Arches

6

The Romans were the first to use arches as major structural elements, employing them, mainly in semicircular form, in bridge and aqueduct construction and for roof supports, particularly the barrel vault. Their choice of the semicircular shape was due to the ease with which such an arch could be set out. Generally these arches, as we shall see, carried mainly compressive loads and were therefore constructed from stone blocks, or *voussoirs*, where the joints were either dry or used weak mortar.

During the Middle Ages, Gothic arches, distinguished by their pointed apex, were used to a large extent in the construction of the great European cathedrals. The horizontal thrust developed at the supports, or *springings*, and caused by the tendency of an arch to 'flatten' under load was frequently resisted by *flying buttresses.* This type of arch was also used extensively in the 19th century.

In the 18th century masonry arches were used to support bridges over the large number of canals that were built in that period. Many of these bridges survive to the present day and carry loads unimagined by their designers.

Today arches are usually made of steel or of reinforced or prestressed concrete and can support both tensile as well as compressive loads. They are used to support bridge decks and roofs and vary in span from a few metres in a roof support system to several hundred metres in bridges. A fine example of a steel arch bridge is the Sydney harbour bridge in which the deck is supported by hangers suspended from the arch (see Fig. 1.6(a) and (b) for examples of bridge decks supported by arches).

Arches are constructed in a variety of forms. Their components may be straight or curved, but generally fall into two categories. The first, which we shall consider in this chapter, is the three-pinned arch which is statically determinate, whereas the second, the two-pinned arch, is statically indeterminate and will be considered in Chapter 16.

Initially we shall examine the manner in which arches carry loads.

6.1 The linear arch

There is a direct relationship between the action of a flexible cable in carrying loads and the action of an arch. In Section 5.1 we determined the tensile forces in the segments of lightweight cables carrying concentrated loads and saw that the geometry of a cable changed under different loading systems; hence, for example, the two geometries of the same cable in Fig. 5.2(a) and (b).

Let us suppose that the cable in Fig. 5.2(a) is made up of three bars or links AC, CD and DB hinged together at C and D and pinned to the supports at A and B. If the loading remains unchanged the deflected shape of the three-link structure will be identical to that of the cable in Fig. 5.2(a) and is shown in Fig. 6.1(a). Furthermore the tension in a link will be exactly the same as the tension in the corresponding segment of the cable. Now suppose that the three-link structure of Fig. 6.1(a) is inverted as shown in Fig. 6.1(b) and that the loads W_1 and W_2 are applied as before. In this situation the forces in the links will be identical in magnitude

to those in Fig. 6.1(a) but will now be compressive as opposed to tensile; the structure shown in Fig. 6.1(b) is patently an arch.

The same argument can be applied to any cable and loading system so that the internal forces in an arch may be deduced by analysing a cable having exactly the same shape and carrying identical loads, a fact first realized by Robert Hooke in the 17th century. As in the example in Fig. 6.1 the internal forces in the arch will have the same magnitude as the corresponding cable forces but will be compressive, not tensile.

It is obvious from the above that the internal forces in the arch act along the axes of the different components and that the arch is therefore not subjected to internal shear forces and bending moments; an arch in which the internal forces are purely axial is called a *linear arch*. We also deduce, from Section 5.2, that the internal forces in an arch whose shape is that of a parabola and which carries a uniform horizontally distributed load are purely axial. Further, it will now have become clear why the internal members of a bowstring truss (Section 4.1) carrying loads of equal magnitude along its upper chord joints carry zero force.

However, there is a major difference between the behaviour of the two structures in Fig. 6.1(a) and (b). A change in the values of the loads W_1 and W_2 will merely result in a change in the geometry of the structure in Fig. 6.1(a), whereas the slightest changes in the values of W_1 and W_2 in Fig. 6.1(b) will result in the collapse of the arch as a mechanism. In this particular case collapse could be prevented by replacing the pinned joint at C (or D) by a rigid joint as shown in Fig. 6.2. The forces in the members remain unchanged since the geometry of the structure is unchanged, but the arch is now stable and has become a *three-pinned arch* which, as we shall see, is statically determinate.

If now the pinned joint at D was replaced by a rigid joint, the forces in the members would remain the same, but the arch has become a *two-pinned arch*. In this case, because of the tension cable equivalence, the arch is statically determinate. It is important to realize, however, that the above arguments only apply for the set of loads W_1 and W_2 which produce the particular shape of cable shown in Fig. 6.1(a). If the loads were repositioned or changed in magnitude, the two-pinned arch would become statically indeterminate and would probably cease to be a linear arch so that bending moments and shear forces would be induced.
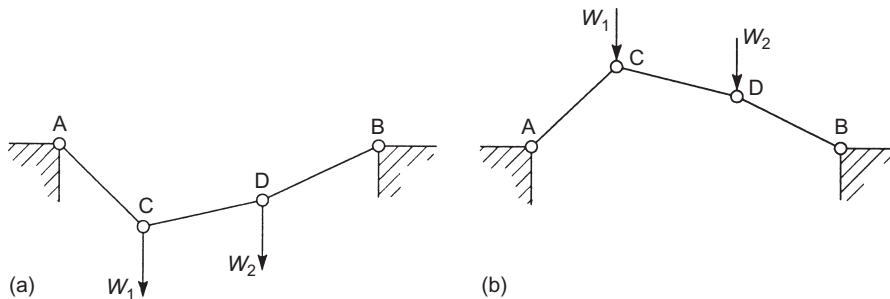

FIGURE 6.1
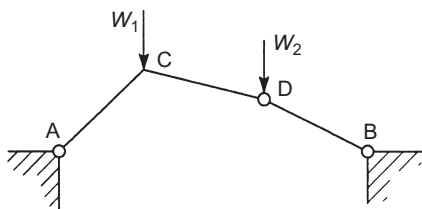
Equivalence of cable and arch structures.

FIGURE 6.2

Linear three-pinned arch.

The three-pinned arch of Fig. 6.2 would also become non-linear if the loads were repositioned or changed in magnitude.

In the above we have ignored the effect on the geometry of the arch caused by the shortening of the members. The effect of this on the three-pinned arch is negligible since the pins can accommodate the small changes in angle between the members which this causes. This is not the case in a two-pinned arch or in an arch with no pins at all (in effect a portal frame) so that bending moments and shear forces are induced. However, so long as the loads (W_1 and W_2 in this case) remain unchanged in magnitude and position, the corresponding stresses are 'secondary' and will have little effect on the axial forces.

The linear arch, in which the internal forces are purely axial, is important for the structural designer since the linear arch shape gives the smallest stresses. If, however, the thrust line is not axial, bending stresses are induced and these can cause tension on the inner or outer faces (the *intrados* and *extrados*) of the arch. In a masonry arch in which the joints are either dry or made using a weak mortar, this can lead to cracking and possible failure. Furthermore, if the thrust line lies outside the faces of the arch, instability leading to collapse can also occur. We shall deduce in Section 9.2 that for no tension to be developed in a rectangular cross section, the compressive force on the section must lie within the middle third of the section.

In small-span arch bridges, these factors are not of great importance since the greatest loads on the arch come from vehicular traffic. These loads vary with the size of the vehicle and its position on the bridge, so that it is generally impossible for the designer to achieve a linear arch. On the other hand, in large-span arch bridges, the self-weight of the arch forms the major portion of the load the arch has to carry. In Section 5.2 we saw that a cable under its own weight takes up the shape of a catenary. It follows that the ideal shape for an arch of constant thickness is an inverted catenary. However, in the analysis of the three-pinned arch we shall assume a general case in which shear forces and bending moments, as well as axial forces, are present.

6.2 **The three-pinned arch**

A three-pinned arch would be used in situations where there is a possibility of support displacement; this, in a two-pinned arch, would induce additional stresses. In the analysis of a three-pinned arch the first step, generally, is to determine the support reactions.
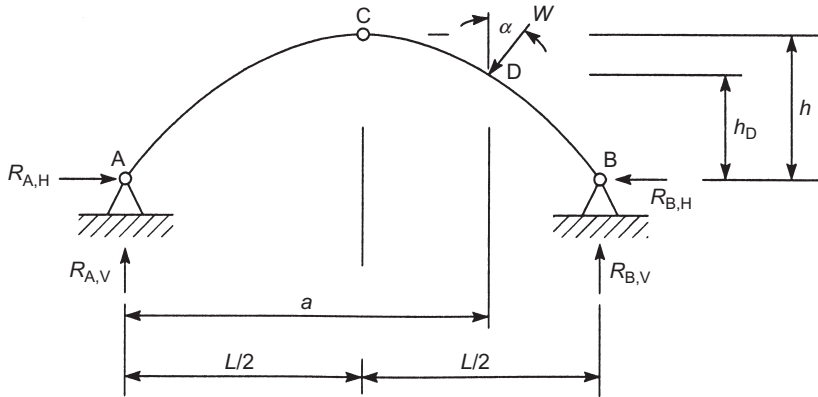
Support reactions – supports on same horizontal level

Consider the arch shown in Fig. 6.3. It carries an inclined concentrated load, W, at a given point D, a horizontal distance a from the support point A. The equation of the shape of the arch will generally be known so that the position of specified points on the arch, say D, can be obtained. We shall suppose that the third pin is positioned at the crown, C, of the arch, although this need not necessarily be the case; the height or *rise* of the arch is h.

The supports at A and B are pinned but neither can be a roller support or the arch would collapse. Therefore, in addition to the two vertical components of the reactions at A and B, there will be horizontal components $R_{A,H}$ and $R_{B,H}$. Thus, there are four unknown components of reaction but only three equations of overall equilibrium (Eq. (2.10)) so that an additional equation is required. This is obtained from the fact that the third pin at C is unable to transmit bending moments although, obviously, it is able to transmit shear forces.

Then, from the overall vertical equilibrium of the arch in Fig. 6.3, we have

$$R_{A,V} + R_{B,V} - W \cos \alpha = 0 \tag{6.1}$$

FIGURE 6.3

Three-pinned arch.

and from the horizontal equilibrium

$$R_{A,H} - R_{B,H} - W \sin \alpha = 0 \tag{6.2}$$

Now taking moments about, say, B,

$$R_{A,V}L - W \cos \alpha (L - a) - W \sin \alpha h_D = 0 \tag{6.3}$$

The internal moment at C is zero so that we can take moments about C of forces to the left or right of C. A slightly simpler expression results by considering forces to the left of C, i.e.

$$R_{A,V}\frac{L}{2} - R_{A,H}h = 0 \tag{6.4}$$

Equations (6.1)–(6.4) enable the four components of reaction to be found; the normal force, shear force and bending moment at any point in the arch follow.

EXAMPLE 6.1

Calculate the normal force, shear force and bending moment at the point X in the semicircular arch shown in Fig. 6.4.

In this example we can find either vertical component of reaction directly by taking moments about one of the support points. Hence, taking moments about B, say,

$$R_{A,V} \times 12 - 60 \left(6 \cos 30° + 6\right) - 100 \left(6 \sin 30° + 6\right) = 0$$
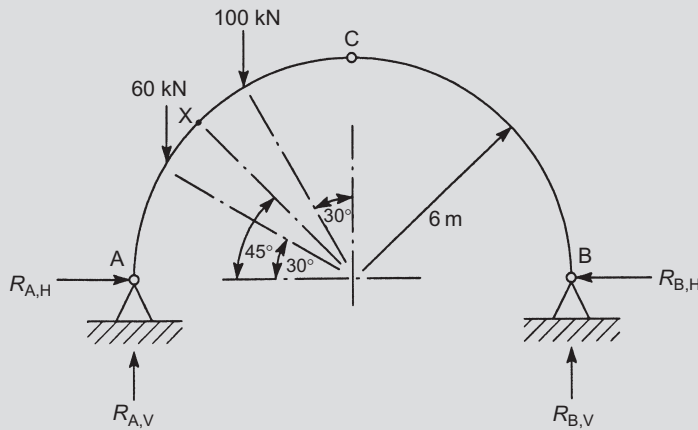
which gives

$$R_{A,V} = 131.0 \text{ kN}$$

Now resolving forces vertically

$$R_{B,V} + R_{A,V} - 60 - 100 = 0$$

which, on substituting for $R_{A,V}$, gives

$$R_{B,V} = 29.0 \text{ kN}$$

FIGURE 6.4

Three-pinned arch of Ex. 6.1.

Since no horizontal loads are present, we see by inspection that

$$R_{A,H} = R_{B,H}$$

Finally, taking moments of forces to the right of C about C (this is a little simpler than considering forces to the left of C) we have

$$R_{B,H} \times 6 - R_{B,V} \times 6 = 0$$

from which

$$R_{B,H} = 29.0 \text{ kN} = R_{A,H}$$

The normal force at the point X is obtained by resolving the forces to one side of X in a direction tangential to the arch at X. Thus, considering forces to the left of X and taking tensile forces as positive

$$N_X = -R_{A,V} \cos 45° - R_{A,H} \sin 45° + 60 \cos 45°$$

so that

$$N_X = -70.7 \text{ kN}$$

and is compressive.

The shear force at X is found by resolving the forces to one side of X in a direction perpendicular to the tangent at X. We shall take a positive shear force as acting radially inwards when it is to the left of a section. So, considering forces to the left of X

$$S_X = -R_{A,V} \sin 45° + R_{A,H} \cos 45° + 60 \sin 45°$$

which gives

$$S_X = -29.7 \text{ kN}$$

Now taking moments about X for forces to the left of X and regarding a positive moment as causing tension on the underside of the arch, we have

$$M_X = R_{A,V} (6 - 6 \cos 45°) - R_{A,H} \times 6 \sin 45° - 60 (6 \cos 30° - 6 \cos 45°)$$

Continued

from which

$$M_X = +50.0 \text{ kNm}$$

Note that in Ex. 6.1 the sign conventions adopted for normal force, shear force and bending moment are the same as those specified in Chapter 3.

Support reactions – supports on different levels

In the three-pinned arch shown in Fig. 6.5 the support at B is a known height, h_B, above A. Let us suppose that the equation of the shape of the arch is known so that all dimensions may be calculated. Now, resolving forces vertically gives

$$R_{A,V} + R_{B,V} - W \cos \alpha = 0 \tag{6.5}$$

and horizontally we have

$$R_{A,H} - R_{B,H} - W \sin \alpha = 0 \tag{6.6}$$

Also, taking moments about B, say,

$$R_{A,V}L - R_{A,H}h_B - W \cos \alpha (L - a) - W \sin \alpha (h_D - h_B) = 0 \tag{6.7}$$

Note that, unlike the previous case, the horizontal component of the reaction at A is included in the overall moment equation (Eq. (6.7)).

Finally we can take moments of all the forces to the left or right of C about C since the internal moment at C is zero. In this case the overall moment equation (Eq. (6.7)) includes both components, $R_{A,V}$ and $R_{A,H}$, of the support reaction at A. If we now consider moments about C of forces to the left of C, we shall obtain a moment equation in terms of $R_{A,V}$ and $R_{A,H}$. This equation, with Eq. (6.7), provides two simultaneous
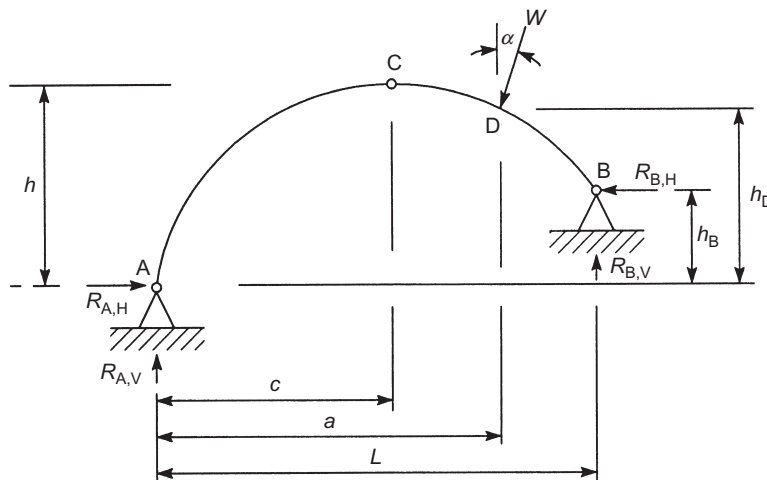

FIGURE 6.5

Three-pinned arch with supports at different levels.

equations which may be solved for $R_{A,V}$ and $R_{A,H}$. Alternatively if, when we were considering the overall moment equilibrium of the arch, we had taken moments about A, Eq. (6.7) would have been expressed in terms of $R_{B,V}$ and $R_{B,H}$. Then we would obtain the fourth equation by taking moments about C of the forces to the right of C and the two simultaneous equations would be in terms of $R_{B,V}$ and $R_{B,H}$. Theoretically this approach is not necessary but it leads to a simpler solution. Referring to Fig. 6.5

$$R_{A,V}c - R_{A,H}h = 0 \tag{6.8}$$

The solution of Eqs (6.7) and (6.8) gives $R_{A,V}$ and $R_{A,H}$, then $R_{B,V}$ and $R_{B,H}$ follow from Eqs (6.5) and (6.6), respectively.

EXAMPLE 6.2

The parabolic arch shown in Fig. 6.6 carries a uniform horizontally distributed load of intensity 10 kN/m over the portion AC of its span. Calculate the values of the normal force, shear force and bending moment at the point D.

Initially we must determine the equation of the arch so that the heights of B and D may be calculated. The simplest approach is to choose the origin of axes at C so that the equation of the parabola may be written in the form

$$y = kx^2 \tag{i}$$

in which k is a constant. At A, $y=7$ m when $x=-15$ m. Hence, from Eq. (i)

$$7 = k \times (-15)^2$$

whence

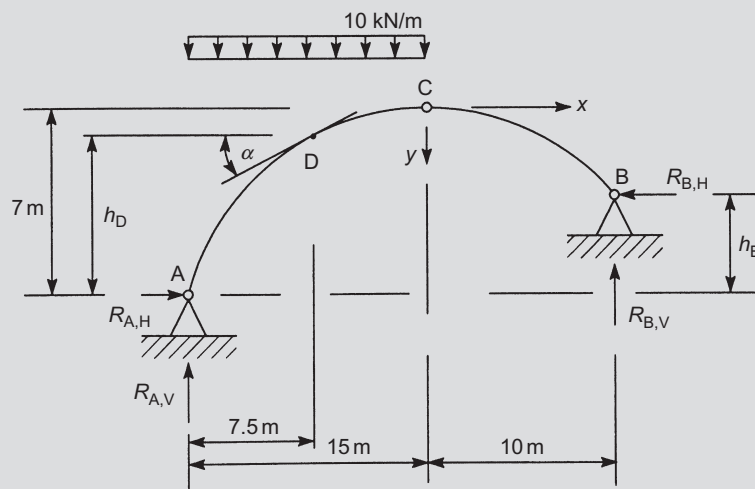$$k = 0.0311$$

and Eq. (i) becomes

$$y = 0.0311x^2 \tag{ii}$$

FIGURE 6.6

Parabolic arch of Ex. 6.2.

EXAMPLE 6.2 CONT'D

Then

$$y_B = 0.0311 \times (10)^2 = 3.11\,\text{m}$$

Hence

$$h_B = 7 - 3.11 = 3.89\,\text{m}$$

Also

$$y_D = 0.0311 \times (-7.5)^2 = 1.75\,\text{m}$$

so that

$$h_D = 7 - 1.75 = 5.25\,\text{m}$$

Taking moments about A for the overall equilibrium of the arch we have

$$R_{B,V} \times 25 + R_{B,H} \times 3.89 - 10 \times 15 \times 7.5 = 0$$

which simplifies to

$$R_{B,V} + 0.16R_{B,H} - 45.0 = 0 \qquad\qquad\qquad\text{(iii)}$$

Now taking moments about C for the forces to the right of C we obtain

$$R_{B,V} \times 10 - R_{B,H} \times 3.11 = 0$$

which gives

$$R_{B,V} - 0.311R_{B,H} = 0 \qquad\qquad\qquad\text{(iv)}$$

The simultaneous solution of Eqs (iii) and (iv) gives

$$R_{B,V} = 29.7\ \text{kN} \quad R_{B,H} = 95.5\ \text{kN}$$

From the horizontal equilibrium of the arch we have

$$R_{A,H} = R_{B,H} = 95.5\ \text{kN}$$

and from the vertical equilibrium

$$R_{A,V} + R_{B,V} - 10 \times 15 = 0$$

which gives

$$R_{A,V} = 120.3\ \text{kN}$$

To calculate the normal force and shear force at the point D we require the slope of the arch at D. From Eq. (ii)

$$\left(\frac{dy}{dx}\right)_D = 2 \times 0.0311 \times (-7.5) = -0.4665 = -\tan\alpha$$

Hence

$$\alpha = 25.0°$$

Now resolving forces to the left (or right) of D in a direction parallel to the tangent at D we obtain the normal force at D. Hence

$$N_D = -R_{A,V} \sin 25.0° - R_{A,H} \cos 25.0° + 10 \times 7.5 \sin 25.0°$$

which gives

$$N_D = -105.7 \text{ kN (compression)}$$

The shear force at D is then

$$S_D = -R_{A,V} \cos 25.0° + R_{A,H} \sin 25.0° + 10 \times 7.5 \cos 25.0°$$

so that

$$S_D = -0.7 \text{ kN}$$

Finally the bending moment at D is

$$M_D = R_{A,V} \times 7.5 - R_{A,H} \times 5.25 - 10 \times 7.5 \times \frac{7.5}{2}$$

from which

$$M_D = +119.6 \text{ kNm}$$

6.3 A three-pinned parabolic arch carrying a uniform horizontally distributed load

In Section 5.2 we saw that a flexible cable carrying a uniform horizontally distributed load took up the shape of a parabola. It follows that a three-pinned parabolic arch carrying the same loading would experience zero shear force and bending moment at all sections. We shall now investigate the bending moment in the symmetrical three-pinned parabolic arch shown in Fig. 6.7.
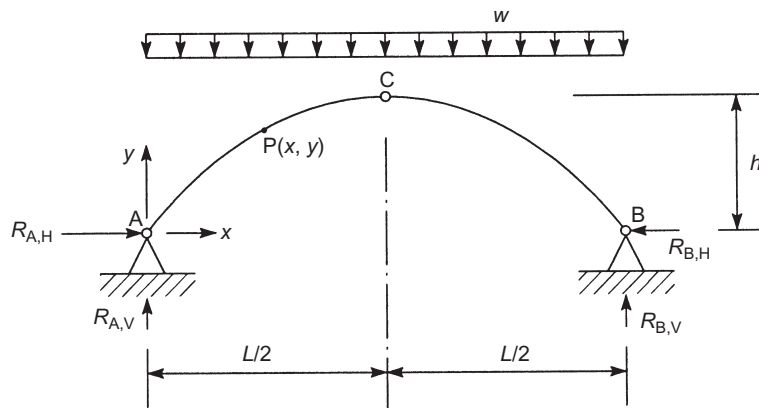

FIGURE 6.7

Parabolic arch carrying a uniform horizontally distributed load.

The vertical components of the support reactions are, from symmetry,

$$R_{A,V} = R_{B,V} = \frac{wL}{2}$$

Also, in the absence of any horizontal loads

$$R_{A,H} = R_{B,H}$$

Now taking moments of forces to the left of C about C,

$$R_{A,H}h - R_{A,V}\frac{L}{2} + \frac{wL}{2}\frac{L}{4} = 0$$

which gives

$$R_{A,H} = \frac{wL^2}{8h}$$

With the origin of axes at A, the equation of the parabolic shape of the arch may be shown to be

$$y = \frac{4h}{L^2}(Lx - x^2)$$

The bending moment at any point P(x,y) in the arch is given by

$$M_P = R_{A,V}x - R_{A,H}y - \frac{wx^2}{2}$$

or, substituting for $R_{A,V}$ and $R_{A,H}$ and for y in terms of x,

$$M_P = \frac{wL}{2}x - \frac{wL^2}{8h}\frac{4h}{L^2}(Lx - x^2) - \frac{wx^2}{2}$$

Simplifying this expression

$$M_P = \frac{wL}{2}x - \frac{wL}{2}x + \frac{wx^2}{2} - \frac{wx^2}{2} = 0$$

as expected.

The shear force may also be shown to be zero at all sections of the arch.

6.4 Bending moment diagram for a three-pinned arch carrying concentrated loads and having supports on the same horizontal level

For the particular case of a three-pinned arch having supports on the same horizontal level and carrying concentrated loads only, a semi-graphical approach may be used to construct the complete bending moment diagram.

Consider the arch shown in Fig. 6.8; we shall suppose that the equation of the arch referred to the xy axes is known. The load W is applied at a given point D(x_D, y_D) and the support reactions may be calculated by the methods previously described. The bending moment, M_{P1}, at any point P$_1(x,y)$ between A and D is given by

$$M_{P1} = R_{A,V}x - R_{A,H}y \tag{6.9}$$

and the bending moment, M_{P2}, at the point P$_2$, (x,y) between D and B is

$$M_{P2} = R_{A,V}x - W(x - x_D) - R_{A,H}y \tag{6.10}$$

Now let us consider a simply supported beam AB having the same span as the arch and carrying a load, W, at the same horizontal distance, x_D, from the left-hand support (Fig. 6.9(a)). The vertical reactions, R_A and R_B will have the same magnitude as the vertical components of the support reactions in the arch. Note that this would not be the case if the arch supports were on different levels since the vertical components of the arch support reactions would then include contributions from the horizontal components of the support reactions, which do not, of course, exist in the case of a simply supported beam carrying vertical loads. Thus the bending moment at any point between A and D and a distance x from A is

$$M_{AD} = R_A x = R_{A,V} x \tag{6.11}$$

Also the bending moment at any point between D and B a distance x from A is

$$M_{DB} = R_A x - W(x - x_D) = R_{A,V} x - W(x - x_D) \tag{6.12}$$

giving the bending moment diagram shown in Fig. 6.9(b). Comparing Eqs (6.11) and (6.12) with Eqs (6.9) and (6.10), respectively, we see that Eq. (6.9) may be written

FIGURE 6.8

Determination of the bending moment diagram for a three-pinned arch.
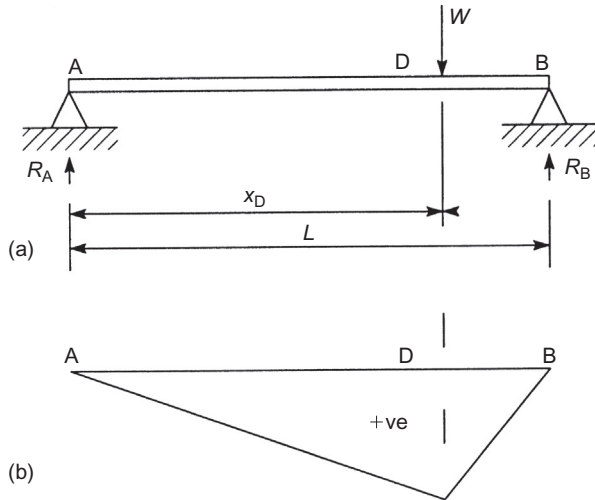

(a)

(b)

FIGURE 6.9

Bending moment diagram for a simply supported beam (tension on undersurface of beam).

$$M_{P1} = M_{AD} - R_{A,H}y \qquad (6.13)$$

and Eq. (6.10) may be written

$$M_{P2} = M_{DB} - R_{A,H}y \qquad (6.14)$$

Therefore, the complete bending moment diagram for the arch may be regarded as the sum of a 'simply supported beam' bending moment diagram and an 'arch' bending moment diagram in which the 'arch' diagram has the same shape as the arch itself, since its ordinates are equal to a constant multiplied by y. The two bending moment diagrams may be superimposed as shown in Fig. 6.10 to give the complete bending moment diagram for the arch. Note that the curve of the arch forms the baseline of the bending moment diagram and that the bending moment at the crown of the arch where the third pin is located is zero.

In the above it was assumed that the mathematical equation of the curve of the arch is known. However, in a situation where, say, only a scale drawing of the curve of the arch is available, a semigraphical procedure may be adopted if the loads are vertical. The 'arch' bending moment at the crown C of the arch is $R_{AH}h$ as shown in Fig. 6.10. The magnitude of this bending moment may be calculated so that the scale of the bending moment diagram is then fixed by the rise (at C) of the arch in the scale drawing. Also this bending moment is equal in magnitude but opposite in sign to the 'simply supported beam' bending moment at this point. Other values of 'simply supported beam' bending moment may be calculated at, say, load positions and plotted on the complete bending moment diagram to the already determined scale. The diagram is then completed, enabling values of bending moment to be scaled off as required.

In the arch of Fig. 6.8 a simple construction may be used to produce the complete bending moment diagram. In this case the arch shape is drawn as in Fig. 6.10 and this, as we have seen, fixes the scale of the bending moment diagram. Then, since the final bending moment at C is zero and is also zero at
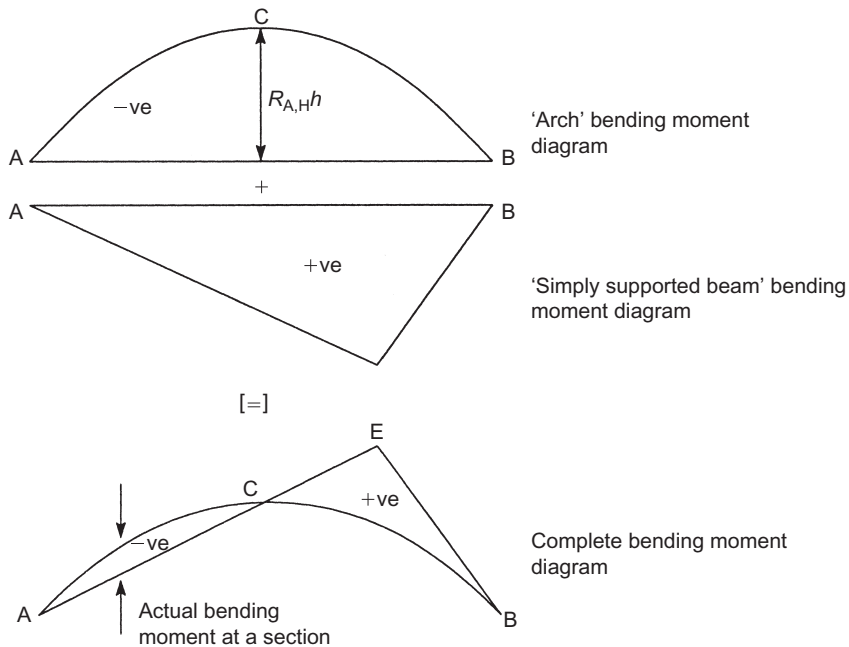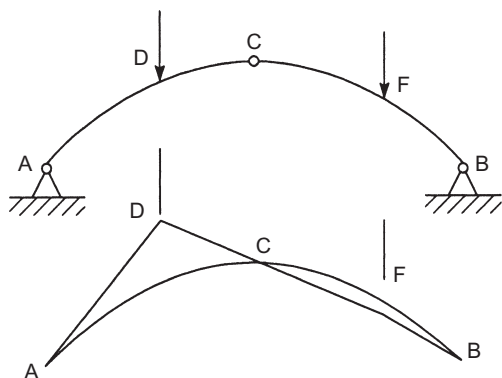

FIGURE 6.10

Complete bending moment diagram for a three-pinned arch.

FIGURE 6.11

Bending moment diagram for a three-pinned arch carrying two loads.

A and B, a line drawn from A through C to meet the vertical through the point of application of the load at E represents the 'simply supported beam' bending moment diagram between A and D. The bending moment diagram is then completed by drawing the line EB.

This construction is only possible when the arch carries a single load. In the case of an arch carrying two or more loads as in Fig. 6.11, the 'simply supported beam' bending moments must be calculated at D and F and their values plotted to the same scale as the 'arch' bending moment diagram. Clearly the bending moment at C remains zero.

We shall consider the statically indeterminate two-pinned arch in Chapter 16.

EXAMPLE 6.3

Construct the bending moment diagram for the semi-circular three-pinned arch shown in Fig. 6.12.
   The equation of the arch referred to the xy axes system shown in Fig. 6.12 is

$$x^2 + y^2 = 8^2$$

Therefore, when $x = -5$m, $y = 6.2$ m and when $x = +4$m, $y = 6.9$ m.
   Taking moments about B

$$R_{A,V} \times 16 - 5 \times 13 - 10 \times 4 = 0,$$

which gives $R_{A,V} = 6.6$ kN.
   Taking moments about C

$$R_{A,H} \times 8 - 6.6 \times 8 + 5 \times 5 = 0$$

from which $R_{A,H} = 3.5$ kN
   Therefore, the simply supported beam bending moment at C is given by

$$M_C(\text{ss beam}) = R_{A,V} \times 8 - 5 \times 5$$

i.e.

$$M_C(\text{ss beam}) = 6.6 \times 8 - 25 = 27.8 \,\text{kNm}$$

The "simply supported beam" bending moment at C is equal and opposite to the "arch" bending moment at C since the final bending moment at C must be zero. Therefore, we draw a semicircle whose radius is 27.8 kNm to some convenient scale as in Fig. 6.13. To complete the bending moment diagram, we now calculate the simply supported beam bending moment at D (or E).

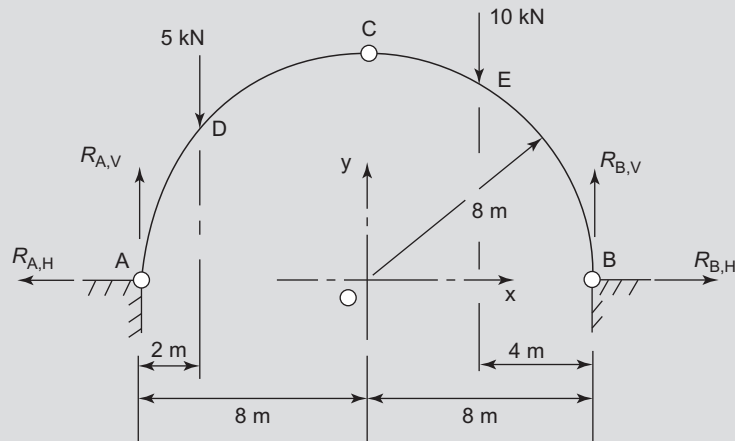*Continued*

EXAMPLE 6.3   CONT'D

FIGURE 6.12

Semi-circular arch of Ex. 63.

$$M_D(\text{ss beam}) = 6.6 \times 3 = 19.8\,\text{kNm}$$

We now measure a vertical from the diameter of the semi-circle of 19.8 kNm to the same scale as before. This locates the point D in the bending moment diagram. Since the "arch" and "simply supported beam" bending moment diagrams must intersect at C and since D has been located, we can now complete the bending moment diagram as shown in Fig. 6.13. The bending moment at any point in the arch may now be scaled off where the semi-circle is the baseline.

FIGURE 6.13

Bending moment diagram for the arch of Ex. 6.3.

PROBLEMS

P.6.1 Determine the value of the bending moment in the loaded half of the semicircular three-pinned arch shown in Fig. P.6.1 at a horizontal distance of 5 m from the left-hand support.

Ans. 67.0 kNm (sagging).

20 kN/m

C

10 m

A

B

FIGURE P.6.1

P.6.2 Figure P.6.2 shows a three-pinned arch of radius 12 m. Calculate the normal force, shear force and bending moment at the point D.
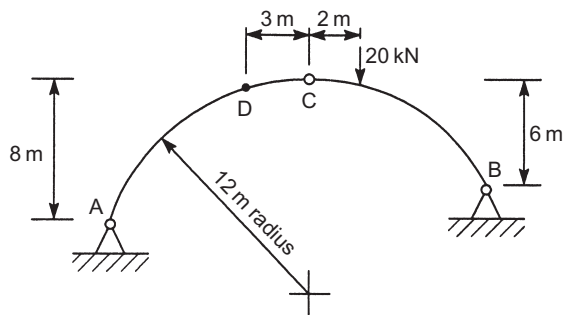
Ans. 14.4 kN (compression), 5.5 kN, 21.6 kNm (hogging).

3 m   2 m

20 kN

D   C

6 m

8 m

12 m radius

A

B

FIGURE P.6.2

P.6.3 The three-pinned arch shown in Fig. P.6.3 is parabolic in shape. If the arch carries a uniform horizontally distributed load of intensity 40 kN/m over the part CB, calculate the bending moment at D.

Ans. 140.9 kNm (sagging).

4 m

C

D

3 m

A

B

10 m   7 m

FIGURE P.6.3

P.6.4 In the three-pinned arch ACB shown in Fig. P.6.4 the portion AC has the shape of a parabola with its origin at C, while CB is straight. The portion AC carries a uniform horizontally distributed load of intensity 30 kN/m, while the portion CB carries a uniform horizontally distributed load of intensity 18 kN/m. Calculate the normal force, shear force and bending moment at the point D.
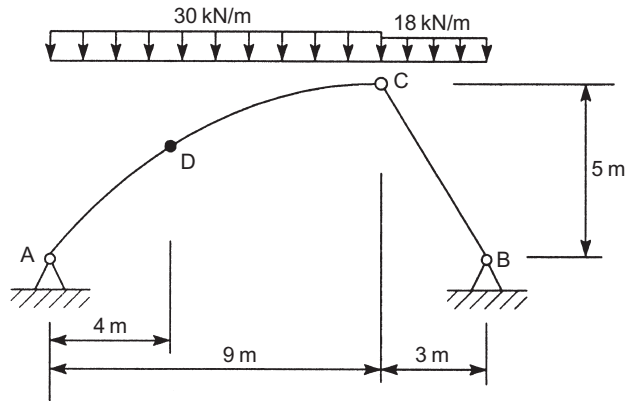
Ans. 91.2 kN (compression), 9.0 kN, 209.8 kNm (sagging).

FIGURE P.6.4

P.6.5 Draw normal force, shear force and bending moment diagrams for the loaded half of the three-pinned arch shown in Fig. P.6.5.

Ans. $N_{BD}=26.5$ kN, $N_{DE}=19.4$ kN, $N_{EF}=N_{FC}=15$ kN (all compression).
$S_{BD}=5.3$ kN, $S_{DE}=-1.8$ kN, $S_{EF}=2.5$ kN, $S_{FC}=-7.5$ kN.
$M_D=11.3$ kNm, $M_E=7.5$ kNm, $M_F=11.3$ kNm (sagging).
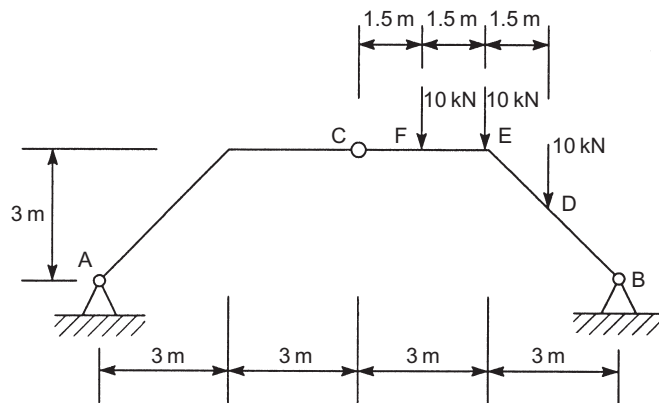

FIGURE P.6.5

P.6.6 Calculate the components of the support reactions at A and D in the three-pinned arch shown in Fig. P.6.6 and hence draw the bending moment diagram for the member DC; draw the diagram on the tension side of the member. All members are 1.5 m long.

Ans. $R_{A,V}=6.46$ kN, $R_{A,H}=11.13$ kN, $R_{D,V}=21.46$ kN, $R_{D,H}=3.87$ kN.
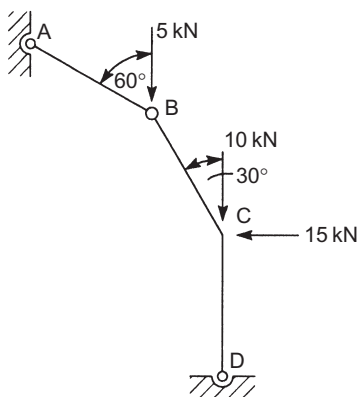$M_D=0$, $M_C=5.81$ kNm (tension on left of CD).

FIGURE P.6.6

P.6.7 Calculate the reactions at the supports A and D in the three-pinned arch shown in Fig. P.6.7 and obtain the bending moment diagram for the members AB and BC. In each case draw the diagram on the tension side of the member.

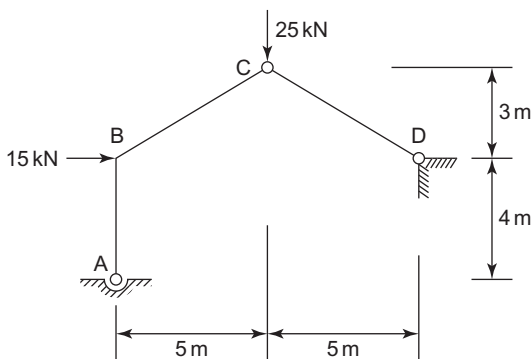*Ans.* $M_A = M_C = 0$, $M_B = 14.1$ kNm.

FIGURE P.6.7

P.6.8 The three-pinned arch shown in Fig. P.6.8 carries loads of 60 kN and 120 kN at the points E and D in addition to a distributed load due to its self-weight of 10 kN/m of its true length. Calculate the bending moments at the points E and D and also at the mid-point of AC.

Ans. $M_D = 186.3$ kNm, $M_E = 150.0$ kNm, $M_{mid\ AC} = 3.95$ kNm.

P.6.9 Construct the bending moment diagram for the three-pinned parabolic arch shown in Fig. P.6.9 and hence determine the bending moment in the arch 1 m from the left-hand support.
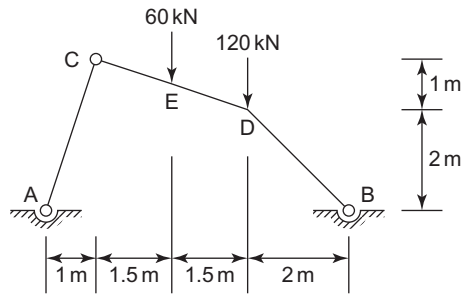
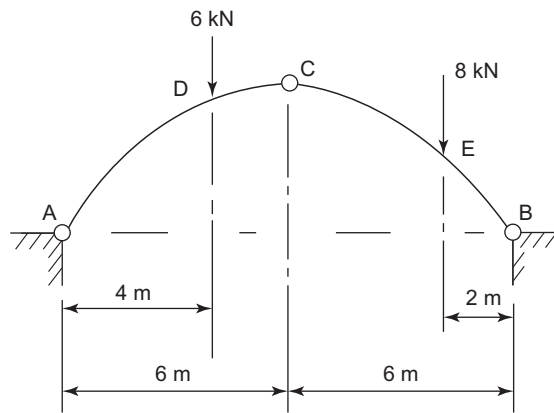*Ans.* -6.0 kNm (hogging).

FIGURE P.6.8

FIGURE P.6.9