# Signals, Systems and **Control**

## Contents

# 3. Control

## 3.1. Introduction to Feedback Control Systems

**What is a System?**

A **system** can be thought of as a "black box" with **inputs** and **outputs**. The system has a deterministic mapping between the inputs and the outputs, so if the input is known, then the output can be known. Systems can be connected together, where the output of one system becomes the input to another, forming a larger, more complex system.

**What is Control?**

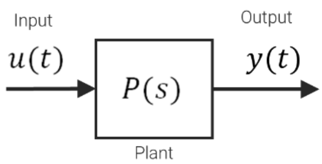
**Control** is the use of algorithms and feedback in engineered systems. Automatic control is essential in many fields of engineering and science, including space-vehicle systems, robotic systems, modern manufacturing systems, and industrial operations.

**What is a Control System?**

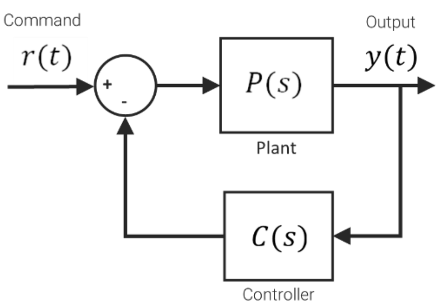
A **control system** exists to regulate or track a variable (energy, information, or other quantities) in a desired manner. It comprises a set of devices, including a plant, controller/processor, and sensors, that provide the desired response.

**Types of Control Systems**

- **Open-loop system:** Control action is independent of the output. It is sensitive to disturbance. Feedforward control is form of open-loop control.




- **Closed-loop system:** Control action depends on the output; also known as a **feedback control system**. There is a direct comparison between the desired output response and the current output (measurement) response.



## 3.2. The basic components of a feedback control loop

The basic components of a feedback control loop are:

- **Plant:** The system to be controlled. The plant is a given system whose outputs are to be controlled.
- **Sensors:** Devices that measure the outputs of the plant. An output transducer, or sensor, measures the output response and converts it into a form used by the controller.
- **Controller:** A subsystem that generates the input to the plant or process. It is a system that calculates the necessary values of the plant inputs to achieve desired values of the plant outputs. The controller's role is to drive the plant in such a way that the plant output tracks some desired behaviour.
- **Actuators:** Devices that set the values of the plant inputs to those dictated by the controller.
- **Reference input:** The desired or command signal for the system.
- **Disturbance :** Depending on the modelling , this could be environment effects or any other perturbation that is not considered in the model of other components of the loop.
- **Noise:**
- **Output:** The actual response of the system.
- **Error signal:** The difference between the reference input and the output, which is fed back to the controller to reduce the error. The controller uses this error signal to adjust the input to the plant.

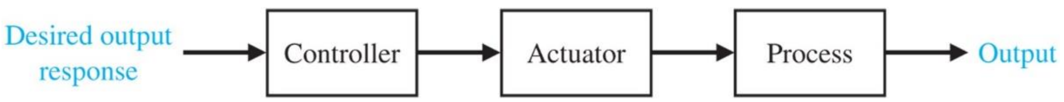
These components are interconnected in a **closed loop**, where the output is fed back and compared to the reference input, and the difference (error) is used by the controller to adjust the plant's behaviour.

## 3.3. Open-loop vs. closed-loop control

**Open-loop control systems** and **closed-loop control systems** represent two distinct approaches to system control, each with its own characteristics, advantages, and disadvantages. The choice between them depends on the specific application and the desired performance.

**Open-Loop Control Systems:**

- In an **open-loop control system**, the control action is independent of the output. The system operates in a straightforward chain of causality, where the input determines the output, without any feedback or monitoring of the actual results.
- The output is **neither measured nor fed back** for comparison with the input.
- An open-loop system **cannot compensate for disturbances** that add to the controller's driving signal or affect the output directly.
- Any control system that operates on a **time basis** is open loop.
- **Accuracy** depends on calibration.
- Open-loop control can be used only if the **relationship between the input and output is known** and if there are **neither internal nor external disturbances**.
- Open-loop systems are typically reserved for **simple processes** that have well-defined input to output behaviours.




**Advantages of Open-Loop Control Systems:**

- **Simple construction and ease of maintenance**
- **Less expensive** than a corresponding closed-loop system
- **No stability problem**
- **Convenient** when output is hard to measure or measuring the output precisely is expensive

**Disadvantages of Open-Loop Control Systems:**

- **Sensitive to disturbances**
- **Cannot correct for disturbances**
- The **output has no effect on the control action**.

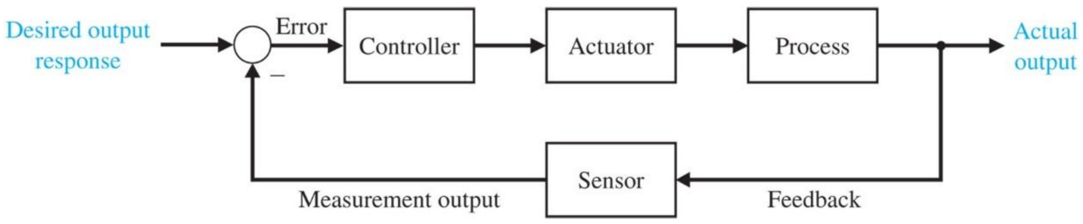
**Examples of Open-Loop Control Systems:**

- A washing machine that operates on a time basis

- Traffic control by means of signals operated on a time basis
- An open-loop toaster
- Dishwashers

**Closed-Loop Control Systems (Feedback Control Systems):**

- A **closed-loop control system** maintains a prescribed relationship between the output and the reference input by comparing them and using the difference as a means of control.
- **Feedback** means the input of the system is a function of its output, creating a loop.
- The actuating error signal, which is the difference between the input signal and the feedback signal, is fed to the controller to reduce the error and bring the system output to a desired value.
- Closed-loop systems compensate for disturbances by measuring the output response, feeding that measurement back through a feedback path, and comparing that response to the input.
- If there is any difference between the two responses, the system drives the plant, via the actuating signal, to make a correction.
- **Feedback changes the dynamics of the system** and its underlying behaviour.




**Advantages of Closed-Loop Control Systems:**

- **Increased accuracy**
- The use of feedback makes the **system response relatively insensitive to external disturbances** and internal variations in system parameters.
- Transient response and steady-state error can be controlled more conveniently and with greater flexibility.
- **Less sensitive to the disturbance**

**Disadvantages of Closed-Loop Control Systems:**

- More complex and expensive than open-loop systems
- Stability is a major problem. Closed-loop control system may tend to overcorrect errors and thereby can cause oscillations of constant or changing amplitude.
- The number of components used is more than that for a corresponding open-loop control system.
- Requires sensors

**Examples of Closed-Loop Control Systems:**

- A room-temperature control system
- A closed-loop toaster oven

**Key Differences and Trade-offs:**

- **Disturbance Rejection:** Closed-loop systems are better at disturbance rejection due to feedback. Open loop systems are more sensitive to disturbances.
- **Parameter Sensitivity:** Closed-loop systems are less sensitive to changes in system parameters.
- **Stability:** Open-loop systems are easier to build because system stability is not a major problem. Stability is a major problem in closed-loop control systems.
- **Complexity and Cost:** Open-loop systems are generally simpler and less expensive. Closed-loop systems are more complex and costly due to the need for additional components like sensors and feedback mechanisms.
- **When to Use:** For systems with inputs known ahead of time and without disturbances, open-loop control is suitable. Closed-loop control is advantageous when unpredictable disturbances and variations in system components are present.

**Combined Approach:**

A proper combination of open-loop and closed-loop controls is usually less expensive and will give satisfactory overall system performance.

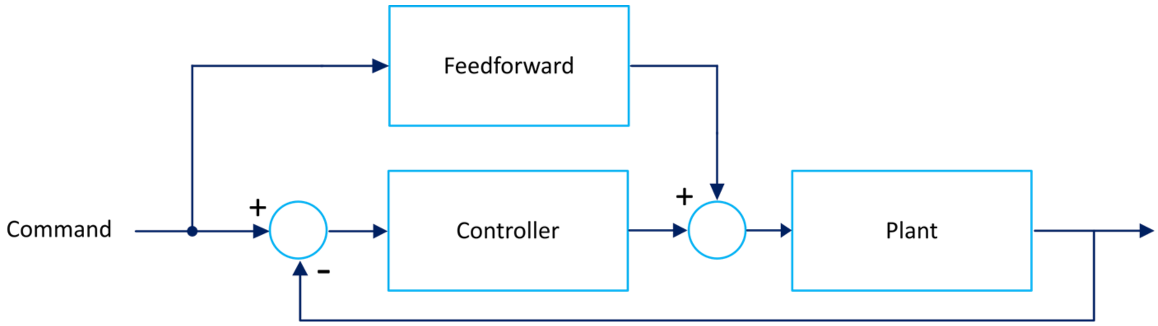



*Figure 1 Feedforward and Feedback Control Loop*

## 3.4. What is Feedback

**Feedback** is a process where a signal travels through a chain of causal relations to re-affect itself. Feedback control is a fundamental principle underlying self-regulating systems. The output of a system influences its input.

Feedback offers several advantages:

- It makes systems relatively insensitive to external disturbances.
- It offers new degrees of freedom to a designer by exploiting sensing, actuation, and computation.
- Feedback can be used to create modularity and shape well-defined relations between inputs and outputs in a structured hierarchical manner.

**Negative feedback** is a control mechanism that counteracts changes in a system, promoting stability, accuracy, and robustness.

**Properties of Negative Feedback:**

- **Increased accuracy**: Reduces steady-state errors, allowing the system to faithfully reproduce the input.
- **Enhanced Stability**: Improves stability and reduces oscillations, preventing potential instability.
- **Reduced sensitivity to variations**: Decreases the sensitivity of the output-to-input ratio to changes in system parameters and characteristics, making the system more robust.
- **Attenuation of disturbances**: Reduces the effects of external disturbances or noise on the system's output, improving overall performance.
- **Linearity**: Increases the range of linearity of a system.
- **Stabilisation**: Can stabilise systems that are inherently unstable.
- **Modularity:** Can be used to create modularity and shape well-defined relations between inputs and outputs in a structured hierarchical manner.

**Positive feedback** is a reinforcing, compounding, or amplifying process.

**Properties of Positive Feedback:**

- **Effect on the range of linearity**: Positive feedback reduces the linear range of the system.
- **Increases sensitivity**: Systems become highly sensitive, requiring careful gain adjustments to avoid oscillations.
- **Potential for instability**: Can cause exponential growth, potentially leading to instability.

### Feedforward Control

**Feedforward control** measures disturbances before they affect the system and compensates for them proactively.

**Benefits of Feedforward Control:**

- **Proactive disturbance rejection:** Corrective action is taken before the system is affected by disturbances.
- **Improved setpoint tracking**: Reduces error from setpoint changes.
- **Complements feedback control**: Reduces setpoint and disturbance errors, allowing the feedback controller to manage noise.
- **No risk of instability**: Does not create dynamic instabilities in a system.

**Feedback vs. Feedforward**

| **Feature** | **Feedback Control** | **Feedforward Control** |
|---|---|---|
| **Loop Type** | Closed-loop | Open-loop |
| **Action Based On** | Deviations from desired output | Plans and anticipated disturbances |
| **Response** | Reactive (corrects errors after they occur) | Proactive (compensates before errors occur) |
| **Sensitivity to Models** | Robust to model uncertainty | Sensitive to model uncertainty |
| **Risk of Instability** | Possible | No risk |
| **Disturbance Measurement** | Measures the output to determine the effect of disturbances | Measures the disturbance directly |

# 3.5. Transfer Functions

A transfer function represents the relationship **between the input and output** of a linear time-invariant (LTI) system. It is defined as the ratio of the Laplace transform of the **output** to the Laplace transform of the **input**, assuming **zero initial conditions**.

Transfer functions can be represented in block diagrams, where each block contains a mathematical function.

Transfer functions simplify the representation of physical systems by converting differential equations into **algebraic expressions**.

**MATLAB Commands**: MATLAB provides commands to create, manipulate, and analyse transfer functions.

- **tf(num,den):** Creates a continuous-time transfer function with numerator and denominator specified by num and den.
- The other way to define transfer function or any laplace operation , the ‘s’ can be defined by using tf command as **s = tf(‘s’)** . Then , the MATLAB recognize s as a Laplace variable.
- **zp2tf(z,p,k)**: Determines the numerator and denominator of the transfer function from zeros, poles, and gain.
- **series(G1,G2,...,Gn)**: Defines a system of cascaded subsystems.
- **feedback( G1, H1, -1)** : Negative feedback over H1 of G1

**System Interconnections and Transfer Functions:**

- **Series Connection**: The transfer function of systems connected in series is the product of their individual transfer functions.
- **Parallel Connection**: Complex systems can be defined by subsystems connected in parallel.
- **Feedback Connection**: Feedback control is used to make systems behave the way you want them to.

## 3.6. Block Diagrams

**Using Block diagrams** is the method to represent and analyse control systems, especially when dealing with multiple interconnected subsystems.

A block diagram is a clear representation of the functions performed by each component of a system. Systems are assembled into cascade, parallel, and feedback forms. **Each block** represents the system elements such as transfer functions, gain , integrators etc. Each block is connected by arrows which indicate the direction of the **signal flow**. There are two main components namely summing point and pickoff (branch) point. Summing points is algebraic sum of signals , while branch points are used to pick the signal to create new flow.

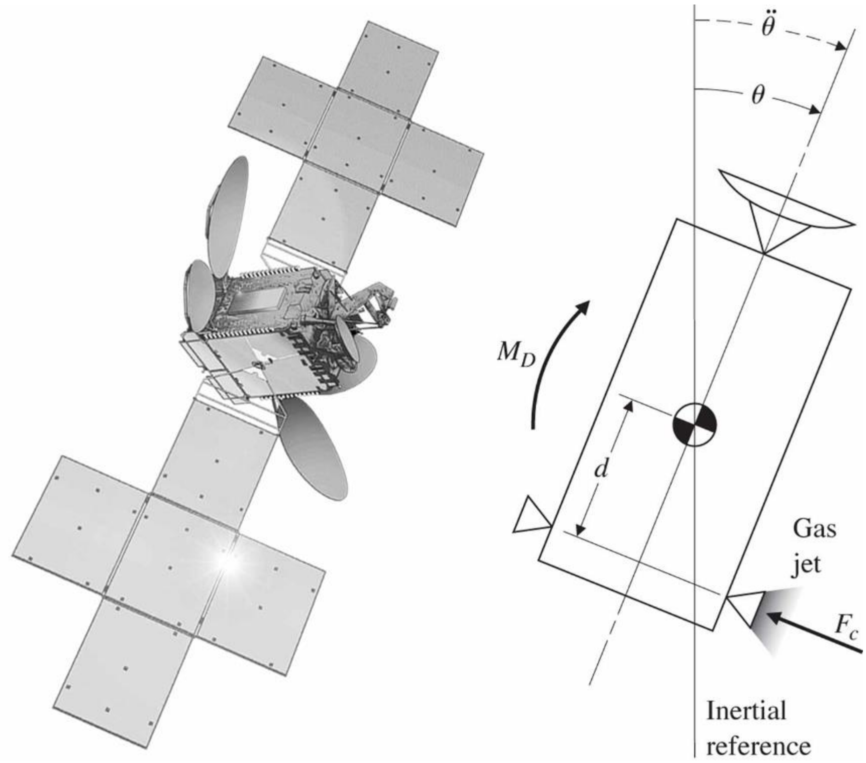



*Figure 2 a) Communication Satellite Source: Courtesy Space Systems /Loral[1] b) Satellite Control Schematic*

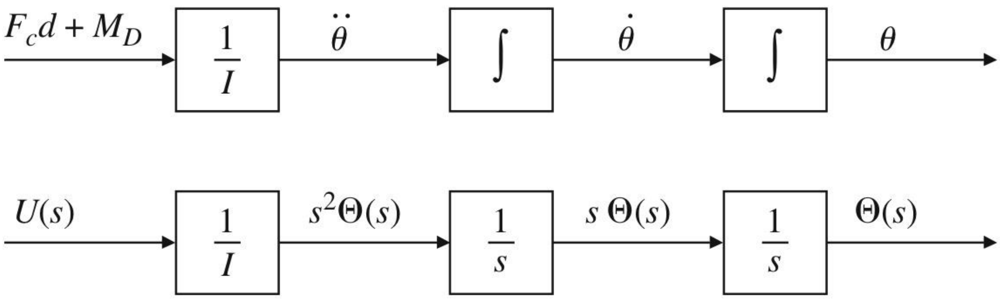



*Figure 3 Block Diagram Representation (Expanded)*

[1] [1] Franklin, G. F., Emami-Naeini, A., & Powell, J. D. (2010). Feedback Control of Dynamic Systems (6th ed.). Pearson Education

One of the most important uses of the Block Diagram is to simplify the complex system and its interaction by using algebraic rules to find the overall transfer function. This is called **Block Diagram Reduction**.

Some of the Block Diagram Operations are given as follows:


*Figure 4 Cascade Connection*

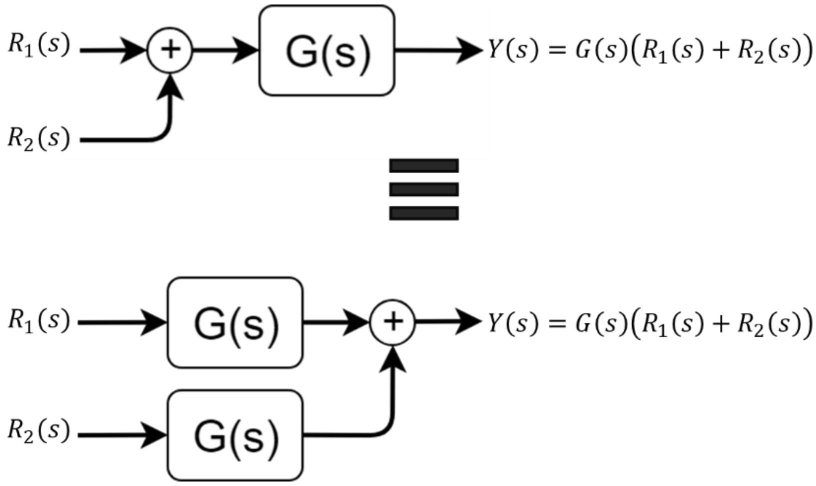



*Figure 5 Moving a summing joint after a block*

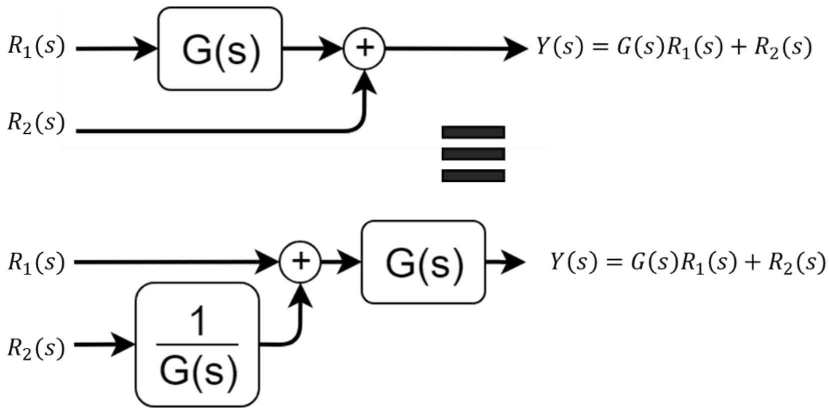



*Figure 6 Moving a summing joint before a block*

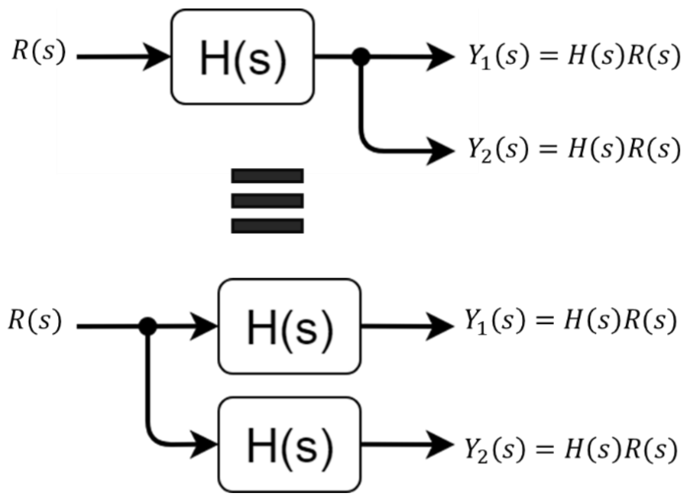



*Figure 7 Moving pickoff point ahead of block*

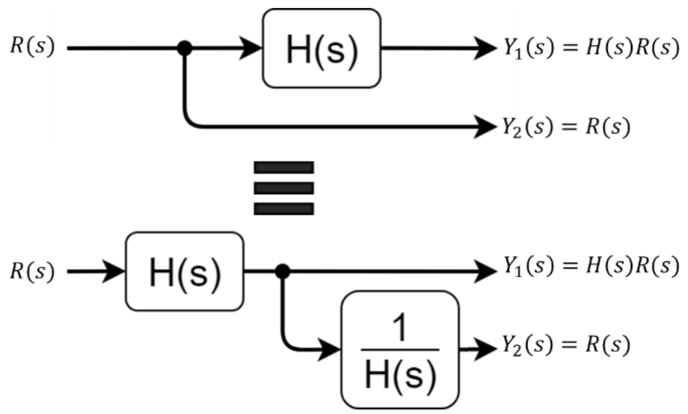



*Figure 8 Moving pickoff point after a block*

These operations are commonly used to simplify the complex block diagrams and obtain the overall transfer functions. The following block diagram reduction is about feedback loop system to obtain the closed loop overall transfer functions as

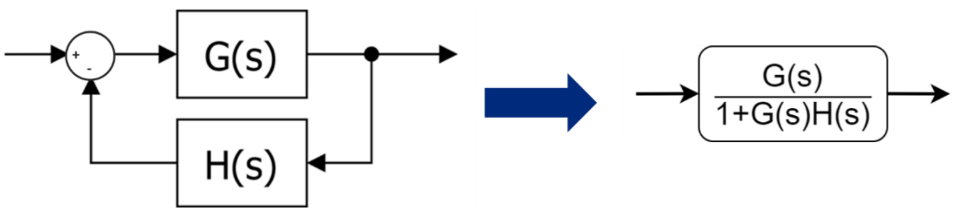



*Figure 9 Closed-Loop Block Diagram Reduction*

**Please note that Figure 9 Closed-Loop Block Diagram Reduction is the most important block diagram operation.**

### 3.6.1. Example: DC Motor Feedback Loop

Find the closed loop overall transfer function of DC Motor Feedback loop given below:

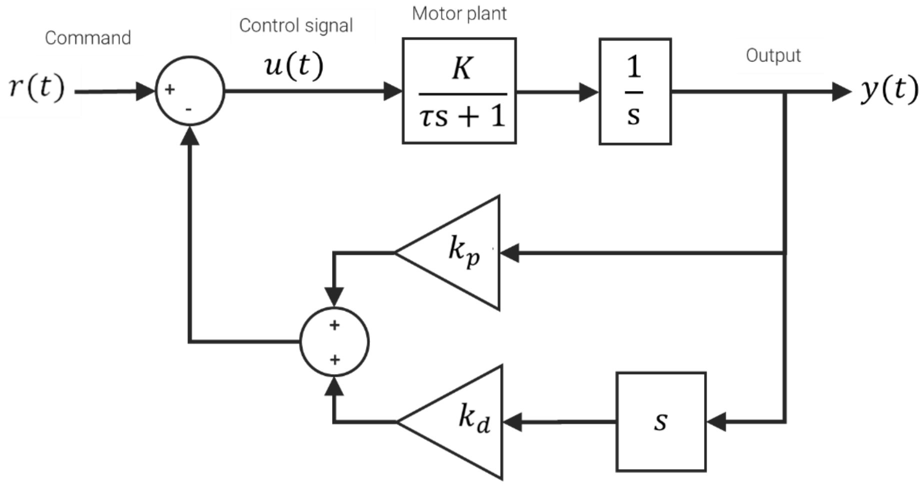
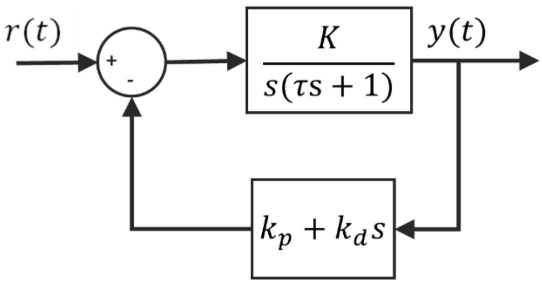
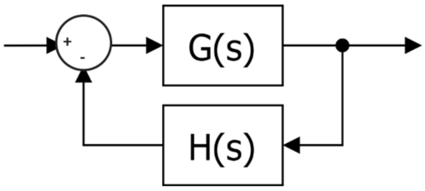



*Figure 10 DC Motor Feedback loop block diagram*

First step is to combine the block on the same path (forward and backward path) :



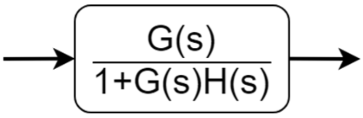

This is equivalent to




$$G(s) = \frac{K}{s(\tau s + 1)}$$

$$H(s) = k_p + k_d s$$

Now, the block diagram is simplified to the one that closed loop block diagram reduction operation given in Figure 9 can be applied as




$$\frac{G(s)}{1 + H(s)G(s)} = \frac{K}{s(\tau s + 1) + \left(k_p + k_d s\right)K}$$

$$= \frac{K}{\tau s^2 + (k_d K + 1)s + k_p K}$$

The overall closed loop DC Motor transfer function is obtained and can be represented in a single block as

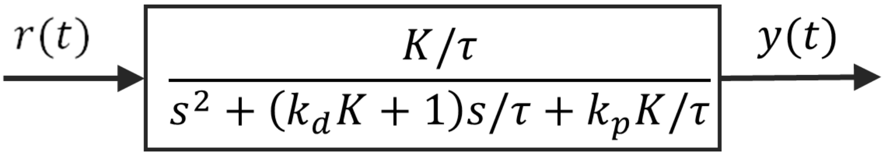



*Figure 11 Closed loop DC Motor Transfer Function*

# 3.7. Stability

**Stability** is the one of the most important concepts in feedback control systems, ensuring that the system's response remains bounded and predictable over time.

The stability of a linear system can be represented as the sum of the natural response and the forced response:

- The natural response is the response due to initial conditions
- The forced response is how the system responds to inputs

The formal definition of stability for a linear time-invariant system based on its natural response is

- A system is stable if the response goes to zero as time goes to infinity
- A system is unstable if the response eventually goes to infinity as time goes to infinity
- A system is marginally stable if the response remains constant or oscillates

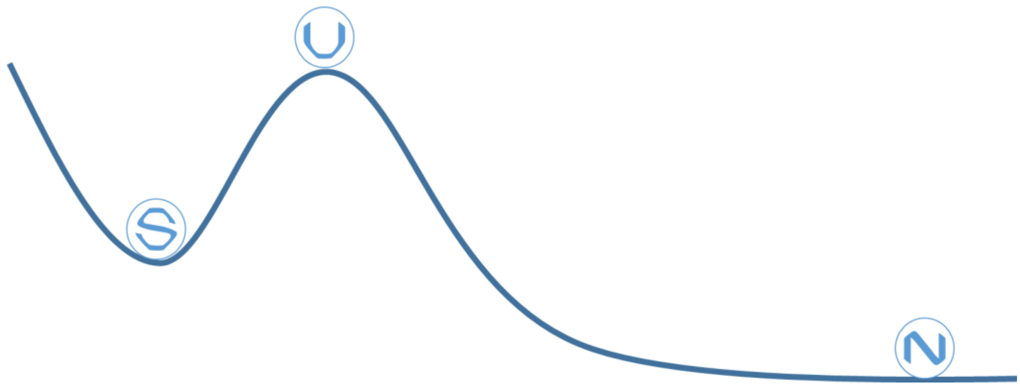



*Figure 12 Stability Illustration*

**Importance of Stability**:

- **Stability is the most important system specification**.
- Without stability, transient response and steady-state errors are meaningless.
- A system must be stable to achieve the desired transient and steady-state responses.

## 3.7.1. BIBO Stability

The most basic stability metric used in control is Bounded -Input Bounded-Output stability which is defined as:

1. A system is **stable** if every bounded input yields a bounded output
2. A system is **unstable** if *any* bounded input yields an unbounded output

The BIBO stability of a system is often determined by observing the response of the system experimentally, though if the system model is known it can be determined using the final value theorem.

It is never a good idea to test the stability of a system that might be unstable if it could result in damage to the system or users.

Assessing the BIBO stability of a system, especially marginal stability, can be impractical depending on the type of system and range of potential inputs.

**Though conceptually BIBO stability is simple, assessment typically involves testing a system which can be difficult and time consuming**

### 3.7.2. Pole Stability

If a mathematical model of a linear time-invariant system is known, the stability of the system can be determined using the location of the poles of the system on a root locus plot.

The definition of stability based on pole locations is:

1. The system is **stable** if all its poles are in the left-hand plane
2. The system is **unstable** if any poles are in the right-hand plane and/or any *repeated* poles are on the imaginary axis
3. The system is **marginally stable** if it has *non-repeated* poles on the imaginary axis, and other poles in the left-hand plane.

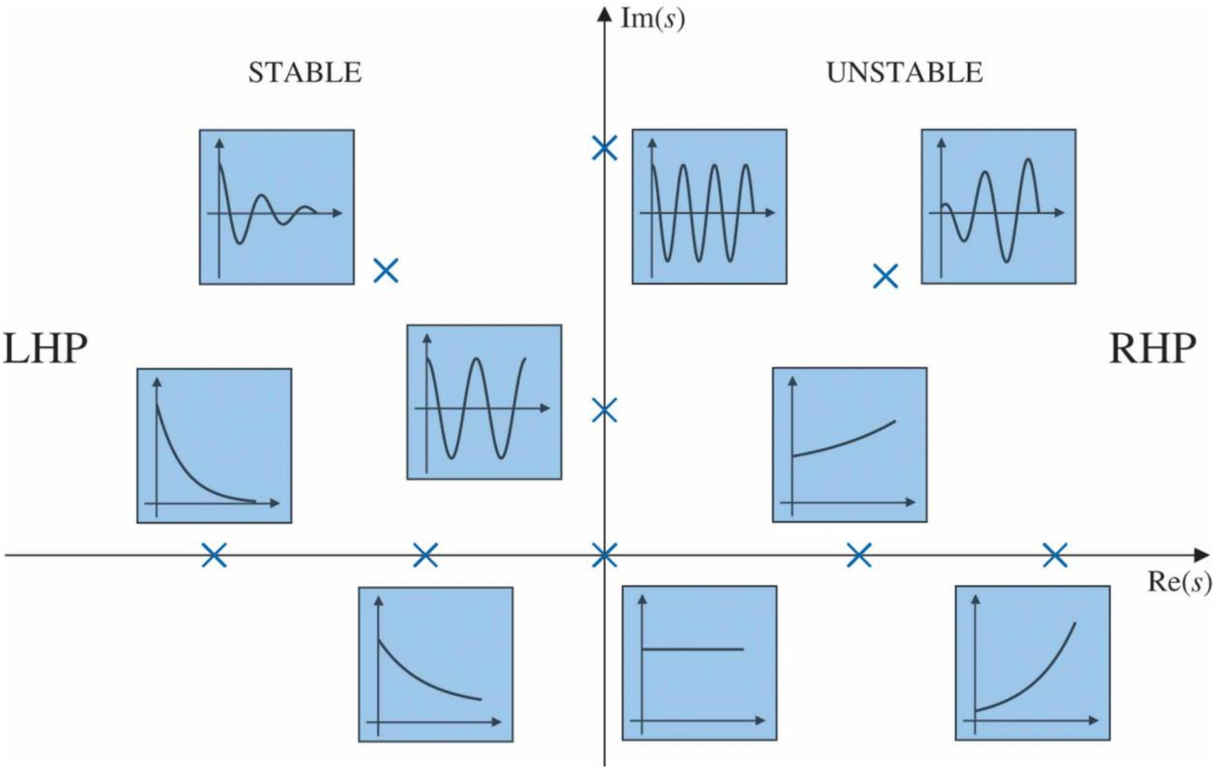



*Figure 13 Time functions associated with points in the s-plane (LHP, left half-plane; RHP, right half-plane)*[2]

A **stable system** is one where the **natural response** approaches zero as time approaches infinity. In a stable system, any bounded input yields a bounded output (BIBO stability). If all closed-loop poles lie in the left-half s-plane, then the system is stable.

[2] Franklin, G. F., Emami-Naeini, A., & Powell, J. D. (2010). Feedback Control of Dynamic Systems (6th ed.). Pearson Education

An **unstable system's** response grows **without bound** as time approaches infinity. An unstable system can be identified if any bounded input results in an unbounded output.

The presence of any closed-loop poles in the right-half s plane indicates instability. Also, note that closed-loop transfer functions with only imaginary axis poles of multiplicity 2 are unstable.

A **marginally stable system** has a natural response that neither decays nor grows but remains constant or oscillates as time approaches infinity.

- Marginally stable systems have closed-loop transfer functions with only imaginary axis poles of multiplicity 1 and poles in the left half-plane.
- Marginally stable systems are considered unstable under the BIBO definition because there will be a bounded input that results in an unbounded output.

### 3.7.3. Determining Stability:

- **Location of Closed-Loop Poles**: The location of the closed-loop poles in the s-plane determines stability.
  - Left-half plane: Stable.
  - Right-half plane: Unstable.
  - Imaginary axis with multiplicity 1: Marginally stable.
- **Routh-Hurwitz Criterion**: This criterion determines the number of closed-loop poles in the left-half plane, right-half plane, and on the jw-axis without solving for the poles explicitly.( without having to know exact pole locations)
- **Nyquist Criterion**: The Nyquist criterion relates open-loop frequency response to closed-loop stability, especially useful for systems with time delays.
- **Bode Plots**: Bode plots can be used to assess stability via gain and phase margins.
- **Root Locus**: The root locus graphically shows the location of closed-loop poles as a system parameter (usually gain) is varied, providing insights into stability.

### 3.7.4. Factors Affecting Stability

- **Feedback**: While feedback can stabilise unstable systems, it can also cause instability if not properly designed.
- **Gain**: Increasing the gain can sometimes lead to instability.
- **Time Delays**: Time delays in the system can reduce stability or cause instability.
- **Unmodelled Dynamics**: Ignoring dynamics of sensors and actuators or other components that were not accounted for in the equation can lead to instability.

## 3.8. Performance Metrics

**Performance metrics** are essential for evaluating and designing control systems, focusing on how well a system meets its desired objectives. These metrics can be categorised into **steady-state response** and **transient response**.

**Design Objectives** can be defined in this manner such that control system design involves achieving the desired transient response, reducing steady-state error, and ensuring stability. By considering these performance metrics, control systems can be analysed and designed to meet specific requirements, balancing transient and steady-state responses while ensuring stable operation.

### 3.8.1. Transient Response:.

- Refers to the system's behaviour as it transitions from an initial state to a final state.
- Transient response is important for several reasons. For example, a lift's slow or excessively rapid transient response can affect passenger comfort.
- **Rise time** indicates the time required for the response to reach a certain percentage (e.g., 10% to 90% or 0% to 100%) of its final value.
- **Peak time** is the time required for the response to reach the first peak of the overshoot.
- **Maximum overshoot** is the maximum peak value of the response curve, measured from the final value. It often indicates the relative stability of the system.
- **Settling time** is the time required for the response to reach and stay within a specified percentage (usually 2% or 5%) of the final value.

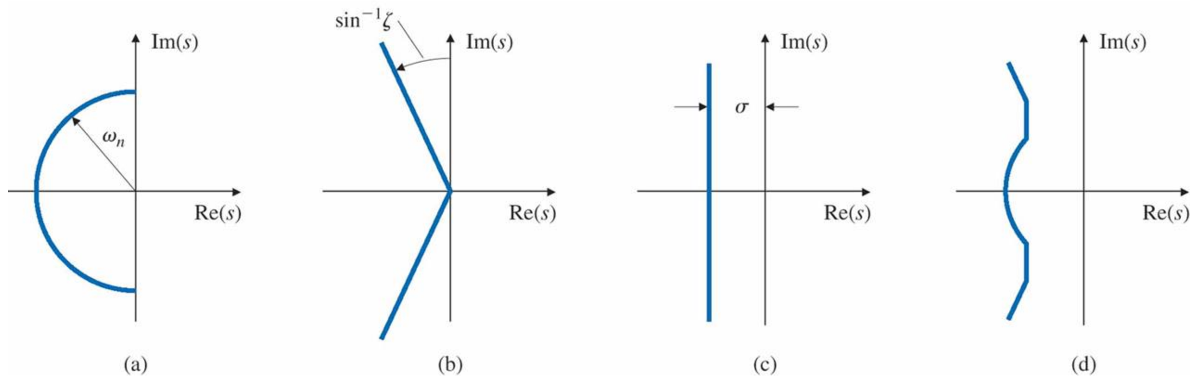
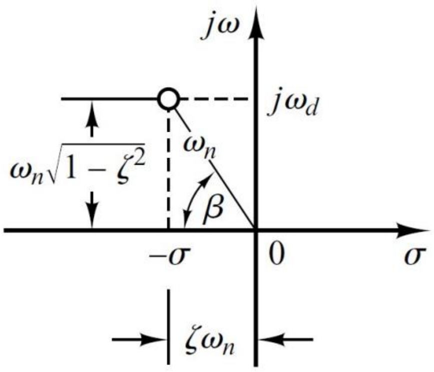



*Figure 14: Graphs of regions in the s-plane delineated by certain transient requirements: (a) rise time; (b) overshoot;
(c) settling time; (d) composite of all three requirements[3]*

[3] Franklin, G. F., Emami-Naeini, A., & Powell, J. D. (2010). Feedback Control of Dynamic Systems (6th ed.). Pearson Education

| | |
|---|---|
| Poles | $p_{1,2} = -\zeta\omega_n \pm j\omega_d$ |
| Damped Natural Frequency | $\omega_d = \omega_n\sqrt{(1-\zeta^2)}$ |
| Rise Time | $t_r = \frac{\pi - \beta}{\omega_d}, \ \beta = tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$ |
| Peak Time | $t_p = \frac{\pi}{\omega_d}$ |
| Peak Overshoot | $M_p = 100e^{-\zeta\omega_n t_p}(\%)$ |
| Settling Time[4] | $t_s \cong \frac{4.5}{\zeta\omega_n}$ |




## 3.8.2. Steady-State Response

- Steady State Response describes the behaviour of the system as time approaches infinity.
- **Steady-state error** is the difference between the input and output after the transients have died out. It indicates the accuracy of the system.
- **Static error constants** such as position constant ($K_p$), velocity constant ($K_v$), and acceleration constant ($K_a$), are used to evaluate steady-state errors. Higher static error constants reduce steady-state error.
- **System type** refers to the number of pure integrations in the forward path of a unity feedback system and affects the steady-state error for different types of inputs.

## 3.8.3. Stability

- A fundamental requirement for control systems.
- An unstable system has a response that grows without bound.
- Relative stability, gain margin, and phase margin are important considerations.

## 3.8.4. Test Signals

- Commonly used test inputs include step functions, ramp functions, acceleration functions, impulse functions, sinusoidal functions, and white noise.
- **Step inputs** are useful for evaluating a system's ability to position itself relative to a stationary target.
- **Ramp inputs** test the system’s ability to track a constant-velocity target.
- **Impulse functions** are appropriate for systems subjected to shock inputs.

[4] Please refer to the textbook , Fig. 5-11 of Ogata, 5th Ed. , . there is no single equation for settling time. Sometimes it can be assumed $t_s \cong \frac{4}{\zeta\omega_n}$ , for (%2 error band) depending on the damping ratio.

### 3.8.5. Error Analysis

- Errors can arise from changes in the reference input, imperfections in system components, and the system's inability to follow particular types of inputs.
- **Steady-state error** can be calculated using the final value theorem.

Let's define error as a function of time:

$$e(t) = r(t) - y(t)$$

In laplace domain:

$$E(s) = R(s) - Y(s)$$

The final value theorem is then

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)$$

- **System Type and Steady-State Error:**.
  - **Type 0 systems** have a finite steady-state error for a step input and infinite error for ramp and acceleration inputs.
  - **Type 1 systems** have zero steady-state error for a step input, finite error for a ramp input, and infinite error for an acceleration input.
  - **Type 2 systems** have zero steady-state error for step and ramp inputs but finite error for an acceleration input.

## 3.9. Real-world examples of feedback control systems

Feedback control systems are pervasive in both natural and engineered environments. They maintain stability, increase accuracy, and reduce sensitivity to disturbances, making them essential in a wide array of applications.

Here are some real-world examples of feedback control systems:

- **Physiological Control Systems**: The human body employs feedback mechanisms to maintain internal conditions. Examples include body temperature regulation and blood pressure maintenance.
- **Thermostats**: **Room temperature control systems** use a thermostat to measure the actual temperature and compare it to the desired temperature, adjusting heating or cooling equipment accordingly.
- **Toilets**: A toilet is a mechanical feedback system that automatically refills the tank after a flush.
- **Cruise Control**: Cruise control in vehicles maintains a set speed by monitoring the vehicle's velocity and adjusting the throttle.
- **Manufacturing**: Feedback control is used in **automation of production machinery**.
- **Pacemakers**: Electronic pacemakers regulate the speed of the heart pump.
- **Power Networks**: Feedback control systems maintain the environment, lighting and power in buildings and factories.
- **Hybrid Vehicles**: Control systems manage the complex interactions within hybrid electric drivetrains.
- **Disk Drives**: Feedback systems control the read/write head displacement in floppy disk drives.
- **Robotics**: PID-based position control is used in automated assembly lines.
- **Active Suspension**: Active suspensions in vehicles use feedback to improve ride quality.
- **Satellites**: Feedback increases the precision of instruments such as telescopes.

These examples illustrate how feedback control enhances system performance and stability across various domains.

## 3.10. Test Yourself

- What is the key difference between open-loop and closed-loop control?
- Give an example of an open-loop system
- What is the primary goal of negative feedback ?
- What is BIBO stability ?
- **How does locations of poles affect the system stability?**
- **How to analysis Transient Response ( How to calculate the corresponding metric)**
- **Define steady-state error.**
- Describe three real-world applications of feedback control systems.
- **Get familiar with block diagram operation and block diagram reductions**