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Aerodynamics and Numerical Simulation Methods

Introduction to Turbulent Boundary Layers

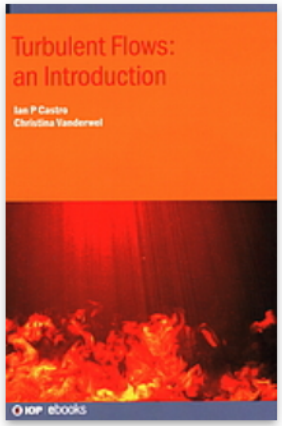
University of BRISTOL

Topics for today

- What's turbulence?
- Reynolds time-averaging concept
- Reynolds Averaged Navier-Stokes equations for turbulent boundary layers
- Turbulence modelling

Course Materials

- Access from: https://www.bristol.ac.uk/library/
- Search for "**Turbulent flows : an introduction**" in the search tool (eBook available)
- Live chat option available if you are stuck



Turbulent flows : an introduction

Authors: Castro P Ian (Author), Vanderwel Christina (Author)

eBook 2021
Bristol : IOP Publishing, 2021.

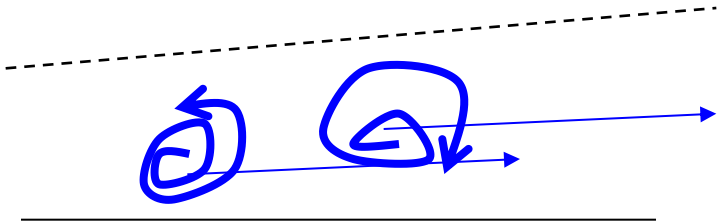
Abstract: This book presents an introduction to the fundamentals of **turbulent flow**. Its focus is on understanding and simplifying the equations of motion for various classes of **flow**, so as to elucidate the most crucial and practically important aspects of the physics.

Turbulence

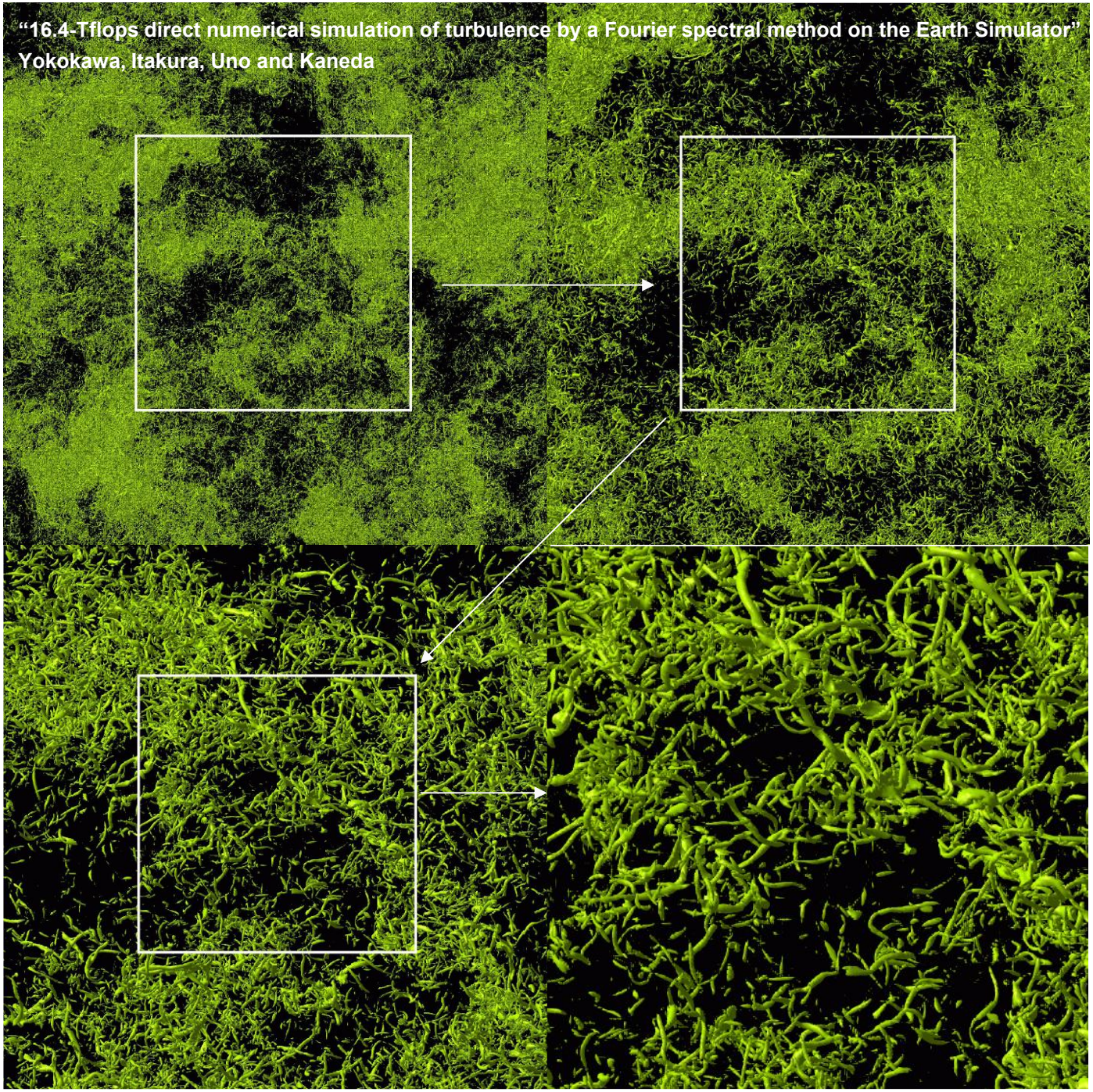
- Turbulence is very complex
  - rapid fluctuations
  - ‘random’ motion
  - **many scales of interactions**
- In turbulent flows there are rapid fluctuations locally even in steady flows
- Last great scientific challenge in aerodynamics?
- Fully resolving turbulence with CFD at realistic Re unlikely – so the challenge may be more to do with dealing with the uncertainty rather than removing it

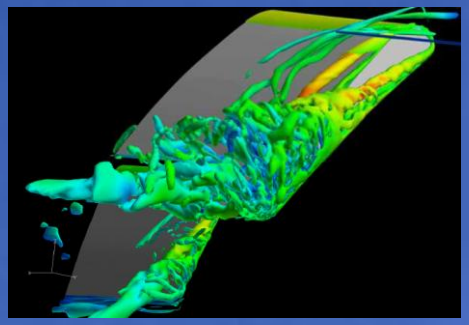
What causes turbulence?

- Basically, turbulence is caused by the presence of *turbulent eddies* in the flow



- Fortunately these are cohesive
- Occur when inertial forces dominate viscous dissipation, i.e. high Reynolds numbers.
- Will return to the process of change between the two (*transition*) later in the course.


"16.4-Tflops direct numerical simulation of turbulence by a Fourier spectral method on the Earth Simulator"
Yokokawa, Itakura, Uno and Kaneda



‘Turbulence’

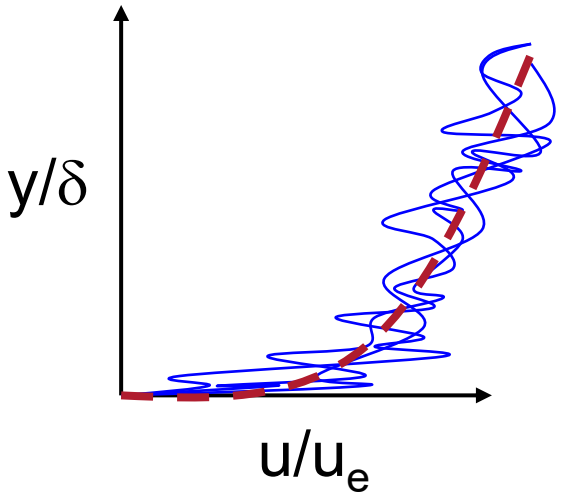
- Attached flows may have laminar or turbulent boundary layers. Usually, aircraft operate with attached flows and turbulent boundary layers
- Flow separation (such as when stalling) is separate, but it does also result in large scale turbulence. However, **turbulence does not imply separation!**

- Turbulence has as a very wide range of time- and length-scales which makes it particularly challenging to resolve
- Atmospheric ‘turbulence’ may consist of large roughly uniform air currents while ‘wake turbulence’ consists of clearly identifiable vortices.

Time Scales:

- The turbulent eddies introduce very small variations (i.e. fluctuations) in each velocity component (i.e. *u,v*)
- Fluctuations change with time, but do so much faster than e.g. the time scale of a smooth manoeuvre
- Hence, do not need to know exactly what's going on, only what effect the fluctuations have over a specific time scale

$$u = \hat{u} + u'$$



The Turbulent Boundary Layer Equations

- As with laminar boundary layers, we have

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

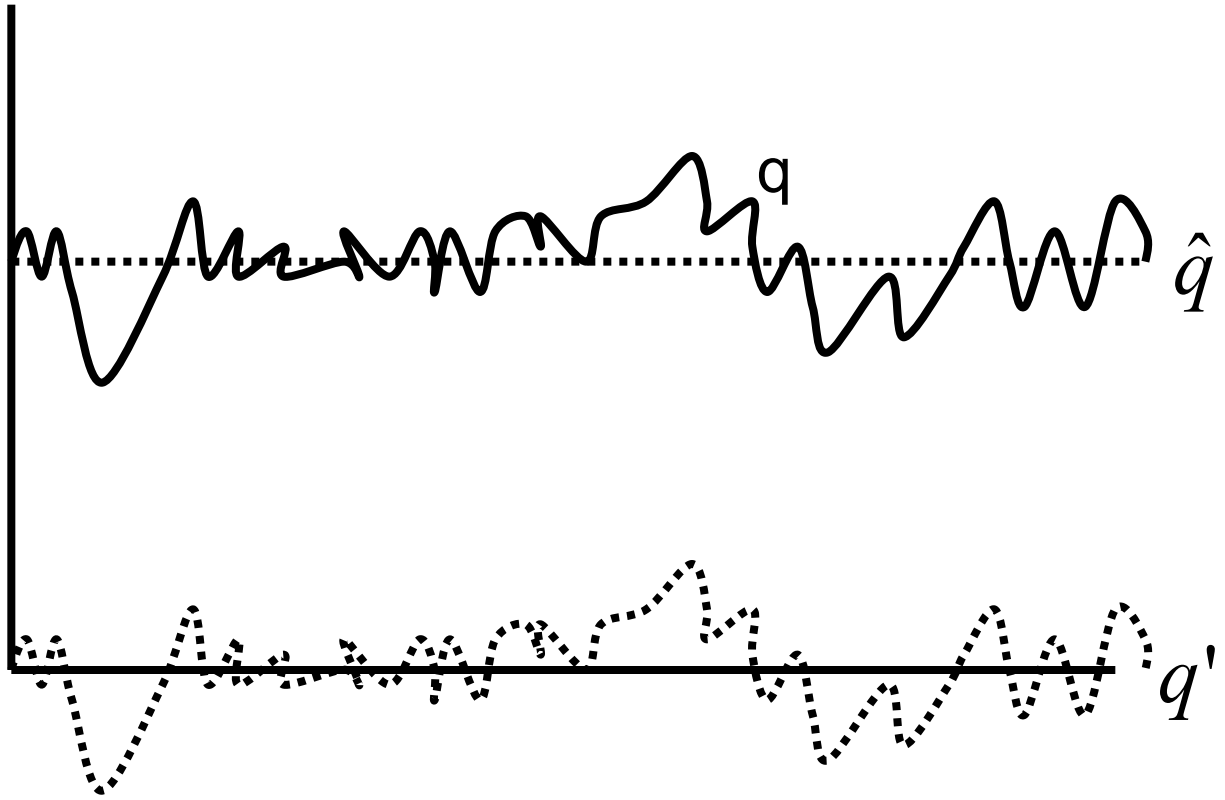
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp}{dx}+\nu\frac{\partial^2 u}{\partial y^2}$$

- But now we represent the flow properties *u,v* etc. as a sum of mean plus turbulent fluctuation, i.e.

$$u=\hat{u}+u'$$

Reference: Frank White Page 360 (Reynolds time-averaging concept)

Mean and Fluctuating Components



integral of q’ over time is 0

Continuity Equation

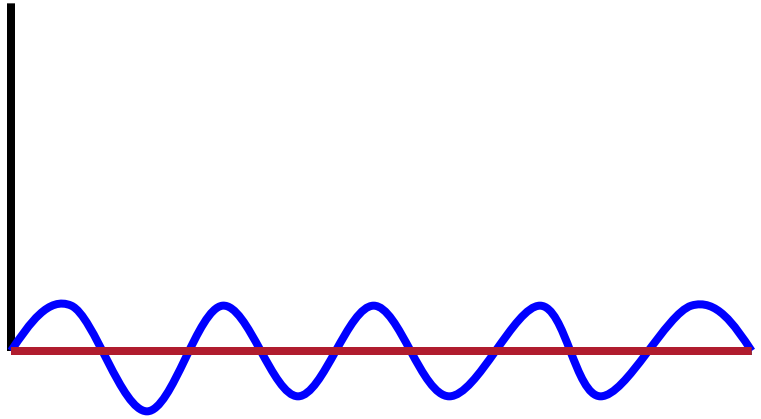
- Substituting in these Reynolds decomposed variables into the continuity equation gives:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=\frac{\partial(\hat{u}+u')}{\partial x}+\frac{\partial(\hat{v}+v')}{\partial y}=0$$

- Next, we would like to perform Reynolds averaging
- If we average over some time much longer than the timescale of the fluctuation, but less than that of anything we are interested in (denote this by an overline), then…

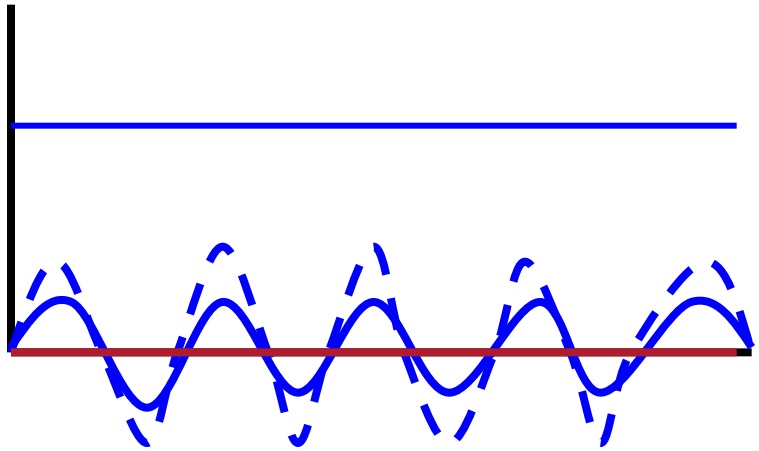
Properties of time averaged variables:

$$\overline{\hat{q}} = \hat{q}$$

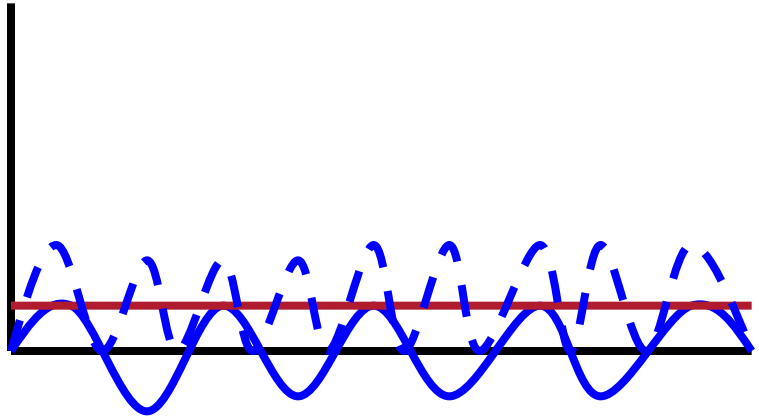


$$\overline{q'} = 0$$

Properties of time averaged variables:



$$\overline{\hat{q}s'} = 0$$



$$\overline{q's'} \neq 0$$

Finally

$$\overline{\frac{\partial \hat{q}}{\partial x}} = \frac{\partial \overline{\hat{q}}}{\partial x} = \frac{\partial \hat{q}}{\partial x}$$

Mean of a gradient = gradient of a mean

and

$$\overline{\frac{\partial q'}{\partial x}} = \frac{\partial \overline{q'}}{\partial x} = 0$$

Time averaging and differentiation are commutative

So, returning to the continuity equation, and time averaging:

$$\overline{\frac{\partial(\hat{u}+u')}{\partial x}+\frac{\partial(\hat{v}+v')}{\partial y}}=0$$

$$\Rightarrow \quad \frac{\overline{\partial\hat{u}}}{\partial x}+\frac{\overline{\partial u'}}{\partial x}+\frac{\overline{\partial\hat{v}}}{\partial y}+\frac{\overline{\partial v'}}{\partial y}=0$$

but, using our rules from earlier, the time average of the fluctuating differentials are zero, hence

$$\Rightarrow \quad \frac{\partial\hat{u}}{\partial x}+\frac{\partial\hat{v}}{\partial y}=0$$

Similarly, we can show that the x-momentum equation becomes

$$\hat{u}\frac{\partial \hat{u}}{\partial x}+\hat{v}\frac{\partial \hat{u}}{\partial y}=-\frac{1}{\rho}\frac{d\hat{p}}{dx}+\frac{\partial}{\partial y}\left(\nu\frac{\partial \hat{u}}{\partial y}-\overline{u'v'}\right)-\frac{\partial \overline{u'^2}}{\partial x}$$

where the last two terms are called ***Reynolds Stresses***. We sometimes ignore the last one due to the thin layer assumption (=gradients in normal direction dominant).

Overall, the continuity and momentum equations are the same in turbulent flow as laminar, as long as we use time averaged values, and include this extra term involving *u'v'.*

We now have the RANS equations (Reynolds Averaged NS).

The Boussinesq Hypothesis:

- The extra stress acts just like a viscous stress, but is a function of local flow variables. This is in analogy with how the momentum transfer caused by the molecular motion in a gas can be described by a molecular viscosity i.e.

$$\tau_t = -\rho \overline{u'v'} = \mu_t \frac{\partial \hat{u}}{\partial y}, \qquad \mu_t = F(u, v, etc.) \rightarrow \text{eddy viscosity}$$

- In turn this means that the boundary layer equations become (dropping the hat symbol, it is implicit that time averaged values are meant)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + (\nu + \nu_t)\frac{\partial^2 u}{\partial y^2}$$

This is almost always how turbulence modelling is done in CFD!

Finally…

- The problem now is that we have more unknowns than equations. Finding additional closing eqns (e.g. $\mu_t = F(u, v, etc.)$) that give 'good' results is the field of turbulence modelling.
- The additional equations often (but not always) mimic physical ones and are largely empirical. Sometimes further unknowns are introduced to model more complicated effects.
- Remember they are semi-empirical=semi-fictional! They do **not** have the solid Newtonian foundations of the Navier-Stokes equations
- Nevertheless, often they look similar, and their construction is based on sound experimental observation and experience

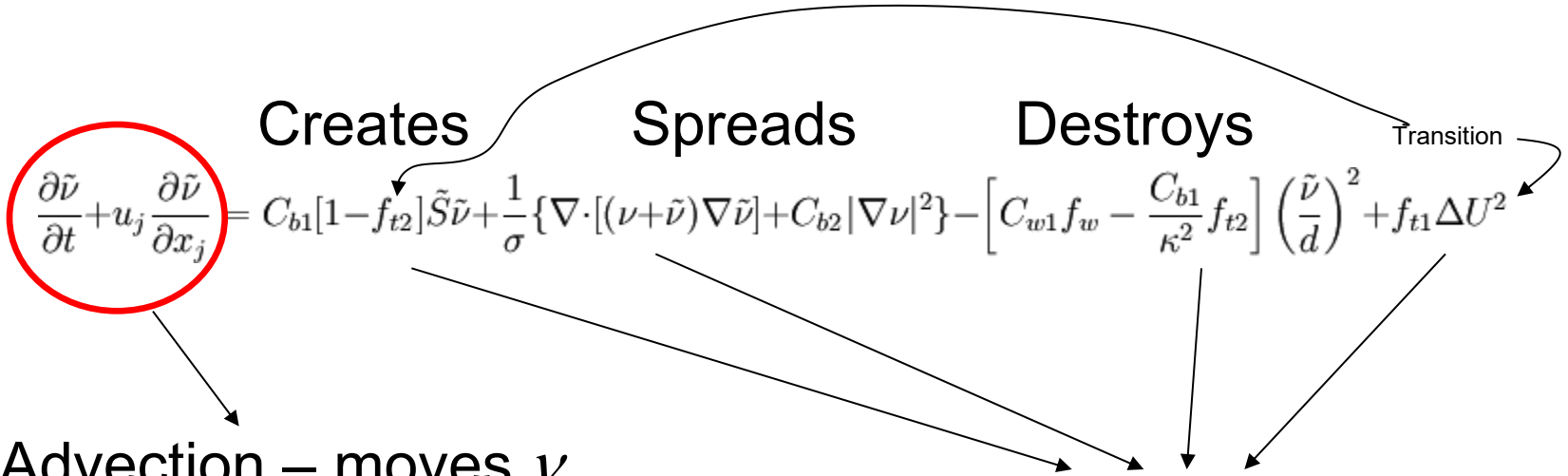
Common turbulence models

- Spalart–Allmaras (S–A)
  - One equation transport model that solves a transport equation for the Spalart-Allmaras variable
  - Low Re model, requires y+ calculation near the wall
  - Performs well for wall-bounded aerospace applications
- k–ε (k–epsilon)
  - Two equation transport model that describes the TKE and the dissipation rate
  - Generally performs well for fully turbulent flow without separation, e.g. shear layers, high Re or freestream flows
  - Poor for complex flows with separation, adverse pressure gradients, strong streamline curvatures, stability issues for problems with thick viscous sublayer

Common turbulence models

- k–ω (k–omega)
  - Two equation transport model that describes the TKE and the specific dissipation rate
  - Requires y+ calculation near the wall
  - Performs well for wall-bounded flow or low Re-flow where near-wall treatment needs to be more accurate, free shear flow, turbomachinery
- SST (Menter's Shear Stress Transport)
  - Combines k–ω for inner region of boundary layer and the k–ε for away from the wall using a blending function
  - Performs well for high accuracy boundary layer simulations

Spalart-Allmaras (1 equation model)



Advection – moves ν around and analogous to $\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}$ in Navier-Stokes.

Moderately ‘physical’

These ‘source’ terms create/destroy/spread depending the time averaged values.

Very empirical

Just an example –

don’t memorise!

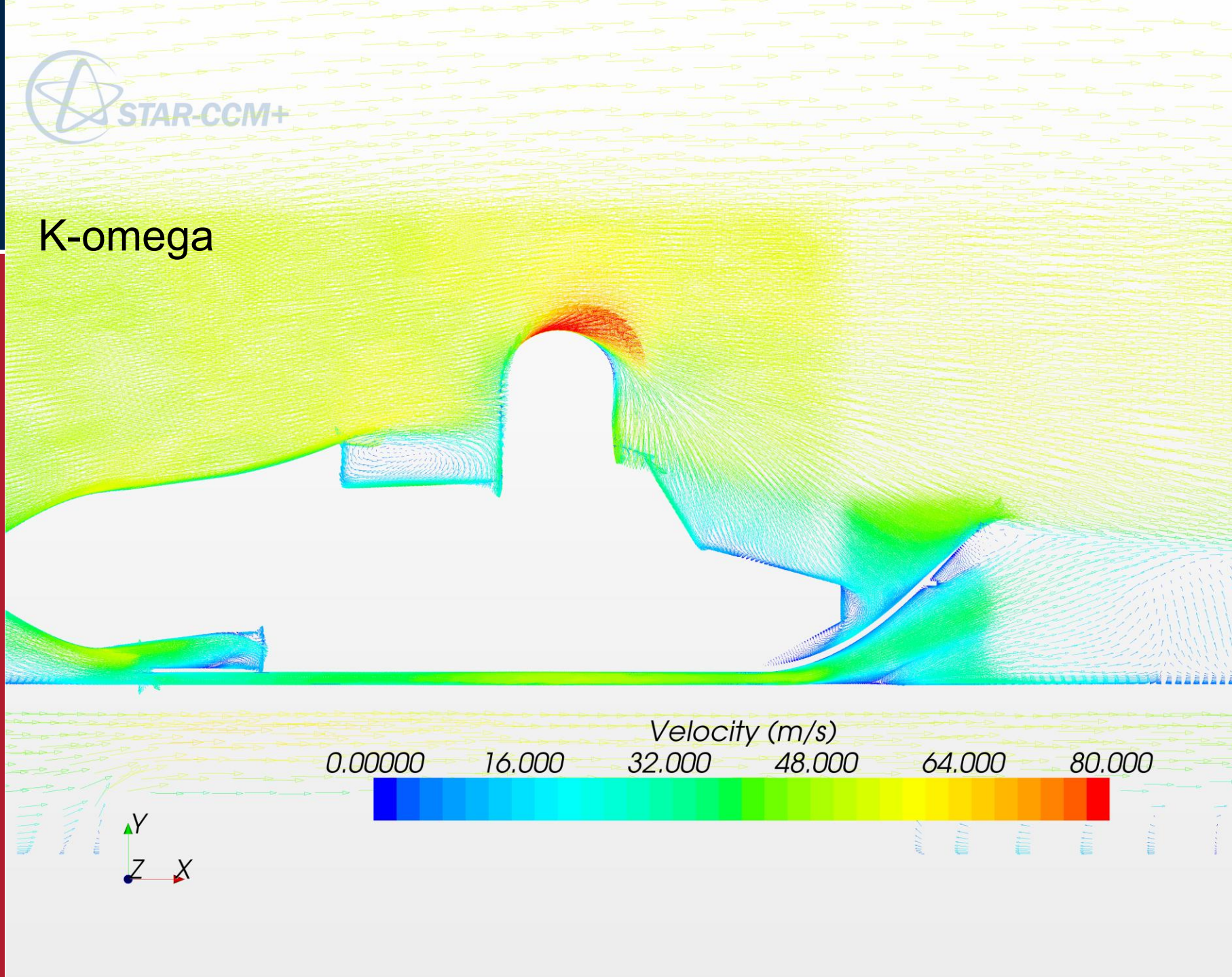
Example 1 – Small Race car

- Full NS model + turbulence model
- Simulated at wind tunnel Re
- Variations in the following flows are only a result of **differing turbulence models**
- Similar effects carry through to the forces (eg drag, downforce)

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp}{dx}+\left(\nu+\nu_t\right)\frac{\partial^2 u}{\partial y^2}$$

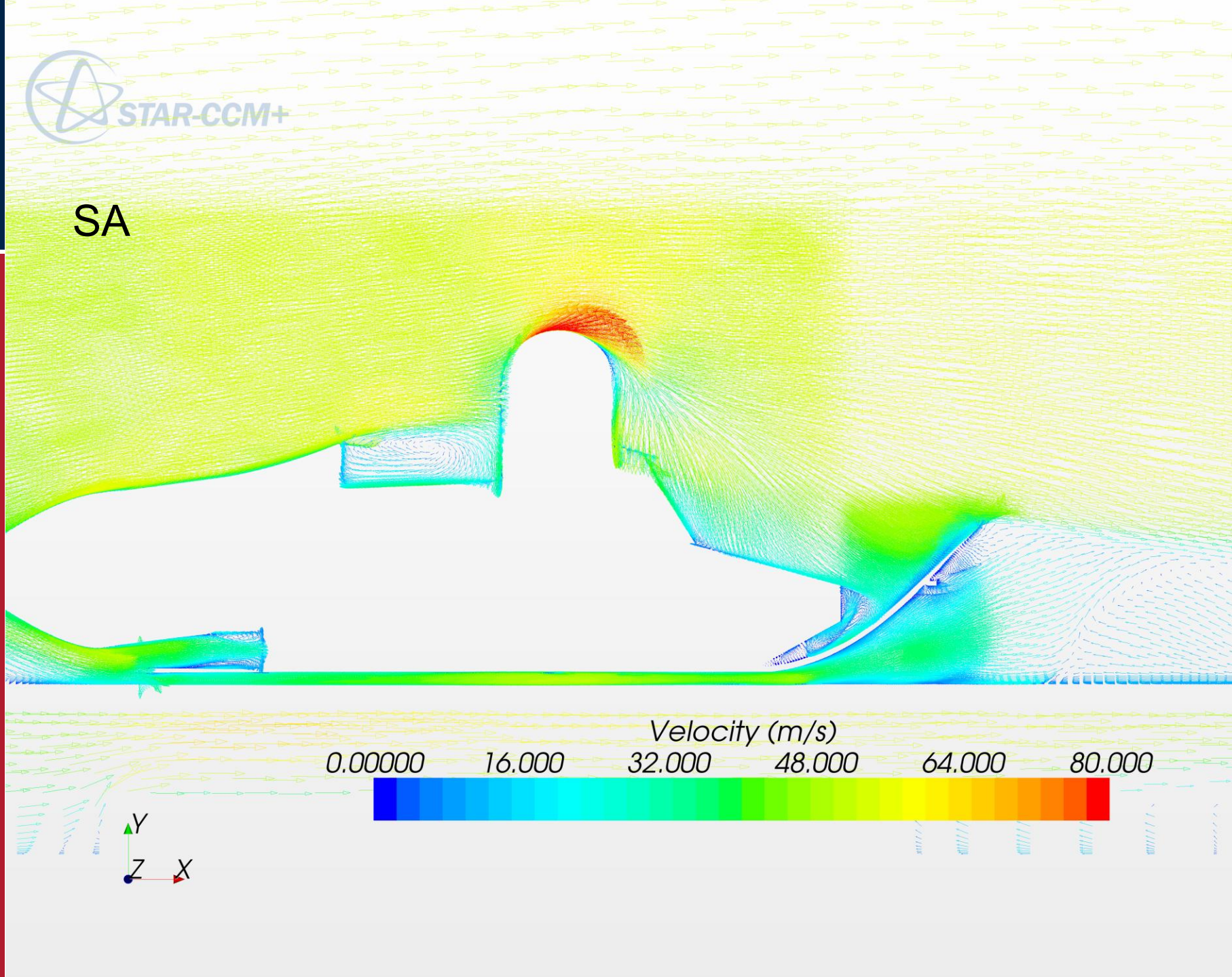
STAR-CCM+

K-omega



STAR-CCM+

SA



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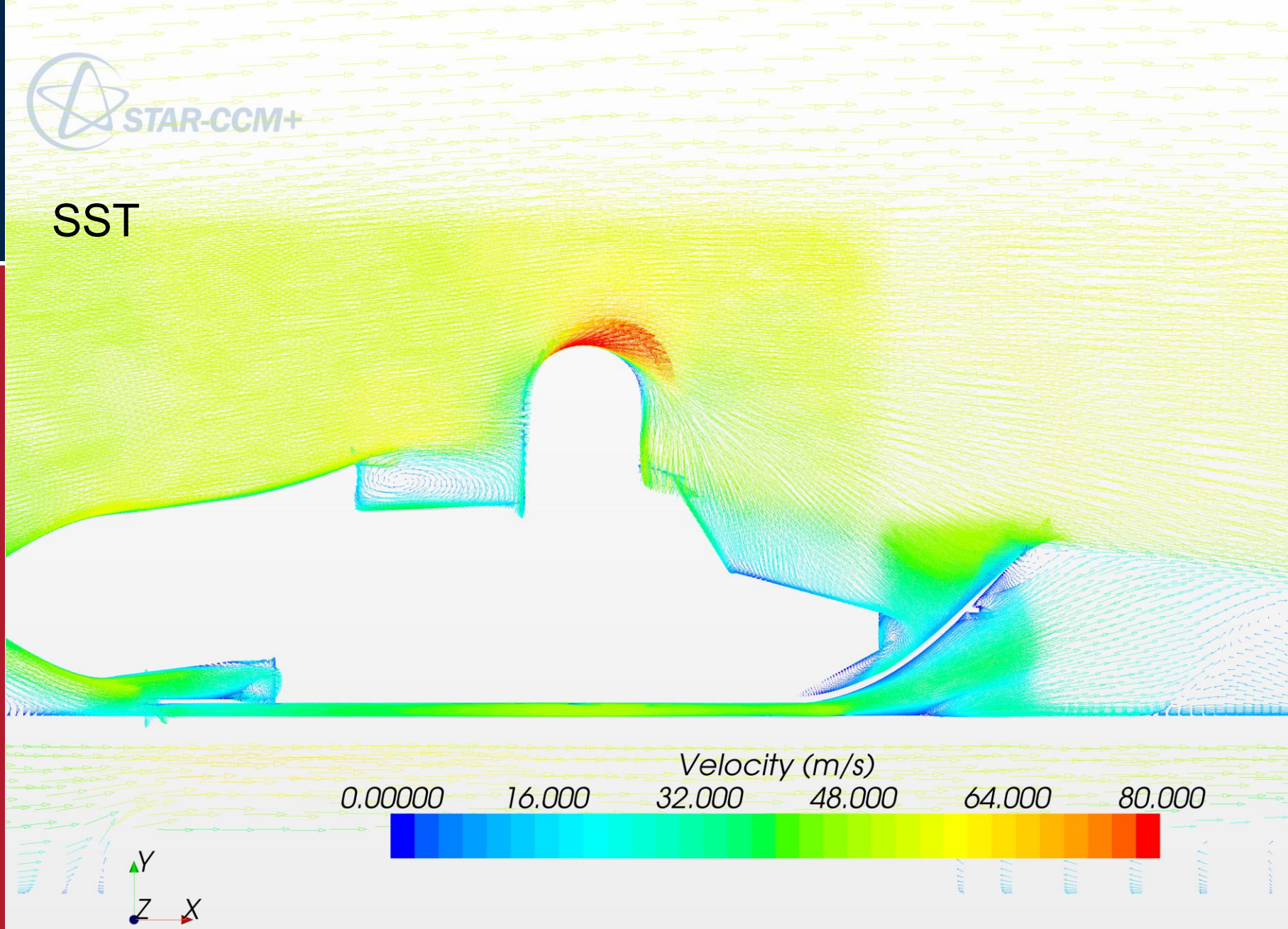
SST