AIRCRAFT DYNAMIC MODES

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Bristol and Gloucestershire Gliding Club

Tug Aircraft: 141hp EuroFox 2K

Aircraft Dynamic Modes

Airbus A350-1000

https://www.britishairways.com

Flight Dynamics Principles – Book References

- Chapter 5: The Solution of the Equations of Motion
  - *Section 5.1 – Methods of solution*
  - *Section 5.6 – The state space method*
  - *Section 5.7 – State space model augmentation*
- Chapter 6: Longitudinal Dynamics
  - *Section 6.1 – Response to controls*
  - *Section 6.2 – The dynamic stability modes*
  - *Section 6.5 – Flying and handling qualities*
  - *Section 6.6 – Mode excitation*
- Chapter 7: Lateral-Directional Dynamics
  - *Section 7.1 – Response to controls*
  - *Section 7.2 – The dynamic stability modes*
  - *Section 7.5 – Flying and handling qualities*
  - *Section 7.6 – Mode excitation*

- *This session*
- *Next session*

Note that references are suggested starting points. Other sections might also be helpful!

Flight Dynamics Principles: A Linear Systems Approach to Aircraft Stability and Control (Elsevier Aerospace Engineering) Hardcover – 9 Aug 2007

The story so far….

- Nonlinear equations of motion (FD01 – FD07)
- Trimming the aircraft, static & manoeuvre stability (FD08 – FD11)
    - Find the required control inputs to balance forces and moments
- Linearisation (FD12)
    - Numerical linearisation to form state-space model
- Find the eigenvalues and eigenvectors of the ***A*** matrix (FD11 – FD13)
    - These represent the dynamic modes of motion which describe the response of the aircraft
    - Assume for conventional aircraft that longitudinal and lateral-directional response is de-coupled
    - Five characteristic modes of motion, two longitudinal and three lateral-directional
- Aircraft flying and handling qualities (FD14)
- Flight test (FD15 – FD16)
- Control

Longitudinal and Lateral-directional motion

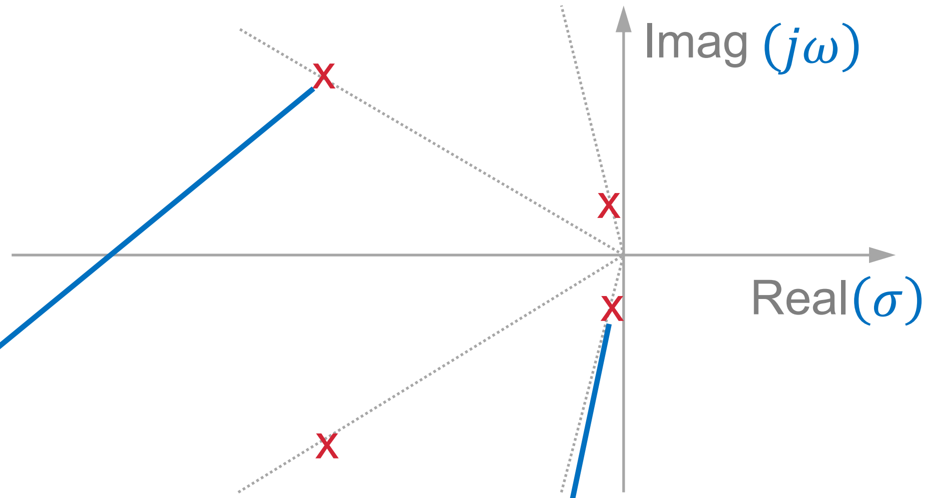
- The distinction between lateral freedoms and longitudinal freedoms of motion is based on what happens to the vertical plane of symmetry that passes through the centre of the fuselage.
- For longitudinal motions, the original orientation and disturbed orientation of that plane remain the same.
- For lateral motions, this vertical plane is displaced.

Typical Aircraft Response

- Aircraft Modes – Longitudinal
  - Phugoid & Short-period
- Aircraft Modes – Lateral-directional
  - Roll Subsidence, Dutch Roll and Spiral

Longitudinal Modes

- For a conventional aircraft configuration, the characteristic roots / eigenvalues will normally occur in two complex pairs, as shown below.



Short-period pair (higher frequency and more heavily damped) σ is much more negative

Phugoid pair (low frequency and low damping ratio) i.e. σ is not large and can even be positive for some flight conditions.

The Short-Period Mode

- The disturbance velocity in the transverse direction, w, helps to define the total incidence during the disturbance because of $\Delta\alpha = {}^{w}/_{U}$.

- Thus, whatever steady flight incidence there had been, a transverse disturbance of w will alter it, and the longitudinal balance of forces will be lost.

- Similarly, a disturbance of the pitch balance will lead to pitching action and a pitch rate q. These two disturbances cause the wing and tail lifts to depart from the trimmed values and thus the consequent short-period mode is essentially a seeking of that trimmed state again. It does not take long to achieve and is generally quite heavily damped.

The Short-Period Mode

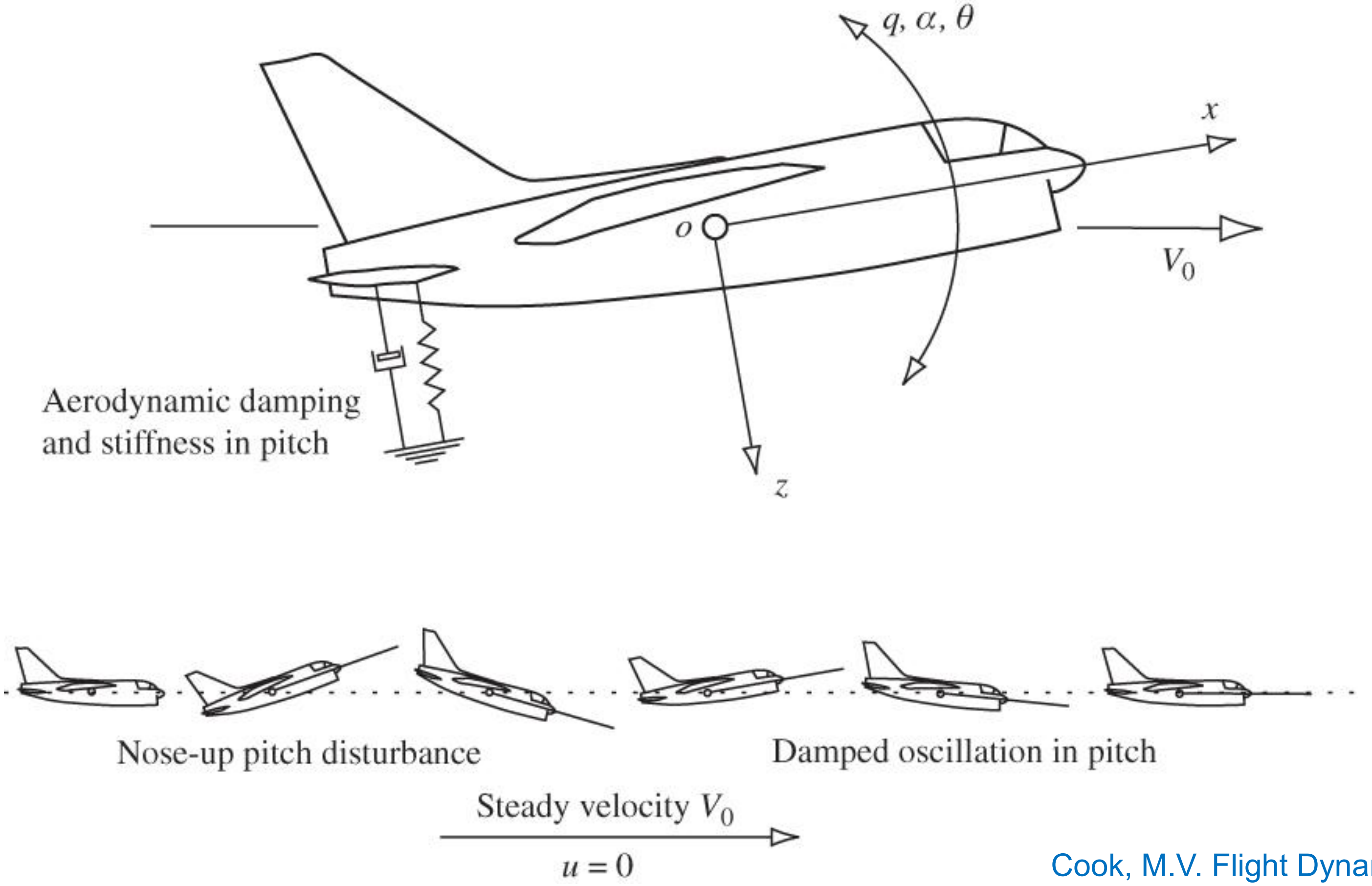
- The short period motion is thus composed largely of w (or α) and q with virtually no change in forward speed ($u \approx 0$ because $U \approx$ constant);

- the weather-cock action in the vertical plane is over so quickly that no significant changes in drag can occur to alter U.

- A typical period would be 1-4 seconds;

- This motion is important in Handling Quality evaluations because the pilot feels it and can react quickly enough to contribute. There is thus the possibility of a PIO (pilot-induced oscillation).

Stable Short Period Pitching Oscillation



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The Phugoid Mode

- This is primarily u and θ motion with small q and nearly zero w. The u and θ action can be envisaged as follows:

  a. a pitch disturbance ($\Delta\theta$) causes the aircraft to climb slowly,
  b. the forward speed U is caused to drop (u becomes negative)
  c. for a stable aircraft, such a loss of speed will cause a pitching moment (nose down) that tends to restore the trimmed condition, so the positive disturbance ($\Delta\theta$) tends to zero and then becomes negative after the aircraft reaches a peak departure in altitude from its former flight level,

The Phugoid Mode

d. the inevitable overshoot in θ because of the vehicle's pitch inertia, i.e. after $\Delta\theta$ goes negative, causes the aircraft to "start going downhill",

e. the speed slowly picks up (u now becoming obviously positive),

f. again, for the same reason as in c. above, the overall pitching moment becomes positive and begins to bring the nose up, so the negative θ turns to zero and eventually to positive values and the aircraft "starts going uphill",

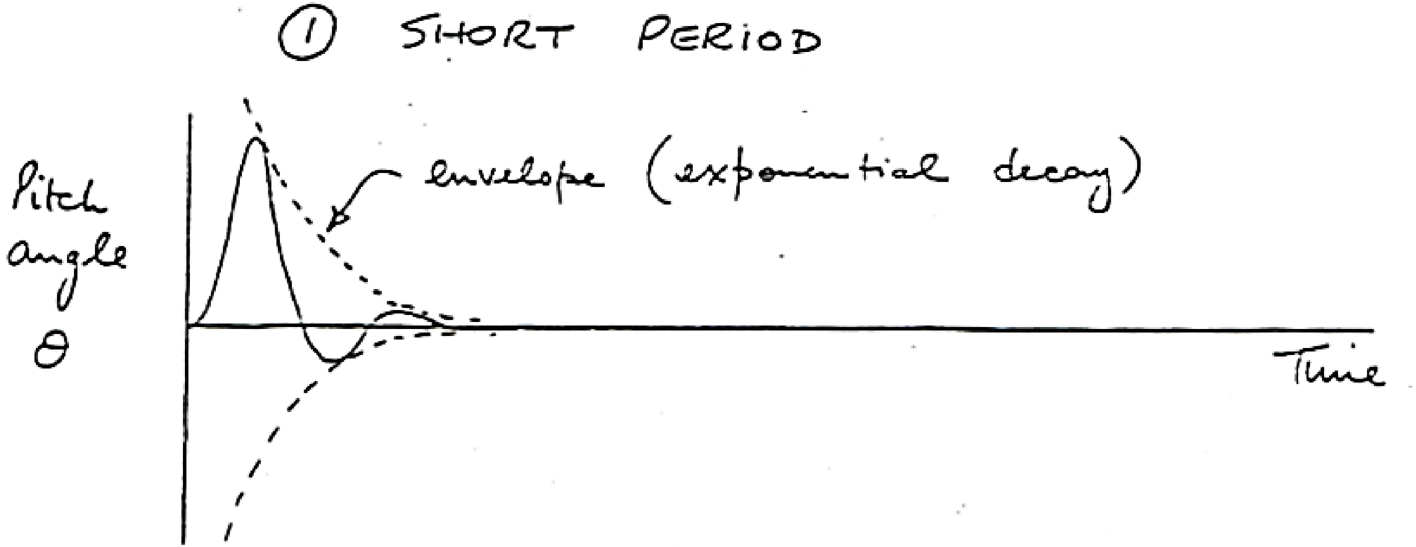
The Phugoid Mode

g. there will have been a noticeable loss of altitude and the climb back to original height begins, at small positive θ.

- The full cycle can involve changes in altitude of some hundreds of feet for a large aircraft, but the pitch action and the changes of forward speed are quite slow and the "roller-coaster" action is not necessarily obvious to a passenger.
- Periods are typically 15-100s and, even if $\sigma>0$ the unstable motion is controllable by the pilot because of the long period (small frequency); in practice, an autopilot loop often suppresses the motion by raising the damping artificially.
- Kinetic vs Potential energy.

Typical Longitudinal Response

- Aircraft response to a disturbance
- Note individual components
- Two sets of roots / eigenvalues are well separated in the complex plane



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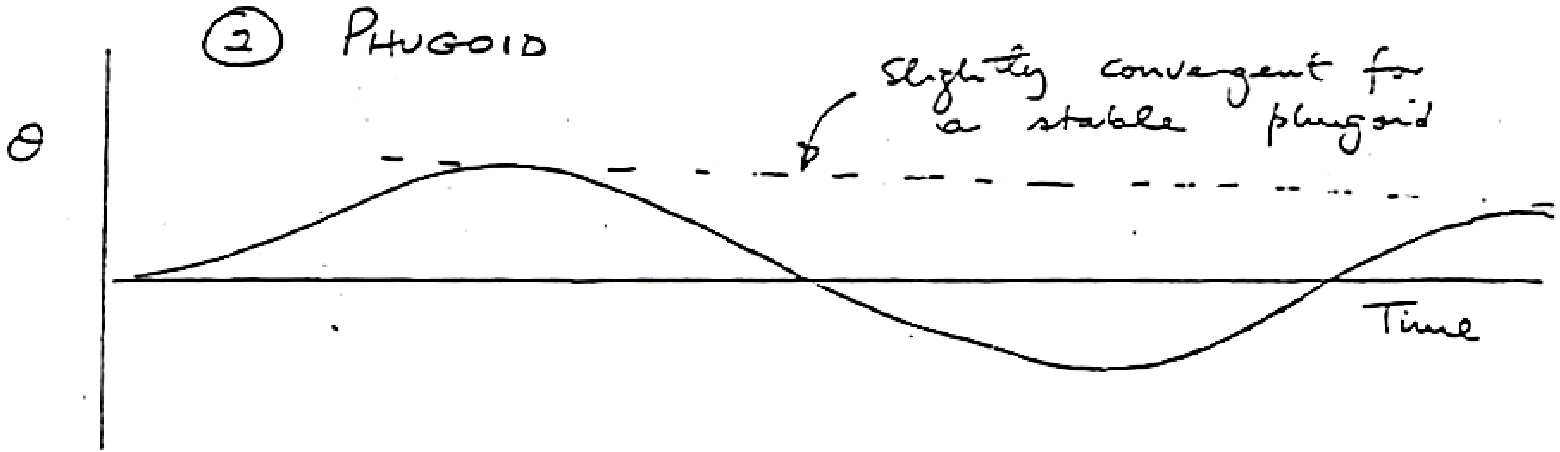

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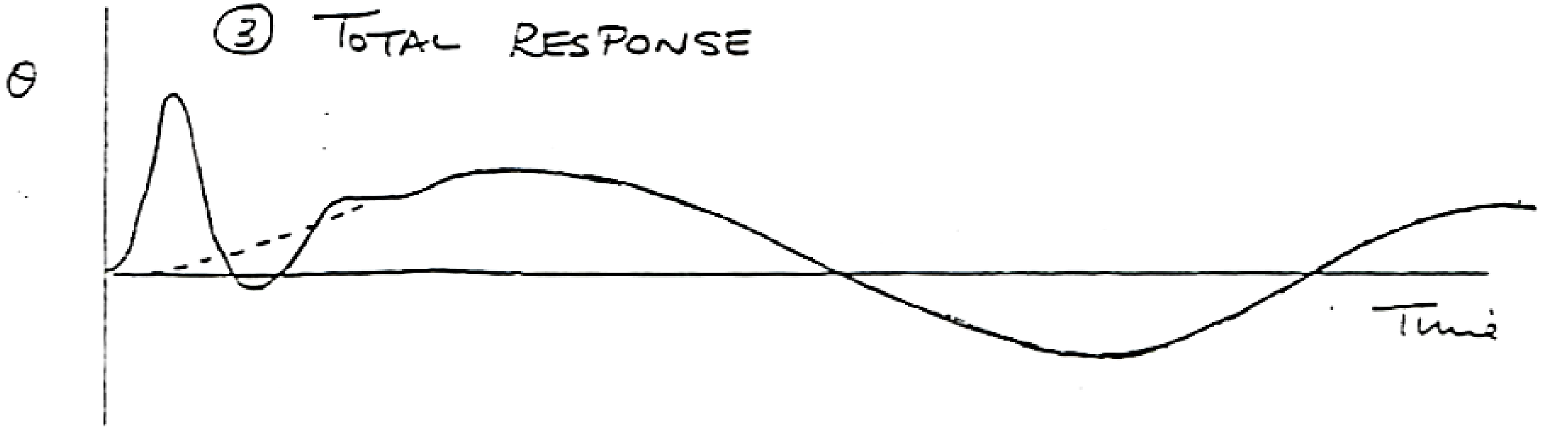
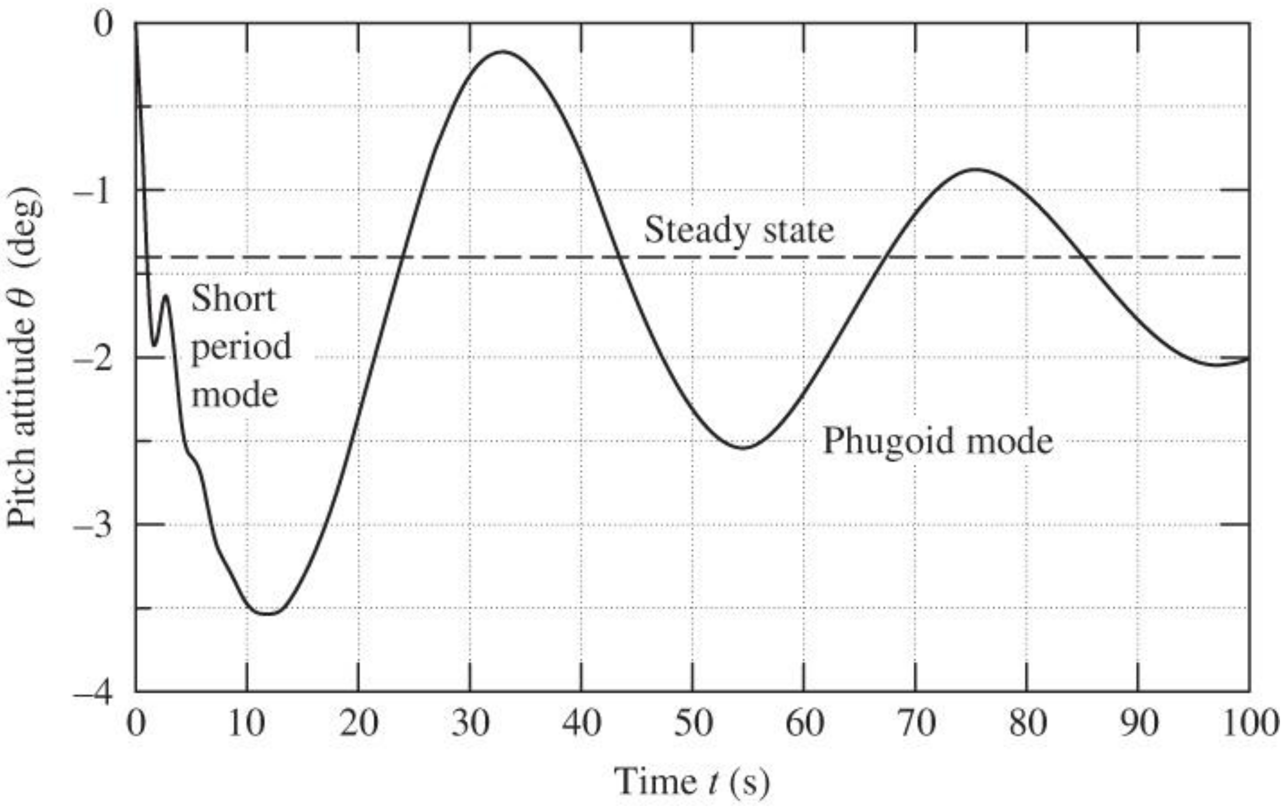


Pitch Attitude Response of the F-104 to a 1-degree Elevator Step Input



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UoB
G-UOB
University of BRISTOL

Aircraft Dynamic Modes

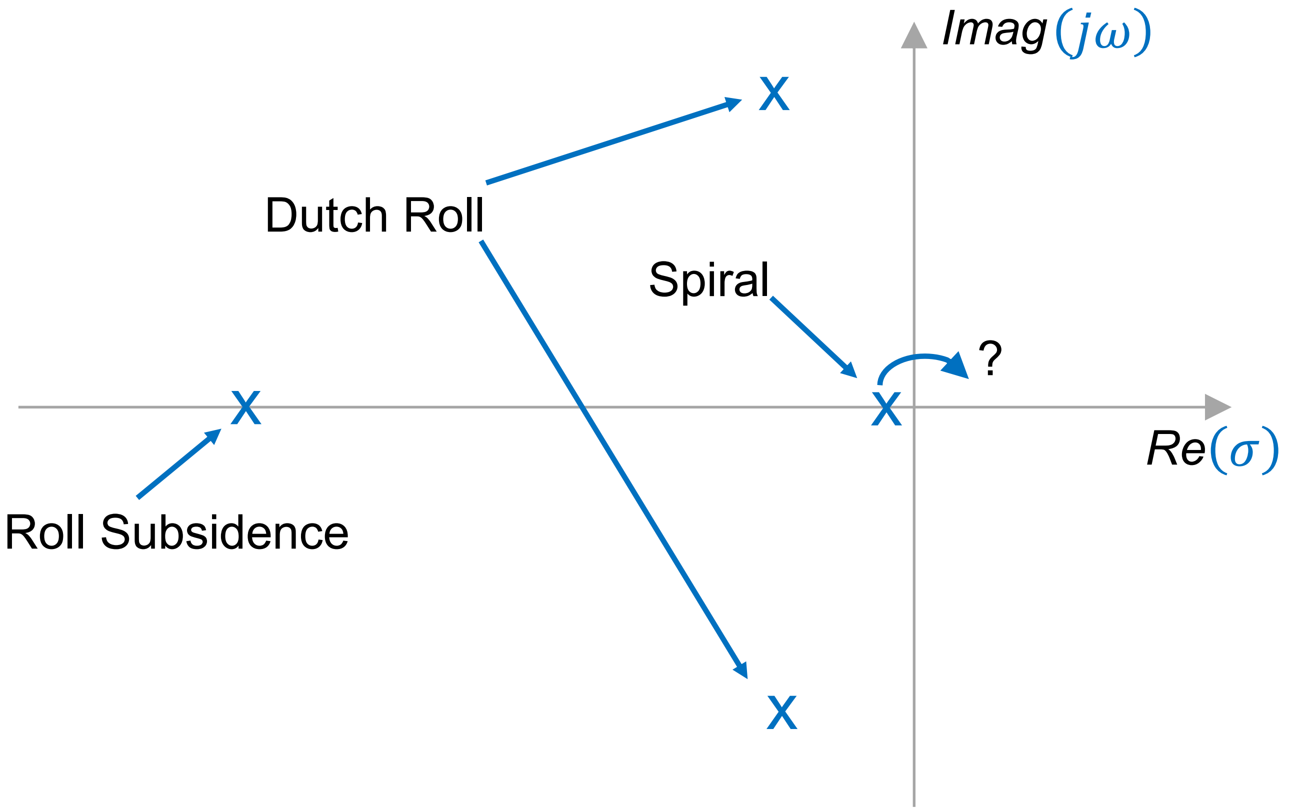
- Short Period
- Phugoid
- Roll Subsidence
- Spiral
- Dutch Roll

Longitudinal

Lateral

Variables?

Lateral Modes



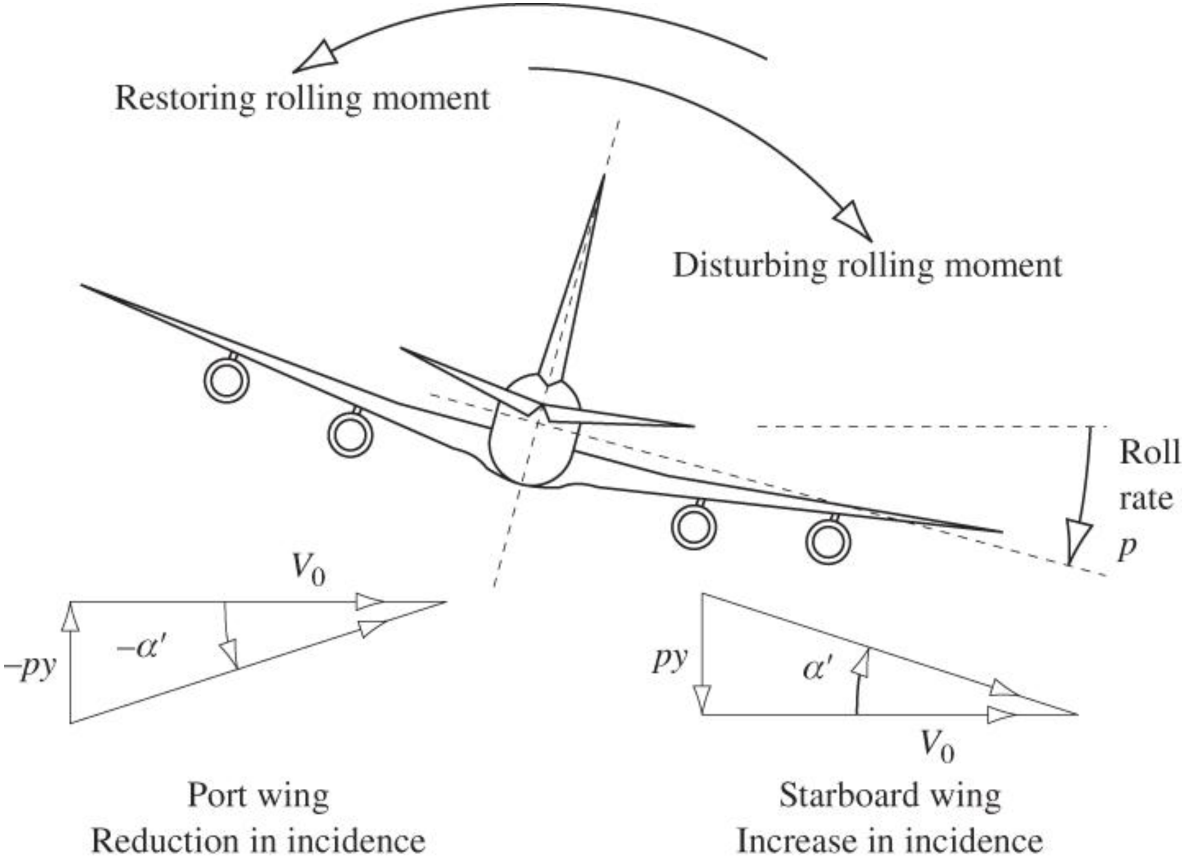
Lateral Modes

1. Large negative real root - the **roll convergence** (***or roll subsidence***).

   This implies almost pure rolling motion and of course cannot last long in reality because after the first 90^o you "fall out of the sky"

   A more realistic attitude suggests that if a roll "pulse" hits the aircraft (e.g. a brief up-gust on one wing), the consequent response in roll would be heavily damped.

Roll Subsidence Mode



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Lateral Modes

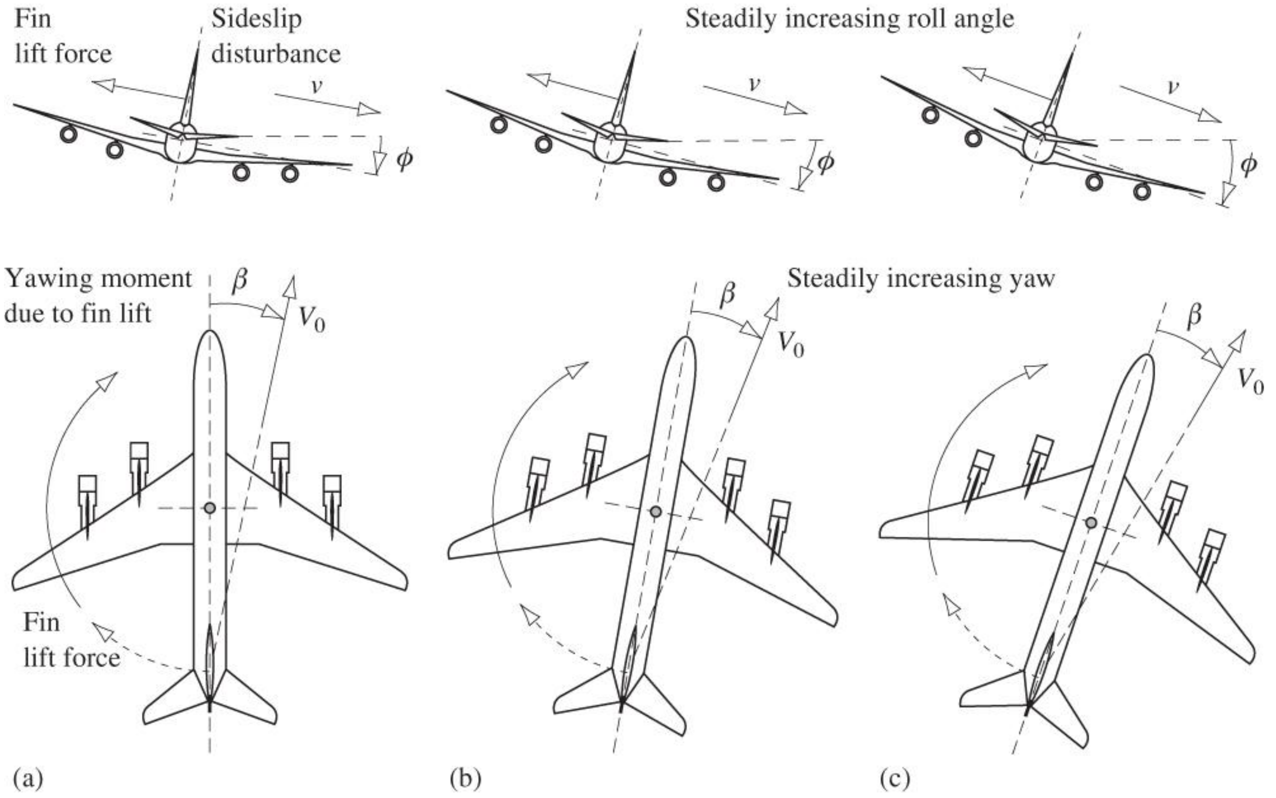
2. Small real root, of either sign - the **spiral mode**.

   This would be, for an unstable case, a slow divergence in yaw (say nose to starboard) while a roll angle built up (rolling to starboard) and thus also a sideslip would develop.

   The later stage would be a tightening spiral dive with all three motion variables involved.

   In practice, a pilot can control an unstable spiral mode (*slow!*).

The Spiral Mode Development



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Lateral Modes

3. The complex pair - **Dutch Roll**.

   Strictly speaking, all freedoms are active here, in an oscillatory sense, and out of phase with each other.

   This mode can be badly (though positively) damped and will affect Handling Qualities. The frequency is probably lower (the period a bit longer) than the longitudinal short period mode.

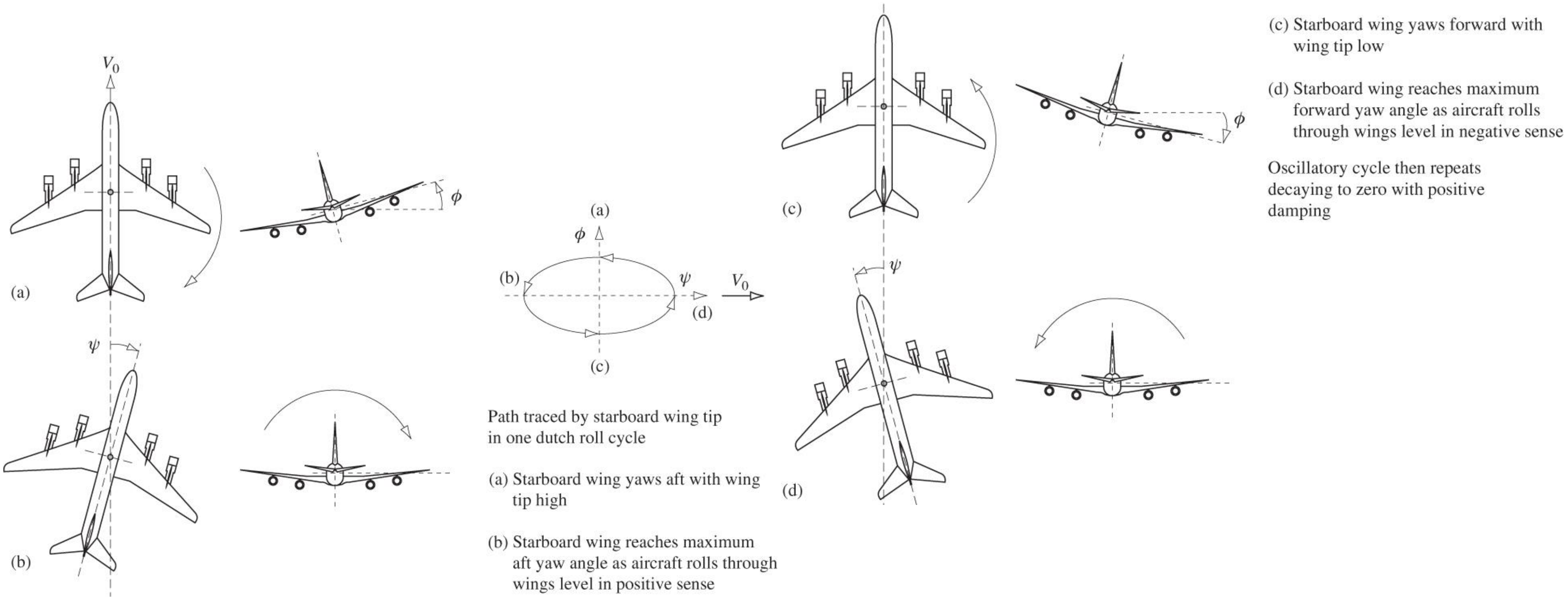
   Can be unpleasant if poorly damped! (nausea)

   Often poorly damped on swept wing aircraft

   Excited with the rudder or aileron

   (Yaw damper)

The Oscillatory Dutch Roll Mode



Aircraft Stability

Airbus A350-1000

https://www.britishairways.com

System Stability

Introduction

- Measures of transient performance may include steady state errors, overshoot, settling time, etc…
- The dynamic characteristics which govern the response cannot be separated from the stability of the system as implied by its characteristic roots / eigenvalues

System Stability

- Tau: time constant
- zeta: damping ratio
- wn: natural frequency

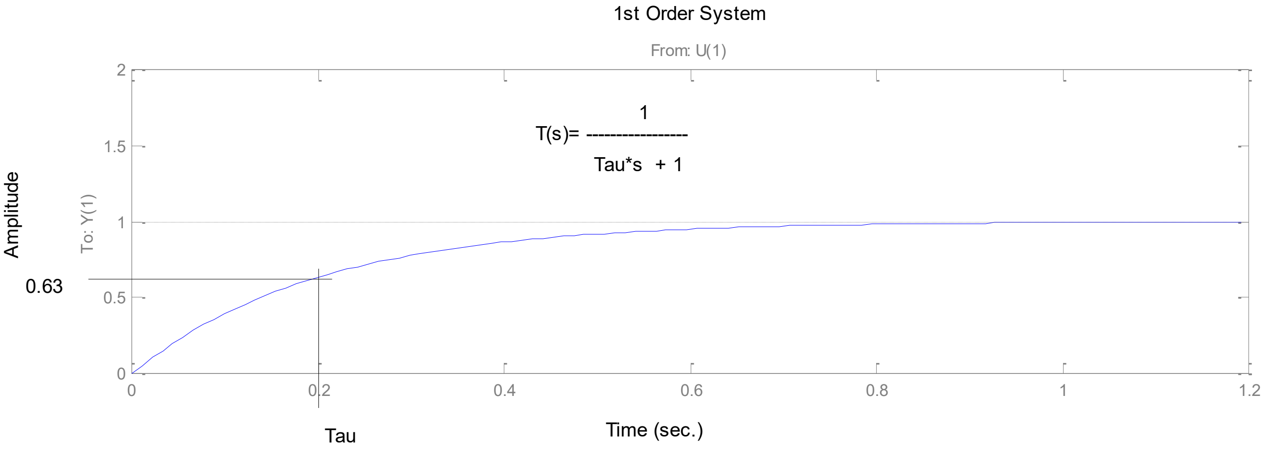
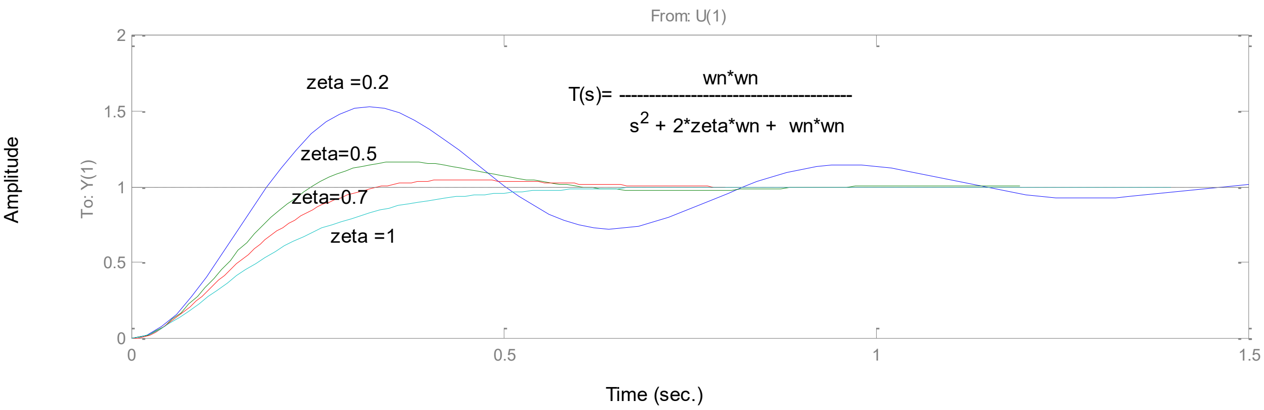
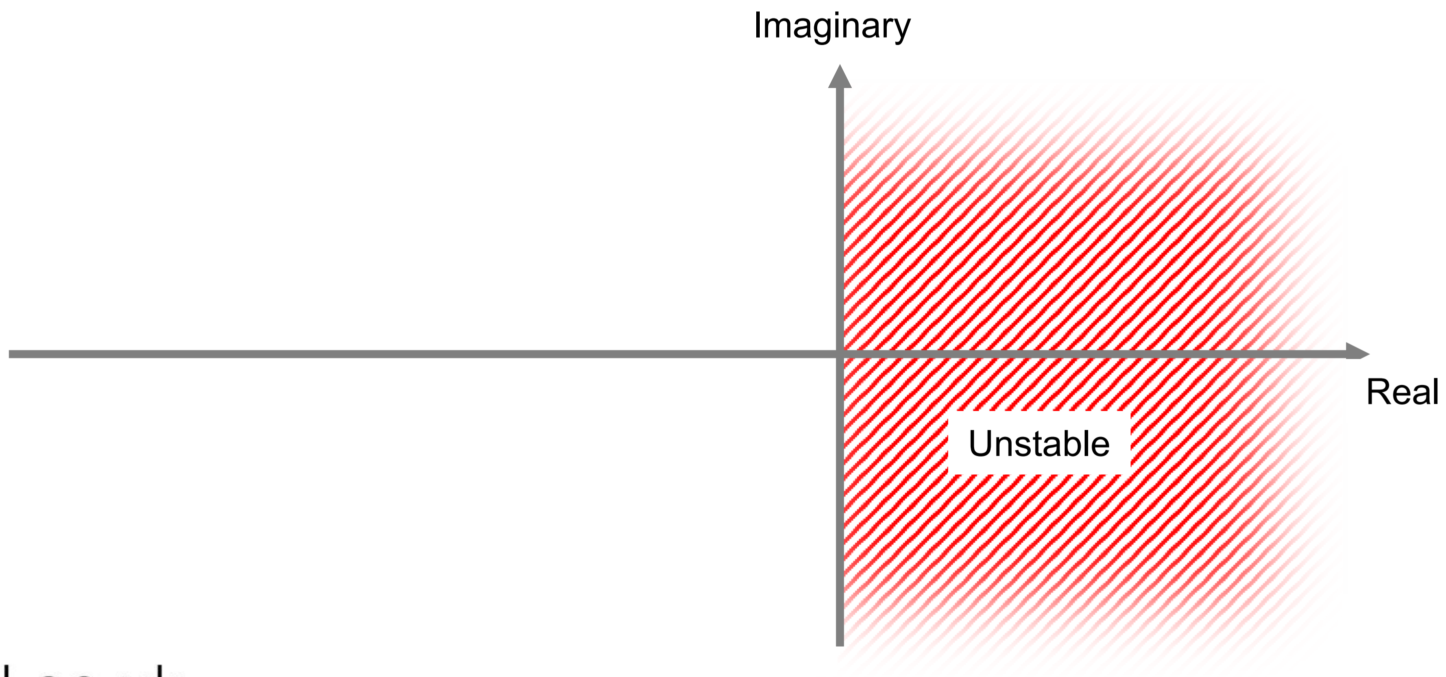


Fig.2



System Stability

- A system is said to be stable if its impulse response tends to a finite value (often, but not necessarily, zero) as $t \rightarrow \infty$.

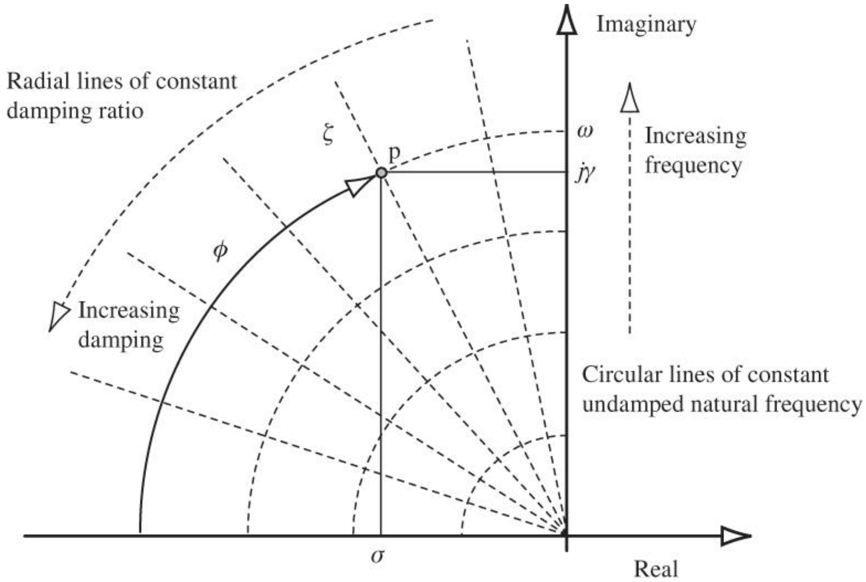


System Stability

- Stability is ensured if all the characteristic roots / eigenvalues have negative real parts, which commits them to a position in the left half of the complex plane and for the general case,

$$\lambda = \sigma \pm j\omega$$

- the **boundary** between stability and instability is $\sigma = 0$ for any root, real or complex



Flight Dynamics Principles

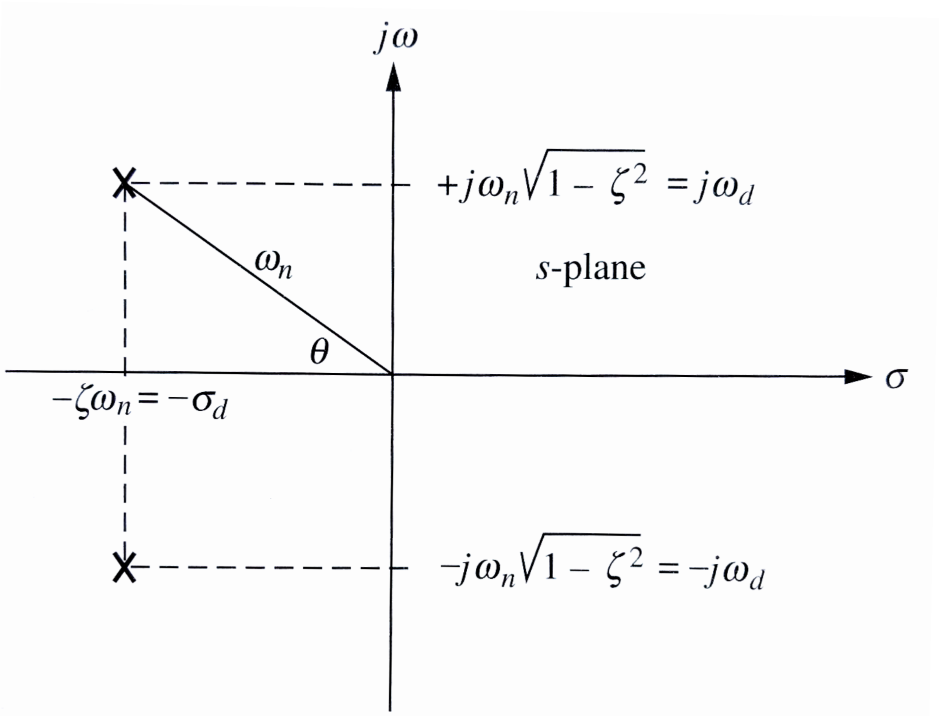
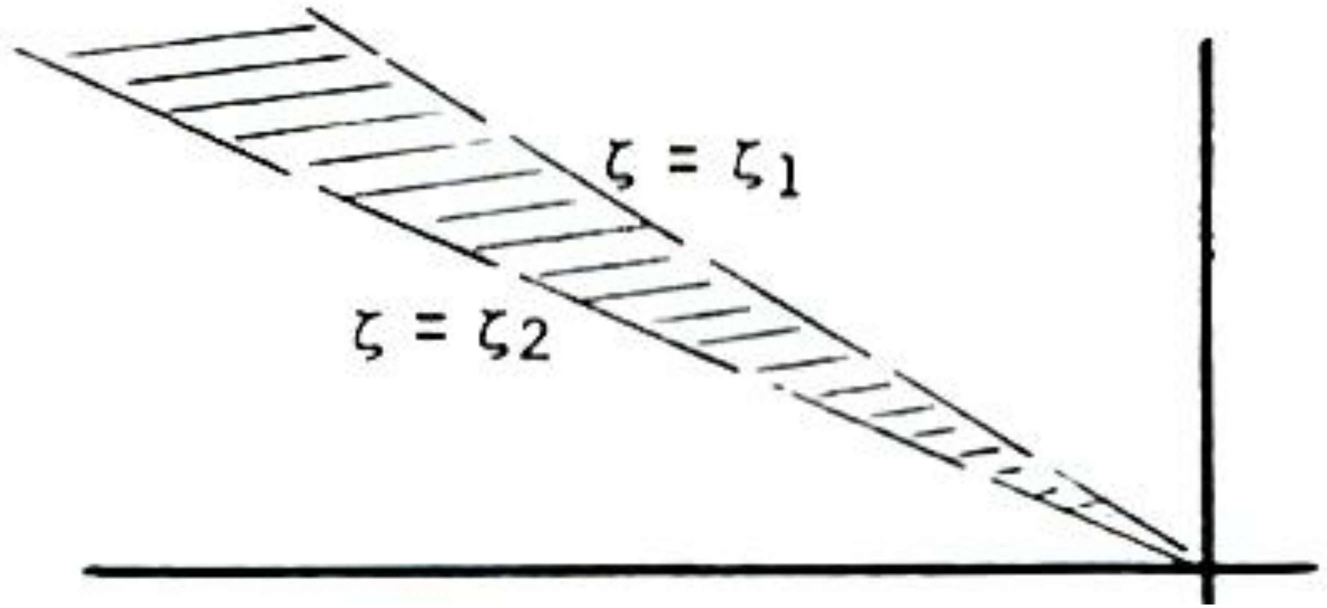


FIGURE 4.17 Pole plot for an underdamped second-order system

Control Systems Engineering

Norma S. Nise.

Constant Damping Ratio ζ



- We can show that: $\zeta = \sin\theta$
- *(be careful with the definition for θ)*
- *Note for the assignment: logarithmic decrement*

Constant Damping Ratio ζ

- For ζ within a required range, we must expect the characteristic roots / eigenvalues to fall within a fan-shaped' band emanating from the origin.

- Note, the so-called "optimum damping" of an oscillatory component requires a pair of characteristic roots / eigenvalues on the lines which are at 45° above and below the negative real axis, because

$$\zeta_{opt} = \frac{1}{\sqrt{2}}$$

Constant Natural Frequency

- The general expression for a pair of complex roots i.e.

$$\lambda = \sigma \pm j\omega = -\zeta\omega_o \pm j\omega_o\sqrt{1-\zeta^2}$$

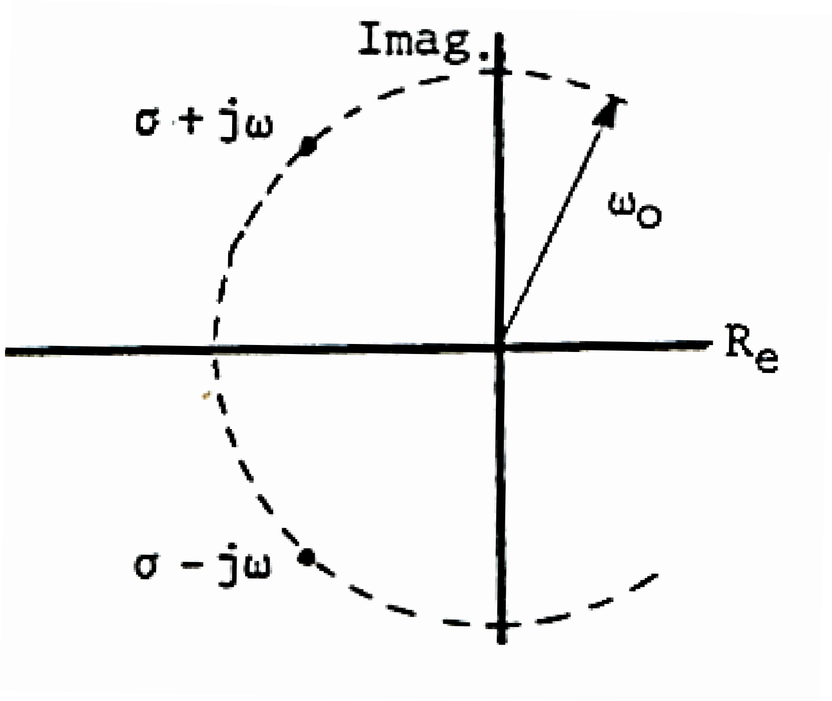
- leads naturally to:

$$\sigma^2 + \omega^2 = \zeta^2\omega_o^{\ 2} + \omega_o^{\ 2}\left(1-\zeta^2\right) = \omega_o^{\ 2}$$

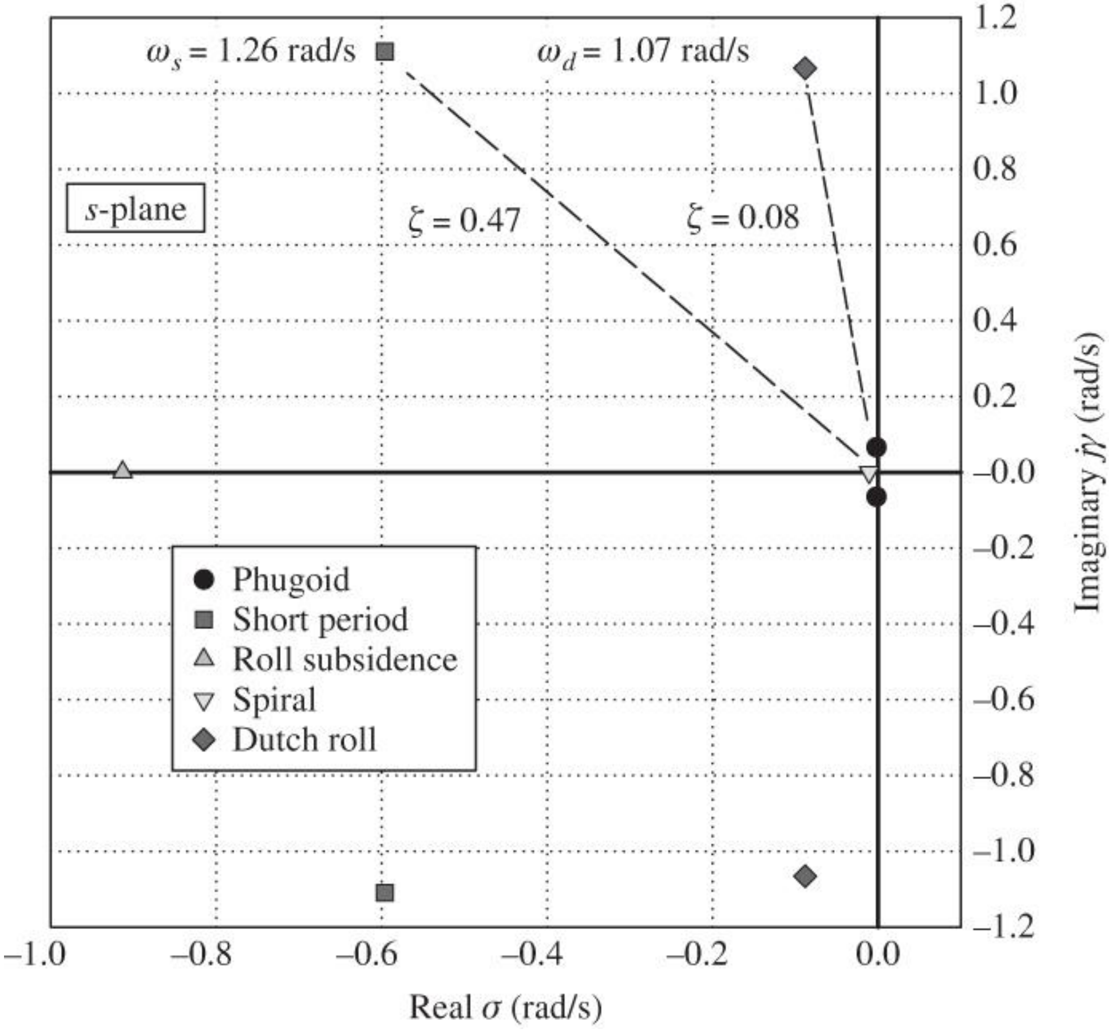
- which is the equation of a circle in the root plane, centred at the origin and having a radius of ω_0.

Constant Natural Frequency

- Thus, the locus of root positions for a complex pair, when only their damping ratio ζ is varied, is a (semi)circular arc of radius ω_0.



Boeing B-747



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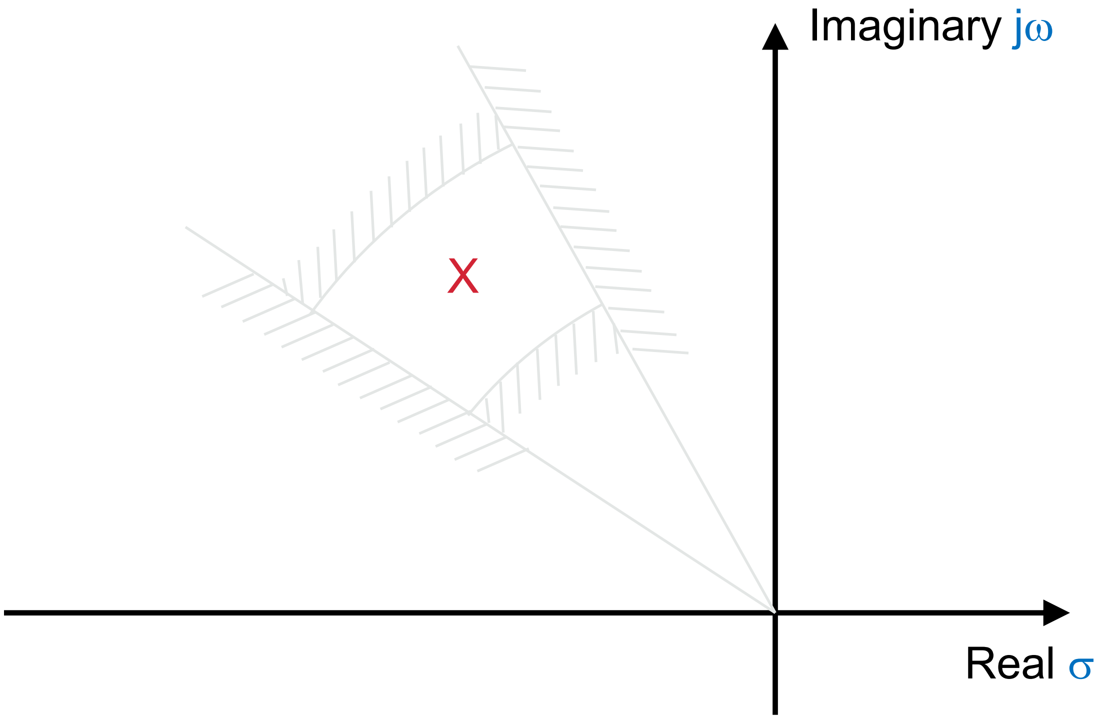
Desired region?



Next Session
Handling Qualities

- NASA and Lockheed Martin achieve first flight of X-59 quiet supersonic demonstrator
- www.flightglobal.com/
- 28 October 2025

ANY QUESTIONS

Prof. Mark Lowenberg & Prof. Tom Richardson

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