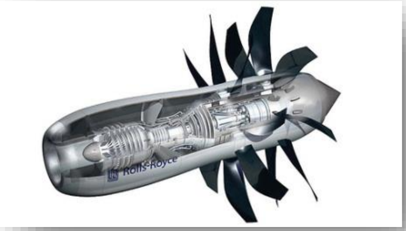
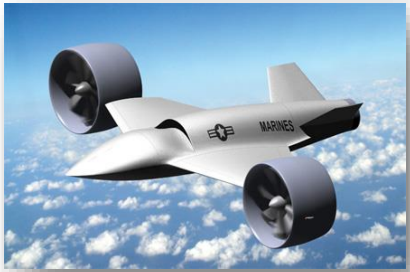
Propellers and Ducted Fans

Equation Summary





Key

- Equations/terms you need to remember
- Equations/terms you don’t need to remember but you REALLY need to understand
- Equations you need to be able to derive

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- Equations/terms you don’t need to remember but you need to understand. Note other equations/terms not shown in these slides would fall in this category.

Analysis of Propeller Aerodynamics

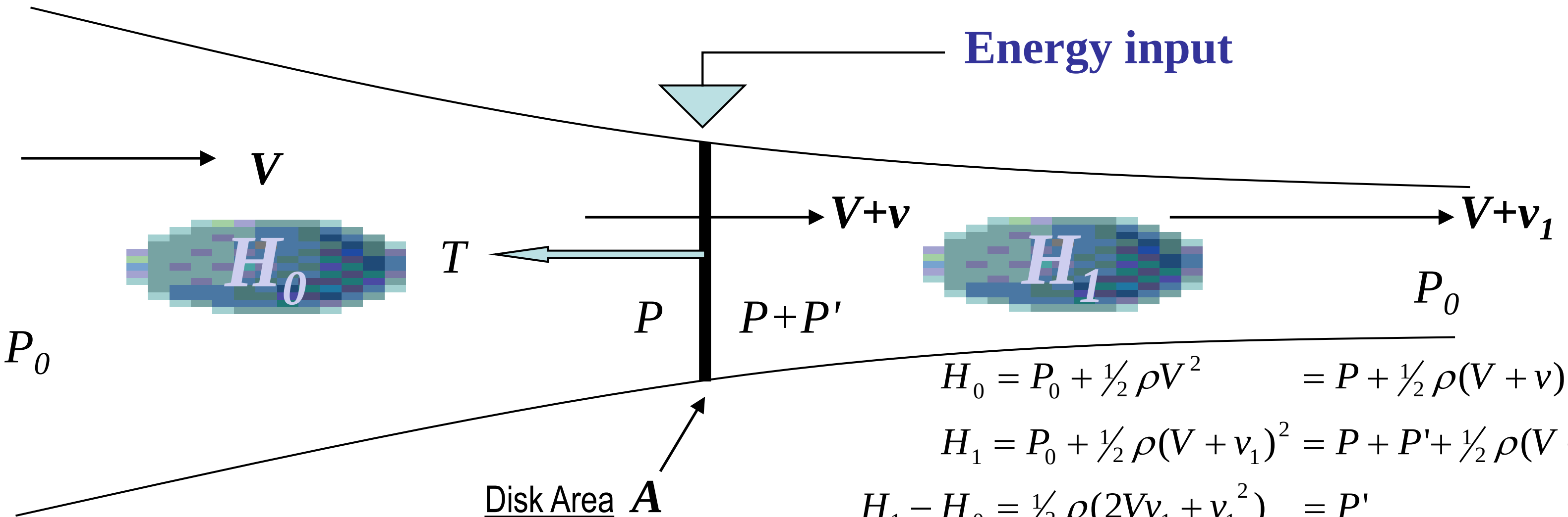
Lecture 1

Notes in Blackboard: https://www.ole.bris.ac.uk

Actuator Disc Theorem

The <u>Propeller</u> or <u>Axial Fan</u> .

Mechanical energy (in the form of rotating blades) is used to accelerate ***(a)*** a mass ***(m)*** of air. Newton's law (every action has a reaction), states ***F = ma,*** where ***F***, is the propeller thrust ***(T)***.



$$H_0 = P_0 + \tfrac{1}{2}\rho V^2 = P + \tfrac{1}{2}\rho (V+v)^2$$

$$H_1 = P_0 + \tfrac{1}{2}\rho (V+v_1)^2 = P + P' + \tfrac{1}{2}\rho (V+v)^2$$

$$H_1 - H_0 = \tfrac{1}{2}\rho (2Vv_1 + v_1{}^2) = P'$$

$$= \rho v_1 (V + \frac{v_1}{2}) = P'$$

Momentum Analysis based on Actuator Disk Theorem

Now, $T = \rho A (V+v) v_1, \quad \frac{T}{A} = \rho v_1 (V+v)$

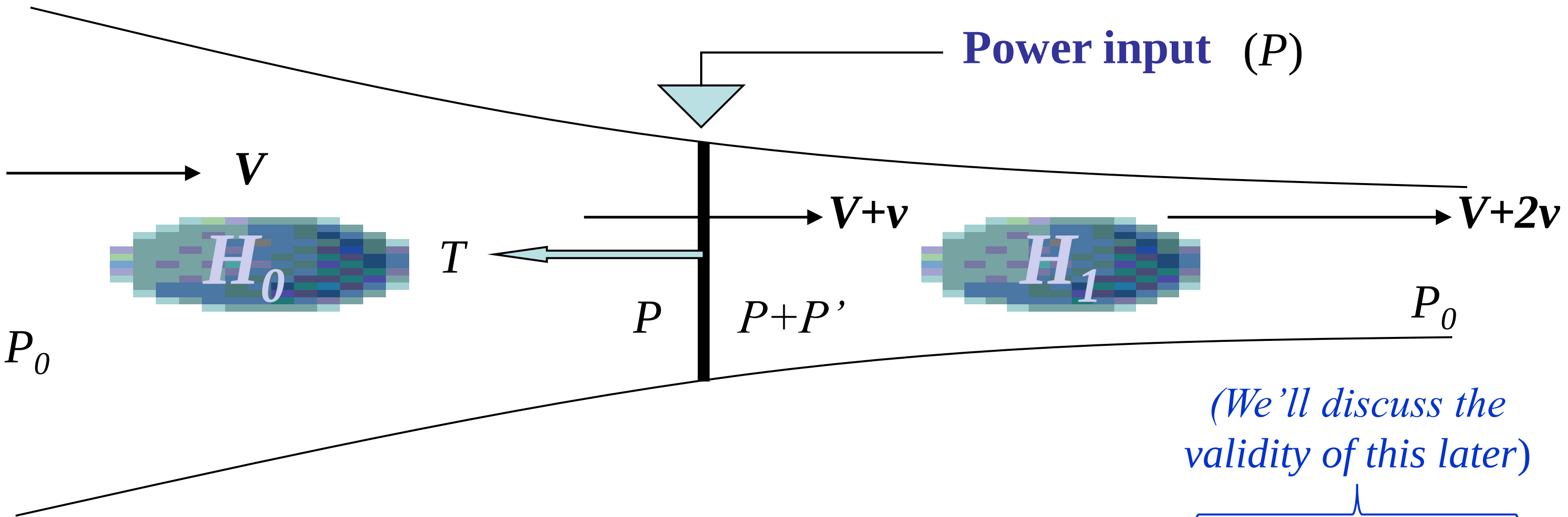
ρ Air density

A Disc area

$$\boxed{so \; \frac{v_1}{2} = v \qquad \therefore T = 2\rho A (V+v) v}$$

Efficiency

Since we have assumed no losses at the actuator disc, then the energy input (generating the pressure rise at the actuator disc) must be equal to the rise in kinetic energy in the fully developed wake.



(We'll discuss the validity of this later)

Kinetic Energy rise in the wake is

$$\Delta KE = \tfrac{1}{2}\rho A(V+v)\overbrace{((V+2v)^2 - V^2)}$$

$$= 2\rho A(V+v)v(V+v)$$

$$\Delta KE = T(V+v)$$ *(and remember this for later)*

If P is power input to the actuator disc, $\eta = \dfrac{T(V+v)}{P}$ = **UNITY** ***in this case***

Thus, efficiency of a thrust generator is $\eta = \dfrac{T(V+v)}{P}$ where:

T is output thrust
V is onset velocity
v is induced velocity
and P is input (shaft) power

For an aircraft in flight cruise, the **propeller** efficiency is $\eta_p = \dfrac{\boldsymbol{TV}}{\boldsymbol{P}}$

Clearly this cannot be used when $\boldsymbol{V} = 0$, the static thrust case. Similarly it cannot be used for lifting propellers (on tilt rotor aircraft) in the hover. For such **rotors**, the work done on the air is more important and the efficiency is $\eta_r = \dfrac{\boldsymbol{Tv}}{\boldsymbol{P}}$

Whereas $\boldsymbol{V}$ is usually known (the aircraft airspeed), $\boldsymbol{v}$ is not, so $\boldsymbol{Tv}$ is replaced by the ideal power (P_{ideal}) and the efficiency is referred to as a "**Figure of Merit**" (*FoM*).

$$\eta_r = \frac{\boldsymbol{P_{ideal}}}{\boldsymbol{P}}$$ *(which we can call rotor efficiency or FoM)*

The **IDEAL** propeller is one with **NO LOSSES**.

Thus all the input power (P) is converted to increased kinetic energy (ΔKE),

$\boldsymbol{P = \Delta KE = T\,(V + v)}$, *(as determined earlier)*

For the ideal propeller, when $\boldsymbol{V \neq 0}$ then, $\eta_p = \dfrac{TV}{T(V+v)} = \dfrac{1}{1+a}$

Where $\boldsymbol{a} = \dfrac{\boldsymbol{v}}{\boldsymbol{V}}$ and is known as the **axial interference factor (inflow ratio)**.

This suggests that maximum efficiency occurs when $\boldsymbol{a}=0$.

Since $\boldsymbol{T = 2\rho A V^2 a(1 + a)}$ then when $\boldsymbol{a}=0$, $\boldsymbol{T}=0$.

(This is the trivial case and has no practical use)

The value of "$\boldsymbol{a}$ " can be found analytically in terms of a thrust coefficient $\boldsymbol{T_c}$

Where $\boldsymbol{T_c} = \dfrac{\boldsymbol{T}}{\rho \boldsymbol{V}^2 \boldsymbol{D}^2} = \dfrac{2\rho(\pi D^2/4)V^2 a(1+a)}{\rho V^2 D^2} = \dfrac{\pi a(1+a)}{2}$ *(D is propeller diameter)*

Taking only the positive root the quadratic equation $\pi a^2 + \pi a - 2T_c = 0$

$$a = \tfrac{1}{2}\left\{\sqrt{1 + \frac{8T_c}{\pi}} - 1\right\}$$

The Static Thrust of an IDEAL propeller

The general thrust coefficient $T_c = \dfrac{T}{\rho V^2 D^2}$ has limited value for propellers as it has an infinite value at $V=0$.

A more suitable **thrust coefficient** is $C_T = \dfrac{T}{\rho n^2 D^4}$ giving a static thrust coefficient based on rotational speed n (revs/sec) and the propeller diameter D.

We can refer to this parameter nD as the **reference velocity**.

It is also normal to express the forward speed (V) of the propeller relative to this reference velocity and this is called the **Advance Ratio J**. Thus $J = \dfrac{V}{nD}$

Substituting $V^2 = n^2 D^2 J^2$ into $T_c = \dfrac{T}{\rho V^2 D^2}$

gives $T_c = \dfrac{T}{\rho n^2 D^4 J^2} = \dfrac{C_T}{J^2}$

The Static Thrust of an IDEAL propeller

Recall $\boxed{a = \dfrac{v}{V} \quad \text{is known as the } \textbf{axial interference factor (inflow ratio)}.}$

The inflow ratio (***a***) also has limited value in propeller analysis as it too has an infinite value at ***V***=*0*.

For **Static Thrust** the actual induced velocity (***v***) is required.

$$T = 2\rho A(V + v)v = 2\rho A v^2 \quad \text{so that} \qquad v = \sqrt{\frac{T}{2\rho A}}$$

The POWER of an IDEAL propeller.

The Power is non-dimensionalised in a similar manner to thrust but with an additional ***nD*** term as $P \propto TV$ (and ***nD*** is the reference velocity parameter)

Thus **power coefficient** $C_P = \dfrac{P}{\rho n^3 D^5}$

The propeller efficiency $\eta_p = \dfrac{TV}{P}$

Thus $\eta_p P = \dfrac{\pi}{2} D^2 \rho V^3 (1+a)a = \dfrac{\pi}{2} D^2 \rho V^3 \dfrac{(1-\eta_p)}{\eta_p{}^2}$

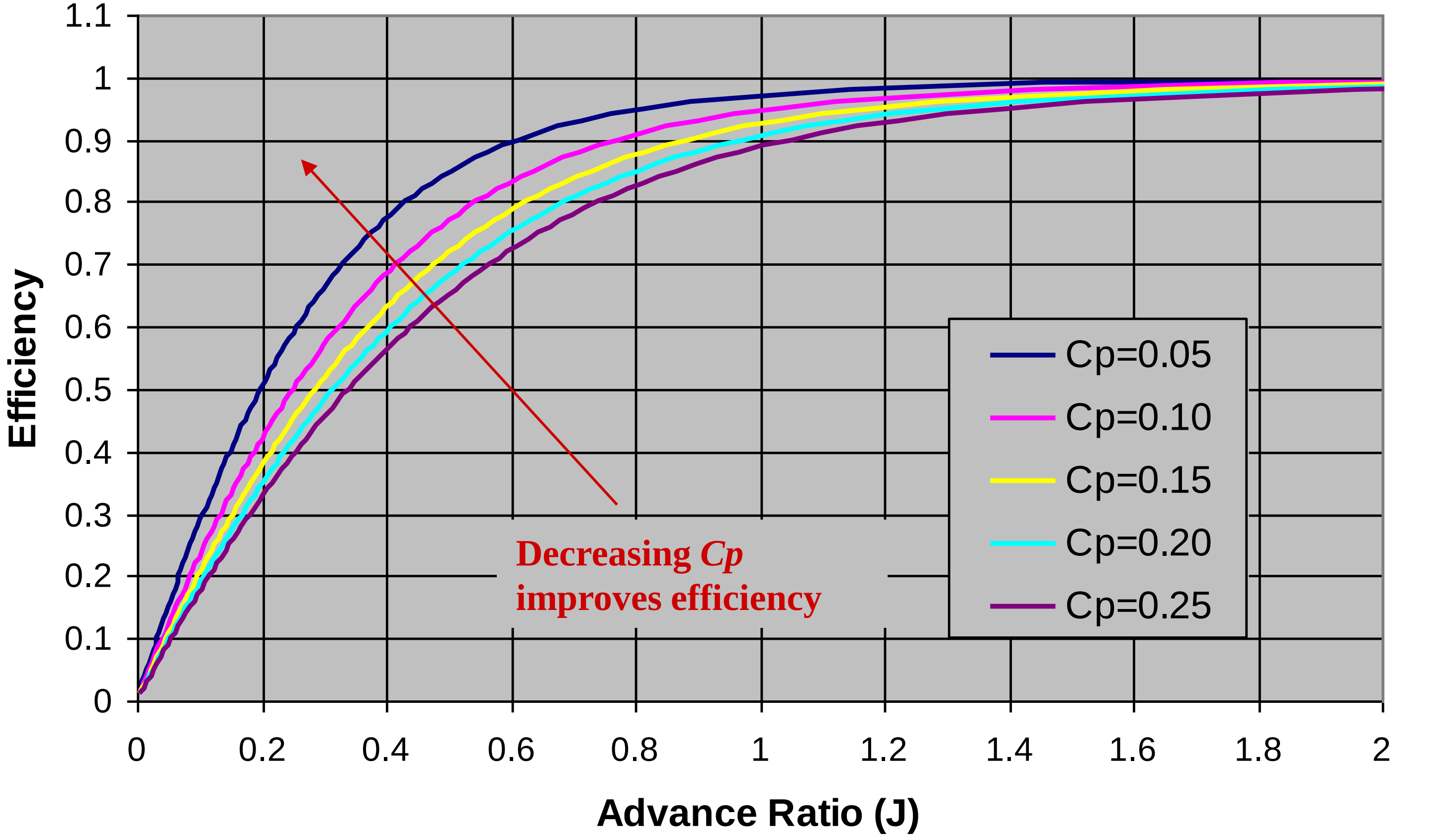
and $$\boxed{\frac{1-\eta_p}{\eta_p{}^3} = \frac{2P}{\pi \rho D^2 V^3} = \frac{2C_P}{\pi J^3}}$$

Of academic interest only (ideal rotor) but gives an indication of parameter sensitivity.

Ideal Propeller

(for a range of Cp values)

$$\frac{1-\eta_p}{\eta_p^{\,3}} = \frac{2P}{\pi \rho D^2 V^3} = \frac{2C_P}{\pi J^3}$$



The Propeller in Action

Ideally

In flight cruise, propeller efficiency is $\eta_p = \frac{TV}{P} \left(= \frac{TV}{T(V+v)} \right)$ and v must be small.

In a similar manner to conventional Lift & Drag coefficients we have a Thrust coefficient:

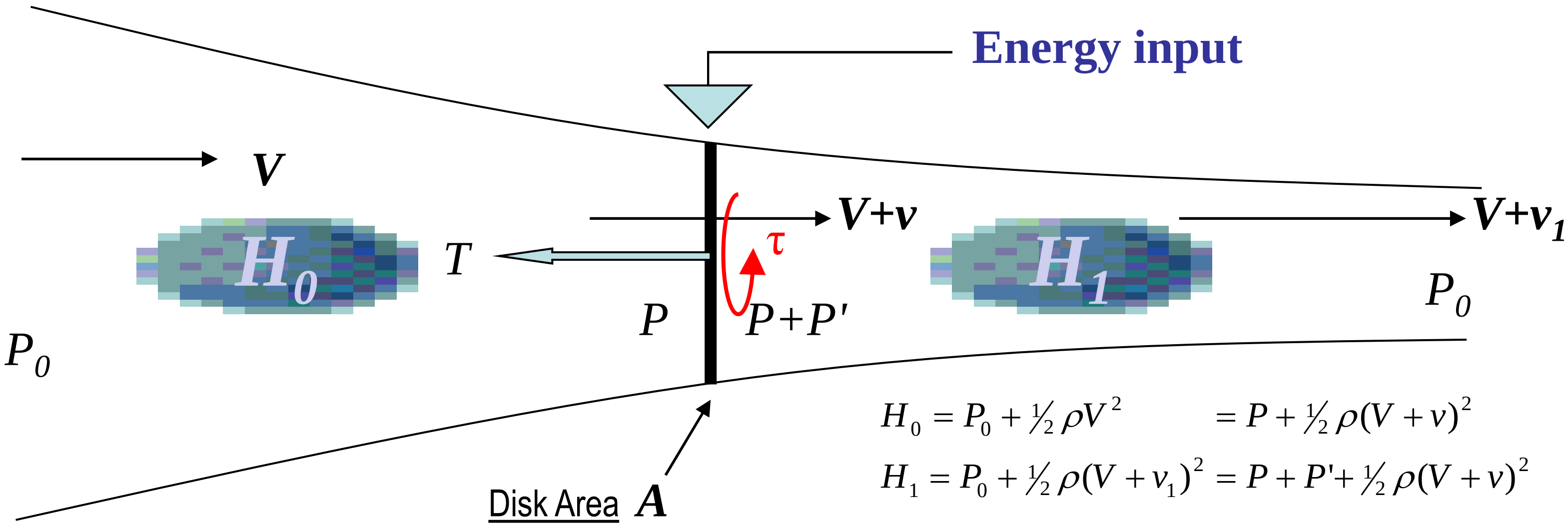
$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S} \qquad C_D = \frac{D}{\frac{1}{2}\rho V^2 S} \qquad \boxed{C_T = \frac{T}{\rho n^2 D^4}}$$

Analysis of Propeller Aerodynamics

Lecture 2

Notes in Blackboard: https://www.ole.bris.ac.uk

The actuator disk can add only static pressure to the flow and <u>nothing else</u>



$$H_0 = P_0 + \tfrac{1}{2}\rho V^2 \qquad = P + \tfrac{1}{2}\rho (V+v)^2$$

$$H_1 = P_0 + \tfrac{1}{2}\rho (V+v_1)^2 = P + P' + \tfrac{1}{2}\rho (V+v)^2$$

More generally in both H_0 and H_1 states;

$$\overline{P} + \tfrac{1}{2}\rho \overline{V}^2 = \text{contant, thus } \frac{\overline{P}}{\rho} + \tfrac{1}{2}\overline{V}^2 = \text{constant}$$

$$\text{or } \frac{\overline{P}}{\rho} + \tfrac{1}{2}\overline{V}^2 + \underline{\frac{K^2}{A}} = \text{constant, where } K = \frac{\tau}{\dot{m}}$$

["K" *is a* "swirl parameter" based on a free vortex

and τ is fan torque and $\dot{m}$ is the mass flow rate]

Momentum Analysis based on Actuator Disk Theorem can be extended to include swirl, though strictly speaking it can no longer be referred to as an Actuator Disk.

The REALity is that IDEAL PROPELLERS don't exist, REAL PROPELLERS do!

In analysing and predicting propeller / rotor performance we need to consider:

- Viscosity of the fluid
- Rotor blade tip effects
- Non-uniform inflow
- Blade vortex interaction

$$\boxed{\eta_p = \frac{TV}{P}\left(= \frac{TV}{T(V+v) + \underline{\text{Losses}}} \right)}$$

Needless to say, these result in energy loss and therefore reduced efficiency.

In order to obtain a more detailed knowledge of the behaviour of a rotor:

- It is necessary to analyse the forces on the rotor blades.
- The blade has to be considered as a number of separate aerofoil elements.
- Elements are then integrated to represent the characteristics of the whole propeller.

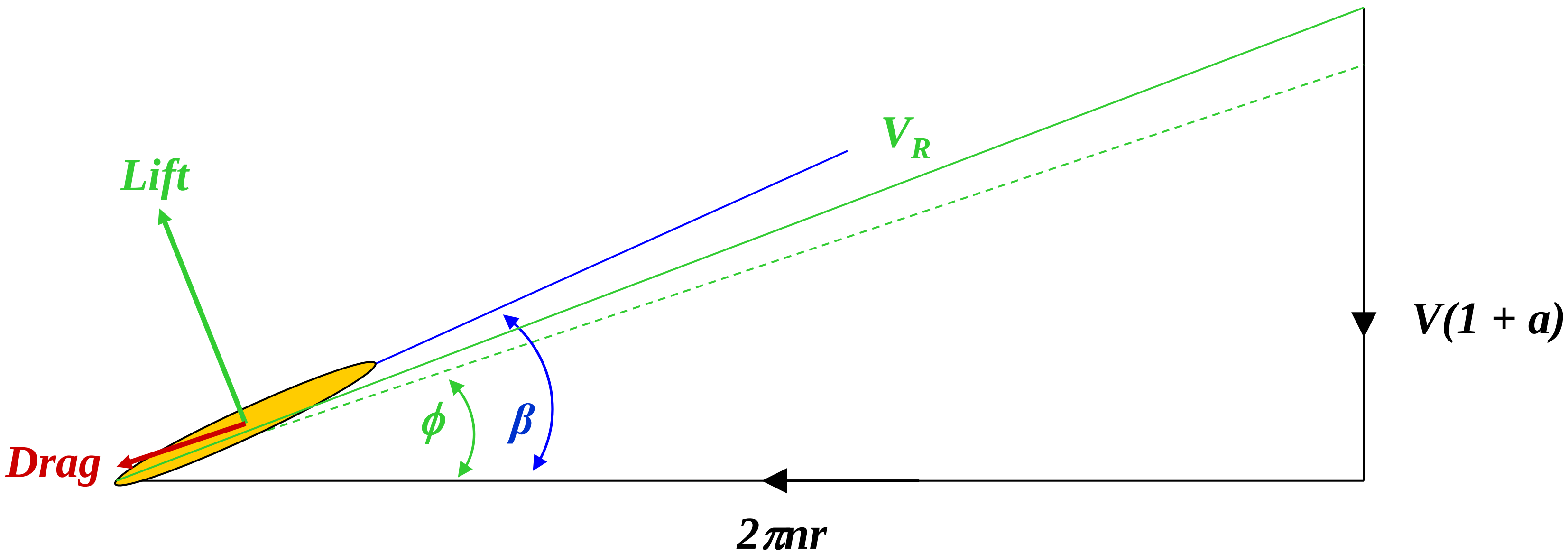
This is achieved by BLADE ELEMENT (often called STRIP ANALYSIS)

Interference Flows

These are the axial and rotational flows that are in addition to those of the rotor.

Axial

The axial interference flow (also referred to as the "induced" velocity $\boldsymbol{v}$) has been determined previously by momentum analysis. It adds to the onset velocity $\boldsymbol{V}$. This increases the inflow angle, tilts the lift vector backward resulting in induced drag and reduces the angle of incidence given by $\boxed{\alpha = (\beta - \phi).}$

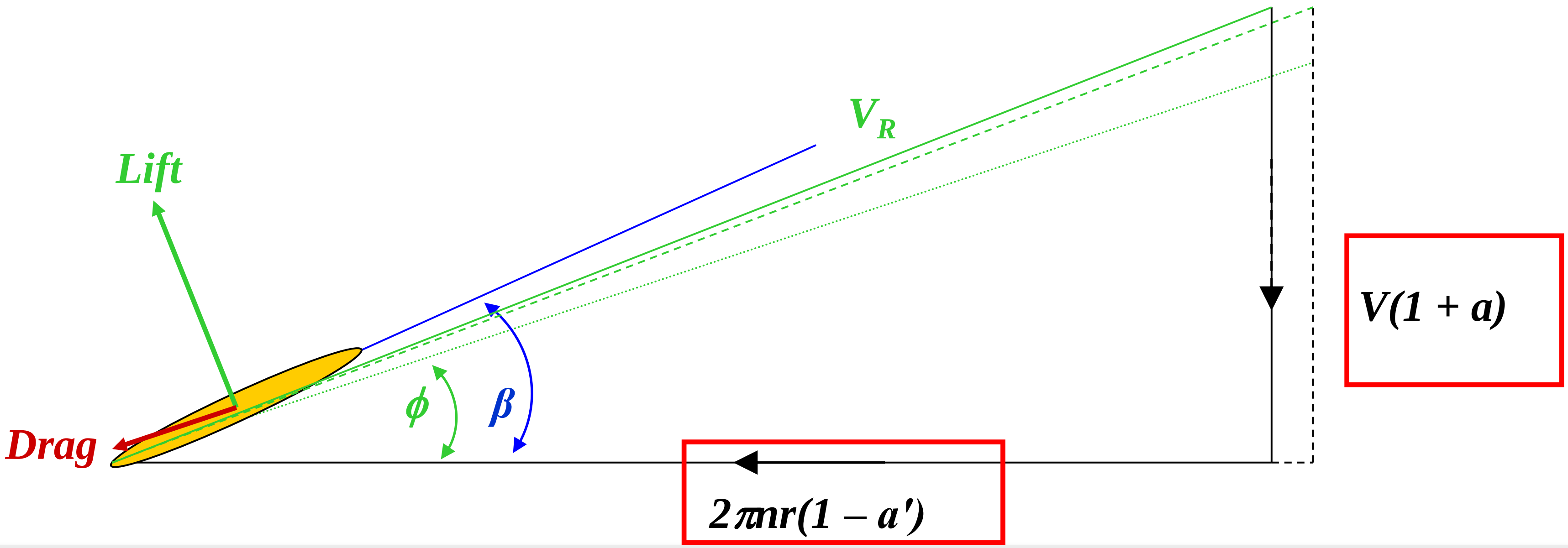


Interference Flows

Radial

The rotational interference flow (which we shall call ***w***) can be determined from vortex theory and is physically represented by the swirl of the rotor wake. It is suffice to say that it has a value ***w*** at the rotor and ***2w*** in the far wake. Unlike the axial interference flow, it has no value upstream of the rotor and it is not wholly induced, as it contains a contribution from the viscous drag of the rotor blades.

This also increases the inflow angle, tilting the lift vector backward and further increasing induced drag and reducing the angle of incidence given by $\alpha = (\beta - \phi)$.

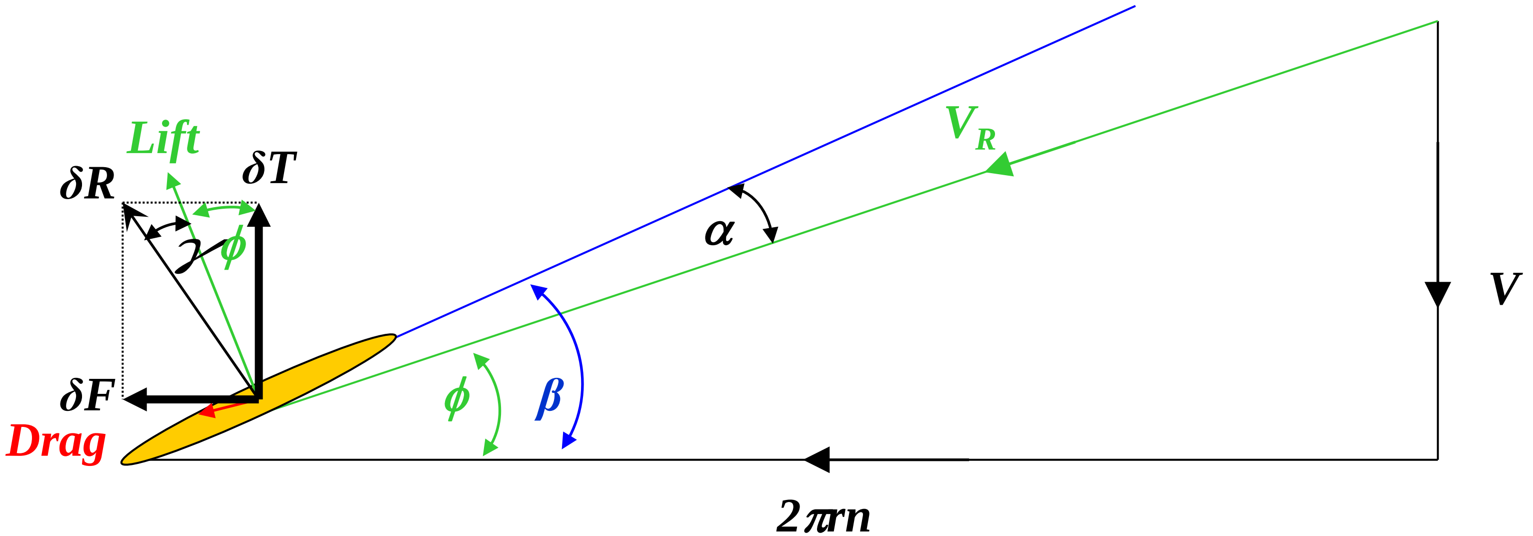


If γ is the angle between the lift component and the resultant force then:

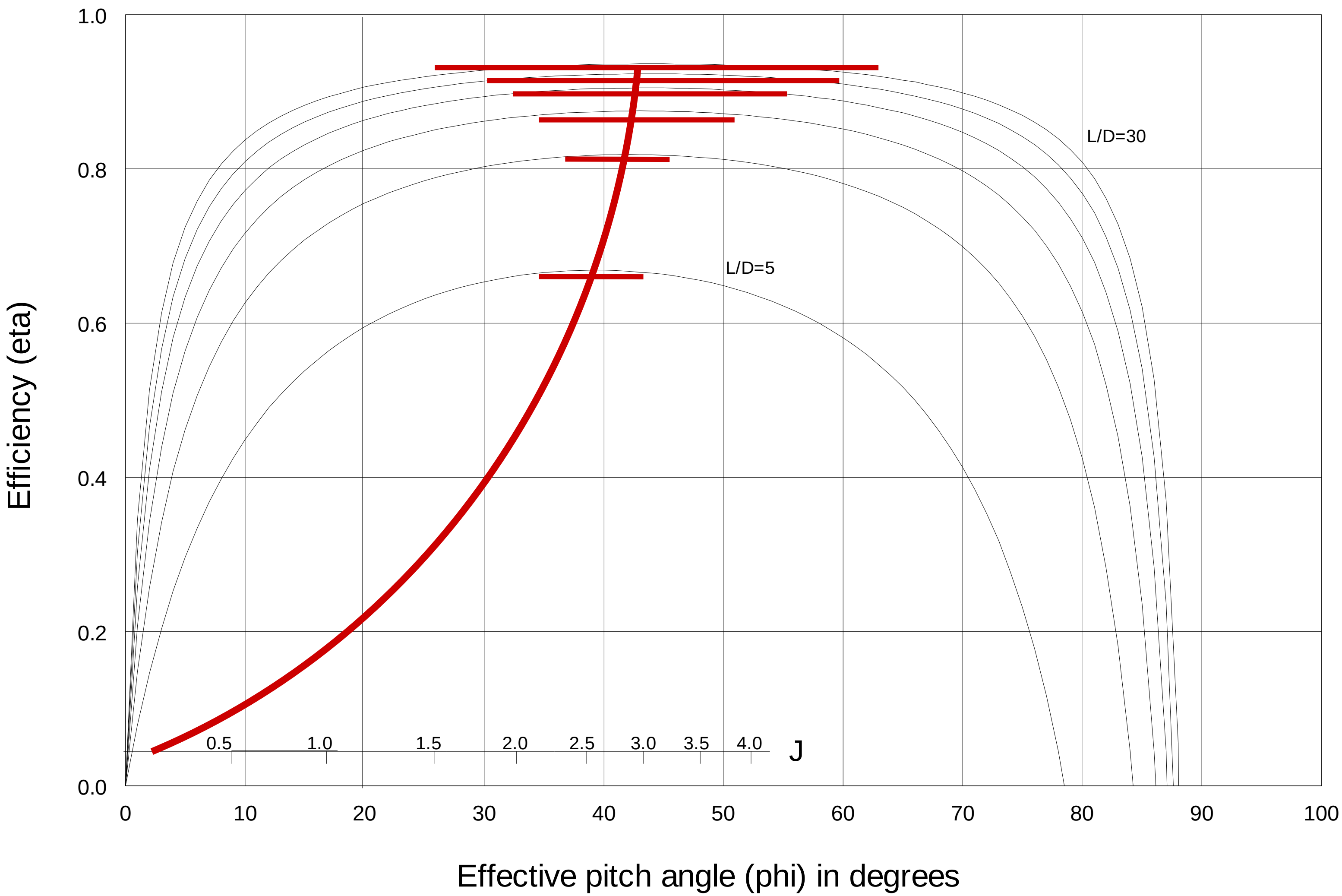
$$\boxed{\tan\gamma = \frac{D}{L} = \frac{C_D}{C_L}}$$

$$\boxed{\boxed{\eta' = \frac{\delta T V}{\delta F\, 2\pi r n}} = \frac{\delta R \cos(\phi+\gamma) V}{\delta R \sin(\phi+\gamma) 2\pi r n} = \frac{\tan\phi}{\tan(\phi+\gamma)}}$$

… using $\tan\phi = \frac{V}{2\pi r n}$



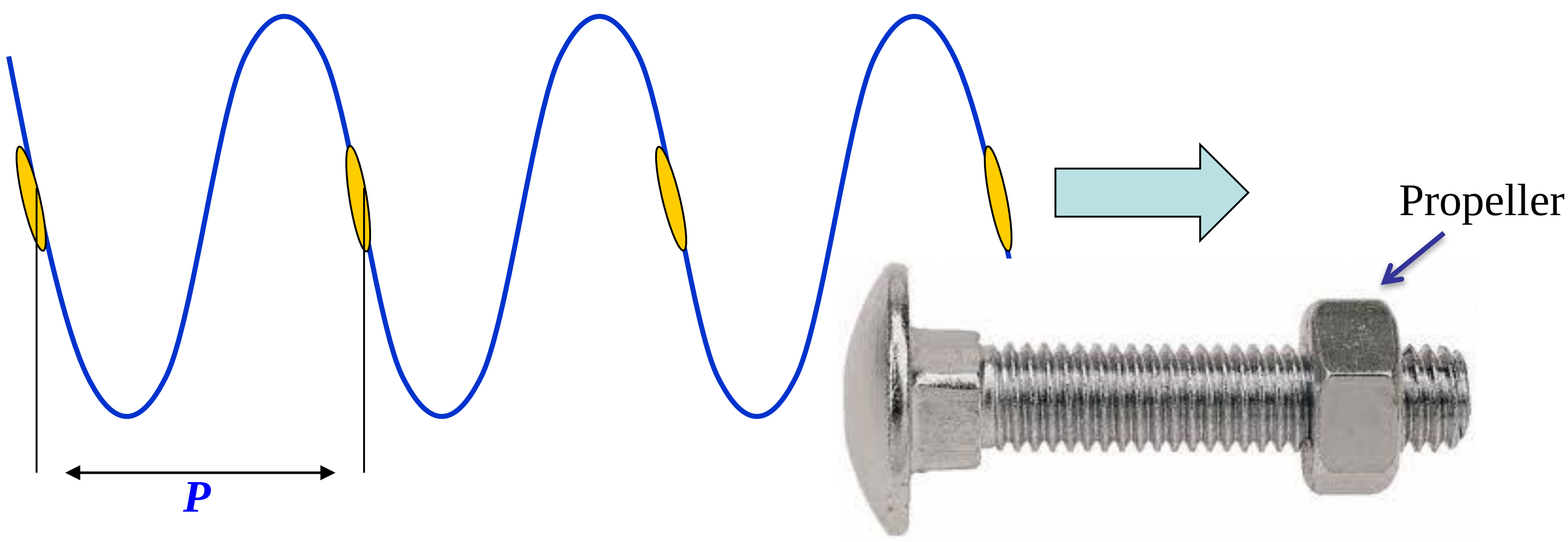
Blade-element efficiency as a function of effective pitch angle



Propeller Pitch

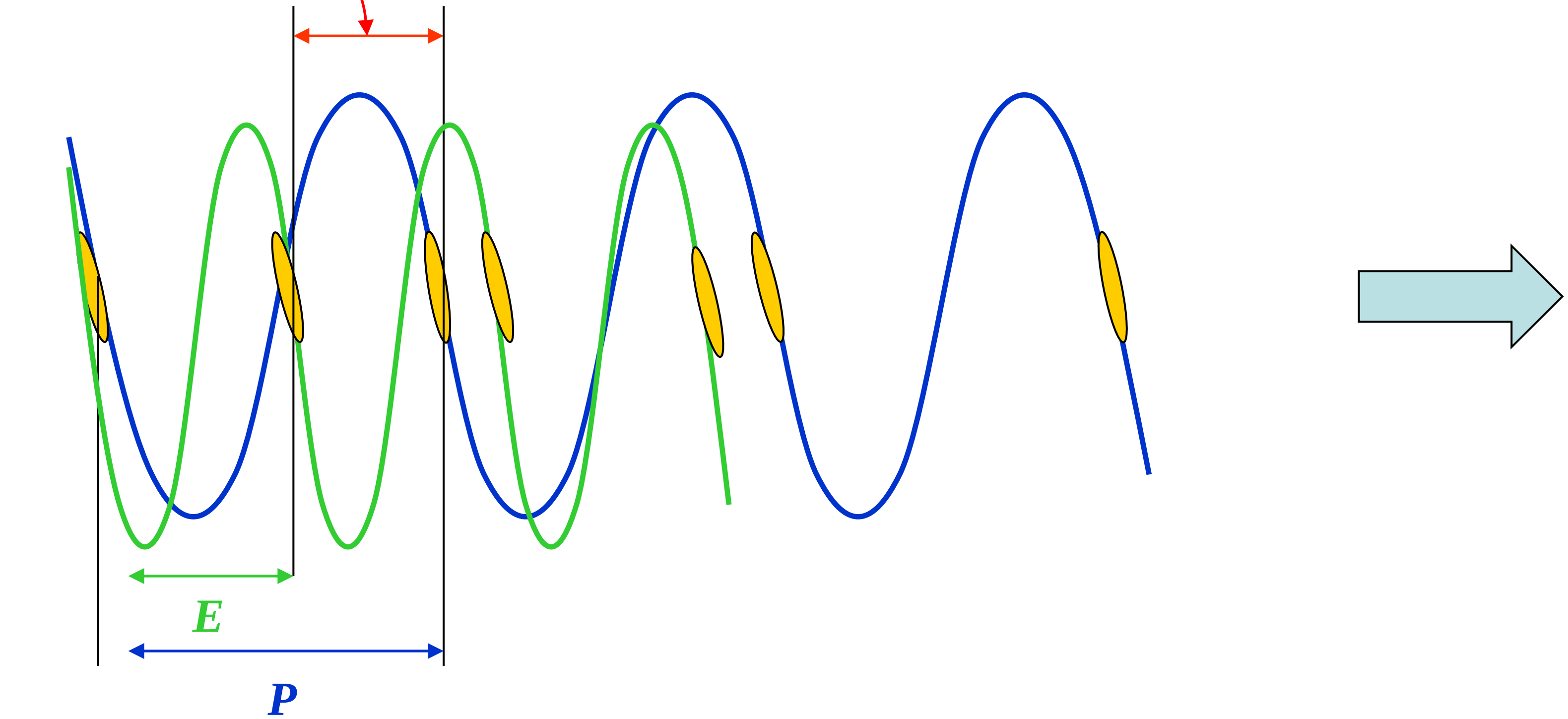
Whilst Propeller Pitch Angle (β) has been referred to in the previous illustrations, Propeller Pitch is a linear distance not an angle and if Propeller Pitch Angle is quoted it should state the radial station at which it is measured (e.g. $\beta_{0.75R}$)

The Geometric Pitch (***P***) of a propeller is the axial displacement of the propeller prescribed by the geometric chord in one revolution. This is analogous with the mechanical screw thread (which is why propellers were originally called "airscrews").



Propeller Pitch

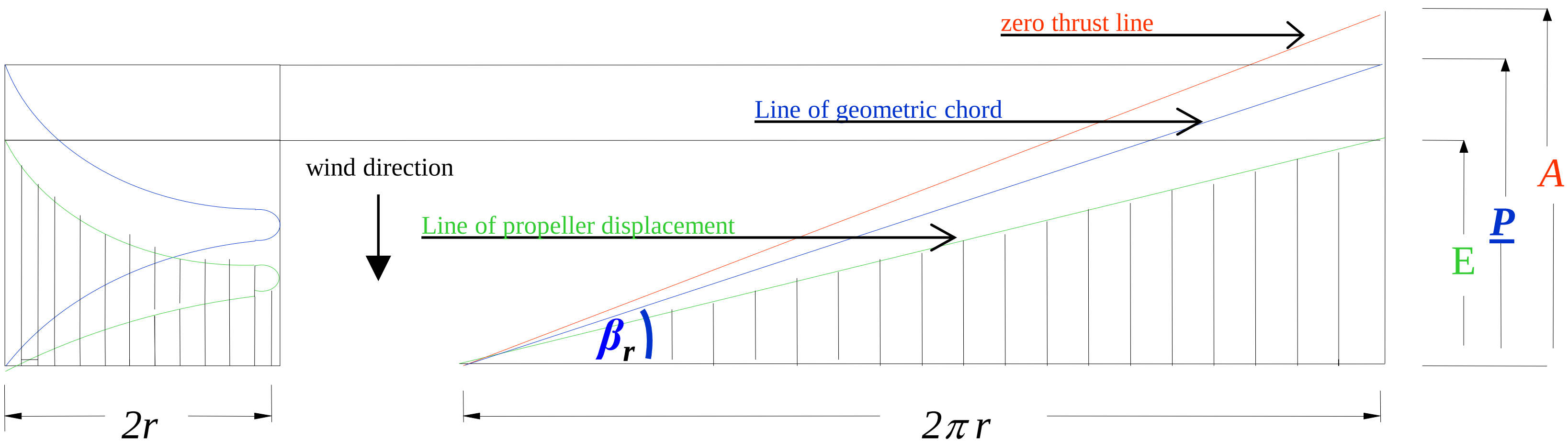
Whilst the Geometric Pitch (***P***) of a propeller is the theoretical axial displacement of the propeller prescribed by the geometric chord in one revolution, the Effective Pitch (***E***) is the actual axial displacement of the propeller. The difference between these two lengths is called the Propeller Slip. If thrust is to be produced there must be a finite value of Propeller Slip.



Propeller Pitch

The Geometric Pitch (***P***) of a propeller cannot be easily measured and this is why use is often made of the Propeller Pitch Angle (***β***) as it can be determined with a protractor. The Effective Pitch (***E***) is similarly difficult to measure as a length but in unit time it represents the velocity of the aircraft.

Thus the “developed view” of the propeller geometry is analogous of the velocity diagrams previously described.



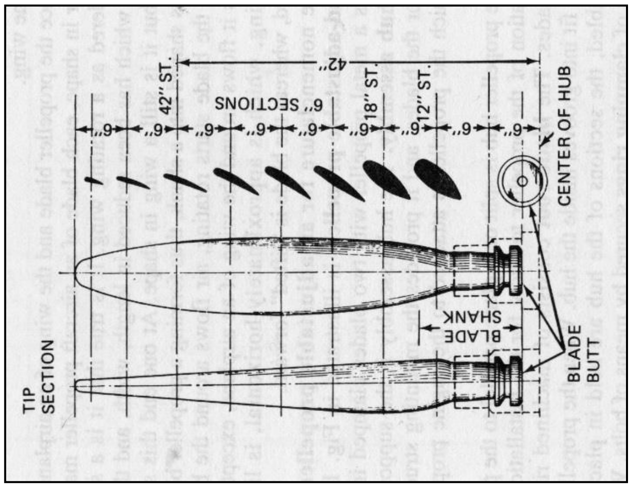
E = Effective Pitch

P = Geometric Pitch

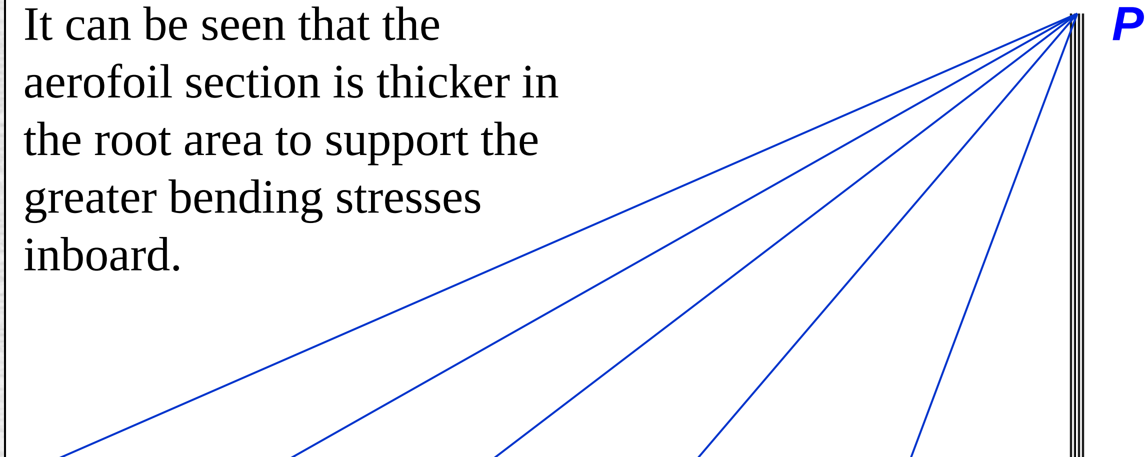
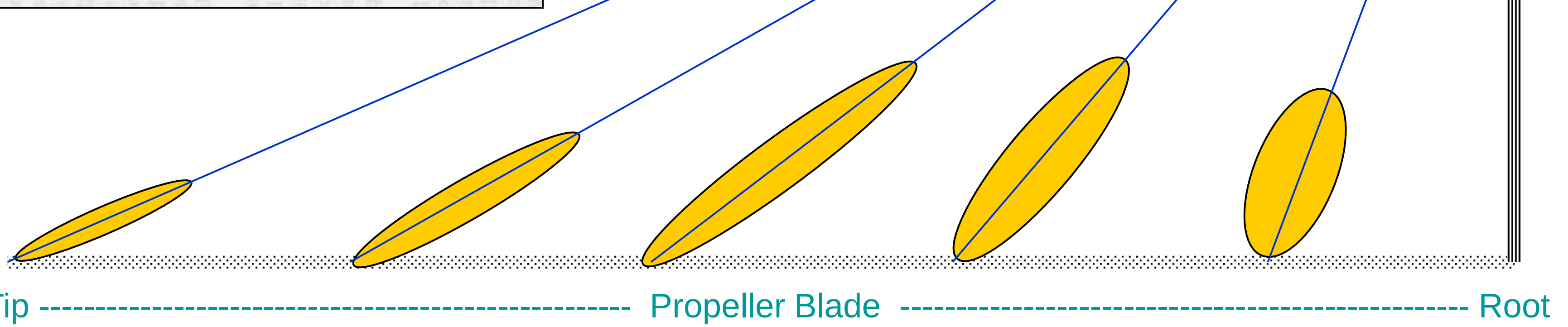
A = Aerodynamic Pitch

Propeller Pitch

The discussion so far has referred to the Geometric Pitch (***P***) of an element of the propeller at one radial station. If the propeller blade has a radial twist such that all stations have the same Geometric Pitch the propeller is described as a Constant Pitch Propeller, as shown below.

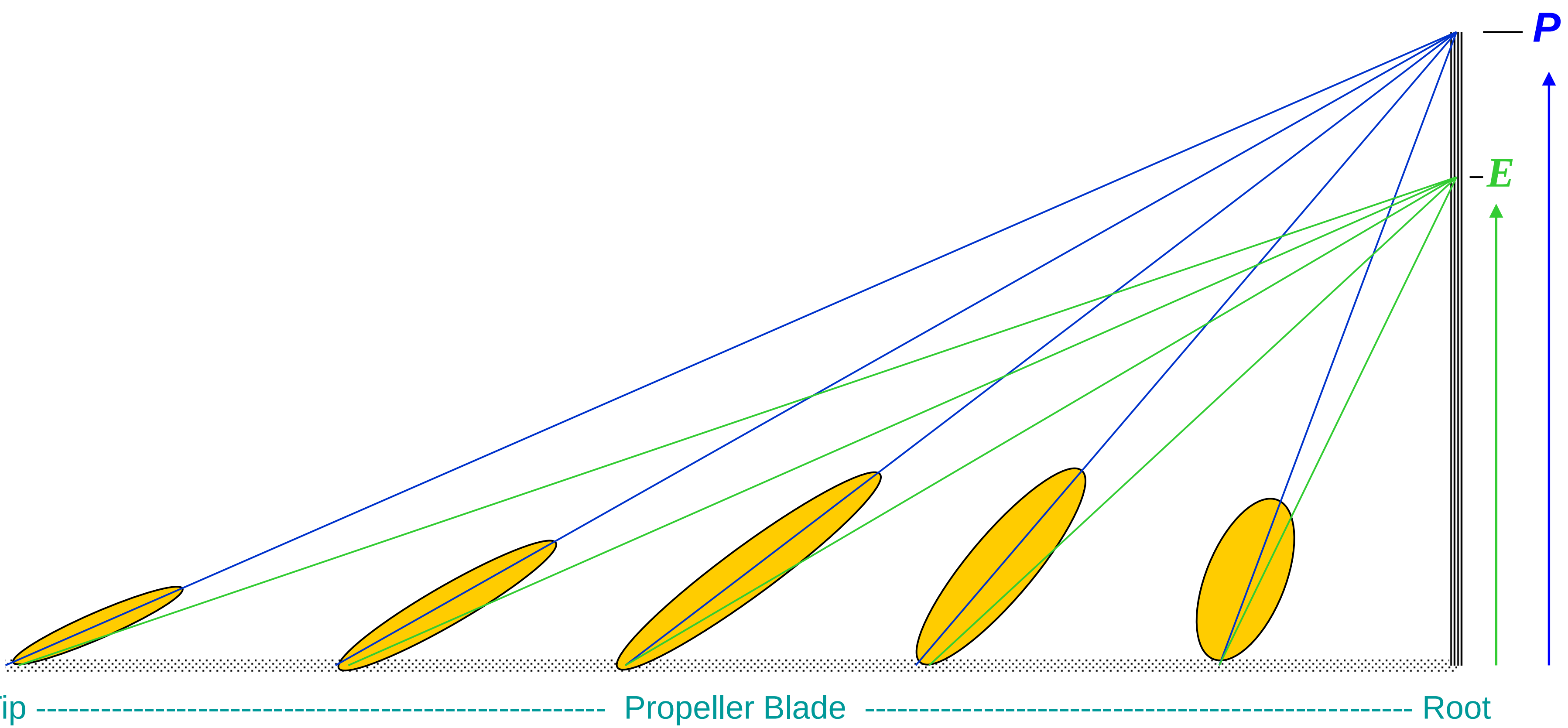


It can be seen that the aerofoil section is thicker in the root area to support the greater bending stresses inboard.

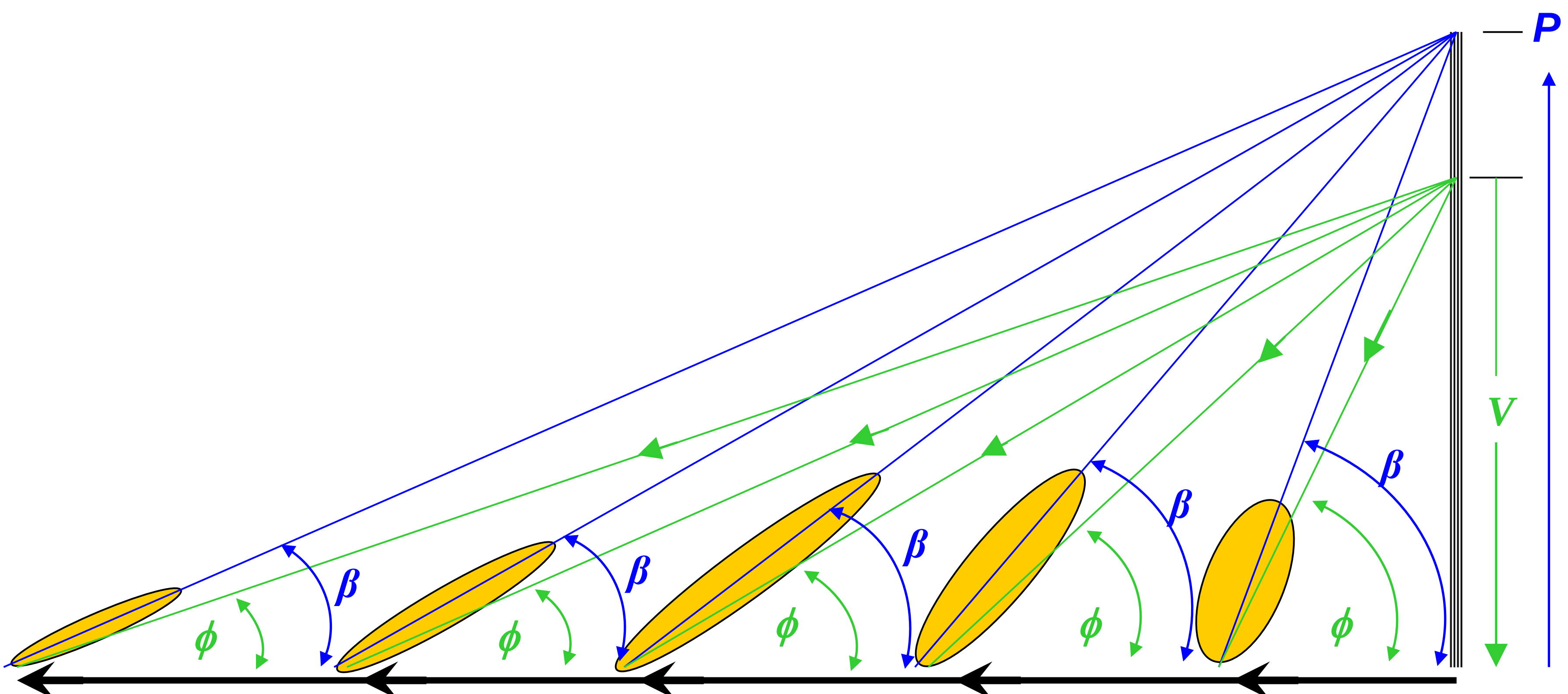
Propeller Pitch

The radial twist shown is normal for propellers. Whilst the rotational speed "***n***" is constant, the velocity in the plane of rotation increases with radius. Thus the static analysis that indicates Geometric Pitch (***P***) and Effective Pitch (***E***)



Propeller Pitch

The radial twist shown is normal for propellers. Whilst the rotational speed "***n***" is constant, the velocity in the plane of rotation increases with radius. Thus the static analysis that indicates Geometric Pitch (***P***) and Effective Pitch (***E***) <u>is analogous to the dynamic analysis of velocities and inflow angles.</u>



Propeller Performance and Design

Lecture 3

Notes in Blackboard: https://www.ole.bris.ac.uk

The combination of Momentum Theory and Blade Element Analysis allows for some basic design parameters to be determined.

For Constant Pitch Propeller

$$\boxed{\tan\beta = \frac{P}{2\pi r}}$$

The Geometric Pitch (***P***) is the axial displacement of the propeller prescribed by the physical shape of the blades.

The angle (β) is the angle of the blade to the plane of

Since all blade radial locations must have the same value of ***E***, then:

$$\boxed{\tan\phi = \frac{E}{2\pi r}}$$

The Effective Pitch (*E*) is the axial displacement of the propeller relative to the general air mass (i.e. the progress of the aircraft).

The angle (ϕ) is the angle of the resultant airflow to the plane of rotation.

But since this is not a static condition, it is more convenient to replace ***E*** with ***V*** and modify the rotational displacement accordingly then:

$$\boxed{\tan\phi = \frac{V}{2\pi\, rn} \quad \left(= \frac{V}{2\pi\, rn}\frac{D}{D} = \frac{JD}{2\pi\, r} \right)}$$

Now since the angle of incidence $\boxed{\alpha = \beta - \phi}$ then

$$\frac{5}{2\pi(0.75R)} = \frac{5}{4.7} = 1.06 --> 46.6\text{ degs}$$

$$\boxed{\alpha = \arctan\frac{P}{2\pi r} - \arctan\frac{JD}{2\pi r}}$$

$$\frac{2x2}{2\pi(0.75R)} = \frac{4}{4.7} = 0.848 --> 40.3\text{ degs}$$

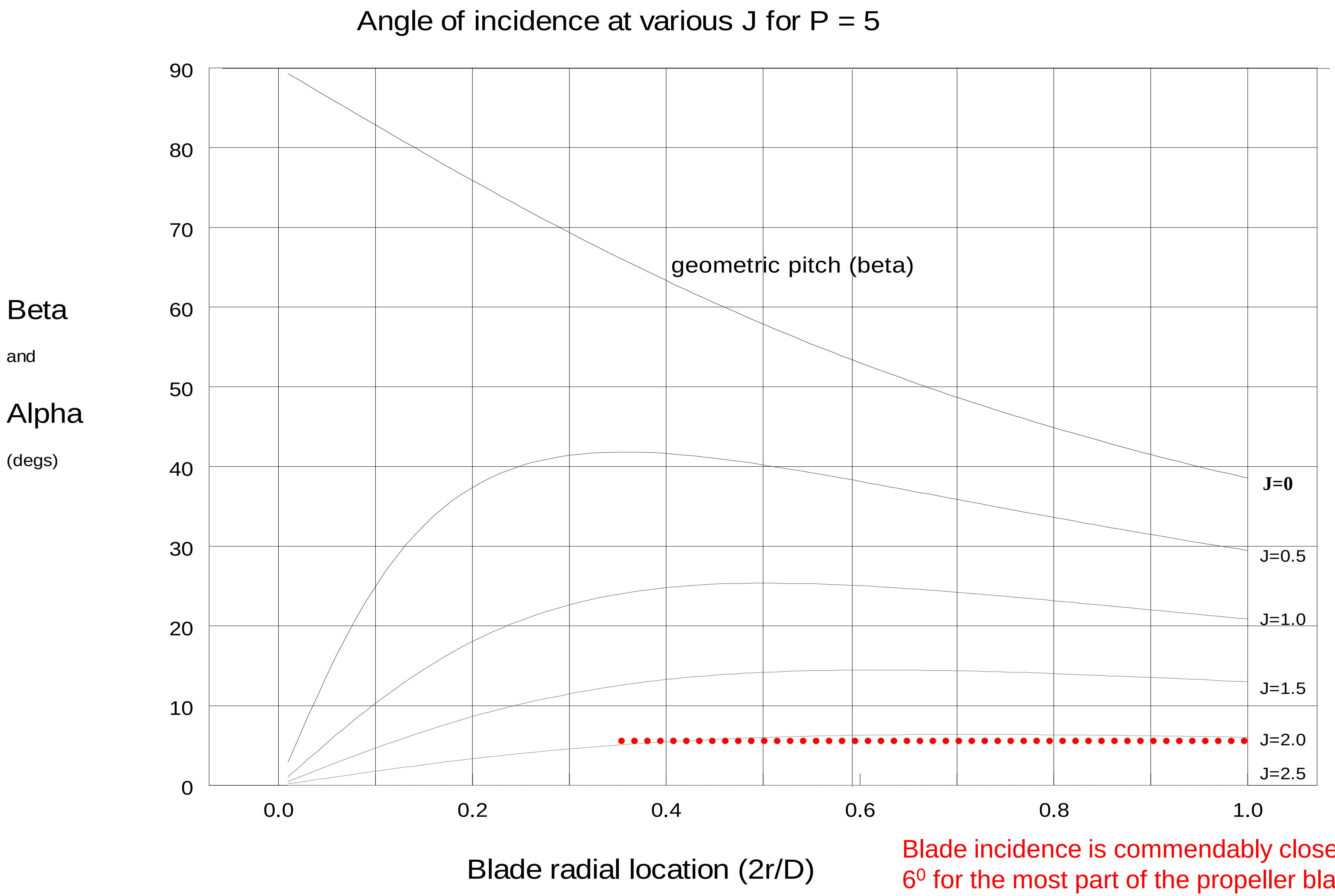
It can be seen that when $P = JD$ $\left(\text{ie. when } J = \dfrac{P}{D}\right)$ then the incidence along the entire blade is zero and the blade produces zero lift.

N.B. - In practice it is unusual for all the radial locations of a blade to have the same pitch (referred to as Constant Pitch).

For example; If $\boldsymbol{P}$=5*m* and $\boldsymbol{D}$=2*m*, the angle of incidence (α) can be determined over a range of velocities (shown as values of ***J***). What about ***J***=2?

The values for ***P*** and ***D*** were chosen to give an angle of incidence of 6 degrees at $\boldsymbol{\beta}_{0.75}$ (at J=2). This angle of incidence is typical of that for best lift/drag of an aerofoil section.

PROPELLER DESIGN – Example



Blade incidence is commendably close to 6^0 for the most part of the propeller blade.

Design for Optimum Incidence

In practice, it is unusual for all the radial locations of a blade to have the same pitch (Constant Pitch).

In theory the radial twist of the blade can be such that the angle of incidence is constant along it's length (for a particular ***J***) so that the entire blade can operate at it's best L/D.

For the incidence to remain constant along the length of the blade, the Geometric Blade Pitch Angle (***β***) for each radial station can be determined as follows:

$$\beta = \alpha + \phi \,, \qquad \tan\phi = \frac{V}{2\pi r n} = \frac{JnD}{2\pi r n} = \frac{JD}{2\pi r}$$

thus $$\tan\beta = \tan(\alpha + \phi) = \frac{\tan\alpha + \tan\phi}{1 - \tan\alpha\tan\phi}$$

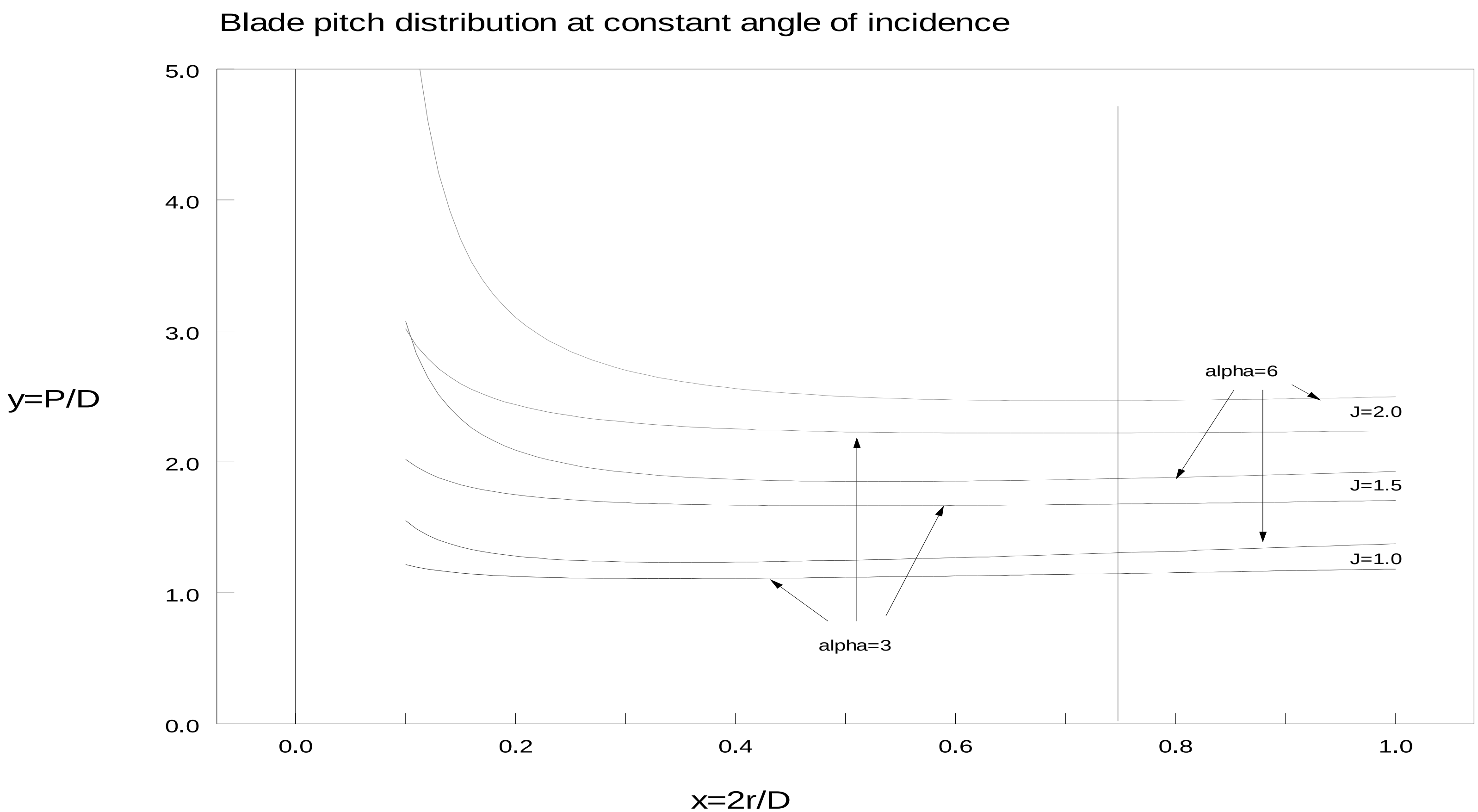
but $$\tan\beta = \frac{P}{2\pi r},$$ so $$P = 2\pi r\,\frac{\tan\alpha + JD/2\pi r}{1 - (JD/2\pi r)\tan\alpha}$$

Using the dimensionless quantities $$y = \frac{P}{D} \quad \text{and} \quad x = \frac{2r}{D}$$

$$\boxed{y = \pi x\,\frac{\tan\alpha + J/\pi x}{1 - J\tan\alpha/\pi x} = J\,\frac{(1 + \pi x\tan\alpha/J)}{(1 - J\tan\alpha/\pi x)}}$$

Example

This has been plotted for two angles of incidence, $\alpha = 3^0$ and $\alpha = 6^0$ and for three values of advance ratio, $J = 1.0, J = 1.5$ and $J = 2.0$ and this is shown below:



PROPELLER PERFORMANCE

Recalling the Coefficients used in the analysis of propeller performance

$$J = \frac{V}{nD} \text{ (where } J \text{ in the Advance Ratio)}$$

$$C_P = \frac{P}{\rho n^3 D^5}$$

$$C_T = \frac{T}{\rho n^2 D^4}$$

$$\eta_p = \frac{TV}{P} = \frac{C_T \rho n^2 D^4 V}{C_P \rho n^3 D^5} = \frac{C_T J}{C_P}$$

Where ρ = air density, n = propeller rotational speed (revs/sec), D = propeller diameter

Propellers vs. Ducted Fans

Lecture 4

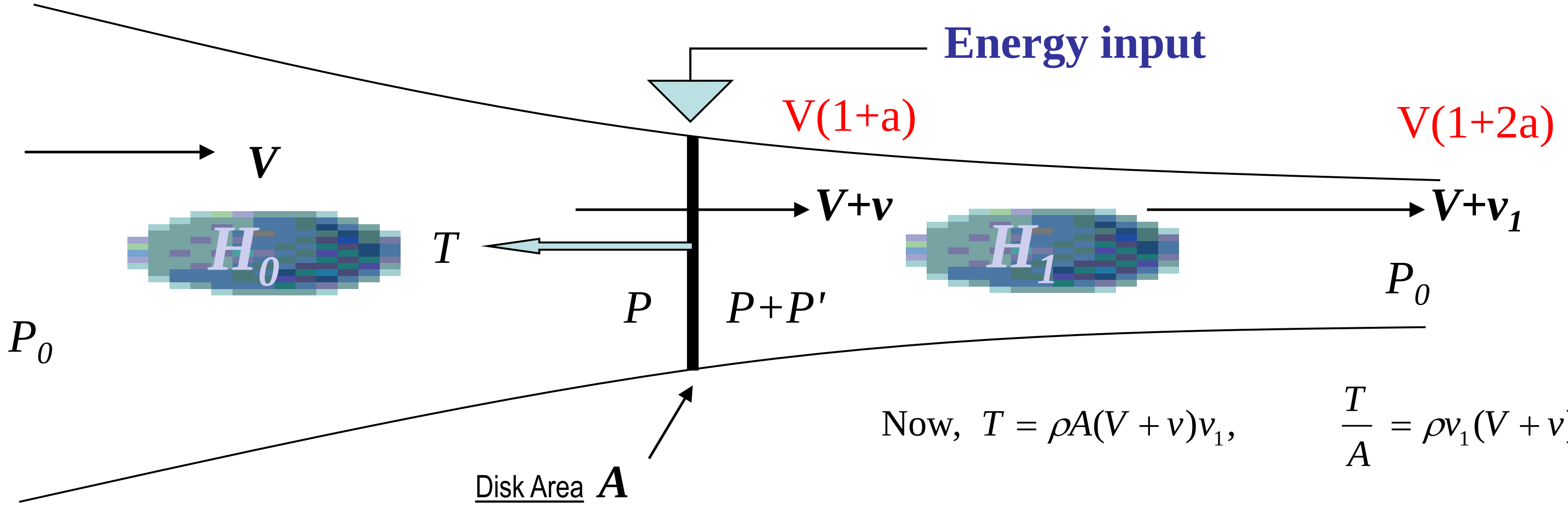
Notes in Blackboard: https://www.ole.bris.ac.uk

Tractor

Pusher

Tractor and Pusher

Open Propellers (Recall)



Now, $T = \rho A(V+v)v_1$, $\quad \dfrac{T}{A} = \rho v_1 (V+v)$

$so\ \dfrac{v_1}{2} = v \qquad \therefore T = 2\rho A(V+v)v$

ρ Air density

A Disc area

$$remembering\ \ that\ \ a = v/V$$

$$T = 2\rho A(V+v)v \ = \ 2\rho AV^2(1+a)a$$

$$or\ \ (1+2a)^2 = 1 + \frac{2T}{\rho AV^2}$$

Open Propeller (Tractor propeller drag effects)

$$T = 2\rho A(V+v)v = 2\rho AV^2(1+a)a \quad (\textit{remembering that } a = v/V)$$

$$or \;\; (1+2a)^2 = 1 + \frac{2T}{\rho AV^2}$$

The aircraft body (in wake) experiences this increased velocity *V(1+2a)* and hence the drag D which it would experience in a stream of velocity V is increased to a higher value D^*_{biw} according to the equation:

$$D^*_{biw} = D(1+2a)^2 = D\left[1 + \frac{2T}{\rho AV^2}\right]$$

Propulsive thrust T_P is the apparent thrust T minus the increased drag $(D^* - D)$ of the aircraft due to the interference of the propeller:

$$T_P = T - (D^* - D) = T\left[1 - \frac{2D}{\rho AV^2}\right]$$

Since the drag D $(\textit{and } D^*)$ is proportional to the square of the velocity V, the propulsive thrust is a constant fraction of the apparent thrust.