University of BRISTOL

TRIMMING AND LINEARISATION

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Bristol and Gloucestershire Gliding Club

Tug Aircraft: 141hp EuroFox 2K

Flight Dynamics Principles – Book References (Sessions 12, 13 & 14)

- Chapter 3: Static equilibrium and trim
  - Section 3.6 – Calculation of aircraft trim condition
- Chapter 4: The equations of motion
  - *Section 4.1 – The equations of motion of a rigid symmetric aircraft*
  - *Section 4.4 – Alternative forms of the equations of motion*
- Chapter 5: The Solution of the Equations of Motion
  - *Section 5.1 – Methods of solution*
  - *Section 5.6 – The state space method*
  - *Section 5.7 – State space model augmentation*
- Chapter 6: Longitudinal Dynamics
  - *Section 6.1 – Response to controls*
  - *Section 6.2 – The dynamic stability modes*
- Chapter 7: Lateral-Directional Dynamics
  - *Section 7.1 – Response to controls*
  - *Section 7.2 – The dynamic stability modes*

Flight Dynamics Principles: A Linear Systems Approach to Aircraft Stability and Control (Elsevier Aerospace Engineering) Hardcover – 9 Aug 2007

Trimming and Linearisation

Airbus A350-1000

https://www.britishairways.com

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6DOF Equations of Motion

Equations (8), (16), (17) and (18) form the 12 equations of motion for general atmospheric 6 degree-of-freedom flight.

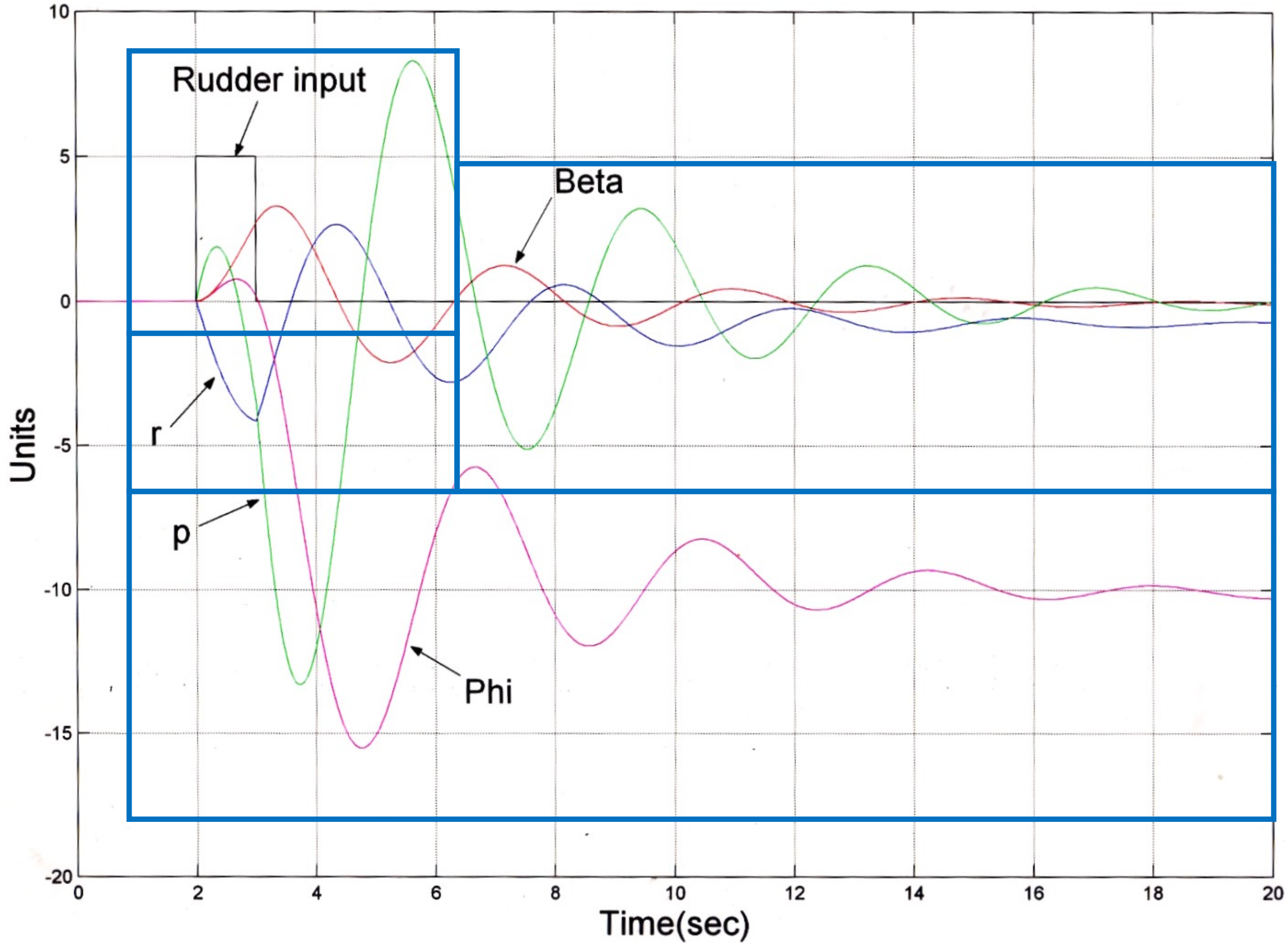
They are all first-order differential equations that can be expressed in nonlinear form as:

$$\dot{x} = f(x, \delta) \tag{19}$$

where $x = [U, V, W, p, q, r, \psi, \theta, \phi, x_E, y_E, z_E]'$ (state vector)

and δ is the vector of input parameters of interest. These are often aerodynamic or propulsion system inputs such as aileron, elevator, rudder deflection or thrust (upon which the formulation of the forces X, Y and Z will depend).

How can you define a trimmed condition?



Rudder input
Beta
r
p
Phi
Units
Time(sec)
10
5
0
-5
-10
-15
-20
0
2
4
6
8
10
12
14
16
18
20

Alternative Form of the Equations

TABLE 2.4-1. The Flat-Earth, Body Axes 6-DOF Equations

Force Equations

$$\dot{U} = RV - QW - g_0' \sin\theta + \frac{F_x}{m}$$

$$\dot{V} = -RU + PW + g_0' \sin\phi \cos\theta + \frac{F_y}{m} \qquad (2.4\text{-}2)$$

$$\dot{W} = QU - PV + g_0' \cos\phi \cos\theta + \frac{F_z}{m}$$

Kinematic Equations

$$\dot{\phi} = P + \tan\theta(Q \sin\phi + R\cos\phi)$$

$$\dot{\theta} = Q\cos\phi - R\sin\phi \qquad (2.4\text{-}3)$$

$$\dot{\psi} = \frac{Q\sin\phi + R\cos\phi}{\cos\theta}$$

Reference: Aircraft Control & Simulation, Stevens & Lewis, Wiley (1st edition)

Trimming: Introduction

- ‘The object of trimming is to bring the forces and moments acting on the aircraft into a state of equilibrium. That is the condition when the axial, normal and side forces, and the roll, pitch and yaw moments are all zero.’
- ‘Provided that the aircraft is stable it will then stay in equilibrium until it is disturbed by pilot control inputs or by external influences such as turbulence.’
- ‘The transient motion following such a disturbance is characterised by the dynamic stability characteristics and the stable aircraft will eventually settle into its equilibrium state once more.’
- ‘For a given aircraft mass, cg position, altitude and airspeed, symmetric trim is described by the aerodynamic operating condition, namely angle of attack, thrust, pitch attitude, elevator angle and flight path angle.’

Flight Dynamics Principles: A Linear Systems Approach to Aircraft Stability and Control (Elsevier Aerospace Engineering) Hardcover – 9 Aug 2007

Linearisation: Introduction

- ‘The equations (of motion for an aircraft) are non-linear and their solution by analytical means is not generally practicable.’ (TSR PhD)
- ‘In order to proceed with the development of the equations of motion for analytical purposes, they must be linearised.’
- ‘..for the vast majority of aeroplanes when small perturbation transient motion only is considered, as is the case here, longitudinal–lateral coupling is usually negligible..’
- ‘For small perturbations, the aeroplane is a classical example of a linear dynamic system and frequently the solution of its equations of motion is a prelude to flight control system design and analysis..’

Flight Dynamics Principles: A Linear Systems Approach to Aircraft Stability and Control (Elsevier Aerospace Engineering) Hardcover – 9 Aug 2007

Trimming & Linearisation

$$\dot{x} = f(x, \delta)$$

Numerical Trimming
e.g. Matlab 'trim' function

$\dot{x} = ?$ e.g. find the control inputs required for a specified trim condition

Simplify Analytically

Numerical Linearisation – e.g.

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.068 & -0.011 & -0.049 & -9.81 \\ 0.023 & -2.10 & 366.0 & 0 \\ 0.011 & -0.160 & -9.52 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} -0.41 \\ -77.0 \\ -61.0 \\ 0 \end{bmatrix} \eta$$

Small Perturbation Equations – e.g.

$$\begin{bmatrix} ms - X_u & -X_w & mg - X_q s \\ -Z_u & ms - Z_w & -mUs - Z_q s \\ -M_u & -M_{\dot{w}} s - M_w & I_{yy} s^2 - M_q s \end{bmatrix} \begin{bmatrix} u \\ w \\ \theta \end{bmatrix} = \begin{bmatrix} X_\eta & X_\delta \\ Z_\eta & Z_\delta \\ M_\eta & M_\delta \end{bmatrix} \begin{bmatrix} \eta \\ \delta \end{bmatrix} - w_g \begin{bmatrix} X_w \\ Z_w \\ M_w \end{bmatrix}$$

Analysis of Aircraft Modes
and Handling Qualities

e.g. Eigenvalues & Eigenvectors

Cranfield University Jetstream: Example From ‘Flight Dynamics Principles’

- To illustrate the use of ‘AeroTrim’ it is applied to the Cranfield University Jetstream 31 flying laboratory aircraft.
- The sources of data used include manufacturer’s published technical information, flight manual, limited original wind tunnel test data and data obtained from flight experiments. Aerodynamic data not provided by any of these sources were estimated using the ESDU Aerodynamics Series (2006) and refined by reference to observed flight behaviour.
- The chosen operating condition is typical for the aircraft, and the speed range was chosen to vary from the stall, at around 100 kt, to 250 kt in 15 kt increments.
- Running the programme returns trim data for the chosen operating flight condition, of which a reduced selection is shown.
- Trimmed Elevator & Alpha for a given Airspeed

Cook, M.V. Flight Dynamics Principles. Arnold, London, 2012.

Cranfield University Jetstream: Example From 'Flight Dynamics Principles'

Flight condition

	Units	Value
Aircraft weight	kN	61.8
Altitude	ft	6562
Flight path angle	deg	0
cg position		0.29
Neutral point		0.412
Static margin		0.122
Minimum drag speed	kt	150
Stall speed	kt	116

Is this aircraft stable?

Cook, M.V. Flight Dynamics Principles. Arnold, London, 2012.

Cranfield University Jetstream: Example From 'Flight Dynamics Principles'

Example trim data

(e = equilibrium)

Is this aircraft stable?

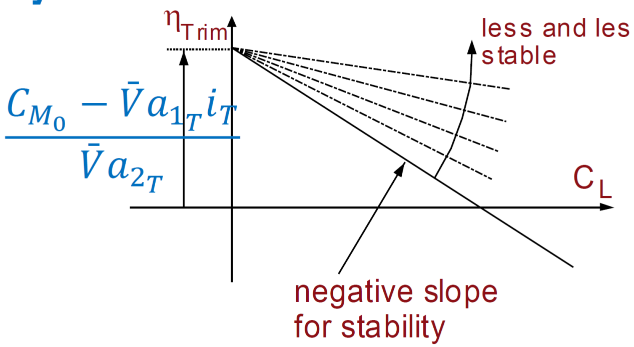
V_{true} (knots)	C_L	C_D	C_τ	L/D	α_e (deg)	η_e (deg)	L (kN)	D (kN)	τ_e (kN)
100	1.799	0.174	0.181	9.409	15.105	−1.208	60.23	5.834	6.042
115	1.374	0.114	0.116	11.017	10.885	−0.460	60.83	5.053	5.146
130	1.081	0.082	0.083	12.106	7.970	0.100	61.15	4.643	4.688
145	0.872	0.064	0.064	12.603	5.885	0.521	61.34	4.494	4.518
160	0.717	0.053	0.053	12.573	4.346	0.842	61.46	4.535	4.548
175	0.600	0.046	0.046	12.154	3.181	1.091	61.54	4.722	4.729
190	0.510	0.042	0.042	11.496	2.277	1.287	61.60	5.025	5.029
205	0.438	0.039	0.039	10.720	1.564	1.444	61.65	5.424	5.426
220	0.381	0.036	0.036	9.912	0.990	1.572	61.70	5.907	5.908
235	0.334	0.035	0.035	9.123	0.523	1.677	61.74	6.465	6.465
250	0.295	0.034	0.034	8.383	0.136	1.764	61.79	7.089	7.089

Cook, M.V. Flight Dynamics Principles. Arnold, London, 2012.

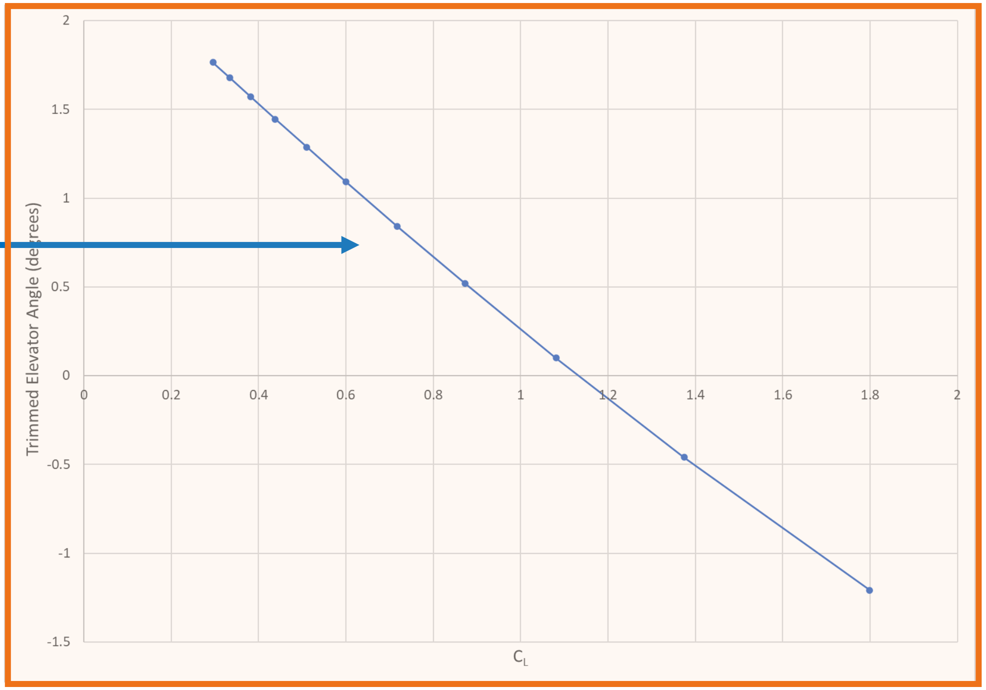
Cranfield University Jetstream: Example From ‘Flight Dynamics Principles’

From Week 4

Static Stability



- Because the bracketed terms in Equations (3) & (4) are the same, the *boundary* between stable and unstable flight is shown by a zero value for that term and thus a horizontal line for the η_{trim} vs C_L graph.
- C.G. change = data for another line (moving significant masses!)



Pick a trim condition =>

Cook, M.V. Flight Dynamics Principles. Arnold, London, 2012.

F16 Aircraft

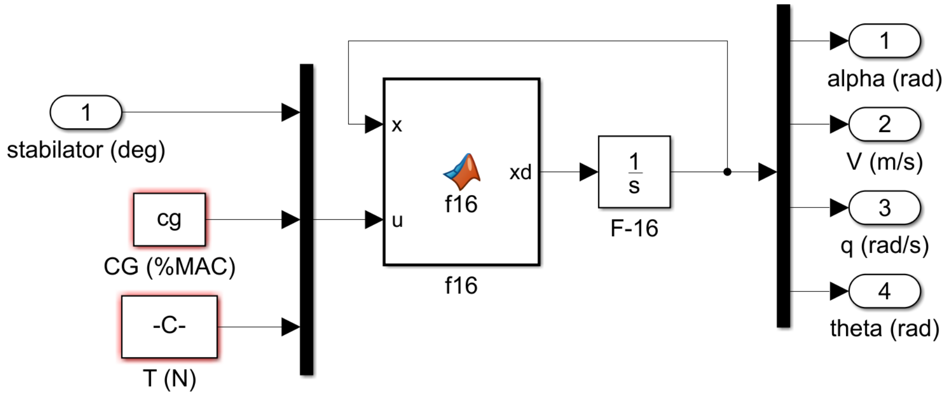
F-16 Air to Air with Stefan "VADOR" Darte
Belgian Display Pilot 2018-2021 Air 2 Air

https://www.youtube.com/watch?v=b9Q5R9oBrmo&ab_channel=Marc%27sBestAirshowVideosbyMarcTalloen

F16 Aircraft: MATLAB Version – Longitudinal Trim

```
%% Initialise
% Define the trim condition
cg       = 25; % CG position (%MAC)
gamma    = 0; % flight path angle (deg)
Vtrim    = 200; % Velocity (m/s)

% Guess the trim solution
alpha_guess = 5; % angle of attack (deg)
dstab       = -5;  % stabilator (deg)
T           = 10000; % thrust (N)
```

```
%% Find the trim point
open_system('f16_trim')
set_param('f16_trim','InitInArrayFormatMsg','None');
feval('f16_trim',[],[],[],'compile');

% options = optimset('TolFun',1.0e-06,'TolX',1.0e-06,
'MaxIter',6000,'MaxFunEvals',6000);
[trimvar,fval]=fminsearch(@cost,[dstab, T, alpha0]);

feval('f16_trim',[],[],[],'term');
close_system('f16_trim')
```



Week 3 – Longitudinal Balance

Week 3 – Elevator Angle to Trim

[4] Nguyen, L.T. *et al*, 1979. Simulator study of stall/post stall characteristics of a fighter aircraft with relaxed longitudinal stability.

[5] Dr Duc H Nguyen, University of Bristol – Matlab Version of the F-16 Model

F16 Aircraft: MATLAB Version – Longitudinal Trim

```
function J = cost(trimvar0)
```

```
global cg gamma Vtrim x_eval
```

(not recommended!)

```
u_eval(1)  = trimvar0(1); % stabilator (deg)
u_eval(2)  = cg;          % cg (%MAC)
u_eval(3)  = trimvar0(2); % thrust (N)

x_eval(1)  = trimvar0(3);                % alpha (rad)
x_eval(2)  = Vtrim;                      % V (m/s)
x_eval(3)  = 0;                          % q (rad/s)
x_eval(4)  = trimvar0(3)+gamma/180*pi;   % theta (rad)
```

Define inputs and states

```
o=f16_trim(0,x_eval(1:4),u_eval(1:3),'outputs');
f=f16_trim(0,x_eval(1:4),u_eval(1:3),'derivs');
```

Evaluate

```
J = f(1)^2 + 0.01*f(2)^2 + f(3)^2 + f(4)^2 ;
%   alphadot^2 + 0.01 Vdot^2 + qdot^2 + thetadot^2;
```

Cost function

Note the weighting

How would you define and solve for other trim conditions?

[4] Nguyen, L.T. *et al*, 1979. Simulator study of stall/post stall characteristics of a fighter aircraft with relaxed longitudinal stability.

[5] Dr Duc H Nguyen, University of Bristol – Matlab Version of the F-16 Model

F16 Aircraft: MATLAB Version – Longitudinal Trim

- Longitudinal trim for sea level flight conditions at 160 m/s
- MATLAB trim output

```
TRIM CONDITION:
    gamma      = 0 deg
    CG         = 25% MAC
    velocity   = 160 m/s
TRIM SOLUTION:
    alpha      = 3.4693 deg
    theta      = 3.4693 deg
    stabilator = -5.0899 deg
    thrust     = 16005.1855 N
```

(compare the trim conditions)

```
TRIM CONDITION:
    gamma      = 0 deg
    CG         = 35% MAC
    velocity   = 160 m/s
TRIM SOLUTION:
    alpha      = 3.2343 deg
    theta      = 3.2343 deg
    stabilator = -3.1406 deg
    thrust     = 15742.7645 N
```

(Note: number of decimal places could be reduced!)

[4] Nguyen, L.T. *et al*, 1979. Simulator study of stall/post stall characteristics of a fighter aircraft with relaxed longitudinal stability.

[5] Dr Duc H Nguyen, University of Bristol – Matlab Version of the F-16 Model

F16 Aircraft: Linearisation

linmod

Extract continuous-time linear state-space model around operating point

In continuous-time, a state-space model is of the following form:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

```
%% Linearise at trim point
sys=linmod('f16_linearise',[trimvar(3), Vtrim, 0, trimvar(3)+gamma/180*pi],trimvar(1));
sys=ss(sys.a, sys.b, sys.c, sys.d);
sys.StateName = {'alpha', 'V',  'q',     'theta'};
sys.StateUnit = {'rad',   'm/s','rad/s', 'rad'};
sys.InputName = {'stabilator'};
sys.InputUnit = {'deg'};
```

Here, x, u and y represent the states, inputs and outputs respectively, while A, B, C and D are the state-space matrices. The `ss` object represents a state-space model in MATLAB® storing A, B, C and D along with other information such as sample time, names and delays specific to the inputs and outputs.

`linmod` compute a linear state-space model by linearizing each block in a model individually.

The default algorithm uses preprogrammed analytic block Jacobians for most blocks which should result in more accurate linearization than numerical perturbation of block inputs and states. A list of blocks that have preprogrammed analytic Jacobians is available in the Simulink Control Design documentation along with a discussion of the block-by-block analytic algorithm for linearization.

So linearising the model for both CG positions….

[4] Nguyen, L.T. *et al*, 1979. Simulator study of stall/post stall characteristics of a fighter aircraft with relaxed longitudinal stability.

[5] Dr Duc H Nguyen, University of Bristol – Matlab Version of the F-16 Model

F16 Aircraft: CG 25% at 160 m/s

Week 7 – Aircraft Modes

- Phugoid & Short Period modes

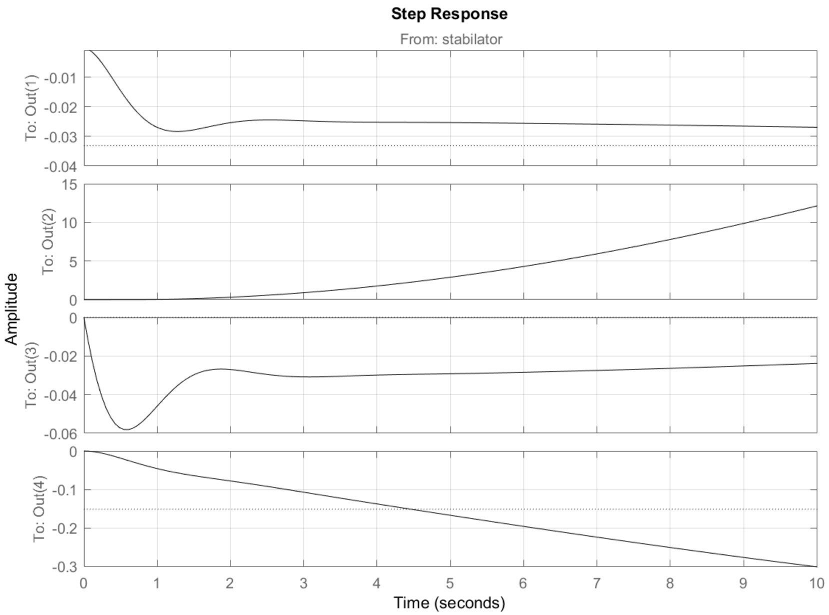
A =

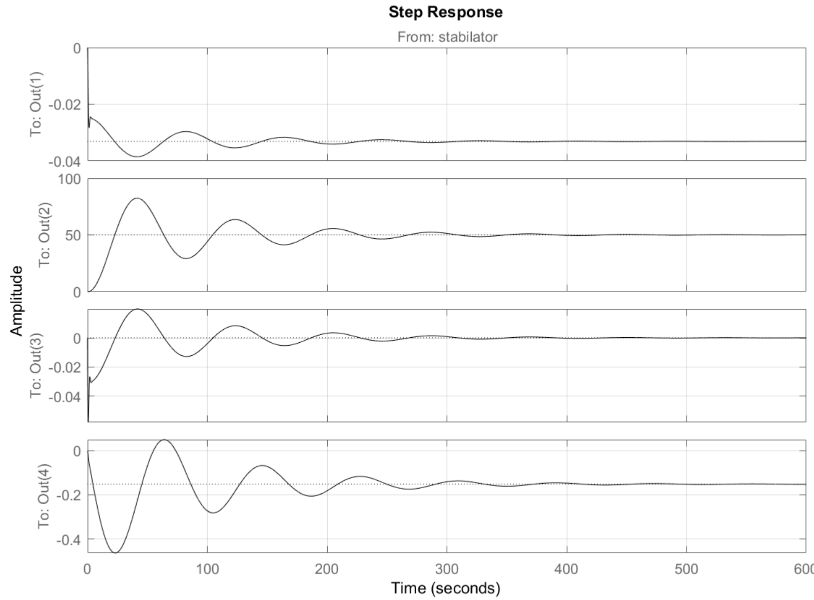
	alpha	V	q	theta
alpha	-1.219	-0.0007583	0.9042	0
V	13.34	-0.02149	0.2331	-9.81
q	-6.707	-3.453e-12	-1.821	0
theta	0	0	1	0

B =

	stabilator
alpha	-0.002432
V	0.03282
q	-0.2222
theta	0

```
% Linear analysis
figure; pzmap(sys,'k');
figure; step(sys,'k-');
figure; bode(sys,'k-');
```





Additional Examples: Linearisation

- Longitudinal: Lockheed F-104 Starfighter
  - Note the use of different states
  - Eigenvalues & eigenvectors
- Longitudinal: Ling-Temco-Vought A-7A Corsair II Aircraft
  - What are the similarities to the Starfighter?
  - Longitudinal time response
- Lateral-Directional: Douglas DC-8 Aircraft
  - Note aircraft states and inputs used
  - Response to the rudder input

Flight Dynamics Principles: A Linear Systems Approach to Aircraft Stability and Control (Elsevier Aerospace Engineering) Hardcover – 9 Aug 2007

Lockheed F-104 Starfighter

$$[U, \cancel{V}, W, \cancel{p}, q, \cancel{r}, \cancel{\phi}, \theta, \cancel{\psi}, \cancel{x_E}, \cancel{y_E}, \cancel{z_E}]$$

$$\boxed{\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix}} = \boxed{\begin{bmatrix} -0.0352 & 0.1070 & 0 & -32.2 \\ -0.2140 & -0.4400 & 305 & 0 \\ 1.198 \times 10^{-4} & -0.0154 & -0.4498 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}} \boxed{\begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix}} + \begin{bmatrix} 0 \\ -22.1206 \\ -4.6580 \\ 0 \end{bmatrix} \boxed{\eta}$$

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

Professor Tom Richardson

Wikipedia

bristol.ac.uk

Cook, M.V. Flight Dynamics Principles. Arnold, London, 2012.

Lockheed F-104 Starfighter

which defines the matrices **A** and **B** and the vectors $\mathbf{x}(t)$ and $\mathbf{u}(t)$. Using the computer software package *MATLAB* interactively the diagonal eigenvalue matrix is calculated:

$$\boldsymbol{\Lambda} = \begin{bmatrix} -0.4459+2.1644j & 0 & 0 & 0 \\ 0 & -0.4459-2.1644j & 0 & 0 \\ 0 & 0 & -0.0166+0.1474j & 0 \\ 0 & 0 & 0 & -0.0166-0.1474j \end{bmatrix}$$

$$\equiv \begin{bmatrix} \lambda_s & 0 & 0 & 0 \\ 0 & \lambda_s^* & 0 & 0 \\ 0 & 0 & \lambda_p & 0 \\ 0 & 0 & 0 & \lambda_p^* \end{bmatrix}$$

Wikipedia

Cook, M.V. Flight Dynamics Principles. Arnold, London, 2012.

Lockheed F-104 Starfighter

and the $\boxed{\text{corresponding eigenvector matrix}}$ is calculated:

$$\mathbf{V} = \begin{bmatrix} 0.0071 - 0.0067j & 0.0071 + 0.0067j & -0.9242 - 0.3816j & -0.9242 + 0.3816j \\ 0.9556 - 0.2944j & 0.9556 + 0.2944j & 0.0085 + 0.0102j & 0.0085 - 0.0102j \\ 0.0021 + 0.0068j & 0.0021 - 0.0068j & -0.0006 - 0.0002j & -0.0006 + 0.0002j \\ 0.0028 - 0.0015j & 0.0028 + 0.0015j & -0.0012 + 0.0045j & -0.0012 - 0.0045j \end{bmatrix}$$

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix}$$

Check relative magnitudes of the eigenvectors.

Wikipedia

Cook, M.V. Flight Dynamics Principles. Arnold, London, 2012.

Ling-Temco-Vought A-7A Corsair II Aircraft

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0.00501 & 0.00464 & -72.90000 & -31.34000 \\ -0.08570 & -0.54500 & 309.00000 & -7.40000 \\ 0.00185 & -0.00767 & -0.39500 & 0.00132 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 5.63000 \\ -23.80000 \\ -4.51576 \\ 0 \end{bmatrix} \eta$$

Useful MATLAB Commands:

[Wn,Z] = **damp**(SYS) returns vectors Wn and Z containing the natural frequencies and damping factors of the linear system SYS. For

E = **eig**(A) produces a column vector E containing the eigenvalues of a square matrix A.

[V,D] = **eig**(A) produces a diagonal matrix D of eigenvalues and a full matrix V whose columns are the corresponding eigenvectors so that A*V = V*D.

Wikipedia

Cook, M.V. Flight Dynamics Principles. Arnold, London, 2012.

Ling-Temco-Vought A-7A Corsair II Aircraft

```
Corsair =

    0.0050    0.0046  -72.9000  -31.3400
   -0.0857   -0.5450  309.0000   -7.4000
    0.0019   -0.0077   -0.3950    0.0013
         0         0    1.0000         0
```

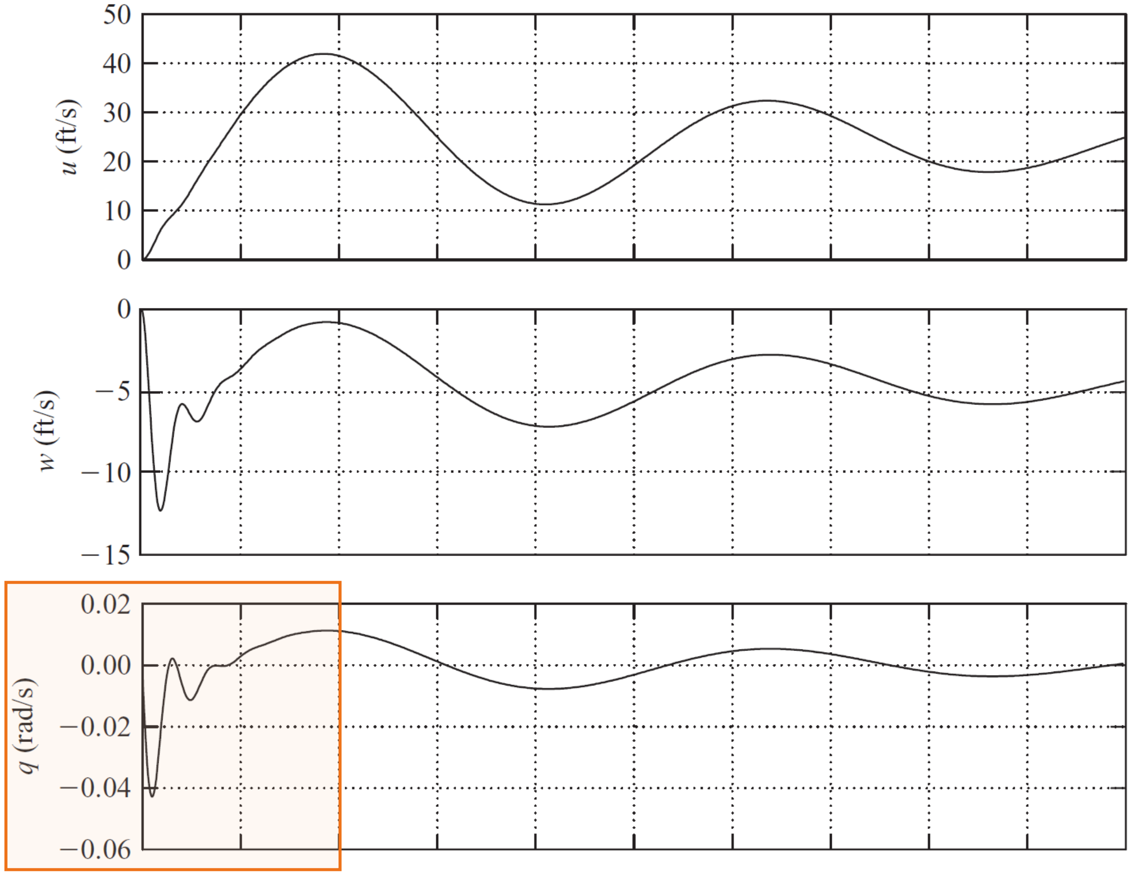
```
>> damp(Corsair)

         Pole              Damping       Frequency       Time Constant
                                       (rad/TimeUnit)     (TimeUnit)

 -4.51e-01 + 1.57e+00i     2.76e-01       1.63e+00          2.22e+00
 -4.51e-01 - 1.57e+00i     2.76e-01       1.63e+00          2.22e+00
 -1.66e-02 + 1.39e-01i     1.19e-01       1.40e-01          6.01e+01
 -1.66e-02 - 1.39e-01i     1.19e-01       1.40e-01          6.01e+01
```

```
>> eig(Corsair)

ans =

  -0.4509 + 1.5689i
  -0.4509 - 1.5689i
  -0.0166 + 0.1394i
  -0.0166 - 0.1394i
```

Wikipedia

Cook, M.V. Flight Dynamics Principles. Arnold, London, 2012.

Ling-Temco-Vought A-7A Corsair II Aircraft



Aircraft response to 1° elevator step input.

Wikipedia

bristol.ac.uk

Cook, M.V. Flight Dynamics Principles. Arnold, London, 2012.

Ling-Temco-Vought A-7A Corsair II Aircraft

$$
\begin{array}{cc}
 & \begin{array}{cc} \text{Short period mode} & \text{Phugoid mode} \end{array} \\
\mathbf{V} = & \left[\begin{array}{cc|cc}
-0.1682 - 0.1302j & -0.1682 + 0.1302j & 0.1467 + 0.9677j & 0.1467 - 0.9677j \\
0.2993 + 0.9301j & 0.2993 - 0.9301j & 0.0410 + 0.2008j & 0.0410 - 0.2008j \\
-0.0046 + 0.0018j & -0.0046 - 0.0018j & 0.0001 + 0.0006j & 0.0001 - 0.0006j \\
0.0019 + 0.0024j & 0.0019 - 0.0024j & 0.0041 - 0.0013j & 0.0041 + 0.0013j
\end{array}\right]
\begin{array}{l} : u \\ : w \\ : q \\ : \theta \end{array}
\end{array}
\tag{6.9}
$$

To facilitate interpretation of the eigenvector matrix, the magnitude of each component eigenvector is calculated as follows:

$$
\left[\begin{array}{cc|cc}
0.213 & 0.213 & 0.979 & 0.979 \\
0.977 & 0.977 & 0.204 & 0.204 \\
0.0049 & 0.0049 & 0.0006 & 0.0006 \\
0.0036 & 0.0036 & 0.0043 & 0.0043
\end{array}\right]
\begin{array}{l} : u \\ : w \\ : q \\ : \theta \end{array}
$$

Wikipedia

Cook, M.V. Flight Dynamics Principles. Arnold, London, 2012.

Douglas DC-8 Aircraft

$$[\cancel{U}, V, \cancel{W}, p, \cancel{q}, r, \phi, \cancel{\theta}, \cancel{\psi}, \cancel{x_E}, \cancel{y_E}, \cancel{z_E}]$$

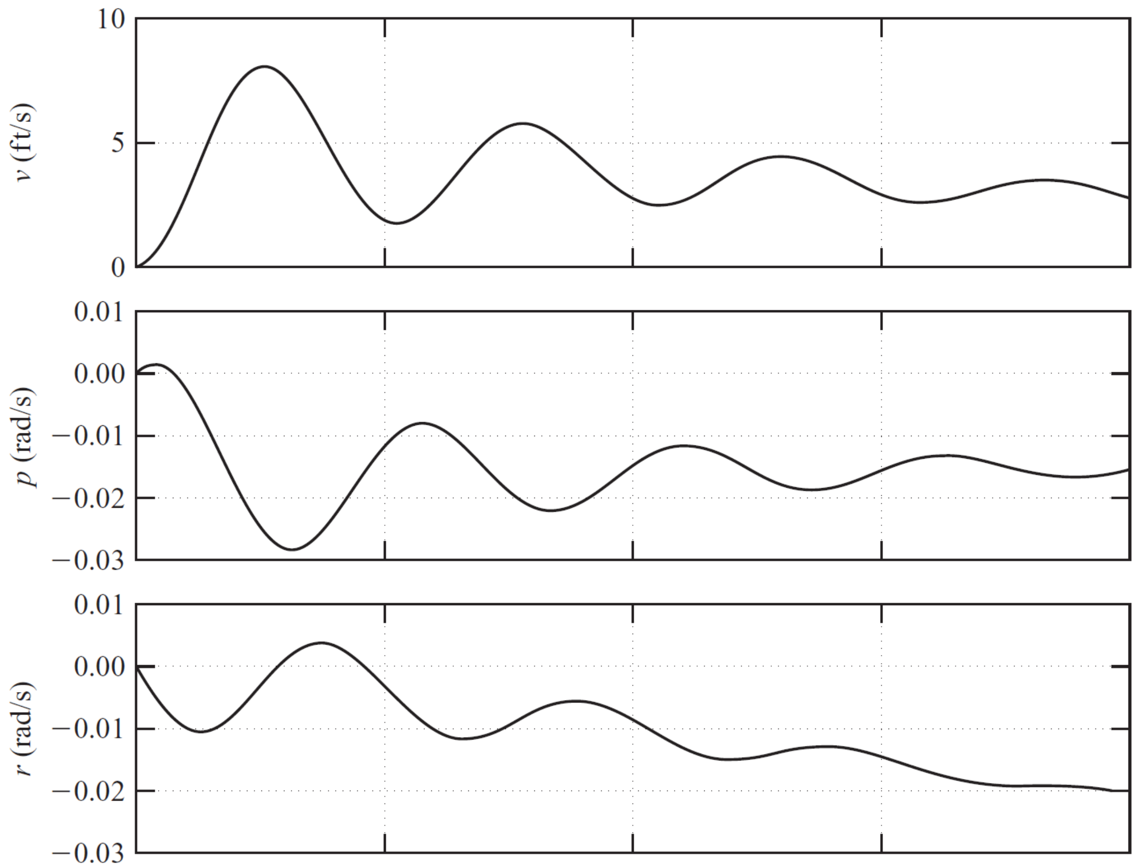
$$\boxed{\begin{matrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{matrix}} = \begin{bmatrix} -0.1008 & 0 & -468.2 & 32.2 \\ -0.00579 & -1.232 & 0.397 & 0 \\ 0.00278 & -0.0346 & -0.257 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \boxed{\begin{matrix} v \\ p \\ r \\ \phi \end{matrix}}$$

$$+ \begin{bmatrix} 0 & 13.48416 \\ -1.62 & 0.392 \\ -0.01875 & -0.864 \\ 0 & 0 \end{bmatrix} \boxed{\begin{matrix} \xi \\ \zeta \end{matrix}}$$

Wikipedia

Cook, M.V. Flight Dynamics Principles. Arnold, London, 2012.

Douglas DC-8 Aircraft



Aircraft response to 1° rudder step input.

Wikipedia

Cook, M.V. Flight Dynamics Principles. Arnold, London, 2012.

Douglas DC-8 Aircraft

```
>> damp(DC8)

           Pole                Damping       Frequency       Time Constant
                                           (rad/TimeUnit)     (TimeUnit)

 -1.27e-01 + 1.19e+00i        1.06e-01      1.20e+00          7.87e+00
 -1.27e-01 - 1.19e+00i        1.06e-01      1.20e+00          7.87e+00
 -1.33e+00                    1.00e+00      1.33e+00          7.52e-01
 -6.49e-03                    1.00e+00      6.49e-03          1.54e+02
```

The first real root describes the *spiral mode* with time constant

$$T_s = \frac{1}{0.0065} \cong 154\,\mathrm{s}$$

the second real root describes the *roll subsidence mode* with time constant

$$T_r = \frac{1}{1.329} = 0.75\,\mathrm{s}$$

and the pair of complex roots describe the oscillatory *dutch roll mode* with characteristics

$$\text{Damping ratio } \zeta_d = 0.11$$
$$\text{Undamped natural frequency } \omega_d = 1.2\,\mathrm{rad/s}$$

Wikipedia

Cook, M.V. Flight Dynamics Principles. Arnold, London, 2012.

Next Session

Aircraft Modes

Handling Qualities

https://www.youtube.com/watch?v=wncRFPd69rg

https://www.autoevolution.com/news/pivotal-blackfly-becomes-the-world-s-first-evtol-to-fly-1000-times-with-a-pilot-on-board-258701.html#agal_0

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ANY QUESTIONS?

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