Appendix 8: Approximate Expressions for the Dimensionless Aerodynamic Stability and Control Derivatives

Table A8.1 Longitudinal Aerodynamic Stability Derivatives

Small Perturbation Derivatives Referred to Aircraft Wind Axes			
Derivative	**Description**	**Expression**	**Comments**
X_u eq. (13.16)	Axial force due to velocity	$-2C_D - V_0\dfrac{\partial C_D}{\partial V} + \dfrac{1}{\frac{1}{2}\rho V_0 S}\dfrac{\partial \tau}{\partial V}$	Drag and thrust effects due to velocity perturbation
X_w eq. (13.27)	Axial force due to "incidence"	$C_L - \dfrac{\partial C_D}{\partial \alpha}$	Lift and drag effects due to incidence perturbation
X_q eq. (13.46)	Axial force due to pitch rate	$-\overline{V}_T\dfrac{\partial C_{D_T}}{\partial \alpha_T}$	Tailplane drag effect, usually negligible
$X_{\dot{w}}$ eq. (13.68)	Axial force due to downwash lag	$-\overline{V}_T\dfrac{\partial C_{D_T}}{\partial \alpha_T}\dfrac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha} \equiv X_q\dfrac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}$	Tailplane drag due to downwash lag effect (added mass effect)
Z_u eq. (13.21)	Normal force due to velocity	$-2C_L - V_0\dfrac{\partial C_L}{\partial V}$	Lift effects due to velocity perturbation
Z_w eq. (13.30)	Normal force due to "incidence"	$-C_D - \dfrac{\partial C_L}{\partial \alpha}$	Lift and drag effects due to incidence perturbation
Z_q eq. (13.51)	Normal force due to pitch rate	$-\overline{V}_T a_1$	Tailplane lift effect
$Z_{\dot{w}}$ eq. (13.72)	Normal force due to downwash lag	$-\overline{V}_T a_1\dfrac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha} = Z_q\dfrac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}$	Tailplane lift due to downwash lag effect (added mass effect)
M_u eq. (13.34)	Pitching moment due to velocity	$V_0\dfrac{\partial C_m}{\partial V}$	Mach dependent, small at low speed
M_w eq. (13.39)	Pitching moment due to "incidence"	$\dfrac{\mathrm{d}C_m}{\mathrm{d}\alpha} = -aK_n$	Pitch stiffness, dependent on static margin
M_q eq. (13.55)	Pitching moment due to pitch rate	$-\overline{V}_T a_1\dfrac{l_T}{\overline{\overline{c}}} \equiv Z_q\dfrac{l_T}{\overline{\overline{c}}}$	Pitch damping, due mainly to tailplane
$M_{\dot{w}}$ eq. (13.76)	Pitching moment due to downwash lag	$-\overline{V}_T a_1\dfrac{l_T}{\overline{\overline{c}}}\dfrac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha} \equiv M_q\dfrac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}$	Pitch damping due to downwash lag effect at tailplane

Table A8.2 Lateral-Directional Aerodynamic Stability Derivatives

Small Perturbation Derivatives Referred to Aircraft Wind Axes			
Derivative	**Description**	**Expression**	**Comments**
Y_v eq. (13.82)	Side force due to sideslip	$\left(\frac{S_B}{S}y_B - \frac{S_F}{S}a_{1_F}\right)$	Always negative and hence stabilising
L_v eq. (13.92) eq. (13.105) eq. (13.108)	Rolling moment due to sideslip	Wing with dihedral $-\frac{1}{Ss}\int_0^s c_y a_y \Gamma y \mathrm{d}y$ Wing with aft sweep $-\frac{2C_L \tan\Lambda_{1/4}}{Ss}\int_0^s c_y y \mathrm{d}y$ Fin contribution $-a_{1_F}\overline{V}_F\frac{h_F}{l_F}$	Lateral static stability, determined by total dihedral effect; many contributions, most of which are difficult to estimate reliably; most accessible approximate contributions given
N_v eq. (13.112)	Yawing moment due to sideslip	Fin contribution $a_{1_F}\overline{V}_F$	Natural weathercock stability, dominated by fin effect
Y_p eq. (13.117)	Side force due to roll rate	Fin contribution $-\frac{1}{Sb}\int_0^{H_F} a_h c_h h \mathrm{d}h$	Fin effect dominates, often negligible
L_p eq. (13.128)	Rolling moment due to roll rate	Wing contribution $-\frac{1}{2Ss^2}\int_0^s (a_y + C_{D_y})c_y y^2 \mathrm{d}y$	Roll damping; wing effects dominate but fin and tailplane contribute
N_p eq. (13.137)	Yawing moment due to roll rate	Wing contribution $-\frac{1}{2Ss^2}\int_0^s \left(C_{L_y} - \frac{\mathrm{d}C_D}{\mathrm{d}\alpha_y}\right)c_y y^2 \mathrm{d}y$	
Y_r eq. (13.142)	Side force due to yaw rate	Fin contribution $\overline{V}_F a_{1_F}$	Many contributions but often negligible
L_r eq. (13.150) eq. (13.155)	Rolling moment due to yaw rate	Wing contribution $\frac{1}{Ss^2}\int_0^s C_{L_y} c_y y^2 \mathrm{d}y$ Fin contribution $a_{1_F}\overline{V}_F\frac{h_F}{b} \equiv -L_{v(fin)}\frac{l_F}{b}$	
N_r eq. (13.162) eq. (13.167)	Yawing moment due to yaw rate	Wing contribution $-\frac{1}{Ss^2}\int_0^s C_{D_y} c_y y^2 \mathrm{d}y$ Fin contribution $-a_{1_F}\overline{V}_F\frac{l_F}{b} = -\frac{l_F}{b}N_{v(fin)}$	Yaw damping, for large aspect ratio rectangular wing wing contribution approximately $C_D/6$

Table A8.3 Longitudinal Aerodynamic Control Derivatives

Small Perturbation Derivatives Referred to Aircraft Wind Axes			
Derivative	**Description**	**Expression**	**Comments**
X_η eq. (13.174)	Axial force due to elevator	$-2\dfrac{S_T}{S}k_T C_{L_T} a_2$	Usually insignificantly small
Z_η eq. (13.178)	Normal force due to elevator	$-\dfrac{S_T}{S}a_2$	
M_η eq. (13.181)	Pitching moment due to elevator	$-\overline{V}_T a_2$	Principal measure of pitch control power

Table A8.4 Lateral-Directional Aerodynamic Control Derivatives

Small Perturbation Derivatives Referred to Aircraft Wind Axes			
Derivative	**Description**	**Expression**	**Comments**
Y_ξ	Side force due to aileron		Insignificant for conventional aeroplanes
L_ξ eq. (13.188)	Rolling moment due to aileron	$-\dfrac{1}{Ss}a_{2_A}\displaystyle\int_{y_1}^{y_2} c_y y\mathrm{d}y$	Principal measure of roll control power
N_ξ eq. (13.193)	Yawing moment due to aileron	$\dfrac{1}{Ss}\displaystyle\int_{y_1}^{y_2}\left(\dfrac{\partial C_{D_y}}{\partial \xi}\right)c_y y\mathrm{d}y$	Adverse yaw due to aileron
Y_ζ eq. (13.197)	Side force due to rudder	$\dfrac{S_F}{S}a_{2_R}$	
L_ζ eq. (13.200)	Rolling moment due to rudder	$\overline{V}_F\dfrac{h_F}{l_F}a_{2_R}$	Principal measure of yaw control power
N_ζ eq. (13.203)	Yawing moment due to rudder	$-\overline{V}_F a_{2_R}$	Adverse roll due to rudder