

# AVDASI 3

(CADE 30007)

## Gas Turbine Propulsion

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Week 16: Lecture 4

Off-Design Conditions  
(2 of 2)

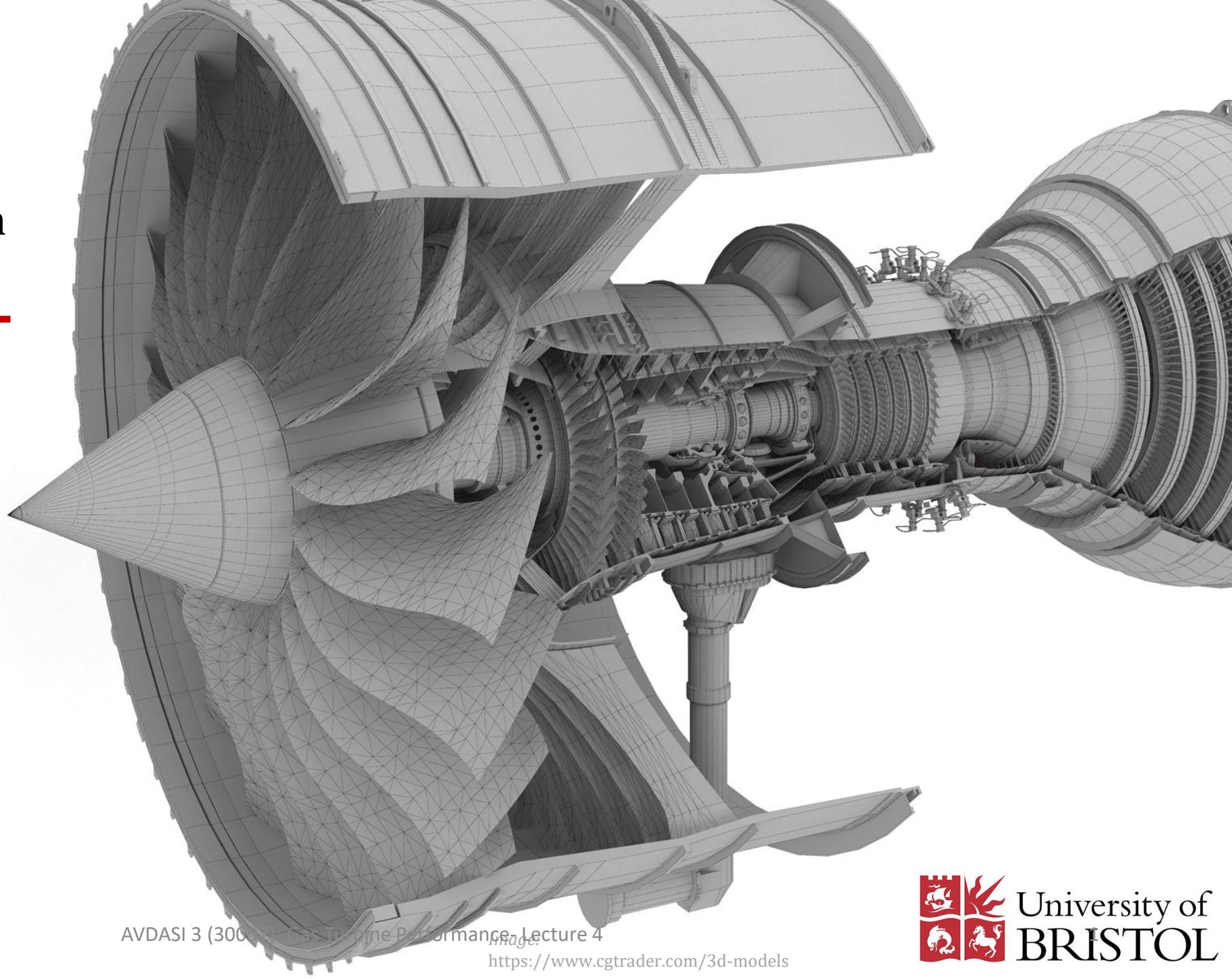
**Lecturer:**

Dr Daniele Zagaglia

[daniele.zagaglia@bristol.ac.uk](mailto:daniele.zagaglia@bristol.ac.uk)

**Unit Director:**

Dr. Samudra Dasgupta



## Objective ~ Lecture 4

*To show how the detailed performance of a propulsion system can be analysed.*

- **The Design Point of an Engine**

- Where all of the components are matched at their *Design Conditions*.
- Design Conditions: where each unit achieves design pressure ratio & peak efficiency at maximum flow.
- Also sometimes called *Synthesis Matching Point*
- The Altitude & Mach Number conditions are usually those which are critical for the Aircraft/Engine Requirements.

# Design Point & Off-Design Performance

- **Subsonic Passenger Aircraft:**

- Typically the Design Point will be at the top of Climb.
- $M = 0.8$ ,  $h = 35,000$  ft.  $T_1 = 240K$ .

- **Military Aircraft:**

- The Design Point will be at the critical condition for manoeuvring in Combat.
- $M = 1.8$ ,  $h = 40,000$  ft.  $T_1 = 357K$ .

- **Helicopter:**

- The Design Point will be at hovering at sea level.
- $M = 0$ ,  $h = \text{at Sea level}$   $T_1 = 288 K$ .
- All other conditions are ***Off-Design***, where the pressure ratio, efficiency and flow are different from those at the design point.

# Dimensional Relationships

Group	Non-Dimensional	Quasi-dimensionless
1. Rotational Speed	$\frac{N \cdot D}{\sqrt{R \cdot T_o}}$	$\frac{N}{\sqrt{T_o}}$
2. Mass Flow	$\frac{\dot{m} \sqrt{R \cdot T_o}}{A \cdot P_o}$	$\frac{\dot{m} \sqrt{T_o}}{P_o}$
3. Flow Velocity	$\frac{C}{\sqrt{\gamma R T}} (M)$	$\frac{C}{\sqrt{T}}$

Nomenclature:  $T_o$  Total Temperature,  $T$  Static Temperature

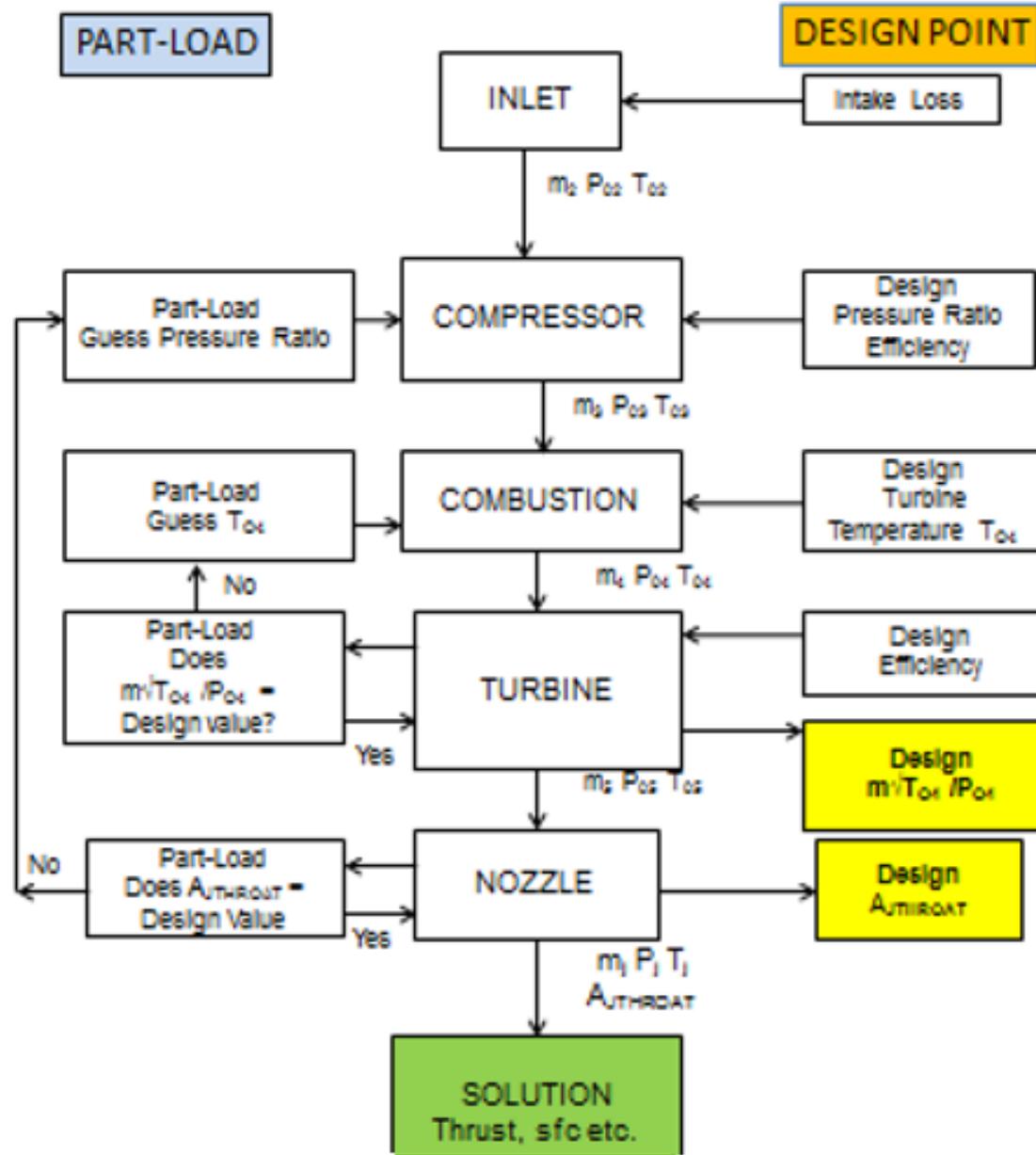
$\dot{m}$  Mass flow

$C$  Flow velocity

$M$  Mach Number

$D$  a “reference” dimension

$N$  Rotational Speed



# Flow chart for simple design point & part-load performance calculation

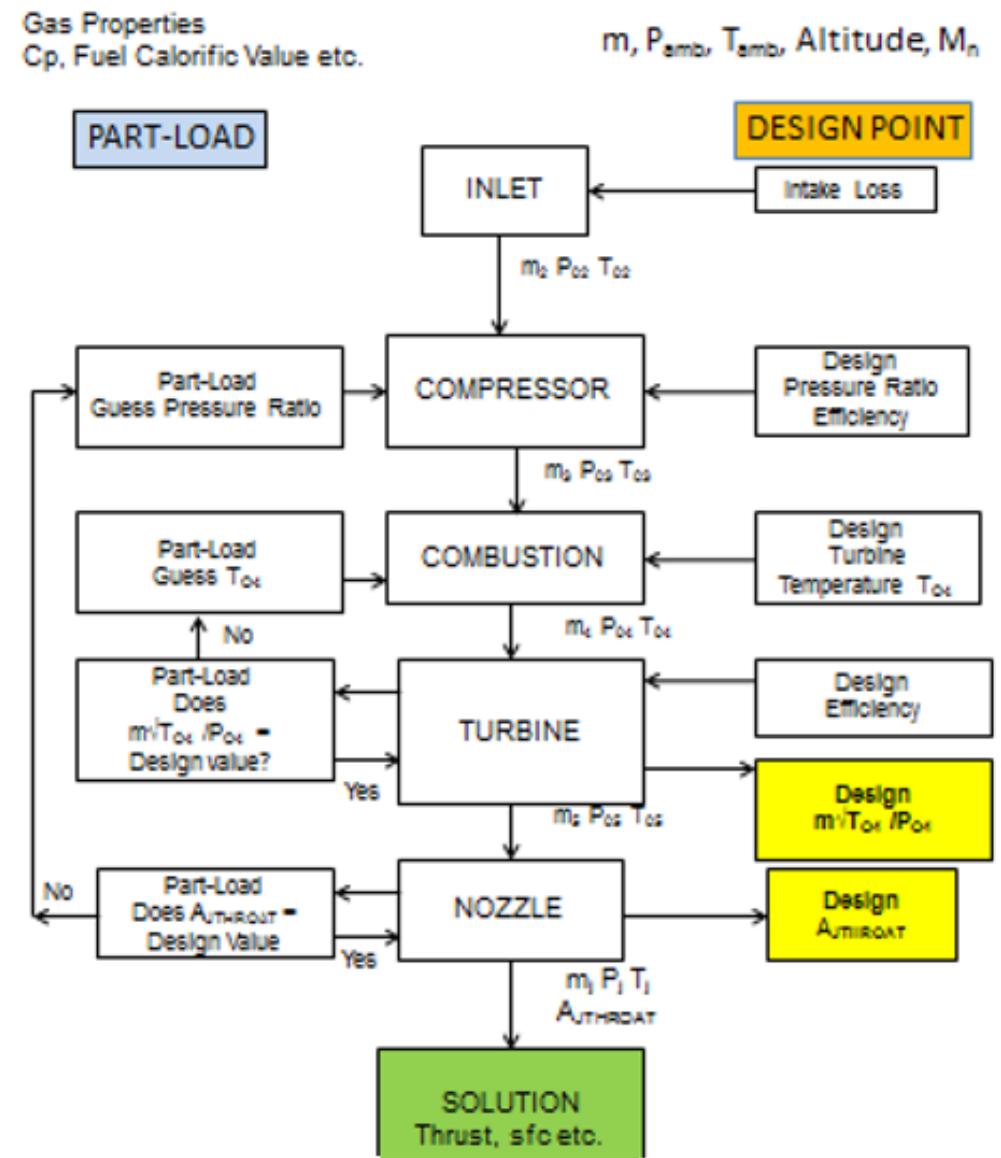
The Design point fixes:

- $\frac{m_1 \sqrt{T_{0,4}}}{P_{0,4}}$
- $A_{THROAT}$

# Part-load iteration procedure

- Iteration carried out on  $P_{03}/P_{02}$  and combustor temperature  $T_{04}$  until the turbine "swallowing capacity" i.e. non-dimensional  $\frac{\dot{m}_4 \sqrt{T_{04}}}{P_{04}}$  and nozzle throat area are the same as the engine design point values.
- From first guess, the combustor temperature  $T_{04}$  is varied at constant compressor delivery pressure ratio until the design value of  $\frac{\dot{m}_4 \sqrt{T_{04}}}{P_{04}}$  is obtained.
- This is repeated for another value of compression ratio and so on until the final solution is reached.

Note: *The first guess would be valid if the engine could be designed with a variable turbine throat area and a variable nozzle throat area and provided that there is sufficient compressor surge margin.*



# Turbojet Design Point example

Turbojet Design Point ~Pressure ratio 10:1

*Design Point M = 0.8, 20000 ft*

## Cycle parameters:

- **Inlet**

- Airflow 100kg/s
- Total pressure loss 5%

- **Compressor**

- Pressure Ratio 10:1
- Efficiency (isentropic) 89%

- **Combustor**

- T4 1200K
- Pressure loss 8%

- **Turbine**

- Efficiency (isentropic) 86%
- *No nozzle or transmission losses*

# Turbojet Design Point example

## Design Point & Part-load Performance Calculation

Single Spool Turbojet M = 0.8 20,000 ft

	$\dot{m}$	kg/s	Design	First	Final
			Point	Guess	Iteration
Inlet Airflow	$\dot{m}$	kg/s	100	70	70
Ambient Temperature	$T_a$	kg/s	248.5	248.5	248.5
Ambient Pressure	$p_a$	kPa	46.6	46.6	46.6
Speed of Sound		m/s	316.2	316.2	316.2
Compressor Pressure Ratio			10:01	7:1	5.86:1
Combustion Temperature	$T_{04}$	K	1200	1000	855
INLET & AIR INTAKE	$T_{01}$	K	280.3	280.3	280.3
	$P_{01}$	kPa	71	71	71
	$C_0$	m/s	252.6	252.6	252.6
	$T_{02}$	K	280.3	280.3	280.3
	$P_{02}$	kPa	67.4	67.4	67.4
COMPRESSOR	$\frac{\dot{m}_2 \sqrt{T_{02}}}{P_{02}}$		24.83	17.38	17.38
	$P_{03}$	kPa	674.2	472	399.1
	$T_{03}$	K	573.4	514.6	487.4
	Power	kW	29457	16479	14565
COMBUSTION	$T_{04}$	K	1200	1000	855
	Fuel/Air Ratio		0.0173	0.0128	0.0093
	Fuel Flow	kg/s	1.73	0.896	0.651
	$P_{04}$	kPa	620.3	434.2	365.5

# Turbojet Design Point example

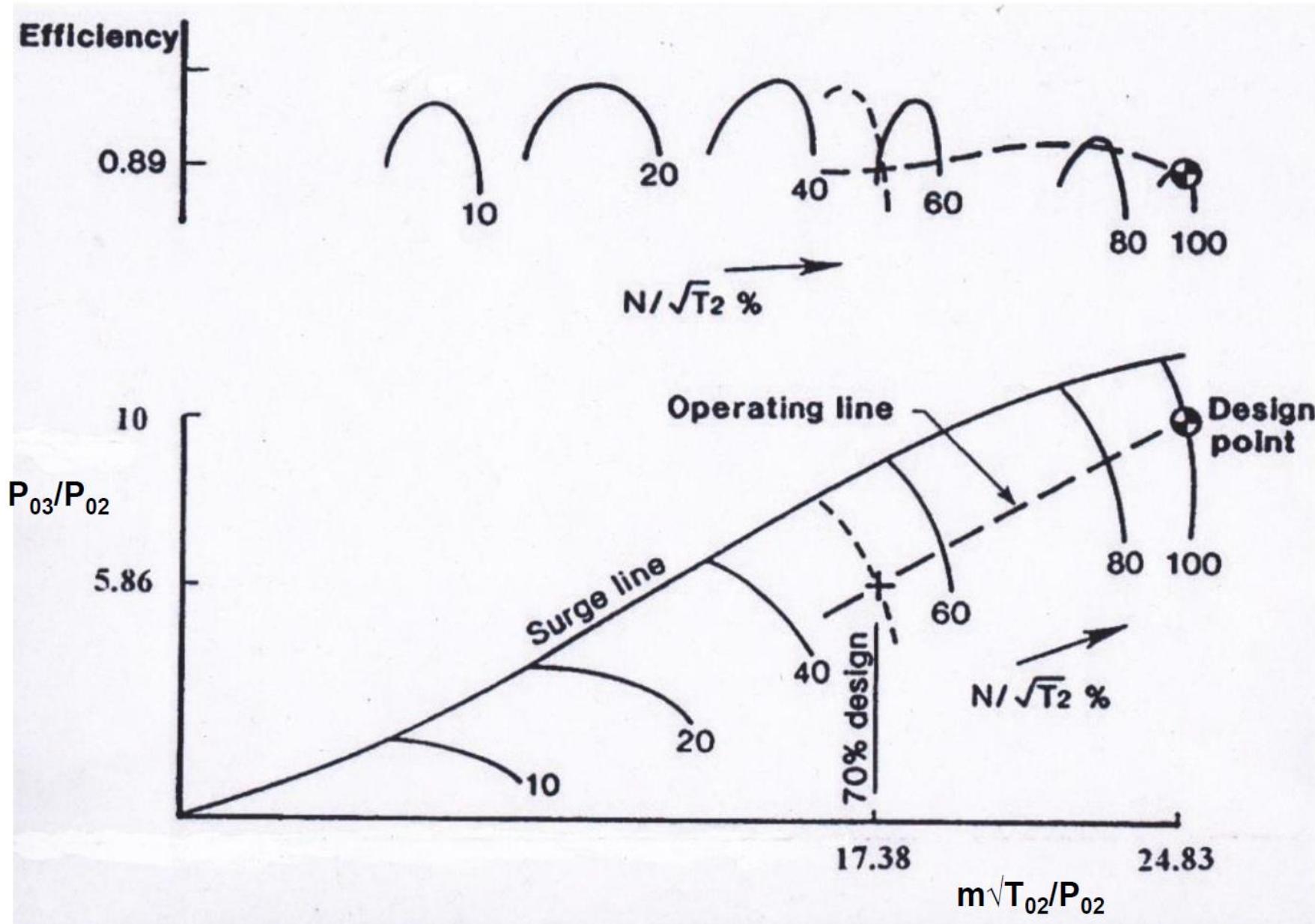
## Design Point & Part-load Performance Calculation

Sheet 2

**Single Spool Turbojet @ M = 0.8, 20,000ft**

			Design Point	First Guess	Final Iteration
<b>TURBINE</b>					
<i>Design Value</i>	$\frac{\dot{m}_4 \sqrt{T_{o4}}}{P_{o4}}$		5.681	5.163	5.683
	T <sub>05</sub>	K	947.5	797.5	675.3
	P <sub>05</sub>	kPa	202.2	148.2	118.5
<b>CONVERGENT NOZZLE</b>	$P_{05}/P^*$		4.336	3.184	2.544
	$P_{05}/P_{oN^*}$		1.853	1.853	1.853
	P <sub>N*</sub>	kPa	109	80.1	64
	T <sub>N*</sub>	K	812.3	683.6	578.9
	C <sub>N</sub>	m/s	557.5	511.5	470.7
<i>Design Value</i>	A <sub>JTHROAT</sub>	m <sup>2</sup>	0.39	0.34	0.39
<b>PERFORMANCE</b>	$\dot{m}C_N$	kN	56.7	-	33.2
<b>(Convergent Nozzle)</b>	A <sub>J</sub> (P <sub>N*</sub> - P <sub>a</sub> )	kN	24.4	-	6.8
	Gross Thrust	kN	81.1	-	40
	Mom. Drag	kN	25.3	-	17.7
	Nett Thrust	kN	55.8	-	22.3
	SFC	kg/hr/N	0.112	-	0.105
<b>CON-DI NOZZLE</b>	P <sub>05</sub> /P <sup>*</sup>		4.336	-	2.544
<b>Fully Expanded Temperature</b>	T <sub>NFE</sub>	K	656.6	-	534.7
<b>Fully Expanded Velocity</b>	C <sub>JFE</sub>	m/s	817.3	-	568.2
<b>Fully Expanded Area</b>	A <sub>JFE</sub>	m <sup>2</sup>	0.504	-	0.41
<b>Ideal Gross Thrust</b>		kN	83.1	-	40.1

# Typical Compressor Operating Line



# Turbojet Design Point example

## Turbojet Design Point ~Pressure ratio 20:1

*Design Point: Sea level static ISA*

### Inlet

Airflow 100kg/s

Inlet Pressure recovery MIL-E-5007D

### Compressor

Pressure Ratio 20:1

Efficiency (isentropic) 80%

### Combustor

Inlet Temperature  $T_{03}$  758K

Outlet Temperature  $T_{04}$  1500K

Pressure loss 8.6%

### Turbine

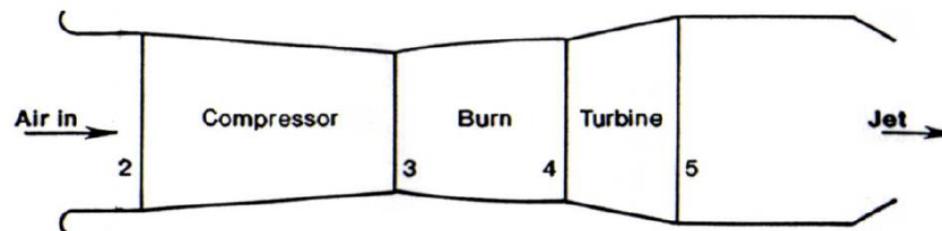
Expansion ratio  $P_{04}/P_{05}$  6.1:1

Efficiency(isentropic) 86%

Cooling bleeds (% of compressor flow) 19%

### Nozzle

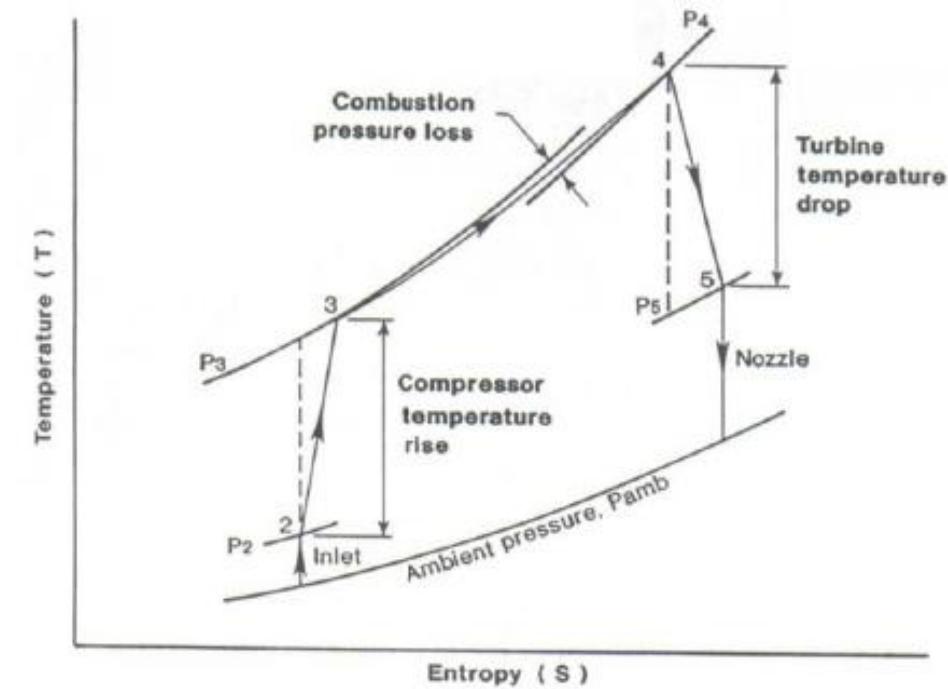
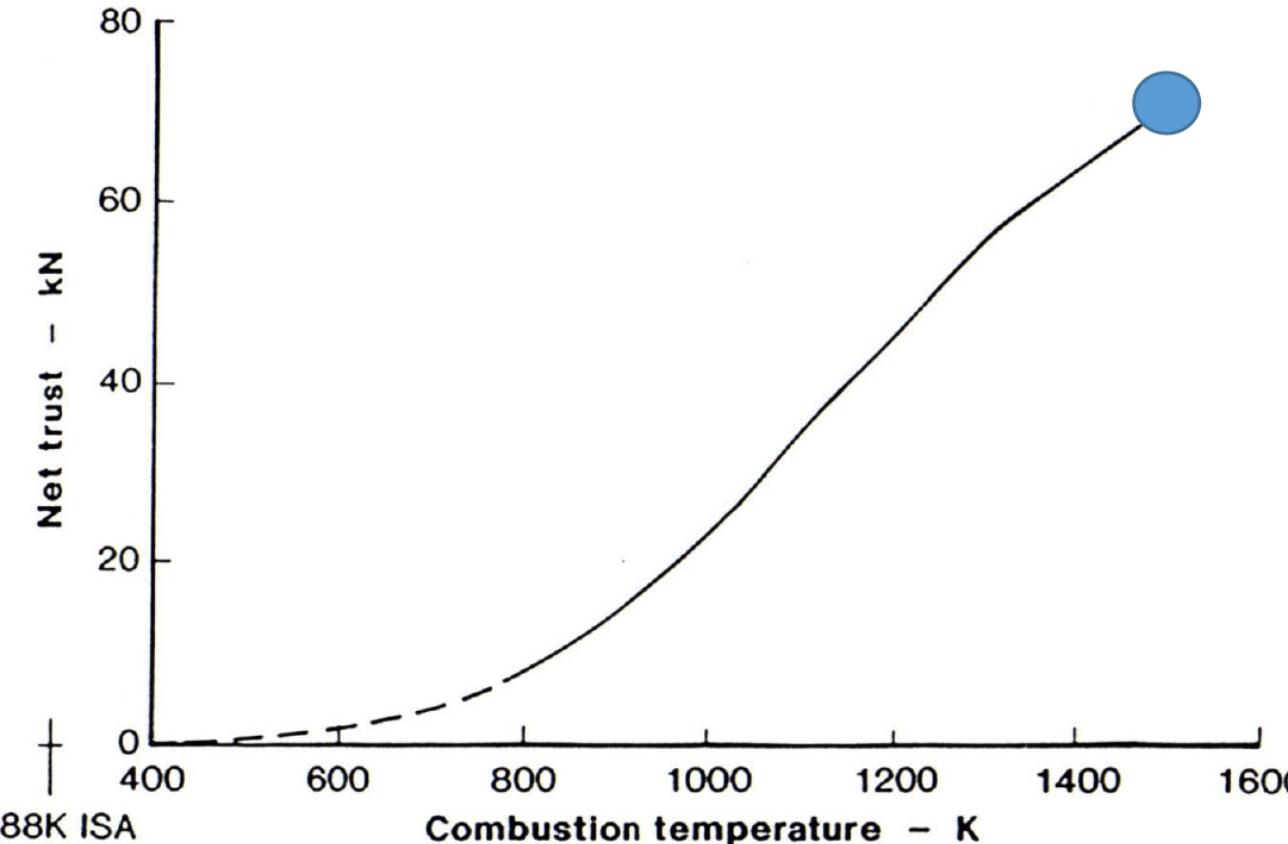
Thrust coefficient 0.96



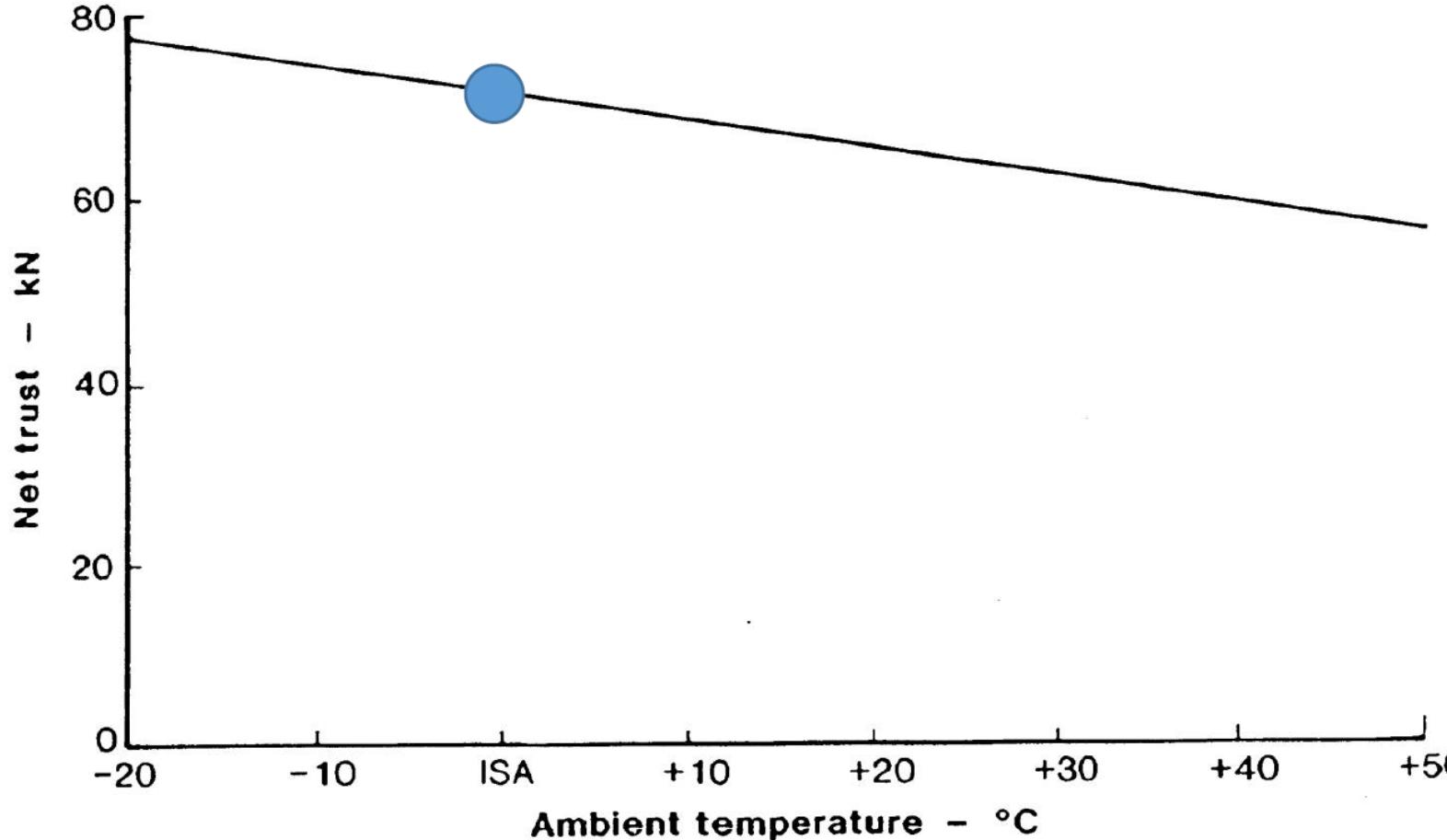
$Thrust = 71.7 \text{ kN}$
$Fuel \text{ flow} = 1.76 \text{ kg/s}$
$SFC = 0.0245 \text{ kg/s/kN}$

# Turbojet Thrust variation with Combustion Temperature

Sea level static, ISA  
MIL-E-5007D Intake pressure recovery



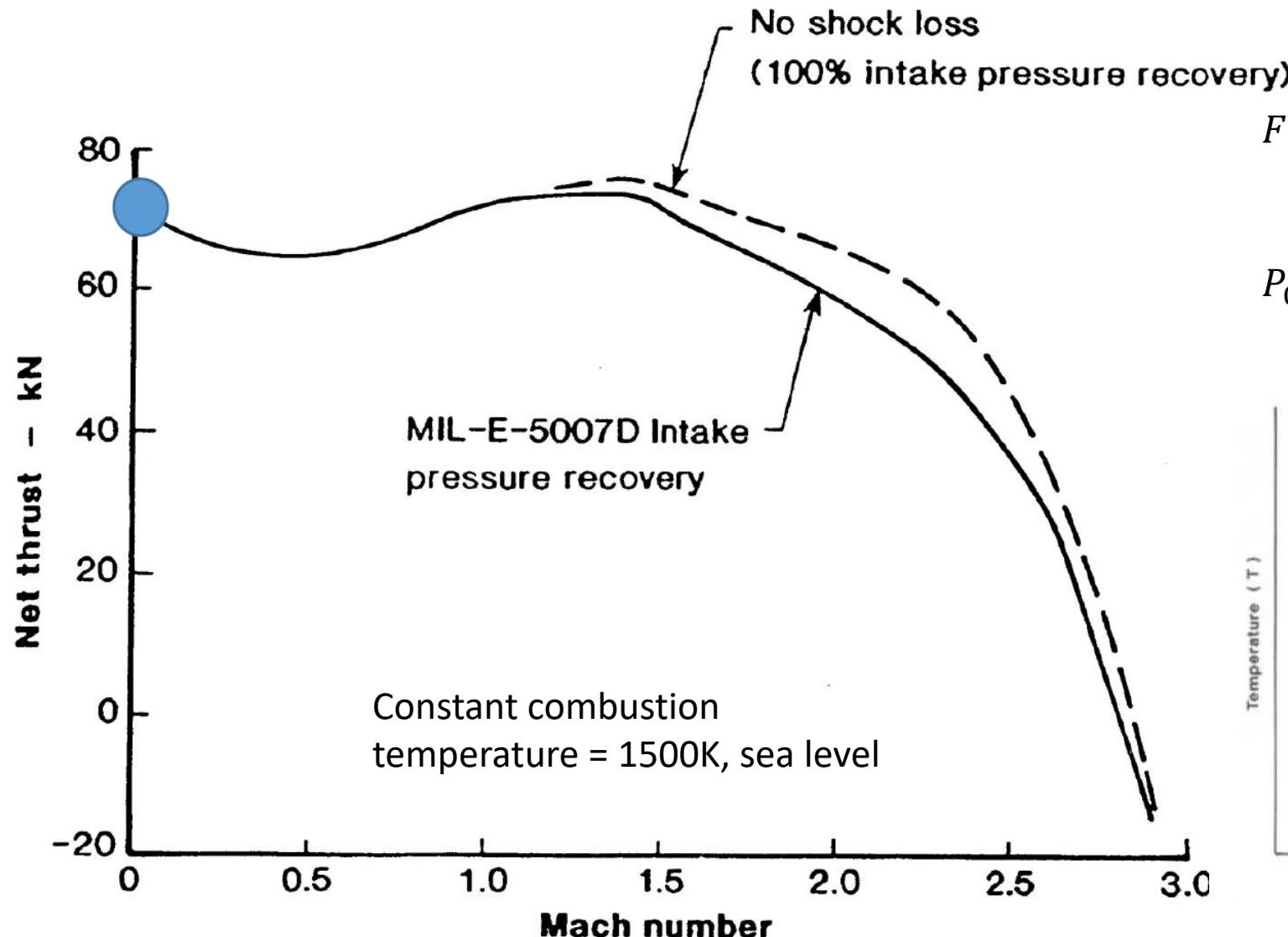
# Turbojet Thrust variation with Ambient Temperature



- If  $T_a$  increases:
- $\rho_a$  decreases, less  $\dot{m} = \rho_a A C_a$

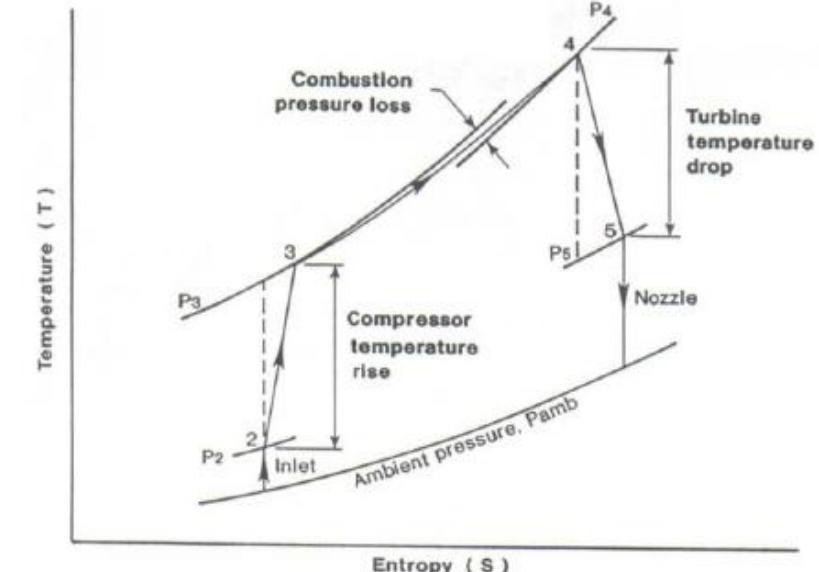
Constant combustion temperature = 1500K  
MIL-E-5007D Intake pressure recovery

# Turbojet Thrust variation with Mach number

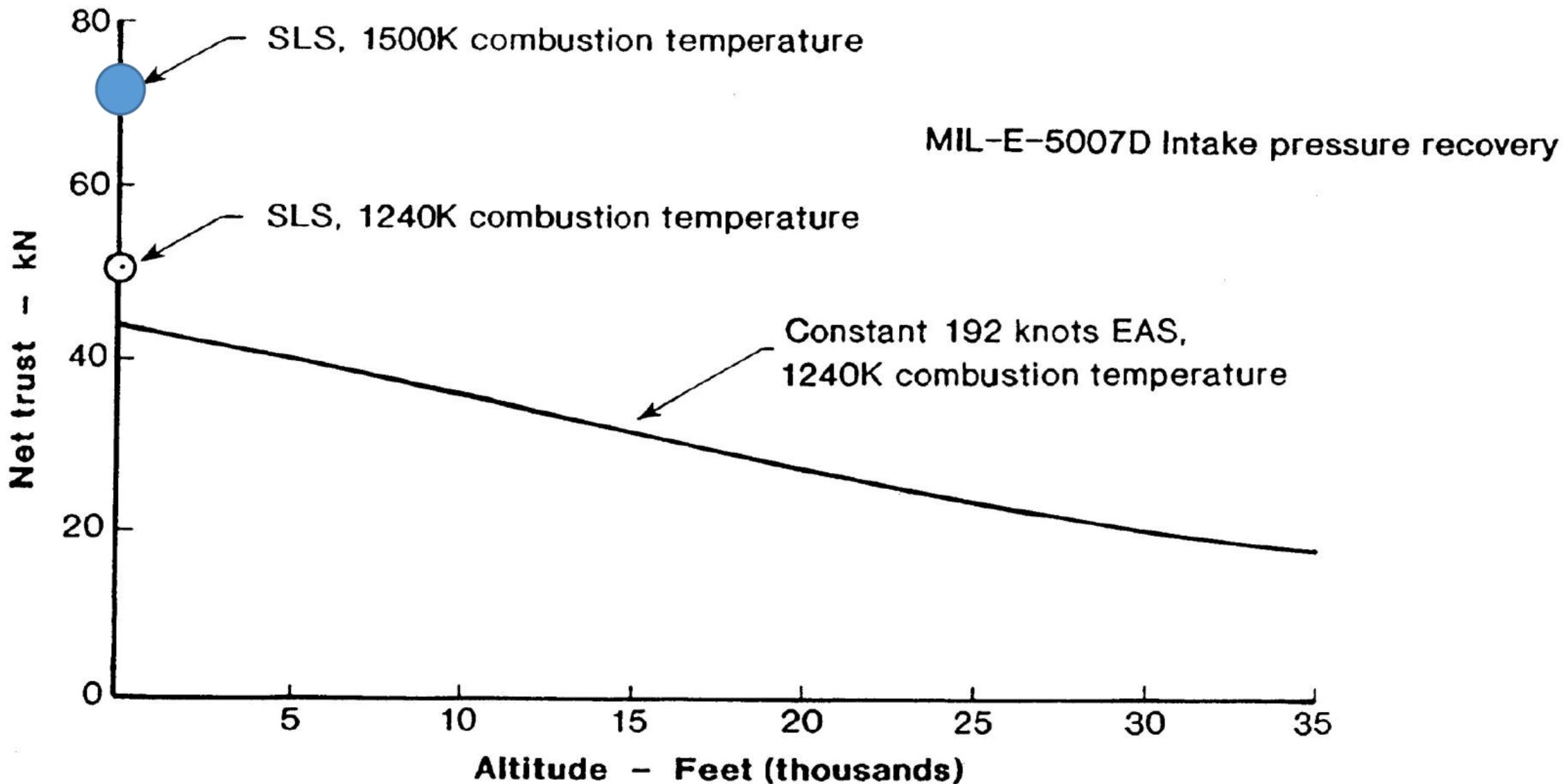


$$F = \dot{m}(C_J - C_a) + A_j(P_j - P_a)$$

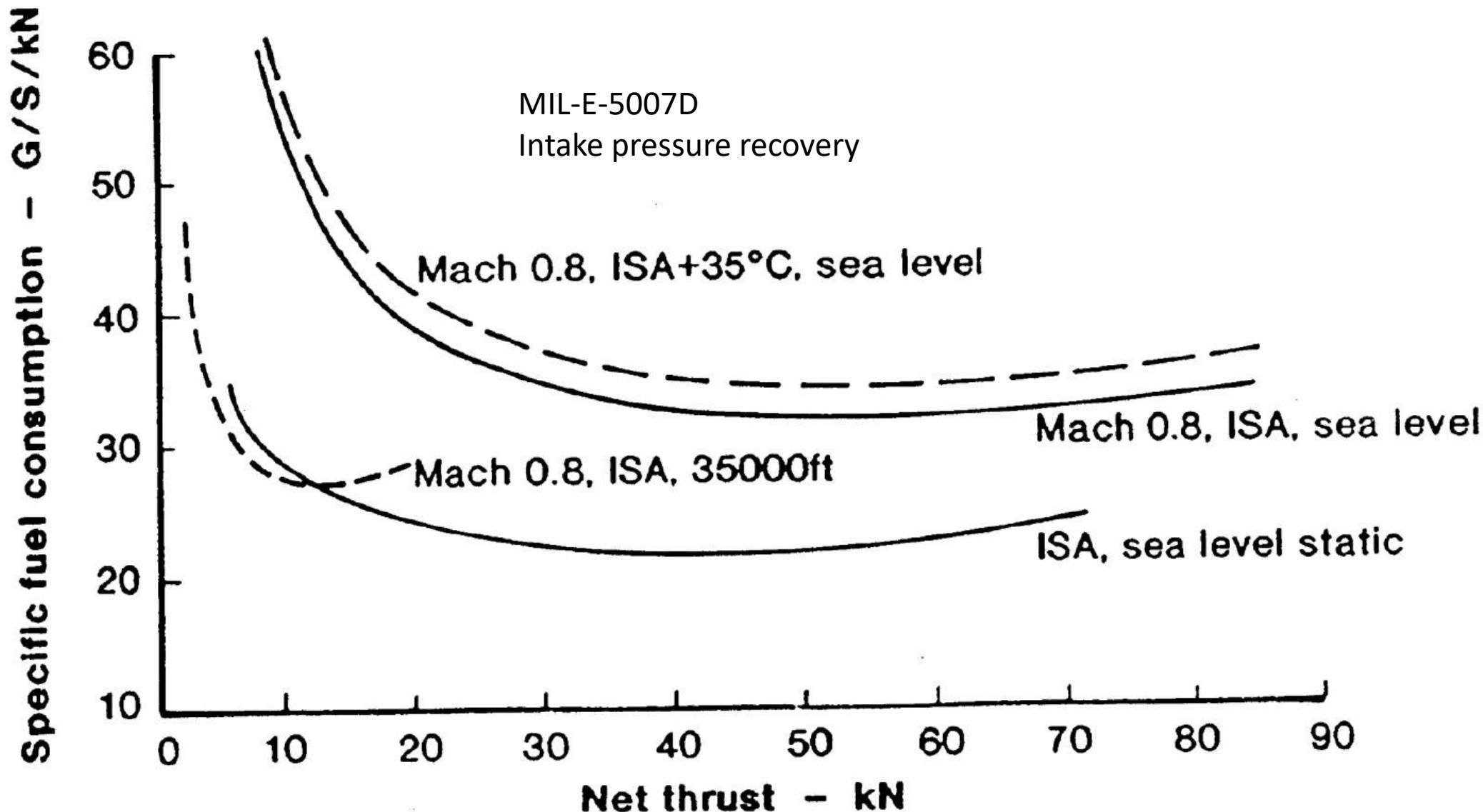
$$P_0 = P \left( 1 + \frac{\gamma - 1}{2} M^2 \right)^{\frac{\gamma}{\gamma - 1}}$$



# Turbojet Climb Thrust Characteristics

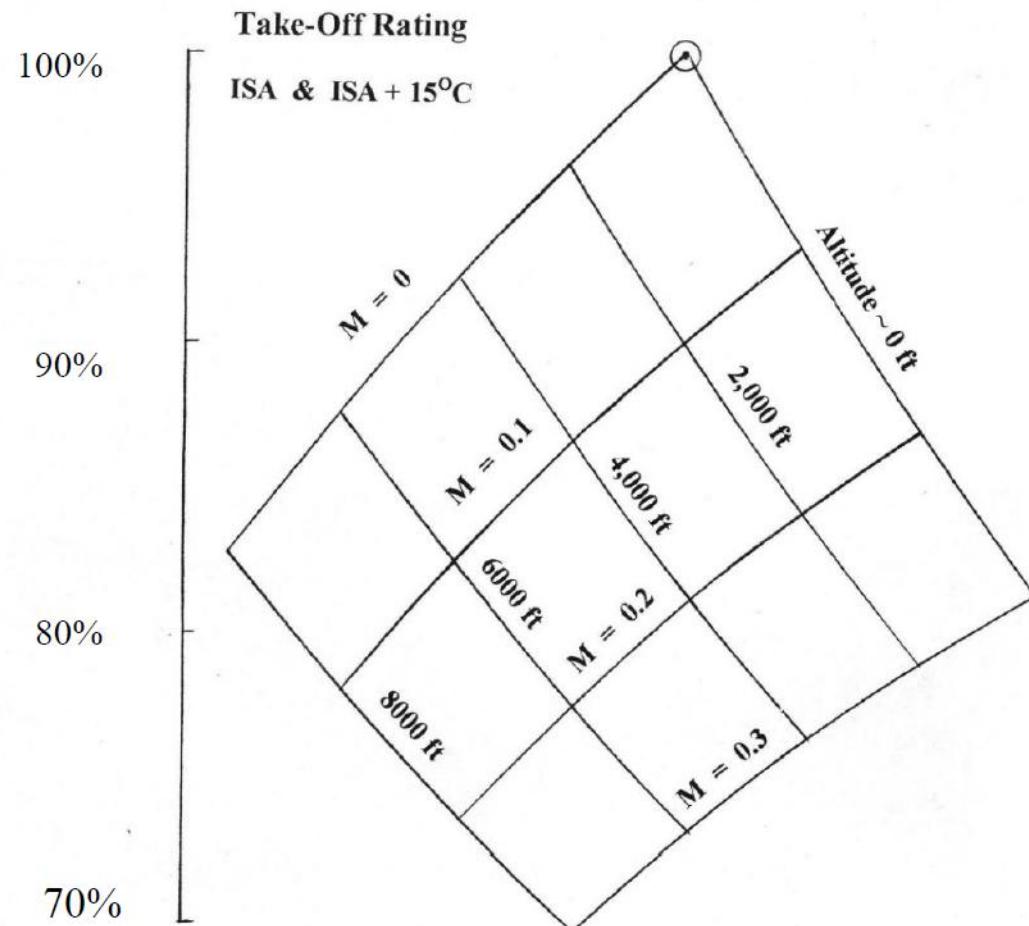


# Turbojet Fuel Consumption Characteristics

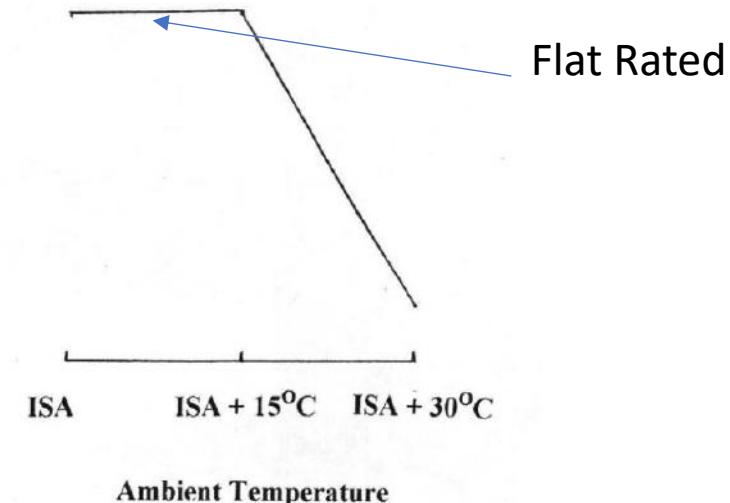


# Typical engine data

## *Subsonic Transport Aircraft ~ Take-off*



*Typical High By-pass Ratio Turbofan  
Take-off performance*



# Overall Performance Characteristics

The major points of interest are:

- As combustion temperature and spool speed increase at fixed atmospheric conditions, the thrust increase is non-linear – the thrust increases rapidly at the higher values of combustion temperature & spool speed. The gas turbine has a different characteristic to that of the reciprocating engine.
- At a fixed combustion temperature, thrust falls almost linearly with atmospheric conditions due to falling density and quantity of inlet airflow.
- Initially thrust decreases with forward speed (at fixed atmospheric conditions & combustor temperature). This is due to increasing momentum drag. Subsequently thrust rises as inlet pressure rises giving higher values of jet pipe pressure & jet velocity. Finally thrust falls as compressor delivery temperature rises towards the combustor temperature.

# Key take-aways of Lecture 4

- Design Points for different types of platforms
- Fundamental Dimensionless relationships
- The calculation of Off-design Performance
- How thrust & fuel consumption varies with inlet conditions i.e. Altitude, Mach Number & throttle setting

# What's on next week?

- How the airframe affects the engine and how the engine affects the airframe
- Concept of “standard” thrust drag
- Examine the issues arising from installing the propulsion system into an aircraft