CADE30001

Advanced Structures and Materials

Lecture 3: Stress based methods in fracture mechanics

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Course Content

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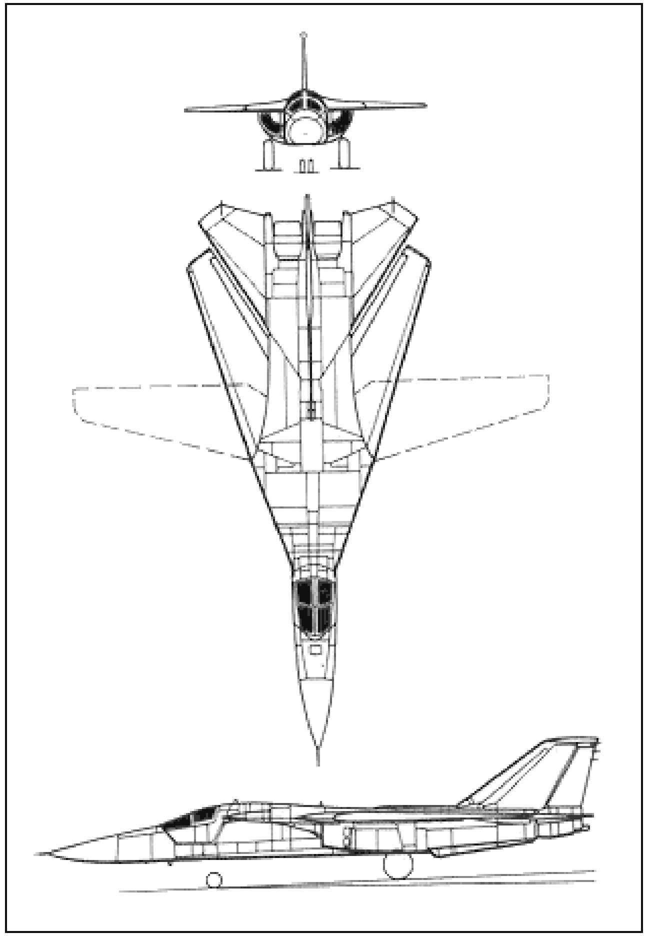
• Why study failure? • Concept of strain energy and toughness • Ductile, brittle failure • Fractography • Factors affecting ductile to brittle transition	• Historical Examples • Design philosophies	• Introduction to fracture mechanics • Theoretical stress approach to fracture • Stress intensity factor	• Griffith's energy balance approach • Irwin's energy balance approach	• Fracture process zone and geometrical considerations • Measuring toughness • Anisotropic materials

Previously...



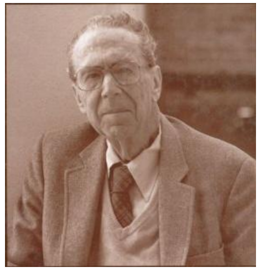
Cheat notes

Titan submersible	Carbon fibre hull, hydrostatic pressure, implosion, delamination
Liberty ships	Ductile to brittle transition, T27 charpy test, cold temperatures of Atlantic
Comet	High altitude, cabin pressurization, stress concentration, square windows, no crack arrestors
F111	Swing wing, manufacturing flaw in pivot, damage tolerance
Boeing 707	Fail safe design, additional chord, no fatigue testing, Testing pyramid
Aloha	Island hopping, multi site damage, no inspections, aging of aircraft
C130 A	New loading spectra

Milestones in Fracture Mechanics

Charles Augustin de Coulomb (1736 – 1806)

Alan Arnold Griffith (1893 – 1963)

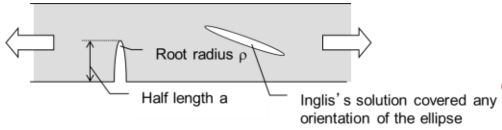


Irwin

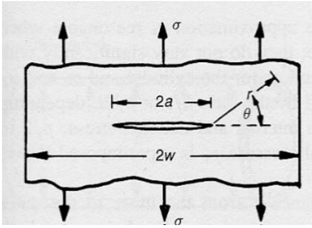
Stone Age → Middle Age → Coulomb 1776 → Inglis 1913 → Griffith Model 1920 → Irwin's Model 1957

Flint Knapping

Crack propagation in stones under compression



$$\sigma_f = \sqrt{\frac{E\gamma}{4a}}$$



$$\sigma_f = \sqrt{\frac{2E\gamma}{\pi a}}$$

Irwin line-crack $\sigma_f = \sqrt{\frac{2E\gamma}{\pi a(1-\nu^2)}}$

Fracture Energy $G = \frac{P^2}{2B}\frac{dC}{da}$

Stress Intensity Factor $K = \sigma\sqrt{\pi a}$

Observe

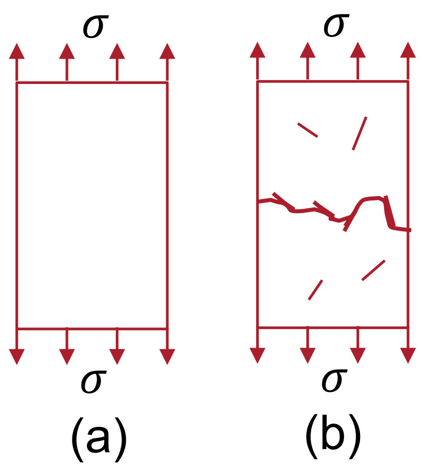
bristol.ac.uk

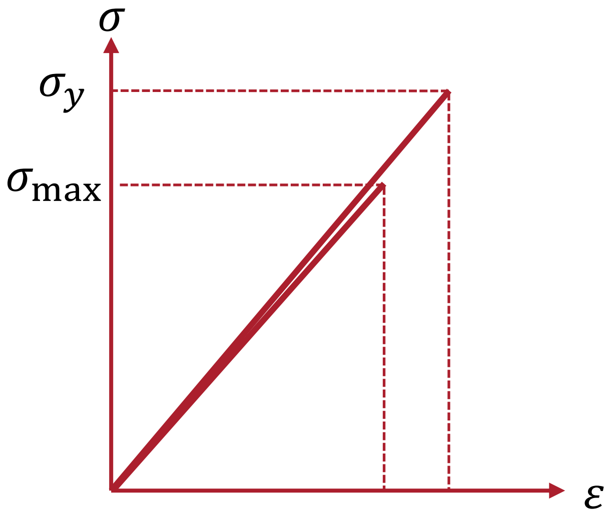
https://youtu.be/30Lbgge_kkM?t=58

https://www.youtube.com/shorts/_E1ab2LFVqc?feature=share

Introduction to Fracture Mechanics

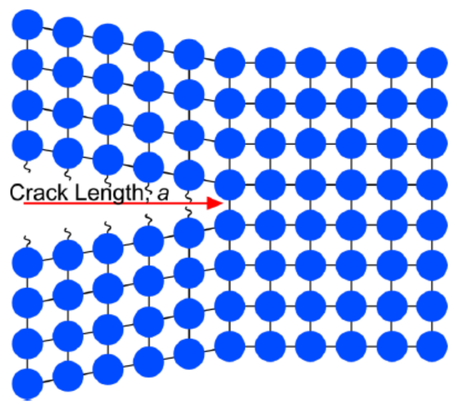
- The presence of defects modify the local stress field in the material
- Therefore, elastic stress analyses assuming perfectly homogeneous and flawless materials are not suitable for designs using high-strength materials
- When a crack reaches a certain critical length, it can propagate catastrophically through the structure
- This can happen at gross stresses which are much less than the yield stress of the material

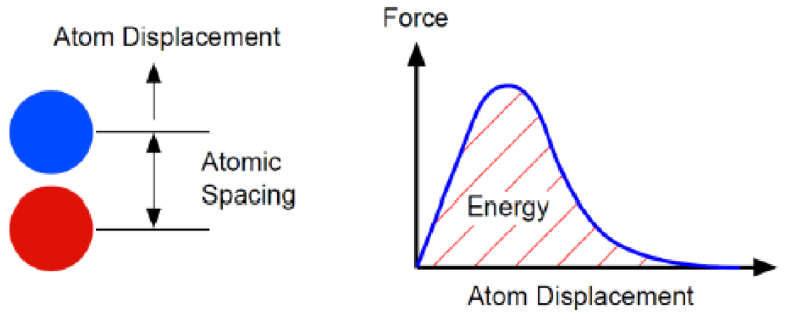




Atomic view of fracture

- The bond strength is supplied by the attractive forces between atoms.
- A material fractures when sufficient stress and work are applied at the atomic level to break the bonds that hold atoms together.
- A tensile force is required to increase the separation distance from the equilibrium value, this force must exceed the cohesive force to sever the bond.





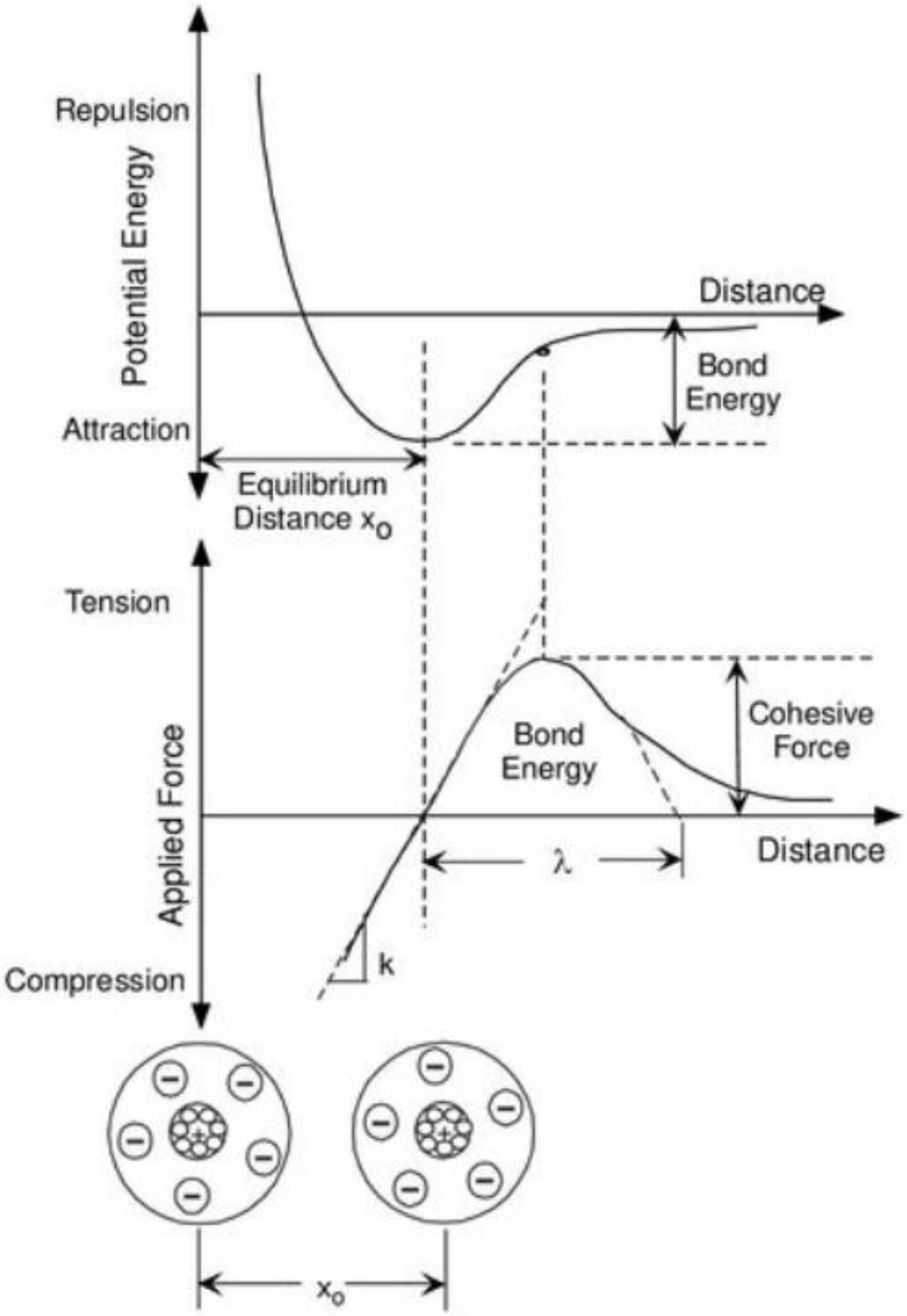
Fracture stress

$$P = P_c \sin\left(\frac{\pi x}{\lambda}\right)$$

$$k = P_c\left(\frac{\pi}{\lambda}\right)$$

$$\frac{n k x_o}{A(=1)} = \frac{n P_c x_o}{A(=1)}\left(\frac{\pi}{\lambda}\right)$$

$$E = \sigma_c\left(\frac{\pi x_o}{\lambda}\right) \Rightarrow \sigma_c = \frac{E\lambda}{\pi x_o}$$



Early fracture mechanics

- Fracture mechanics was developed during WWI by English aeronautical engineer A. A. Griffith to explain the following observations:
- The theoretical stress needed for breaking atomic bonds is approximately 10,000 MPa
- The stress needed to fracture bulk glass is around 100 MPa
- Experiments on glass fibers that Griffith himself conducted: the fracture stress increases as the fiber diameter decreases => Hence the uniaxial tensile strength, which had been used extensively to predict material failure before Griffith, could not be a specimen-independent material property.

Size effect

- Griffith suggested that the low fracture strength observed in experiments, as well as the size-dependence of strength, was due to the presence of **microscopic flaws** in the bulk material.

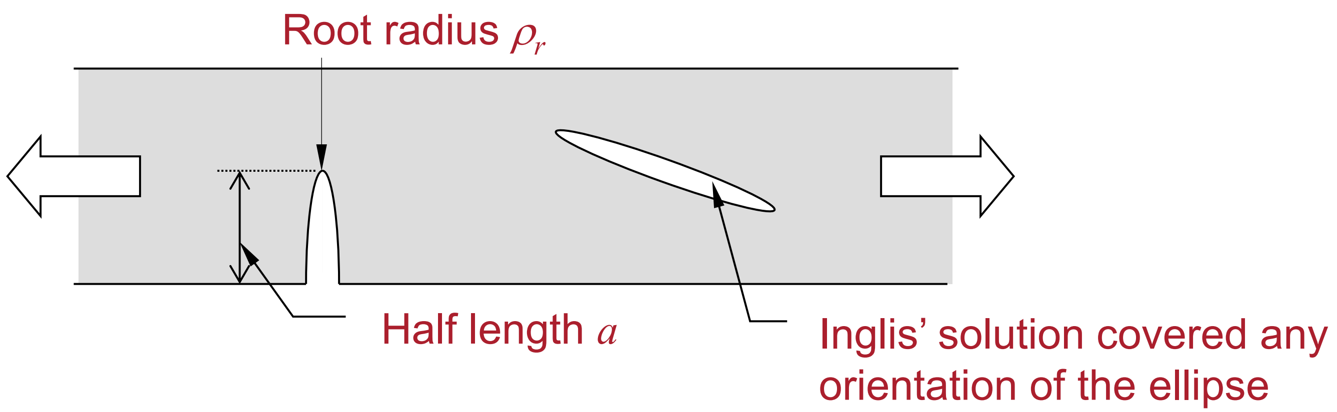
TABLE 1.1. Strength of glass fibers according to Griffith's experiments.

Diameter (10^{-3} in)	Breaking stress (lb/in^2)	Diameter (10^{-3} in)	Breaking stress (lb/in^2)
40.00	24 900	0.95	117 000
4.20	42 300	0.75	134 000
2.78	50 800	0.70	164 000
2.25	64 100	0.60	185 000
2.00	79 600	0.56	154 000
1.85	88 500	0.50	195 000
1.75	82 600	0.38	232 000
1.40	85 200	0.26	332 000
1.32	99 500	0.165	498 000
1.15	88 700	0.130	491 000

Fracture mechanics in daily life

Effect of flaws - Inglis' Elliptical Notch

- In 1913 Charles Inglis (OBE, FRS) derived the stress field around an elliptical opening at any orientation in a plate subjected to a tensile stress:



- Inglis showed that the maximum stress at the tip of the notch has the form:

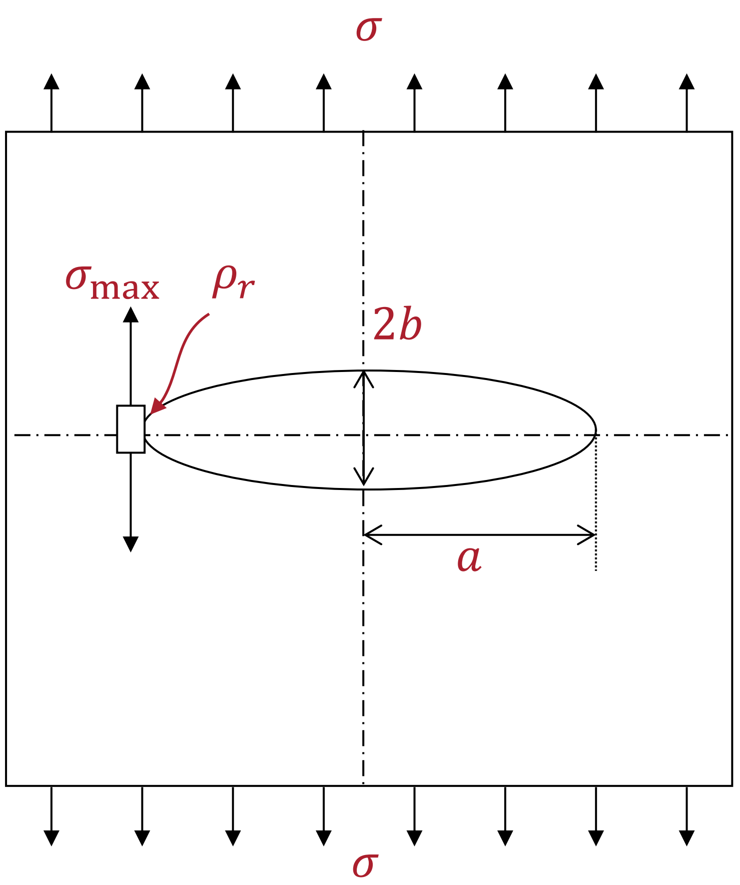
$$\sigma_{max} = R\left(1 + 2\sqrt{\frac{a}{\rho_r}}\right)$$

- where R depends on the orientation of the notch and the details of the stress field etc.

Elliptical Notch - Normal Loading

Key slide!

For an elliptical notch normal to the direction of a uniform ('far field') tensile stress (*s*) in an infinite plate:



$$\sigma_{max} = \sigma\left(1 + 2\sqrt{\frac{a}{\rho_r}}\right) \qquad \rho_r = \frac{b^2}{a}$$

e.g. for a circular hole where *b* = *a*

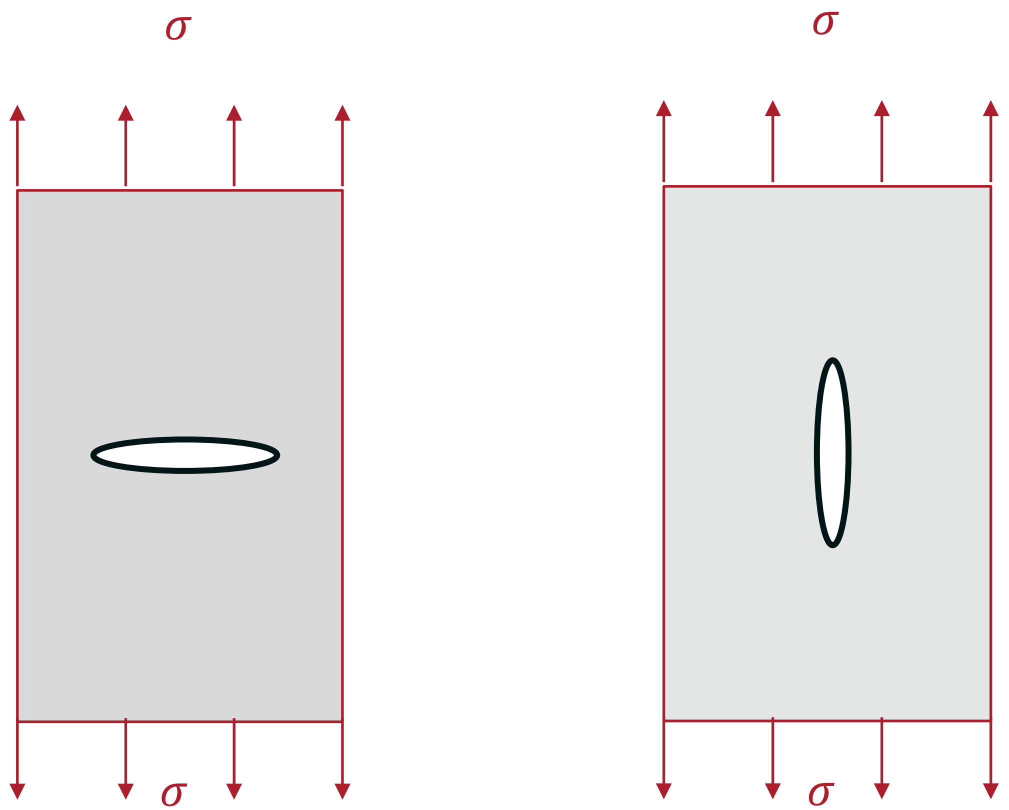
$$\sigma_{\max} = \sigma\left(1 + 2\sqrt{1}\right) = 3\sigma$$

The ratio of maximum stress to the nominal applied stress is known as the stress concentration factor, $\boldsymbol{k_t}$

$$k_t = \frac{\sigma_{\max}}{\sigma}$$

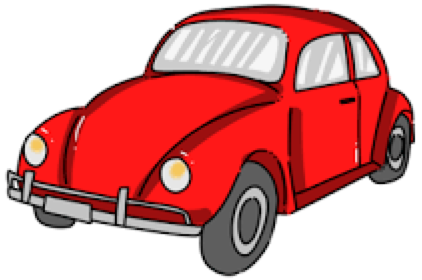
Elliptical Notch – Example (3 minutes)

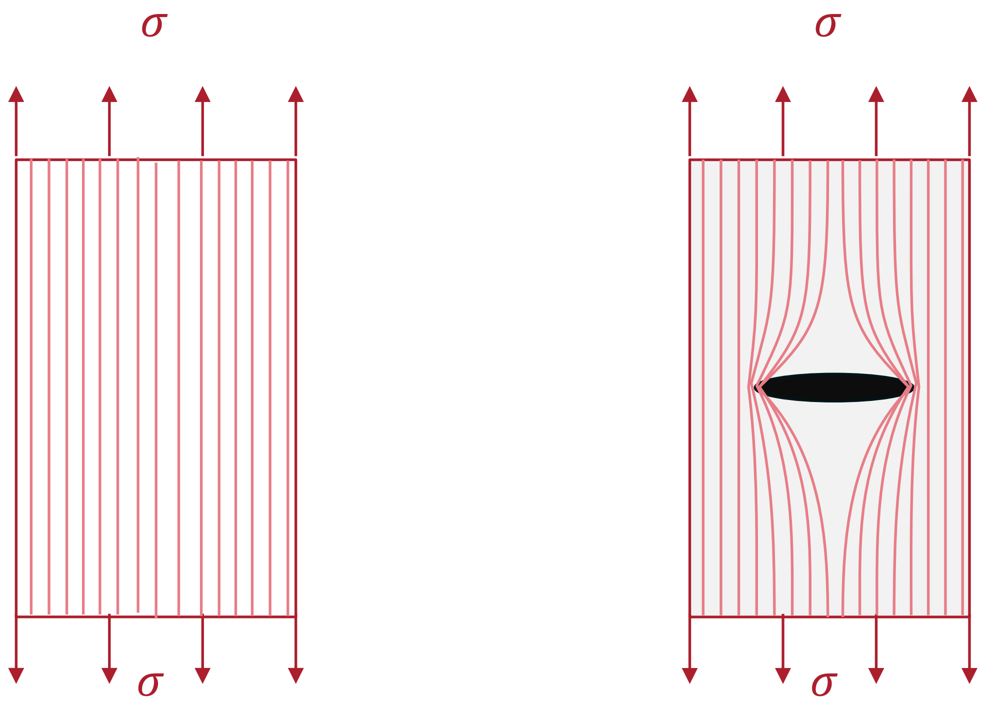
- Consider these two cases below where the ellipse is 30 mm long and 10 mm wide. What are their respective concentration factors k_t?



$$\sigma_{max} = \sigma\left(1 + 2\sqrt{\frac{a}{\rho_r}}\right)$$

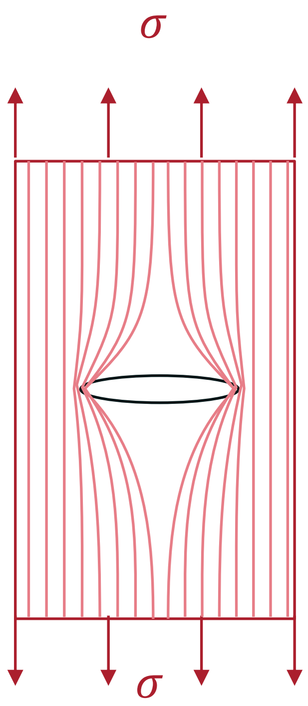
Stress lines



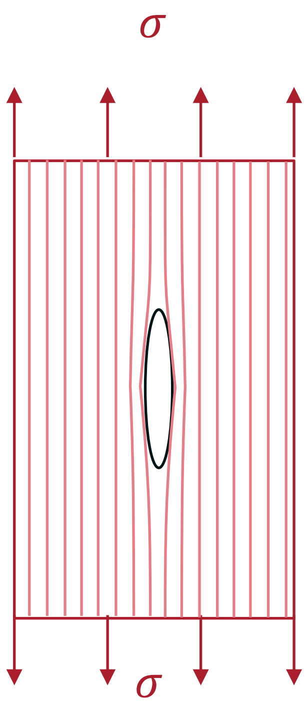


Elliptical Notch - Example

- Consider these two cases below where the ellipse is 30 mm long and 10 mm wide. What are their respective concentration factors k_t?



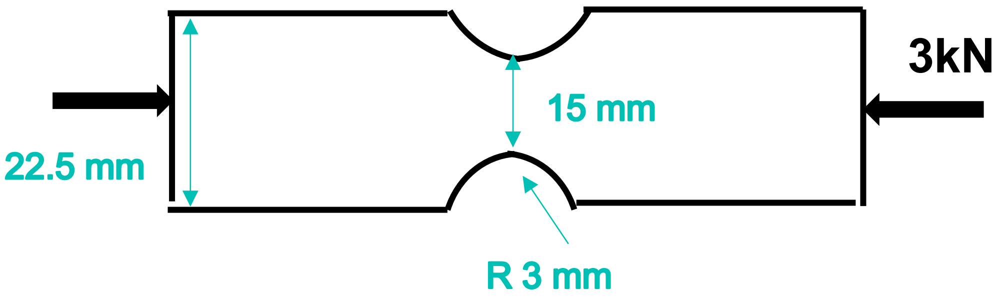
$$k_t = 1 + 2\sqrt{\frac{15}{25/15}} = 7$$



$$k_t = 1 + 2\sqrt{\frac{5}{45}} = 1.67$$

Example

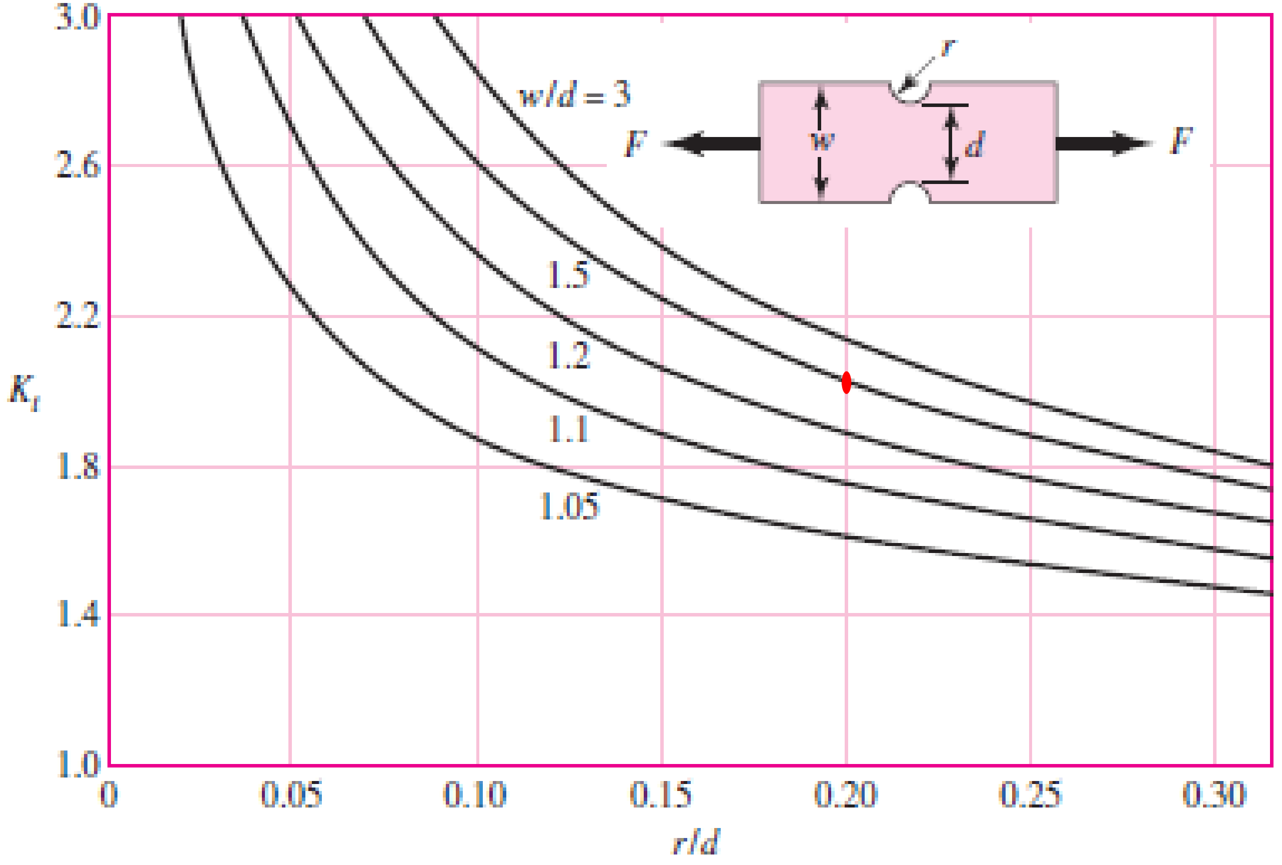
- Find the maximum stress in the notched rectangular bar of 4 mm thickness with 3000 N compressive load



Lookup charts

Figure A–15–3

Notched rectangular bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.



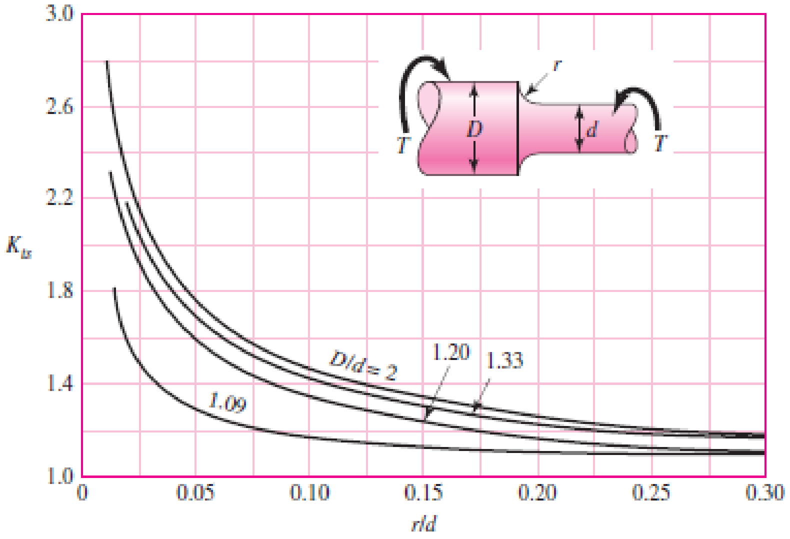
Solution

- P= 3000 N
- r/d= 3/15=0.2
- w/d= 22.5/15 = 1.5
- σ = P/A = P/dt = 3000/(15*4)= 50 N/mm^2
- K_t= 2.1
- σ_{max}= 2.1*50 = 105 MPa

Other cases

Figure A–15–8

Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.



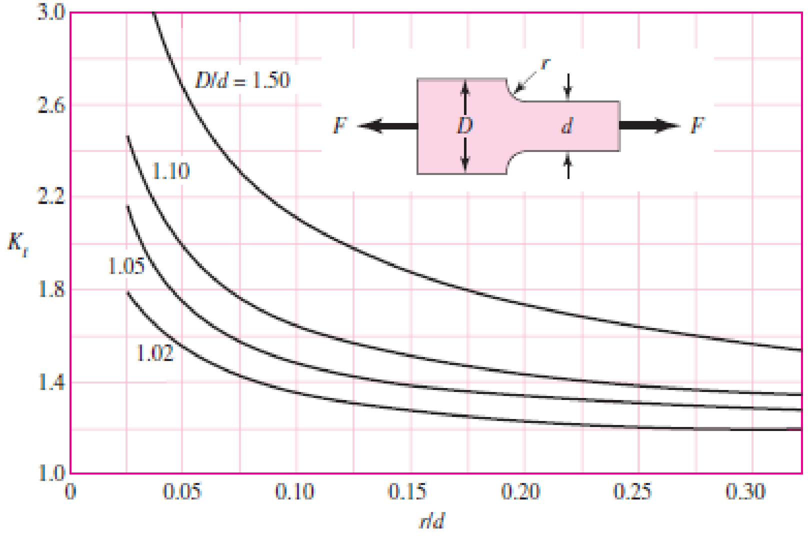
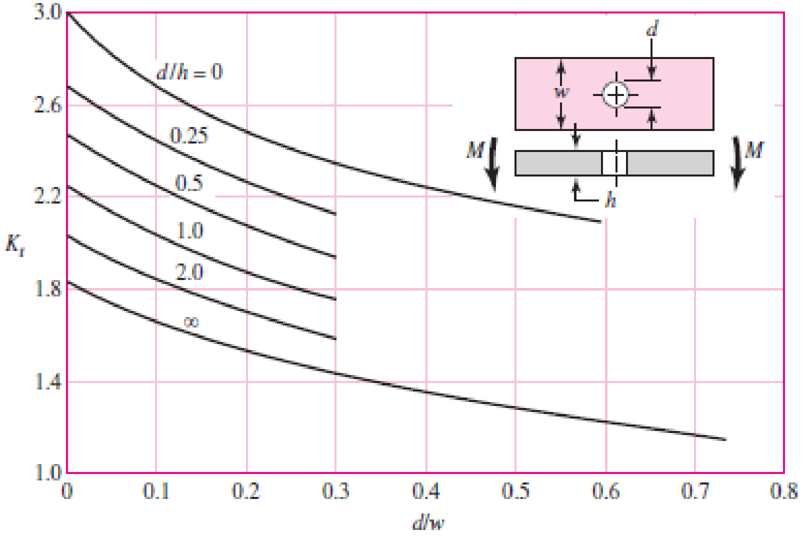


Figure A–15–2

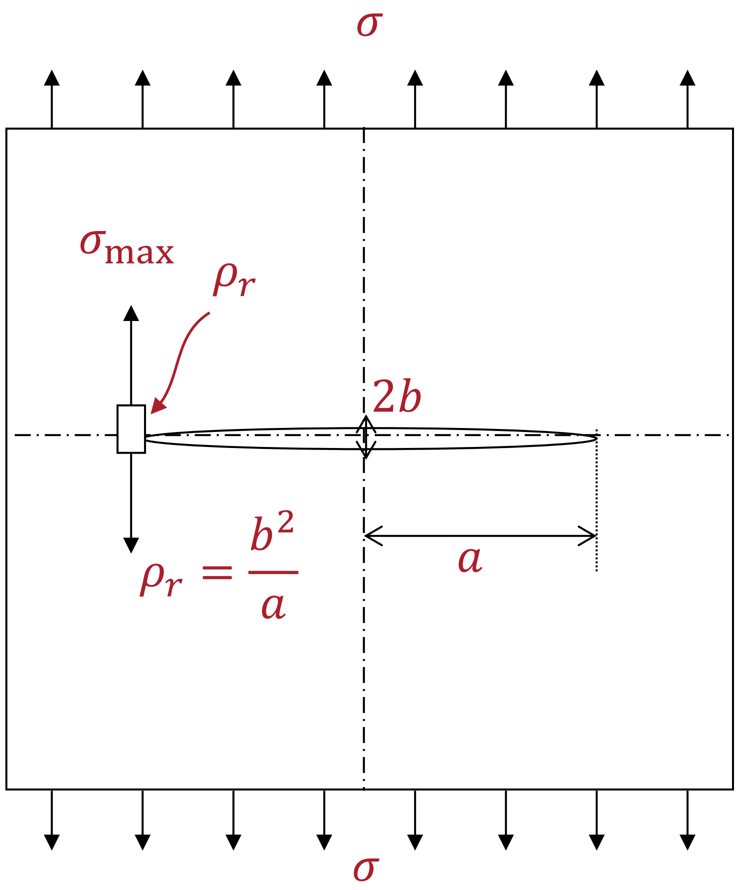
Rectangular bar with a transverse hole in bending. $\sigma_0 = Mc/I$, where $I = (w - d)h^3/12$.



Sharp Notches

- Recalling the elliptical notch normal to the direction of a uniform tensile stress:

$$\sigma_{max} = \sigma\left(1 + 2\sqrt{\frac{a}{\rho_r}}\right)$$



As b becomes smaller, $a/\rho_r \gg 1$

Therefore the equation above can be simplified to:

$$\sigma_{tip} \approx 2\sigma\sqrt{\frac{a}{\rho_r}}$$

When crack becomes infinitely sharp, ρ_r vanishes (i.e. tends towards zero)

Crack Tip Singularity

At the limit of a perfectly sharp crack, the stresses tend to be infinite at the crack tip – the stress field becomes singular

Using such a result would predict that materials would have near-zero strength!

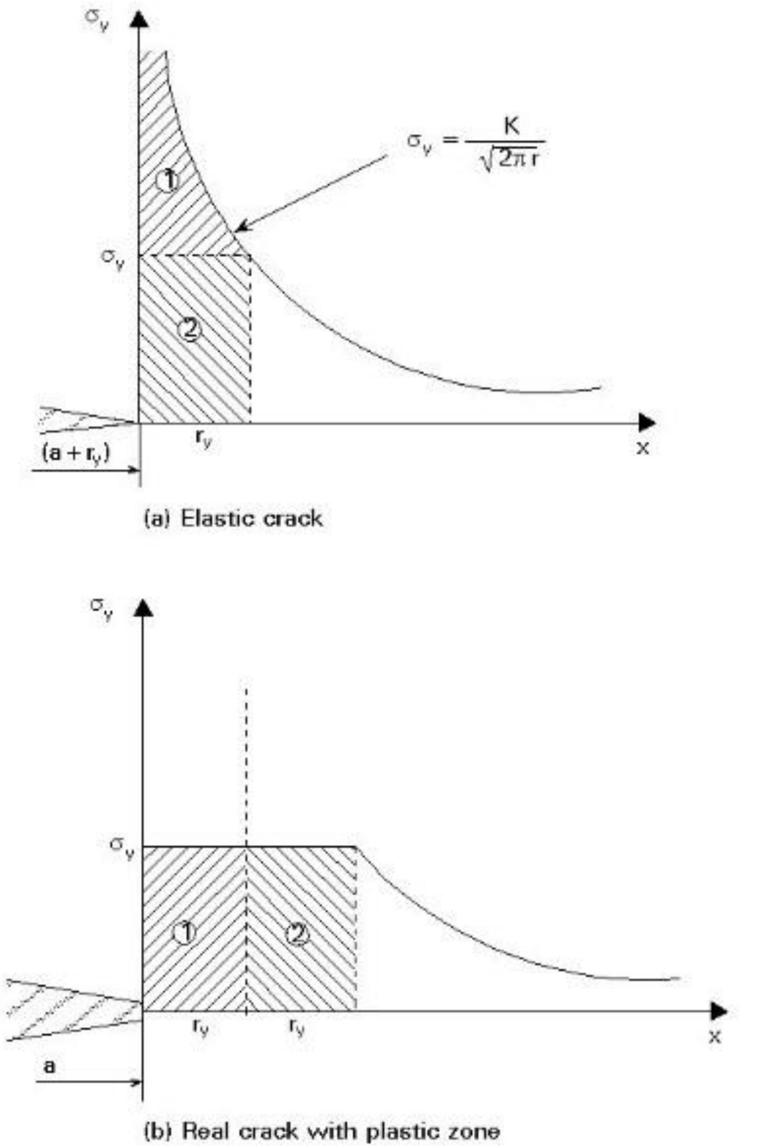
This is obviously non-physical

In reality the material generally undergoes **local yielding** which **blunts** the crack tip

Crack tip yielding

Near the crack tip, material will yield, thus the maximum stress will have a limit.

During fracture large part of the energy is associated to this plastic flow near known as ‘plastic zone’



(a) Elastic crack

(b) Real crack with plastic zone

Figure 6 Plastic zone correction

Stress Intensity Factor

Measure that captures the singular nature of the stress field

Singularity

$$\frac{1}{\sqrt{2\pi r}} . \sigma\sqrt{\pi a} . f(\theta)$$

The angular dependence

Geometrical relationship and the applied stress which **George R. Irwin** defined (in 1957) as the **stress intensity factor**, K

$$K = \sigma\sqrt{\pi a}$$

- For a specific crack geometry and applied stress, K will be a constant

Key slide!

Stress Intensity Factor

- For the initial case of an infinite plate with internal crack of length $2a$ subjected to uniform stresses, stress intensity factor is:

$$K = \sigma\sqrt{\pi a}$$

- However in general K is highly dependent on the geometry of the cracked body, so it usual to express it as:

$$K = \beta\sigma\sqrt{\pi a}$$

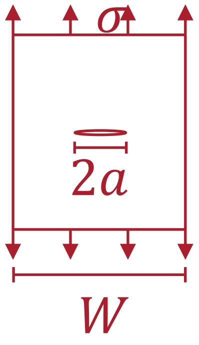
- Where β is a function determined depending on the geometrical configuration of the cracked body.

Stress Concentration vs Stress Intensity Factor

- Stress concentration refers to the increase in stress around discontinuities or geometric irregularities in a material, such as holes, notches, grooves, or sudden changes in cross-section. It is usually described by the stress concentration factor (Kt)
- Stress intensity factor quantifies the stress state near the tip of a crack. It is an important parameter in fracture mechanics and is used to predict crack growth and failure.
- The stress intensity factor (K) depends on the applied stress, crack size, and geometry of the cracked structure.

Shape factor β for common geometries

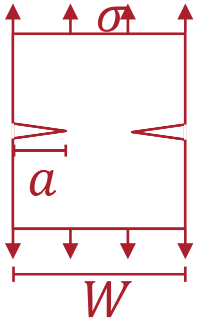
Centre Cracked Plate



$$K = \beta\sigma\sqrt{\pi a}$$

$$\beta = \sqrt{sec\frac{\pi a}{W}}$$

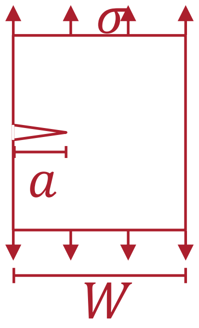
Double Edge Notched Plate



$$K = \beta\sigma\sqrt{\pi a}$$

$\beta = 1.12$ for small cracks

Single Edge Notched Plate

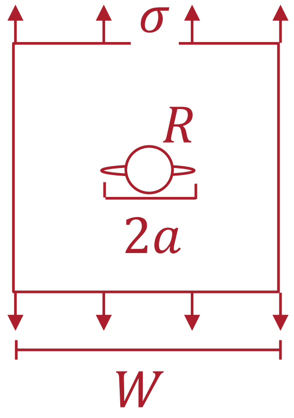


$$K = \beta\sigma\sqrt{\pi a}$$

$\beta = 1.12$

for small cracks

Cracked Hole



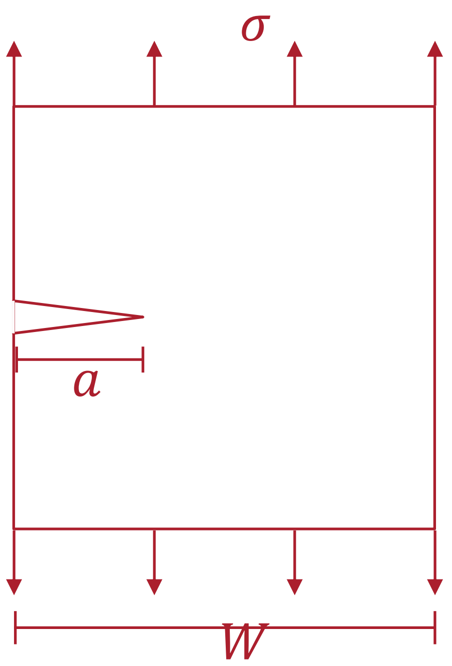
$$K = \beta\sigma\sqrt{\pi a}$$

$$\beta = f(R, a, W)$$

Example problem

Find the Stress intensity factor for a Single Edge Notched Plate with following dimensions and loading

- Plate Width: W=100 mm
- Plate Thickness: B=10 mm
- Crack Length: a=2.5 mm
- Applied Tensile Stress: σ=100 MPa



Solution

- $K = \beta\sigma\sqrt{\pi a}$
  $\beta = 1.12$

- $K = 1.12 * 100 * \sqrt{\pi * 2.5} = 313.87\ MPa.mm^{1/2}$

Fracture toughness K_c

- Within the fracture mechanics, the simplest way to account for the fracture process is to stipulate that the crack extends when the stress intensity factor *K* reaches a critical value, referred to as Fracture Toughness, *K*c

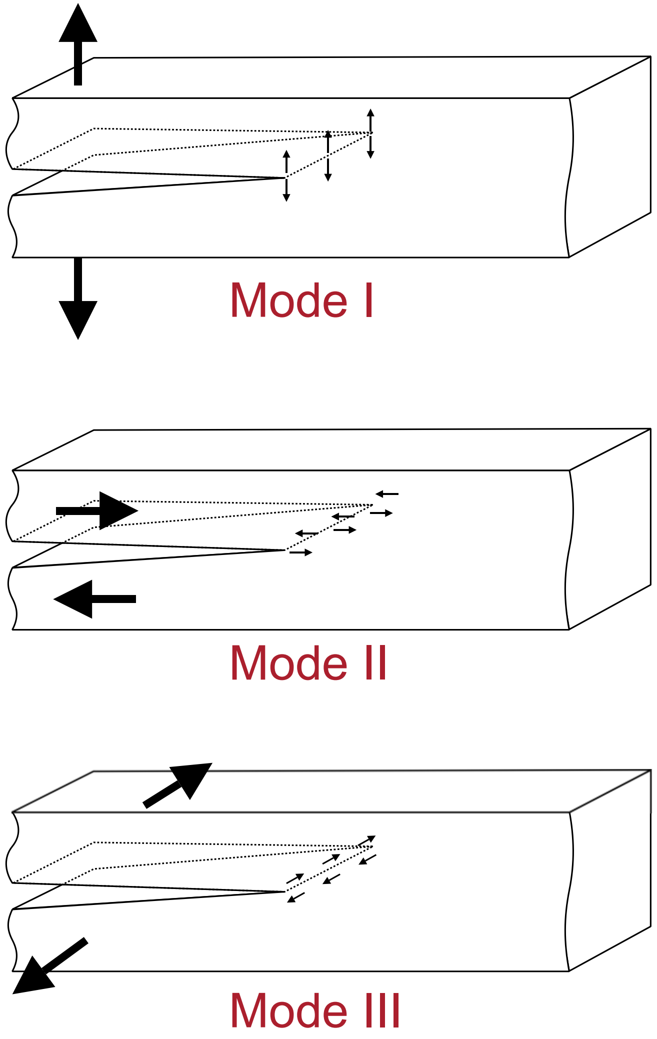
$$K_c = \beta \sigma_f \sqrt{\pi a}$$

- The stress intensity factor is a *loading parameter*. The toughness is a *material parameter*.
- For a given material, *Kc* is determined by a fracture test.
- Representative values of toughness:
  - Glass: K*c* = 1 MPa $m^{1/2}$
  - Steel: K*c* = 100 MPa $m^{1/2}$
  - Epoxy: K*c* = 1 MPa $m^{1/2}$

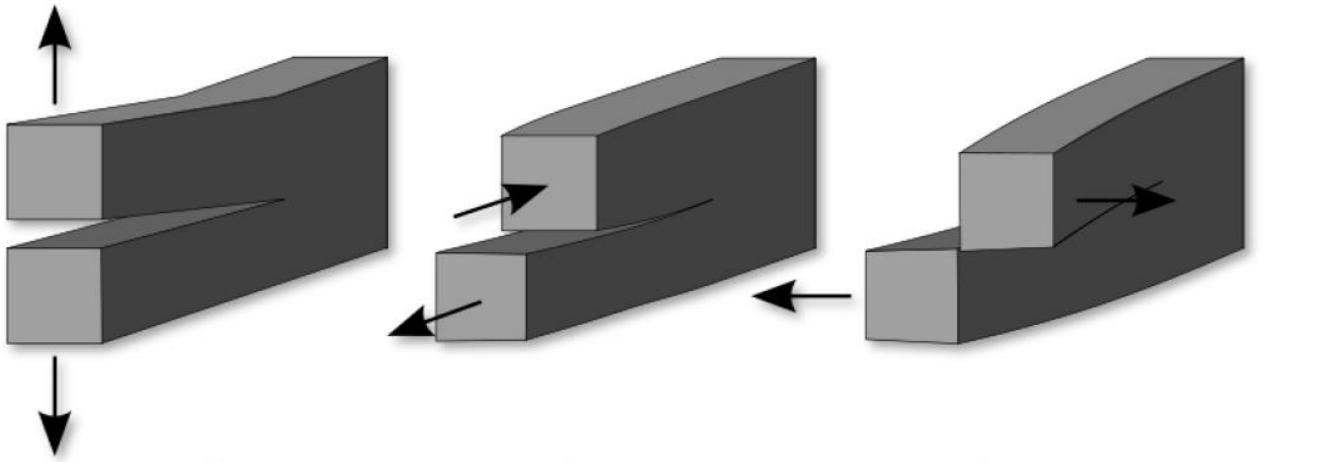
Loading modes

- The stress at the tip of the crack can undergo three modes of loading:
  - I. Tensile opening,
  - II. Shear sliding
  - III. Tearing

- Many real fractures in components are a mixture of these three modes, but mode I and mode II tend to dominate



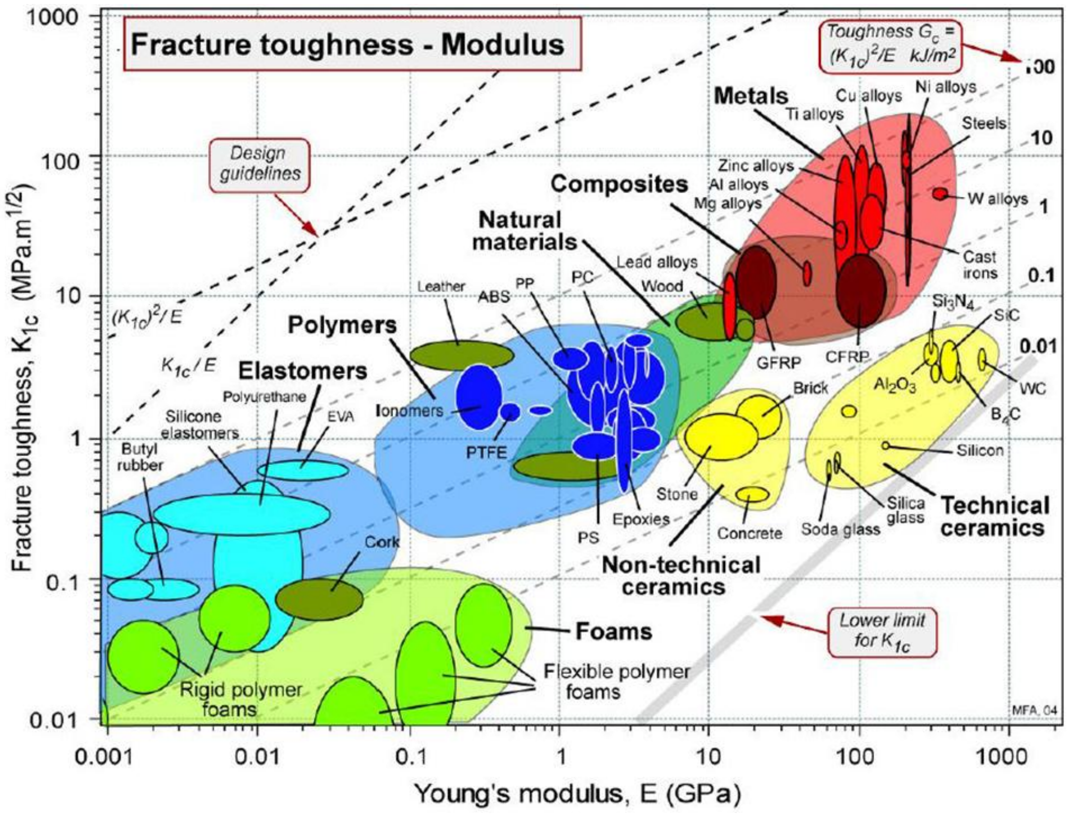
Effect of loading mode

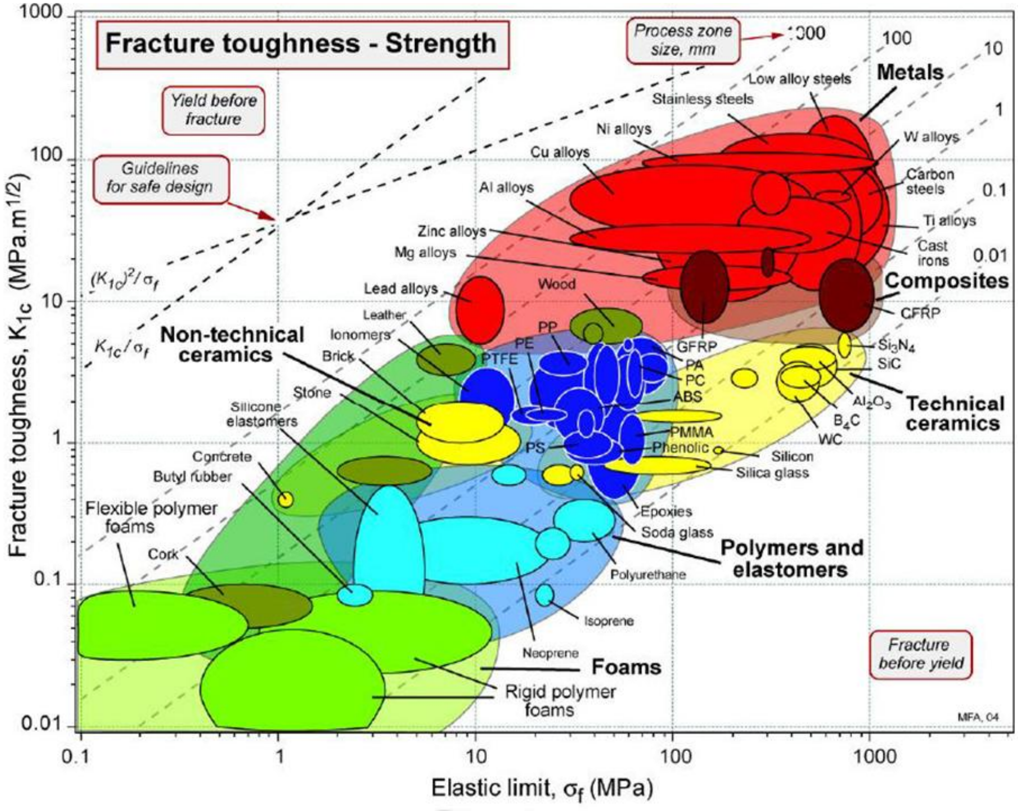


$$K_{Ic} \neq K_{IIc} \neq K_{IIIc}$$

- Most materials are susceptible to fracture by normal stresses than by shear
- Mode I is given higher importance
- K_{IIc} and K_{IIIc} are higher than K_{Ic}

Material selection – Fracture toughness





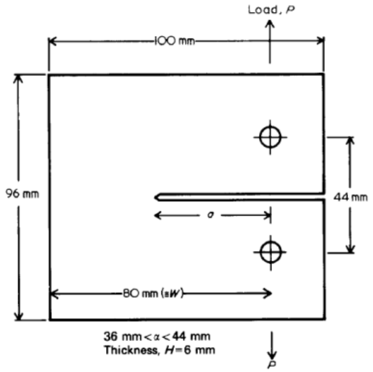
https://www.proxom.net/material-selection/

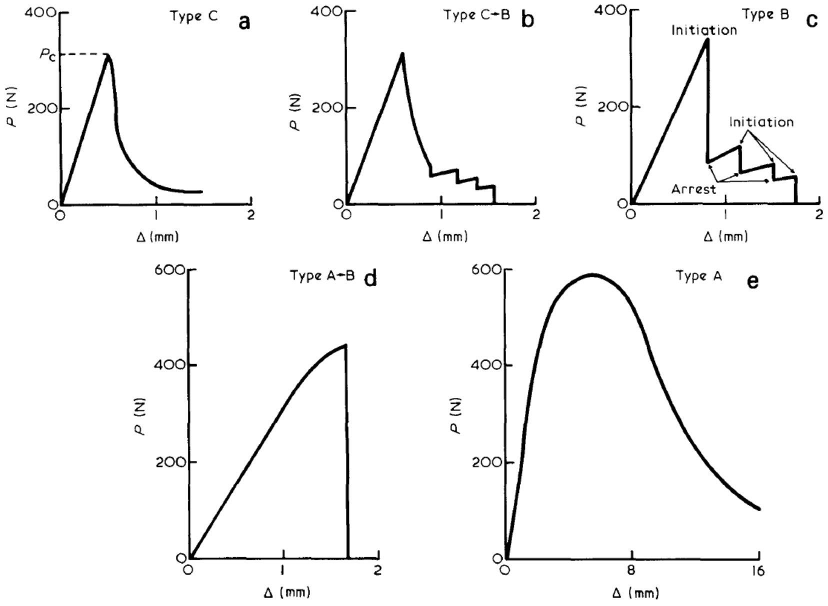
Homework

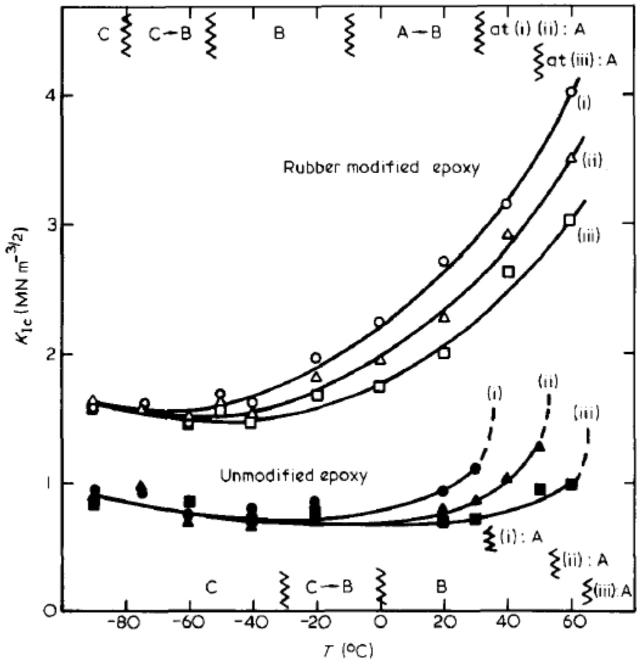
Deformation and fracture behaviour of a rubber-toughened epoxy: 1. Microstructure and fracture studies

A. J. Kinloch, S. J. Shaw and D. A. Tod
Ministry of Defence (PE), Propellants, Explosives and Rocket Motor Establishment, Waltham Abbey, Essex, UK

and D. L. Hunston
National Bureau of Standards, Polymer Division, Washington DC, USA
(*Received* 9 *March* 1983)

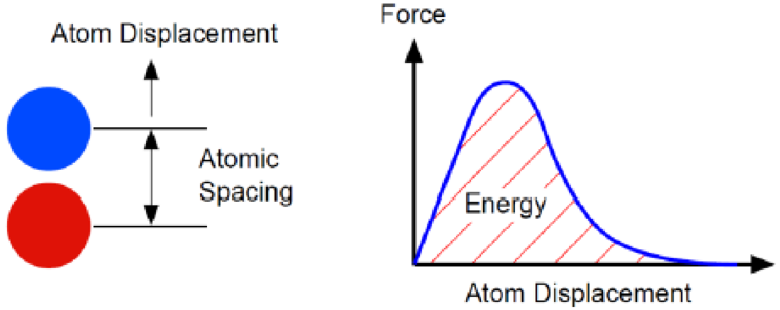


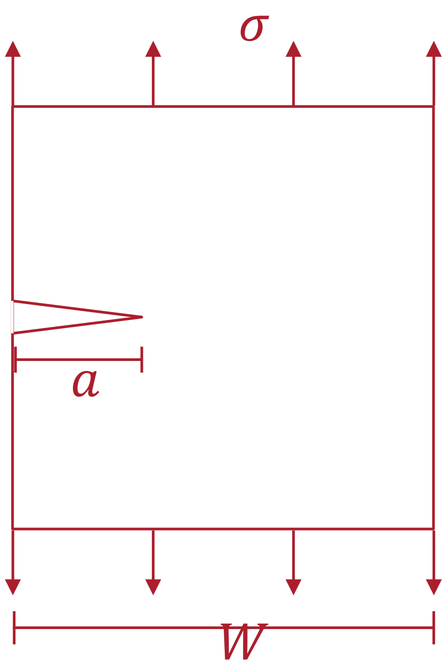


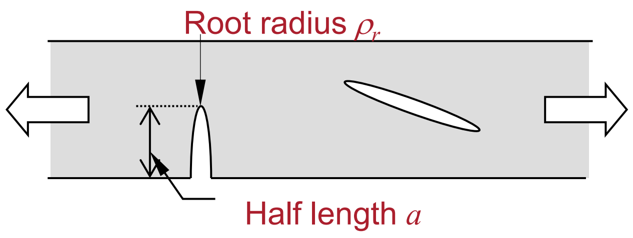


Summary

- Atomic view of fracture
- Inglis work on elliptical hole
- Stress concentration
- Singularity at sharp crack tip
- Stress Intensity factor
- Fracture toughness
- Loading modes







Next week

- Energy based approaches