CHAPTER

Properties of Engineering Materials

8

It is now clear from the discussion in Chapter 7 that the structural designer requires a knowledge of the behaviour of materials under different types of load before he/she can be reasonably sure of designing a safe and, at the same time, economic structure.

One of the most important properties of a material is its strength, by which we mean the value of stress at which it fractures. Equally important in many instances, particularly in elastic design, is the stress at which yielding begins. In addition, the designer must have a knowledge of the stiffness of a material so that he/she can prevent excessive deflections occurring that could cause damage to adjacent structural members. Other factors that must be taken into consideration in design include the character of the different loads. For example, it is common experience that a material, such as cast iron fractures readily under a sharp blow whereas mild steel merely bends.

In Chapter 1 we reviewed the materials that are in common use in structural engineering; we shall now examine their properties in detail.

8.1 Classification of engineering materials

Engineering materials may be grouped into two distinct categories, ductile materials and brittle materials, which exhibit very different properties under load. We shall define the properties of ductility and brittleness and also some additional properties which may depend upon the applied load or which are basic characteristics of the material.

Ductility

A material is said to be *ductile* if it is capable of withstanding large strains under load before fracture occurs. These large strains are accompanied by a visible change in cross-sectional dimensions and therefore give warning of impending failure. Materials in this category include mild steel, aluminium and some of its alloys, copper and polymers.

Brittleness

A brittle material exhibits little deformation before fracture, the strain normally being below 5%. Brittle materials therefore may fail suddenly without visible warning. Included in this group are concrete, cast iron, high-strength steel, timber and ceramics.

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Elastic materials

A material is said to be *elastic* if deformations disappear completely on removal of the load. All known engineering materials are, in addition, *linearly elastic* within certain limits of stress so that strain, within these limits, is directly proportional to stress.

Plasticity

A material is perfectly *plastic* if no strain disappears after the removal of load. Ductile materials are *elastoplastic* and behave in an elastic manner until the *elastic limit* is reached after which they behave plastically. When the stress is relieved the elastic component of the strain is recovered but the plastic strain remains as a *permanent set.*

Isotropic materials

In many materials the elastic properties are the same in all directions at each point in the material although they may vary from point to point; such a material is known as *isotropic.* An isotropic material having the same properties at all points is known as *homogeneous*, e.g. mild steel.

Anisotropic materials

Materials having varying elastic properties in different directions are known as *anisotropic.*

Orthotropic materials

Although a structural material may possess different elastic properties in different directions, this variation may be limited, as in the case of timber which has just two values of Young's modulus, one in the direction of the grain and one perpendicular to the grain. A material whose elastic properties are limited to three different values in three mutually perpendicular directions is known as *orthotropic.*

8.2 Testing of engineering materials

The properties of engineering materials are determined mainly by the mechanical testing of specimens machined to prescribed sizes and shapes. The testing may be static or dynamic in nature depending on the particular property being investigated. Possibly the most common mechanical static tests are tensile and compressive tests which are carried out on a wide range of materials. Ferrous and non-ferrous metals are subjected to both forms of test, while compression tests are usually carried out on many non-metallic materials, such as concrete, timber and brick, which are normally used in compression. Other static tests include bending, shear and hardness tests, while the toughness of a material, in other words its ability to withstand shock loads, is determined by impact tests.

Tensile tests

Tensile tests are normally carried out on metallic materials and, in addition, timber. Test pieces are machined from a batch of material, their dimensions being specified by Codes of Practice. They are commonly circular in cross section, although flat test pieces having rectangular cross sections are used when the batch of material is in the form of a plate. A typical test piece would have the dimensions specified in Fig. 8.1. Usually the

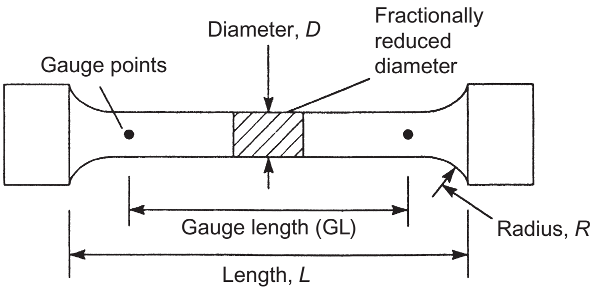

FIGURE 8.1

Standard cylindrical test piece.

diameter of a central portion of the test piece is fractionally less than that of the remainder to ensure that the test piece fractures between the gauge points.

Before the test begins, the mean diameter of the test piece is obtained by taking measurements at several sections using a micrometer screw gauge. Gauge points are punched at the required gauge length, the test piece is placed in the testing machine and a suitable strain measuring device, usually an extensometer, is attached to the test piece at the gauge points so that the extension is measured over the given gauge length. Increments of load are applied and the corresponding extensions recorded. This procedure continues until yield (see Section 8.3) occurs, when the extensometer is removed as a precaution against the damage which would be caused if the test piece fractured unexpectedly. Subsequent extensions are measured by dividers placed in the gauge points until, ultimately, the test piece fractures. The final gauge length and the diameter of the test piece in the region of the fracture are measured so that the percentage elongation and percentage reduction in area may be calculated. The two parameters give a measure of the ductility of the material.

A stress–strain curve is drawn (see Figs 8.8 and 8.12), the stress normally being calculated on the basis of the original cross-sectional area of the test piece, i.e. a *nominal stress* as opposed to an *actual stress* (which is based on the actual area of cross section). For ductile materials there is a marked difference in the latter stages of the test as a considerable reduction in cross-sectional area occurs between yield and fracture. From the stress–strain curve the ultimate stress, the yield stress and Young's modulus, E, are obtained (see Section 7.7).

There are a number of variations on the basic tensile test described above. Some of these depend upon the amount of additional information required and some upon the choice of equipment. Thus there is a wide range of strain measuring devices to choose from, extending from different makes of mechanical extensometer, e.g. Huggenberger, Lindley, Cambridge, to the electrical resistance strain gauge. The last would normally be used on flat test pieces, one on each face to eliminate the effects of possible bending. At the same time a strain gauge could be attached in a direction perpendicular to the direction of loading so that lateral strains are measured. The ratio lateral strain/longitudinal strain is Poisson's ratio, ν, (Section 7.8).

In the past, testing machines were driven hydraulically or the loads were applied mechanically by weights. With the development of computers and electronically controlled load cells, modern testing machines record load and extension on screen as the test proceeds, a distinct advantage when investigating the behaviour, say, of mild steel at yield.

Compression tests

A compression test is similar in operation to a tensile test, with the obvious difference that the load transmitted to the test piece is compressive rather than tensile. This is achieved by placing the test piece between the platens of the testing machine and reversing the direction of loading. Test pieces are normally cylindrical and are limited in length to eliminate the possibility of failure being caused by instability (Chapter 21). Again contractions are measured over a given gauge length by a suitable strain measuring device.

Variations in test pieces occur when only the ultimate strength of the material in compression is required. For this purpose concrete test pieces may take the form of cubes having edges approximately 10 cm long, while mild steel test pieces are still cylindrical in section but are of the order of 1 cm long.

Bending tests

Many structural members are subjected primarily to bending moments. Bending tests are therefore carried out on simple beams constructed from the different materials to determine their behaviour under this type of load.

Two forms of loading are employed the choice depending upon the type specified in Codes of Practice for the particular material. In the first a simply supported beam is subjected to a 'two-point' loading system as shown in Fig. 8.2(a). Two concentrated loads are applied symmetrically to the beam, producing zero shear force and constant bending moment in the central span of the beam (Fig. 8.2(b) and (c)). The condition of pure bending is therefore achieved in the central span (see Section 9.1).

The second form of loading system consists of a single concentrated load at mid-span (Fig. 8.3(a)) which produces the shear force and bending moment diagrams shown in Fig. 8.3(b) and (c).

The loads may be applied manually by hanging weights on the beam or by a testing machine. Deflections are measured by a dial gauge placed underneath the beam. From the recorded results a load–deflection diagram is plotted.

For most ductile materials the test beams continue to deform without failure and fracture does not occur. Thus plastic properties, e.g. the ultimate strength in bending, cannot be determined for such materials. In the case of brittle materials, including cast iron, timber and various plastics, failure does occur, so that plastic properties can be evaluated. For such materials the ultimate strength in bending is defined by the *modulus of rupture*. This is taken to be the maximum direct stress in bending, $\sigma_{x,u}$, corresponding to the ultimate moment M_u, and is assumed to be related to M_u by the elastic relationship

$$\sigma_{x,u} = \frac{M_u}{I} y_{max} \text{ (see Eq. (9.9))}$$

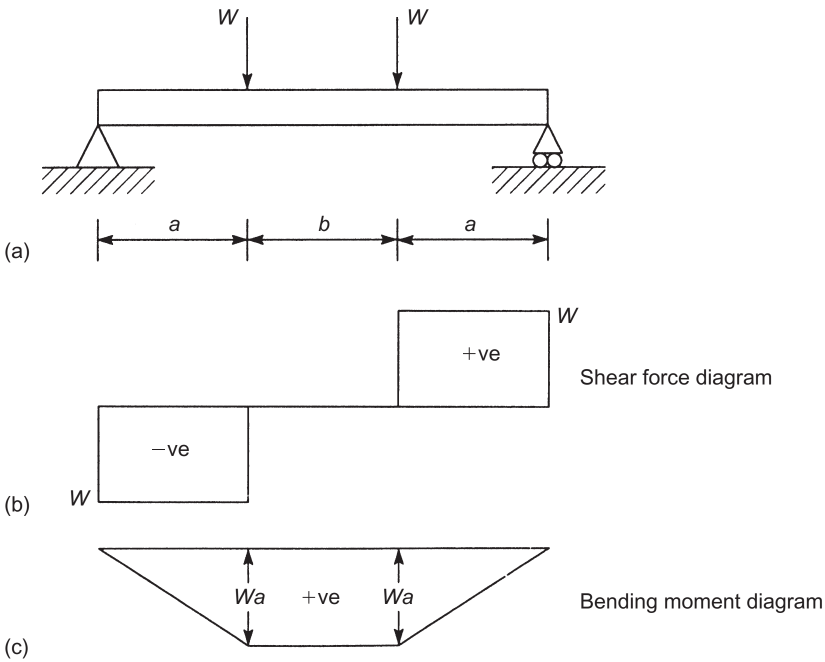

FIGURE 8.2

Bending test on a beam, 'two-point' load.

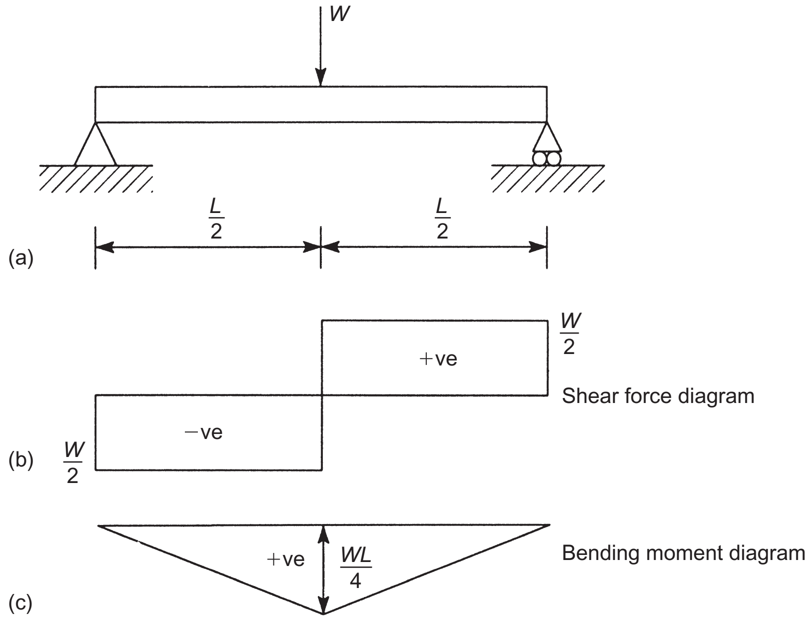

FIGURE 8.3

Bending test on a beam, single load.

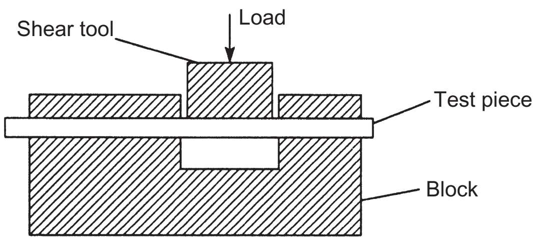

FIGURE 8.4

Shear test.

Other bending tests are designed to measure the ductility of a material and involve the bending of a bar round a pin. The angle of bending at which the bar starts to crack is then taken as an indication of its ductility.

Shear tests

Two main types of shear test are used to determine the shear properties of materials. One type investigates the direct or transverse shear strength of a material and is used in connection with the shear strength of bolts, rivets and beams. A typical arrangement is shown diagrammatically in Fig. 8.4 where the test piece is clamped to a block and the load is applied through the shear tool until failure occurs. In the arrangement shown the test piece is subjected to double shear, whereas if it is extended only partially across the gap in the block it would be subjected to single shear. In either case the average shear strength is taken as the maximum load divided by the shear resisting area.

The other type of shear test is used to evaluate the basic shear properties of a material, such as the shear modulus, G (Eq. (7.9)), the shear stress at yield and the ultimate shear stress. In the usual form of test a solid circular-section test piece is placed in a torsion machine and twisted by controlled increments of torque. The corresponding angles of twist are recorded and torque–twist diagrams plotted from which the shear properties of the material are obtained. The method is similar to that used to determine the tensile properties of a material from a tensile test and uses relationships derived in Chapter 11.

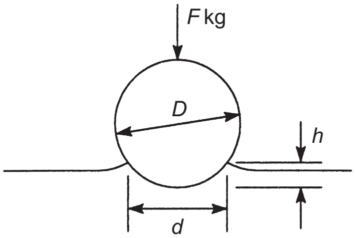

FIGURE 8.5

Brinell hardness test.

Hardness tests

The machinability of a material and its resistance to scratching or penetration are determined by its 'hardness'. There also appears to be a connection between the hardness of some materials and their tensile strength so that hardness tests may be used to determine the properties of a finished structural member where tensile and other tests would be impracticable. Hardness tests are also used to investigate the effects of heat treatment, hardening and tempering and of cold forming. Two types of hardness test are in common use: *indentation tests* and *scratch and abrasion tests.*

Indentation tests may be subdivided into two classes: static and dynamic. Of the static tests the *Brinell* is the most common. In this a hardened steel ball is pressed into the material under test by a static load acting for a fixed period of time. The load in kg divided by the spherical area of the indentation in mm^2 is called the *Brinell Hardness Number* (BHN). Thus in Fig. 8.5, if D is the diameter of the ball, F the load in kg, h the depth of the indentation, and d the diameter of the indentation, then

$$\mathrm{BHN} = \frac{F}{\pi D h} = \frac{2F}{\pi D\left[D - \sqrt{D^2 - d^2}\right]}$$

In practice the hardness number of a given material is found to vary with F and D so that for uniformity the test is standardized. For steel and hard materials $F = 3000$ kg and $D = 10$ mm while for soft materials $F = 500$ kg and $D = 10$ mm; in addition the load is usually applied for 15 s.

In the Brinell test the dimensions of the indentation are measured by means of a microscope. To avoid this rather tedious procedure, direct reading machines have been devised of which the *Rockwell* is typical. The indenting tool, again a hardened sphere, is first applied under a definite light load. This indenting tool is then replaced by a diamond cone with a rounded point which is then applied under a specified indentation load. The difference between the depth of the indentation under the two loads is taken as a measure of the hardness of the material and is read directly from the scale.

A typical dynamic hardness test is performed by the *Shore Scleroscope* which consists of a small hammer approximately 20 mm long and 6 mm in diameter fitted with a blunt, rounded, diamond point. The hammer is guided by a vertical glass tube and allowed to fall freely from a height of 25 cm onto the specimen, which it indents before rebounding. A certain proportion of the energy of the hammer is expended in forming the indentation so that the height of the rebound, which depends upon the energy still possessed by the hammer, is taken as a measure of the hardness of the material.

A number of tests have been devised to measure the 'scratch hardness' of materials. In one test, the smallest load in grams which, when applied to a diamond point, produces a scratch visible to the naked eye on a polished specimen of material is called its hardness number. In other tests the magnitude of the load required to produce a definite width of scratch is taken as the measure of hardness. Abrasion tests, involving the shaking over a period of time of several specimens placed in a container, measure the resistance to wear of some materials. In some cases there appears to be a connection between wear and hardness number although the results show no level of consistency.

Impact tests

It has been found that certain materials, particularly heat-treated steels, are susceptible to failure under shock loading whereas an ordinary tensile test on the same material would show no abnormality. Impact tests measure the ability of materials to withstand shock loads and provide an indication of their *toughness.* Two main tests are in use, the *Izod* and the *Charpy.*

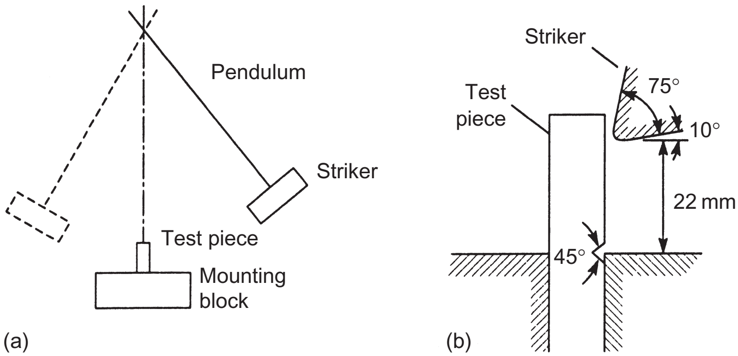

FIGURE 8.6

Izod impact test.

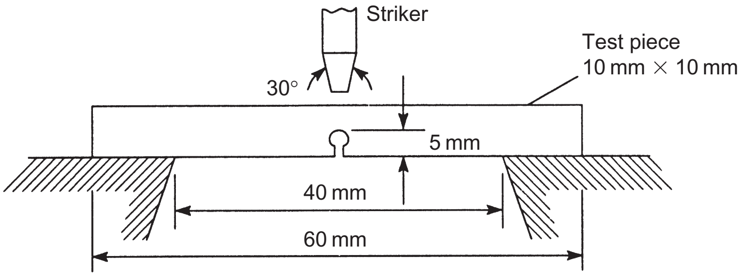

FIGURE 8.7

Charpy impact test.

Both tests rely on a striker or weight attached to a pendulum. The pendulum is released from a fixed height, the weight strikes a notched test piece and the angle through which the pendulum then swings is a measure of the toughness of the material. The arrangement for the Izod test is shown diagrammatically in Fig. 8.6(a). The specimen and the method of mounting are shown in detail in Fig. 8.6(b). The Charpy test is similar in operation except that the test piece is supported in a different manner as shown in the plan view in Fig. 8.7.

8.3 Stress–strain curves

We shall now examine in detail the properties of the different materials used in civil engineering construction from the viewpoint of the results obtained from tensile and compression tests.

Low carbon steel (mild steel)

A nominal stress–strain curve for mild steel, a ductile material, is shown in Fig. 8.8. From 0 to 'a' the stress–strain curve is linear, the material in this range obeying Hooke's law. Beyond 'a', the *limit of proportionality*, stress is no longer proportional to strain and the stress–strain curve continues to 'b', the *elastic limit*, which is defined as the maximum stress that can be applied to a material without producing a permanent plastic

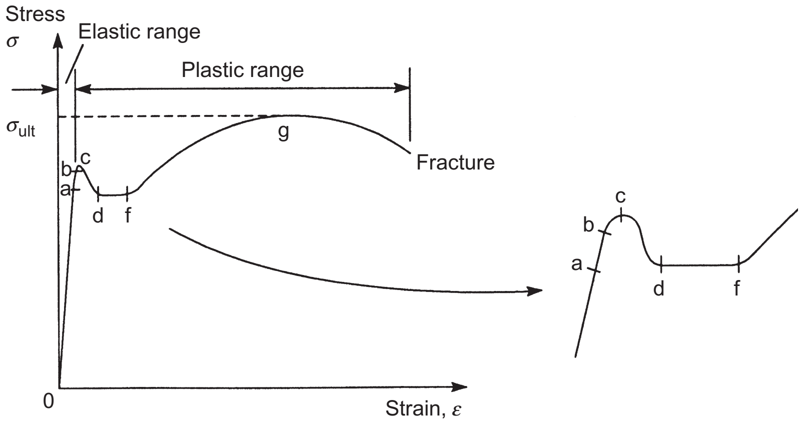

FIGURE 8.8

Stress–strain curve for mild steel.

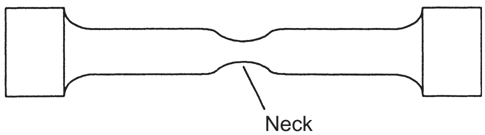

FIGURE 8.9

'Necking' of a test piece in the plastic range.

deformation or *permanent set* when the load is removed. In other words, if the material is stressed beyond 'b' and the load then removed, a residual strain exists at zero load. For many materials it is impossible to detect a difference between the limit of proportionality and the elastic limit. From 0 to 'b' the material is said to be in the *elastic range* while from 'b' to fracture the material is in the *plastic range*. The transition from the elastic to the plastic range may be explained by considering the arrangement of crystals in the material. As the load is applied, slipping occurs between the crystals which are aligned most closely to the direction of load. As the load is increased, more and more crystals slip with each equal load increment until appreciable strain increments are produced and the plastic range is reached.

A further increase in stress from 'b' results in the mild steel reaching its *upper yield point* at 'c' followed by a rapid fall in stress to its *lower yield point* at 'd'. The existence of a lower yield point for mild steel is a peculiarity of the tensile test wherein the movement of the ends of the test piece produced by the testing machine does not proceed as rapidly as its plastic deformation; the load therefore decreases, as does the stress. From 'd' to 'f' the strain increases at a roughly constant value of stress until *strain hardening* (see Section 8.4) again causes an increase in stress. This increase in stress continues, accompanied by a large increase in strain to 'g', the *ultimate stress*, σ_{ult}, of the material. At this point the test piece begins, visibly, to 'neck' as shown in Fig. 8.9. The material in the test piece in the region of the 'neck' is almost perfectly plastic at this stage and from this point, onwards to fracture, there is a reduction in nominal stress.

For mild steel, yielding occurs at a stress of the order of 300 N/mm^2. At fracture the strain (i.e. the elongation) is of the order of 30%. The gradient of the linear portion of the stress–strain curve gives a value for Young's modulus in the region of 200 000 N/mm^2.

The characteristics of the fracture are worthy of examination. In a cylindrical test piece the two halves of the fractured test piece have ends which form a 'cup and cone' (Fig. 8.10). The actual failure planes in this case are inclined at approximately 45° to the axis of loading and coincide with planes of maximum shear stress (Section 14.2). Similarly, if a flat tensile specimen of mild steel is polished and then stressed, a pattern of fine lines appears on the polished surface at yield. These lines, which were first discovered by Lüder in 1854, intersect approximately at right angles and are inclined at 45° to the axis of the specimen, thereby

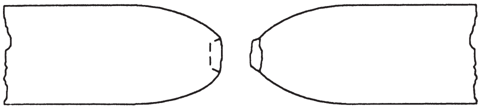

FIGURE 8.10

'Cup-and-cone' failure of a mild steel test piece.

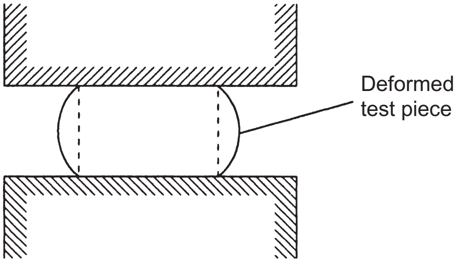

FIGURE 8.11

'Barrelling' of a mild steel test piece in compression.

coinciding with planes of maximum shear stress. These forms of yielding and fracture suggest that the crystalline structure of the steel is relatively weak in shear with yielding taking the form of the sliding of one crystal plane over another rather than the tearing apart of two crystal planes.

The behaviour of mild steel in compression is very similar to its behaviour in tension, particularly in the elastic range. In the plastic range it is not possible to obtain ultimate and fracture loads since, due to compression, the area of cross section increases as the load increases producing a 'barrelling' effect as shown in Fig. 8.11. This increase in cross-sectional area tends to decrease the true stress, thereby increasing the load resistance. Ultimately a flat disc is produced. For design purposes the ultimate stresses of mild steel in tension and compression are assumed to be the same.

The ductility of mild steel is often an advantage in that structures fabricated from mild steel do not generally suffer an immediate and catastrophic collapse if the yield stress of a member is exceeded. The member will deform in such a way that loads are redistributed to other adjacent members and at the same time will exhibit signs of distress thereby giving a warning of a probable impending collapse.

Higher grades of steel have greater strengths than mild steel but are not as ductile. They also possess the same Young's modulus so that the higher stresses are accompanied by higher strains.

Steel structures are very susceptible to rust which forms on surfaces exposed to oxygen and moisture (air and rain) and this can seriously weaken a member as its cross-sectional area is eaten away. Generally, exposed surfaces are protected by either *galvanizing*, in which they are given a coating of zinc, or by painting. The latter system must be properly designed and usually involves shot blasting the steel to remove the loose steel flakes, or millscale, produced in the hot rolling process, priming, undercoating and painting. Cold-formed sections do not suffer from millscale so that protective treatments are more easily applied.

Aluminium

Aluminium and some of its alloys are also ductile materials, although their stress–strain curves do not have the distinct yield stress of mild steel. A typical stress–strain curve is shown in Fig. 8.12. The points 'a' and 'b' again mark the limit of proportionality and elastic limit, respectively, but are difficult to determine experimentally. Instead a *proof stress* is defined which is the stress required to produce a given permanent strain on removal of the load. In Fig. 8.12, a line drawn parallel to the linear portion of the stress–strain curve from a strain of 0.001 (i.e. a strain of 0.1%) intersects the stress–strain curve at the 0.1% proof stress. For elastic design this, or the 0.2% proof stress, is taken as the working stress.

Beyond the limit of proportionality the material extends plastically, reaching its ultimate stress, σ_{ult}, at 'd' before finally fracturing under a reduced nominal stress at 'f'.

A feature of the fracture of aluminium alloy test pieces is the formation of a 'double cup' as shown in Fig. 8.13, implying that failure was initiated in the central portion of the test piece while the outer surfaces remained intact. Again considerable 'necking' occurs.

In compression tests on aluminium and its ductile alloys similar difficulties are encountered to those experienced with mild steel. The stress–strain curve is very similar in the elastic range to that obtained in

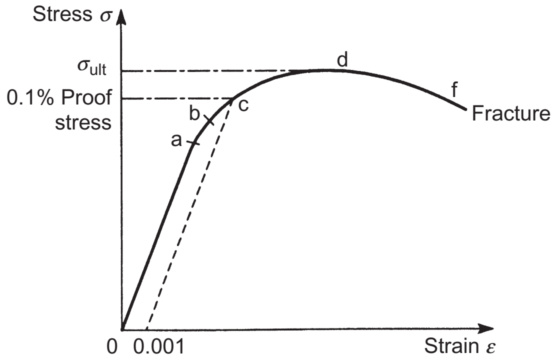

FIGURE 8.12

Stress–strain curve for aluminium.

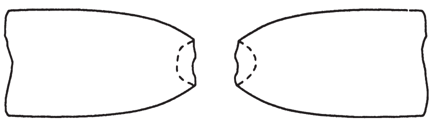

FIGURE 8.13

'Double-cup' failure of an aluminium alloy test piece.

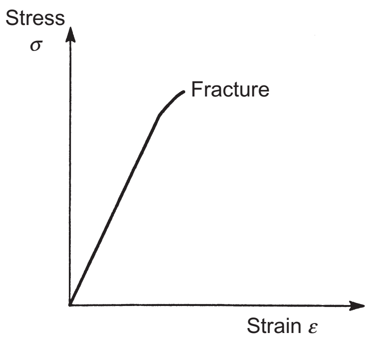

FIGURE 8.14

Stress–strain curve for a brittle material.

a tensile test but the ultimate strength in compression cannot be determined; in design its value is assumed to coincide with that in tension.

Aluminium and its alloys can suffer a form of corrosion particularly in the salt laden atmosphere of coastal regions. The surface becomes pitted and covered by a white furry deposit. This can be prevented by an electrolytic process called *anodizing* which covers the surface with an inert coating. Aluminium alloys will also corrode if they are placed in direct contact with other metals, such as steel. To prevent this, plastic is inserted between the possible areas of contact.

Brittle materials

These include cast iron, high-strength steel, concrete, timber, ceramics, glass, etc. The plastic range for brittle materials extends to only small values of strain. A typical stress–strain curve for a brittle material under tension is shown in Fig. 8.14. Little or no yielding occurs and fracture takes place very shortly after the elastic limit is reached.

The fracture of a cylindrical test piece takes the form of a single failure plane approximately perpendicular to the direction of loading with no visible 'necking' and an elongation of the order of 2–3%.

In compression the stress–strain curve for a brittle material is very similar to that in tension except that failure occurs at a much higher value of stress; for concrete the ratio is of the order of 10: 1. This is thought to be due to the presence of microscopic cracks in the material, giving rise to high stress concentrations which are more likely to have a greater effect in reducing tensile strength than compressive strength.

The form of the fracture of brittle materials under compression is clear and visible. For example, a cast-iron cylinder cracks on a diagonal plane as shown in Fig. 8.15(a) while failure of a concrete cube is shown in Fig. 8.15(b) where failure planes intersect at approximately 45° along each vertical face. Figure 8.15(c) shows a typical failure of a rectangular block of timber in compression. Failure in all these cases is due primarily to a breakdown in shear on planes inclined to the direction of compression.

Brittle materials can suffer deterioration in hostile environments although concrete is very durable and generally requires no maintenance. Concrete also provides a protective cover for the steel reinforcement in beams where the amount of cover depends on the diameter of the reinforcing bars and the degree of exposure of the beam. In some situations, e.g. in foundations, concrete is prone to chemical attack from sulphates

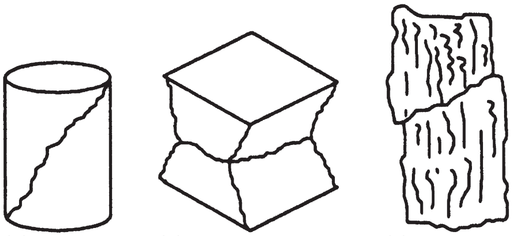

FIGURE 8.15

Failure of brittle materials.

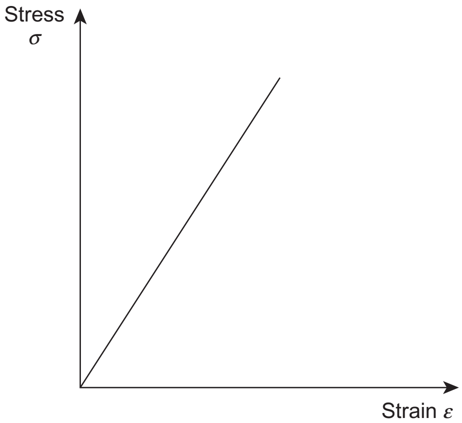

FIGURE 8.16

Stress–strain curve for a fibre composite.

contained in groundwater although if these are known to be present sulphate resisting cement can be used in the concrete.

Brick and stone are durable materials and can survive for hundreds of years as evidenced by the many medieval churches and Jacobean houses which still exist. There are, of course, wide variations in durability. For example, granite is extremely hard whereas the much softer sandstone can be worn away over periods of time by the combined effects of wind and rain, particularly acid rain which occurs when sulphur dioxide, produced by the burning of fossil fuels, reacts with water to form sulphuric acid. Bricks and stone are vulnerable to repeated wetting and freezing in which water, penetrating any surface defect, can freeze causing parts of the surface to flake off or *spall.* Some protection can be provided by masonry paints but these require frequent replacement. An alternative form of protection is a sealant which can be sprayed onto the surface of the masonry. The disadvantage of this is that, while preventing moisture penetrating the building, it also prevents water vapour from leaving. The ideal solution is to use top quality materials, do not apply any treatment and deal with any problem as it arises.

Timber, as we noted in Chapter 1, can be protected from fungal and insect attacks by suitable treatments.

Composites

Fibre composites have stress–strain characteristics which indicate that they are brittle materials (Fig. 8.16). There is little or no plasticity and the modulus of elasticity is less than that of steel and aluminium alloy. However, the fibres themselves can have much higher values of strength and modulus of elasticity than the composite. For example, carbon fibres have a tensile strength of the order 2400 N/mm^2 and a modulus of elasticity of 400 000 N/mm^2.

Fibre composites are highly durable, require no maintenance and can be used in hostile chemical and atmospheric environments; vinyls and epoxy resins provide the best resistance.

All the stress–strain curves described in the preceding discussion are those produced in tensile or compression tests in which the strain is applied at a negligible rate. A rapid strain application would result in significant changes in the apparent properties of the materials giving possible variations in yield stress of up to 100%.

8.4 Strain hardening

The stress–strain curve for a material is influenced by the *strain history*, or the loading and unloading of the material, within the plastic range. For example, in Fig. 8.17 a test piece is initially stressed in tension beyond the yield stress at, ‘a’, to a value at ‘b’. The material is then unloaded to ‘c’ and reloaded to ‘f’ producing an

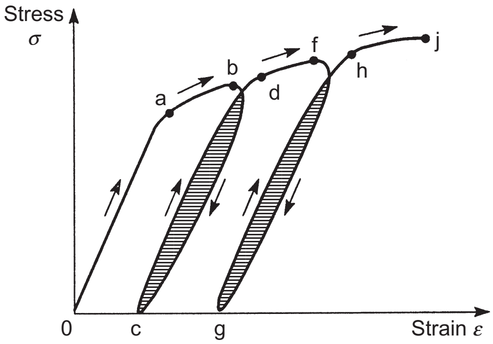

FIGURE 8.17

Strain hardening of a material.

increase in yield stress from the value at 'a' to the value at 'd'. Subsequent unloading to 'g' and loading to 'j' increases the yield stress still further to the value at 'h'. This increase in strength resulting from the loading and unloading is known as *strain hardening*. It can be seen from Fig. 8.17 that the stress–strain curve during the unloading and loading cycles forms loops (the shaded areas in Fig. 8.17). These indicate that strain energy is lost during the cycle, the energy being dissipated in the form of heat produced by internal friction. This energy loss is known as *mechanical hysteresis* and the loops as *hysteresis loops*. Although the ultimate stress is increased by strain hardening it is not influenced to the same extent as yield stress. The increase in strength produced by strain hardening is accompanied by decreases in toughness and ductility.

8.5 Creep and relaxation

We have seen in Chapter 7 that a given load produces a calculable value of stress in a structural member and hence a corresponding value of strain once the full value of the load is transferred to the member. However, after this initial or 'instantaneous' stress and its corresponding value of strain have been attained, a great number of structural materials continue to deform slowly and progressively under load over a period of time. This behaviour is known as *creep*. A typical creep curve is shown in Fig. 8.18.

Some materials, such as plastics and rubber, exhibit creep at room temperatures but most structural materials require high temperatures or long-duration loading at moderate temperatures. In some 'soft' metals, such as zinc and lead, creep occurs over a relatively short period of time, whereas materials such as concrete may be subject to creep over a period of years. Creep occurs in steel to a slight extent at normal temperatures but becomes very important at temperatures above 316°C.

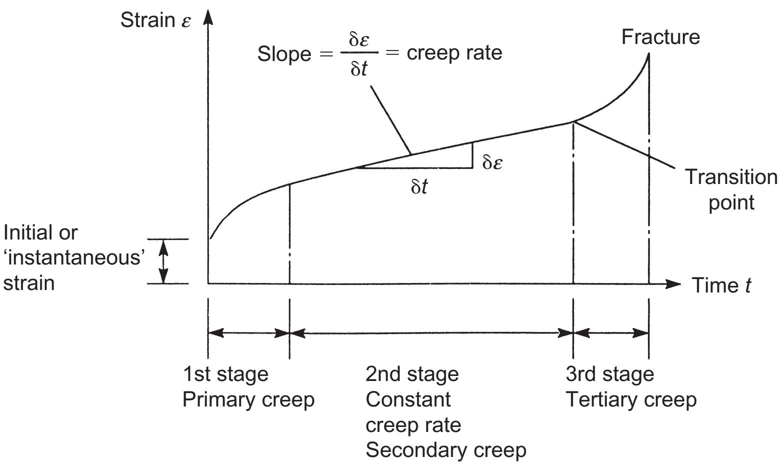

FIGURE 8.18

Typical creep curve.

Closely related to creep is *relaxation.* Whereas creep involves an increase in strain under constant stress, relaxation is the decrease in stress experienced over a period of time by a material subjected to a constant strain.

8.6 Fatigue

Structural members are frequently subjected to repetitive loading over a long period of time. For example, the members of a bridge structure suffer variations in loading possibly thousands of times a day as traffic moves over the bridge. In these circumstances a structural member may fracture at a level of stress substantially below the ultimate stress for non-repetitive static loads; this phenomenon is known as *fatigue.*

Fatigue cracks are most frequently initiated at sections in a structural member where changes in geometry, e.g. holes, notches or sudden changes in section, cause *stress concentrations.* Designers seek to eliminate such areas by ensuring that rapid changes in section are as smooth as possible. Thus at re-entrant corners, fillets are provided as shown in Fig. 8.19.

Other factors which affect the failure of a material under repetitive loading are the type of loading (fatigue is primarily a problem with repeated tensile stresses due, probably, to the fact that microscopic cracks can propagate more easily under tension), temperature, the material, surface finish (machine marks are potential crack propagators), corrosion and residual stresses produced by welding.

Frequently in structural members an alternating stress, σ_{alt}, is superimposed on a static or mean stress, σ_{mean}, as illustrated in Fig. 8.20. The value of σ_{alt} is the most important factor in determining the number of cycles of load that produce failure. The stress, σ_{alt}, that can be withstood for a specified number of cycles is called the *fatigue strength* of the material. Some materials, such as mild steel, possess a stress level that can be withstood for an indefinite number of cycles. This stress is known as the *endurance limit* of the material; no such limit has been found for aluminium and its alloys. Fatigue data are frequently presented in the form of an *S–n* curve or stress-endurance curve as shown in Fig. 8.21.

The stress-endurance curves shown in Fig. 8.21 are produced by testing a number of specimens at the same stress amplitude and determining the number of cycles, *N*, to failure. An average value of *N* is then obtained since at each stress amplitude the ratio of maximum *N* to minimum *N* may be as high as 10:1. Two other curves may therefore be drawn as shown in Fig. 8.22 enveloping all, or nearly all, the experimental results; these curves are known as the *confidence limits*. If 99.9 % of all the results lie between the curves, that is, only 1 in 1000 falls outside, they represent the 99.9% confidence limits. If 99.99999% of results lie between the curves only 1 in 10^7 results falls outside them and they represent the 99.99999% confidence limits.

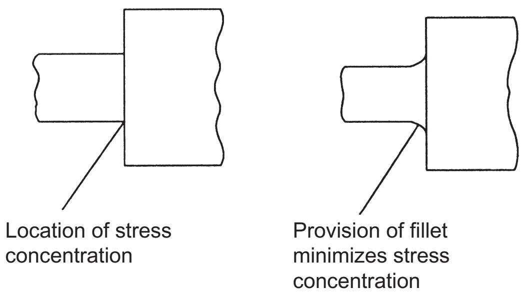

FIGURE 8.19

Stress concentration location.

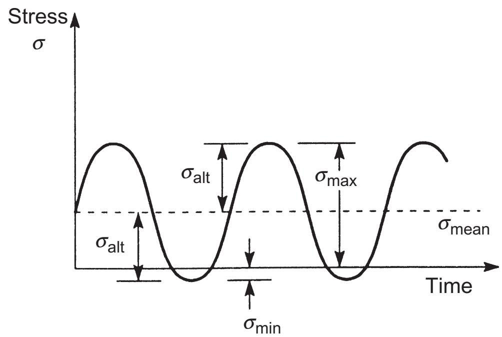

FIGURE 8.20

Alternating stress in fatigue loading.

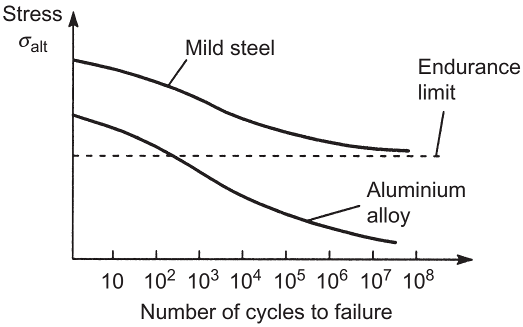

FIGURE 8.21

Stress-endurance curves.

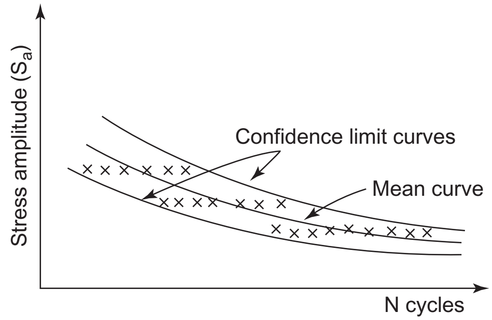

FIGURE 8.22

S-N diagram

The results from tests on a number of specimens may be represented as histograms in which the number of specimens failing within certain ranges R of N is plotted against N. Then, if N_{av} is the average value of N at a given stress amplitude, the probability of failure occurring at N cycles is given by

$$p(N) = \left[1/\left(\sigma\sqrt{2\pi}\right)\right] \exp\left\{-[(N - N_{av})/\sigma]^2/2\right\} \tag{8.1}$$

in which σ is the standard deviation of the whole population of N values. The derivation of Eq. (8.1) depends on the histogram approaching the profile of a continuous function close to the *normal distribution* which it does as the interval of N_{av}/R becomes smaller and the number of tests increases. The *cumulative probability*, which gives the probability that a particular specimen fails at or below N cycles, is defined as

$$P(N) = \int_{-\infty}^{N} p(N)\,\mathrm{d}N \tag{8.2}$$

The probability that a specimen endures more than N cycles is then $1 - P(N)$. The normal distribution allows negative values of N which is clearly impossible in a fatigue testing situation. Other distributions, *extreme value distributions*, are more realistic and allow the existence of minimum fatigue endurances and fatigue limits.

The damaging portion of a fluctuating load cycle occurs when the stress is tensile; this causes cracks to open and grow. Therefore, if a steady tensile stress is superimposed on a cyclic stress the maximum tensile stress during the cycle increases and the number of cycles to failure decreases. An approximate method of assessing the effect of a steady mean value of stress is provided by a Goodman diagram shown in Fig. 8.23. This shows the cyclic stress amplitudes which can be superimposed upon different mean stress levels to give a constant fatigue life. In Fig. 8.23 S_a is the allowable stress amplitude, $S_{a,0}$ is the stress amplitude required to produce fatigue at N cycles

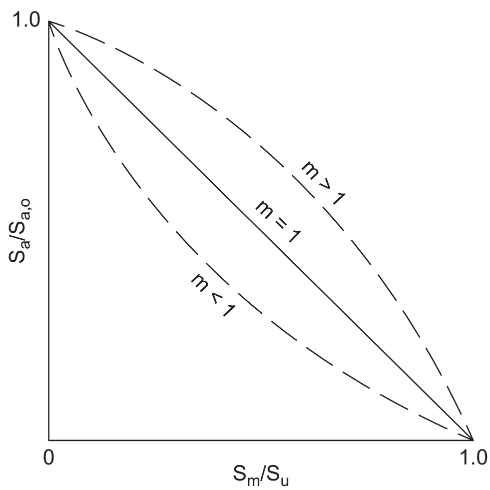

FIGURE 8.23

Goodman diagram

with zero mean stress, S_m is the mean stress and S_u the ultimate tensile stress. If $S_m = S_u$ any cyclic stress causes failure while if $S_m = 0$ the allowable stress amplitude is $S_{a,0}$. The equation of the straight-line portion of the diagram is

$$(S_a/S_{a,0}) = [1 - (S_m/S_u)] \tag{8.3}$$

Experimental evidence suggests a non-linear relationship for particular materials. Equation (8.3) then becomes

$$(S_a/S_{a,0}) = [1 - (S_m/S_u)^m] \tag{8.4}$$

where m lies between 0.6 and 2. The relationship for the case of $m = 2$ is known as the *Gerber parabola.*

EXAMPLE 8.1

The allowable stress amplitude for a low-carbon steel is ±225 N/mm^2 and its ultimate tensile stress is 750 N/mm^2. The steel is subjected to a repeated cycle of stress in which the minimum stress is zero. Calculate the safe range of stress based on the Goodman and Gerber predictions.

For the Goodman prediction Eq. (8.3) applies in which $S_{a,0} = 450$ N/mm^2 and $S_m = S_a/2$. Then

$$S_a = 450[1 - (S_a/2 \times 750)]$$

from which

$$S_a = 346\,\text{N/mm}^2$$

For the Gerber prediction $m = 2$ in Eq. (8.4). Then

$$S_a = 450[1 - (S_a/2 \times 750)^2]$$

which simplifies to

$$S_a^2 + 5000S_a - 2.25 \times 10^6 = 0$$

Solving gives

$$S_a = 415\,\text{N/mm}^2$$

EXAMPLE 8.2

If the steel in Ex. 8.1 is subjected to an alternating cycle of tensile stress about a mean stress of 180 N/mm^2 calculate the safe range of stress based on the Goodman prediction and also the maximum stress values.

From Eq. (8.3)

$$S_a = 450[1 - (180/750)]$$

which gives

$$S_a = 342\,\text{N/mm}^2$$

Now

$$S_a = \sigma_{max} - \sigma_{min} \tag{i}$$

and

$$S_m = (\sigma_{max} + \sigma_{min})/2 \tag{ii}$$

Adding Eqs (i) and (ii)

$$S_a + 2S_m = 2\sigma_{max}$$

that is

$$342 + 2 \times 180 = 2\sigma_{max}$$

so that

$$\sigma_{max} = 351\ \mathrm{N/mm^2}$$

Then, from Eq. (i)

$$\sigma_{min} = 351 - 342 = 9\ \mathrm{N/mm^2}$$

In many practical situations the amplitude of the alternating stress varies and is frequently random in nature. The $S-n$ curve does not, therefore, apply directly and an alternative means of predicting failure is required. *Miner's cumulative damage theory* suggests that failure will occur when

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \cdots + \frac{n_r}{N_r} = 1 \tag{8.5}$$

where $n_1, n_2, \ldots, n_r$ are the number of applications of stresses σ_{alt}, σ_{mean} and $N_1, N_2, \ldots, N_r$ are the number of cycles to failure of stresses σ_{alt}, σ_{mean}.

EXAMPLE 8.3

A structural member is subjected to a series of cyclic loads, which produce different levels of alternating stress as listed in Table Ex. 8.3. Determine whether or not a fatigue failure occurs.

Table Ex. 8.3

Loading	1	2	3	4	5
No. of cycles	10^5	2×10^4	10^5	1.5×10^6	10^7
No. of cycles to failure	3×10^6	10^5	2×10^6	10^7	1.5×10^8

From Eq. (8.5)

$$\frac{10^5}{3 \times 10^6} + \frac{2 \times 10^4}{10^5} + \frac{10^5}{2 \times 10^6} + \frac{1.5 \times 10^6}{10^7} + \frac{10^7}{1.5 \times 10^8} = 0.5,$$

which shows that fatigue failure does not occur.

EXAMPLE 8.4

In a fatigue test a steel specimen is subjected to a reversed cyclic loading in a continuous sequence of four stages as follows:

200 cycles at ± 150 N/mm^2
250 cycles at ± 125 N/mm^2
400 cycles at ± 120 N/mm^2
550 cycles at ± 100 N/mm^2

If the loading is applied at the rate of 80 cycles/hour and the fatigue lives at these stress levels are 10^4, 10^5, 1.5×10^5 and 2×10^5, respectively, calculate the life of the specimen.

Suppose that the specimen fails after P sequences of the four stages. Then, from Eq. (8.6)

$$P[(200/10^4) + (250/10^5) + (400/1.5\times10^5) + (550/2\times10^5)] = 1$$

which gives

$$P = 35.8$$

The total number of cycles in the four stages is 1400 so that for a loading rate of 80 cycles/hour the total number of hours to fracture is given by

$$\text{Total hours to fracture} = 35.8 \times 1400/80 = 626.5$$

Crack propagation

We have noted that the fatigue life of a structural member can be severely compromised by the presence of cracks. It is useful, therefore, for a designer to be able to predict the rate at which a perceived crack will propagate until it reaches proportions at which the member will fail.

There are three basic modes of crack growth and these are shown in Fig. 8.24.

Generally, the stress field in the region of the crack tip is described by a two-dimensional model which may be used as an approximation for many practical three-dimensional loading cases. Texts on fracture mechanics suggest that the stress system at a distance r ($r \leq a$) from the tip of the crack of length $2a$, as shown in Fig. 8.25, can be expressed in the form

$$S_r, S_\theta, S_{r,\theta} = [K/(2\pi r)^{1/2}]f(\theta) \tag{8.6}$$

in which $f(\theta)$ is a different function for each of the three stresses and K is the *stress concentration factor*; K is a function of the nature and magnitude of the applied stress levels and also of the crack size. The terms $(2\pi r)^{1/2}$ and $f(\theta)$ map the stress field in the vicinity of the crack and are the same for all cracks under external loads that cause crack openings of the same type.

Equation (8.6) applies to all modes of crack opening with K having different values depending on the geometry of the structure, the nature of the applied loads and the type of crack.

Experimental data show that crack growth and residual strength data are better correlated using K than any other parameter. K may be expressed as a function of the nominal applied stress, S, and the crack length in the form

$$K = S(\pi a)^{1/2}\alpha \tag{8.7}$$

in which α is a non-dimensional coefficient usually expressed as the ratio of crack length to any convenient local dimension in the plane of the component; for a crack in an infinite plate under an applied uniform stress

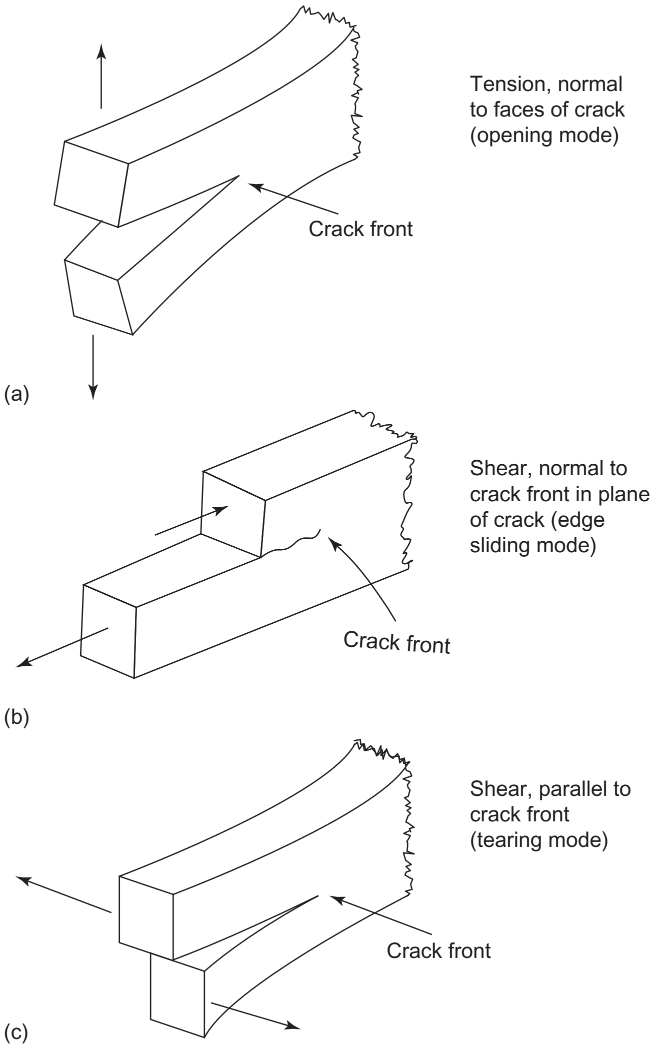

FIGURE 8.24

Basic modes of crack growth

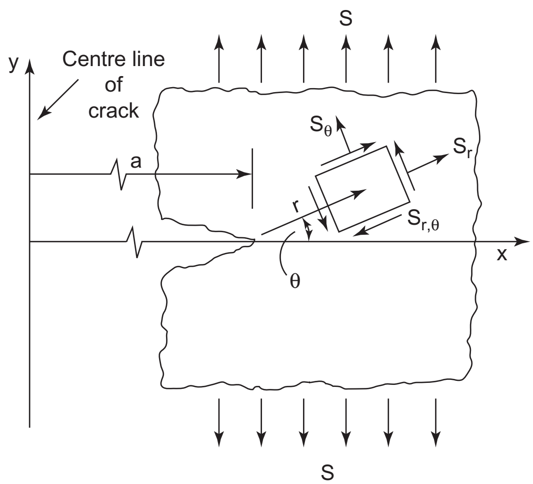

FIGURE 8.25

Stress field in the vicinity of a crack

level, S, remote from the crack, $\alpha = 1.0$. Alternatively, in cases where opposing loads, P, are applied at points close to the crack

$$K = P\alpha/(\pi a)^{1/2} \tag{8.8}$$

in which P is the load/unit thickness. Equations (8.7) and (8.8) may be rewritten as

$$K = K_0\alpha \tag{8.9}$$

where K_0 is a reference value of the stress concentration factor which depends on the loading. For the simple case of a remotely loaded plate in tension

$$K_0 = S(\pi a)^{1/2} \tag{8.10}$$

and Eqs (8.9) and (8.7) are identical so that, for a given ratio of crack length to plate width, α is the same in both formulations. In more complex cases, for example the in-plane bending of a plate of width $2b$ and having a central crack of length $2a$

$$K_0 = (3Ma/4b^3)(\pi a)^{1/2} \tag{8.11}$$

in which M is the bending moment per unit thickness. Comparing Eqs (8.11) and (8.7) we see that $S = 3Ma/4b^3$ which is the value of direct stress given by basic bending theory at a point a distance $\pm\, a/2$ from the central axis (see Chapter 9). However, if S is specified as the bending stress in the outer fibres of the plate, that is at $\pm\, b$, then $S = 3M/2b^2$; clearly the different specifications of S require different values of α. On the other hand the final value of K must be independent of the form of presentation used. Use of Eqs (8.7) – (8.9) depends on the form of the solution for K_0 and care must be taken to ensure that the formula used and the way in which the nominal stress is defined are compatible with those used in the derivation of α.

A number of methods are available for determining the values of K and α. In one method the solution for a structural member subjected to more than one type of loading is obtained from available standard solutions using superposition or, if the geometry is not covered, two or more standard solutions may be compounded. Alternatively a finite element analysis may be used.

The coefficient α in Eq. (8.7) has, as we have noted, different values depending on the plate and crack geometries. The following are values of α for some of the more common cases.

i. A semi-infinite plate having an edge crack of length a; $\alpha = 1.12$.

ii. An infinite plate having an embedded circular crack or a semi-circular crack each of radius a and lying in a plane normal to the applied stress; $\alpha = 0.64$.

iii. An infinite plate having an embedded elliptical crack of axes $2a$ and $2b$ or a semi-elliptical crack of width $2b$ in which the depth a is less than half the plate thickness, each lying in a plane normal to the applied stress; $\alpha = 1.12\Phi$ in which Φ varies with the ratio b/a as follows:

b/a	0	0.2	0.4	0.6	0.8
Φ	1.0	1.05	1.15	1.28	1.42

For $b/a = 1$ the situation is identical to case (ii).

iv. A plate of finite width w having a central crack of length $2a$ where $a \leq 0.3w$;

$$\alpha = [\sec(a\pi/w)]^{1/2}.$$

v. For a plate of finite width w having two symmetrical edge cracks each of depth $2a$, Eq. (8.7) becomes

$$K = S[w\tan(\pi a/w) + (0.1w)\sin(2\pi a/w)]^{1/2}$$

From Eq. (8.6) it can be seen that the stress concentration at a point ahead of a crack can be expressed in terms of the parameter K. Failure occurs when K reaches a critical value K_c. This is known as the *fracture toughness* of the material and has units $MN/m^{3/2}$ or $N/mm^{3/2}$.

EXAMPLE 8.5

An infinite plate has a fracture toughness of 3300 $N/mm^{3/2}$. If the plate contains an embedded circular crack of 3 mm radius calculate the maximum allowable stress that could be applied around the boundary of the plate.

In this case Eq. (8.7) applies with $\alpha = 0.64$. Then,

$$S = 3300/[(\pi \times 3)^{1/2} \times 0.64]$$

which gives

$$S = 1680\,\mathrm{N/mm^2}$$

EXAMPLE 8.6

If the steel plate of Ex. 8.5 develops an elliptical crack of length 6 mm and width 2.4 mm calculate the allowable stress that could be applied around the boundary of the plate.

In this case $b/a = 1.2/3 = 0.4$. Then $\alpha = 1.12 \times 1.15 = 1.164$ and from Eq. (8.7)

$$S = 3300/[(\pi \times 3)^{1/2} \times 1.164]$$

so that

$$S = 931.5\,\mathrm{N/mm^2}$$

EXAMPLE 8.7

Suppose that the plate of Ex. 8.5 has a finite width of 50 mm and develops a central crack of length 6 mm. What then is the allowable stress that could be applied around the boundary of the plate?

For this case

$$\alpha = [\sec(\pi a/w)]^{1/2} = [\sec(\pi \times 3/50)]^{1/2} = 1.018$$

so that

$$S = 3300/[(\pi \times 3)^{1/2} \times 1.018]$$

which gives

$$S = 1056\,\mathrm{N/mm^2}$$

Having obtained values of the stress concentration factor and the coefficient α, fatigue propagation rates may be estimated. From these the life of a structure containing cracks or crack-like defects may be determined. Alternatively the loading condition may be modified or inspection periods arranged so that the crack is detected before failure. The following summarises the results detailed in specialised texts on fracture mechanics.

Under constant amplitude loading the rate of crack propagation may be represented graphically by curves described in general terms by the law

$$(\mathrm{d}a/dN) = f(R, \Delta K) \tag{8.12}$$

in which ΔK is the stress concentration factor range and $R = S_{\min}/S_{\max}$. If Eq. (8.7) is used

$$\Delta K = (S_{\max} - S_{\min})(\pi a)^{1/2}\alpha \tag{8.13}$$

The curves represented by Eq. (8.12) may be divided into three regions. The first corresponding to a very slow crack growth rate ($<10^{-8}$ m/cycle) where the curves approach a threshold value of stress concentration factor ΔK^{th} corresponding to 4×10^{-11} m/cycle, that is, no crack growth. In the second region (10^{-8}–10^{-6} m/cycle) much of the crack life takes place and, for small ranges of ΔK, Eq. (8.12) may be represented by

$$(\mathrm{d}a/dN) = C(\Delta K)^n \tag{8.14}$$

in which C and n depend on the material properties; over small ranges of $\mathrm{d}a/\mathrm{dN}$ and ΔK, C and n remain approximately constant. The third region corresponds to crack growth rates $>10^{-6}$ m/cycle where instability and final failure occur.

An attempt has been made to describe the complete set of curves by the relationship

$$(\mathrm{d}a/dN) = C\,(\Delta K)^n/[(1-R)K_c - \Delta K] \tag{8.15}$$

in which K_c is the fracture toughness of the material obtained from toughness tests. Integration of Eq. (8.14) or (8.15) analytically or graphically gives an estimate of the crack growth life of the structure, that is the number of cycles required for a crack to grow from an initial size to an unacceptable length or the crack growth rate for failure whichever is the design criterion. Thus, for example, integration of Eq. (8.14) gives, for an infinite width plate for which $\alpha = 1.0$

$$[N]_{Ni}^{Nf} = \left[\left(a^{(1-n/2)}\right)/(1-n/2)\right]_{ai}^{af} / C[(S_{\max} - S_{\min})\pi^{1/2}]^n \tag{8.16}$$

and for which $n > 2$. An analytical integration may only be carried out if n is an integer and α is in the form of a polynomial otherwise graphical or numerical techniques must be employed. Substituting the limits in Eq. (8.16) and taking $N_i = 0$, the number of cycles to failure is given by

$$N_f = 2[(1/a_i{}^{(n-2)/2}) - (1/a_f{}^{(n-2)/2})]/\{C(n-2)[(S_{\max} - S_{\min})\pi^{1/2}]^n\} \tag{8.17}$$

EXAMPLE 8.8

An infinite plate contains a crack having an initial length of 0.2 mm and is subjected to a cyclic repeated stress range of 175 N/mm^2. If the fracture toughness of the plate is 1708 N/mm$^{3/2}$ and the rate of crack growth is 40×10^{-15} $(\Delta K)^4$ mm/cycle determine the number of cycles to failure.

The crack length at failure is given by Eq. (8.7) in which $\alpha = 1$, $K = 1708$ N/mm$^{3/2}$ and $S = 175$ N/mm^2. Then

$$a_f = 1708^2/(\pi\times175^2) = 30.3\,\mathrm{mm}$$

Also, it can be seen from Eq. (8.14) that $C = 40 \times 10^{-15}$ and $n = 4$. Substituting the relevant parameters in Eq. (8.17) gives

$$N_f = \{1/[40 \times 10^{-15}(175 \times \pi^{1/2})^4\}[(1/0.1) - (1/30.3)]$$

From which

$$N_f = 26919 \text{ cycles}$$

8.7 Design methods

In Section 8.3 we examined stress–strain curves for different materials and saw that, generally, there are two significant values of stress: the yield stress, σ_Y, and the ultimate stress, σ_{ult}. Either of these two stresses may be used as the basis of design which must ensure, of course, that a structure will adequately perform the role for which it is constructed. In any case the maximum stress in a structure should be kept below the elastic limit of the material otherwise a permanent set will result when the loads are applied and then removed.

Two design approaches are possible. The first, known as *elastic design*, uses either the yield stress (for ductile materials), or the ultimate stress (for brittle materials) and establishes a *working* or *allowable stress* within the elastic range of the material by applying a suitable factor of safety whose value depends upon a number of considerations. These include the type of material, the type of loading (fatigue loading would require a larger factor of safety than static loading which is obvious from Section 8.6) and the degree of complexity of the structure. Therefore for materials such as steel, the working stress, σ_w, is given by

$$\sigma_w = \frac{\sigma_Y}{n} \tag{8.18}$$

where n is the factor of safety, a typical value being 1.65. For a brittle material, such as concrete, the working stress would be given by

$$\sigma_w = \frac{\sigma_{ult}}{n} \tag{8.19}$$

in which n is of the order of 2.5.

Elastic design has been superseded for concrete by *limit state* or *ultimate load* design and for steel by *plastic design* (or limit, or ultimate load design). In this approach the structure is designed with a given factor of safety against complete collapse which is assumed to occur in a concrete structure when the stress reaches σ_{ult} and occurs in a steel structure when the stress at one or more points reaches σ_Y (see Section 9.10). In the design process working or actual loads are determined and then factored to give the required ultimate or collapse load of the structure. Knowing σ_{ult} (for concrete) or σ_Y (for steel) the appropriate section may then be chosen for the structural member.

The factors of safety used in ultimate load design depend upon several parameters. These may be grouped into those related to the material of the member and those related to loads. Thus in the ultimate load design of a reinforced concrete beam the values of σ_{ult} for concrete and σ_Y for the reinforcing steel are factored by *partial safety factors* to give *design strengths* that allow for variations of workmanship or quality of control in manufacture. Typical values for these partial safety factors are 1.5 for concrete and 1.15 for the reinforcement. Note that the design strength in both cases is less than the actual strength. In addition, as stated above, design loads are obtained in which the actual loads are increased by multiplying the latter by a partial safety factor which depends upon the type of load being considered.

As well as strength, structural members must possess sufficient stiffness, under normal working loads, to prevent deflections being excessive and thereby damaging adjacent parts of the structure. Another consideration related to deflection is the appearance of a structure which can be adversely affected if large deflections cause cracking of protective and/or decorative coverings. This is particularly critical in reinforced concrete beams where the concrete in the tension zone of the beam cracks; this does not affect the strength of the beam since the tensile stresses are withstood by the reinforcement. However, if deflections are large the crack widths will be proportionately large and the surface finish and protection afforded by the concrete to the reinforcement would be impaired.

Codes of Practice limit deflections of beams either by specifying maximum span/depth ratios or by fixing the maximum deflection in terms of the span. A typical limitation for a reinforced concrete beam is that the total deflection of the beam should not exceed span/250. An additional proviso is that the deflection that takes place after the construction of partitions and finishes should not exceed span/350 or 20 mm, whichever is the lesser. A typical value for a steel beam is span/360.

It is clear that the deflections of beams under normal working loads occur within the elastic range of the material of the beam no matter whether elastic or ultimate load theory has been used in their design. Deflections of beams, therefore, are checked using elastic analysis.

8.8 Material properties

Table 8.1 lists some typical properties of the more common engineering materials.

Table 8.1

Material	Density (kN/m^3)	Modulus of Elasticity, E (N/mm^2)	Shear Modulus, G (N/mm^2)	Yield Stress, σ_Y (N/mm^2)	Ultimate Stress, σ_{ult} (N/mm^2)	Poisson's Ratio ν
Aluminium alloy	27.0	70 000	40 000	290	440	0.33
Brass	82.5	103 000	41 000	103	276	
Bronze	87.0	103 000	45 000	138	345	
Cast iron	72.3	103 000	41 000		552 (compression) 138 (tension)	0.25
Concrete (medium strength)	22.8	21 400			20.7 (compression)	0.13
Copper	80.6	117000	41 000	245	345	
Steel (mild)	77.0	200 000	79 000	250	410–550	0.27
Steel (high carbon)	77.0	200 000	79 000	414	690	0.27
Prestressing wire		200 000			1570	
Timber						
softwood		7000			16	
hardwood	6.0	12000			30	
Composite (glass fibre)		20 000			250	

PROBLEMS

P.8.1. Describe a simple tensile test and show, with the aid of sketches, how measures of the ductility of the material of the specimen may be obtained. Sketch typical stress–strain curves for mild steel and an aluminium alloy showing their important features.

P.8.2. A bar of metal 25 mm in diameter is tested on a length of 250 mm. In tension the following results were recorded:

Table P.8.2(a)

Load (kN)	10.4	31.2	52.0	72.8
Extension (mm)	0.036	0.089	0.140	0.191

A torsion test gave the following results:

Table P.8.2(b)

Torque (kNm)	0.051	0.152	0.253	0.354
Angle of twist (degrees)	0.24	0.71	1.175	1.642

Represent these results in graphical form and hence determine Young's modulus, E, the modulus of rigidity, G, Poisson's ratio, ν, and the bulk modulus, K, for the metal.
(Note: see Chapter 11 for torque–angle of twist relationship).

Ans. $E \simeq 205\,000$ N/mm^2, $G \simeq 80\,700$ N/mm^2, $\nu \simeq 0.27$, $K \simeq 148\,500$ N/mm^2.

P.8.3. The actual stress–strain curve for a particular material is given by $\sigma = C\varepsilon^n$ where C is a constant. Assuming that the material suffers no change in volume during plastic deformation, derive an expression for the nominal stress–strain curve and show that this has a maximum value when $\varepsilon = n/(1-n)$.

Ans. σ (nominal) $= C\varepsilon^n/(1+\varepsilon)$.

P.8.4. A structural member is to be subjected to a series of cyclic loads which produce different levels of alternating stress as shown below. Determine whether or not a fatigue failure is probable.

Ans. Not probable $(n_1/N_1 + n_2/N_2 + \cdots = 0.39)$.

Table P.8.4

Loading	Number of Cycles	Number of Cycles to Failure
1	10^4	5×10^4
2	10^5	10^6
3	10^6	24×10^7
4	10^7	12×10^7

P.8.5. A material has a fatigue limit of ± 230 N/mm^2 and an ultimate tensile strength of 870 N/mm^2. If the safe range of stress is determined by the Goodman prediction calculate its value.

Ans. 363 N/mm^2.

P.8.6. A more accurate estimate for the safe range of stress for the material of P.8.5 is given by the Gerber prediction. Calculate its value.

Ans. 432 N/mm^2.

P.8.7. A steel component is subjected to a reversed cyclic loading of 100 cycles/day over a period of time in which ± 160 N/mm^2 is applied for 200 cycles, ± 140 N/mm^2 is applied for 200 cycles and ± 100 N/mm^2 is applied for 600 cycles. If the fatigue life of the material of the component at each of these stress levels is 10^4, 10^5 and 2×10^5 cycles respectively, estimate the life of the component using Miner's law.

Ans. 400 days.

P.8.8. An infinite steel plate has a fracture toughness 3320 $N/mm^{3/2}$ and contains a 4 mm long crack. Calculate the maximum allowable design stress that could be applied around the boundary of the plate.

Ans. 1324 N/mm^2.

P.8.9. A semi-infinite plate has an edge crack of length 0.4 mm. If the plate is subjected to a cyclic repeated stress loading of 180 N/mm^2, its fracture toughness is 1800 $N/mm^{3/2}$ and the rate of crack growth is 30×10^{-15} $(\Delta K)^4$ mm/cycle determine the crack length at failure and the number of cycles to failure.

Ans. 25.4 mm, 7916 cycles.

P.8.10. A steel plate 50 mm wide is 5 mm thick and carries an in-plane bending moment. If the plate develops an elliptical crack of length 6 mm and width 2.4 mm calculate the maximum bending moment the plate can withstand if the fracture toughness of the steel is 3500 $N/mm^{3/2}$.

Ans. 4300 Nmm.