowenberg & Prof. Tom Richardson
Email: thomas.richardson@bristol.ac.uk

bristol.ac.uk

Bristol and Gloucestershire Gliding Club
Tug Aircraft: 141hp EuroFox 2K

Flight Dynamics Principles – Book References

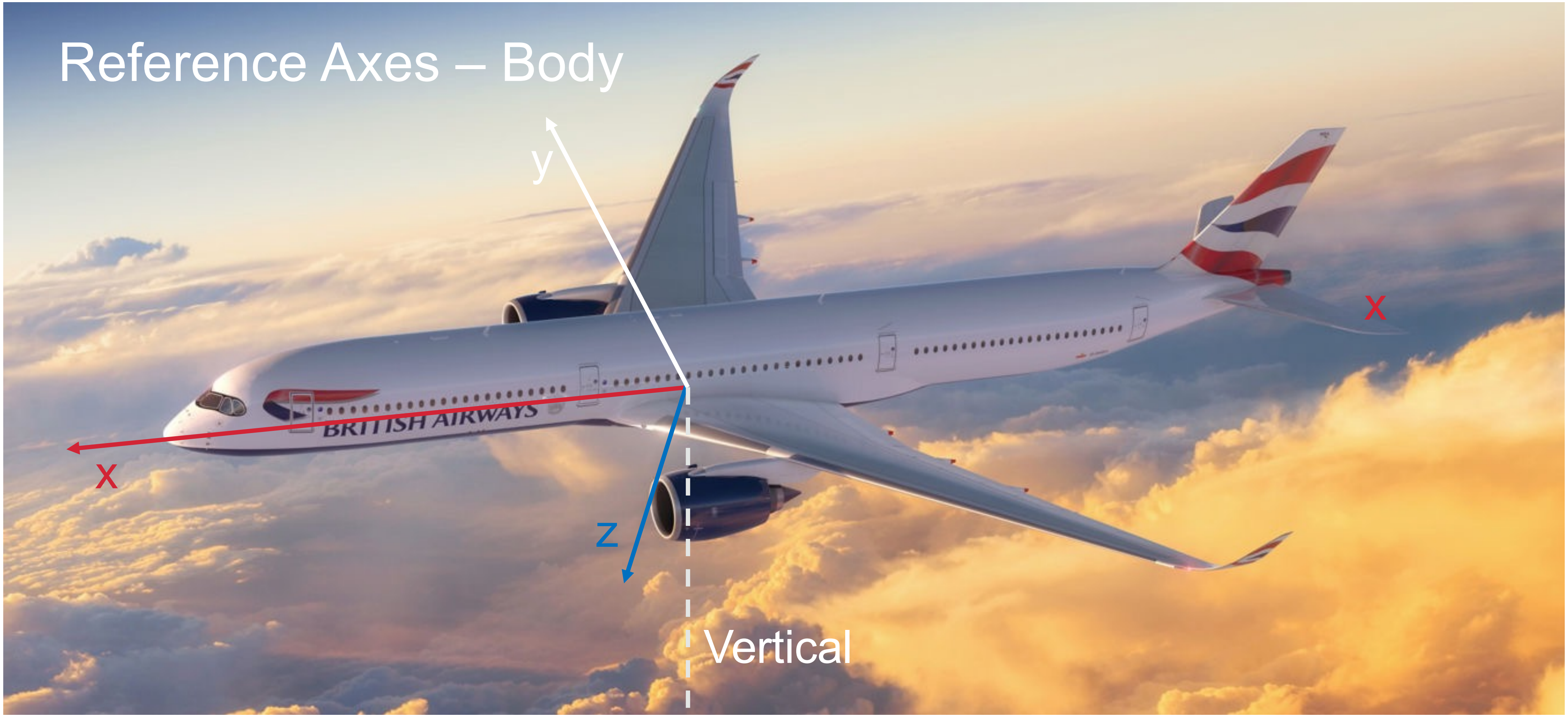
- Note: some of the nomenclature and derivations differ from those we use in lectures.
- These references are for the 2007 edition.

  Flight Dynamics Principles: A Linear Systems Approach to Aircraft Stability and Control (Elsevier Aerospace Engineering) Hardcover – 9 Aug 2007

- Chapter 3: Static equilibrium and trim
  - *Section 3.1 – Trim equilibrium*
  - *Section 3.2 – The pitching moment equation*
  - *Section 3.3 – Longitudinal static stability*
  - Section 3.4 – Lateral static stability
  - Section 3.5 – Directional static stability
  - Section 3.6 – Calculation of aircraft trim condition

- Chapter 4: The equations of motion
  - *Section 4.1 – The equations of motion of a rigid symmetric aircraft*
  - Section 4.2 – The linearised equations of motion
  - Section 4.3 – The decoupled equations of motion
  - Section 4.4 – Alternative forms of the equations of motion

Reference Axes – Body



Airbus A350-1000

https://www.britishairways.com

Balance vs Stability - Introduction

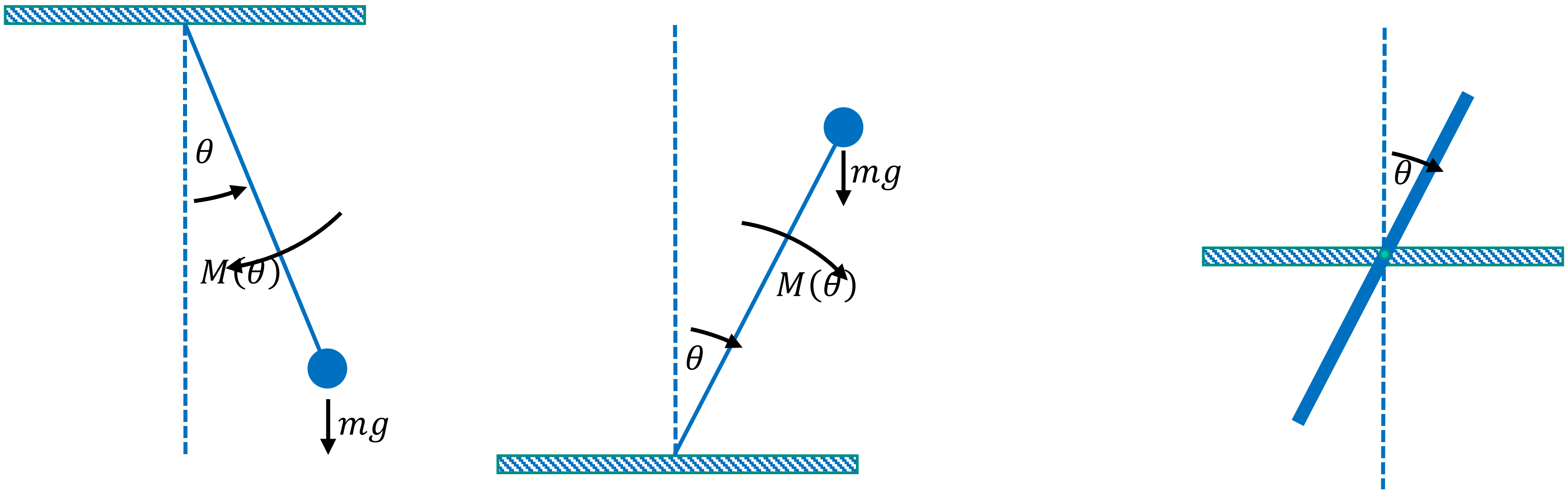
- Trim Equilibrium – *Flight Dynamics Principles References*
    - *'In steady non-accelerating flight the aircraft is in equilibrium and the forces and moments acting on the airframe are in balance and sum to zero. This initial condition is usually referred to as trimmed equilibrium.'*
- 'In normal flight it is usual for the pilot to adjust the controls of an aircraft such that on releasing the controls it continues to fly at the chosen flight condition….. The aircraft is then said to be trimmed, and the trim state defines the initial condition about which the dynamics of interest may be studied.'
- 'The static stability of an aircraft is commonly interpreted to describe its tendency to converge on the initial equilibrium condition following a small disturbance from trim. Dynamic stability, on the other hand, describes the transient motion involved in the process of recovering equilibrium following the disturbance.'

Balance vs Stability - Introduction

- ‘Dynamic stability is also important of course, and largely determines the characteristics of the transient motion, following a disturbance, about a trimmed flight condition.’
- ‘The object(*ive*) of trimming is to bring the forces and moments acting on the aircraft into a state of equilibrium. That is the condition when the axial, normal and side forces, and the roll, pitch and yaw moments are all zero.’
- ‘Provided that the aircraft is stable it will then stay in equilibrium until it is disturbed by pilot control inputs or by external influences such as turbulence.’
- ‘The transient motion following such a disturbance is characterised by the dynamic stability characteristics and the stable aircraft will eventually settle into its equilibrium state once more.’

Balance vs Stability - Introduction

- Flight Balance vs Flight Stability
- Consider a normal pendulum vs an inverted pendulum:



Both cases display zero in-plane moment when $\theta = 0$.

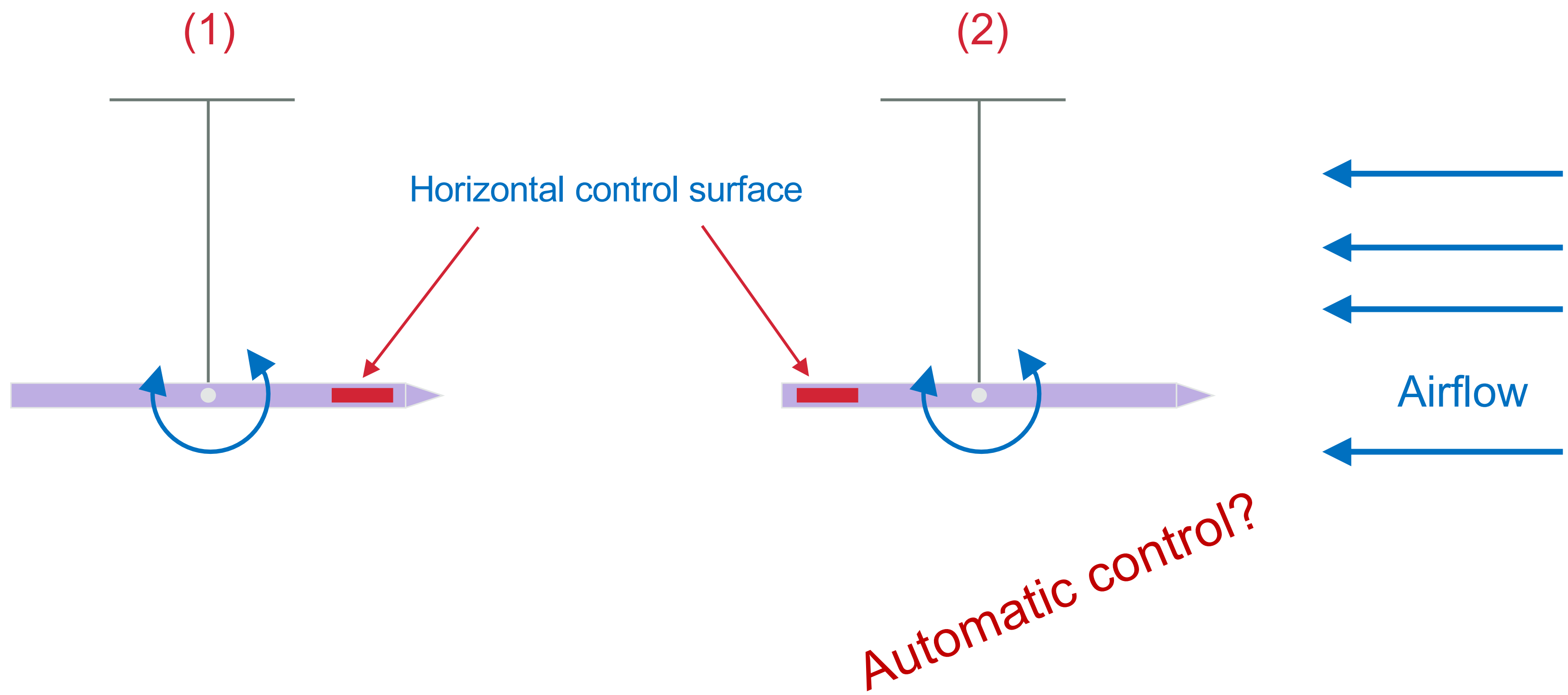
Balance vs Stability - Introduction

- Consider the moment $M(\theta)$ due created as a result of either pendulum being moved from its position of equilibrium.
- Normal pendulum – restoring moment that acts toward the original equilibrium orientation.
- Inverted pendulum – the moment created acts in a direction that would cause the pendulum to move further away (depart) from the original equilibrium position.

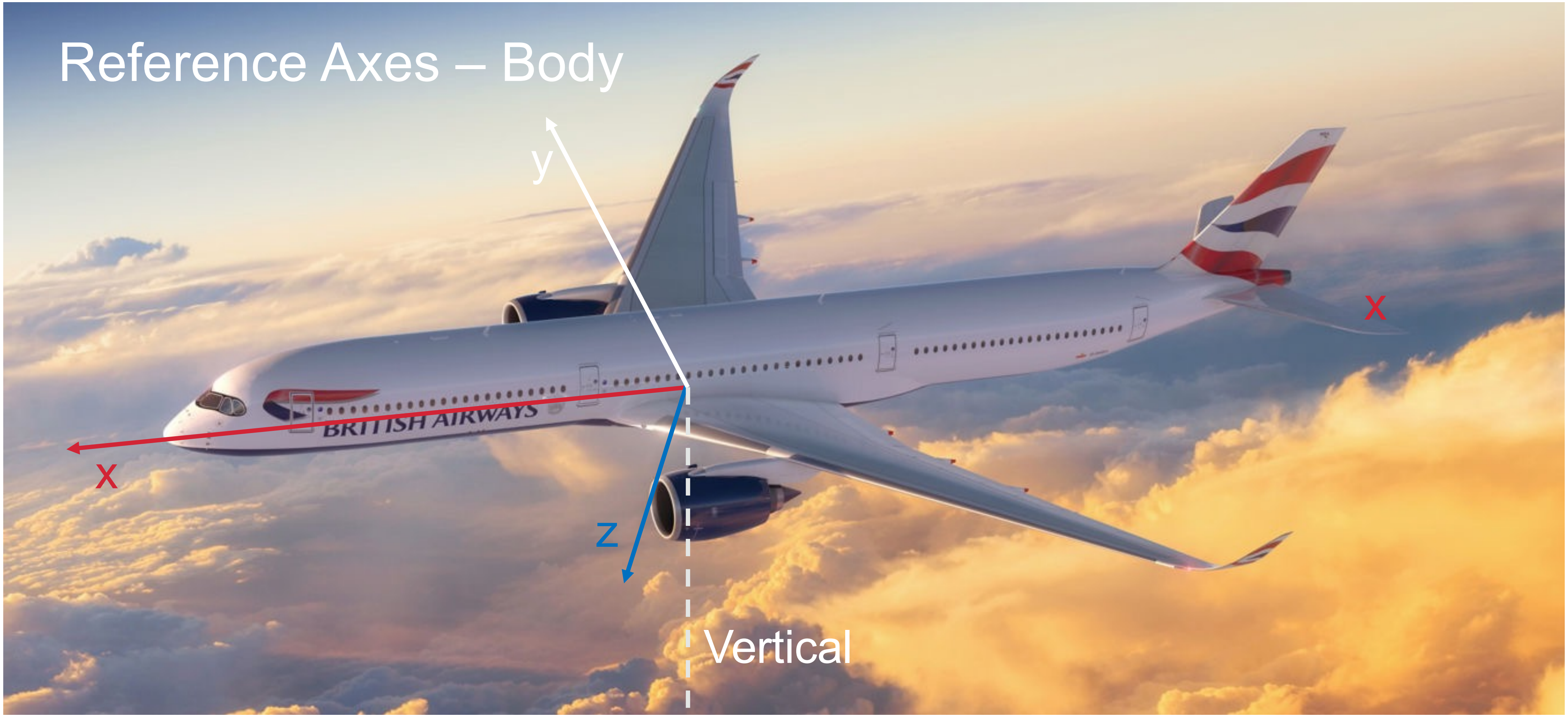
- Define this as positive static stability in the case of the normal pendulum and negative static stability for the inverted pendulum.
- Neutral stability would occur if a slender bar were supported at its mid-point such that the c.g. was at the fulcrum. In this case every value of θ would be an equilibrium position.

Balance vs Stability - Introduction

For a simple missile model, free to rotate in pitch in a wind tunnel, consider the following two situations:



Reference Axes – Body



Airbus A350-1000

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Longitudinal balance

Simplest version of the longitudinal model (assumes axial forces – thrust and drag – in balance):

- we need enough lift to support the aircraft in the trimmed condition.
- we need to distribute the aerodynamic forces such that overall pitching moment is zero.

Challenge:

- To arrange for both of these to be achieved simultaneously.
- To establish and maintain this balance throughout flight e.g. with careful adjustments to the throttle and elevator.
- We will consider straight and level flight, however we could easily extend the analysis to include steady climbing flight. i.e. non-zero, constant flight path angle γ and even turning flight.

Longitudinal balance

Note: always include a force/moment diagram

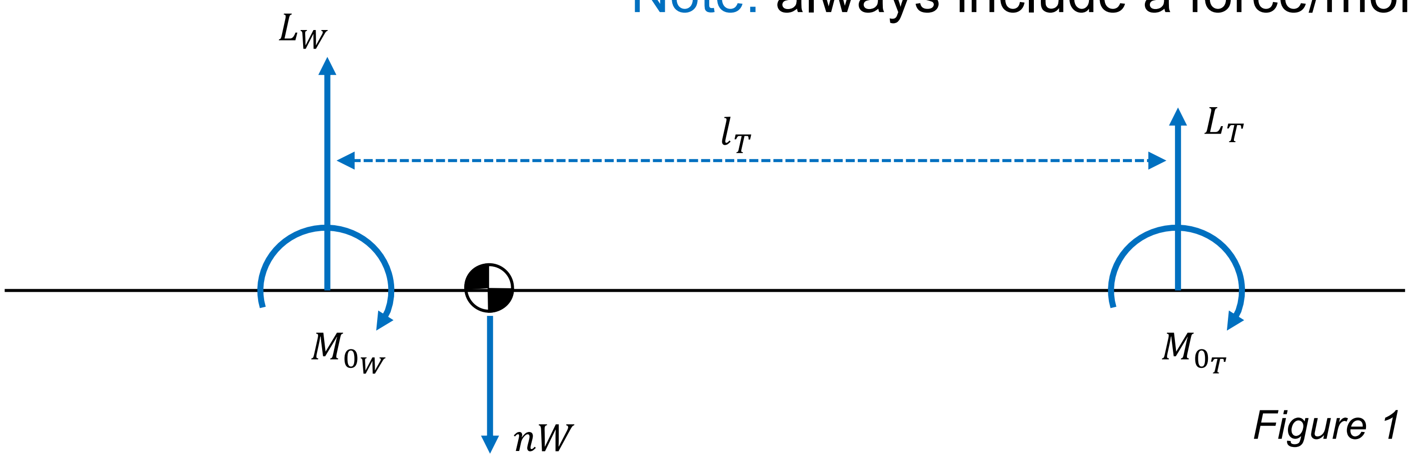


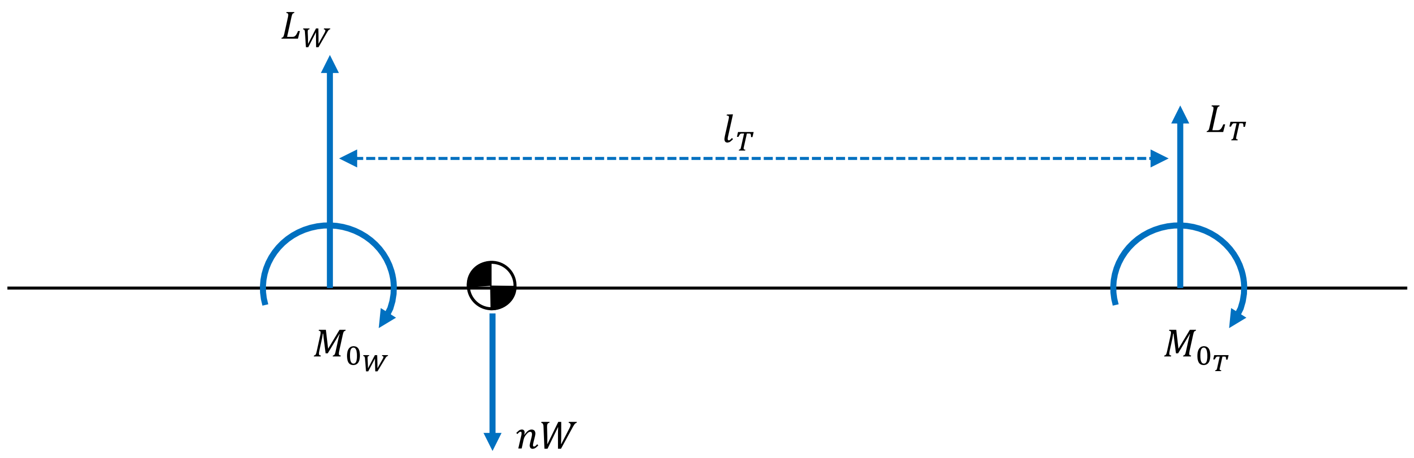
Figure 1

Hence equation (1) where a balance of transverse forces is sought to ensure that:

$$\sum F_{vertical} = 0 \qquad (1)$$

- The longitudinal plane of symmetry will contain the lift vectors for both wing and tail.
- There will also be aerodynamic moments which act in the pitch sense.
- For zero roll angle, the longitudinal plane will also contain the gravity (weight) vector.

Longitudinal balance



- A need for balance in pitch leads to:

$$\sum M_{pitch} = 0 \tag{2}$$

(positive nose-up, about any point)

- Since the moment $L_T\, l_T$ is very large compared with M_{o_T} we can ignore M_{o_T} relative to $L_T\, l_T$, though M_{o_W} must be retained.

Longitudinal balance

An alternative configuration:

Figure 2

where the wing must produce $L_W = nW + L_T$ (3)

Efficient?

Longitudinal balance

- For the configuration shown in *Figure 1*:

$$\begin{aligned} L_W &= \bar{q}\ S\ C_{L_W} \\ L_T &= \bar{q}\ S_T\ C_{L_T}\ , \end{aligned} \tag{4}$$

  and yet we must retain $(L_W + L_T) = nW$.

- The simple expression $L = \bar{q}Sa_1\alpha$ suggests that if speed (or $\bar{q}$) drops, we must simply increase the α to keep L constant,
- *but* the two separate lift coefficients C_{L_W} and C_{L_T} will be seen later to be separately related to α and we cannot simply alter the aircraft incidence and expect both L_W and L_T altered by the same factor.
- Therefore, we must expect the *relative* values of L_W and L_T to change even if their sum remains at nW. (i.e. the need to trim/balance an aircraft for a given airspeed).
- This typically requires adjustment of both elevator (or stabiliser) and throttle.

Longitudinal balance – Aerodynamic Centre

Credits: NASA Photo / Tom MIller

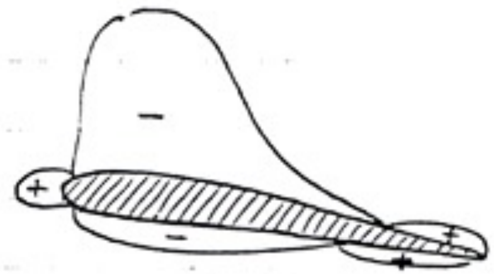
https://www.nasa.gov/centers/armstrong/news/FactSheets/FS-098-DFRC.html

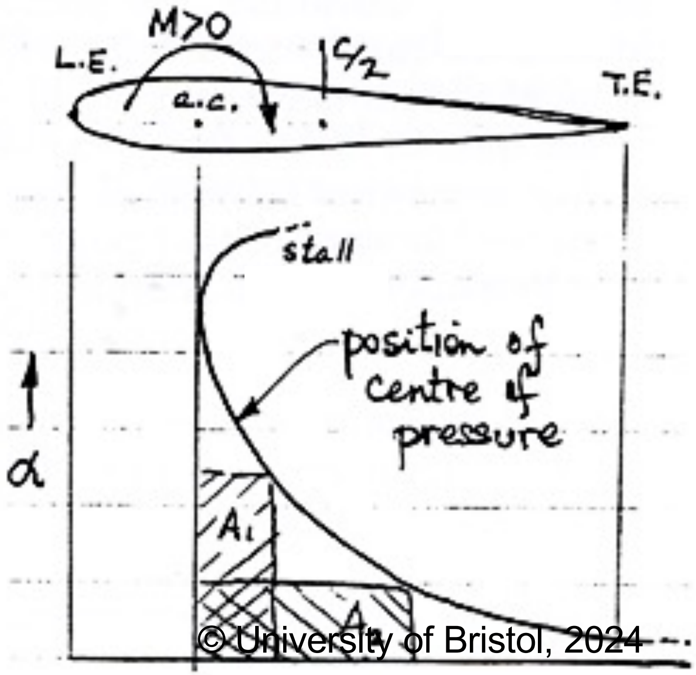
Longitudinal balance – Aerodynamic Centre

- The centre of pressure for an aerofoil can move quite noticeably over the chord as the incidence varies.
- For a *reasonable range of α*, can we find a point (the aerodynamic centre) on the aerofoil about which the moment generated will be constant. i.e.

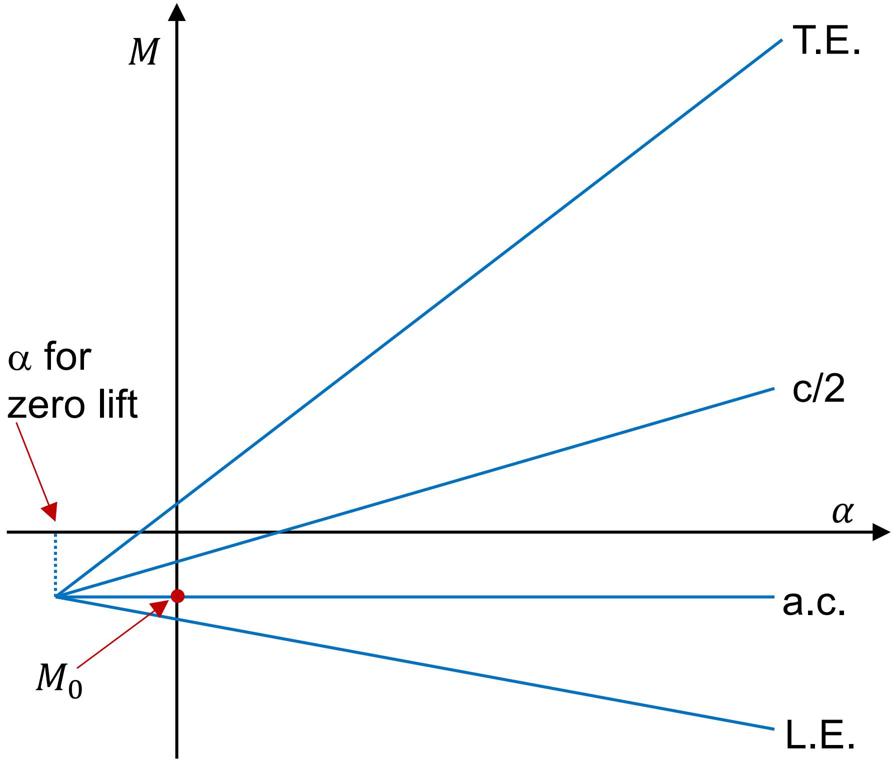
$$\text{lift force} \times \text{arm about a.c.} = \text{constant.} \qquad (5)$$

- Define the pitching moment M_0 that is taken to be constant for a range of α and is not altered when there is a Δ*Lift.*





Longitudinal balance – Aerodynamic Centre



- As the incidence changes, the value of lift changes and the centre of its action also changes (moment arm from c.p. to any chosen point).
- Clearly, for most values of α there must be a positive moment about the T.E. and a negative moment about the L.E.
- What also becomes evident is that for some position near $\frac{1}{4}\bar{c}$ the pitching moment can be independent of α though not necessarily of zero value.

A Simple model for Longitudinal Balance

Credits: NASA Photo / Tom MIller

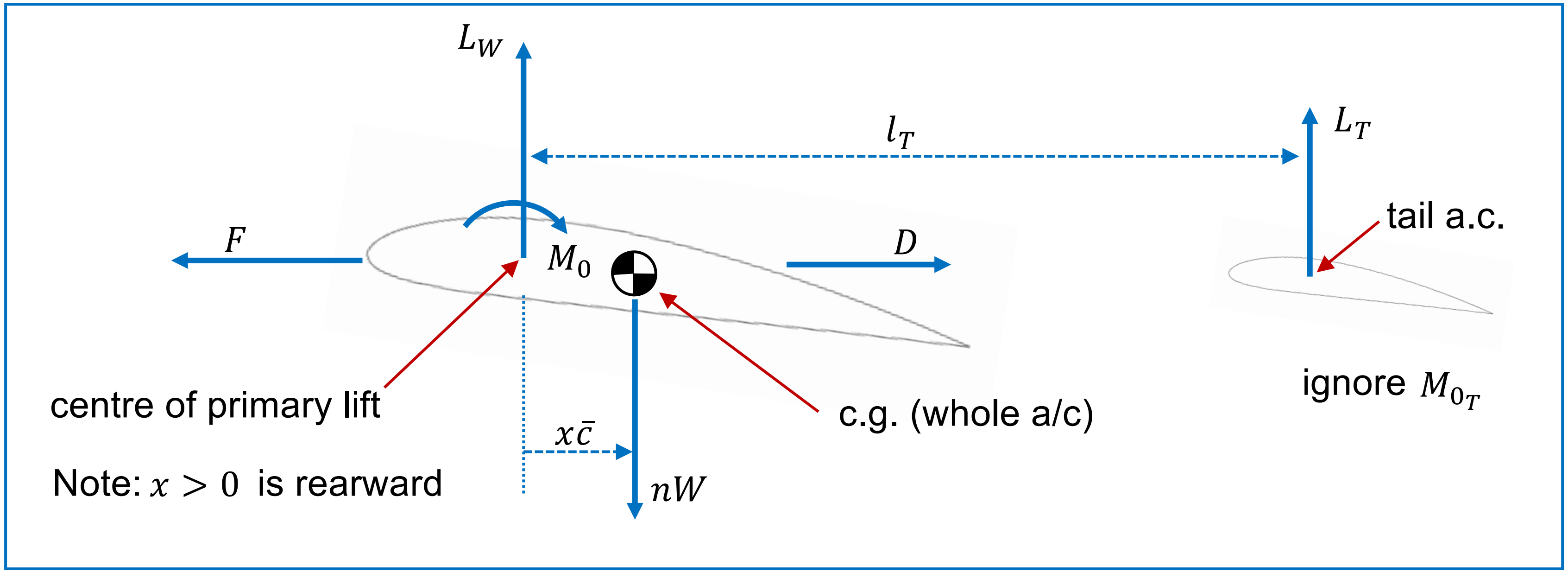
https://www.nasa.gov/centers/armstrong/news/FactSheets/FS-098-DFRC.html

Longitudinal balance

Note: definition of "wing, fuselage and pod/nacelles" – L_{WFP}:

- For ease of use, we can '*lump together*' all lift contributions *other* than those due to the tail and thus refer to the primary lift as L_{WFP} which means lift on the wing, fuselage and pods (engine nacelles).
- We may also refer to this as simply L_W.
- We still consider tail lift separately.
- There would still be a moment that is independent of lift but it would act about an aerodynamic centre which is not now at the wing quarter-chord (due to the contribution of the fuselage).

Simple model for longitudinal balance



Simple model for longitudinal balance

Assumptions:

- The pitching moments due to thrust F and all drag components D are often considered to be quite small, compared with other moments;
- i.e. we initially ignore the pitching effect of F and D.
- The two lift forces act normal to the horizontal wind vector which itself is aligned with F and D. Clearly, this is true for D but the thrust line is fixed by the engine installation and will not in general be aligned with the wind vector. The difference of alignment will not be large, hence the assumption given above.

Simple model for longitudinal balance

Assumptions:

- The weight vector nW acts vertically, normal to the body x axis. In other words, we are confining the analysis to the fuselage being horizontal.
- Note that whilst we might choose to neglect L_T in the transverse equation, if $L_T \ll L_w$, we cannot ignore its major contribution to pitching moment because of its long moment arm l_T.
- The tail is not *shown* to have an elevator: for now, we include such effects in any expression for C_{L_T}.

Simple model for longitudinal balance

The three equations valid in the plane of symmetry are :

- transverse: $L_w + L_T = nW$ (6)
- axial: $F = D$ (7)
- pitch: for the moment we shall choose to take moments about the a.c. of the wing, leading to:

$$\begin{aligned} M_{ac} &= M_0 + nWx\bar{c} - L_T l_T \\ &= 0 \quad \text{for equilibrium.} \end{aligned} \tag{8}$$

- We can non-dimensional the equations using $\bar{q}S$ for forces and $\bar{q}S\bar{c}$ for moments.

Simple model for longitudinal balance

With $\bar{q} = \frac{1}{2}\rho U^2 = \frac{1}{2}\rho_0 {U_E}^2$, from Eqn. (6):

$$\frac{L_W}{\bar{q}S} + \frac{S_T}{S}\frac{L_T}{\bar{q}S_T} = \frac{nW}{\bar{q}S}$$

or $$C_{L_W} + \frac{S_T}{S} C_{L_T} = \frac{nW}{\bar{q}S} \qquad (9)$$

and from Eqn. (8)

$$\frac{M_{ac}}{\bar{q}S\bar{c}} = \frac{M_0}{\bar{q}S\bar{c}} + \frac{nWx\bar{c}}{\bar{q}S\bar{c}} - \frac{S_T}{S}\frac{L_T l_T}{\bar{q}S_T\bar{c}} \qquad (10)$$

Simple model for longitudinal balance

If we distinguish between a total C_L (no subscript on L) and the wing coefficient C_{L_W}, i.e. if we choose to use, for convenience:

$$C_L = \frac{(L_W + L_T)}{\bar{q}S} \tag{11}$$

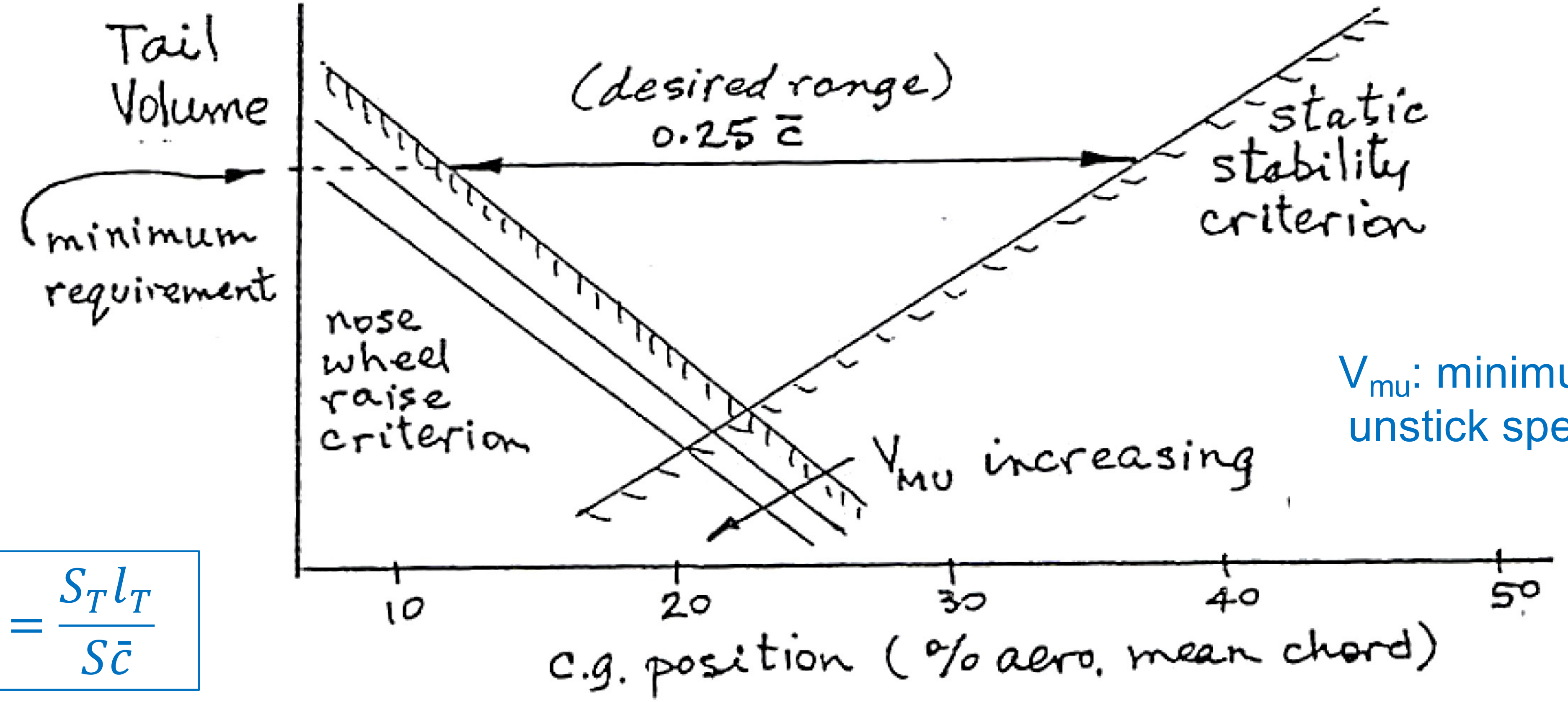
then Eqn. (10) becomes:

$$\begin{aligned} C_{M_{ac}} &= C_{M_0} + xC_L - \frac{S_T l_T}{S\bar{c}} C_{L_T} \\ &= C_{M_0} + xC_L - \bar{V} C_{L_T} \end{aligned} \tag{12}$$

where $\bar{V} = \dfrac{S_T l_T}{S\bar{c}}$ (13)

is the 'tail volume' coefficient.

Simple model for longitudinal balance – design aspects



V_{mu}: minimum-unstick speed

$$\bar{V} = \frac{S_T l_T}{S\bar{c}}$$

CG / Balance Diagram

Design requirements. Note the practical limits on C.G. movement, namely about 11%-36%; i.e. must ensure C.G. of A320 lies within ~1 metre.

The Loading Loop:

- There is a need to derive the C.G's at different Loaded Configurations.
- Can define a 'reasonable' passenger loading procedure, i.e. how the passengers will be distributed throughout the cabin when they are all on board.
- The effects of the holds and the fuel are shown as added to the case with full passengers but there might be a partly loaded configuration.

CG / Balance Diagram

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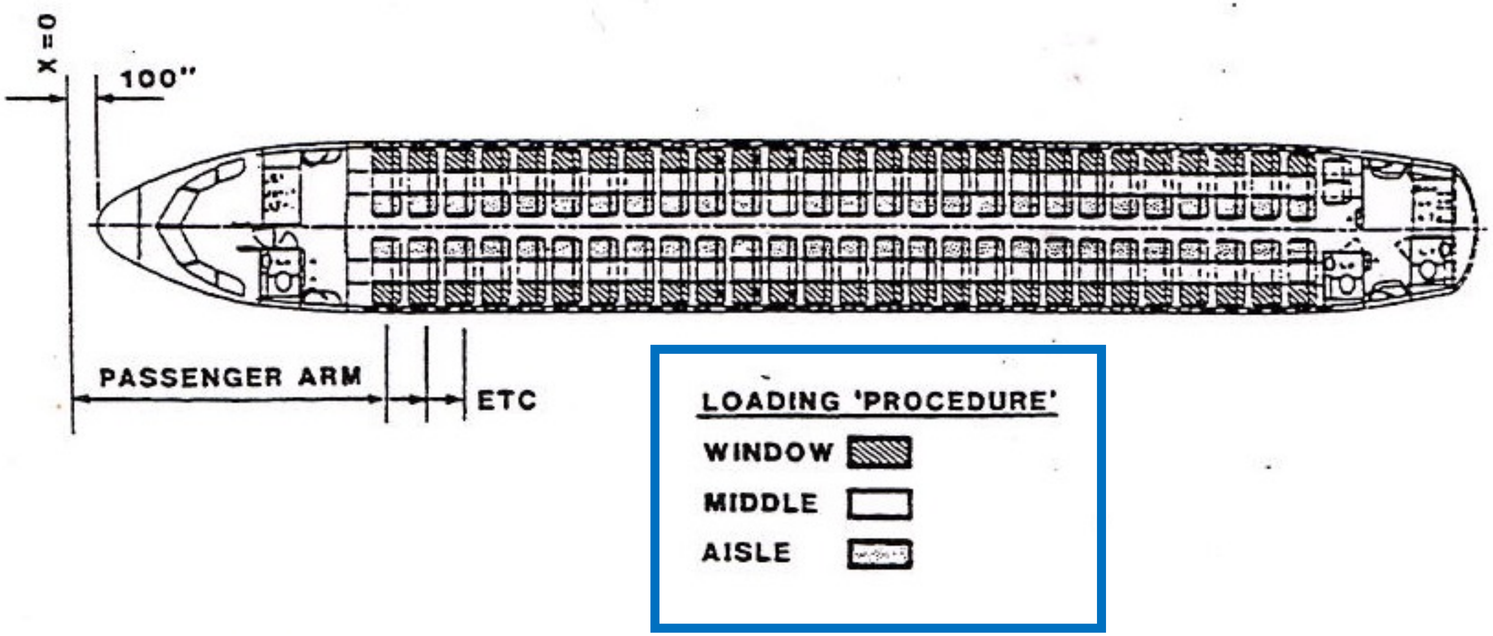
DESIGN PROJECT

12. UB90 Cabin Layout - VG 6



CG / Balance Diagram

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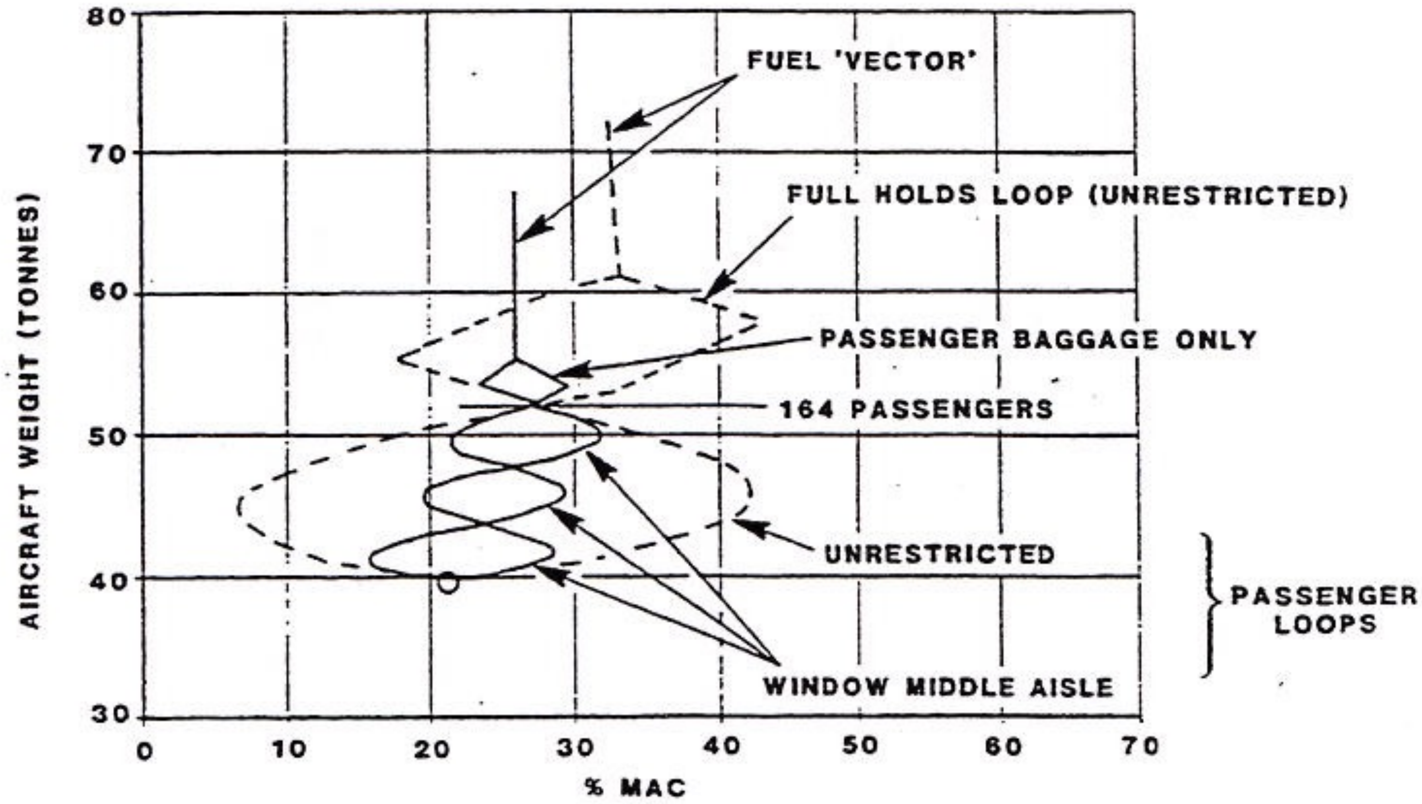
DESIGN PROJECT

11. Passenger/Holds/Fuel Loading Loops - VG 5



4.15

CG / Balance Diagram

Example of a C.G. Limit:

- Note the rear take-off limit, which is a function of the required load on the nosewheel of the aircraft during take-off, to ensure adequate steering.
- One requirement might be that the load on the nosewheel shall be no less than 2.5% of the total aircraft weight, with full static engine thrust.

CG / Balance Diagram

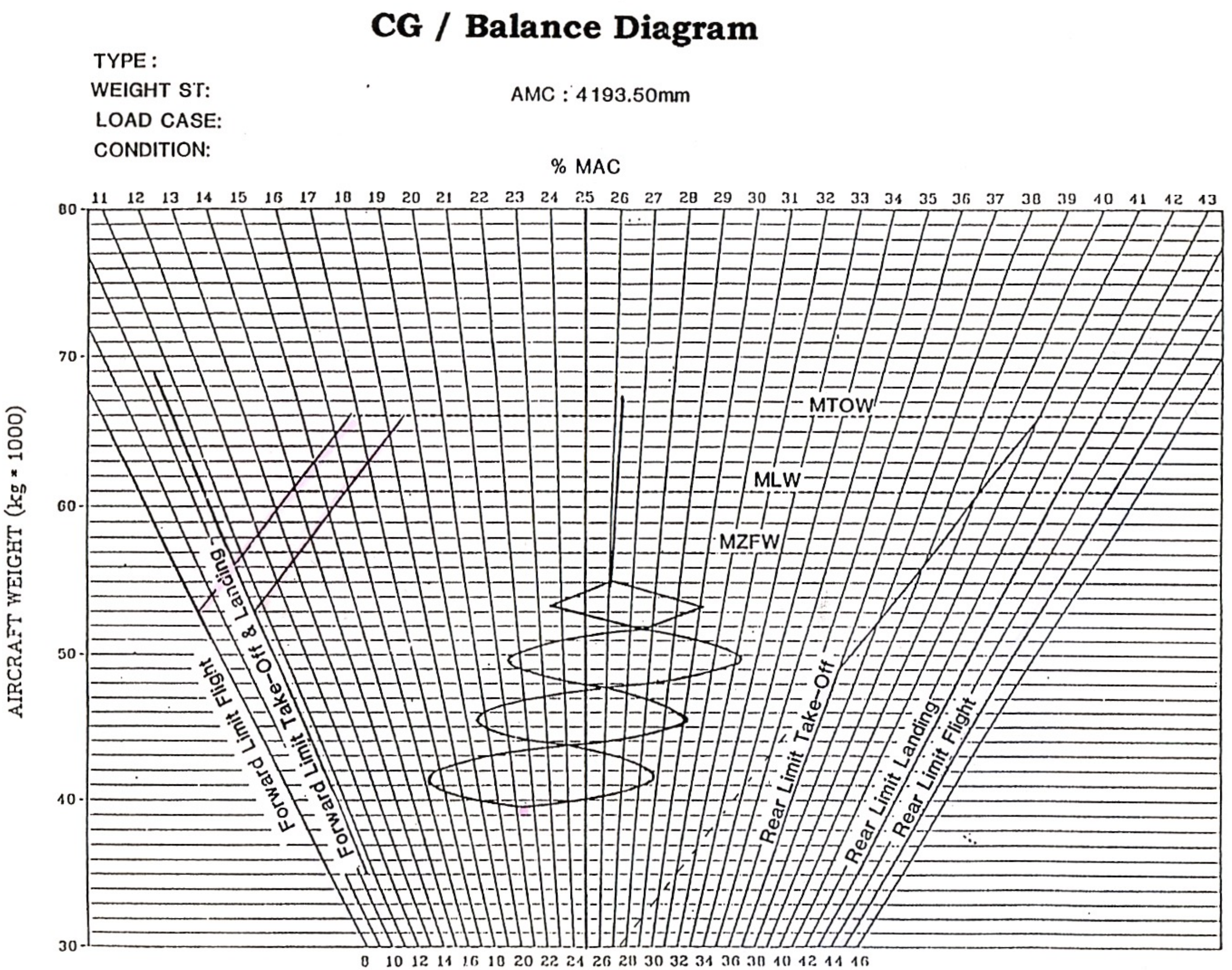
MTOW: max take-off weight

MLW: max landing weight

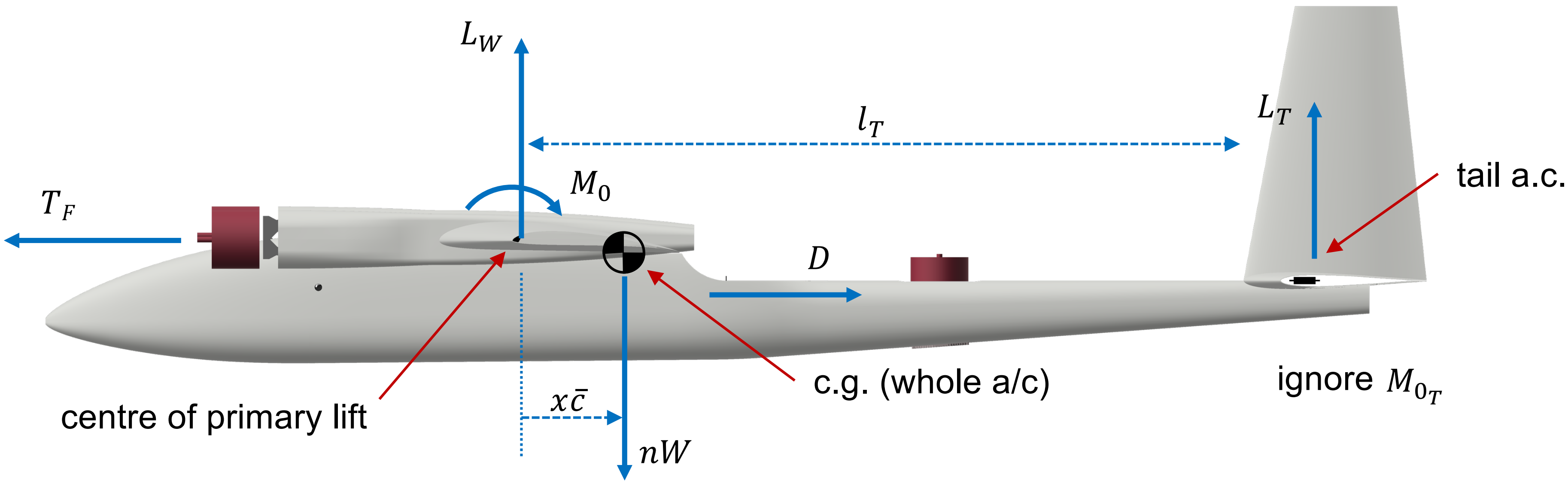
MZFW: max zero-fuel weight

Note: forward and rear limits in different flight phases.

Limits can be driven by a range of criteria including nose-wheel raise, stability and steering requirements.

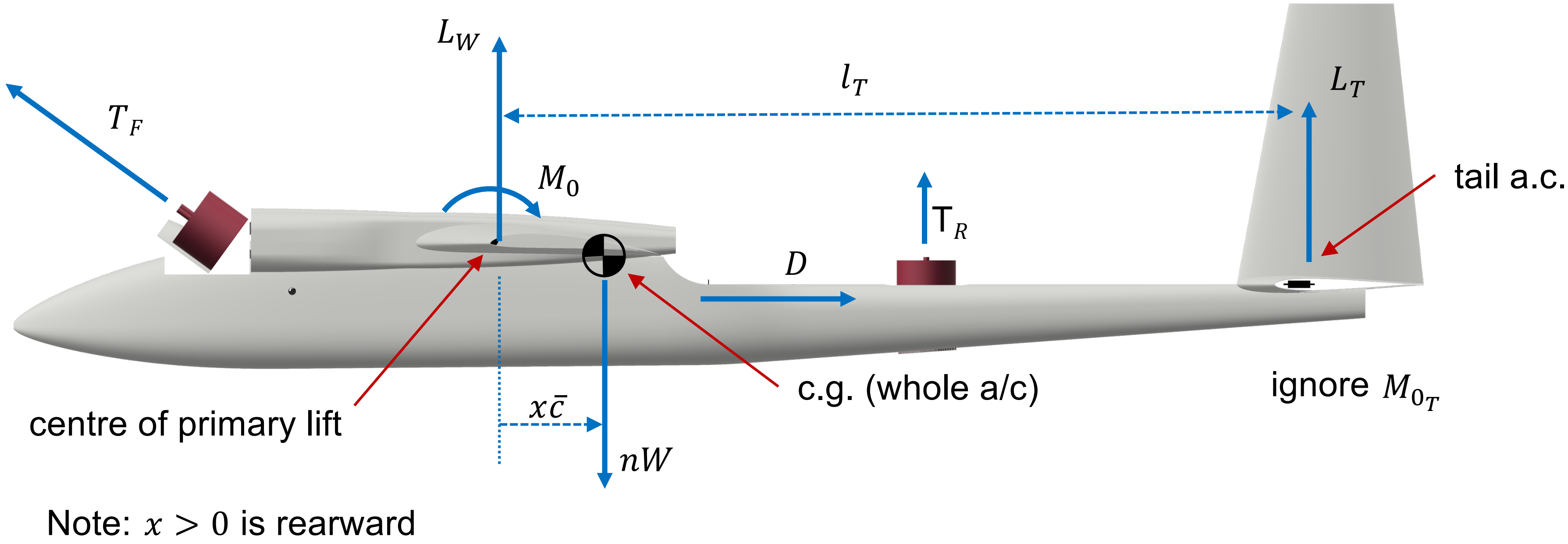


‘BUDDI’ in conventional fixed wing flight



Note: $x > 0$ is rearward

‘BUDDI’ in low speed flight?



Contribution of the motor thrust

to force and moment equations?

Next Session
Elevator Angle to Trim

The Starship SN10 prototype after a ‘safe’ landing. SPACEX

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