

Propellers and Ducted Fans

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Room 2.40 QB



Content of Lectures

- Introduction to propellers and ducted fans
- Aerodynamics aspects of propellers and ducted fans
- Propeller coefficients and performance charts.
- Propeller design
- Comparison between propellers and ducted fans
- Revision (typical exam question)

Notes in Blackboard: <https://www.ole.bris.ac.uk>



Introduction to Propellers (Air Screws)

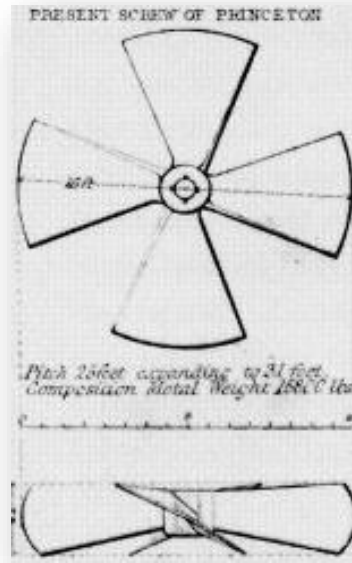
Lecture 1

The Applications (and a bit of history)

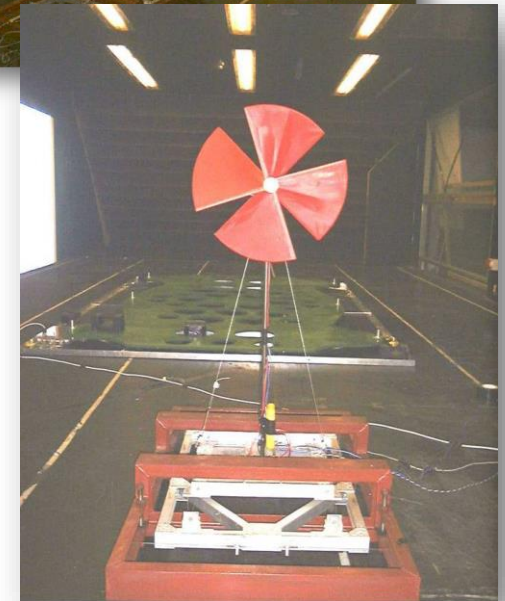
The first propellers operated in water, not air. This is the “prop” of the first propeller driven steam ship –

Brunel’s

“S.S.Great Britain”



The propeller of **Stringfellow’s “Flyer”** model aircraft of 1848 is very similar in form, as can be seen from this replica on test in the Bristol University wind tunnel in 2003.

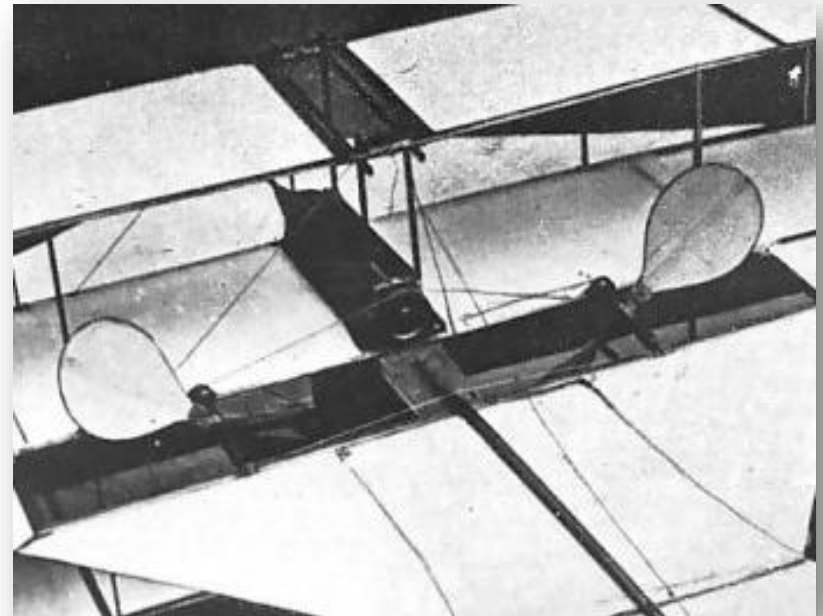


The Applications (and a bit of history)

Early Propellers – *Some modest development !*



1848

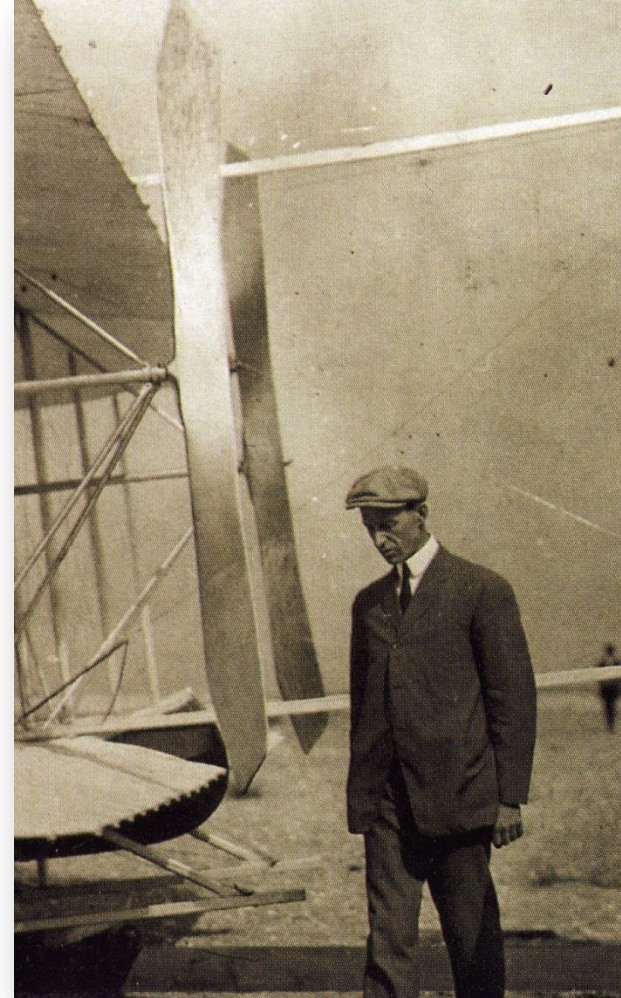


1868

The Applications (and a bit of history)

For the first powered manned aircraft, the 1903 **Wright Flyer** was fitted with two propellers. It can be seen that the propeller is of a higher aspect ratio, setting the trend for all the propeller designs that were to follow.

(This is a “fixed pitch” propeller - being formed from one single piece of wood)



The Applications (and a bit of history)

This is also a “fixed pitch” propeller, manufactured from a single piece of aluminium, but here the similarity with the Wright Flyer ends.

This Propeller absorbs all the 2600-hp from the Rolls-Royce supercharged R Engine to take this 1931 Supermarine S6b Schneider Cup winner to 400mph.



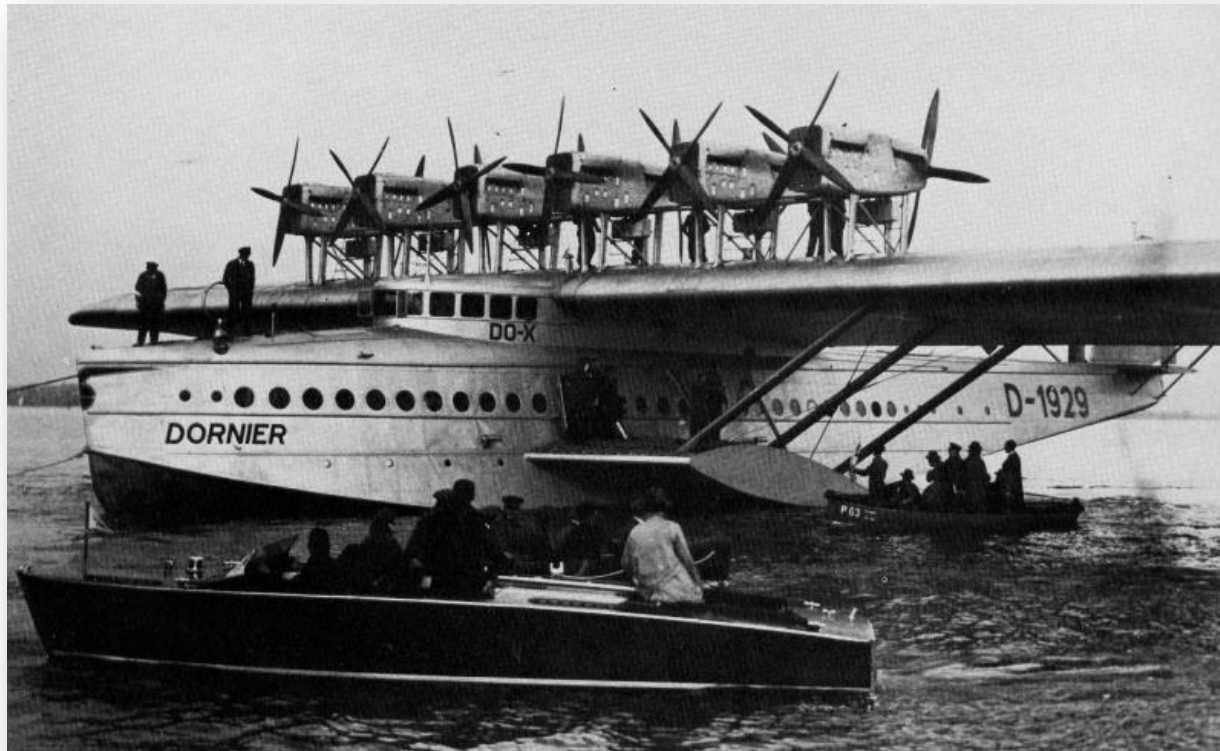
The Applications (and a bit of history)

Schneider successes led to the most famous single propeller aircraft, the **Supermarine “Spitfire”**.

The high performance was a result of a powerful water cooled engine (with low frontal area), an efficient propeller and a very low airframe drag coefficient.



The Applications (and a bit of history)



Not all aircraft were so blessed – this Dornier DO-X with 12 Curtiss Conqueror powered propellers took longer than Christopher Columbus to cross the Atlantic on it's first scheduled flight in 1931 - the same year as S6b won the Schneider Cup.

The Applications (and a bit of history)

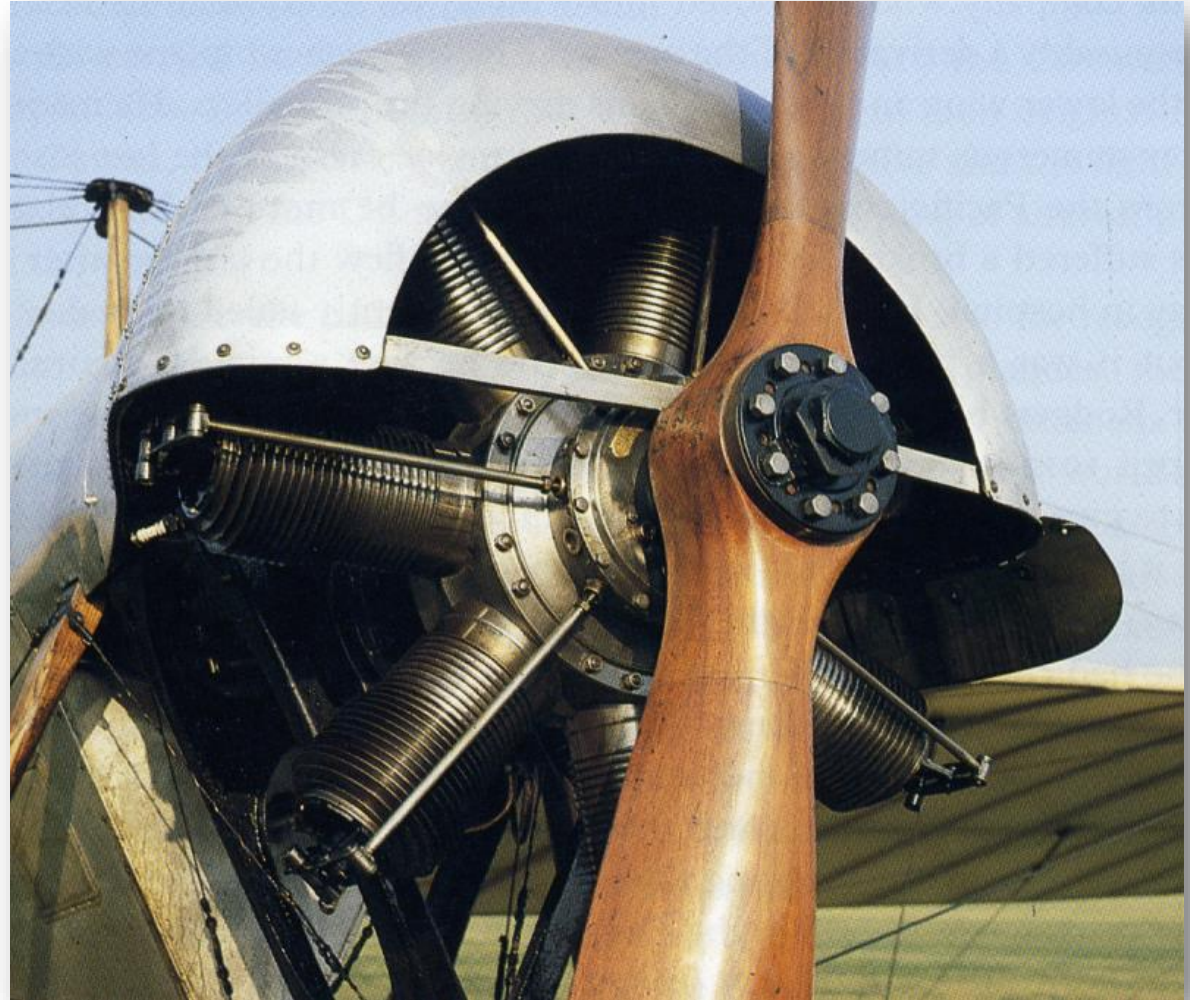


Of course the propeller will also function as a fan when the aircraft is stationary. The large air-cooled radial engine in this 1932 GeeBee racer would benefit from the pre-flight cooling afforded by the propeller.

The Applications (and a bit of history)

A novel solution to engine cooling was the **“Rotary Engine”**

The propeller was attached to the engine casing (and cylinders) and the non-rotating crankshaft was affixed to the frame



The Applications (and a bit of history)



Not all propellers are so obviously installed. This Edgley Optica has an enclosed propeller system, often referred to as a ducted propeller or fan.

The Applications (and a bit of history)



Here again the propeller system is not always visible. This is a “get you home” device for a sailplane should the thermals become elusive.

The Applications (and a bit of history)

There are still some new
thoughts on propellers!



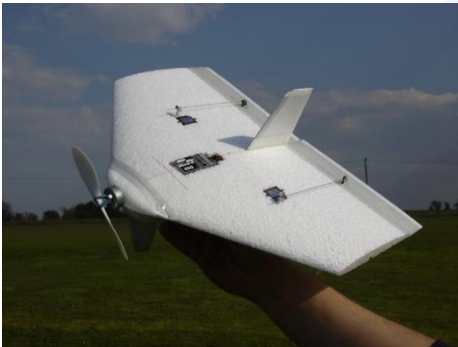
The Applications (and a bit of history)

Open Rotor



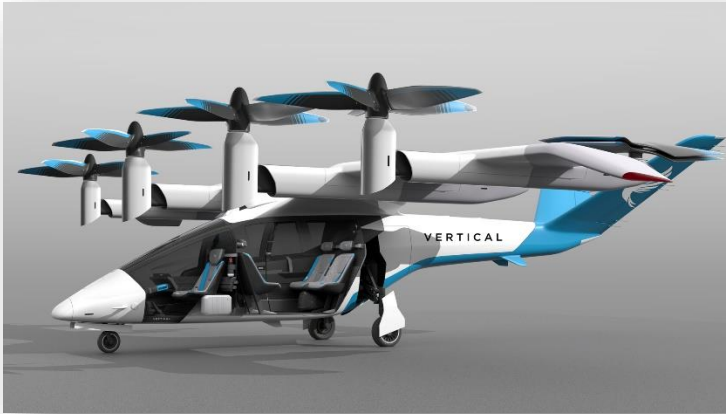
The Applications (and a bit of history)

UAVs and MAVs



The Applications (and a bit of history)

Urban Air Mobility



The Analysis

This is normally achieved through combinations of

Momentum Theory



Blade Element
Analysis

OR

Vortex Lifting Line Theory



Blade Element
Analysis

MOMENTUM THEORY

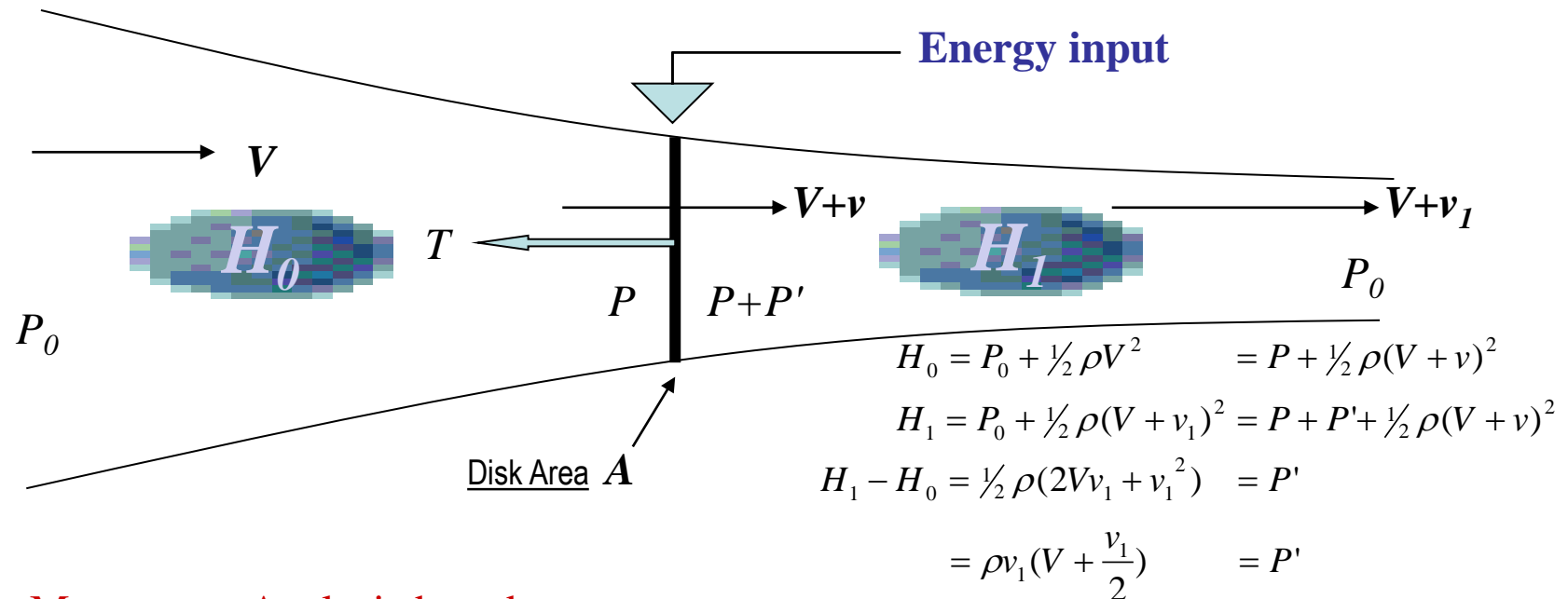


Actuator Disc Theorem

The Propeller or Axial Fan .

Mechanical energy (in the form of rotating blades) is used to accelerate (***a***) a mass (***m***) of air.

Newton's law (every action has a reaction), states ***F = ma***, where ***F***, is the propeller thrust (***T***).



**Momentum Analysis based on
Actuator Disk Theorem**

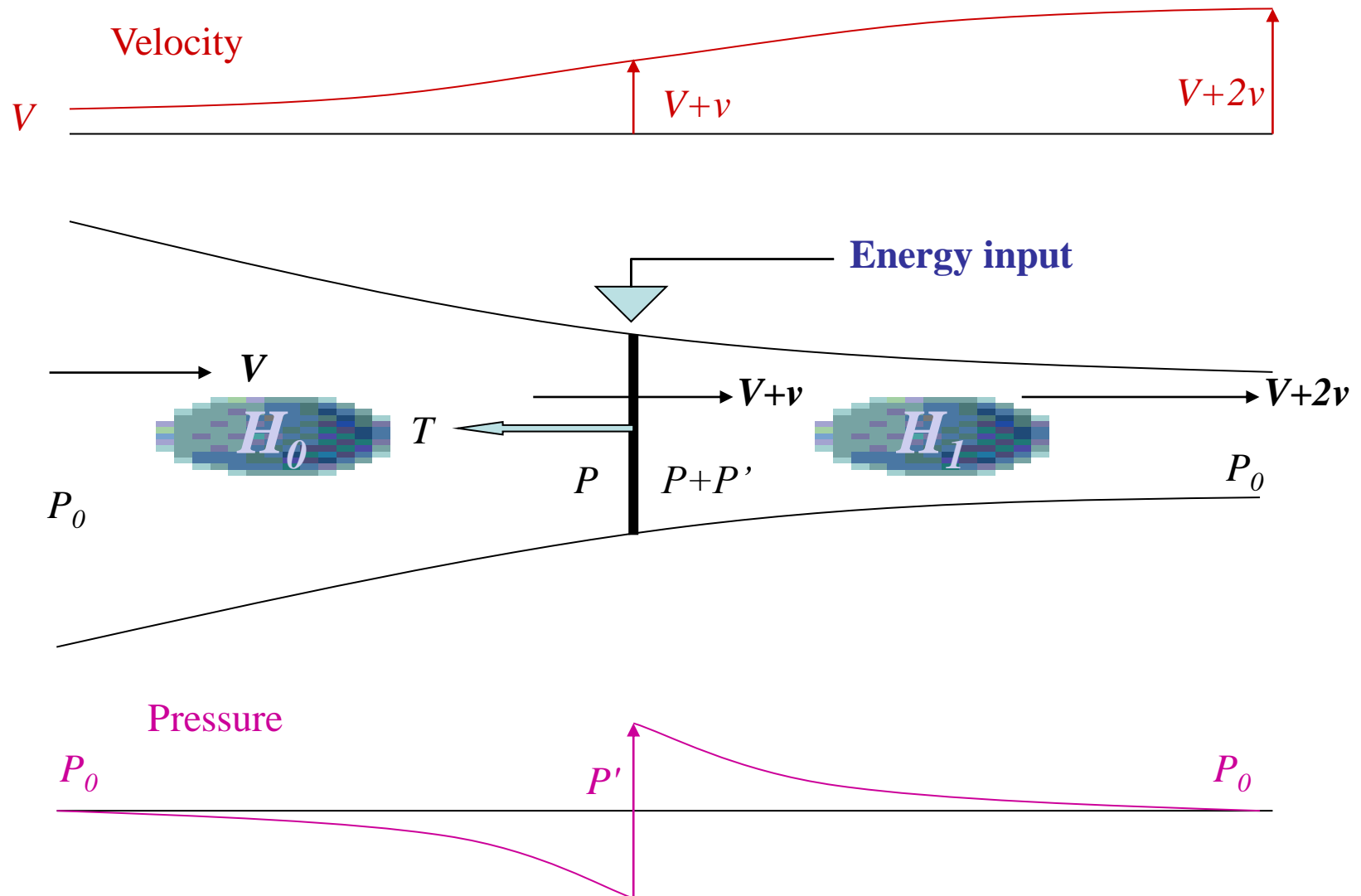
$$\text{Now, } T = \rho A (V + v) v_1, \quad \frac{T}{A} = \rho v_1 (V + v)$$

ρ Air density
 A Disc area

$$\text{so } \frac{v_1}{2} = v$$

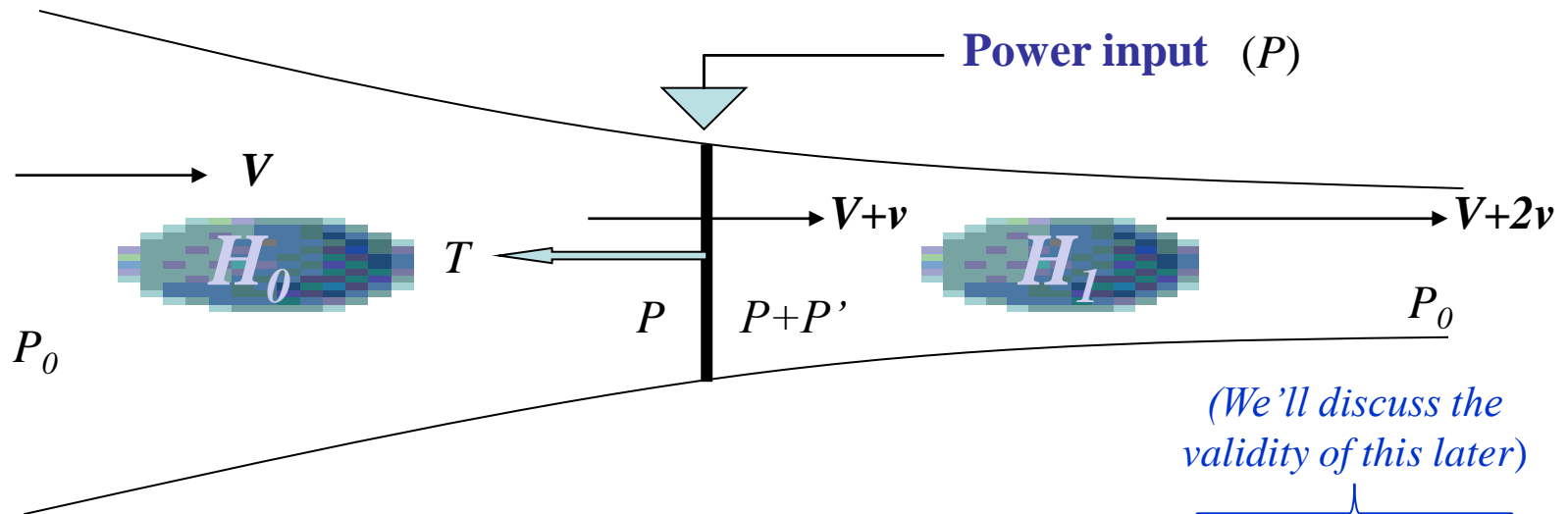
$$\therefore T = 2 \rho A (V + v) v$$

Momentum Theory



Efficiency

Since we have assumed no losses at the actuator disc, then the energy input (generating the pressure rise at the actuator disc) must be equal to the rise in kinetic energy in the fully developed wake.



Kinetic Energy rise in the wake is

$$\begin{aligned} \frac{\partial \Delta KE}{\partial t} &= \frac{1}{2} \rho A (V + v) ((V + 2v)^2 - V^2) \\ &= 2 \rho A (V + v) v (V + v) \\ &= T (V + v) \text{ (and remember this for later)} \end{aligned}$$

If P is power input to the actuator disc,

$$\eta = \frac{T(V + v)}{P} = \text{UNITY in this case}$$

Thus, efficiency of a thrust generator is $\eta = \frac{T(V + v)}{P}$ where:

T is output thrust
 V is onset velocity
 v is induced velocity
and P is input (shaft) power

For an aircraft in flight cruise, the **propeller** efficiency is $\eta_p = \frac{TV}{P}$

Clearly this cannot be used when $V = 0$, the static thrust case. Similarly it cannot be used for lifting propellers (on tilt rotor aircraft) in the hover. For such **rotors**, the work done on the air is more important and the efficiency is $\eta_r = \frac{Tv}{P}$

Whereas V is usually known (the aircraft airspeed), v is not, so Tv is replaced by the ideal power (P_{ideal}) and the efficiency is referred to as a “**Figure of Merit**” (FoM).

$$\eta_r = \frac{P_{ideal}}{P} \text{ (which we can call rotor efficiency or } FoM \text{)}$$

The **IDEAL** propeller is one with **NO LOSSES**.

Thus all the input power (P) is converted to increased kinetic energy (ΔKE),

$$P = \frac{\partial \Delta KE}{\partial t} = T (V + v), \quad (\text{as determined earlier})$$

For the ideal propeller, when $V \neq 0$ then, $\eta_p = \frac{TV}{T(V+v)} = \frac{1}{1+a}$

Where $a = \frac{v}{V}$ and is known as the **axial interference factor (inflow ratio)**.

This suggests that maximum efficiency occurs when $a=0$.

Since $T = 2\rho A V^2 a(1+a)$ then when $a=0$, $T=0$.

*(This is the trivial case
and has no practical use)*

The value of “ a ” can be found analytically in terms of a thrust coefficient T_c

$$\text{Where } T_c = \frac{T}{\rho V^2 D^2} = \frac{2\rho(\pi D^2/4)V^2 a(1+a)}{\rho V^2 D^2} = \frac{\pi a(1+a)}{2} \quad (D \text{ is propeller diameter})$$

Taking only the positive root the quadratic equation $\pi a^2 + \pi a - 2T_c = 0$

$$a = \frac{1}{2} \left\{ \sqrt{1 + \frac{8T_c}{\pi}} - 1 \right\}$$

The Static Thrust of an IDEAL propeller

The general thrust coefficient $T_c = \frac{T}{\rho V^2 D^2}$ has limited value for propellers as it has an infinite value at $V=0$.

A more suitable **thrust coefficient** is $C_T = \frac{T}{\rho n^2 D^4}$ giving a static thrust coefficient based on rotational speed n (revs/sec) and the propeller diameter D .

We can refer to this parameter nD as the **reference velocity**.

It is also normal to express the forward speed (V) of the propeller relative to this reference velocity and this is called the **Advance Ratio J** . Thus $J = \frac{V}{nD}$

$$\begin{aligned} \text{Substituting } V^2 = n^2 D^2 J^2 \text{ into } T_c = \frac{T}{\rho V^2 D^2} \\ \text{gives } T_c = \frac{T}{\rho n^2 D^4 J^2} = \frac{C_T}{J^2} \end{aligned}$$

The Static Thrust of an IDEAL propeller

Recall $a = \frac{v}{V}$ is known as the **axial interference factor (inflow ratio)**.

The inflow ratio (a) also has limited value in propeller analysis as it too has an infinite value at $V=0$.

For **Static Thrust** the actual induced velocity (v) is required.

$$T = 2\rho A(V + v)v = 2\rho Av^2 \text{ so that } v = \sqrt{\frac{T}{2\rho A}}$$

The POWER of an IDEAL propeller.

The Power is non-dimensionalised in a similar manner to thrust but with an additional nD term as $P \propto TV$ (and nD is the reference velocity parameter)

Thus **power coefficient** $C_P = \frac{P}{\rho n^3 D^5}$

The propeller efficiency $\eta_p = \frac{TV}{P}$

Thus $\eta_p P = \frac{\pi}{2} D^2 \rho V^3 (1+a)a = \frac{\pi}{2} D^2 \rho V^3 \frac{(1-\eta_p)}{\eta_p^2}$

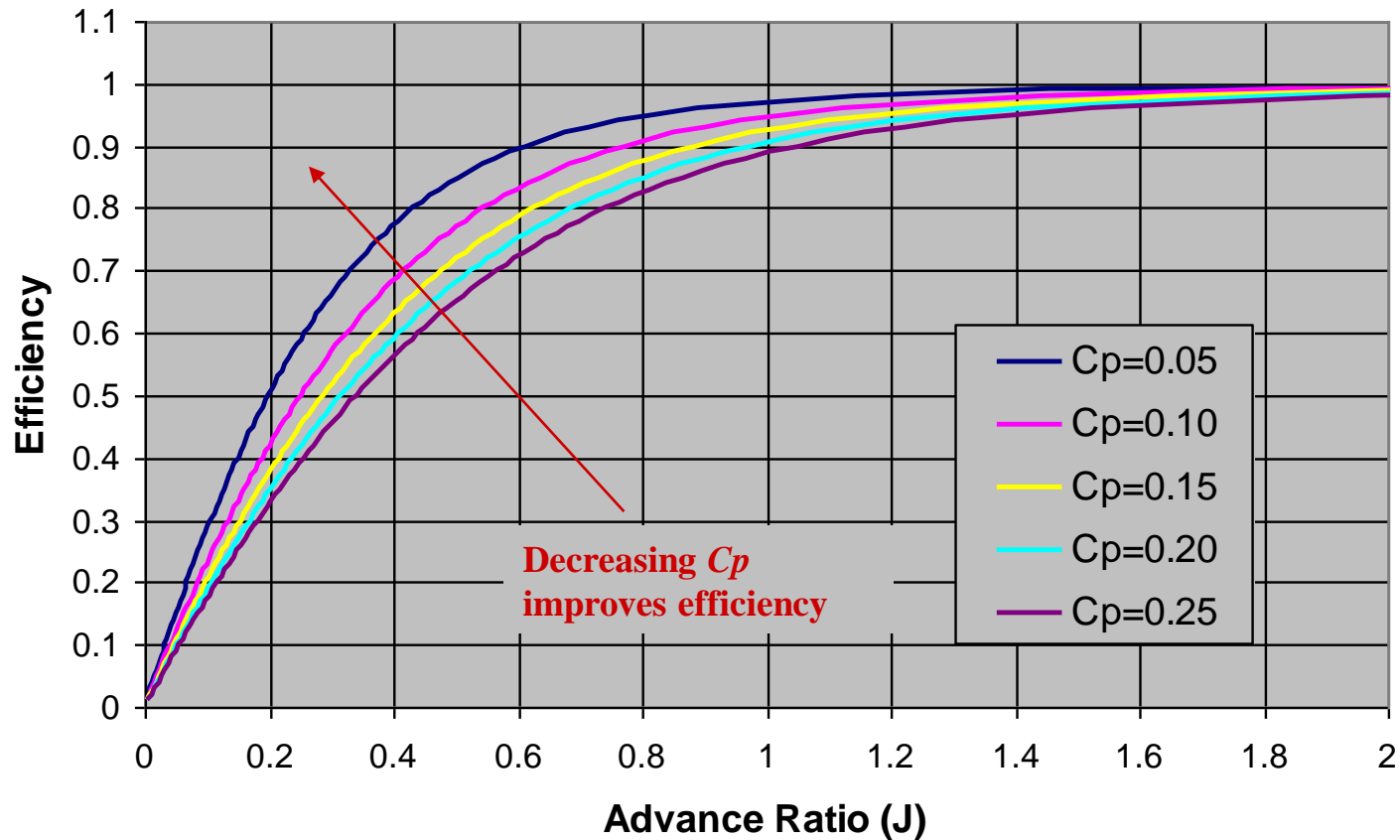
and

$$\frac{1-\eta_p}{\eta_p^3} = \frac{2P}{\pi \rho D^2 V^3} = \frac{2C_P}{\pi J^3}$$

Of academic interest only (ideal rotor) but gives an indication of parameter sensitivity.

Ideal Propeller (for a range of C_p values)

$$\frac{1 - \eta_p}{\eta_p^3} = \frac{2P}{\pi \rho D^2 V^3} = \frac{2C_p}{\pi J^3}$$



The Propeller in Action

In flight cruise, propeller efficiency is $\eta_p = \frac{TV}{P} \left(= \frac{TV}{T(V+v)} \right)$ and v must be small.

Ideally



In a similar manner to conventional Lift & Drag coefficients we have a Thrust coefficient:

$$C_L = \frac{L}{\frac{1}{2} \rho V^2 S} \quad C_D = \frac{D}{\frac{1}{2} \rho V^2 S} \quad C_T = \frac{T}{\rho n^2 D^4}$$

Flying in STILL air



$$T = \rho A (V_{A/C} + v) 2v$$

$$\frac{\partial \Delta KE_{air}}{\partial t} = P_{\Delta KE_{air}} = \frac{1}{2} \rho A (V_{A/C} + v) (2v)^2$$

$$P = T V_{A/C} + P_{\Delta KE_{air}} = 2 \rho A (V_{A/C} + v) v V_{A/C} + \frac{1}{2} \rho A (V_{A/C} + v) (2v)^2$$

$$P = 2 \rho A v [(V_{A/C}^2 + v V_{A/C}) + (V_{A/C} v + v^2)] = 2 \rho A v (V_{A/C} + v)^2$$

Flying with HEADWIND of velocity V



$$T = \rho A (V_A + v) 2v$$

$$\frac{\partial \Delta KE_{air}}{\partial t} = P_{\Delta KE_{air}} = \frac{1}{2} \rho A (V_A + v) ((V_A + 2v)^2 - V_A^2)$$

$$P = TV_{A/C} + P_{\Delta KE_{air}} = 0 + \frac{1}{2} \rho A (V_A + v) ((V_A + 2v)^2 - V_A^2)$$

$$P = \frac{1}{2} \rho A (V_A + v) (V_A^2 + 4V_A v + 4v^2 - V_A^2) = 2\rho A v (V_A + v)^2$$

$$\eta_P = \frac{TV_{A/C}}{P} = 0$$

Flying in a WINDTUNNEL with velocity V



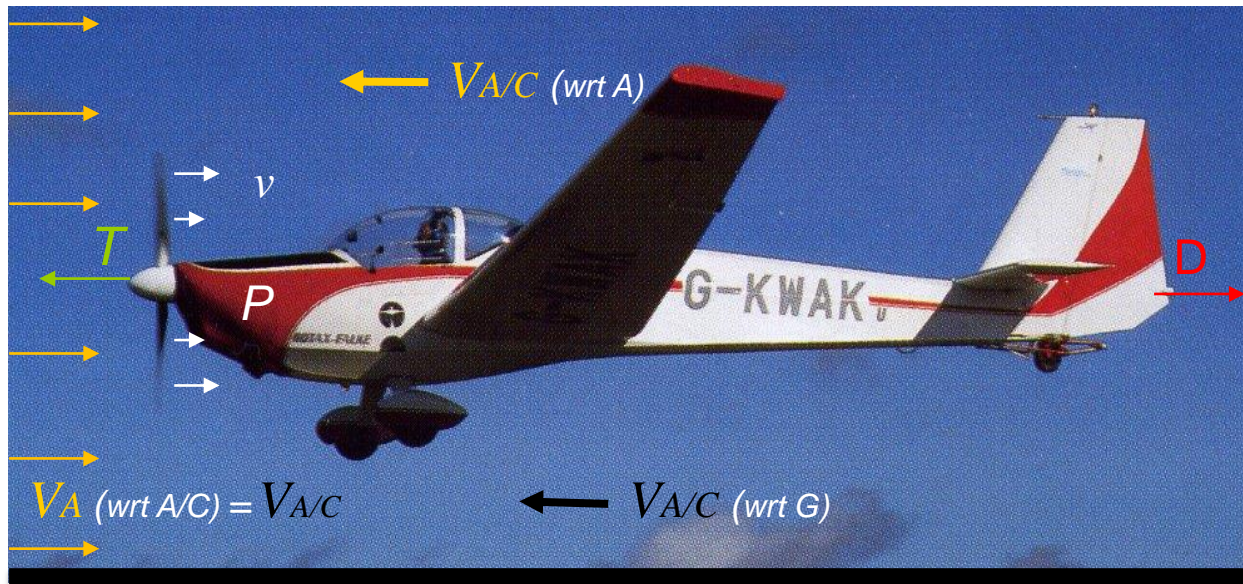
$$T = \rho A(V + v)2v$$

$$\frac{\partial \Delta KE_{air}}{\partial t} = P_{\Delta KE_{air}} = \frac{1}{2} \rho A(V + v)((V + 2v)^2 - V^2)$$

$$P = TV_{A/C} + P_{\Delta KE_{air}} = 0 + \frac{1}{2} \rho A(V + v)((V + 2v)^2 - V^2)$$

$$P = \frac{1}{2} \rho A(V + v)(V^2 + 4Vv + 4Vv^2 - V^2) = 2\rho Av(V + v)^2$$

Frames of Reference



- Be careful when accounting for energy budget
- Similarity between moving aircraft and moving air not so straightforward
- Strictly speaking the propeller efficiency is only valid for still air.. **but for the general analysis of propeller efficiency, consider aircraft relative to air.**