MANOEUVRE STABILITY

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Tug Aircraft: 141hp EuroFox 2K

Airbus A350-1000

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Reminder: static stability/neutral point

In the previous lecture we considered the static stability of an aircraft in steady level flight. It was determined that an aircraft is statically stable if the centre of gravity (CG) is ahead of the **neutral point**.

The expression derived for stick-fixed neutral point position as a ratio of mean aerodynamic chord, $\bar{\bar{c}}$, was

$$h_n = \frac{1}{4} + \bar{V}\frac{a_{1_T}}{a_1}(1-k) \qquad (1)$$

- $\bar{V}$ is tail volume coefficient, $\bar{V} = \frac{S_T l_T}{S\bar{\bar{c}}}$, with S_T and S the tailplane and wing reference areas respectively, l_T the distance between the wing and the tail aerodynamic centres and $\bar{\bar{c}}$ the wing mean aerodynamic chord;
- a_{1_T} is horizontal tailplane lift curve slope;
- a_1 is wing lift curve slope (actually, lift curve slope of whole aircraft minus horiz. tailplane);
- $k = \partial\varepsilon/\partial\alpha$ is rate of change of downwash at the tailplane with aircraft angle of attack, α.

Reminder: static stability/neutral point

As the CG moves aft towards the neutral point the aircraft becomes less stable. The distance through which the CG can move rearward before the aircraft displays neutral static stability known as the ***static margin***.

If h designates the position of the CG relative to the mean aerodynamic chord, $\bar{\bar{c}}$, then the static margin is given by H_n where

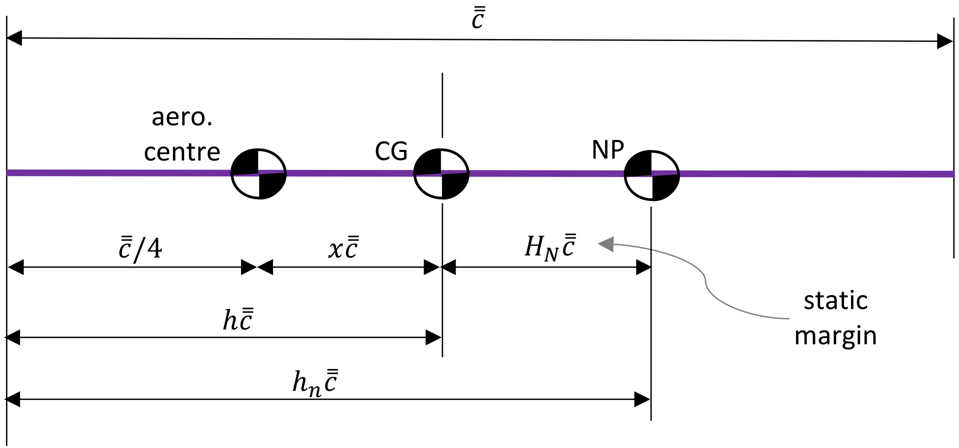
$$H_n = (h_n - h) \tag{2}$$

The static margin is positive if the aircraft is stable. We also saw that

$$\boxed{H_n = -\frac{\partial C_M}{\partial C_L} = (h_n - h) = \bar{V}\frac{a_{1T}}{a_1}(1-k) - x} \tag{3}$$

where x is the distance between the aerodynamic centre of the wing and the CG of the aircraft as a ratio of $\bar{\bar{c}}$.

Reminder: static stability/neutral point



Manoeuvring aircraft

We now wish to consider whether *manoeuvring* the aircraft makes it more or less stable.

In the longitudinal sense this means inducing a non-zero pitch rate, which entails increasing the load factor (n) from 1.

This can be achieved in either a banked horizontal turn or pulling out of a dive.

The latter is easier to treat as it is in the vertical plane and we consider it here.

Flight in a vertical circle

Fig. 1 shows the conditions associated with flight at the bottom of a vertical circular motion (pull-up).

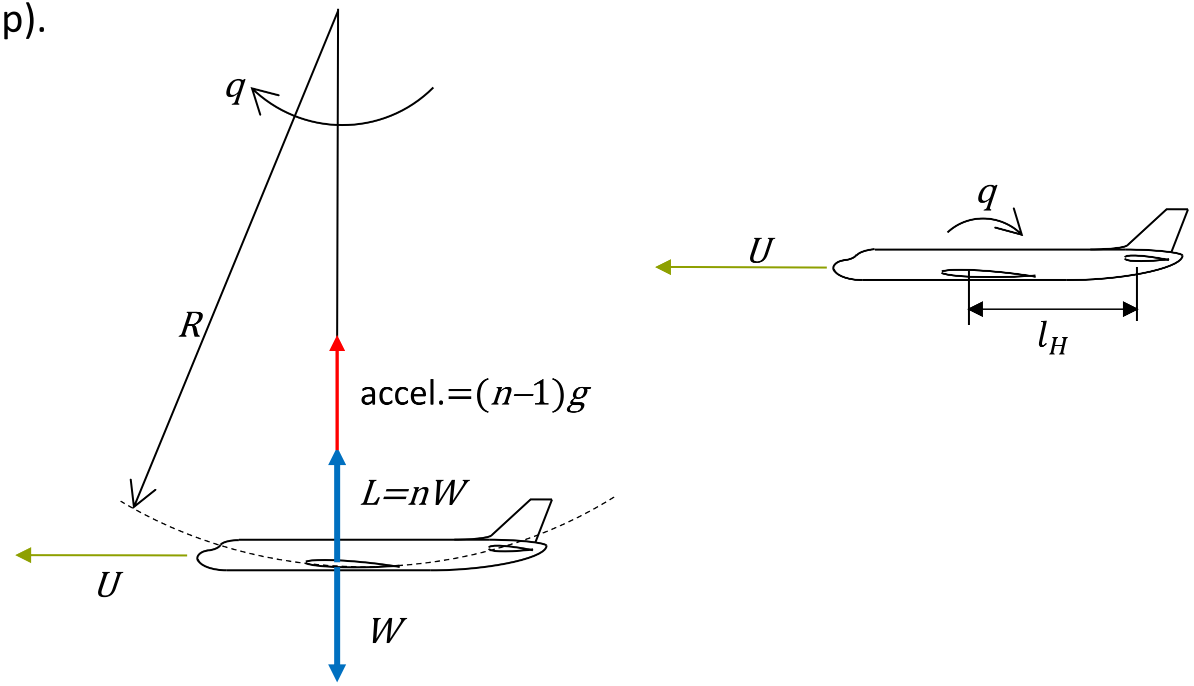


Figure 1: pull-up manoeuvre (circular flight in vertical plane)

Flight in a vertical circle

The forward speed is $U = qR$ where q is the pitch rate and R is the radius of the pull-up manoeuvre. The centripetal acceleration is given by

$$accel. = (n-1)g = \frac{U^2}{R} = q^2R = qU. \tag{4}$$

So the 'g' being pulled can be related to the associated pitch rate by

$$q = \frac{(n-1)}{U}g. \tag{5}$$

Therefore, for $n = 1.0$ we have $q = 0$ as expected.

Flight in a vertical circle

The continuous pitch rate in the manoeuvre adds an increment in incidence to the tailplane of

$$\delta\alpha_T = \frac{q l_H}{U} \tag{6}$$

where l_H is the moment arm from the tailplane aerodynamic centre to the aeroplane CG (see Fig. 1).

Referring to the previous section of notes, the full tail incidence is then

$$\alpha_T = i_T + \alpha(1-k) + \frac{q l_H}{U} \tag{7}$$

where i_T is the tail setting angle relative to the zero-lift line of the wing. Substituting eqn. (5) into eqn. (7) gives

$$\alpha_T = i_T + \alpha(1-k) + \frac{(n-1)}{U^2} g l_H. \tag{8}$$

Flight in a vertical circle

We observe from the previous slide that there is an additional contribution to the tailplane incidence arising from the pitch rate q but negligible additional wing incidence (because the moment arm between the wing centre of lift, approx. the quarter-chord, and the CG is very small).

However, there *is* additional wing lift in this set-up to produce the centripetal acceleration (lift=nW where W is the aircraft weight and n is greater than 1).

The change in balance of lift forces between wing and tailplane to produce the pitching steady state condition is the reason why the new aerodynamic centre of the whole aircraft (the 'neutral point') is different from that in straight-and-level flight.

Stability and manoeuvre point/margin

We now consider the same criterion for testing stability as we did previously for static stability, namely the restoring of aerodynamic stiffness, by looking at

$$\frac{1}{a_1}\frac{\partial C_M}{\partial \alpha} = \frac{\partial C_M}{\partial C_L} \tag{9}$$

and requiring this to be negative as before (because $\frac{\partial C_M}{\partial \alpha}$ represents the restoring moment necessary for stability). Thus we need to consider the pitching moment equation while allowing for the new incidence at the tail and the greater overall lift on the aeroplane.

The lift requirement is

$$Lift = nW = \frac{1}{2}\rho U^2 S\left(C_{L_W} + \frac{S_T}{S}C_{L_T}\right). \tag{10}$$

C_{L_W} is lift coefficient of the wing (actually of the whole aircraft minus horizontal tailplane); C_{L_T} is lift coefficient of the horizontal tail. ρ is the air density.

Stability and manoeuvre point/margin

From (10) we can write

$$\frac{n}{U^2} = \frac{\rho}{2w}\left(C_{L_W} + \frac{S_T}{S} C_{L_T}\right) \tag{11}$$

where w is the wing loading, W/S .

We now deploy the pitching moment equation, as derived in the static margin section:

$$C_M = C_{M_0} + x\left(C_{L_W} + \frac{S_T}{S} C_{L_T}\right) - \bar{V} C_{L_T} \tag{12}$$

where:

- C_M is the pitching moment coefficient of the aeroplane;
- C_{M_0} is the wing zero-lift pitching moment coefficient (again, actually of the whole aircraft minus the horizontal tailplane).

Stability and manoeuvre point/margin

Equation (12) can be rewritten as

$$C_M = C_{M_0} + xC_{L_W} - \left(\bar{V} - x\frac{S_T}{S}\right)C_{L_T} \tag{13}$$

Recall that C_{L_T} can be written as $a_{1_T}\alpha_T + a_{2_T}\eta$, i.e. the lift curve slope of the tailplane multiplied by its angle of attack plus the derivative of tailplane lift w.r.t. the elevator times elevator angle η. (We ignore the elevator trim tab, if there is one, as the pilot would not be attempting to trim the aircraft in a steady pull-up.)

From (8), $\alpha_T = i_T + \alpha(1-k) + \frac{(n-1)}{U^2}gl_H$ which, from (11), can be expressed as

$$\alpha_T = i_T + \frac{C_{L_W}}{a_1}(1-k) - \frac{gl_H}{U^2} + \frac{\rho g l_H}{2w}\left(C_{L_W} + \frac{S_T}{S}C_{L_T}\right) \tag{14}$$

Stability and manoeuvre point/margin

So that we can now write C_{L_T} as

$$a_{1_T}\left[i_T + \frac{C_{L_W}}{a_1}(1-k) - \frac{gl_H}{U^2} + \frac{\rho g l_H}{2w}\left(C_{L_W} + \frac{S_T}{S}C_{L_T}\right)\right] + a_{2_T}\eta \qquad (15)$$

It can be observed that C_{L_T} appears on both the left- and the right-hand sides of eqn. (15).

We could re-arrange the equation but in fact the $\frac{S_T}{S}C_{L_T}$ term is actually very small – typically of order 0.005 – and can be ignored.

From (13) and (15), with the above small term neglected, the trim equation can be written:

$$\left.\begin{aligned} C_M + \left(\bar{V} - x\frac{S_T}{S}\right)a_{2_T}\eta = C_{M_0} - \left(\bar{V} - x\frac{S_T}{S}\right)\left\{a_{1_T}\left(i_T - \frac{gl_H}{U^2}\right)\right\} - \\ \left[\left(\bar{V} - x\frac{S_T}{S}\right)\left\{\frac{a_{1_T}}{a_1}(1-k) + \frac{a_{1_T}\rho g l_H}{2w}\right\} - x\right]C_{L_W} \end{aligned}\right\} \qquad (16)$$

Stability and manoeuvre point/margin

The partial differentiation $\frac{\partial C_M}{\partial C_L}$ results in just the square bracket in eqn. (16), i.e.

$$\frac{\partial C_M}{\partial C_L} = \left(\bar{V} - x\frac{S_T}{S}\right)\left\{\frac{a_{1_T}}{a_1}(1-k) + \frac{a_{1_T}\rho g l_H}{2w}\right\} - x \qquad (17)$$

As we did for static stability/static margin, we use the less accurate version of this formulation by ignoring $x\frac{S_T}{S}$ as this is small.

If we now compare eqn. (17) with eqn. (3) for the static margin, in which the same approximation was made, we have:

$$\boxed{\begin{aligned} -\left.\frac{\partial C_M}{\partial C_L}\right|_{static} &= \bar{V}\left[\frac{a_{1_T}}{a_1}(1-k)\right] - x &&= H_N = (h_n - h) \\ -\left.\frac{\partial C_M}{\partial C_L}\right|_{manoeuvre} &= \bar{V}\left[\frac{a_{1_T}}{a_1}(1-k)\right] - x + \bar{V}\frac{a_{1_T}\rho g l_H}{2w} &&= H_M = (h_m - h) \end{aligned}}$$

Stability and manoeuvre point/margin

H_M is the **manoeuvre margin** and h_m is the **manoeuvre point.**

H_M contains an extra term $(\bar{V}\frac{a_{1_T}\rho g l_H}{2w})$ relative to the static margin.

Note that all the quantities in this extra term are positive so that there is a *larger* margin of stability than in the static case: in other words, the manoeuvre margin $(h_m - \ h)$ is larger than the static margin $(h_n - \ h)$.

This is illustrated schematically in Fig. 2.

The extent to which the manoeuvre point lies behind the neutral point h_n is typically 5–10%$\bar{\bar{c}}$.

We also note that, whereas the static margin is dependent on the geometry of the aircraft and its CG, the manoeuvre margin also depends on the weight and altitude.

Stability and manoeuvre point/margin

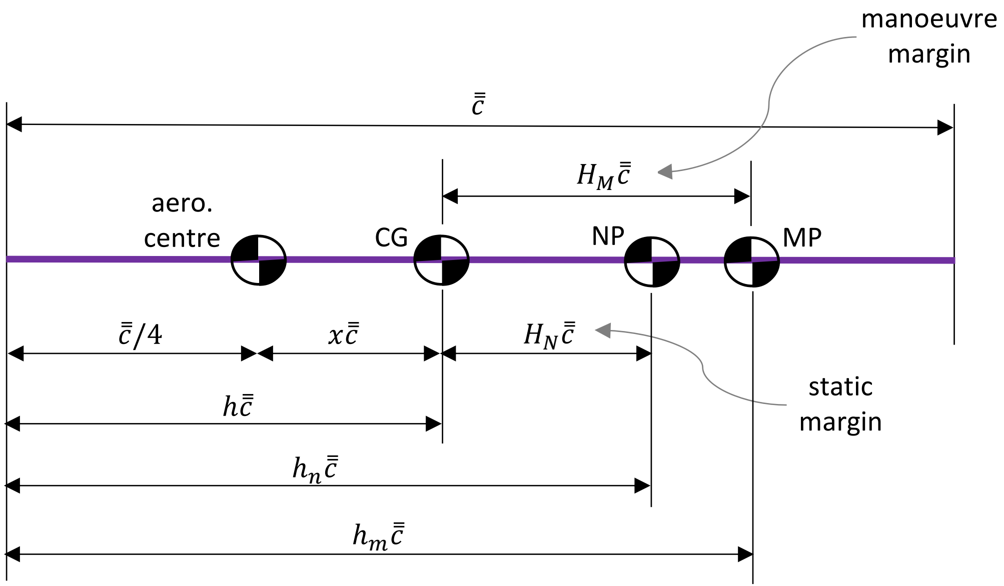


Figure 2: Neutral and manoeuvre points and static and manoeuvre margins in relation to mean aerodynamic chord.

Stability and manoeuvre point/margin

We can explain the meaning of this in terms of stability as follows.

Previously, we referred to a stable aeroplane having a positive aerodynamic stiffness ($\frac{\partial C_M}{\partial \alpha}$ or $\frac{\partial C_M}{\partial C_L}$) offering a restoring moment.

As the CG moves aft towards the neutral point, this aerodynamic stiffness reduces and reaches zero when the CG is *at* the neutral point.

The pilot will feel this because any attempt to pull the nose up involves resistance against the aerodynamic stiffness: the stronger that stiffness is (more stable), the more the elevator will need to be moved to counteract it. In other words, the more stable the aeroplane, the more elevator deflection is needed to produce a nose-up vertical turn – and the same logic applies for any coordinated turn involving a pitch rate.

When *elevator angle per g* drops to zero, there is a complete loss of aerodynamic stiffness and the *manoeuvre point* has been reached, i.e. the CG has gone back to a point where static stability in the vertical circle is lost.

Elevator angle per g

Another way to approach this issue is to consider the *changes* from steady level flight (which we considered to derive the static margin/neutral point) to instantaneous level flight at the bottom of the vertical circular motion.

Eqn. (13) gave the moment balance as $C_M = C_{M_0} + xC_{L_W} - \left(\bar{V} - x\frac{S_T}{S}\right) C_{L_T}$. This is still valid because q is constant – no pitching acceleration.

Therefore, the *change* in pitching moment from the manoeuvring terms in the equations must be zero.

Elevator angle per g

From slide 11, $C_{L_T} = a_{1_T}\alpha_T + a_{2_T}\eta$ and, from eqn. (8), $\alpha_T = i_T + \alpha(1-k) + \frac{(n-1)}{U^2} g l_H$.
Substituting these into eqn. (13) gives

$$C_M = C_{M_0} + xC_{L_W} - \left(\bar{V} - x\frac{S_T}{S}\right)\{a_{1_T}\alpha_T + a_{2_T}\eta\}$$

$$= C_{M_0} + xC_{L_W} - \left(\bar{V} - x\frac{S_T}{S}\right)\left\{a_{1_T}\left[i_T + \alpha(1-k) + \frac{(n-1)}{U^2} g l_H\right]_T + a_{2_T}\eta\right\} \quad (18)$$

Under **static** conditions (i.e. steady level flight as opposed to manoeuvring flight), load factor $n = 1$ so the third term in the α_T expression is zero.

Therefore, from (18), the pitching moment C_M under **static** conditions can be written

$$C_{M_{static}} = C_{M_0} + xC_{L_W} - \left(\bar{V} - x\frac{S_T}{S}\right)\{a_{1_T}[i_T + \alpha(1-k)] + a_{2_T}\eta\} \quad (19)$$

Elevator angle per g

Re-writing eqn. (18) for the **manoeuvring** condition, we add a subscript n to designate factors in the equation that change between level flight (1g, static) and a pull-up (manoeuvring):

$$C_{M_{pull-up}} = C_{M_0} + xC_{L_{W_n}} - \left(\bar{V} - x\frac{S_T}{S}\right)\left\{a_{1_T}\left[i_T + \alpha_n(1-k_n) + \frac{(n-1)}{U^2}gl_H\right]_T + a_{2_T}\eta_n\right\} \quad (20)$$

The factors identified for change in the pull-up relative to steady level flight are C_{L_W}, α, k and η.

Now, subtracting eqn. (19) from eq. (20) gives the *change* in C_M due to the circular manoeuvre:

$$0 = x\left(C_{L_{W_n}} - C_{L_W}\right) - \left(\bar{V} - x\frac{S_T}{S}\right)\left\{a_{1_T}\left[\alpha_n - \alpha - \alpha_n k_n - \alpha k + \frac{(n-1)}{U^2}gl_H\right] + a_{2_T}(\eta_n - \eta)\right\}$$

(21)

Elevator angle per g

We now make further assumptions in order to simplify the expression, namely:

- $x\frac{S_T}{S} \ll \bar{V}$
- $k_n = k$

Also, we define:

- $\delta C_{L_W} = C_{L_{W_n}} - C_{L_W}$ $\qquad \delta\alpha_W = \alpha_n - \alpha$ $\qquad \delta\eta = \eta_n - \eta$

Then we can reduce eqn. (21) to

$$0 = x\delta C_{L_W} - \bar{V}\left\{a_{1_T}\left[\frac{\delta C_{L_W}}{a_1}(1-k) + \frac{(n-1)}{U^2}gl_H\right] + a_{2_T}\delta\eta\right\} \tag{22}$$

Elevator angle per g

Now, rearranging eqn. (22) and noting that the static margin is given by $H_n = \bar{V}\frac{a_{1_T}}{a_1}(1-k) - x$ and that $\delta C_{L_W} \approx (n-1)C_{L_W}$, the *additional* elevator angle (required for the manoeuvre relative to 1g straight and level flight) can be found:

$$\delta\eta = -\frac{a_{1_T}}{a_{2_T}}\frac{(n-1)}{U^2}gl_H - \frac{1}{\bar{V}a_{2_T}}[H_n](n-1)C_{L_W} \qquad (23)$$

where C_{L_W} is the lift coefficient for straight and level 1g flight.

We see from eqn. (23) that to "pull g's" the stick must be pulled back to provide more negative elevator (remember sign convention: +ve elevator $\rightarrow$ nose-down moment).

The desired ***elevator angle per g*** can then be written as

$$\boxed{\frac{\delta\eta}{(n-1)} = -\frac{1}{\bar{V}a_{2_T}}H_n C_{L_W} - \frac{a_{1_T}}{a_{2_T}}\frac{gl_H}{U^2}} \qquad (24)$$

Stick force and determination of manoeuvre point from flight tests

The stick force (which is proportional to hinge moment) is closely related to the elevator deflection.

To the pilot, the stick force is a better indicator than elevator angle. Therefore, the “stick force per g” has been a feature of flight testing to define performance/handling qualities in terms that a pilot can easily imagine.

As mentioned on the previous slide, the negative change in elevator angle required to pull positive g is achieved by pulling the stick back (+ve stick force).

So a positive stick force P is required to increase load factor, n, for a stable aircraft and zero stick force per g would correspond to the CG being at the manoeuvre point – see Fig. 3.

Stick force and determination of manoeuvre point from flight tests

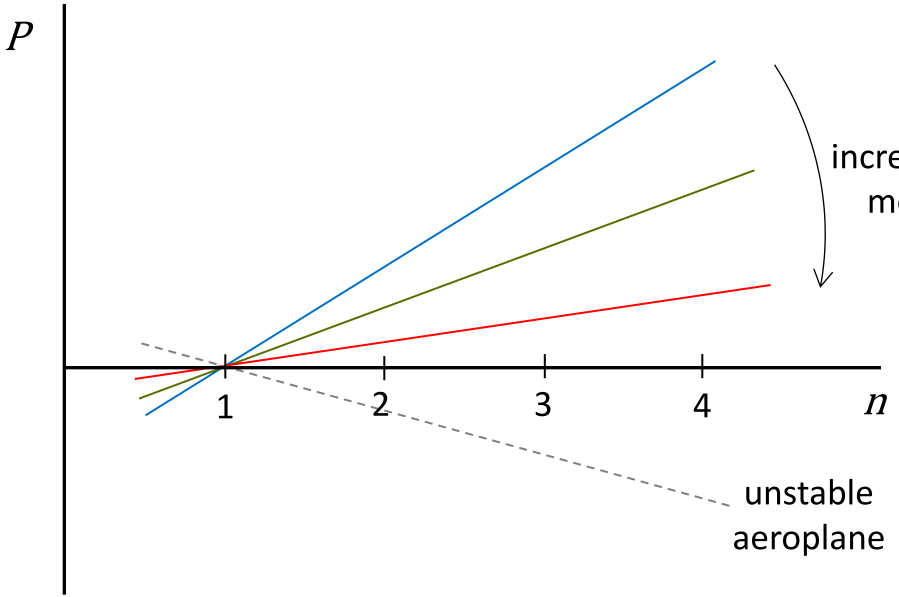


Figure 3: Variation of stick force, P, with load factor, n.

Stick force and determination of manoeuvre point from flight tests

A flight test can be set up in which the stick force is measured for different load factors and repeated for several different CG positions, with the results plotted as in Fig. 3.

The slope of each of these P versus n lines, $\frac{dP}{dn}$, can be determined.

Then a plot of $\frac{dP}{dn}$ versus CG position, h, can be constructed and extrapolated until it intersects the x-axis, i.e. $\frac{dP}{dn} = 0$.

This would be an experimental means of locating the manoeuvre point, $h = h_m$, and is illustrated in Fig. 4

Stick force and determination of manoeuvre point from flight tests

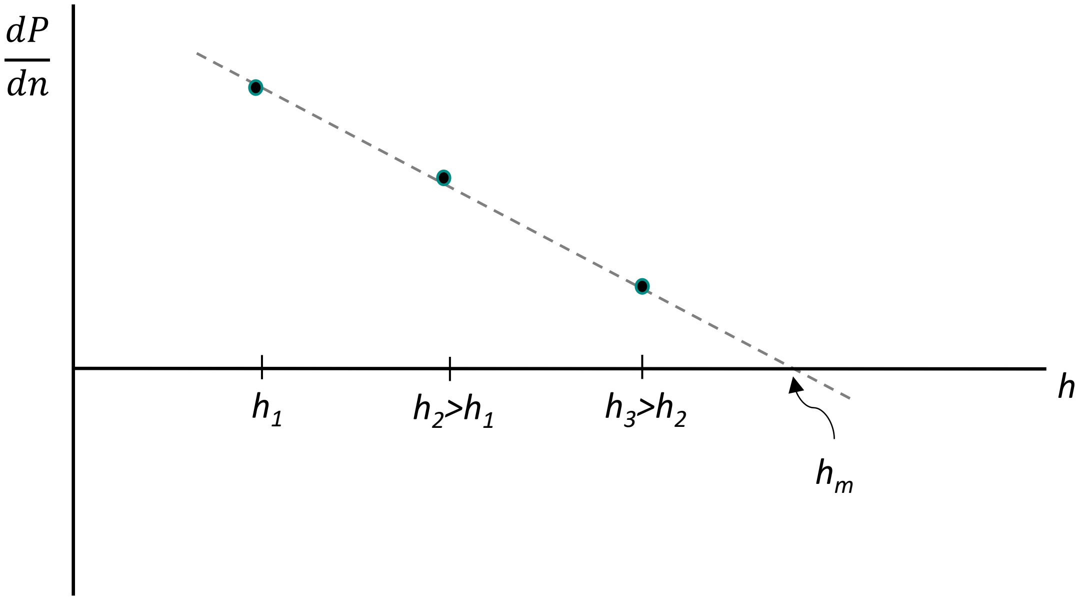


Figure 4: Schematic of flight test determination of h_m.

Influence of static/manoeuvre stability on hydrogen-fuelled airliner concepts?

Figure 5: Airbus Zero-e hydrogen-fuelled configuration concept.

Next Session
Trimming and Linearisation

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