University of BRISTOL

NOTATION AND AXES

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Bristol and Gloucestershire Gliding Club

Tug Aircraft: 141hp EuroFox 2K

References

Cook, M.V. Flight Dynamics Principles. Butterworth-Heinemann, 3rd ed., 2013.
Chapter 1 - Introduction

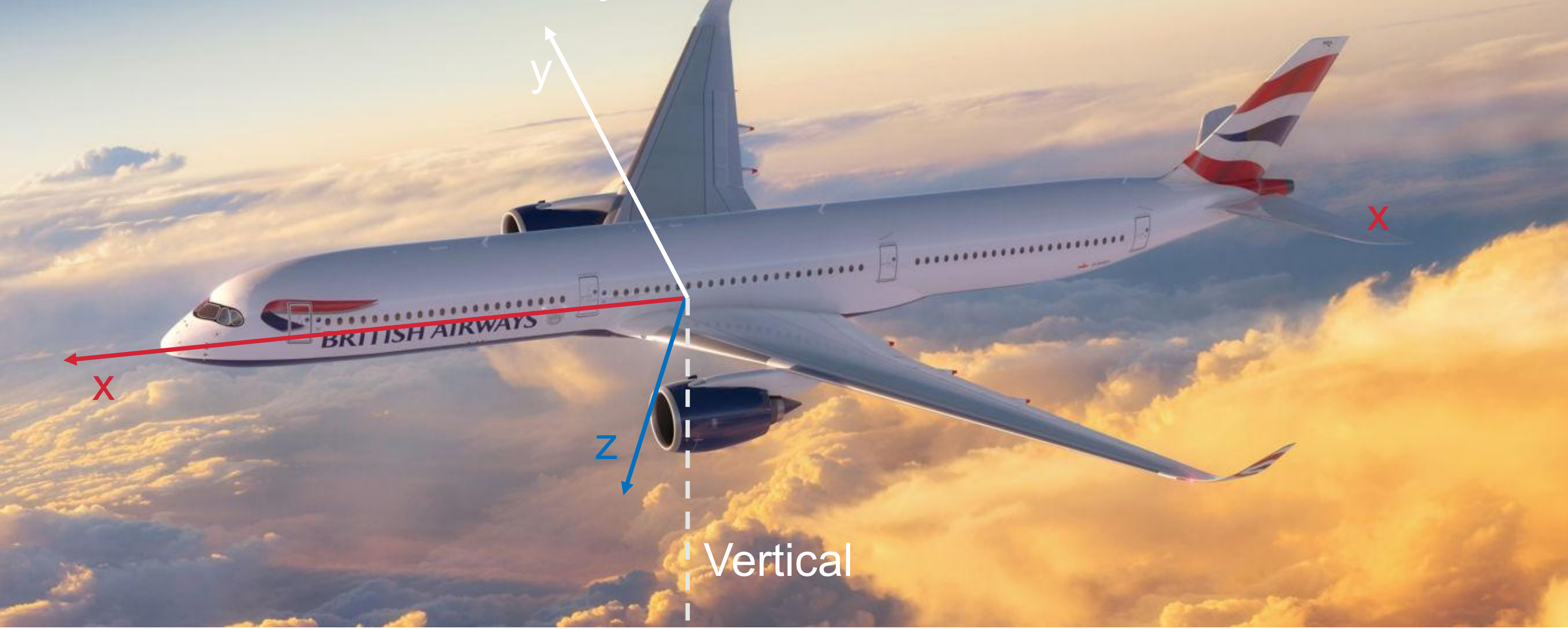
Cook, M.V. Flight Dynamics Principles. Butterworth-Heinemann, 3rd ed., 2013.
Chapter 2 – System of Axes and Notation

‘Flight dynamics in aviation and spacecraft, is the study of the performance, stability, and control of vehicles flying through the air or in outer space. It is concerned with how forces acting on the vehicle determine its velocity and attitude with respect to time.’ *Robert F. Stengel*

Airbus A350-1000

https://www.britishairways.com

Reference Axes – Body



Airbus A350-1000

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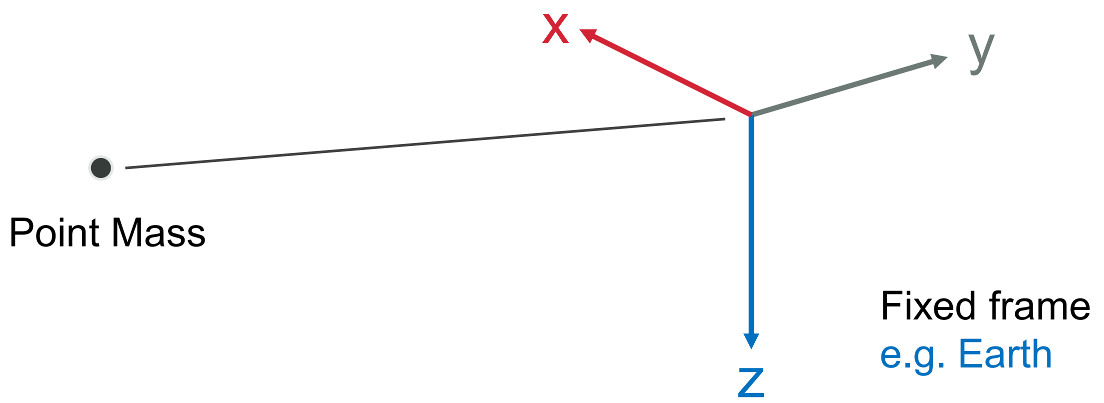
Reference Axes - Earth

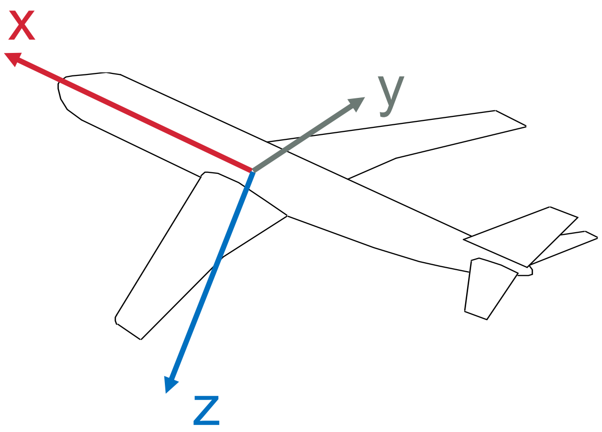
Movements of a point mass do not require a local set of axes attached to the mass.

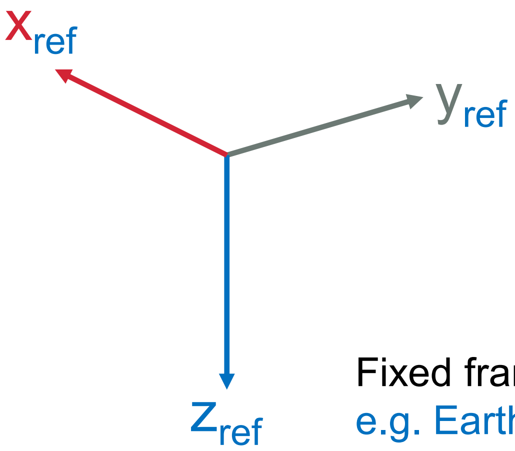


Movements of a rigid body having distributed mass *do* require a local set attached to the body with the second set for reference.

Reference Axes - Body

The six motions of a rigid body – three translational and three rotational are defined all *relative* to a second fixed frame.

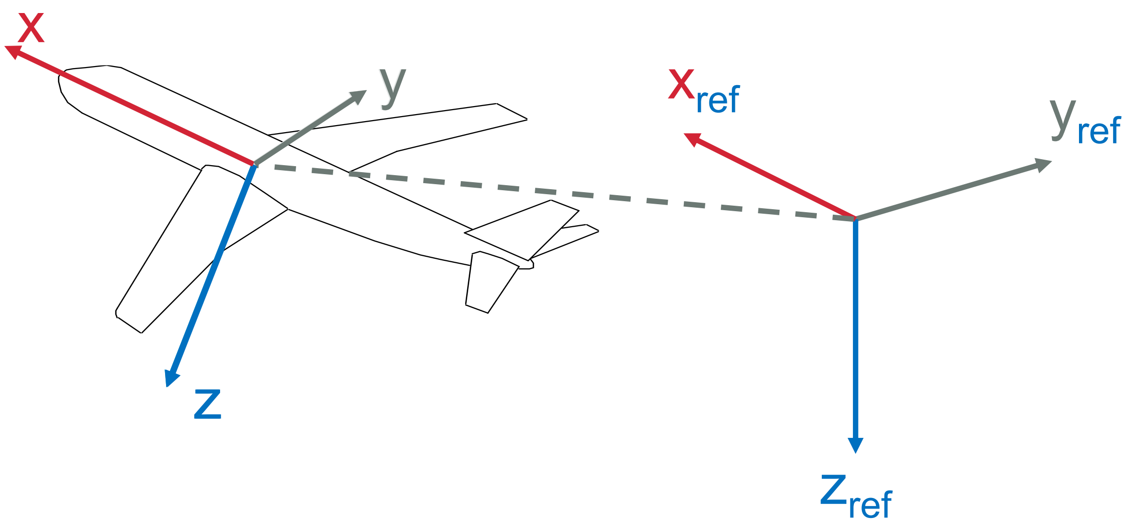




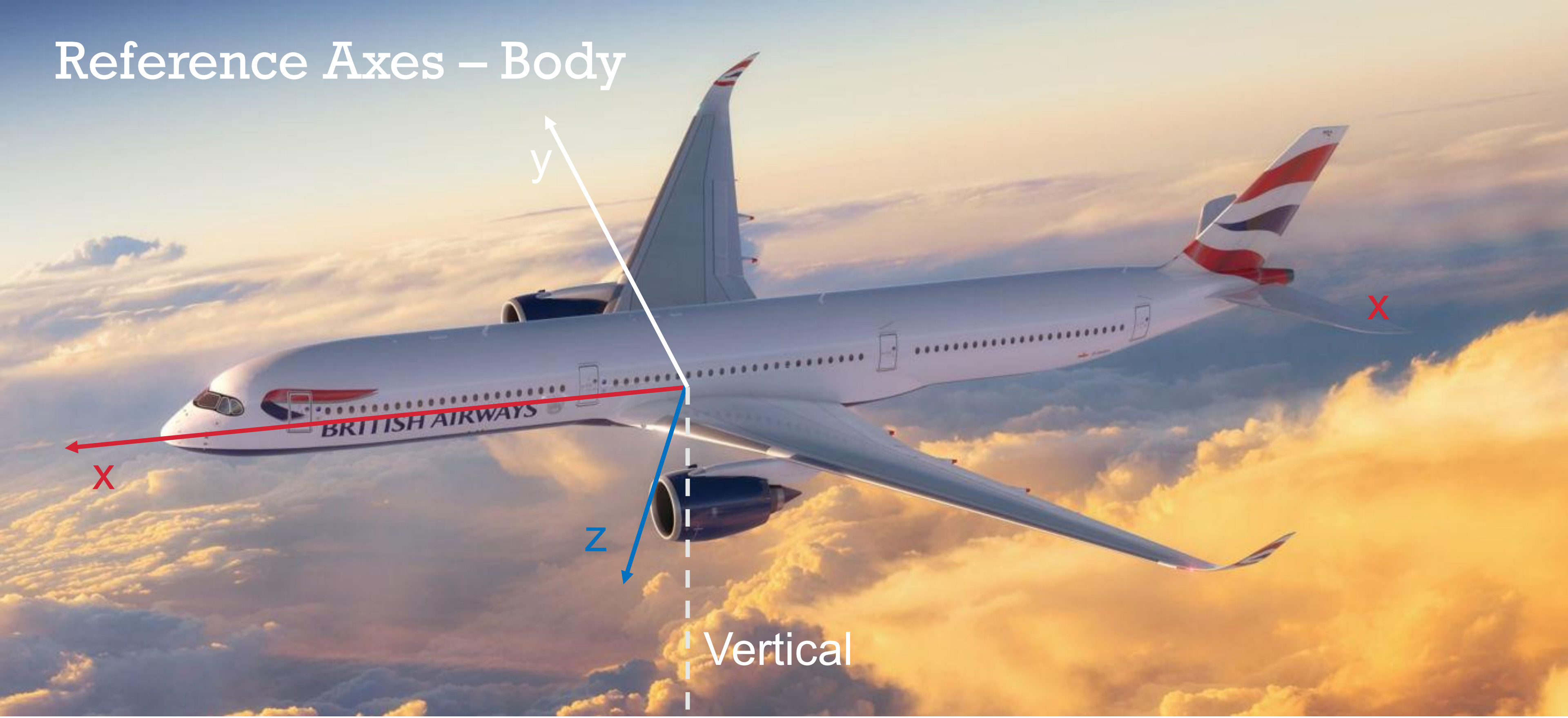
Fixed frame
e.g. Earth

Reference Axes

e.g. all angular motions are measured from the reference set to the local set thereby giving motion relative to where the local axes started, e.g. a local airport.



Reference Axes – Body



Airbus A350-1000

https://www.britishairways.com

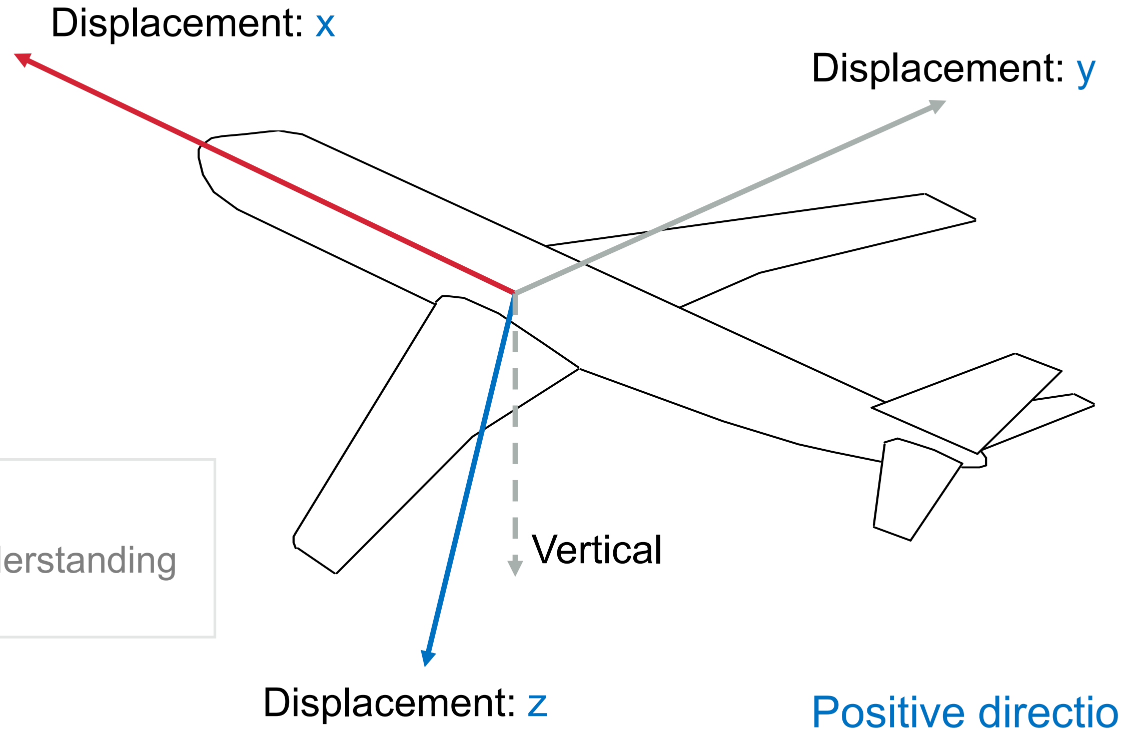
Body Axes

- Body axes (the set fixed to the body) are attached with the orientation often being chosen for the convenience of defining inertias.
- The CG could be chosen for the origin of the body axes, but the CG position may vary and so some other fixed but easily defined point is usually selected.

Body Axes

- The x-axis will be directed forward, usually along the fuselage centre-line (or at least parallel to the fuselage reference).
- The y-axis will be laterally outward (positive to starboard) and perpendicular to the aircraft's plane of symmetry, i.e. not likely to lie fully within the wing though the inner portion will be close to the wing.
- The z-axis is nominally down (perpendicular to both x and y).

Body Axes Notation and Sign Conventions



LEARN
Helps with understanding later lectures!

Positive direction indicated

Body Axes Notation and Sign Conventions

Sets!

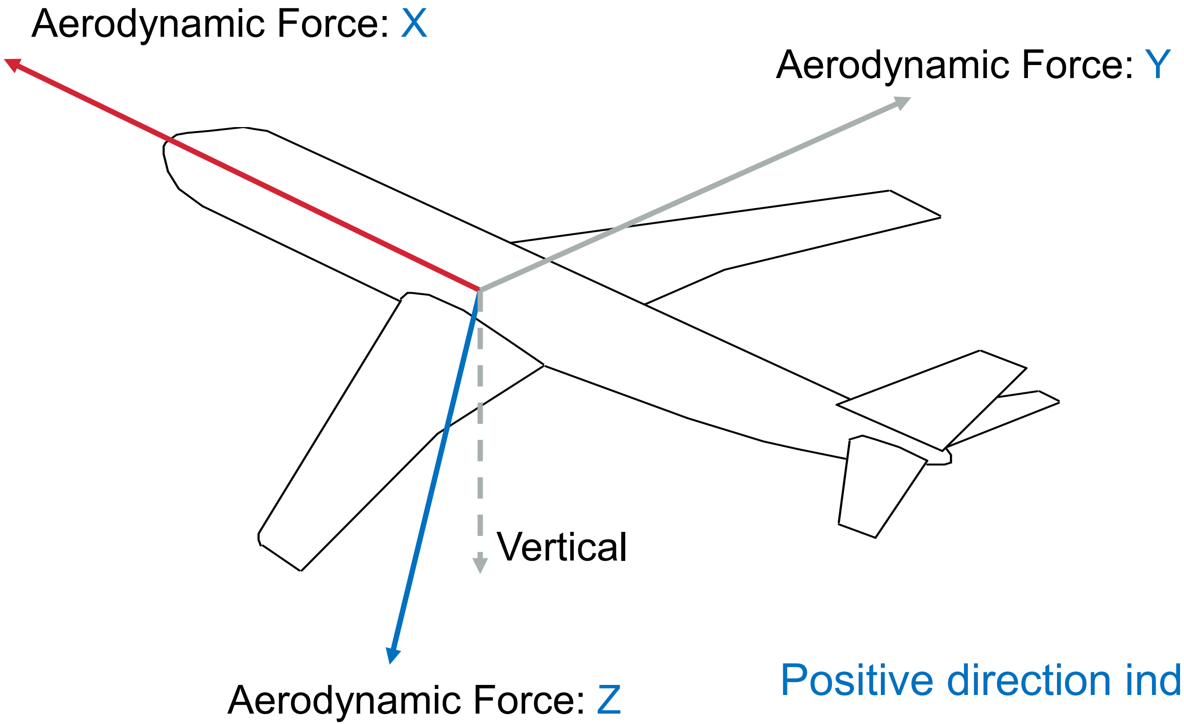
x → y → z

u → v → w

roll → pitch → yaw

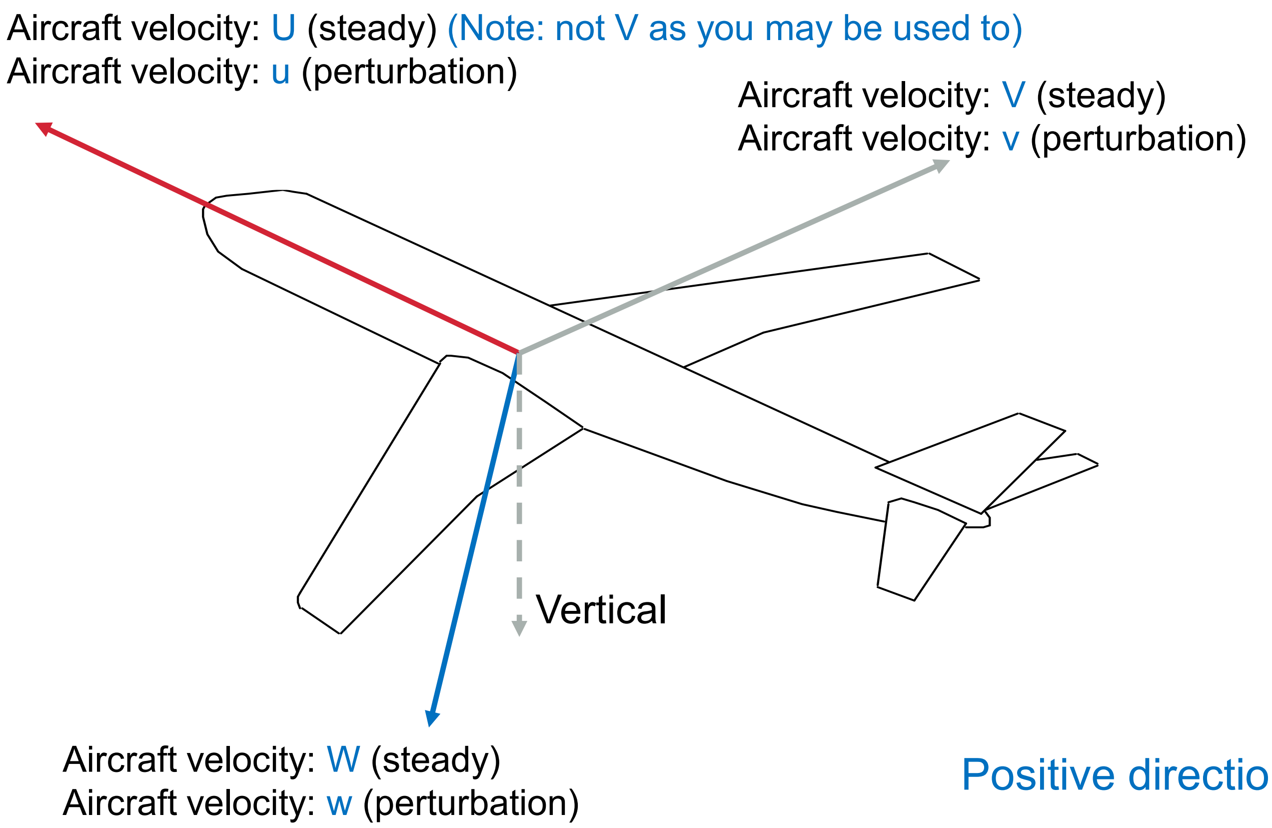
$\phi \rightarrow \theta \rightarrow \psi$

Body Axes Notation and Sign Conventions



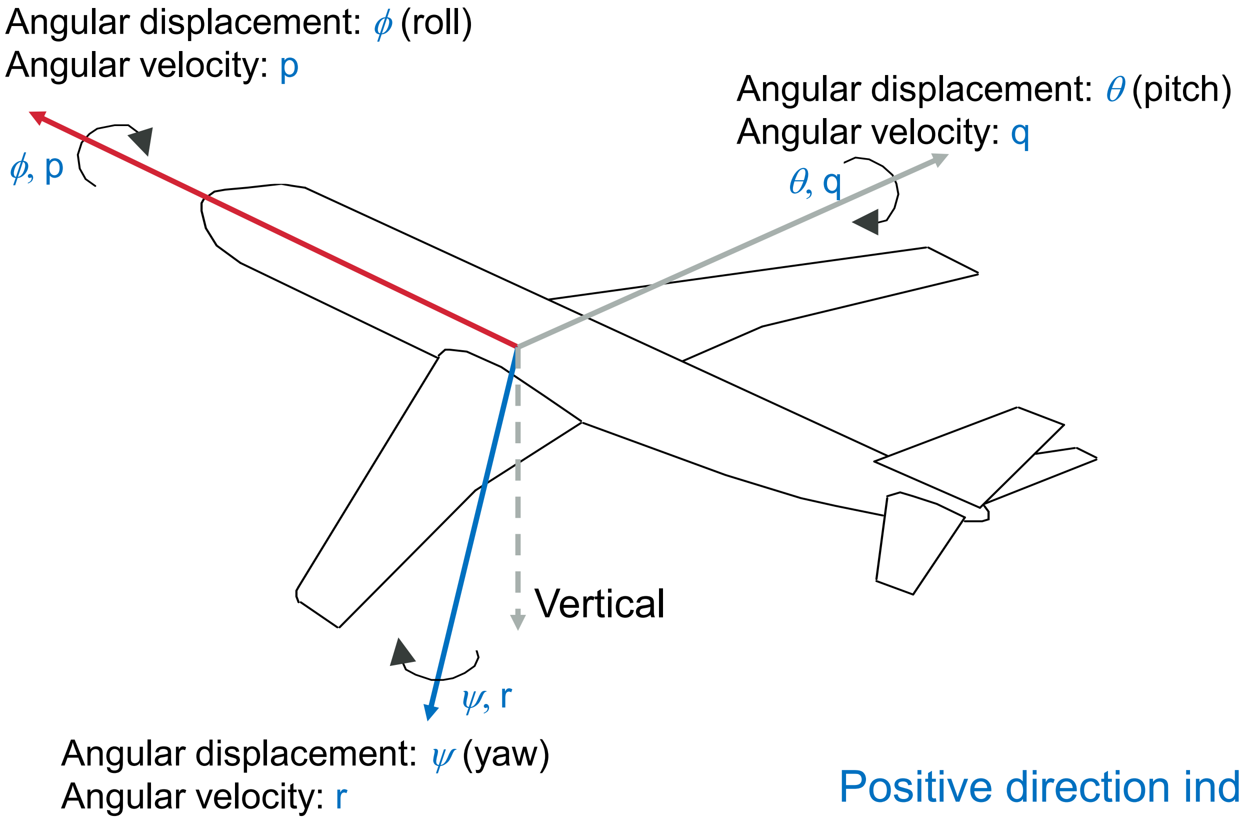
Positive direction indicated

Body Axes Notation and Sign Conventions



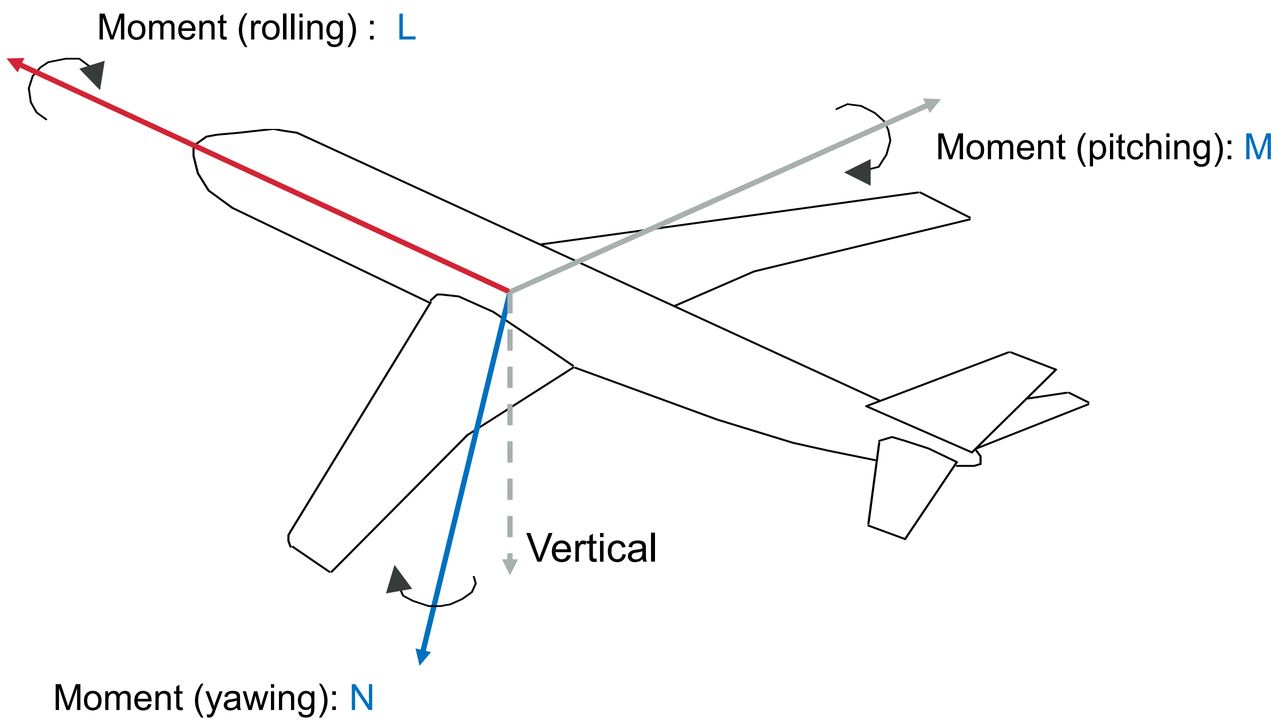
Positive direction indicated

Body Axes Notation and Sign Conventions



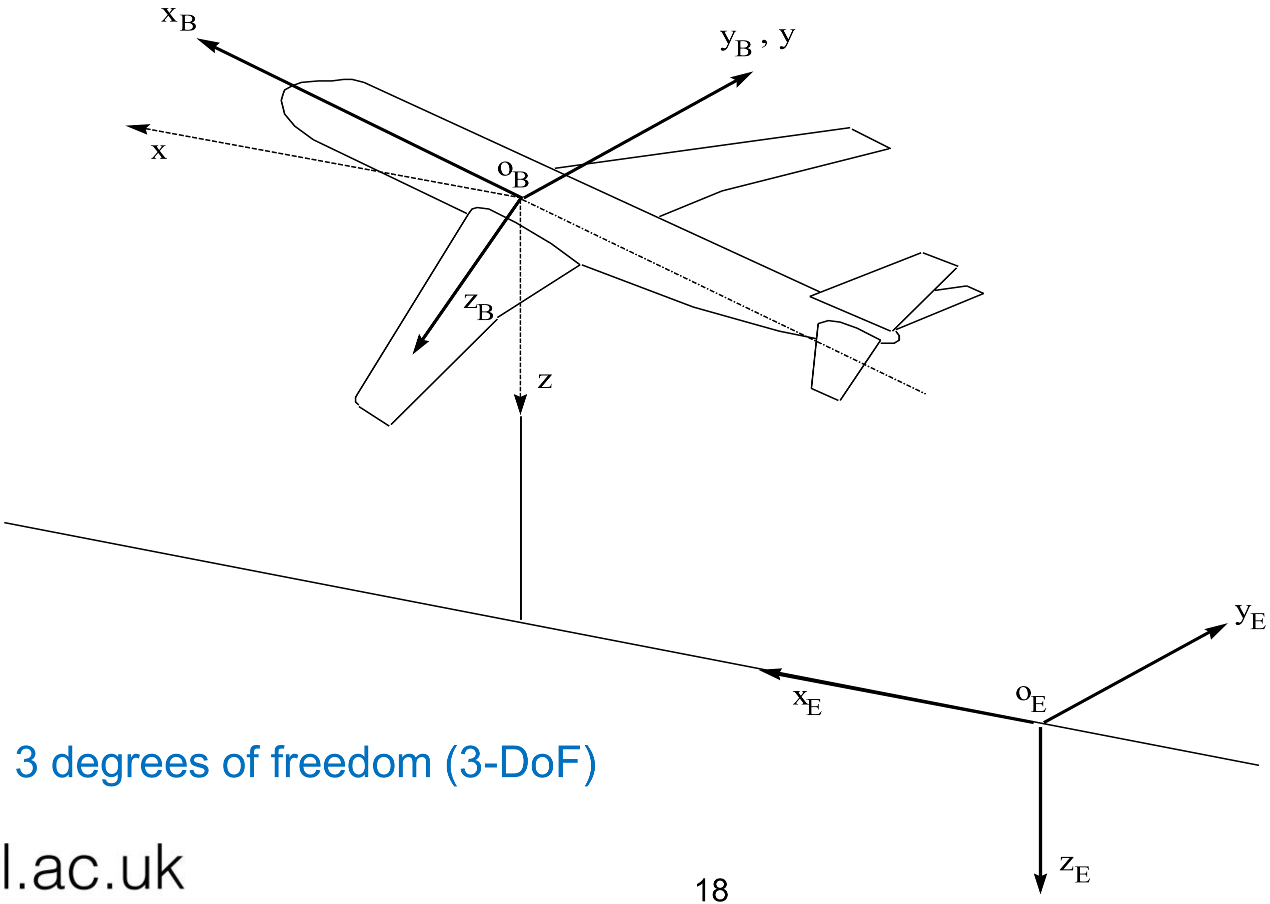
Positive direction indicated

Body Axes Notation and Sign Conventions



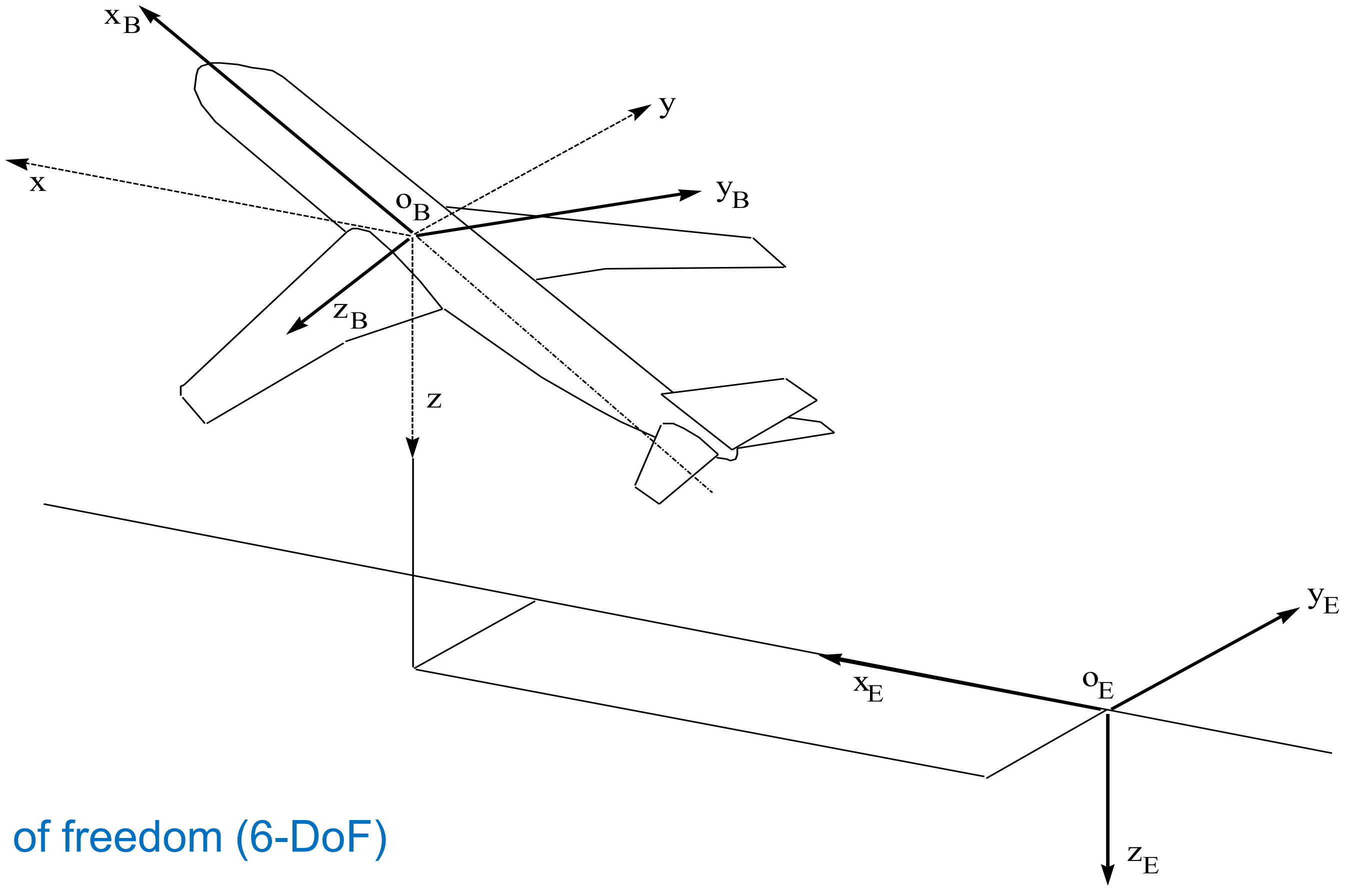
Positive direction indicated

Longitudinal Flight



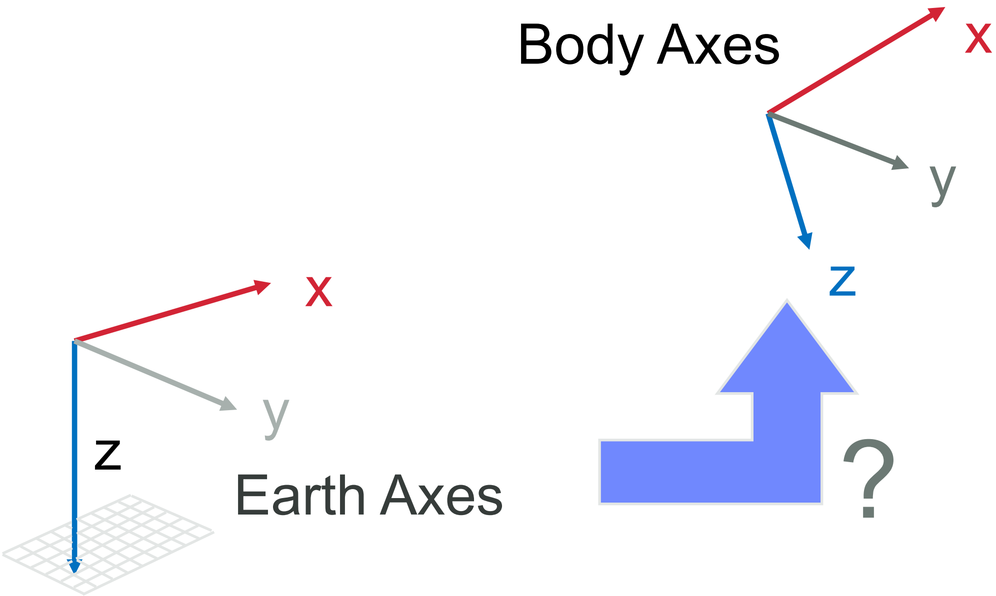
Flight in 3 degrees of freedom (3-DoF)

General Case



Flight in 6 degrees of freedom (6-DoF)

Reference Axes



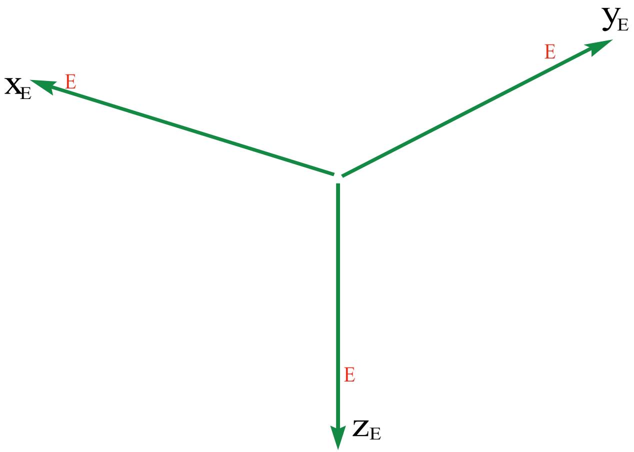
Check the importance of the order of rotation…

The Euler Angles

The following diagrams show four reference frames:

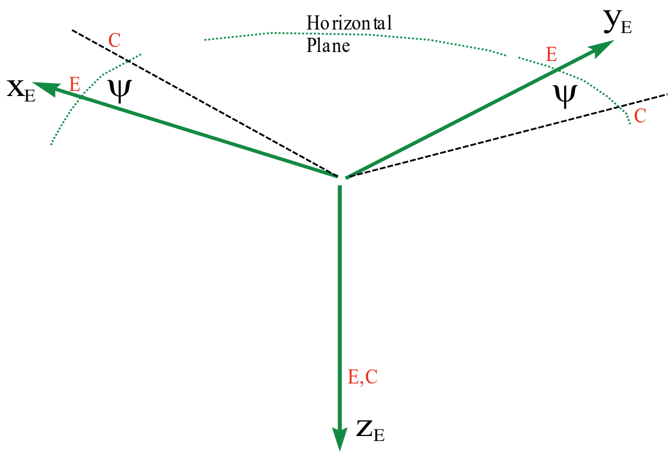
1. Earth Axes Frame F_E
2. Intermediate Frame F_C
3. Intermediate Frame F_D
4. Body Axes Frame F_B

The Euler Angles



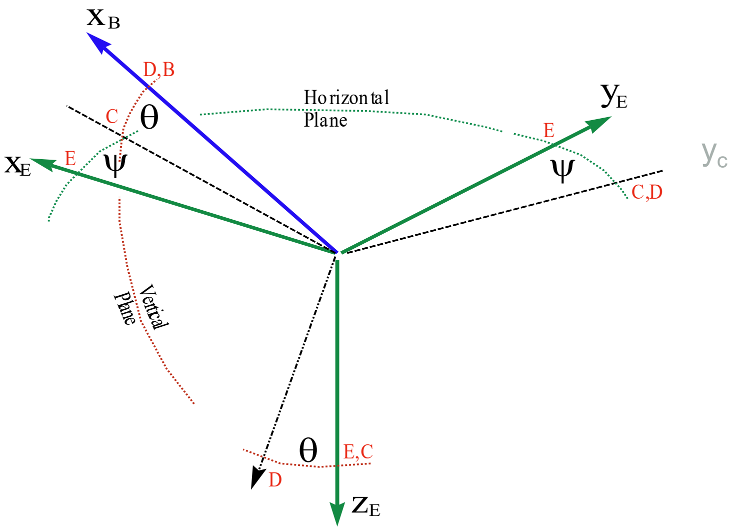
Earth Axes

The Euler Angles



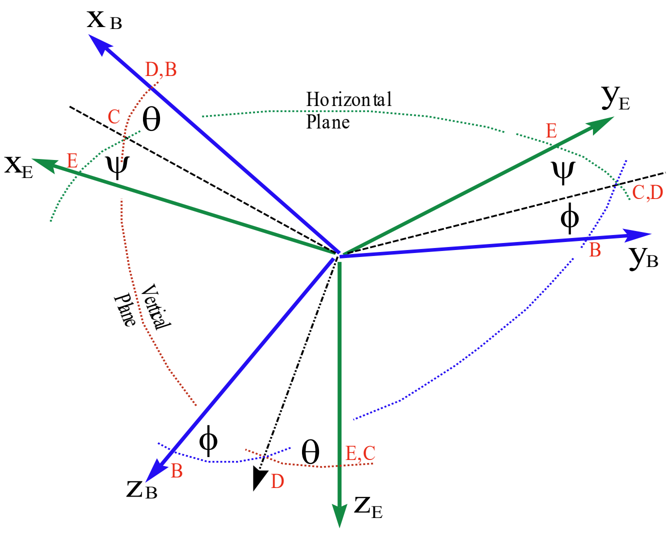
1. Rotation ψ about z_E to Intermediate frame F_C

The Euler Angles



2. Rotation θ about y_c to intermediate frame F_D

The Euler Angles



3. Rotation ϕ about x_B to body axes frame F_B

The Euler Angles

The sequence of rotations is:

1. Rotation ψ about z_E to Intermediate frame F_C
2. Rotation θ about y_c to intermediate frame F_D
3. Rotation ϕ about x_B to body axes frame F_B

check……

(Note: these angles can also be used for small perturbations away from a trimmed state when we linearise a model.)

Reference Axes – Wind

- We need to be able to define the forces acting on the body.

- Most of our "external" forces will be of aerodynamic origin.

- Hence, we need a convenient set of orthogonal axes fixed in the vehicle; then we need to know where the wind vector is, relative to these in order to express the aerodynamic forces mathematically.

Wind axes

- The orientation of the wind axes is usually chosen for the convenience of defining aerodynamic forces.
- As with the body axes, the origin is fixed at the aircraft reference point (probably different from the CG), but the x-axis points into the wind.

Angles α and β

- The angle of attack, α, probably needs no further description – first year.

- You need be clear on the distinction between the two angles

  ψ - yaw angle (psi)

  and β - sideslip angle (beta).

- Similarly, you need be clear on the distinction between

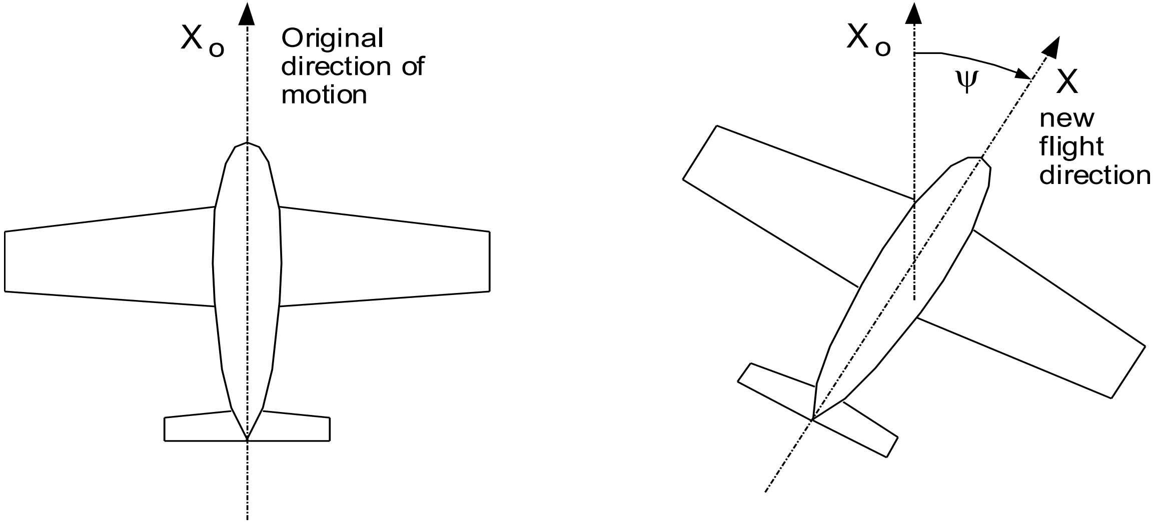
  θ - pitch angle (theta)

  and α - angle of attack (alpha).

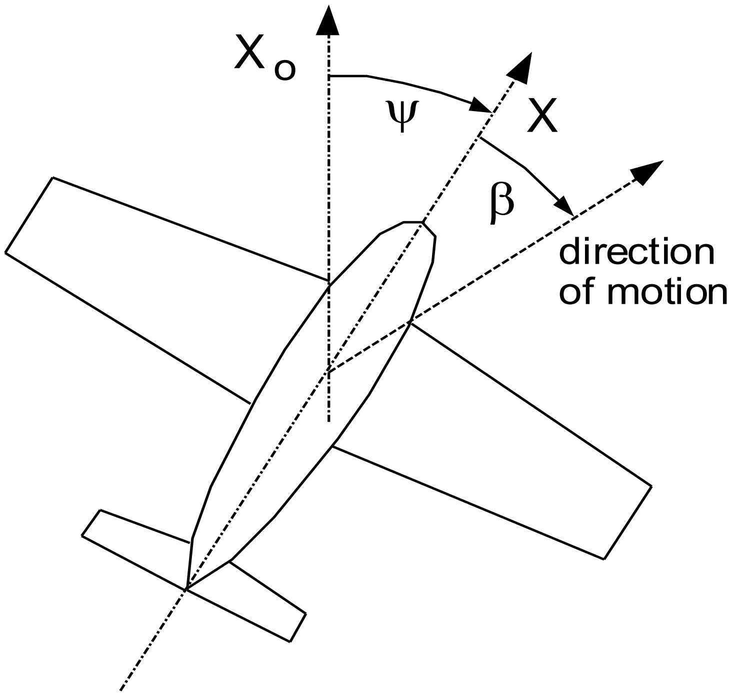
- the aerodynamic forces are based on the two angles α and β, not θ and ψ..

Sideslip angle β

Essentially, ψ is an orientation angle which indicates a pointing direction for the fuselage reference line compared with a previous direction, perhaps relative to a fixed set of fixed earth axes.



Sideslip angle β



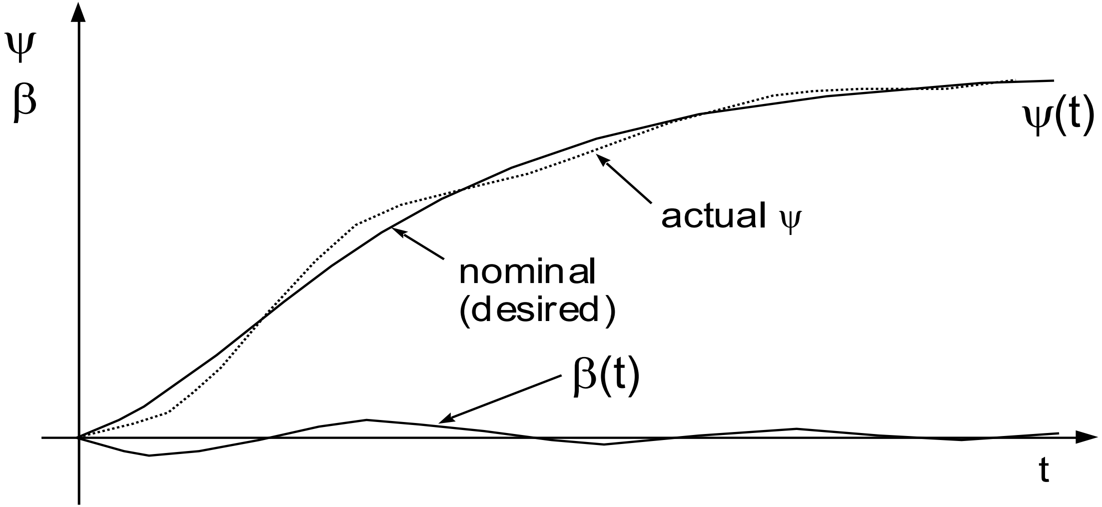
Sideslip implies that the instantaneous wind is not approaching from straight ahead, i.e. the motion of the aircraft is not in the direction of its own longitudinal centre-line.

If the aircraft is apparently "skidding" to the right there is positive sideslip, and the sideslip incidence is given by $\beta = v/U$ (positive v gives positive β). This can be true for zero yaw angle, $\psi = 0$.

Sideslip angle β

ψ(t) and β(t) can be very different, as shown below, with ψ(t) tending to a new steady positive value in a turn whereas β(t) could be nearly zero throughout (coordinated turn).

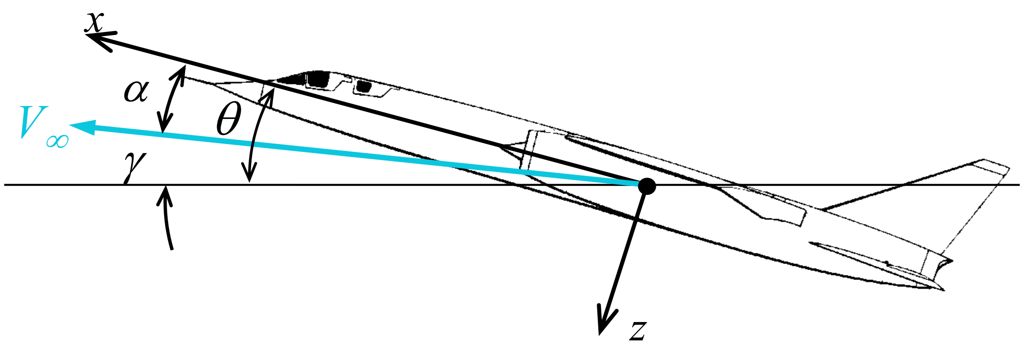
https://en.wikipedia.org/wiki/Yaw_string



Angle of attack α and pitch angle θ

Angle of attack is the angle between the body x-axis and the projection of the velocity vector into the aircraft plane of symmetry.

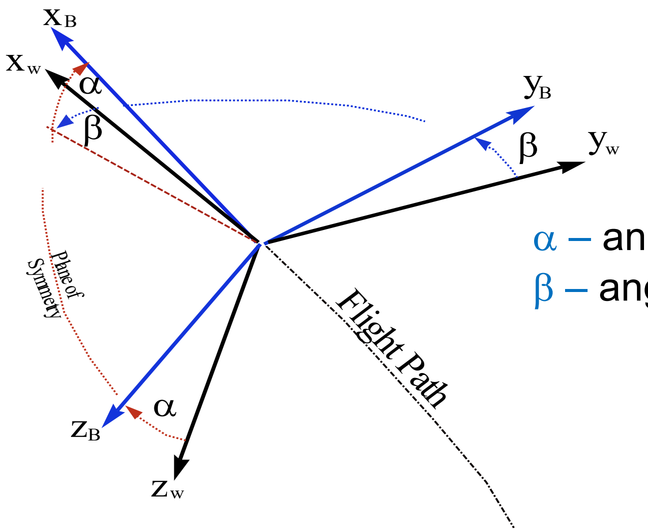
Pitch angle is the angle between the body x-axis and the horizontal.



Wind axes

- If the atmosphere is steady and not moving relative to the earth, then the axes are at a tangent to the flight trajectory of the aircraft relative to earth.
- The other constraint on this set of axes is that the z wind-axis remains in the aircraft plane of symmetry though it may tilt backward.
- Only pitch and yaw departures of the wind vector are necessary to define wind-induced forces.

Wind axes



α – angle of attack
β – angle of sideslip

α – is the angle between the x_B axis and the plane containing the y_B axis and the velocity vector

β – is the angle between the velocity vector and the plane of symmetry (x_B z_B)

State Vector

$$\boxed{\begin{aligned} V_T &= \sqrt{U^2 + V^2 + W^2} \\ \dot{V}_T &= \frac{U\dot{U} + V\dot{V} + W\dot{W}}{V_T} \\ \dot{\beta} &= \frac{\dot{V}V_T - V\dot{V}_T}{V_T^2 \cos\beta} \\ \dot{\alpha} &= \frac{U\dot{W} - W\dot{U}}{U^2 + W^2}. \end{aligned}} \qquad (2.4\text{-}8)$$

The new state vector is

$$X^{\mathrm{T}} = [\boxed{V_T \quad \beta \quad \alpha} \; \boxed{\phi \quad \theta \quad \psi} \; \boxed{P \quad Q \quad R} \; \boxed{p_N \quad p_E \quad h}]. \qquad (2.4\text{-}9)$$

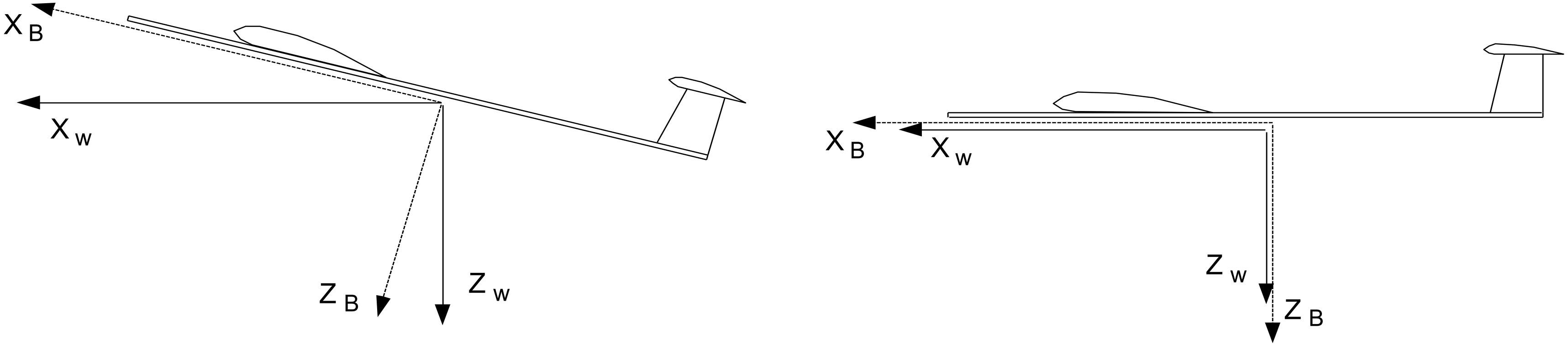
Reference: Aircraft Control & Simulation, Stevens & Lewis, Wiley (First Edition)

Stability Axes

- The relative motion of the body axes away from the wind axes is useful when defining a disturbance, or a response to a control deflection.
- It is convenient to have the two sets coincident just before the start of any study of the dynamics following a disturbance.

Stability Axes

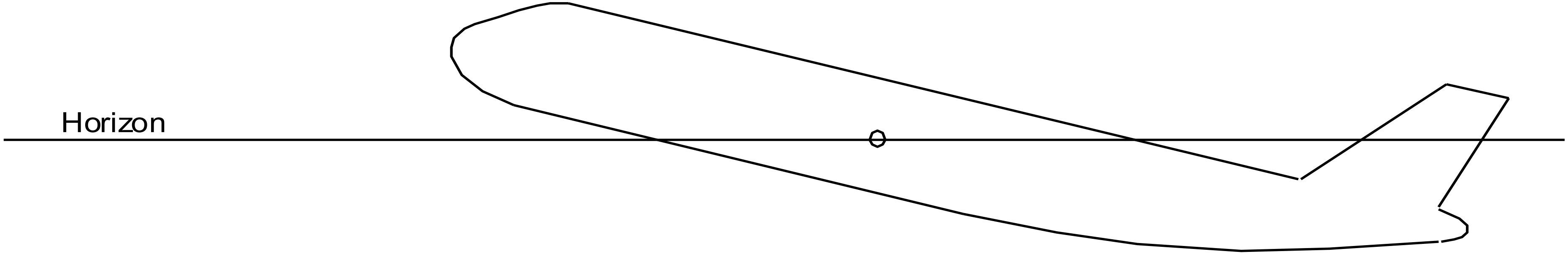
Think of two situations, low-speed (high incidence) and high-speed (low incidence) which suggest (exaggerated):-



Stability Axes

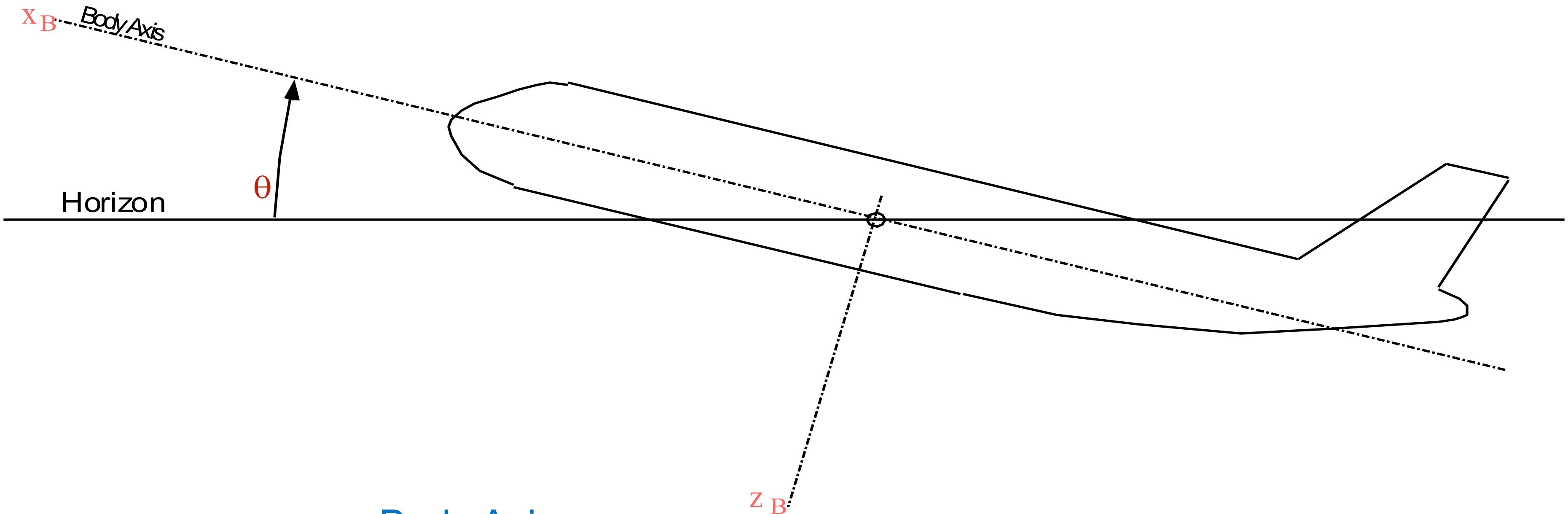
- To allow for initial mis-alignment of the X body-axis and the wind vector we can declare a special set of body axes, fixed to the body and initially aligned with the wind axes.
- This special set of axes, adhering to the requirement for initial alignment, is sometimes called the "stability axes".
- Note: every trimmed flight speed requires a new orientation for this special set of body-axes because the aircraft has a pitch attitude that is uniquely paired with forward speed for fixed weight.

Aircraft Axis System



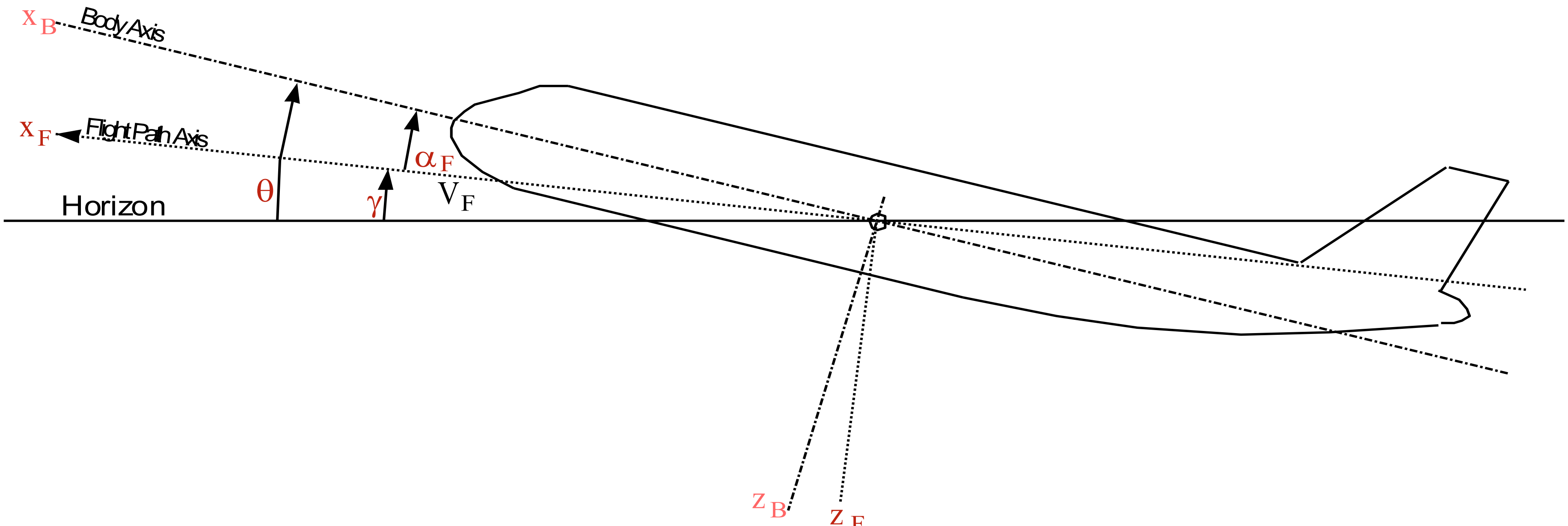
Aircraft + Horizon

Aircraft Axis System



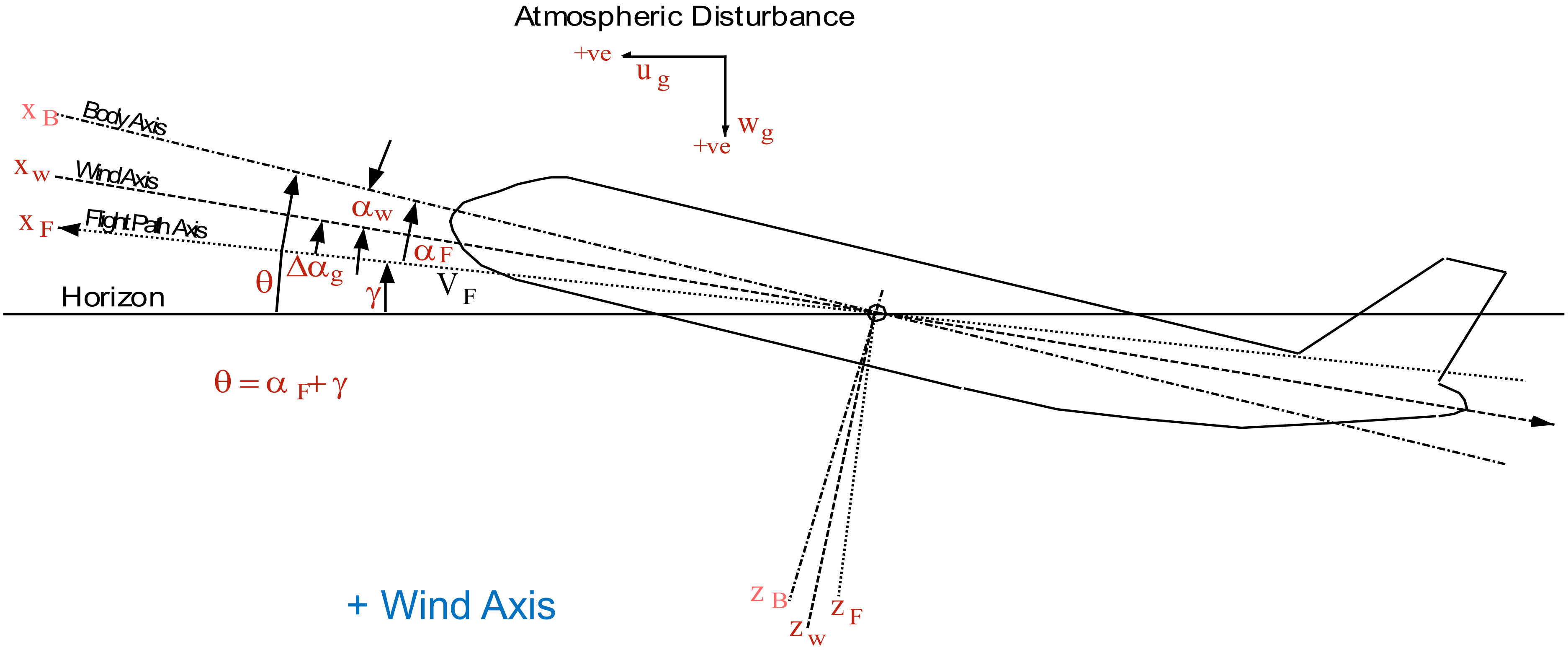
+ Body Axis

Aircraft Axis System



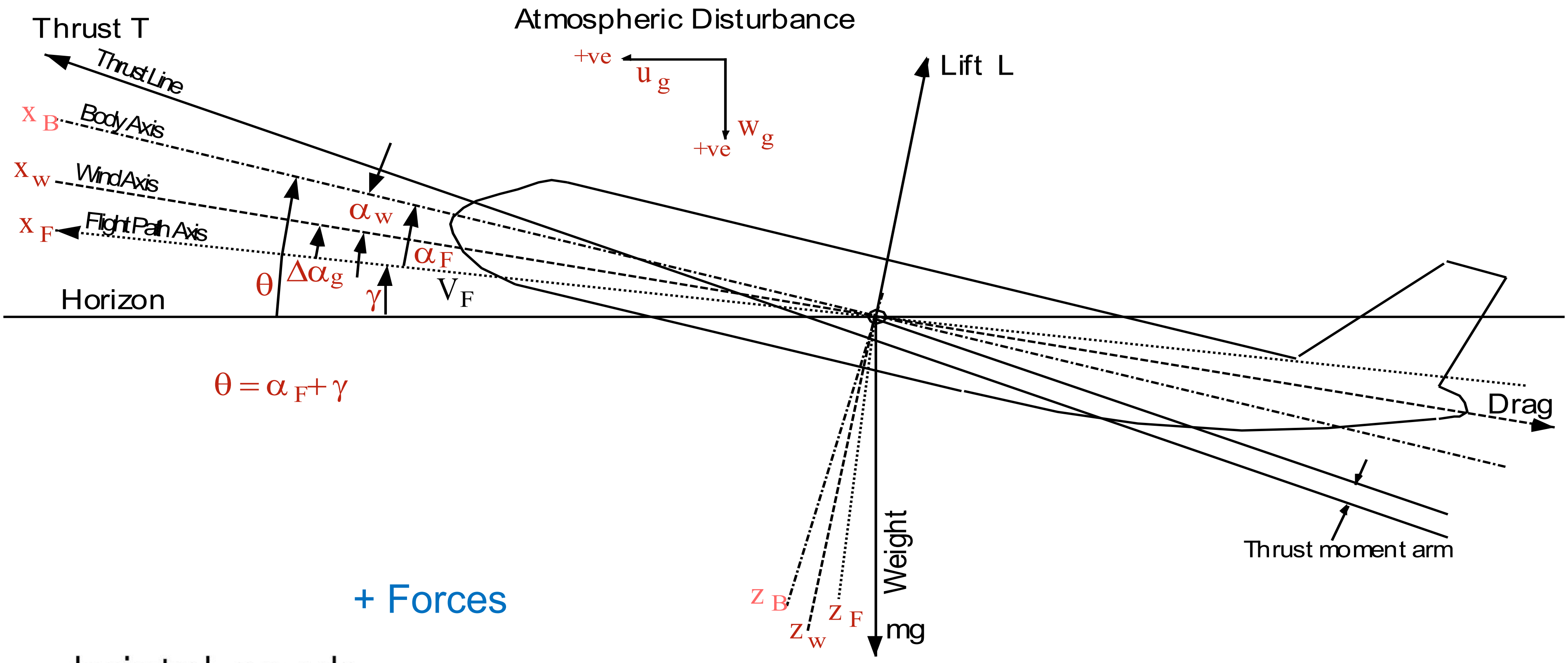
+ Flight Path Axis

Aircraft Axis System



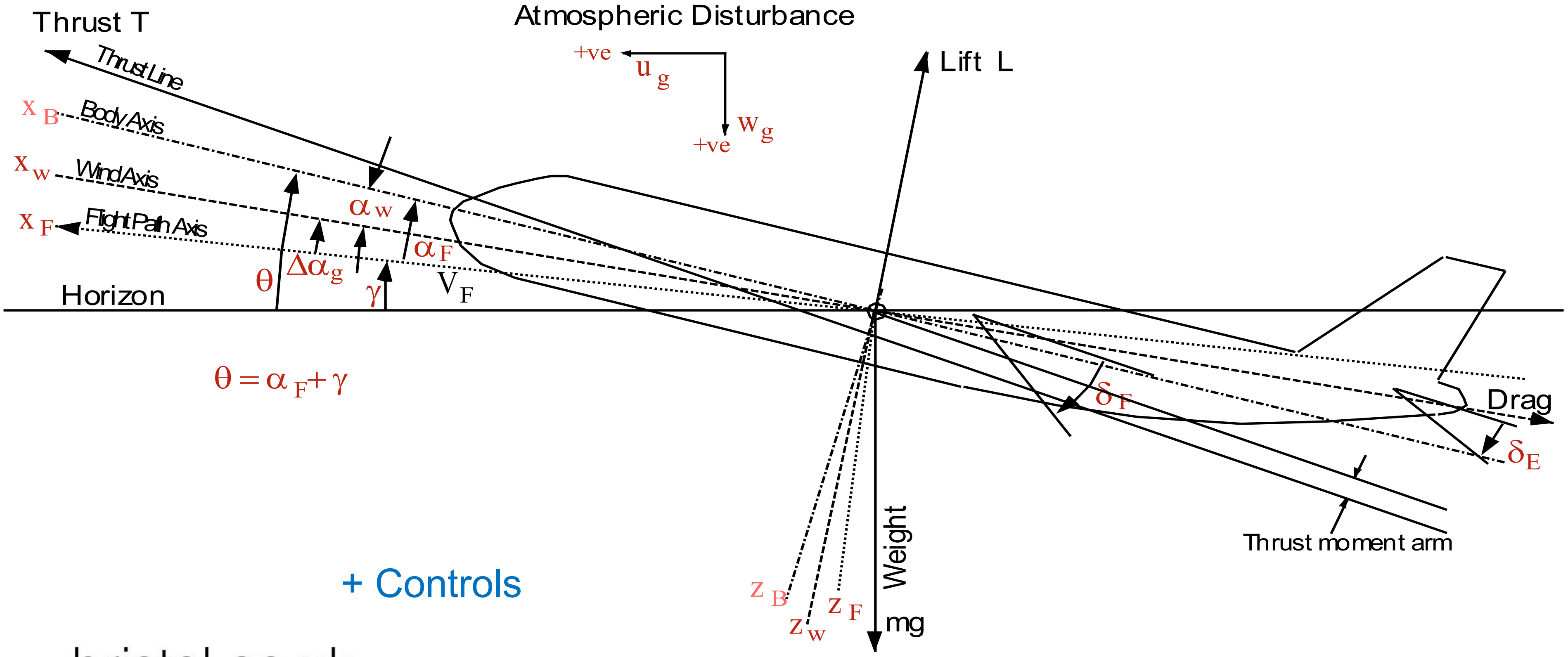
+ Wind Axis

Aircraft Axis System



+ Forces

Aircraft Axis System



+ Controls

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Next Session

Equations of Motion 1

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