STATIC STABILITY

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Tug Aircraft: 141hp EuroFox 2K

Flight Dynamics Principles – book references

- Note: some of the nomenclature and derivations differ from those we use in lectures.
- These references are for the 2007 edition.
  - Flight Dynamics Principles: A Linear Systems Approach to Aircraft Stability and Control (Elsevier Aerospace Engineering) Hardcover – 9 Aug 2007

- Chapter 3: Static equilibrium and trim
  - *Section 3.1 – Trim equilibrium*
  - *Section 3.2 – The pitching moment equation*
  - *Section 3.3 – Longitudinal static stability*
  - *Section 3.4 – Lateral static stability*
  - *Section 3.5 – Directional static stability*
  - Section 3.6 – Calculation of aircraft trim condition
- Chapter 4: The equations of motion
  - *Section 4.1 – The equations of motion of a rigid symmetric aircraft*
  - Section 4.2 – The linearised equations of motion
  - Section 4.3 – The decoupled equations of motion
  - Section 4.4 – Alternative forms of the equations of motion

Airbus A350-1000

https://www.britishairways.com

Session Overview

- Stability – definitions
- Pitching moment equation
- Neutral point
- Static margin

Elevator-Angle-to-Trim *(from a previous session)*

Recall, from previous lecture, that the elevator angle required to obtain this trim is:

$$\eta_{trim} = \frac{1}{\bar{V} a_{2_T}} \left\{ C_{M_0} - \bar{V} a_{1_T} i_T - C_{LW} \left[\bar{V} \frac{a_{1_T}}{a_1} (1 - k) - x \right] \right\}$$

Note that this equation does not guarantee:

- that the necessary angle for η is available; or
- that the trim condition is *stable*.

The Neutral Point and Static Margin

In this lecture, we will discuss:

- how to determine the neutral point, which is the rearmost position of the c.g. before an unstable condition occurs;
- the meaning of the static margin, which is a measure of the distance remaining through which the c.g. could be moved rearward before the aircraft displays neutral static stability (i.e. where the c.g. is at the neutral point);
- how these properties influence aircraft design.

Static Stability

We can consider static stability in terms of stiffness of the system.

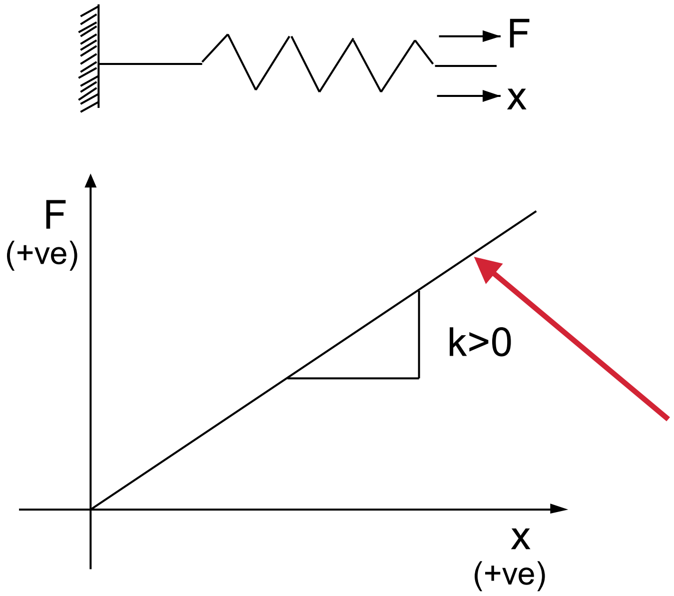


Figure 1

- Consider the simple spring shown in Figure 1.
- What is the relationship between force and displacement?
- Applying force +F results in displacement +x.
- Hence positive slope k.

Static Stability

The force F in Figure 1 was the force *applied* to extend the spring. The spring exerts an *opposing* force: this is the force required to achieve equilibrium in the presence of the applied force, which it counteracts.

Stability deals with the tendency of a system to return to its previous condition after a perturbation (from equilibrium, typically): for a system to be stable, the force it produces in response to the perturbation must tend to restore it to its unperturbed state – as depicted in Figure 2.

- applied +ve displacement x
- results in –ve force
- therefore –ve slope k

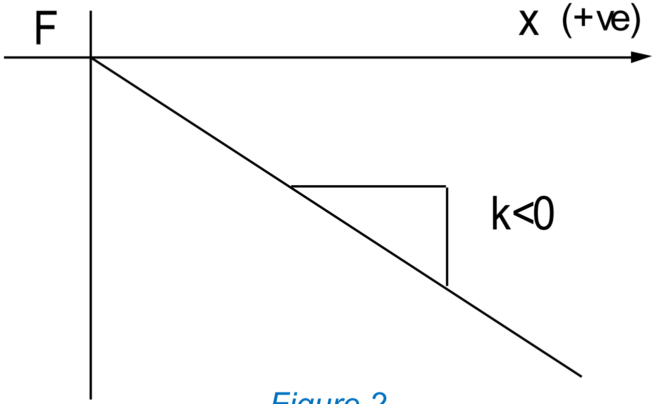


Figure 2

Static Stability

Consider a normal pendulum vs an inverted pendulum

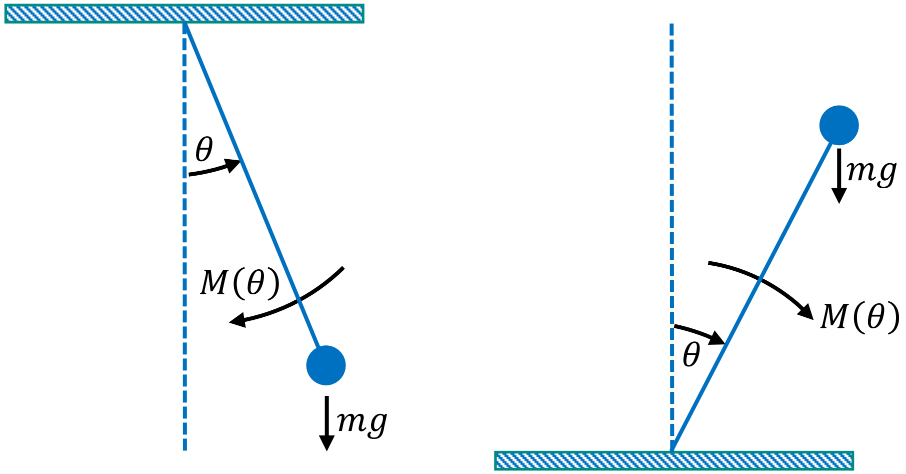


Figure 3

Both cases display zero in-plane moment when $\theta = 0$.

Static Stability

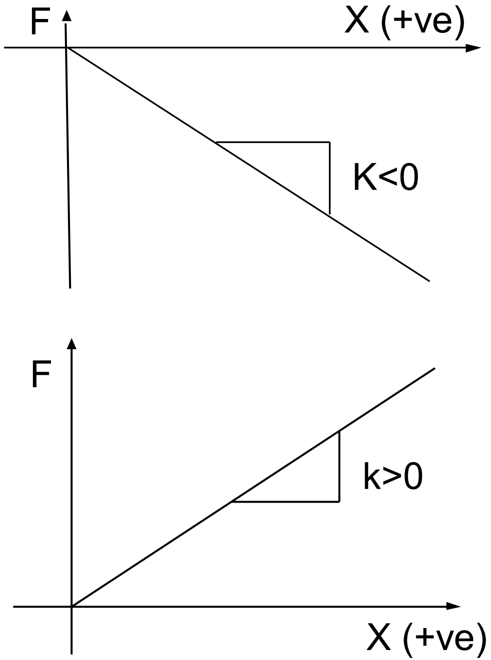
What sign for k would be expected:

- for the stable system (normal pendulum)?
- for the unstable system (inverted pendulum)?

- Normal pendulum:
  - restoring force → stability → –ve k

- Inverted pendulum:
  - diverging force → instability → +ve k

Figure 4



Static Stability

- The flight mechanics case follows this sign convention and displays the displacement/force characteristics given in Figures 3 and 4.
- Our study of stability will consider the pitching moment ΔM produced when there is a departure $\Delta\alpha$ away from the equilibrium flight attitude.

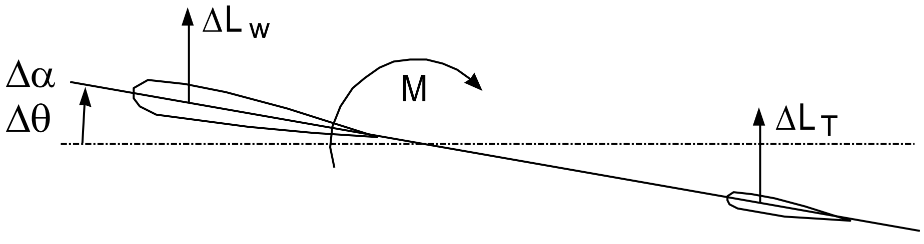
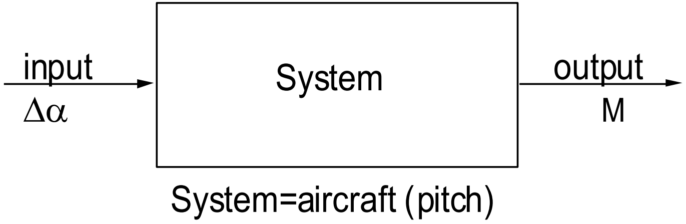


Figure 5



Static Stability

- The "system" is the pitch response of an aircraft; the "input" is a pitch displacement $\Delta\alpha$ and the "output" is the consequent pitching moment ΔM.
- For stable flight we need $\Delta M < 0$ for $\Delta\alpha > 0$ so that there will be a restoring action. This can be seen graphically in Figure 6.

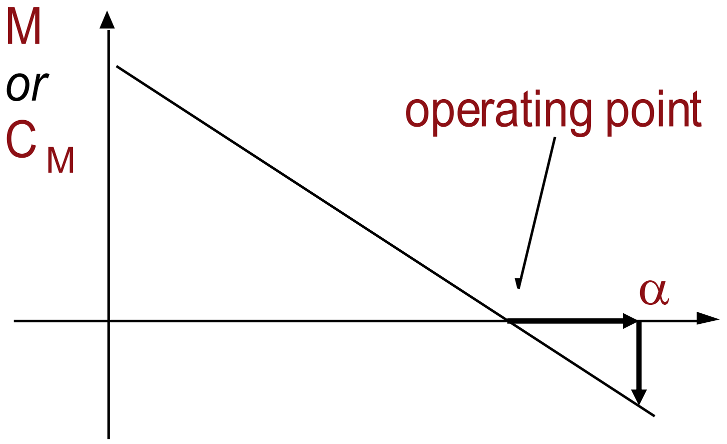
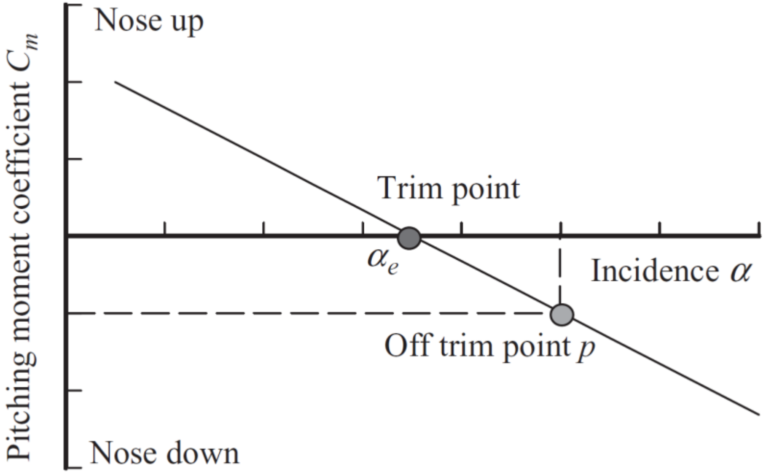


Figure 6



(from Flight Dynamics Principles)

Static Stability

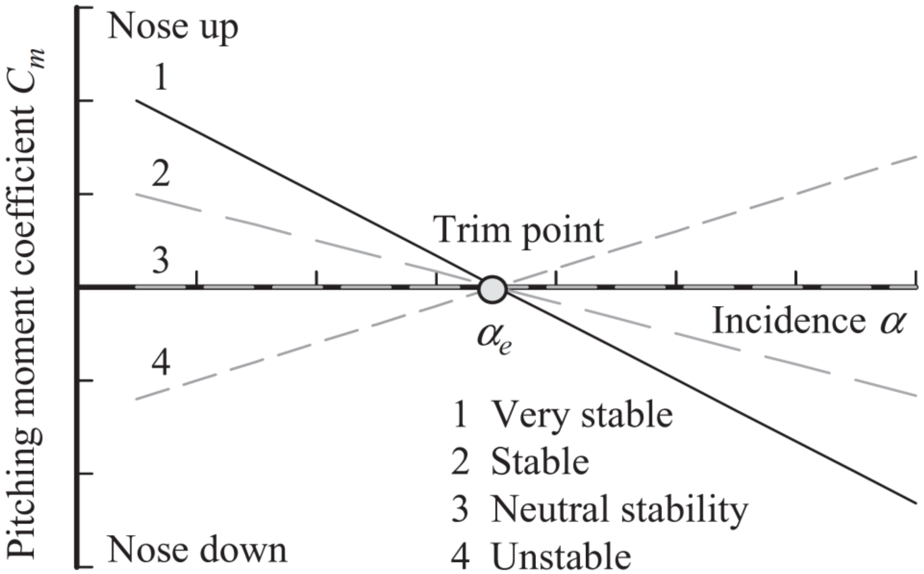
- Consider a perturbation $\Delta\alpha$ away from a trimmed, straight and level operating point? +ve or –ve slope desirable?

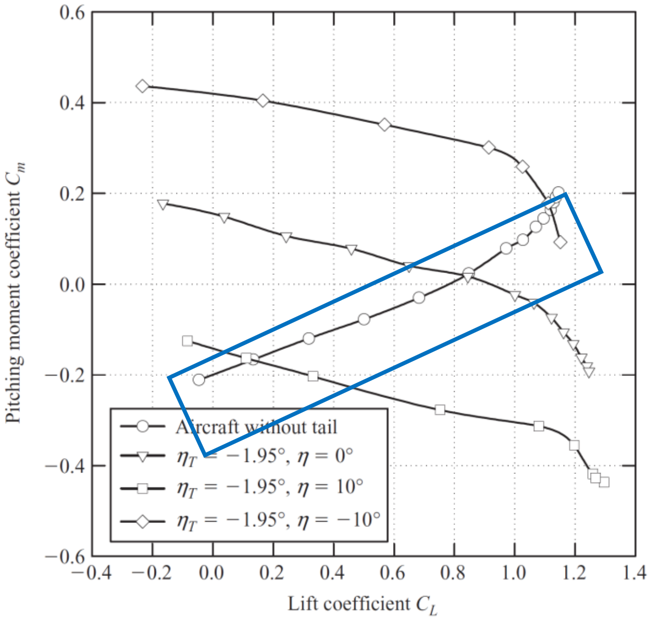
- This implies that whichever sign there is for $\Delta\alpha$ away from the equilibrium flight angle α, the pitching moment caused by $\Delta\alpha$ will be of such a sign as to counteract $\Delta\alpha$ and restore the flight to its equilibrium incidence.

The slope given by $\partial M/\partial\alpha$ must therefore be negative for stability.

Static Stability

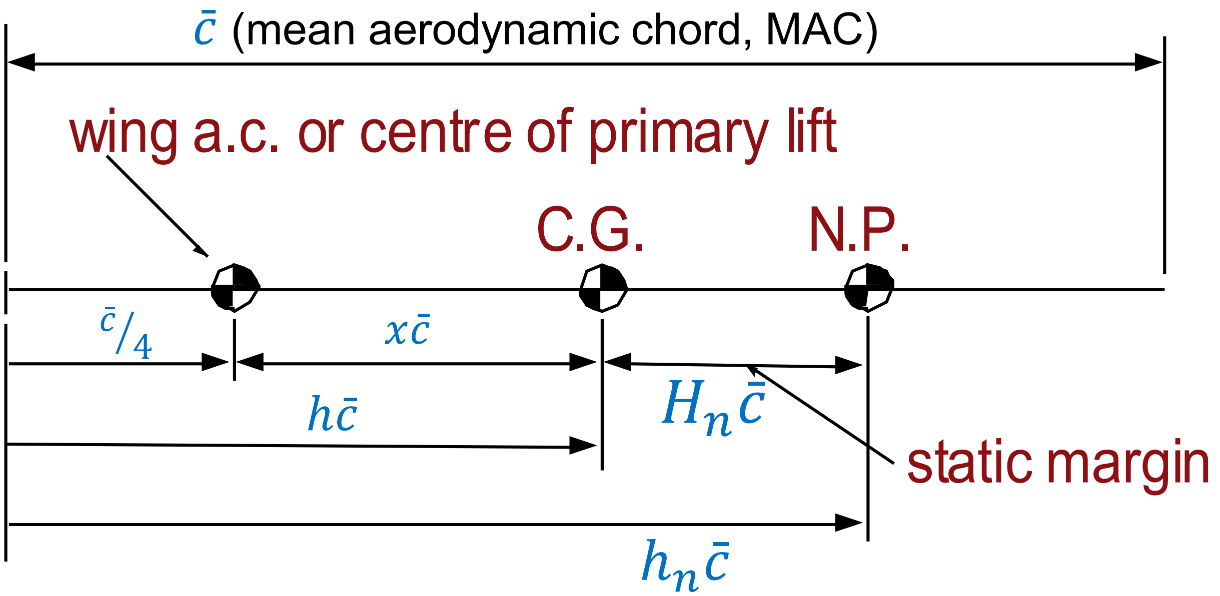
'The static stability of an aircraft is commonly interpreted to describe its tendency to converge on the initial equilibrium condition following a small disturbance from trim. Dynamic stability, on the other hand, describes the transient motion involved in the process of recovering equilibrium following the disturbance.' (Flight Dynamics Principles)





C_m–α *plots for a 1/6th scale model of the Handley Page Jetstream.*

The Neutral Point and Static Margin



Note that our earlier use of c.g. position x (relative to the lift) is replaced here by an alternative measure h from the leading edge of the MAC i.e. or $x = h - ¼$.

Static Stability - Definitions

From Figure 6:

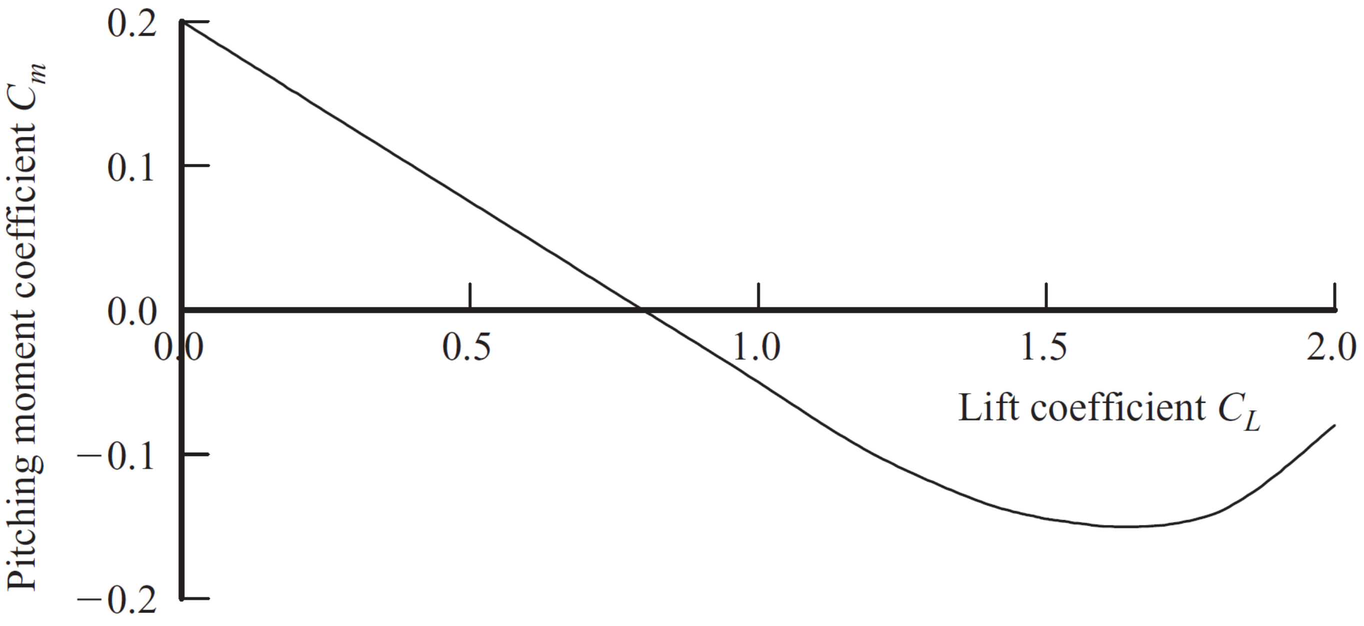
- If the coefficient form $\partial C_M / \partial \alpha$ is negative the aircraft will be stable.
- Remember that α is α_{wing} and that we can put $\alpha_{wing} = C_{L_W} / a_1$.
- Hence there is only a constant factor that would distinguish between $\partial C_M / \partial \alpha$ and $\partial C_M / \partial C_L$.
- Hence if $\partial C_M / \partial C_L$ is negative the aircraft will be stable.

$$a_1 = \frac{\partial C_L}{\partial \alpha}$$

$$C_L = a_1(\alpha - \alpha_0)$$

α_0 is the zero lift angle

Static Stability - Definitions



.6 *Stability reversal at high lift coefficient.* (Flight Dynamics Principles)

Previous Exam Question

Can you think of the directional equivalent?

Q2 (a) Explain what is meant by longitudinal static stability when related to a classical aircraft configuration. Provide an appropriate diagram.

(7 marks)

(b) You have been presented with the plot shown in Figure Q2.1 for an unknown fighter aircraft showing data across a very wide range of α. Using this plot and with the assumption that the aircraft retains lateral-directional stability throughout the angle of attack range given, describe the likely longitudinal response of the aircraft with α. Within your answer, provide a sketch of α vs time for this aircraft as the elevator is ramped in the direction that generates a positive pitching moment.

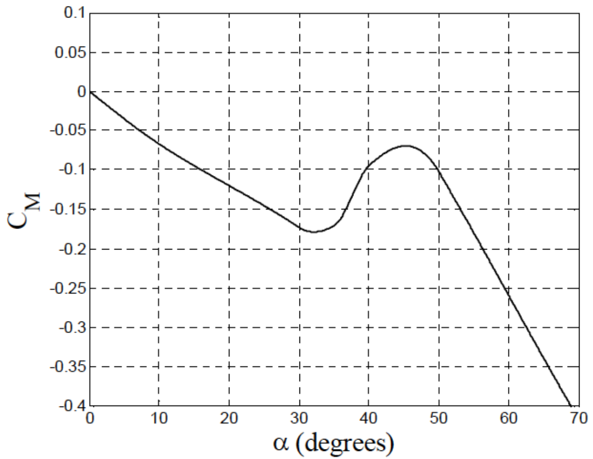


Fig. Q2.1

(13 marks)

(c) For the aircraft described in part (b) suggest how the longitudinal response might be improved for this aircraft when operating at high angles of attack.

(5 marks)

Pitching Moment Equation

Derivatives of the Pitching Moment Equation

The pitching moment expression which we obtained in a previous lecture was:

$$C_M = C_{M_0} - \bar{V} a_{1_T} i_T - C_{L_W}\left[\bar{V}\frac{a_{1_T}}{a_1}(1-k) - x\right] - \bar{V} a_{2_T}\eta \qquad (1)$$

$$\boxed{\bar{V} = \frac{S_T\ l_T}{S\ \bar{c}}}$$

For trimmed flight $C_M = 0$.

Now consider a disturbance in pitch $\Delta\alpha$, perhaps caused by an upward gust, and this $\Delta\alpha$ leads to an associated disturbance $\Delta C_L > 0$.

Static Stability

This $\Delta\alpha$ also leads to an associated disturbance ΔC_M.

On the basis of what we saw above, it should be clear that:

$\Delta C_M > 0$ is adverse $\rightarrow$ unstable aircraft
$\Delta C_M < 0$ is restoring $\rightarrow$ stable aircraft,

Static Stability

and thus in general

$$\frac{1}{a_1}\frac{\partial C_M}{\partial \alpha} \quad \text{or} \quad \frac{\partial C_M}{\partial L} > 0 \qquad \textit{unstable}$$

$$= 0 \qquad \textit{neutrally stable} \qquad (2)$$

$$< 0 \qquad \textit{stable}$$

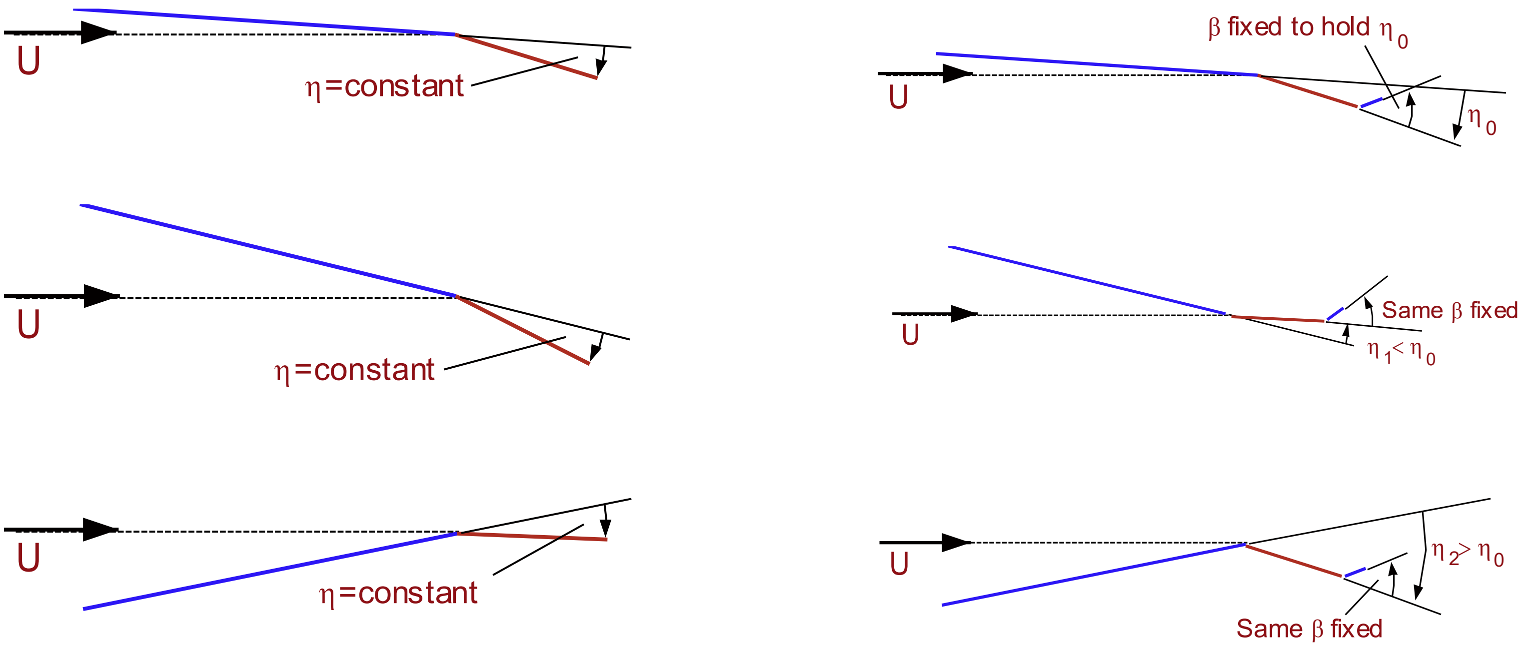
The case for a Fixed Elevator (“Stick-Fixed”)

- We will assume that the elevator is held fixed by the pilot or that there is a “control-fixed” condition imposed by the hydraulics when control demands remain constant.

- With the “stick-free” case, there can be a small difference in the pitch stability if the elevator position is allowed to drift. Under such conditions, the nominal value $\eta = \eta_{trim}$ is not retained when the pitch attitude is disturbed.

- We will only consider the “stick-fixed” case.

‘Stick-Fixed’ ‘Stick-Free’



Static Stability

Consider eqn. (1): $C_M = C_{M_0} - \bar{V} a_{1_T} i_T - C_{L_W}\left[\bar{V}\frac{a_{1_T}}{a_1}(1-k) - x\right] - \bar{V} a_{2_T}\eta$

and recall from previous lecture that $C_L = C_{L_W} + \frac{S_T}{S} C_{L_T}$ and that $\frac{S_T}{S}$ can be neglected.

Then, for the case where the elevator is locked and η = constant (but not necessarily zero) we have, for static stability:

$$\frac{1}{a_1}\frac{\partial C_M}{\partial \alpha} = \frac{\partial C_M}{\partial C_L} = -\left[\bar{V}\frac{a_{1_T}}{a_1}(1-k) - x\right] < 0 \qquad (3)$$

and the factor in the square brackets must therefore be positive for a stable aircraft.

Static Stability

- For k of order 1/2 and with the c.g. behind the primary lift by a *small* distance (i.e. x is small and positive), the terms in the [ ] lead to a positive value.

- This will remain so, as the c.g. moves forward and passes the primary lift to make $x < 0$.

- However, clearly x *could* become sufficiently large and positive for [ ] to become negative and the aircraft would be unstable.

An Alternative Criterion for Pitch Stability?

Another way of looking at the longitudinal stability is to use a logical argument which relates C_L and η_{trim} as follows for balanced flight:

1. If flight is to be at a faster speed, it must also be at a lower C_L, i.e. $\Delta C_L < 0$.

2. To obtain a lower C_L we must push the nose down to obtain lower incidence α.

3. This lower incidence is obtained by a larger upward force at the tail, *via* a more positive η, i.e. $\Delta\eta > 0$, even if $\eta < 0$.

Elevator-Angle-to-Trim – For Reference

and thus the elevator angle required to obtain this trim is:

$$\eta_{trim} = \frac{1}{\bar{V} a_{2_T}} \left\{ C_{M_0} - \bar{V} a_{1_T} i_T - C_{L_W} \left[\bar{V} \frac{a_{1_T}}{a_1}(1-k) - x \right] \right\}$$

Static Stability

- Thus in flight we should expect to have $\Delta\eta_{trim}$ positive as speed is increased while ΔC_L becomes negative, and thus:

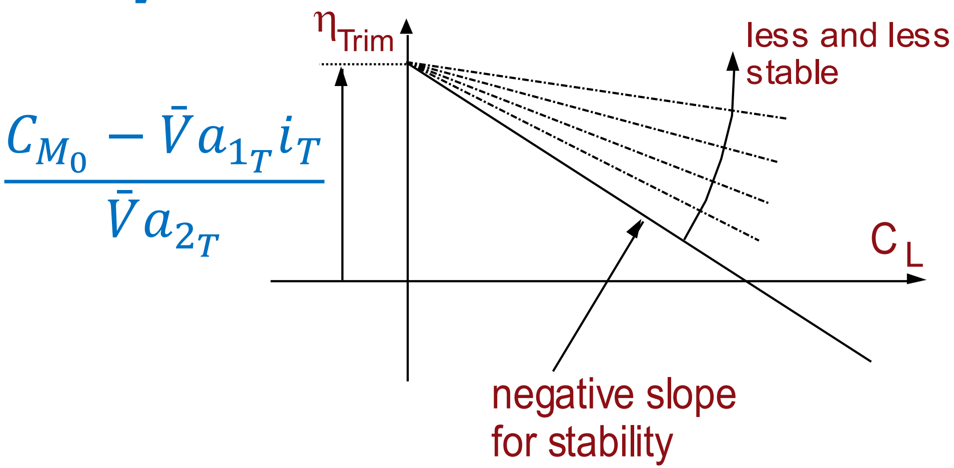
$$\frac{\partial \eta_{trim}}{\partial C_L} < 0$$

- Indeed a differentiation of the expression for η_{trim} will show that this leads to

$$\frac{\partial \eta_{trim}}{\partial C_L} = -\frac{1}{\bar{V}\ a_{2_T}}[\text{same factor in brackets}]. \qquad (4)$$

- Clearly, the RHS will be negative if the bracketed expression is again positive.

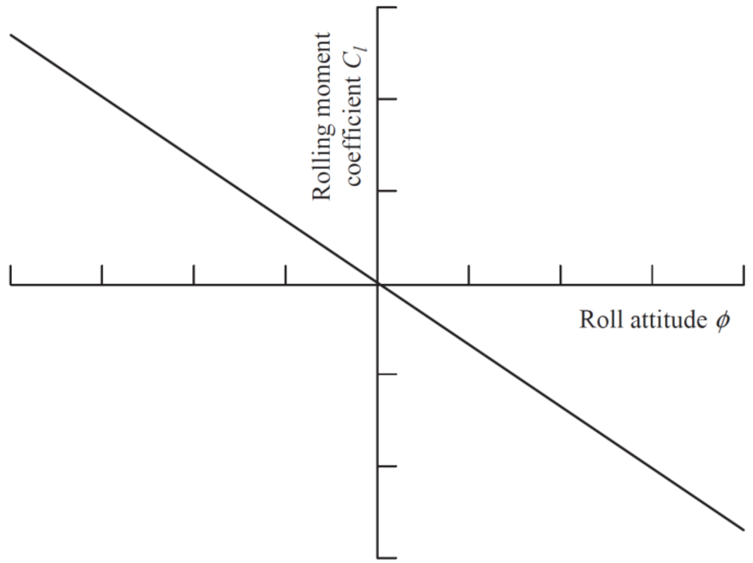
Static Stability



- Because the bracketed terms in Equations (3) & (4) are the same, the *boundary* between stable and unstable flight is shown by a zero value for that term and thus a horizontal line for the η_{trim} vs C_L graph.
- C.G. change = data for another line (moving significant masses!).

Lateral Static Stability

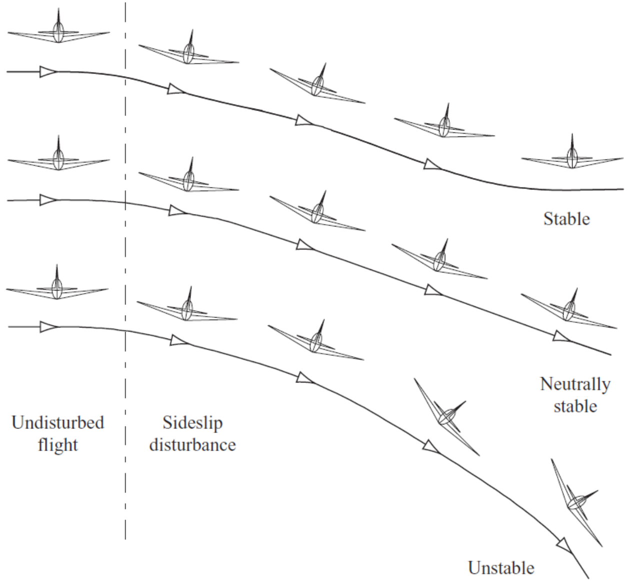
'Lateral static stability is concerned with the ability of the aircraft to maintain wings level equilibrium in the roll sense.' (Flight Dynamics Principles)



C_l–ϕ plot for a stable aircraft.

'Thus, the condition for an aircraft to be laterally stable is that the rolling moment resulting from a positive disturbance in roll attitude must be negative'

$$\frac{dC_l}{d\phi} < 0$$

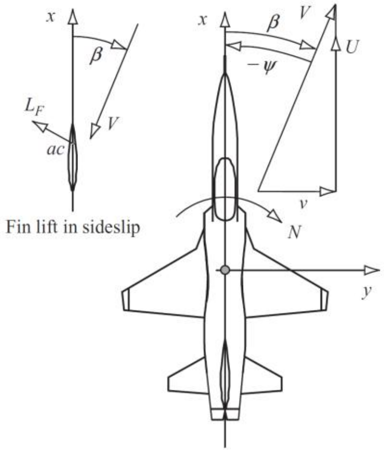


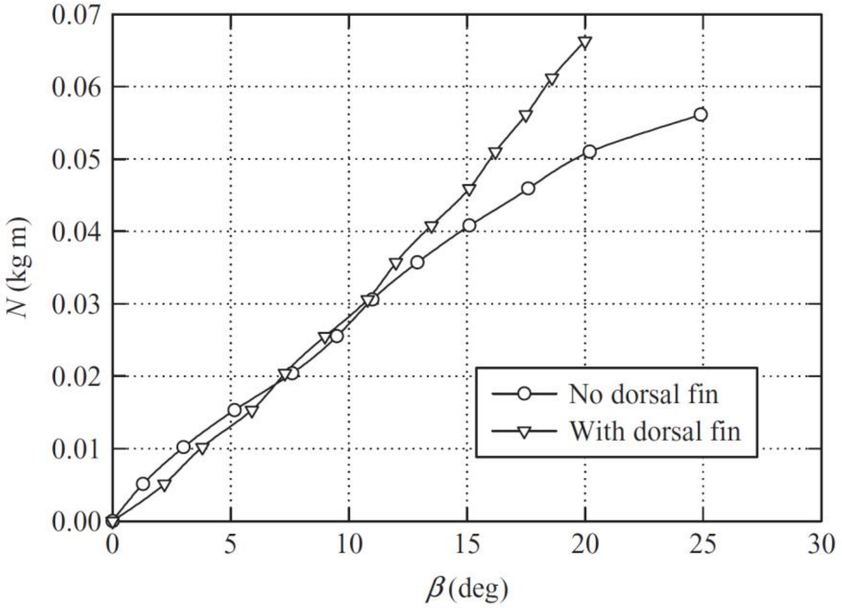
Directional Static Stability

'Directional static stability is concerned with the ability of the aircraft to yaw or weathercock into wind in order to maintain directional equilibrium.' (Flight Dynamics Principles)

'Thus, the condition for an aircraft to be directionally stable is'

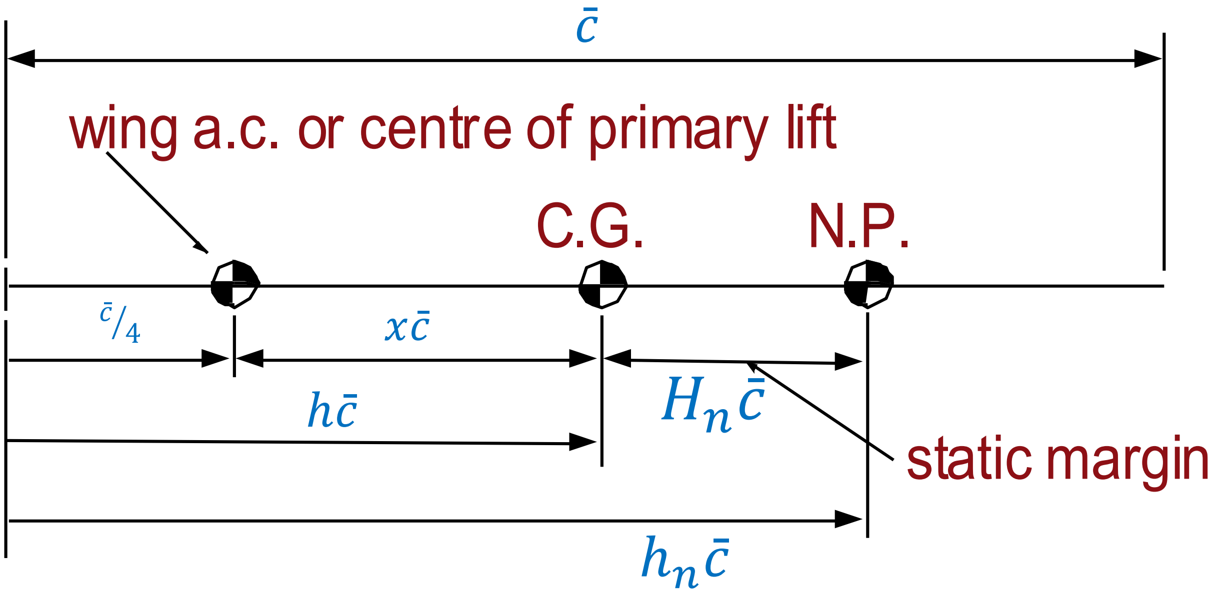
$$\frac{dC_n}{d\psi} < 0 \quad \text{or, equivalently,} \quad \frac{dC_n}{d\beta} > 0$$





Plot of yawing moment against sideslip for a stable aircraft.

The Neutral Point and Static Margin



Note that our earlier use of c.g. position x (relative to the lift) is replaced here by an alternative measure h from the MAC i.e. or $x=h–¼$.

The Neutral Point

- Stability becomes neutral when the c.g. moves just back to the neutral point (N.P.), where:

$$h = h_n \qquad\qquad x = h_n - \frac{1}{4}$$

- The stick-fixed neutral point can be found by setting to zero the derivative that we saw earlier:

$$\frac{1}{a_1}\frac{\partial C_M}{\partial \alpha} = \frac{\partial C_M}{\partial C_L} = -\left[\bar{V}\ \frac{a_{1_T}}{a_1}\ (1-k)\ -\ x\right] < 0 \qquad (5)$$

The Neutral Point

- Alternatively: $$\bar{V}\,\frac{a_{1_T}}{a_1}(1-k)\ - h_n + \frac{1}{4}\ = 0 \qquad (6)$$

- or, to define the length h_n: $$h_n\ = \frac{1}{4} + \ \bar{V}\,\frac{a_{1_T}}{a_1}(1-k) \qquad (7)$$

- and this defines the stick fixed neutral point relative to the MAC i.e. …

Static Margin

- The static margin is a measure of the distance remaining through which the c.g. could be moved rearward before the aircraft displays neutral static stability.
- It is defined as the remaining distance divided by $\bar{c}$, so it is a chord-fraction and is given by

$$(h_n - h) = H_n \qquad (8)$$

- Static Margin is positive if the aircraft is still stable. Note that if in (5) we insert $x = h - 1/4$ we have:

$$\frac{\partial C_M}{\partial C_L} = h - \frac{1}{4} - \bar{V}\frac{a_{1_T}}{a_1}(1-k)$$

The Neutral Point and Static Margin

- but (7) shows that this is just:

$$\frac{\partial C_M}{\partial C_L} = -(h_n - h) = -H_n \qquad (9)$$

- Thus (5) is really just:

$$\frac{\partial C_M}{\partial C_L} = -H_n \qquad (10)$$

- which shows that the static margin must be positive for positive static stability.

Next Session
Manoeuvre Stability

Fairey Rotodyne
First flight - 6 November 1957 - Cancelled 1962

https://www.youtube.com/watch?v=EA3AkvxwS_M

STATIC STABILITY

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