Appendix 2: Definitions of Aerodynamic Stability and Control Derivatives

A2.1 Notes

The derivatives given in Tables A2.5 through A2.8 are referred to **generalised body axes**; $U_e = V_0 \cos\theta_e$ and $W_e = V_0 \sin\theta_e$. In the particular case when the derivatives are referred to wind axes, $\theta_e = 0$ and the following simplifications can be made: $U_e = V_0$, $W_e = 0$, $\sin\theta_e = 0$ and $\cos\theta_e = 1$. The equivalent algebraic expressions in Tables A2.5 to A2.8 were derived with the aid of the computer program Mathcad, which includes a facility for symbolic calculation. In Tables A2.5, A2.6, A2.7, and A2.8 normalised mass and inertias are used. They are defined as follows:

$$m' = \frac{m}{\frac{1}{2}\rho V_0 S}$$

$$I'_x = \frac{I_x}{\frac{1}{2}\rho V_0 S b}$$

$$I'_y = \frac{I_y}{\frac{1}{2}\rho V_0 S \bar{\bar{c}}}$$

$$I'_z = \frac{I_z}{\frac{1}{2}\rho V_0 S b}$$

$$I'_{xz} = \frac{I_{xz}}{\frac{1}{2}\rho V_0 S b}$$

Table A2.1 Longitudinal Aerodynamic Stability Derivatives

Dimensionless	Multiplier	Dimensional
X_u	$\frac{1}{2}\rho V_0 S$	$\overset{\circ}{X}_u$
X_w	$\frac{1}{2}\rho V_0 S$	$\overset{\circ}{X}_w$
$X_{\dot{w}}$	$\frac{1}{2}\rho S \overline{\overline{c}}$	$\overset{\circ}{X}_{\dot{w}}$
X_q	$\frac{1}{2}\rho V_0 S \overline{\overline{c}}$	$\overset{\circ}{X}_q$
Z_u	$\frac{1}{2}\rho V_0 S$	$\overset{\circ}{Z}_u$
Z_w	$\frac{1}{2}\rho V_0 S$	$\overset{\circ}{Z}_w$
$Z_{\dot{w}}$	$\frac{1}{2}\rho S\overline{\overline{c}}$	$\overset{\circ}{Z}_{\dot{w}}$
Z_q	$\frac{1}{2}\rho V_0 S\overline{\overline{c}}$	$\overset{\circ}{Z}_q$
M_u	$\frac{1}{2}\rho V_0 S\overline{\overline{c}}$	$\overset{\circ}{M}_u$
M_w	$\frac{1}{2}\rho V_0 S\overline{\overline{c}}$	$\overset{\circ}{M}_w$
$M_{\dot{w}}$	$\frac{1}{2}\rho S\overline{\overline{c}}^2$	$\overset{\circ}{M}_{\dot{w}}$
M_q	$\frac{1}{2}\rho V_0 S\overline{\overline{c}}^2$	$\overset{\circ}{M}_q$

Table A2.2 Longitudinal Control Derivatives

Dimensionless	Multiplier	Dimensional
X_η	$\frac{1}{2}\rho V_0^2 S$	$\overset{\circ}{X}_\eta$
Z_η	$\frac{1}{2}\rho V_0^2 S$	$\overset{\circ}{Z}_\eta$
M_η	$\frac{1}{2}\rho V_0^2 S\overline{\overline{c}}$	$\overset{\circ}{M}_\eta$
X_τ	1	$\overset{\circ}{X}_\tau$
Z_τ	1	$\overset{\circ}{Z}_\tau$
M_τ	$\overline{\overline{c}}$	$\overset{\circ}{M}_\tau$

Table A2.3 Lateral-Directional Aerodynamic Stability Derivatives

Dimensionless	Multiplier	Dimensional
Y_v	$\frac{1}{2}\rho V_0 S$	$\overset{\circ}{Y}_v$
Y_p	$\frac{1}{2}\rho V_0 Sb$	$\overset{\circ}{Y}_p$
Y_r	$\frac{1}{2}\rho V_0 Sb$	$\overset{\circ}{Y}_r$
L_v	$\frac{1}{2}\rho V_0 Sb$	$\overset{\circ}{L}_v$
L_p	$\frac{1}{2}\rho V_0 Sb^2$	$\overset{\circ}{L}_p$
L_r	$\frac{1}{2}\rho V_0 Sb^2$	$\overset{\circ}{L}_r$
N_v	$\frac{1}{2}\rho V_0 Sb$	$\overset{\circ}{N}_v$
N_p	$\frac{1}{2}\rho V_0 Sb^2$	$\overset{\circ}{N}_p$
N_r	$\frac{1}{2}\rho V_0 Sb^2$	$\overset{\circ}{N}_r$

Table A2.4 Lateral-Directional Aerodynamic Control Derivatives

Dimensionless	Multiplier	Dimensional
Y_ξ	$\frac{1}{2}\rho V_0^2 S$	$\overset{\circ}{Y}_\xi$
L_ξ	$\frac{1}{2}\rho V_0^2 Sb$	$\overset{\circ}{L}_\xi$
N_ξ	$\frac{1}{2}\rho V_0^2 Sb$	$\overset{\circ}{N}_\xi$
Y_ζ	$\frac{1}{2}\rho V_0^2 S$	$\overset{\circ}{Y}_\zeta$
L_ζ	$\frac{1}{2}\rho V_0^2 Sb$	$\overset{\circ}{L}_\zeta$
N_ζ	$\frac{1}{2}\rho V_0^2 Sb$	$\overset{\circ}{N}_\zeta$

Table A2.5 Concise Longitudinal Aerodynamic Stability Derivatives

	Equivalent Expressions	
Concise Derivative	**In Terms of Dimensional Derivatives**	**In Terms of Dimensionless Derivatives**
x_u	$\frac{\mathring{X}_u}{m} + \frac{\mathring{X}_{\dot{w}}\mathring{Z}_u}{m\left(m - \mathring{Z}_{\dot{w}}\right)}$	$\frac{X_u}{m'} + \frac{\frac{\bar{\bar{c}}}{V_0}X_{\dot{w}}Z_u}{m'\left(m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}}\right)}$
z_u	$\frac{\mathring{Z}_u}{m - \mathring{Z}_{\dot{w}}}$	$\frac{Z_u}{m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}}}$
m_u	$\frac{\mathring{M}_u}{I_y} + \frac{\mathring{Z}_u\mathring{M}_{\dot{w}}}{I_y\left(m - \mathring{Z}_{\dot{w}}\right)}$	$\frac{M_u}{I'_y} + \frac{\frac{\bar{\bar{c}}}{V_0}M_{\dot{w}}Z_u}{I'_y\left(m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}}\right)}$
x_w	$\frac{\mathring{X}_w}{m} + \frac{\mathring{X}_{\dot{w}}\mathring{Z}_w}{m\left(m - \mathring{Z}_{\dot{w}}\right)}$	$\frac{X_w}{m'} + \frac{\frac{\bar{\bar{c}}}{V_0}X_{\dot{w}}Z_w}{m'\left(m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}}\right)}$
z_w	$\frac{\mathring{Z}_w}{m - \mathring{Z}_{\dot{w}}}$	$\frac{Z_w}{m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}}}$
m_w	$\frac{\mathring{M}_w}{I_y} + \frac{\mathring{Z}_w\mathring{M}_{\dot{w}}}{I_y\left(m - \mathring{Z}_{\dot{w}}\right)}$	$\frac{M_w}{I'_y} + \frac{\frac{\bar{\bar{c}}}{V_0}M_{\dot{w}}Z_w}{I'_y\left(m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}}\right)}$
x_q	$\frac{\mathring{X}_q - mW_e}{m} + \frac{\left(\mathring{Z}_q + mU_e\right)\mathring{X}_{\dot{w}}}{m\left(m - \mathring{Z}_{\dot{w}}\right)}$	$\frac{\bar{\bar{c}}X_q - m'W_e}{m'} + \frac{(\bar{\bar{c}}Z_q + m'U_e)\frac{\bar{\bar{c}}}{V_0}X_{\dot{w}}}{m'\left(m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}}\right)}$
z_q	$\frac{\mathring{Z}_q + mU_e}{m - \mathring{Z}_{\dot{w}}}$	$\frac{\bar{\bar{c}}Z_q + m'U_e}{m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}}}$
m_q	$\frac{\mathring{M}_q}{I_y} + \frac{\left(\mathring{Z}_q + mU_e\right)\mathring{M}_{\dot{w}}}{I_y\left(m - \mathring{Z}_{\dot{w}}\right)}$	$\frac{\bar{\bar{c}}M_q}{I'_y} + \frac{(\bar{\bar{c}}Z_q + m'U_e)\frac{\bar{\bar{c}}}{V_0}M_{\dot{w}}}{I'_y\left(m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}}\right)}$
x_θ	$-g\cos\theta_e - \frac{\mathring{X}_{\dot{w}}\, g\sin\theta_e}{m - \mathring{Z}_{\dot{w}}}$	$-g\cos\theta_e - \frac{\frac{\bar{\bar{c}}}{V_0}X_{\dot{w}}g\sin\theta_e}{m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}}}$
z_θ	$-\frac{mg\sin\theta_e}{m - \mathring{Z}_{\dot{w}}}$	$-\frac{m'g\sin\theta_e}{m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}}}$
m_θ	$-\frac{\mathring{M}_{\dot{w}}\, mg\sin\theta_e}{I_y\left(m - \mathring{Z}_{\dot{w}}\right)}$	$-\frac{\frac{\bar{\bar{c}}}{V_0}M_{\dot{w}}m'g\sin\theta_e}{I'_y(m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}})}$

Table A2.6 Concise Longitudinal Control Derivatives

	Equivalent Expressions	
Concise Derivative	**In Terms of Dimensional Derivatives**	**In Terms of Dimensionless Derivatives**
x_η	$\frac{\overset{\circ}{X}_\eta}{m} + \frac{\overset{\circ}{X}_{\dot{w}}\overset{\circ}{Z}_\eta}{m\left(m - \overset{\circ}{Z}_{\dot{w}}\right)}$	$\frac{V_0 X_\eta}{m'} + \frac{\frac{\bar{\bar{c}}}{V_0} X_{\dot{w}} Z_\eta}{m'(m' - \frac{\bar{\bar{c}}}{V_0} Z_{\dot{w}})}$
z_η	$\frac{\overset{\circ}{Z}_\eta}{m - \overset{\circ}{Z}_{\dot{w}}}$	$\frac{V_0 Z_\eta}{m' - \frac{\bar{\bar{c}}}{V_0} Z_{\dot{w}}}$
m_η	$\frac{\overset{\circ}{M}_\eta}{I_y} + \frac{\overset{\circ}{M}_{\dot{w}}\overset{\circ}{Z}_\eta}{I_y\left(m - \overset{\circ}{Z}_{\dot{w}}\right)}$	$\frac{V_0 M_\eta}{I'_y} + \frac{\bar{\bar{c}} M_{\dot{w}} Z_\eta}{I'_y(m' - \frac{\bar{\bar{c}}}{V_0} Z_{\dot{w}})}$
x_τ	$\frac{\overset{\circ}{X}_\tau}{m} + \frac{\overset{\circ}{X}_{\dot{w}}\overset{\circ}{Z}_\tau}{m\left(m - \overset{\circ}{Z}_{\dot{w}}\right)}$	$\frac{V_0 X_\tau}{m'} + \frac{\frac{\bar{\bar{c}}}{V_0} X_{\dot{w}} Z_\tau}{m'(m' - \frac{\bar{\bar{c}}}{V_0} Z_{\dot{w}})}$
z_τ	$\frac{\overset{\circ}{Z}_\tau}{m - \overset{\circ}{Z}_{\dot{w}}}$	$\frac{V_0 Z_\tau}{m' - \frac{\bar{\bar{c}}}{V_0} Z_{\dot{w}}}$
m_τ	$\frac{\overset{\circ}{M}_\tau}{I_y} + \frac{\overset{\circ}{M}_{\dot{w}}\overset{\circ}{Z}_\tau}{I_y\left(m - \overset{\circ}{Z}_{\dot{w}}\right)}$	$\frac{V_0 M_\tau}{I'_y} + \frac{\bar{\bar{c}} M_{\dot{w}} Z_\tau}{I'_y(m' - \frac{\bar{\bar{c}}}{V_0} Z_{\dot{w}})}$

Table A2.7 Concise Lateral-Directional Aerodynamic Stability Derivatives

	Equivalent Expressions	
Concise Derivative	**In Terms of Dimensional Derivatives**	**In Terms of Dimensionless Derivatives**
y_v	$\frac{\mathring{Y}_v}{m}$	$\frac{Y_v}{m'}$
y_p	$\frac{\left(\mathring{Y}_p + mW_e\right)}{m}$	$\frac{(bY_p + m'W_e)}{m'}$
y_r	$\frac{\left(\mathring{Y}_r - mU_e\right)}{m}$	$\frac{(bY_r - m'U_e)}{m'}$
y_ϕ	$g\cos\theta_e$	$g\cos\theta_e$
y_ψ	$g\sin\theta_e$	$g\sin\theta_e$
l_v	$\frac{\left(I_z\mathring{L}_v + I_{xz}\mathring{N}_v\right)}{(I_xI_z - I_{xz}^2)}$	$\frac{(I'_zL_v + I'_{xz}N_v)}{(I'_xI'_z - I'^2_{xz})}$
l_p	$\frac{\left(I_z\mathring{L}_p + I_{xz}\mathring{N}_p\right)}{(I_xI_z - I_{xz}^2)}$	$\frac{(I'_zL_p + I'_{xz}N_p)b}{(I'_xI'_z - I'^2_{xz})}$
l_r	$\frac{\left(I_z\mathring{L}_r + I_{xz}\mathring{N}_r\right)}{(I_xI_z - I_{xz}^2)}$	$\frac{(I'_zL_r + I'_{xz}N_r)b}{(I'_xI'_z - I'^2_{xz})}$
l_ϕ	0	0
l_ψ	0	0
n_v	$\frac{\left(I_x\mathring{N}_v + I_{xz}\mathring{L}_v\right)}{(I_xI_z - I_{xz}^2)}$	$\frac{(I'_xN_v + I'_{xz}L_v)}{(I'_xI'_z - I'^2_{xz})}$
n_p	$\frac{\left(I_x\mathring{N}_p + I_{xz}\mathring{L}_p\right)}{(I_xI_z - I_{xz}^2)}$	$\frac{(I'_xN_p + I'_{xz}L_p)b}{(I'_xI'_z - I'^2_{xz})}$
n_r	$\frac{\left(I_x\mathring{N}_r + I_{xz}\mathring{L}_r\right)}{(I_xI_z - I_{xz}^2)}$	$\frac{(I'_xN_r + I'_{xz}L_r)b}{(I'_xI'_z - I'^2_{xz})}$
n_ϕ	0	0
n_ψ	0	0

Table A2.8 Concise Lateral-Directional Control Derivatives

	Equivalent Expressions	
Concise Derivative	**In Terms of Dimensional Derivatives**	**In Terms of Dimensionless Derivatives**
y_ξ	$\frac{\overset{\circ}{Y}_\xi}{m}$	$\frac{V_0 Y_\xi}{m'}$
l_ξ	$\frac{\left(I_z \overset{\circ}{L}_\xi + I_{xz} \overset{\circ}{N}_\xi\right)}{(I_x I_z - I_{xz}^2)}$	$\frac{V_0(I'_z L_\xi + I'_{xz} N_\xi)}{(I'_x I'_z - I'^2_{xz})}$
n_ξ	$\frac{\left(I_x \overset{\circ}{N}_\xi + I_{xz} \overset{\circ}{L}_\xi\right)}{(I_x I_z - I_{xz}^2)}$	$\frac{V_0(I'_x N_\xi + I'_{xz} L_\xi)}{(I'_x I'_z - I'^2_{xz})}$
y_ζ	$\frac{\overset{\circ}{Y}_\zeta}{m}$	$\frac{V_0 Y_\zeta}{m'}$
l_ζ	$\frac{\left(I_z \overset{\circ}{L}_\zeta + I_{xz} \overset{\circ}{N}_\zeta\right)}{(I_x I_z - I_{xz}^2)}$	$\frac{V_0(I'_z L_\zeta + I'_{xz} N_\zeta)}{(I'_x I'_z - I'^2_{xz})}$
n_ζ	$\frac{\left(I_x \overset{\circ}{N}_\zeta + I_{xz} \overset{\circ}{L}_\zeta\right)}{(I_x I_z - I_{xz}^2)}$	$\frac{V_0(I'_x N_\zeta + I'_{xz} L_\zeta)}{(I'_x I'_z - I'^2_{xz})}$