Signals, Systems and Control

Contents

1.1 Signals - some basics

1.1.1 What are signals?

Signals are a representation of some phenomenon that can be described quantitatively. They carry information; sometimes transfer energy. In our block diagrams they are the links between system blocks. Other terms that are synonymous with signals are: 'functions'; 'waves'; 'waveforms'.

The quantity we are measuring is called the 'measurand' and it will exist in its own physical space, (the term 'space' is used here to avoid using the term 'domain' so as not to confuse it with future sections!). For example, the measurand might be a position, a pressure, a light intensity etc.

However, very often signals are converted between physical spaces and represented by other phenomena – we so widely represent signals as variations in voltage that we almost take the conversion for granted. In some case the conversion has some physical origin – for example we will see later in this lecture series how some transducers link a velocity to a voltage due to their physics, but in other cases the conversion is arbitrary. Consider for example thermal imaging – we use an entirely arbitrary colour map to convert temperature to visible light.

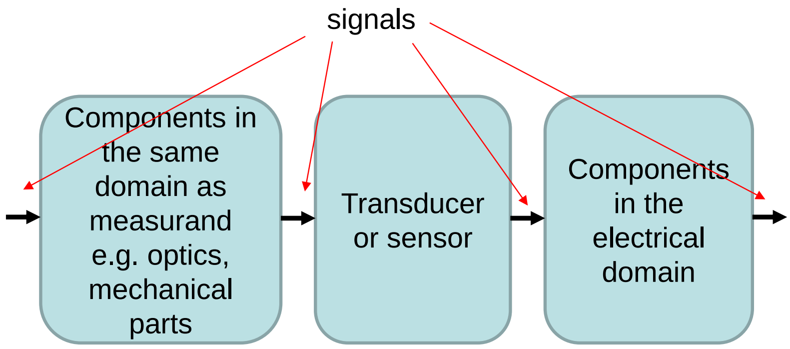


Fig.1 Signals are the links between block in a system diagram

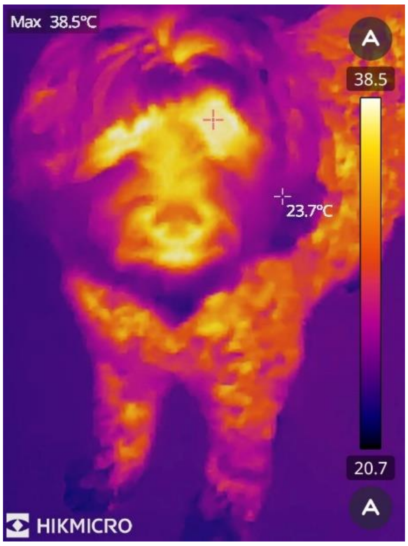


Fig 2. A thermal image uses an arbitrary colour map to represent temperature. The signal exists as an electrical voltage before display. We often use red for hot and blue for cold – but in fact thermal radiation is the opposite of this.

1.1.2 How are signals described?

We mostly experience the world in the temporal (time) domain, that is we experience the world as changes over time. There are some notable exceptions – these tend to be related to signals that vary too quickly for humans to be able to perceive the variation, for example vision or hearing. The idea that we perceive high frequency variations in air pressure as a 'tone' or musical note should give you the first insight that we can represent signals in multiple domains.

In signals theory we will encounter several domains e.g.

- Time (t)
- Discrete time [n]
- Frequency (ω)
- Complex frequency (s)
- Discrete complex frequency (z)

We have mathematical tools to transform our signals (and, as we shall see later in the unit, systems) from one domain to another. This is very useful – operations in some domains are much harder to do than in others. **We transform between domains to help solve engineering problems**.

Fig 4. Do you see a red square and a blue circle or electromagnetic fields varying at 430 THz and 460 THz?

1.1.3 Time and discrete time signals

A concept of continuous signal should be familiar to you. It is defined for all instances of time, $-\infty < t < \infty$, over the full duration of the signal, and can have any value between maximum and minimum limits. These are often described as 'analogue signals'

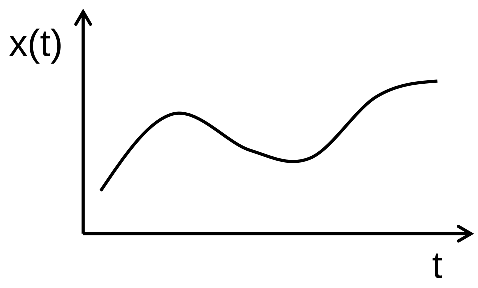


Fig 5. An analogue signal is a continuous function that can take any magnitude value and exists for every instance of time

All this may seem obvious but there are still a couple of useful points we can pull out. First, we can see the origin of calling signals ‘functions’ since the signal x(t) is a function of time. Second, because both time, t, and signal, x, are time varying then by convention we denote then with lower case letters (when a value doesn’t change with time we use a capital letter). We also use curved brackets around the argument ‘t’.

Although all physical signals are continuous, we regularly discretise signals in time – this is a corner stone of digital computing. A discrete signal is only defined for a particular instance of time. This is a mathematical construct – clearly something will exist between these defined instances (even if it is zero), however the maths ignores any values other than at the moment of sampling. The time interval can be uniform or variable, even random. Often discrete signals are produced by sampling a continuous signal.

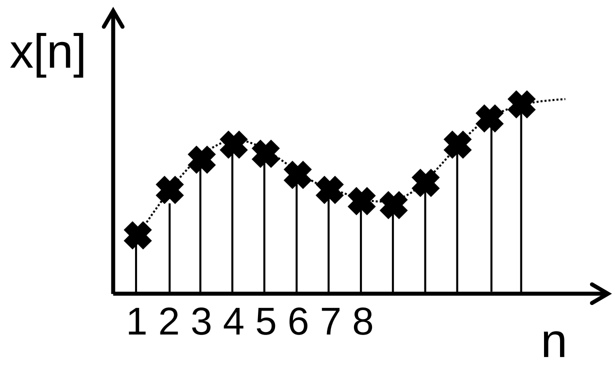


Fig 6. A discrete signal only takes a value an instance of time. Although this signal samples a continuous signal (shown as the dotted line), the discrete signal only exists for the sampling instants.

The signal in the discrete time domain is made up of a series of weighted impulses. ‘n’ is known as the sample number and is always an integer, ranging $0 \le n \le \infty$. ‘n’ is related to time by $t = nT$, where ‘T’ is the sample period, and $1/T$ is the sample frequency. Since this signal has been derived by sampling the continuous time signal, we can state $x[n] = x(nT)$. Here we also see another convention - the use of square brackets to indicate the discrete time domain. We will look at discrete signals in more detail in a future lecture.

1.1.4 Quantised Signals

The sampled signals we have looked at are discretised in the time, but the amplitude is still continuous – it can assume any value over some range. Digital systems cannot cope with this infinite number of possible signal values hence we need to represent our signal with a fixed number of amplitude values.

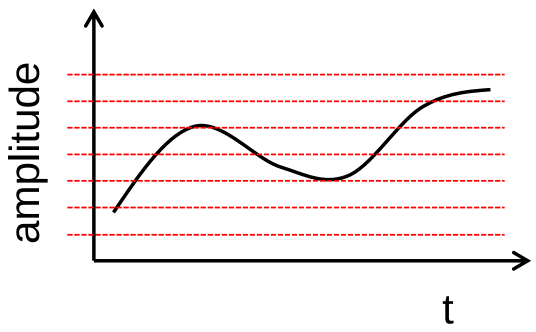


Fig 7. Signals are quantised in magnitude to limit the number of possible values.

But how do we 'map' each amplitude level to a particular sampling instance? We could use regular spaced sampling intervals and assign the closest quantised level to the analogue signal:

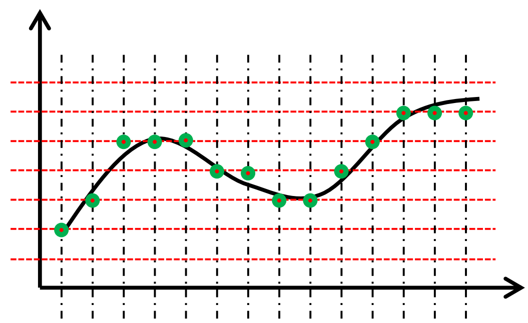


Fig 8. A regularly sampled signal

Or we could take a sample when our signal passes through the quantised amplitude values:

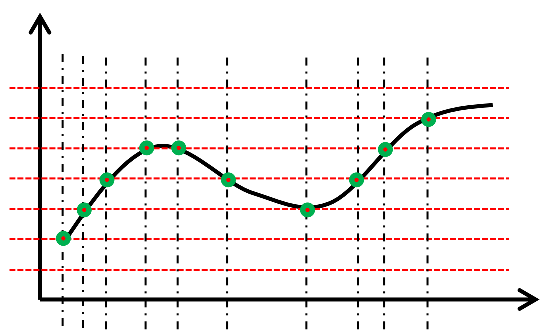


Fig 9. A sample scheme with varying sample period. Hysteretic sampling can produce this type of digital signal but it is a specialist topic beyond this course.

We aren't going to go into the detailed math of each approach in this unit but they will hopefully get you thinking about the issues of representing signals in the digital domain – with regular sampling the quantised values don't actually align with original signal; if we want the sample to 'sit' on the original signal values we end up with a random sample period!

1.1.5 Periodicity

There are several fundamental ways in which signals can be classified, one of them is periodicity. Periodic signals repeat indefinitely with a fixed period:

$x(t)=x(t+T)$ where T is the period of repetition

$x[n]=x[n+N]$ where N is the number of samples each period of repetition

Examples of periodic signals include: Sine waves, Triangular waveform, Series of pulses

Aperiodic signals do not repeat for example: Exponential decay, Impulse function, Random signals.

1.1.6 Elementary Transformations

Consider the two signals x(t) and x[n]. With the addition of four constants, we can illustrate the basic signal transformations of 1) Amplitude scaling; 2) Amplitude shifting; 3) Time scaling; 4) Time shifting;

$$y(t) = \alpha x(\gamma t + \delta) + \beta$$

$$y[n] = \alpha x[\gamma n + \delta] + \beta$$

α – makes the signal larger or smaller

β – shifts the signal up or down on the amplitude axis

δ – shifts the signal back or forth along the time axis

γ – Stretches or shrinks the signal in time

1.1.7 Decibels, Field and Power Quantities

Field and Power Quantities

The signals that we will be dealing with are often representations of real physical phenomena and in some cases, this can affect the math.

We are going to introduce two types of quantity: firstly 'Field' or 'Root-Power' quantities (Field is an old term, and Root-Power the more modern description of the same thing). These quantities should be familiar to you:

- Velocity
- Force
- Current
- Voltage
- Pressure

The square of these quantities is proportional to power in a linear system (hence Root-Power).

The second type of quantity we are considering is simply 'Power', i.e. signals that have SI* units of Watts (or quantities proportional to power – sound intensity or luminous intensity).

*SI describes the 'Internation System of Units'; the SI abbreviation derives from the French *Système international d'unités.*

Decibels

To help us deal with signals that vary over very large comparative values, or to deal with large gains, we turn to logarithms and the unit 'decibel', dB. The decibel is actually 1/10th of a bel, and reportedly named after Alexander Graham Bell (I'm unsure why they dropped one of the 'l's!).

The decibel is a logarithmic unit that express the ratio of two quantities and can conveniently express very large ratios.

It can be used in an absolute sense by making a one of the quantities a prescribed value e.g. 1 mW (one milli-Watt) or 1 V (one Volt); in this case the unit is suffixed: dBm or dBV.

Alternatively, it can be used relatively to capture the ratio between any two signals in which case it expresses gain.

The decibel is an example of where we deal with field and power quantities separately:

- It is 10 times the log to the base 10 of the ratio of two power quantities

or

- It is 10 times the log to the base 10 of the ratio of the square of two field quantities – which is 20 times the log to the base 10 of the field quantities e.g.:

Power: $$\mathrm{dB} = 10log_{10}\frac{P_1}{P_2}$$

Field: $$\mathrm{dB} = 10log_{10}\frac{F_1^2}{F_2^2} = 20log_{10}\frac{F_1}{F_2}$$

To use in an absolute measurement:

1) measure a power relative to 1 mW

dB mW: $$\mathrm{dBm} = 10log_{10}\frac{P(in\ milli{-}Watts)}{1\ mW}$$

so, 500 mW is 27 dBm.

2) measure a voltage relative to 1 V

dB V: $$\mathrm{dB}v = 20log_{10}\frac{V(in\ Volts)}{1V}$$

so, 1mV is -60 dBV

Decibel examples

Sound and hearing is a useful test-case to consider dB's. In fact, the non-linear response of our hearing to changes in Sound Pressure Level (SPL) was one of the driving factors in their development. The threshold of hearing is generally considered to be an SPL of 20 x 10^{-6} Pa. If we make other SPL values relative to this, we use 'dBA' – decibels acoustic. SPL is a field quantity.

A quiet room has a SPL of ~ 40 dBA, so:

$$40 = 20log_{10}\frac{SPL_{40}}{0.00002} \qquad SPL_{40} = 0.002\ Pa$$

Thus, a quiet room has a SPL 100 times greater than the threshold of hearing.

A jet engine at 100m produces ~130 dBA

$$130 = 20log_{10}\frac{SPL_{130}}{0.00002} \qquad SPL_{130} = 63\ Pa$$

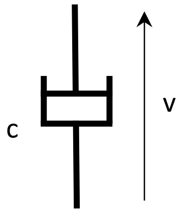
Thus, a jet engine produces a SPL ~30,000 times greater than a quiet room and ~3,000,000 times greater than the threshold of hearing.

Useful dBs (especially for control!)

- Half power is -3dB*
- Doubling of power is 3dB
- 10dB is ten fold increase in power
- -10dB is a ten fold reduction in power

- -6dB is half amplitude for field quantities
- 6dB is a doubling in amplitude for field quantities
- -20dB is a ten fold reduction for field quantities
- 20dB is a ten fold increase for field quantities

A subtlety that is worth reinforcing is that in a linear system reducing a field quantity by 3dB results in half power in a dissipative element (and increasing by 3 dB results in a doubling of power). This is one of the reasons why we define dBs in the way we do. Let's see how that works in practice by considering a car damper with damping 'c' and an applied velocity 'v'.



$$P_{damper} = v^2 c$$

Fig. 9 schematic representation of a damper

If we increase the *field quantity*, v, by 1.414 (or 3dB: $20\log_{10}1.414 = 3$), then the resulting power dissipated in the damper will increase by $1.414^2 = 2$. And we already know that doubling of power is 3 dB increase: $10\log_{10}2 = 3$.

In other words, increasing the magnitude of the field quantity by 3dB results in a doubling of power in the damper: a 3 dB increase always implies a doubling of power in a linear system, whether it is the field quantity or the power that is being increased by 3dB.

1.1.8 Test yourself section 1.1

'Test yourself' questions are designed to check you have picked up on the salient points in each section. There are no model answers provided as you will find them in the handbook.

1) What name do we give to the thing we are measuring?
2) Which physical phenomena do we most commonly use to represent signals?
3) Describe a signal that humans perceive in the time domain?
4) Describe a signal that humans perceive in the frequency domain?
5) How does it help us to mathematically describe signals in different domains?
6) What do square bracket signify in the context: x[ ]?
7) The equation $y[n] = \alpha x[\gamma n + \delta] + \beta$ describes the 4 elementary signal transformations; what is the effect on the signal of alpha, gamma, delta and beta?
8) What is a field quantity? a root power quantity?
9) Why do we use decibels and what are the two ways we commonly use them?
10) If I have a power of 12 mW and I increase it by 3 dB, what is my new power level?