

# AVDASI 3

(CADE 30007)

## Gas Turbine Propulsion

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Week 15: Lecture 3

Off-Design Conditions  
(1 of 2)

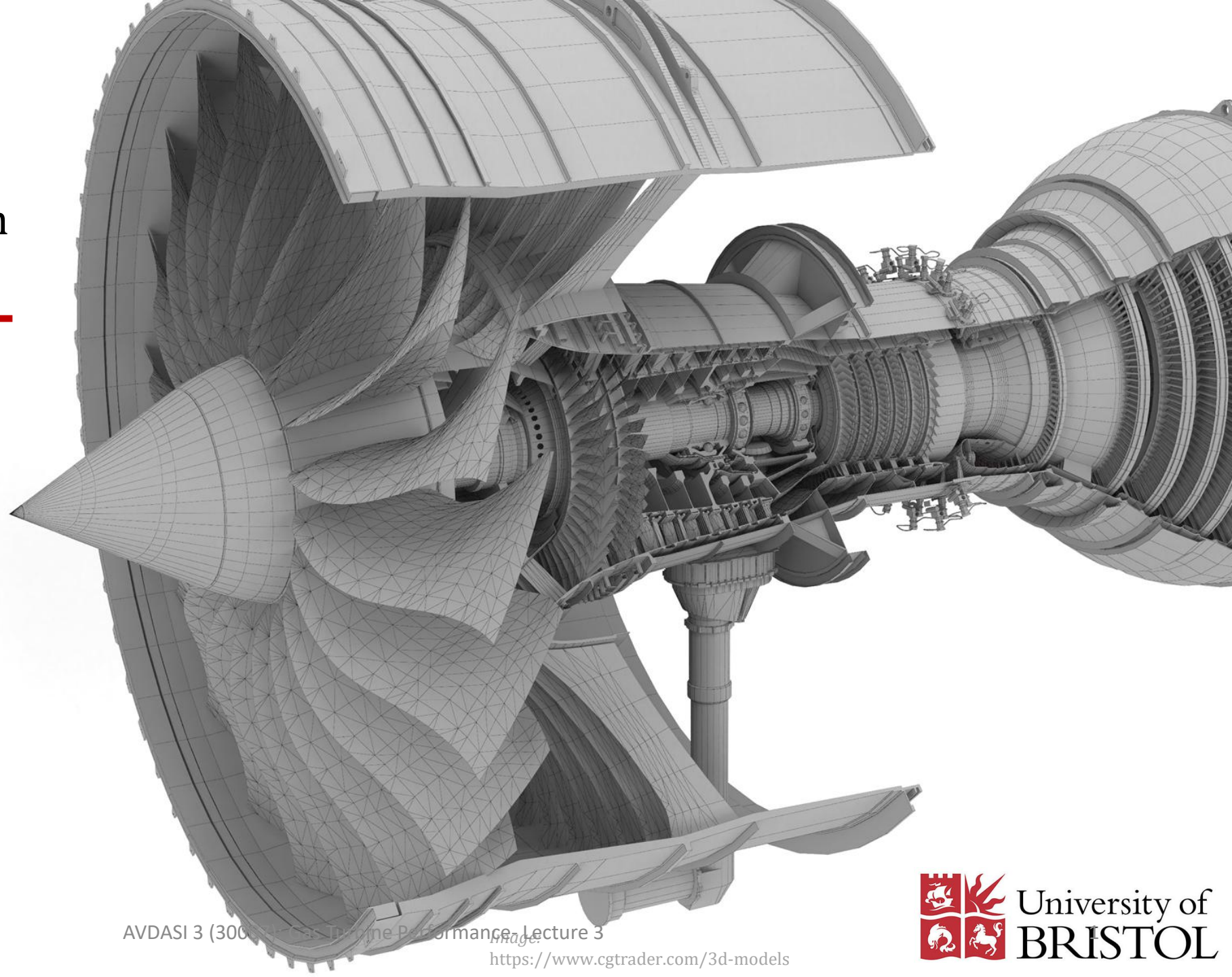
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**Unit Director:**

Dr. Samudra Dasgupta



## **Objective ~ Lecture 3**

***To outline the way that the performance  
of a propulsion system can be  
characterised***

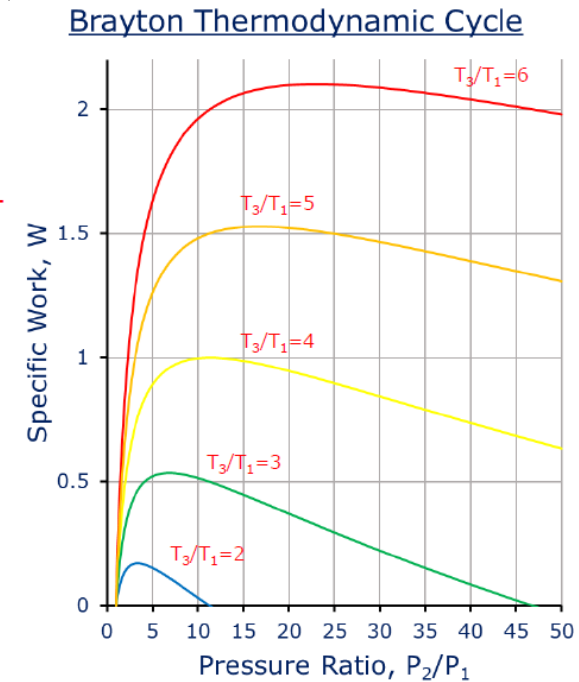
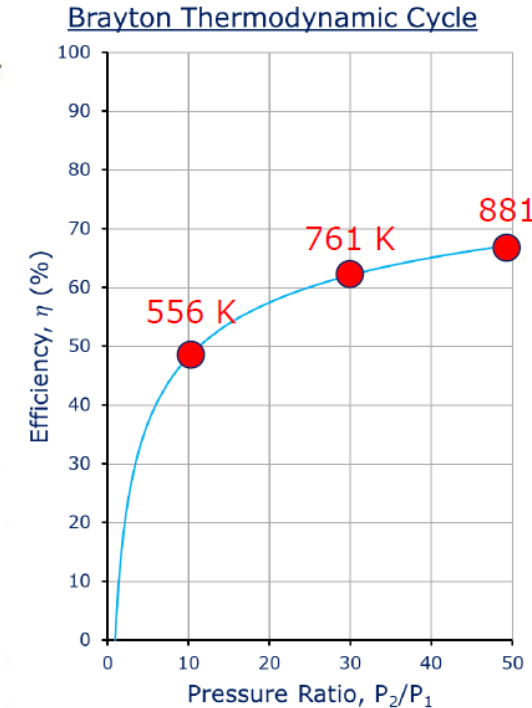
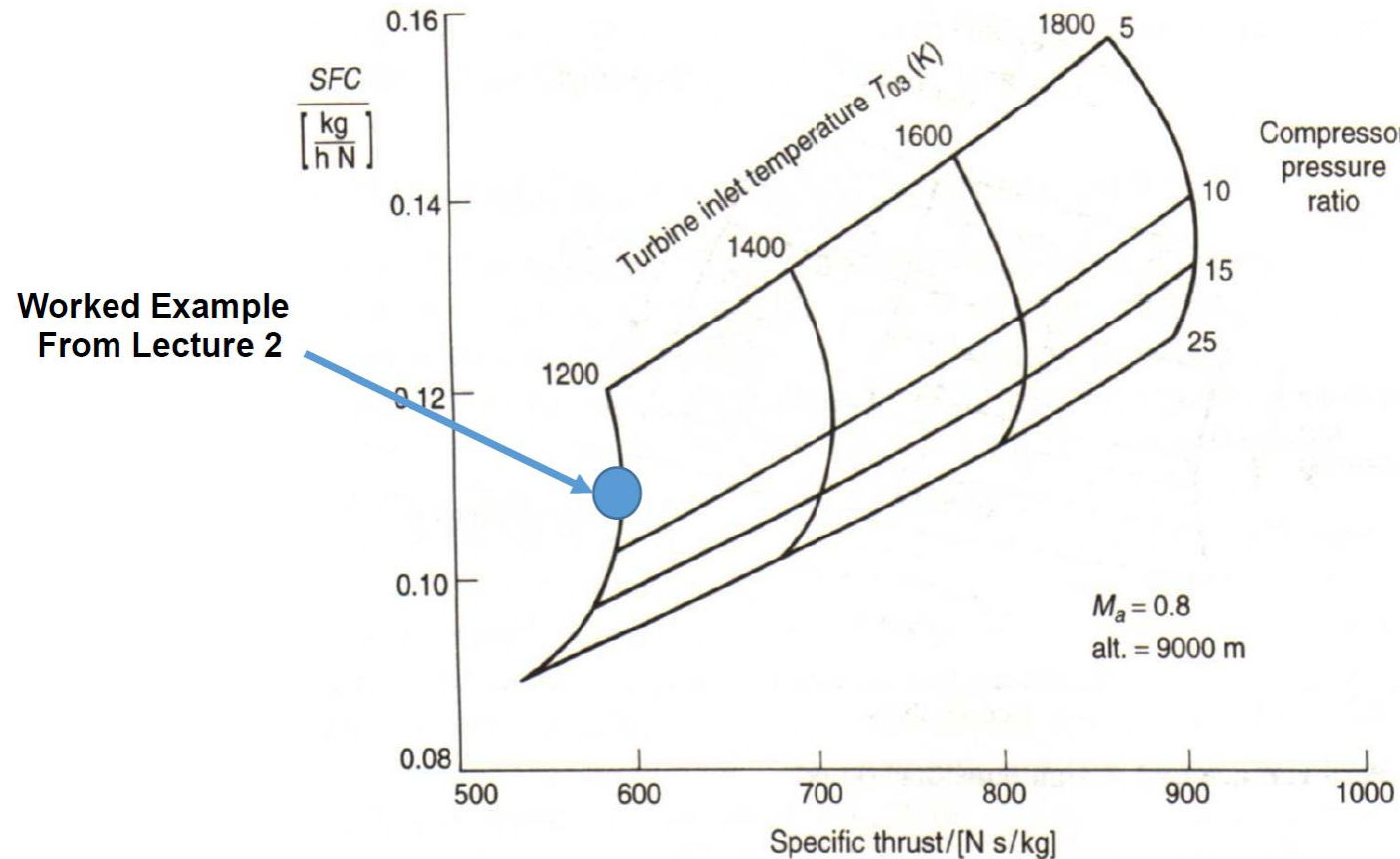
- The Design Point of an Engine is where all of the components are matched at their *Design Conditions* - often where each unit achieves design pressure ratio & peak efficiency at maximum flow.

Also sometimes called *Synthesis Matching Point*

- The Altitude & Mach Number conditions are usually those which are critical for the Aircraft/Engine Requirements.
- Typically the Design Point for a Subsonic Passenger Aircraft will be at the top of Climb i.e.  $M = 0.7 - 0.8$ , 35,000 ft. -  $T_{01} = \sim 240\text{K}$ .
- For a Military Aircraft, the Design Point will be at the critical condition for manoeuvring in Combat i.e.  $M = 1.8$ , 40,000 ft. -  $T_{01} = 357\text{K}$ .
- For a Helicopter the Design Point will be at  $M = 0$  at Sea level -  $T_{01} = 288\text{ K}$ .
- All other conditions are *Off-Design* where the pressure ratio, efficiency and flow are different from those at the design point.

# Performance of a range of Typical Turbojet Cycles

Choice of Cycle characteristics at the Design Point

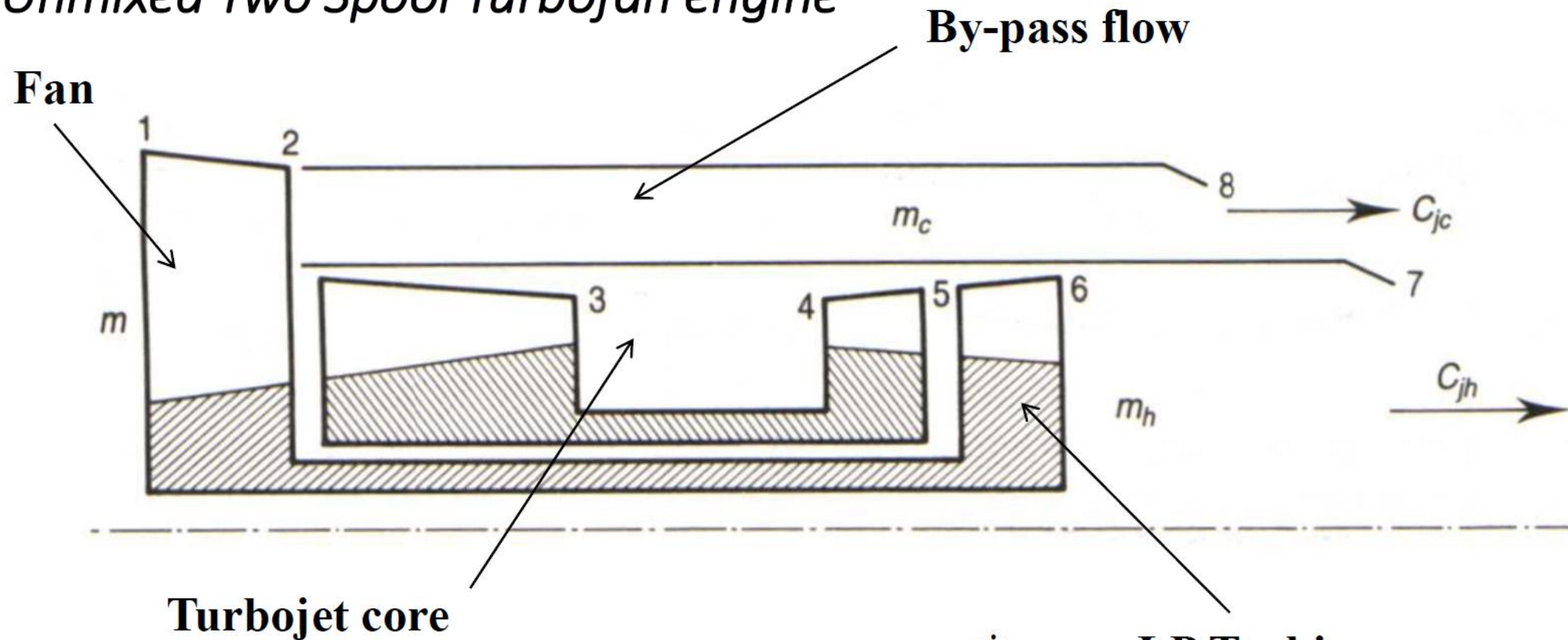


*Design Point calculations ~ each point is a different engine*



# Most “Jet” Engines are By-pass Engines

## *Unmixed Two Spool Turbofan engine*



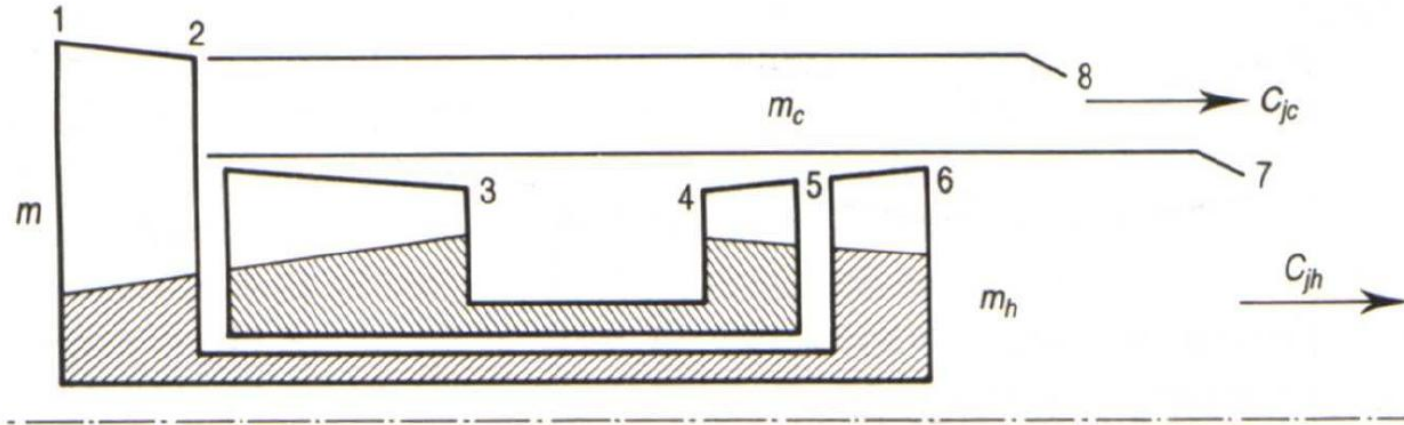
**Bypass ratio**  $\lambda = \frac{\dot{m}_c}{\dot{m}_h}$

$$\dot{m} = \dot{m}_c + \dot{m}_h$$

$$F_H = \dot{m}_H(C_{jH} - C_a) + A_j(P_j - P_a)$$
$$F_C = \dot{m}_C(C_{jC} - C_a)$$
$$F = F_H + F_C$$

# Most “Jet” Engines are By-pass Engines

## Unmixed Two Spool Turbofan engine *Key Cycle Parameters at the Design Point*



- Overall Pressure Ratio
- Fan Pressure Ratio
- Stator Outlet Temperature
- Specific Thrust
- By-pass Ratio

$$OPR = P_{03}/P_{01}$$

$$FPR = P_{02}/P_{01}$$

$$SOTK = T_{04}$$

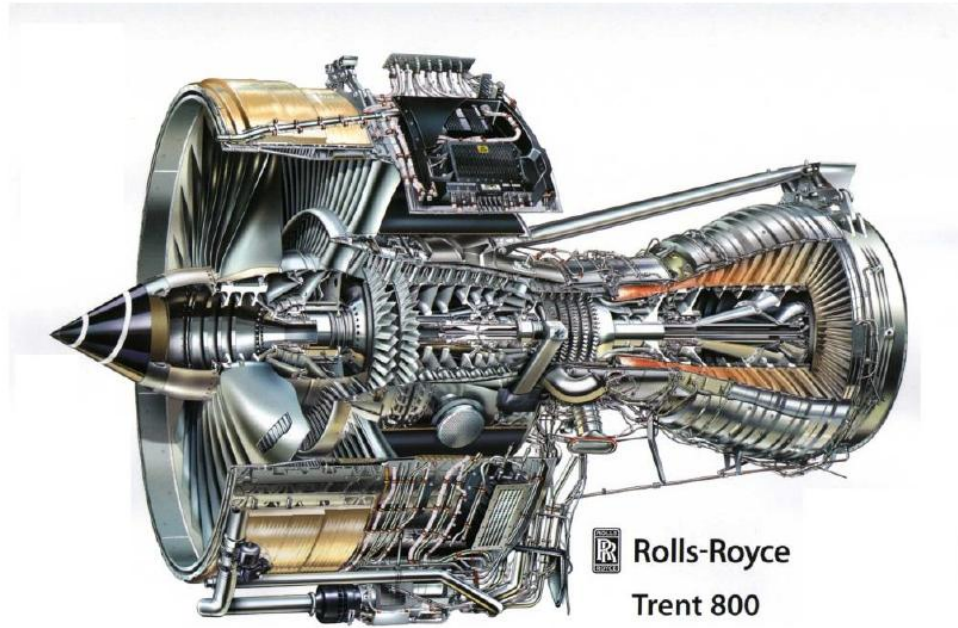
$$ST = F/\dot{m}$$

$$\lambda = \frac{m_c}{m_h} \quad (\text{by-pass flow/core flow})$$

***Note for mixed flow engines  $p_{sh}/p_{sc}$  is also important***



# Propulsion Systems for Transport & Combat Aircraft



## High by-pass ratio Turbofan

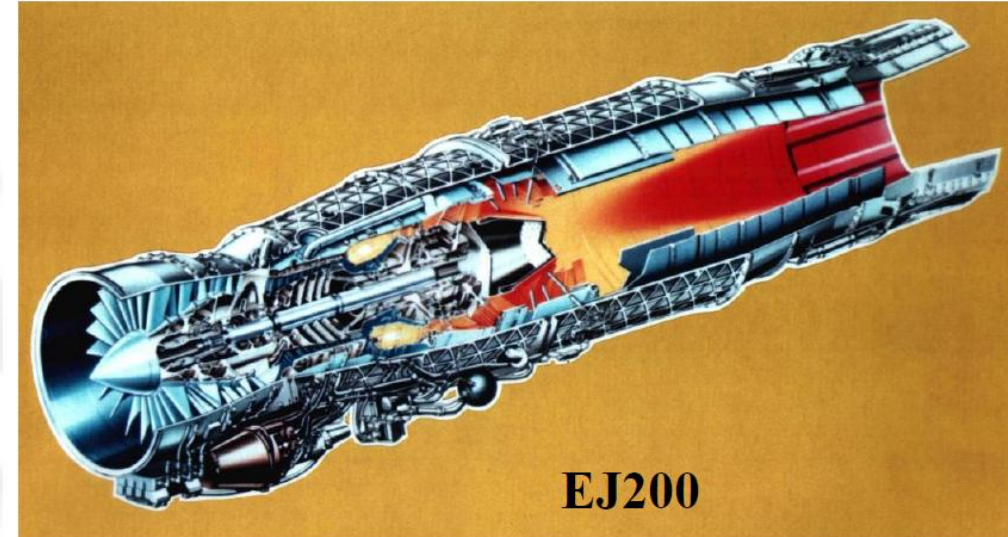
Thrust ~2000 to 100,000 lb

By-pass ratio 4 – 12

OPR ~ > 40

Fan PR ~ 1.9

Specific Thrust ~ 25 – 35 lb/lb/sec  
~ 250 – 350 m/s



## Low by-pass ratio Reheated Turbofan

Thrust ~10,000 to 40,000 lb (inc R/H)

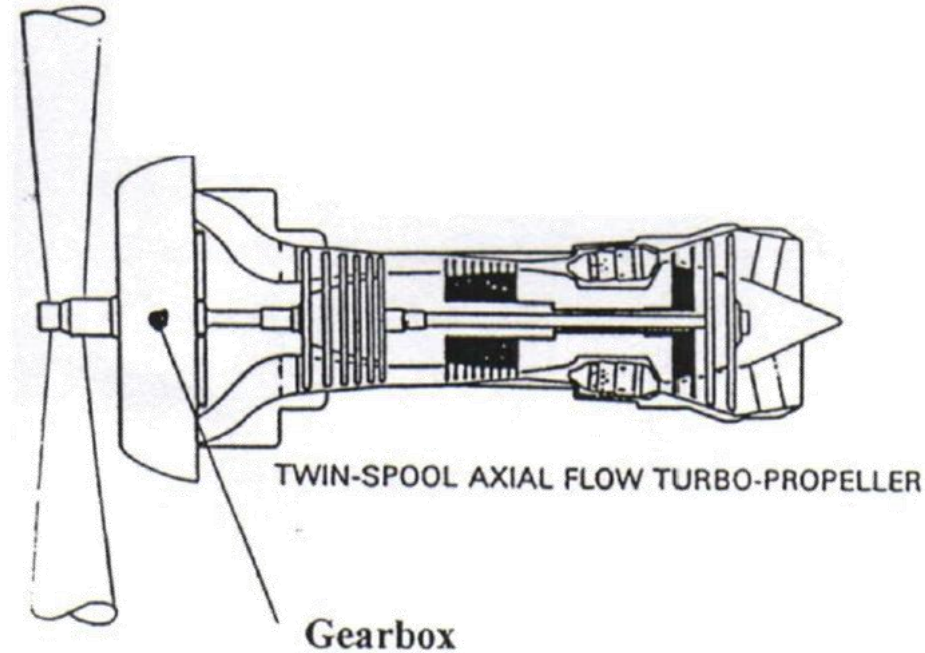
By-pass ratio 0.3 – 1

OPR ~ 25 – 30

Fan PR ~ 3 – 5

Specific Thrust ~ 100 lb/lb/sec (inc R/H)  
1000 m/s

# The turboprop engine



## **Total Thrust:**

Thrust from propeller plus jet thrust

## **Power:**

Shaft Power x Propeller efficiency + Jet thrust x Aircraft Velocity

$$P_{TOT} = \eta_{PROP} Q \Omega + FC_a$$

To compare with a piston engine which has no jet thrust the following is used:

## **Equivalent Shaft Power:**

Shaft Power + (Jet thrust x Aircraft Velocity)/Propeller efficiency



## *Turbofan Engine*

- **Overall Pressure Ratio:**
  - Total pressure at compressor delivery divided by that at entry to first compressor stage
- **Stator Outlet Temperature:**
  - Temperature of gas which does work at first turbine rotor
    - Also called Rotor Inlet Temperature or Turbine Inlet Temperature
- **Fan pressure ratio:**
  - Total pressure at fan delivery divided by that at fan entry

- **By-pass ratio:**
  - Ratio of mass flow of by-pass stream to that passing through combustion system
- **Component performance parameters:**
  - The characteristics of each component in terms of efficiency, flow capacity, pressure etc.
- **Pressure balance in exhaust jets:**
  - The ratio of the static pressure in the two stream should be close to unity.

- The **Intake** can be considered to be an **Adiabatic Duct** with no heat or work transfer.
- **Total Temperature remains constant.**
- The **losses** due to friction, shock waves etc. will be seen as a **reduction in total pressure**
- The main ways of defining Intake Performance are:
  - ***Inlet Pressure Ratio Factor*** (defined as the reduction in stagnation pressure in the inlet)
  - ***Ram Efficiency*** (defined in terms of pressure)
  - ***Isentropic Efficiency*** (defined in terms of temperature)



## Inlet Pressure Ratio Factor

Intake performance is usually quoted in terms of a pressure recovery factor  $\frac{p_{o1}}{p_{o\alpha}}$  i.e. the reduction stagnation pressure in the inlet.

## Ram Efficiency:

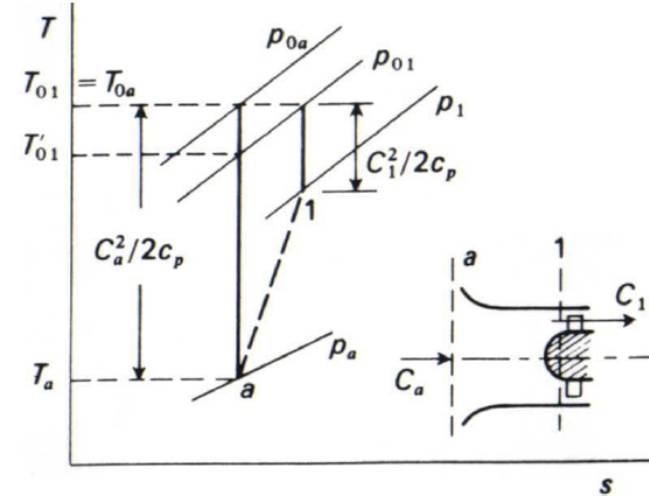
$$\eta_r = \frac{P_{o1} - P_a}{P_{o\alpha} - P_a}$$

# Intake Efficiency: Isentropic

## ISENTROPIC EFFICIENCY:

From T – s diagram:

$$T_{01} = T_{0a} = T + \frac{C_a^2}{2C_p} \quad \& \quad \frac{P_{01}}{P_a} = \left( \frac{T_{01}}{T_a} \right)^{\frac{\gamma}{\gamma-1}}$$



Where  $T'_{01}$  is the temperature that would have been reached after an isentropic compression to  $P_{01}$ .

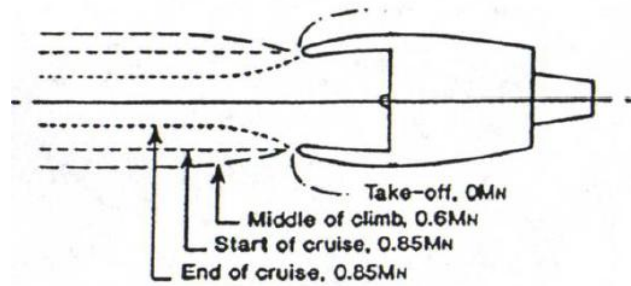
$T'_{01}$  can be related to  $T_{01}$  by Isentropic Efficiency:

$$\eta_{isen} = \frac{T'_{01} - T_{02}}{T_{01} - T_{02}} \quad \& \quad (T'_{01} - T_a) = \eta_{isen} \frac{C_a^2}{2C_p}$$

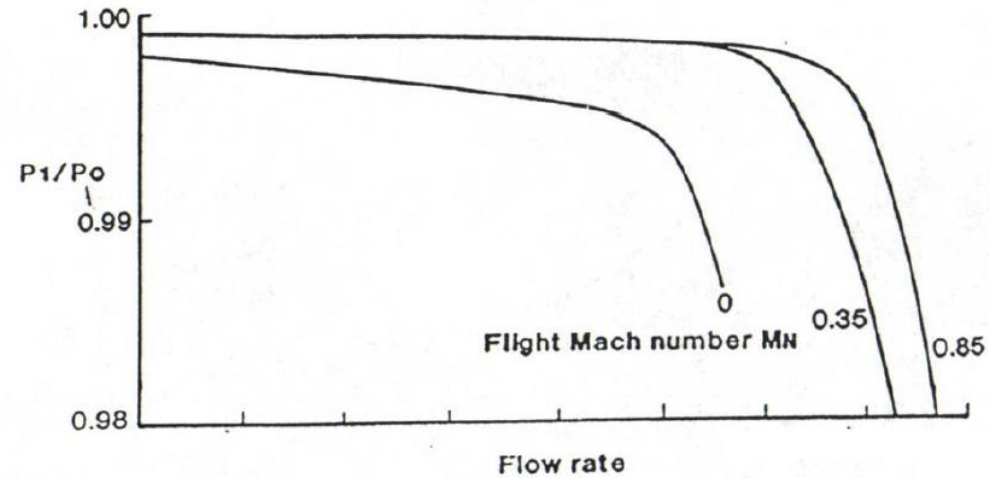
$$\frac{P_{01}}{P_a} = \left[ 1 + \frac{T'_{01} - T_a}{T_a} \right]^{\frac{\gamma}{\gamma-1}} \quad \frac{P_{01}}{P_a} = \left[ 1 + \eta_{isen} \frac{C_a^2}{2C_p T_a} \right]^{\frac{\gamma}{\gamma-1}}$$

$$M = \frac{C}{\sqrt{\gamma R T}} \quad \& \quad \gamma R = C_p(\gamma - 1) \quad \rightarrow \quad \frac{P_{01}}{P_a} = \left[ 1 + \eta_{isen} \frac{(\gamma - 1)}{2} M^2 \right]^{\frac{\gamma}{\gamma-1}}$$

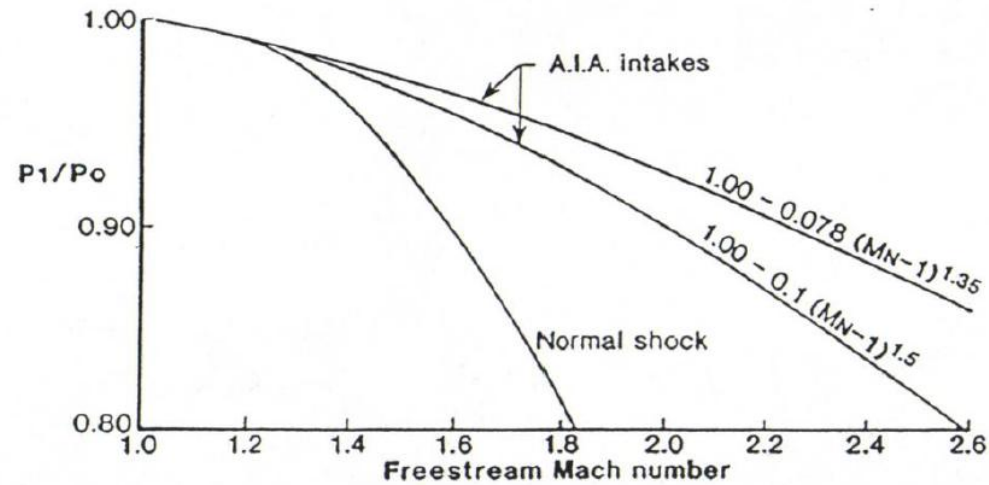
# Intake Performance



Intake flow stream tubes  
(typical subsonic transport)



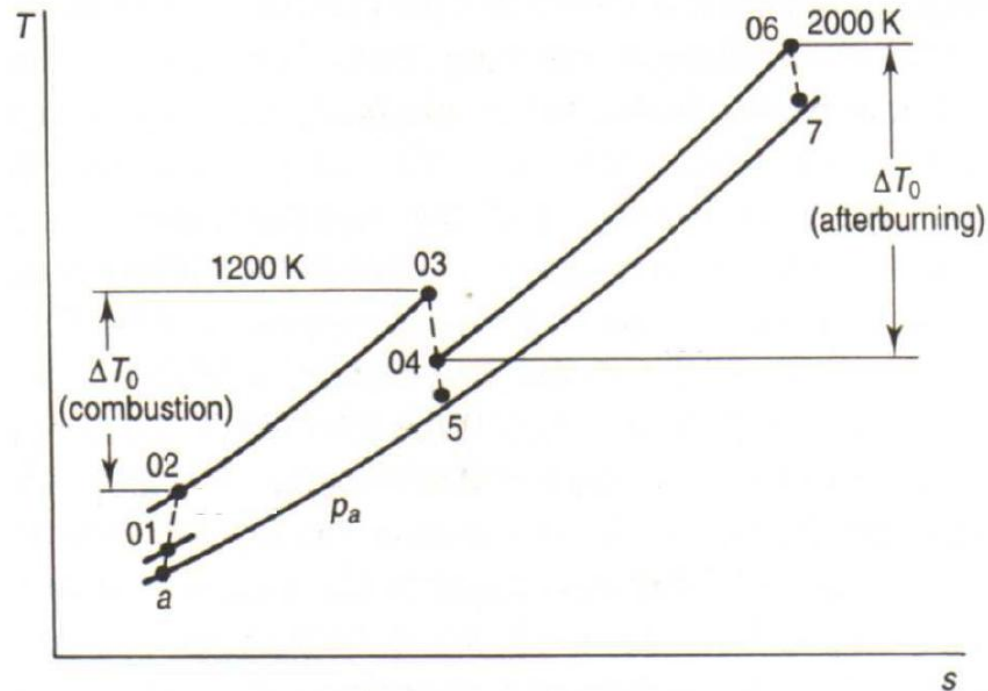
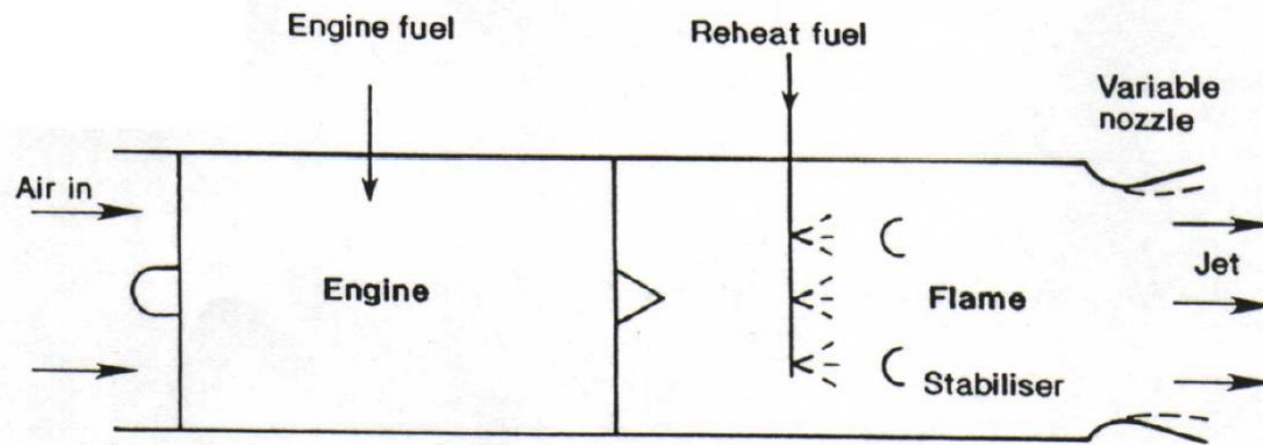
Subsonic flight intake recovery (typical) - Fixed geometry



Supersonic flight intake recovery - Variable geometry



# Thrust Augmentation



- *Net Thrust = Nozzle Gross Thrust – Momentum Drag*

- **NOZZLE GROSS THRUST:**

$$F_{N_{gross}} = \dot{m}_j \cdot C_j + A_j \cdot (P_j - P_a)$$

$\dot{m}C_a$

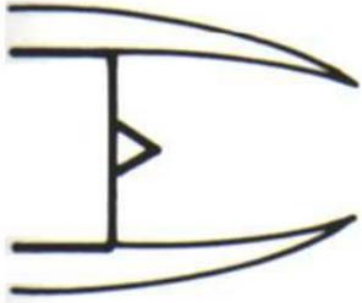
For a Convergent-Divergent or an un-choked Convergent Nozzle, the static pressure at the nozzle exit is equal to the ambient static pressure i.e.  $P_j = P_a$ . Hence the pressure term is 0.

- **IDEAL GROSS THRUST:**

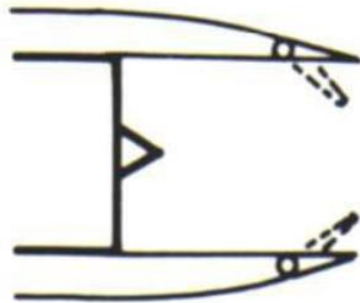
- The ideal gross thrust assuming that the flow entering the nozzle is expanded adiabatically & reversibly to the ambient static pressure i.e. a convergent-divergent nozzle with no losses.

# Nozzle Types

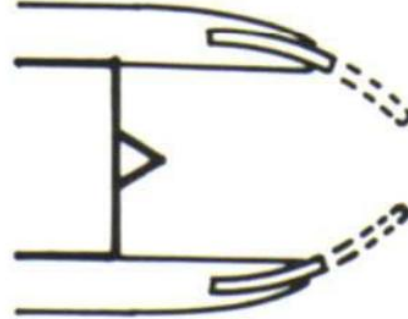
**FIXED  
CONVERGENT**



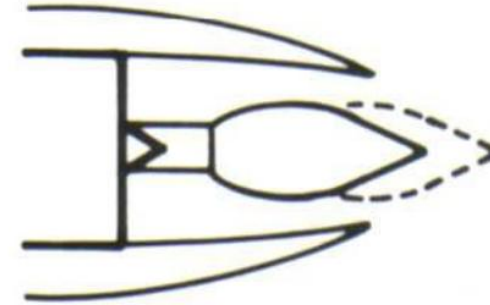
**VARIABLE  
CONVERGENT**



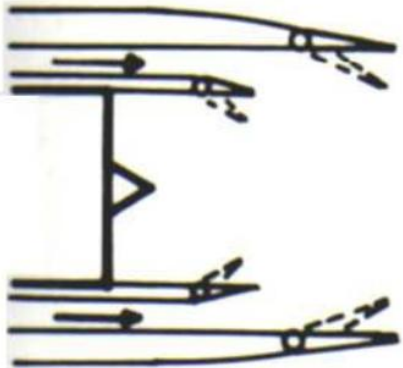
**CONVERGING  
IRIS**



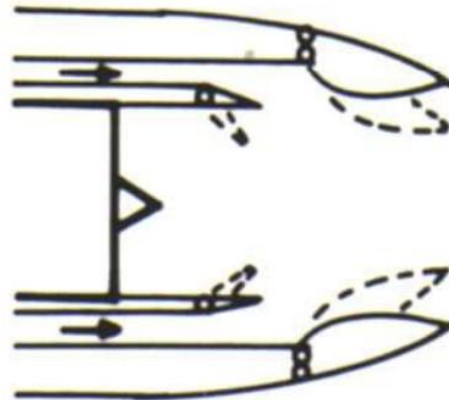
**TRANSLATING  
PLUG**



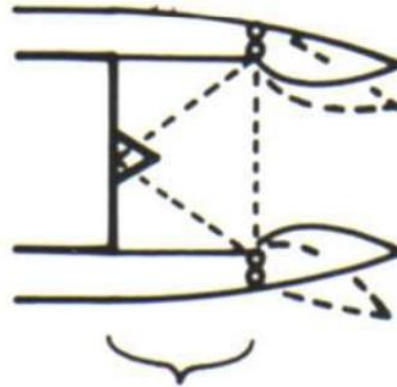
**EJECTOR**



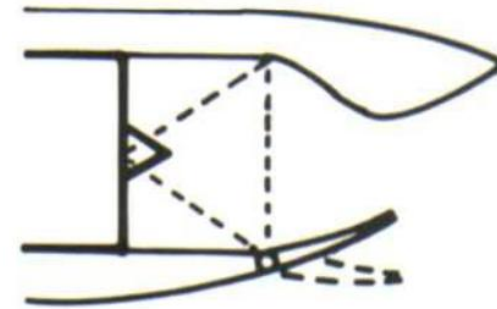
**CONVERGING-DIVERGING  
EJECTOR**



**2-D VECTORING**



**SINGLE  
EXPANSION  
RAMP (SERN)**



**CIRCLE-TO-SQUARE  
ADAPTER**



# Nozzle Types: Supersonic combat aircraft



**RB199 with Convergent Nozzle**



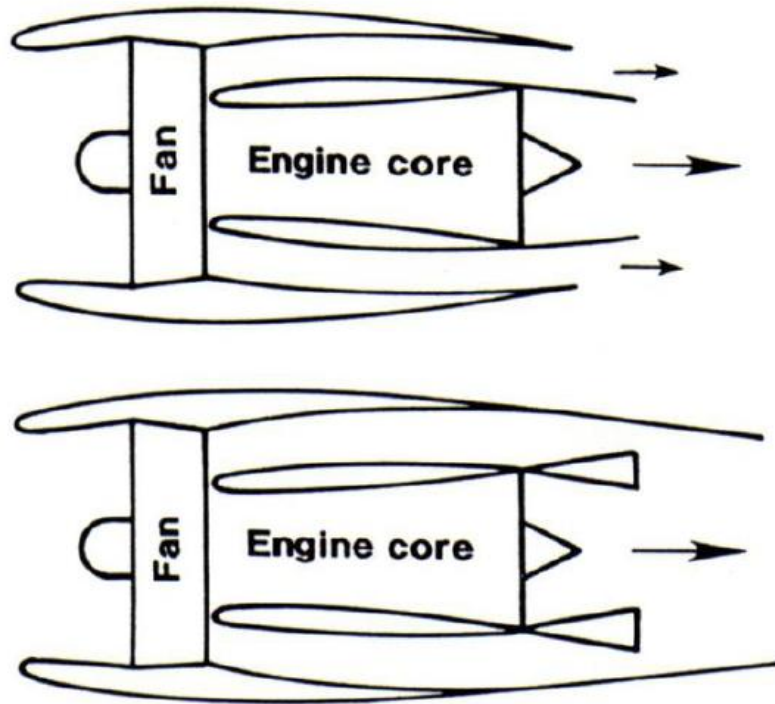
**EJ 200 with Con-di Nozzle**



**F35 with vectoring final Nozzle**

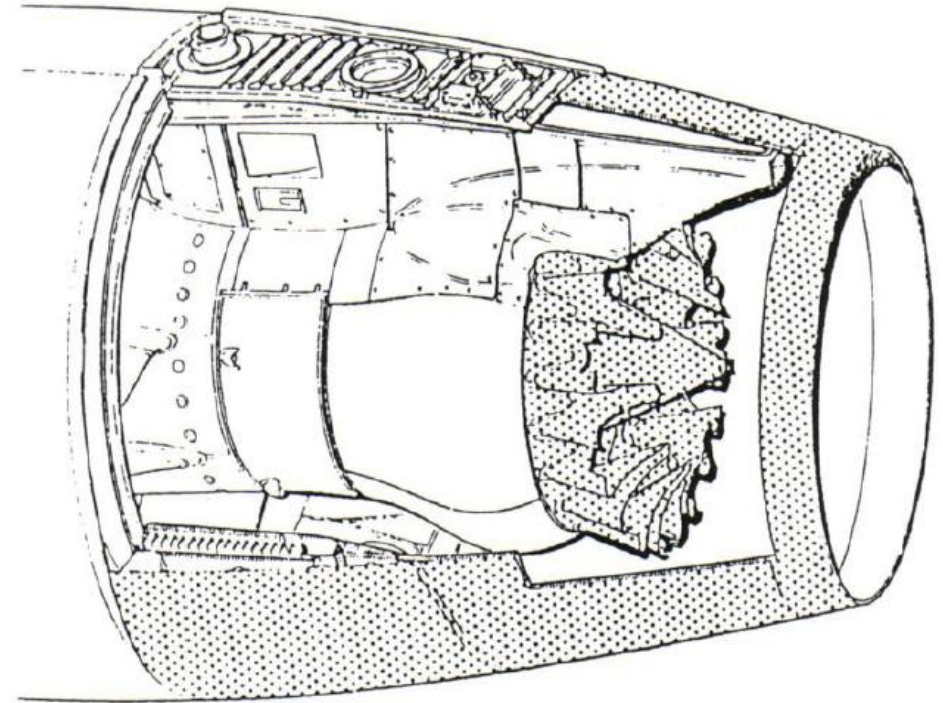
# Nozzle Types:

## Supersonic high-bypass ratio engines



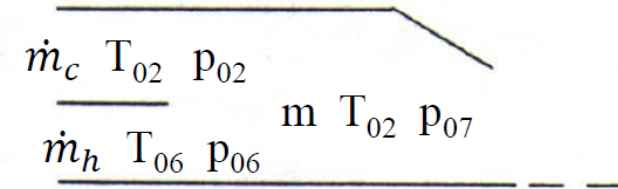
Separate jets

Mixed exhaust



Typical practical mixer

# Mixing of hot and cold streams



## Enthalpy Balance:

$$\dot{m}_c \cdot C_{p_c} \cdot T_{02} + \dot{m}_h \cdot C_{p_h} \cdot T_{06} = \dot{m} \cdot C_{p_m} \cdot T_{07}$$

$$\begin{aligned}\dot{m} &= \dot{m}_c + \dot{m}_h \\ \lambda &= \dot{m}_c / \dot{m}_h\end{aligned}$$

The properties of a mixture of gases to those of its constituents can be written as:

$$C_{p_m} = \frac{\dot{m}_c \cdot C_{p_c} + \dot{m}_h \cdot C_{p_h}}{\dot{m}_c + \dot{m}_h} \quad \& \quad \left( \frac{\gamma}{\gamma - 1} \right)_m = \frac{R_m}{C_{p_m}}$$

$$R_m = \frac{\dot{m}_c \cdot R_c + \dot{m}_h \cdot R_h}{\dot{m}_c + \dot{m}_h}$$

## Momentum Balance:

$$(\dot{m}_c \cdot C_c + P_2 \cdot A_2) + (\dot{m}_h \cdot C_h + P_6 \cdot A_6) = \dot{m} \cdot C_7 + P_7 \cdot A_7$$

(If there is no swirl in the duct then the static pressure will be uniform i.e.  $P_2 = P_6$ )

Continuity gives:  $\dot{m} = \rho_7 \cdot C_7 \cdot A_7$

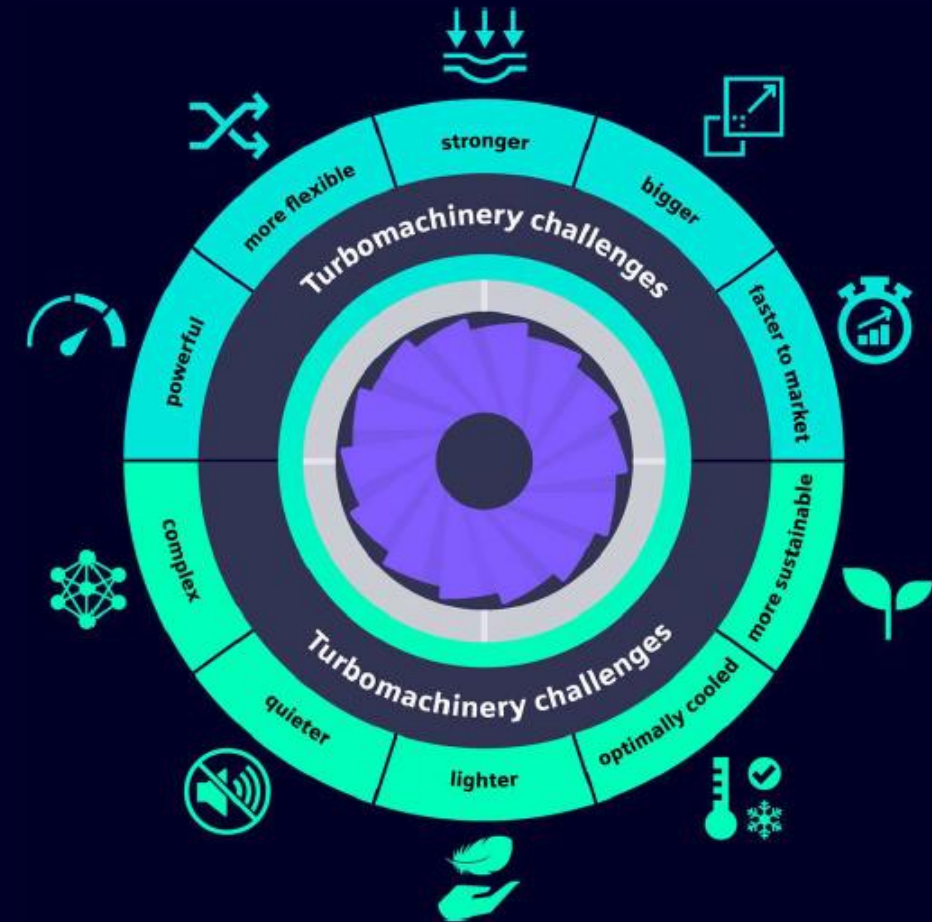
$p_{07}$  is required for the cycle calculation & this is best done using an iterative procedure.



## Design Challenges

For a given Design Point, a number of mechanical design parameters must be kept within limits

- Shaft Speeds
- Compressor delivery temperature and pressure
- Fan flutter
- Vibration levels of rotating components
- Disc and blade stress levels
- Cycling and creep life of major components
- Oxidation levels of components
- Rotating flow instabilities



Courtesy of Siemens DISW

# Key take-aways of Lecture 3

- The difference between Design Point & Off design Performance
- The key parameters for a Turbofan Engine
- How Intake Performance is characterized
- Nozzle types & how their performance is calculated
- Turbo prop & shaft power performance
- The data required for a full performance analysis
- The main Mechanical Design parameters

# What's in Lecture 4?

- Design Points for different types of platforms
- Fundamental Dimensionless relationships
- The calculation of Off design Performance
- How thrust & fuel consumption varies with inlet conditions i.e. Altitude, Mach Number & throttle setting