CHAPTER 4

Analysis of Pin-Jointed Trusses

In Chapter 1 we discussed various structural forms and saw that for moderately large spans, simple beams become uneconomical and may be replaced by trusses. These structures comprise members connected at their ends and are constructed in a variety of arrangements. In general, trusses are lighter, stronger and stiffer than solid beams of the same span; they do, however, take up more room and are more expensive to fabricate.

Initially in this chapter we shall discuss types of truss, their function and the idealization of a truss into a form amenable to analysis. Subsequently, we shall investigate the criterion which indicates the degree of their statical determinacy, examine the action of the members of a truss in supporting loads and, finally, examine methods of analysis of both plane and space trusses.

4.1 Types of truss

Generally the form selected for a truss depends upon the purpose for which it is required. Examples of different types of truss are shown in Fig. 4.1(a)–(f); some are named after the railway engineers who invented them.

For example, the Pratt, Howe, Warren and K trusses would be used to support bridge decks and large-span roofing systems (the Howe truss is no longer used for reasons we shall discuss in Section 4.5) whereas the Fink truss would be used to support gable-ended roofs. The Bowstring truss is somewhat of a special case in that if the upper chord members are arranged such that the joints lie on a parabola and the loads, all of equal magnitude, are applied at the upper joints, the internal members carry no load. This result derives from arch theory (Chapter 6) but is rarely of practical significance since, generally, the loads would be applied to the lower chord joints as in the case of the truss being used to support a bridge deck.

Frequently, plane trusses are connected together to form a three-dimensional structure. For example, in the overhead crane shown in Fig. 4.2, the tower would usually comprise four plane trusses joined together to form a 'box' while the jibs would be constructed by connecting three plane trusses together to form a triangular cross section.

4.2 Assumptions in truss analysis

It can be seen from Fig. 4.1 that plane trusses consist of a series of triangular units. The triangle, even when its members are connected together by hinges or pins as in Fig. 4.3(a), is an inherently stable structure, i.e. it will not collapse under any arrangement of loads applied in its own plane. On the other hand, the rectangular structure shown in Fig. 4.3(b) would be unstable if vertical loads were applied at the joints and would collapse under the loading system shown; in other words it is a mechanism.

Further properties of a pin-jointed triangular structure are that the forces in the members are purely axial and that it is statically determinate (see Section 4.4) so long as the structure is loaded and supported at the joints. The forces in the members can then be found using the equations of statical equilibrium (Eq. (2.10)). It follows

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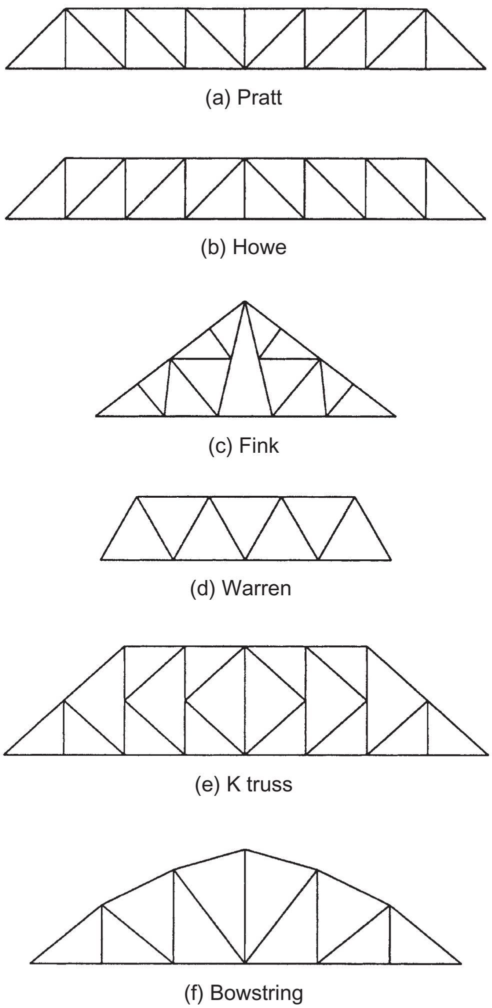

FIGURE 4.1

Types of plane truss.

that a truss comprising pin-jointed triangular units is also statically determinate if the above loading and support conditions are satisfied. In Section 4.4 we shall derive a simple test for determining whether or not a pin-jointed truss is statically determinate; this test, although applicable in most cases is not, as we shall see, foolproof.

The assumptions on which the analysis of trusses is based are as follows:

1. The members of the truss are connected at their ends by frictionless pins or hinges.
2. The truss is loaded and supported only at its joints.
3. The forces in the members of the truss are purely axial.

Assumptions (2) and (3) are interdependent since the application of a load at some point along a truss member would, in effect, convert the member into a simply supported beam and, as we have seen in Chapter 3, generate, in addition to axial loads, shear forces and bending moments; the truss would then become statically indeterminate.

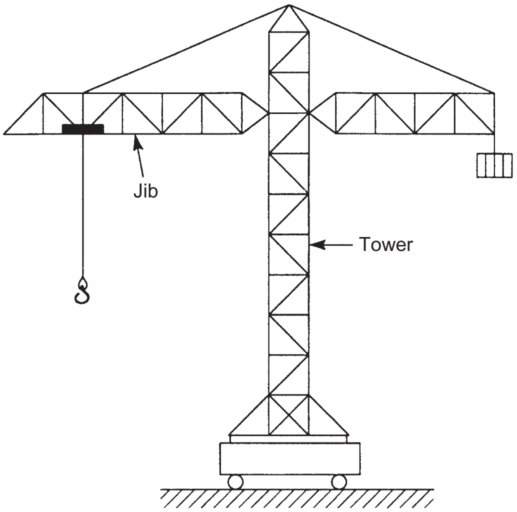

FIGURE 4.2

Overhead crane structure.

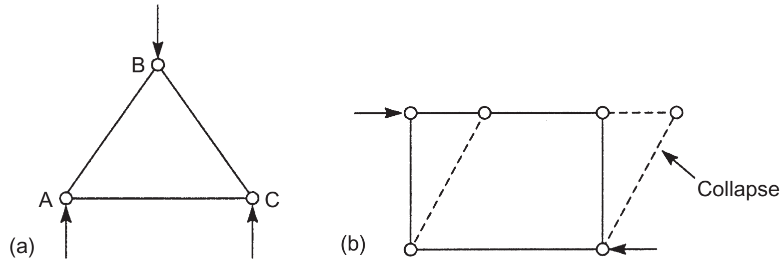

FIGURE 4.3

Basic unit of a truss.

4.3 Idealization of a truss

In practice trusses are not pin-jointed but are constructed, in the case of steel trusses, by bolting, riveting or welding the ends of the members to gusset plates as shown in Fig. 4.4. In a timber roof truss the members are connected using spiked plates driven into their vertical surfaces on each side of a joint. The joints in trusses are therefore semi-rigid and can transmit moments, unlike a frictionless pinned joint. Furthermore, if the loads are applied at points on a member away from its ends, that member behaves as a fixed or built-in beam with unknown moments and shear forces as well as axial loads at its ends. Such a truss would possess a high degree of statical indeterminacy and would require a computer-based analysis.

However, if such a truss is built up using the basic triangular unit and the loads and support points coincide with the member joints then, even assuming rigid joints, a computer-based analysis would show that the shear forces and bending moments in the members are extremely small compared to the axial forces which, themselves, would be very close in magnitude to those obtained from an analysis based on the assumption of pinned joints.

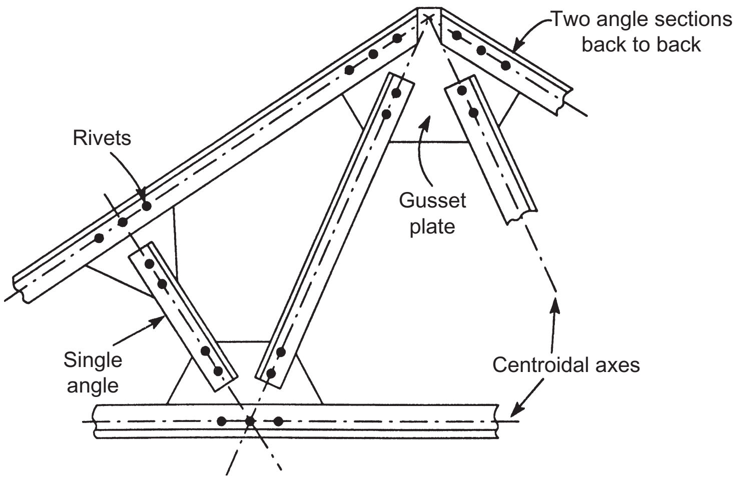

FIGURE 4.4

Actual truss construction.

A further condition in employing a pin-jointed idealization of an actual truss is that the centroidal axes of the members in the actual truss are concurrent, as shown in Fig. 4.4. We shall see in Section 9.2 that a load parallel to, but offset from, the centroidal axis of a member induces a bending moment in the cross-section of the member; this situation is minimized in an actual truss if the centroidal axes of all members meeting at a joint are concurrent.

4.4 Statical determinacy

It was stated in Section 4.2 that the basic triangular pin-jointed unit is statically determinate and the forces in the members are purely axial so long as the loads and support points coincide with the joints. The justification for this is as follows. Consider the joint B in the triangle in Fig. 4.3(a). The forces acting on the actual pin or hinge are the externally applied load and the axial forces in the members AB and BC; the system is shown in the free body diagram in Fig. 4.5. The internal axial forces in the members BA and BC, F_{BA} and F_{BC}, are drawn to show them pulling away from the joint B; this indicates that the members are in tension. Actually, we can see by inspection that both members will be in compression since their combined vertical components are required to equilibrate the applied vertical load. The assumption of tension, however, would only result in negative values in the calculation of F_{BA} and F_{BC} and is therefore a valid approach. In fact we shall adopt the method of initially assuming tension in all members of a truss when we consider methods of analysis, since a negative value for a member force will then always signify compression and will be in agreement with the sign convention adopted in Section 3.2.

Since the pin or hinge at the joint B is in equilibrium and the forces acting on the pin are coplanar, Eq. (2.10) apply. Therefore the sum of the components of all the forces acting on the pin in any two directions at right angles must be zero. The moment equation, $\sum M = 0$, is automatically satisfied since the pin cannot transmit a moment and the lines of action of all the forces acting on the pin must therefore be concurrent. For the joint B, we can write down two equations of force equilibrium which are sufficient to solve for the unknown member forces F_{BA} and F_{BC}. The same argument may then be applied to either joint A or C to solve for the remaining unknown internal force F_{AC} ($= F_{CA}$). We see then that the basic triangular unit is statically determinate.

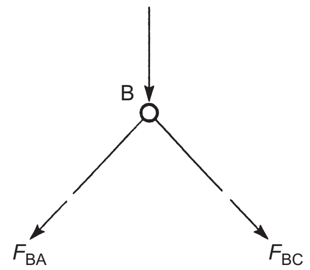

FIGURE 4.5

Joint equilibrium in a triangular structure.

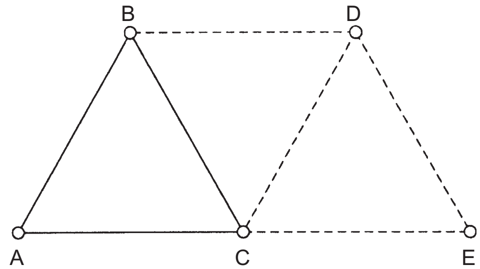

FIGURE 4.6

Construction of a Warren truss.

Now consider the construction of a simple pin-jointed truss. Initially we start with a single triangular unit ABC as shown in Fig. 4.6. A further triangle BCD is created by adding the *two* members BD and CD and the *single* joint D. The third triangle CDE is then formed by the addition of the *two* members CE and DE and the *single* joint E and so on for as many triangular units as required. Thus, after the initial triangle is formed, each additional triangle requires *two* members and a *single* joint. In other words the number of additional members is equal to twice the number of additional joints. This relationship may be expressed quantitatively as follows.

Suppose that m is the total number of members in a truss and j the total number of joints. Then, noting that initially there are three members and three joints, the above relationship may be written

$$m - 3 = 2(j - 3)$$

so that

$$m = 2j - 3 \tag{4.1}$$

If Eq. (4.1) is satisfied, the truss is constructed from a series of statically determinate triangles and the truss itself is statically determinate. Furthermore, if $m < 2j - 3$ the structure is unstable (see Fig. 4.3(b)) or if $m > 2j - 3$, the structure is statically indeterminate. Note that Eq. (4.1) applies only to the internal forces in a truss; the support system must also be statically determinate to enable the analysis to be carried out using simple statics.

EXAMPLE 4.1

Test the statical determinacy of the pin-jointed trusses shown in Fig. 4.7.

In Fig. 4.7(a) the truss has five members and four joints so that $m = 5$ and $j = 4$. Then

$$2j - 3 = 5 = m$$

and Eq. (4.1) is satisfied. The truss in Fig. 4.7(b) has an additional member so that $m = 6$ and $j = 4$. Therefore

$$m > 2j - 3$$

and the truss is statically indeterminate.

The truss in Fig. 4.7(c) comprises a series of triangular units which suggests that it is statically determinate. However, in this case, $m = 8$ and $j = 5$

$$2j - 3 = 7$$

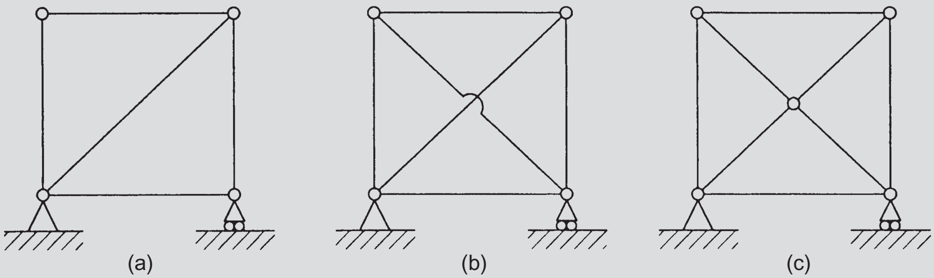

FIGURE 4.7

Statical determinacy of trusses.

so that

$$m > 2j - 3$$

and the truss is statically indeterminate. In fact any single member may be removed and the truss would retain its stability under any loading system in its own plane.

Unfortunately, in some cases, Eq. (4.1) is satisfied but the truss may be statically indeterminate or a mechanism. For example, the truss in Fig. 4.8 has nine members and six joints so that Eq. (4.1) is satisfied. However, clearly the left-hand half is a mechanism and the right-hand half is statically indeterminate. Theoretically, assuming that the truss members are weightless, the truss could support vertical loads applied to the left- and/or right-hand vertical members; this would, of course, be an unstable condition. Any other form of loading would cause a collapse of the left hand half of the truss and consequently of the truss itself.

The presence of a rectangular region in a truss such as that in the truss in Fig. 4.8 does not necessarily result in collapse. The truss in Fig. 4.9 has nine members and six joints so that Eq. (4.1) is satisfied. This does not, as we have seen, guarantee either a stable or statically determinate truss. If, therefore, there is some doubt we can return to the procedure of building up a truss from a single triangular unit as demonstrated in Fig. 4.6. Then, remembering that each additional triangle is created by adding two members and one joint and that the resulting truss is stable and statically determinate, we can examine the truss in Fig. 4.9 as follows.

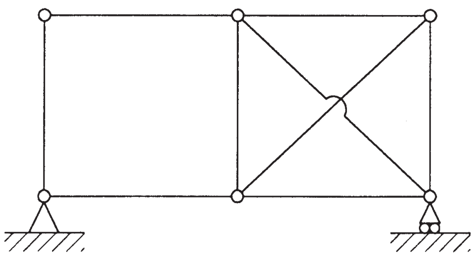

FIGURE 4.8

Applicability of test for statical determinacy.

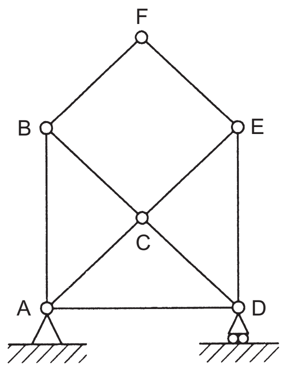

FIGURE 4.9

Investigation into truss stability.

Suppose that ACD is the initial triangle. The additional triangle ACB is formed by adding the two members AB and BC and the single joint B. The triangle DCE follows by adding the two members CE and DE and the joint E. Finally, the two members BF and EF and the joint F are added to form the rectangular portion CBFE. We therefore conclude that the truss in Fig. 4.9 is stable and statically determinate. Compare the construction of this truss with that of the statically indeterminate truss in Fig. 4.7(c).

A condition, similar to Eq. (4.1), applies to space trusses; the result for a space truss having m members and j pinned joints is

$$m = 3j - 6 \tag{4.2}$$

EXAMPLE 4.2

Investigate the determinacy and stability of the trusses shown in Fig. 4.10 under loads applied in their plane.

(a) In this case there are 7 members and 5 joints so that $m = 7$ and $j = 5$. Then

$$2j - 3 = 7$$

and Eq. (4.1) is satisfied and the truss is statically determinate. Also, by inspection, the truss is stable.

(b) For this truss $m = 9$ and $j = 6$ so that $2j - 3 = 9$ and Eq. (4.1) is satisfied but, by inspection, the outer half of the truss is statically indeterminate while the inner half is a mechanism.

(c) In this case $m = 13$ and $j = 9$ so that $2j - 3 = 15$ and $m < 2j - 3$, the truss is therefore a mechanism.

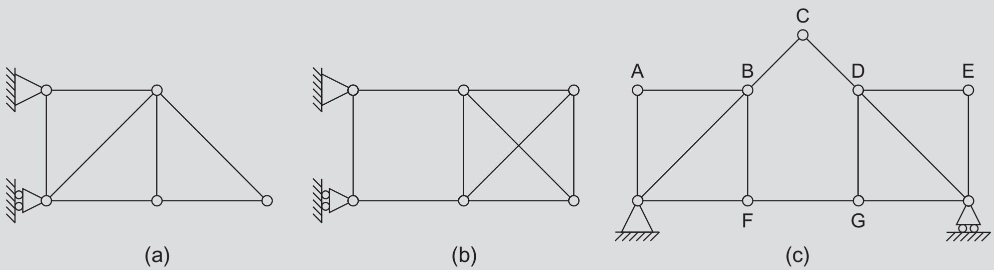

FIGURE 4.10

EXAMPLE 4.3

Suggest two ways in which the truss in Fig. 4.10(c) could be made stable and remain statically determinate.

i. Add members BD and FD (or BG).

ii. Add members CF and CG.

4.5 Resistance of a truss to shear force and bending moment

Although the members of a truss carry only axial loads, the truss itself acts as a beam and is subjected to shear forces and bending moments. Therefore, before we consider methods of analysis of trusses, it will be instructive to examine the manner in which a truss resists shear forces and bending moments.

The Pratt truss shown in Fig. 4.11(a) carries a concentrated load W applied at a joint on the bottom chord at mid-span. Using the methods described in Section 3.4, the shear force and bending moment diagrams for the truss are constructed as shown in Fig. 4.11(b) and (c), respectively.

First we shall consider the shear force. In the bay ABCD the shear force is $W/2$ and is negative. Thus at any section mm between A and B (Fig. 4.12) we see that the internal shear force is $-W/2$. Since the horizontal members AB and DC are unable to resist shear forces, the internal shear force can only be equilibrated by the vertical component of the force F_{AC} in the member AC. Figure 4.12 shows the direction of the internal shear force applied at the section mm so that F_{AC} is tensile. Then

$$F_{AC} \cos 45° = \frac{W}{2}$$

The same result applies to all the internal diagonals whether to the right or left of the mid-span point since the shear force is constant, although reversed in sign, either side of the load. The two outer diagonals are

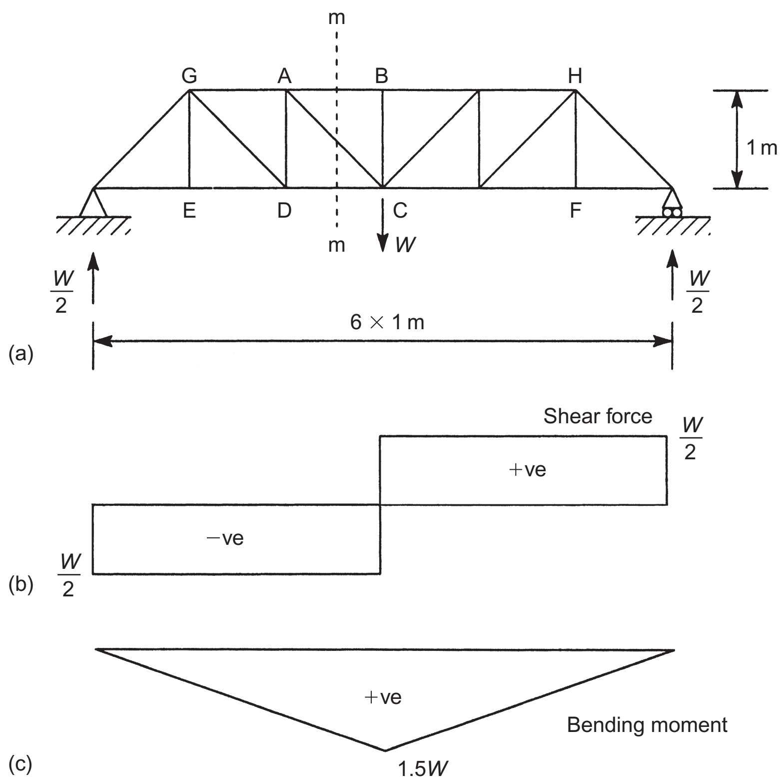

FIGURE 4.11

Shear forces and bending moments in a truss.

in compression since their vertical components must be in equilibrium with the vertically upward support reactions. Alternatively, we arrive at the same result by considering the internal shear force at a section just to the right of the left-hand support and just to the left of the right-hand support.

If the diagonal AC was repositioned to span between D and B it would be subjected to an axial compressive load. This situation would be undesirable since the longer a compression member, the smaller the load required to cause buckling (see Chapter 21). Therefore, the aim of truss design is to ensure that the forces in the longest members, the diagonals in this case, are predominantly tensile. So we can see now why the Howe truss (Fig. 4.1(b)), whose diagonals for downward loads would be in compression, is no longer in use.

In some situations the loading on a truss could be reversed so that a diagonal that is usually in tension would be in compression. To counter this an extra diagonal inclined in the opposite direction is included (spanning, say, from D to B in Fig. 4.13). This, as we have seen, would result in the truss becoming statically indeterminate. However, if it is assumed that the original diagonal (AC in Fig. 4.13) has buckled under the compressive load and therefore carries no load, the truss is once again statically determinate.

We shall now consider the manner in which a truss resists bending moments. The bending moment at a section immediately to the left of the mid-span vertical BC in the truss in Fig. 4.11(a) is, from Fig. 4.11(c), 1.5 W and is positive, as shown in Fig. 4.13. This bending moment is equivalent to the moment resultant, about any point in their plane, of the member forces at this section. In Fig. 4.13, analysis by the method of sections (Section 4.7) gives $F_{BA} = 1.5\ W$ (compression), $F_{AC} = 0.707\ W$ (tension) and $F_{DC} = 1.0\ W$ (tension). Therefore at C, F_{DC} plus the horizontal component of F_{AC} is equal to 1.5 W which, together with F_{BA}, produces a couple of magnitude $1.5\ W \times 1$ which is equal to the applied bending moment. Alternatively, we could take moments of the internal forces about B (or C). Hence

$$M_B = F_{DC} \times 1 + F_{AC} \times 1 \sin 45° = 1.0\ W \times 1 + 0.707\ W \times 1 \sin 45° = 1.5\ W$$

as before. Note that in Fig. 4.13 the moment resultant of the internal force system is *equivalent* to the applied moment, i.e. it is in the same sense as the applied moment.

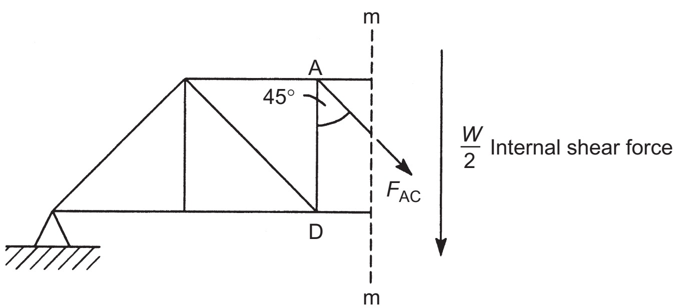

FIGURE 4.12

Internal shear force in a truss.

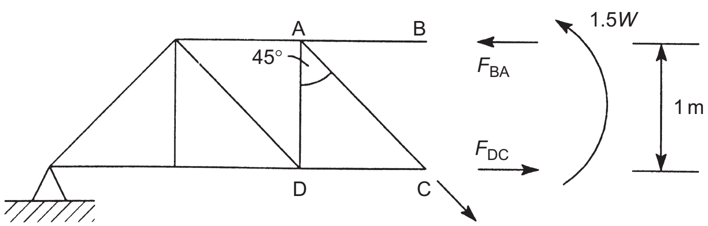

FIGURE 4.13

Internal bending moment in a truss.

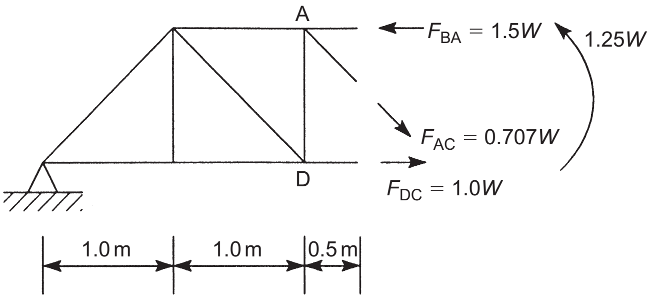

FIGURE 4.14

Resistance of a bending moment at a mid-bay point.

Now let us consider the bending moment at, say, the mid-point of the bay AB, where its magnitude is, from Fig. 4.11(c), 1.25 W. The internal force system is shown in Fig. 4.14 in which F_{BA}, F_{AC} and F_{DC} have the same values as before. Then, taking moments about, say, the mid-point of the top chord member AB, we have

$$M = F_{DC} \times 1 + F_{AC} \times 0.5 \sin 45° = 1.0\ W \times 1 + 0.707\ W \times 0.5 \sin 45° = 1.25\ W$$

the value of the applied moment.

From the discussion above it is clear that, in trusses, shear loads are resisted by inclined members, while all members combine to resist bending moments. Furthermore, positive (sagging) bending moments induce compression in upper chord members and tension in lower chord members.

Finally, note that in the truss in Fig. 4.11 the forces in the members GE, BC and HF are all zero, as can be seen by considering the vertical equilibrium of joints E, B and F. Forces would only be induced in these members if external loads were applied directly at the joints E, B and F. Generally, if three coplanar members meet at a joint and two of them are collinear, the force in the third member is zero if no external force is applied at the joint.

4.6 Method of joints

We have seen in Section 4.4 that the axial forces in the members of a simple pin-jointed triangular structure may be found by examining the equilibrium of their connecting pins or hinges in two directions at right angles (Eq. (2.10)). This approach may be extended to plane trusses to determine the axial forces in all their members; the method is known as the *method of joints* and will be illustrated by the following example.

EXAMPLE 4.4

Determine the forces in the members of the Warren truss shown in Fig. 4.15; all members are 1 m long.

Generally, although not always, the support reactions must be calculated first. So, taking moments about D for the truss in Fig. 4.15 we obtain

$$R_A \times 2 - 2 \times 1.5 - 1 \times 1 - 3 \times 0.5 = 0$$

which gives

$$R_A = 2.75\,\text{kN}$$

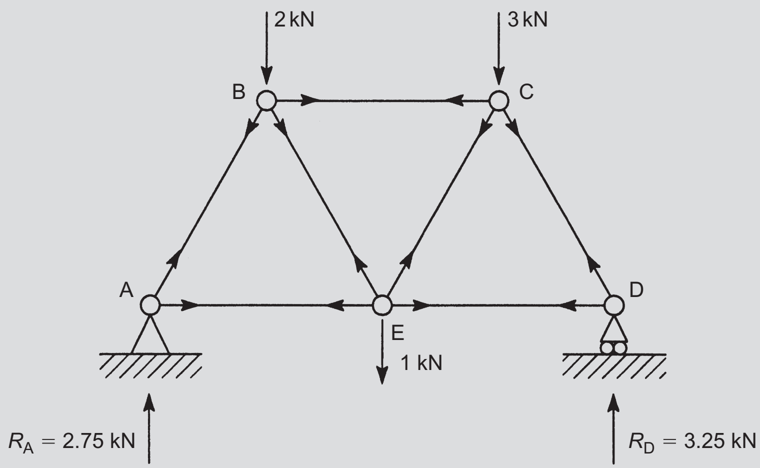

FIGURE 4.15

Analysis of a Warren truss.

Then, resolving vertically

$$R_D + R_A - 2 - 1 - 3 = 0$$

so that

$$R_D = 3.25 \text{ kN}$$

Note that there will be no horizontal reaction at A (D is a roller support) since no horizontal loads are applied.

The next step is to assign directions to the forces *acting on each joint.* In one approach the truss is examined to determine whether the force in a member is tensile or compressive. For some members this is straightforward. For example, in Fig. 4.15, the vertical reaction at A, R_A, can only be equilibrated by the vertical component of the force in AB which must therefore act downwards, indicating that the member is in compression (a compressive force in a member will push towards a joint whereas a tensile force will pull away from a joint). In some cases, where several members meet at a joint, the nature of the force in a particular member is difficult, if not impossible, to determine by inspection. Then a direction must be assumed which, if incorrect, will result in a negative value for the member force. It follows that, in the same truss, both positive and negative values may be obtained for tensile forces and also for compressive forces, a situation leading to possible confusion. Therefore, if every member in a truss is initially assumed to be in tension, negative values will always indicate compression and the solution will then agree with the sign convention adopted in Section 3.2.

We now assign tensile forces to the members of the truss in Fig. 4.15 using arrows to indicate the *action of the force in the member on the joint*; then all arrows are shown to pull away from the adjacent joint.

The analysis, as we have seen, is based on a consideration of the equilibrium of each pin or hinge under the action of *all* the forces at the joint. Thus for each pin or hinge we can write down two equations of equilibrium. It follows that a solution can only be obtained if there are no more than two unknown forces acting at the joint. In Fig. 4.15, therefore, we can only begin the analysis at the joints A or D, since at each of the joints B and C there are three unknown forces while at E there are four.

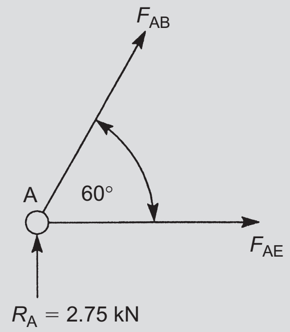

FIGURE 4.16

Equilibrium of forces at joint A.

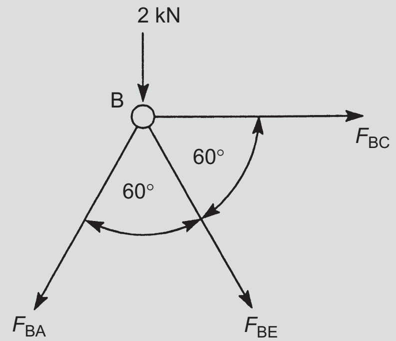

FIGURE 4.17

Equilibrium of forces at joint B.

Consider joint A. The forces acting on the pin at A are shown in the free body diagram in Fig. 4.16. F_{AB} may be determined directly by resolving forces vertically.

Hence

$$F_{AB} \sin 60° + 2.75 = 0 \tag{i}$$

so that

$$F_{AB} = -3.18 \text{ kN}$$

the negative sign indicating that AB is in compression as expected.

Referring again to Fig. 4.16 and resolving forces horizontally

$$F_{AE} + F_{AB} \cos 60° = 0 \tag{ii}$$

Substituting the *negative* value of F_{AB} in Eq. (ii) we obtain

$$F_{AE} - 3.18 \cos 60° = 0$$

which gives

$$F_{AE} = +1.59 \text{ kN}$$

the positive sign indicating that F_{AB} is a tensile force.

Note that it is simpler to retain the assumed tensile direction of a force in the equations of equilibrium and then insert the positive or negative value rather than change the initial assumed direction.

We now inspect the truss to determine the next joint at which there are no more than two unknown forces. At joint E there remain three unknowns since only F_{EA} $(= F_{AE})$ has yet been determined. At joint B there are now two unknowns since F_{BA} $(= F_{AB})$ has been determined; we can therefore proceed to joint B. The forces acting at B are shown in Fig. 4.17. Since F_{BA} is now known we can resolve forces vertically and therefore obtain F_{BE} directly. Thus

$$F_{BE} \cos 30° + F_{BA} \cos 30° + 2 = 0 \tag{iii}$$

Substituting the negative value of F_{BA} in Eq. (iii) gives

$$F_{BE} = +0.87 \text{ kN}$$

which is positive and therefore tensile.

Resolving forces horizontally at the joint B we have

$$F_{BC} + F_{BE}\cos 60^\circ - F_{BA}\cos 60^\circ = 0 \qquad \text{(iv)}$$

Substituting the positive value of F_{BE} and the negative value of F_{BA} in Eq. (iv) gives

$$F_{BC} = -2.03\,\text{kN}$$

the negative sign indicating that the member BC is in compression.

We have now calculated four of the seven unknown member forces. There are in fact just two unknown forces at each of the remaining joints C, D and E so that, theoretically, it is immaterial which joint we consider next. From a solution viewpoint there are three forces at D, four at C and five at E so that the arithmetic will be slightly simpler if we next consider D to obtain F_{DC} and F_{DE} and then C to obtain F_{CE}. At C, F_{CE} could be determined by resolving forces in the direction CE rather than horizontally or vertically. Carrying out this procedure gives

$$F_{DC} = -3.75\,\text{kN (compression)}$$

$$F_{DE} = +1.88\,\text{kN (tension)}$$

$$F_{CE} = +0.29\,\text{kN (tension)}$$

The reader should verify these values using the method suggested above.

It may be noted that in this example we could write down 10 equations of equilibrium, two for each of the five joints, and yet there are only seven unknown member forces. The apparently extra three equations result from the use of overall equilibrium to calculate the support reactions. An alternative approach would therefore be to write down the 10 equilibrium equations which would include the three unknown support reactions (there would be a horizontal reaction at A if horizontal as well as vertical loads were applied) and solve the resulting 10 equations simultaneously. Overall equilibrium could then be examined to check the accuracy of the solution. Generally, however, the method adopted above produces a quicker solution.

4.7 Method of sections

It will be appreciated from Section 4.5 that in many trusses the maximum member forces, particularly in horizontal members, will occur in the central region where the applied bending moment would possibly have its maximum value. It will also be appreciated from Ex. 4.4 that the calculation of member forces in the central region of a multibay truss such as the Pratt truss shown in Fig. 4.1(a) would be extremely tedious since the calculation must begin at an outside support and then proceed inwards joint by joint. This approach may be circumvented by using the *method of sections*.

The method is based on the premise that if a structure is in equilibrium, any portion or component of the structure will also be in equilibrium under the action of any external forces and the internal forces acting between the portion or component and the remainder of the structure. We shall illustrate the method by the following example.

EXAMPLE 4.5

Calculate the forces in the members CD, CF and EF in the Pratt truss shown in Fig. 4.18.

Initially the support reactions are calculated and are readily shown to be

$$R_{A,V} = 4.5\,\text{kN} \quad R_{A,H} = 2\,\text{kN} \quad R_B = 5.5\,\text{kN}$$

We now 'cut' the members CD, CF and EF by a section mm, thereby dividing the truss into two separate parts. Consider the left-hand part shown in Fig. 4.19 (equally we could consider the right-hand part). Clearly, if we actually cut the members CD, CF and EF, both the left- and right-hand parts would collapse. However, the equilibrium of the left-hand part, say, could be maintained by applying the forces F_{CD}, F_{CF} and F_{EF} to the cut ends of the members. Therefore, in Fig. 4.19, the left-hand part of the truss is in equilibrium under the action of the externally applied loads, the support reactions and the forces F_{CD}, F_{CF} and F_{EF} which are, as in the method of joints, initially assumed to be tensile; Eq. (2.10) are then used to calculate the three unknown forces.

Resolving vertically gives

$$F_{CF} \cos 45° + 4 - 4.5 = 0 \tag{i}$$

so that

$$F_{CF} = +0.71\,\text{kN}$$

and is tensile.

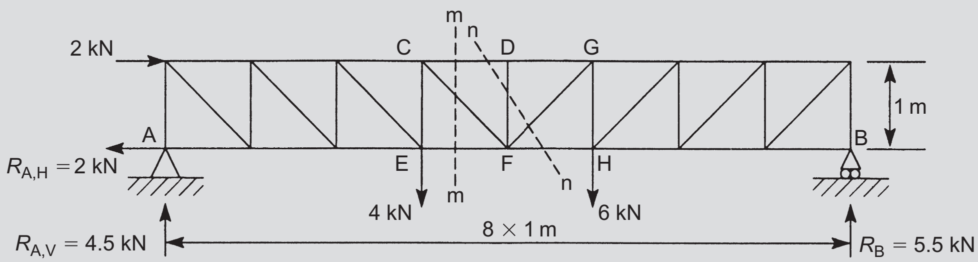

FIGURE 4.18

Calculation of member forces using the method of sections.

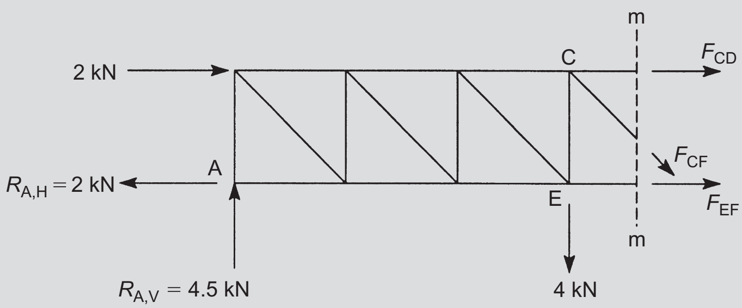

FIGURE 4.19

Equilibrium of a portion of a truss.

Now taking moments about the point of intersection of F_{CF} and F_{EF} we have

$$F_{CD} \times 1 + 2 \times 1 + 4.5 \times 4 - 4 \times 1 = 0 \qquad \text{(ii)}$$

so that

$$F_{CD} = -16\,\text{kN}$$

and is compressive.

Finally F_{EF} is obtained by taking moments about C, thereby eliminating F_{CF} and F_{CD} from the equation. Alternatively, we could resolve forces horizontally since F_{CF} and F_{CD} are now known; however, this approach would involve a slightly lengthier calculation. Hence

$$F_{EF} \times 1 - 4.5 \times 3 - 2 \times 1 = 0 \qquad \text{(iii)}$$

which gives

$$F_{EF} = +15.5\,\text{kN}$$

the positive sign indicating tension.

Note that Eqs (i)–(iii) each include just one of the unknown member forces so that it is immaterial which is calculated first. In some problems, however, a preliminary examination is worthwhile to determine the optimum order of solution.

In Ex. 4.5 we see that there are just three possible equations of equilibrium so that we cannot solve for more than three unknown forces. It follows that a section such as mm which *must divide the frame into two separate parts* must also *not cut through more than three members in which the forces are unknown.* For example, if we wished to determine the forces in CD, DF, FG and FH we would first calculate F_{CD} using the section mm as above and then determine F_{DF}, F_{FG} and F_{FH} using the section nn. Actually, in this particular example F_{DF} may be seen to be zero by inspection (see Section 4.5) but the principle holds.

4.8 Method of tension coefficients

An alternative form of the method of joints which is particularly useful in the analysis of space trusses is the *method of tension coefficients.*

Consider the member AB, shown in Fig. 4.20, which connects two pinned joints A and B whose coordinates, referred to arbitrary xy axes, are (x_A, y_A) and (x_B, y_B) respectively; the member carries a *tensile* force, T_{AB}, is of length L_{AB} and is inclined at an angle α to the x axis. The component of T_{AB} parallel to the x axis at A is given by

$$T_{AB} \cos \alpha = T_{AB} \frac{(x_B - x_A)}{L_{AB}} = \frac{T_{AB}}{L_{AB}} (x_B - x_A)$$

Similarly the component of T_{AB} at A parallel to the y axis is

$$T_{AB} \sin \alpha = \frac{T_{AB}}{L_{AB}} (y_B - y_A)$$

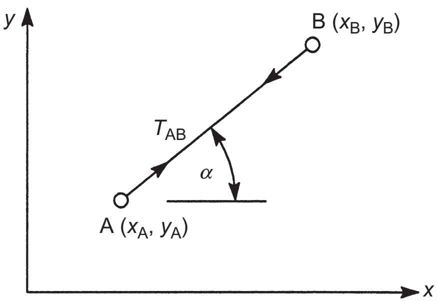

FIGURE 4.20

Method of tension coefficients.

We now define a *tension coefficient* $t_{AB} = T_{AB}/L_{AB}$ so that the above components of T_{AB} become

$$\text{parallel to the } x \text{ axis}: \ t_{AB}(x_B - x_A) \tag{4.3}$$

$$\text{parallel to the } y \text{ axis}: \ t_{AB}(y_B - y_A) \tag{4.4}$$

Equilibrium equations may be written down for each joint in turn in terms of tension coefficients and joint coordinates referred to some convenient axis system. The solution of these equations gives t_{AB}, etc, whence $T_{AB} = t_{AB}L_{AB}$ in which L_{AB}, unless given, may be calculated using Pythagoras' theorem, i.e. $L_{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$. Again the initial assumption of tension in a member results in negative values corresponding to compression. Note the order of suffixes in Eqs (4.3) and (4.4).

EXAMPLE 4.6

Determine the forces in the members of the pin-jointed truss shown in Fig. 4.21.

The support reactions are first calculated and are as shown in Fig. 4.21.

The next step is to choose an *xy* axis system and then insert the joint coordinates in the diagram. In Fig. 4.21 we shall choose the support point A as the origin of axes although, in fact, any joint would suffice; the joint coordinates are then as shown.

Again, as in the method of joints, the solution can only begin at a joint where there are no more than two unknown member forces, in this case joints A and E. Theoretically it is immaterial at which of these joints the analysis begins but since A is the origin of axes we shall start at A. Note that it is unnecessary to insert arrows to indicate the directions of the member forces since the members are assumed to be in tension and the directions of the components of the member forces are automatically specified when written in terms of tension coefficients and joint coordinates (Eqs (4.3) and (4.4)).

The equations of equilibrium at joint A are

$$x \text{ direction}: \ t_{AB}(x_B - x_A) + t_{AC}(x_C - x_A) - R_{A,H} = 0 \tag{i}$$

$$y \text{ direction}: \ t_{AB}(y_B - y_A) + t_{AC}(y_C - y_A) - R_{A,V} = 0 \tag{ii}$$

Substituting the values of $R_{A,H}$, $R_{A,V}$ and the joint coordinates in Eqs (i) and (ii) we obtain, from Eq. (i),

$$t_{AB}(0 - 0) + t_{AC}(1.5 - 0) - 3 = 0$$

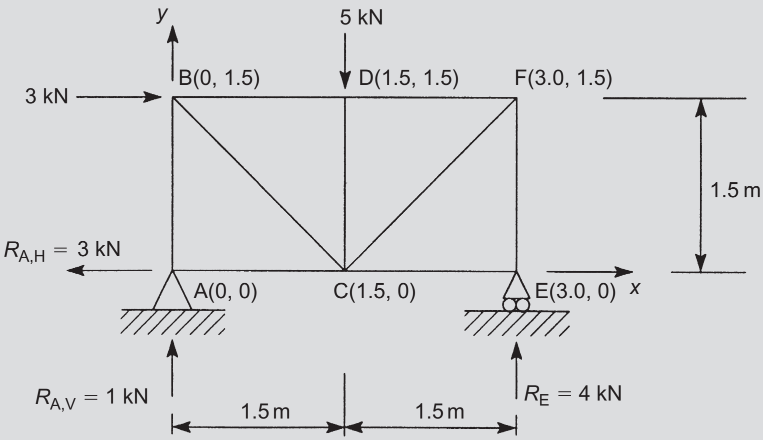

FIGURE 4.21

Analysis of a truss using tension coefficients (Ex. 4.6).

whence

$$t_{AC} = +2.0$$

and from Eq. (ii)

$$t_{AB}(1.5 - 0) + t_{AC}(0 - 0) + 1 = 0$$

so that

$$t_{AB} = -0.67$$

We see from the derivation of Eqs (4.3) and (4.4) that the units of a tension coefficient are force/unit length, in this case kN/m. Generally, however, we shall omit the units.

We can now proceed to joint B at which, since t_{BA} ($= t_{AB}$) has been calculated, there are two unknowns

$$x\text{ direction}: \ t_{BA}(x_A - x_B) + t_{BC}(x_C - x_B) + t_{BD}(x_D - x_B) + 3 = 0 \qquad \text{(iii)}$$

$$y\text{ direction}: \ t_{BA}(y_A - y_B) + t_{BC}(y_C - y_B) + t_{BD}(y_D - y_B) = 0 \qquad \text{(iv)}$$

Substituting the values of the joint coordinates and t_{BA} in Eqs (iii) and (iv) we have, from Eq. (iii)

$$-0.67\,(0 - 0) + t_{BC}(1.5 - 0) + t_{BD}(1.5 - 0) + 3 = 0$$

which simplifies to

$$1.5t_{BC} + 1.5t_{BD} + 3 = 0 \qquad \text{(v)}$$

and from Eq. (iv)

$$-0.67\,(0 - 1.5) + t_{BC}(0 - 1.5) + t_{BD}(1.5 - 1.5) = 0$$

whence

$$t_{BC} = +0.67$$

Hence, from Eq. (v)

$$t_{\mathrm{BD}} = -2.67$$

There are now just two unknown member forces at joint D. Hence, at D

$$x \text{ direction}: \; t_{\mathrm{DB}}(x_{\mathrm{B}} - x_{\mathrm{D}}) + t_{\mathrm{DF}}(x_{\mathrm{F}} - x_{\mathrm{D}}) + t_{\mathrm{DC}}(x_{\mathrm{C}} - x_{\mathrm{D}}) = 0 \tag{vi}$$

$$y \text{ direction}: \; t_{\mathrm{DB}}(y_{\mathrm{B}} - y_{\mathrm{D}}) + t_{\mathrm{DF}}(y_{\mathrm{F}} - y_{\mathrm{D}}) + t_{\mathrm{DC}}(y_{\mathrm{C}} - y_{\mathrm{D}}) - 5 = 0 \tag{vii}$$

Substituting values of joint coordinates and the previously calculated value of t_{DB} $(= t_{\mathrm{BD}})$ in Eqs (vi) and (vii) we obtain, from Eq. (vi)

$$-2.67\,(0 - 1.5) + t_{\mathrm{DF}}(3.0 - 1.5) + t_{\mathrm{DC}}(1.5 - 1.5) - 5 = 0$$

so that

$$t_{\mathrm{DF}} = -2.67$$

and from Eq. (vii)

$$-2.67(1.5 - 1.5) + t_{\mathrm{DF}}(1.5 - 1.5) + t_{\mathrm{DC}}(0 - 1.5) = 0$$

from which

$$t_{\mathrm{DC}} = -3.33$$

The solution then proceeds to joint C to obtain t_{CF} and t_{CE} or to joint F to determine t_{FC} and t_{FE}; joint F would be preferable since fewer members meet at F than at C. Finally, the remaining unknown tension coefficient (t_{EC} or t_{EF}) is found by considering the equilibrium of joint E. Then

$$t_{\mathrm{FC}} = +2.67, \quad t_{\mathrm{FE}} = -2.67, \quad t_{\mathrm{EC}} = 0$$

which the reader should verify.

The forces in the truss members are now calculated by multiplying the tension coefficients by the member lengths, i.e.

$$T_{\mathrm{AB}} = t_{\mathrm{AB}}L_{\mathrm{AB}} = -0.67 \times 1.5 = -1.0\,\mathrm{kN}\,(\text{compression})$$

$$T_{\mathrm{AC}} = t_{\mathrm{AC}}L_{\mathrm{AC}} = +2.0 \times 1.5 = +3.0\,\mathrm{kN}\,(\text{tension})$$

$$T_{\mathrm{BC}} = t_{\mathrm{BC}}L_{\mathrm{BC}}$$

in which

$$L_{\mathrm{BC}} = \sqrt{(x_{\mathrm{B}} - x_{\mathrm{C}})^2 + (y_{\mathrm{B}} - y_{\mathrm{C}})^2} = \sqrt{(0 - 1.5)^2 + (1.5 - 0)^2} = 2.12\,\mathrm{m}$$

Then

$$T_{\mathrm{BC}} = +0.67 \times 2.12 = +1.42\,\mathrm{kN}\,(\text{tension})$$

Note that in the calculation of member lengths it is immaterial in which order the joint coordinates occur in the brackets since the brackets are squared. Also

$$T_{\mathrm{BD}} = t_{\mathrm{BD}}L_{\mathrm{BD}} = -2.67 \times 1.5 = -4.0\,\mathrm{kN}\,(\text{compression})$$

Similarly

$$T_{DF} = -4.0\,\text{kN (compression)}$$
$$T_{DC} = -5.0\,\text{kN (compression)}$$
$$T_{FC} = +5.67\,\text{kN (tension)}$$
$$T_{FE} = -4.0\,\text{kN (compression)}$$
$$T_{EC} = 0$$

4.9 Graphical method of solution

In some instances, particularly when a rapid solution is required, the member forces in a truss may be found using a graphical method.

The method is based upon the condition that each joint in a truss is in equilibrium so that the forces acting at a joint may be represented in magnitude and direction by the sides of a closed polygon (see Section 2.1). The directions of the forces must be drawn in the same directions as the corresponding members and there must be no more than two unknown forces at a particular joint otherwise a polygon of forces cannot be constructed. The method will be illustrated by applying it to the truss in Ex. 4.4.

EXAMPLE 4.7

Determine the forces in the members of the Warren truss shown in Fig. 4.22; all members are 1 m long.

It is convenient in this approach to designate forces in members in terms of the areas between them rather than referring to the joints at their ends. (i.e. Bow's Notation, see Ex. 2.7) Thus, in Fig. 4.22, we number the areas between all forces, both internal and external; the reason for this will become clear when the force diagram for the complete structure is constructed.

The support reactions were calculated in Ex. 4.4 and are shown in Fig. 4.22. We must start at a joint where there are no more than two unknown forces, in this example either A or D; here we select A. The force polygon for joint A is constructed by going round A in, say, a clockwise sense. We must then go round every joint in the same sense.

First we draw a vector 12 to represent the support reaction at A of 2.75 kN to a convenient scale (see Fig. 4.23). Note that we are moving clockwise from the region 1 to the region 2 so that the vector 12 is vertically upwards, the direction of the reaction at A (if we had decided to move round A in an anticlockwise sense the vector would be drawn as 21 vertically upwards). The force in the member AB at A will be represented by a vector 26 in the direction AB or BA, depending on whether it is tensile or compressive, while the force in the member AE at A is represented by the vector 61 in the direction AE or EA depending, again, on whether it is tensile or compressive. The point 6 in the force polygon is therefore located by drawing a line through the point 2 parallel to the member AB to intersect, at 6, a line drawn through the point 1 parallel to the member AE. We see from the force polygon that the direction of the vector 26 is towards A so that the member AB is in compression while the direction of the vector 61 is away from A indicating that the member AE is in tension. We now insert arrows on the members AB and AE in Fig. 4.22 to indicate compression and tension, respectively.

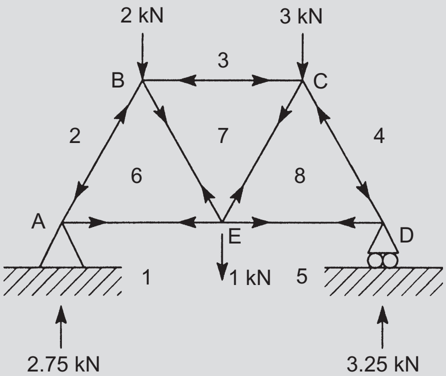

FIGURE 4.22

Analysis of a truss by a graphical method.

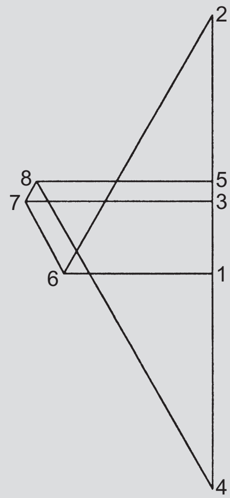

FIGURE 4.23

Force polygon for the truss of Ex. 4.7.

We next consider joint B where there are now just two unknown member forces since we have previously determined the force in the member AB; note that, moving clockwise round B, this force is represented by the vector 62, which means that it is acting towards B as it must since we have already established that AB is in compression. Rather than construct a separate force polygon for the joint B we shall superimpose the force polygon on that constructed for joint A since the vector 26 (or 62) is common to both; we thereby avoid repetition. Thus, through the point 2, we draw a vector 23 vertically downwards to represent the 2 kN load to the same scale as before. The force in the member BC is represented by the vector 37 parallel to BC (or CB) while the force in the member BE is represented by the vector 76 drawn in the direction of BE (or EB); this locates the point 7 in the force polygon. Hence we see that the force in BC (vector 37) acts towards B indicating compression, while the force in BE (vector 76) acts away from B indicating tension; again, arrows are inserted in Fig. 4.22 to show the action of the forces.

Now we consider joint C where the unknown member forces are in CD and CE. The force in the member CB at C is represented in magnitude and direction by the vector 73 in the force polygon. From the point 3 we draw a vector 34 vertically downwards to represent the 3 kN load. The vectors 48 and 87 are then drawn parallel to the members CD and CE and represent the forces in the members CD and CE, respectively. Thus we see that the force in CD (vector 48) acts towards C, i.e. CD is in compression, while the force in CE (vector 87) acts away from C indicating tension; again we insert corresponding arrows on the members in Fig. 4.22.

Finally the vector 45 is drawn vertically upwards to represent the vertical reaction (= 3.25 kN) at D and the vector 58, which must be parallel to the member DE, inserted (since the points 5 and 8 are already located in the force polygon this is a useful check on the accuracy of construction). From the direction of the vector 58 we deduce that the member DE is in tension.

Note that in the force polygon the vectors may be read in both directions. Thus the vector 26 represents the force in the member AB acting at A, while the vector 62 represents the force in AB acting at B.

It should also be clear why there must be consistency in the sense in which we move round each joint; e.g. the vector 26 represents the direction of the force at A in the member AB when we move in a clockwise sense round A. However, if we then move in an anticlockwise sense round the joint B the vector 26 would represent the magnitude and direction of the force in AB at B and would indicate that AB is in tension, but clearly it is not.

4.10 Compound trusses

In some situations simple trusses are connected together to form a compound truss, in which case it is generally not possible to calculate the forces in all the members by the method of joints even though the truss is statically determinate.

Figure 4.24 shows a compound truss comprising two simple trusses AGC and BJC connected at the apex C and by the linking bar GJ; all the joints are pinned and we shall suppose that the truss carries loads at all its joints. We note that the truss has 27 members and 15 joints so that Eq. (4.1) is satisfied and the truss is statically determinate. This truss is, in fact, a Fink truss (see Fig. 4.1(c)).

Initially we would calculate the support reactions at A and B and commence a method of joints solution at the joint A (or at the joint B) where there are no more than two unknown member forces. Thus the magnitudes of F_{AD} and F_{AE} would be obtained. Then, by considering the equilibrium of joint D, we would calculate F_{DE} and F_{DF} and then F_{EF} and F_{EG} by considering the equilibrium of joint E. At this stage, however, the analysis can proceed no further, since at each of the next joints to be considered, F and G, there are three unknown member forces: F_{FG}, F_{FI} and F_{FH} at F, and F_{GF}, F_{GI} and F_{GJ} at G. An identical situation would have arisen if the analysis had commenced in the right-hand half of the truss at B. This difficulty is overcome by taking a section mm to cut the three members HC, IC and GJ and using the method of sections to calculate the corresponding member forces. Having obtained F_{GJ} we can consider the equilibrium of joint G to calculate F_{GI} and F_{GF}. Hence F_{FI} and F_{FH} follow by considering the equilibrium of joint F; the remaining unknown member forces follow. Note that obtaining F_{GI} by taking the section mm allows all the member forces in the right-hand half of the truss to be found by the method of joints.

The method of sections could be used to solve for all the member forces. First we could obtain F_{HC}, F_{IC} and F_{GJ} by taking the section mm and then F_{FH}, F_{FI} and F_{GI} by taking the section nn where F_{GJ} is known, and so on.

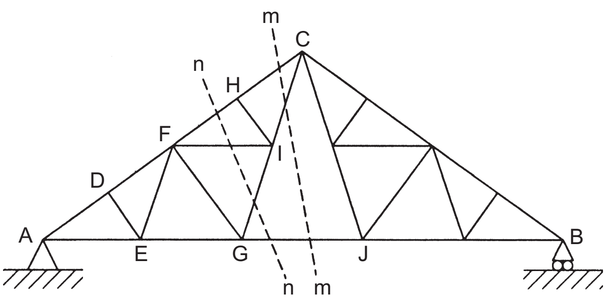

FIGURE 4.24

Compound truss.

4.11 Space trusses

The most convenient method of analysing statically determinate stable space trusses (see Eq. (4.2)) is that of tension coefficients. In the case of space trusses, however, there are three possible equations of equilibrium for each joint (Eq. (2.11)); the moment equations (Eq. (2.12)) are automatically satisfied since, as in the case of plane trusses, the lines of action of all the forces in the members meeting at a joint pass through the joint and the pin cannot transmit moments. Therefore the analysis must begin at a joint where there are no more than three unknown forces.

The calculation of the reactions at supports in space frames can be complex. If a space frame has a statically determinate support system, a maximum of six reaction components can exist since there are a maximum of six equations of overall equilibrium (Eqs (2.11) and (2.12)). However, for the truss to be stable the reactions must be orientated in such a way that they can resist the components of the forces and moments about each of the three coordinate axes. Fortunately, in many problems, it is unnecessary to calculate support reactions since there is usually one joint at which there are no more than three unknown member forces.

EXAMPLE 4.8

Calculate the forces in the members of the space truss whose elevations and plan are shown in Fig. 4.25.

In this particular problem the exact nature of the support points is not specified so that the support reactions cannot be calculated. However, we note that at joint F there are just three unknown member forces so that the analysis may begin at F.

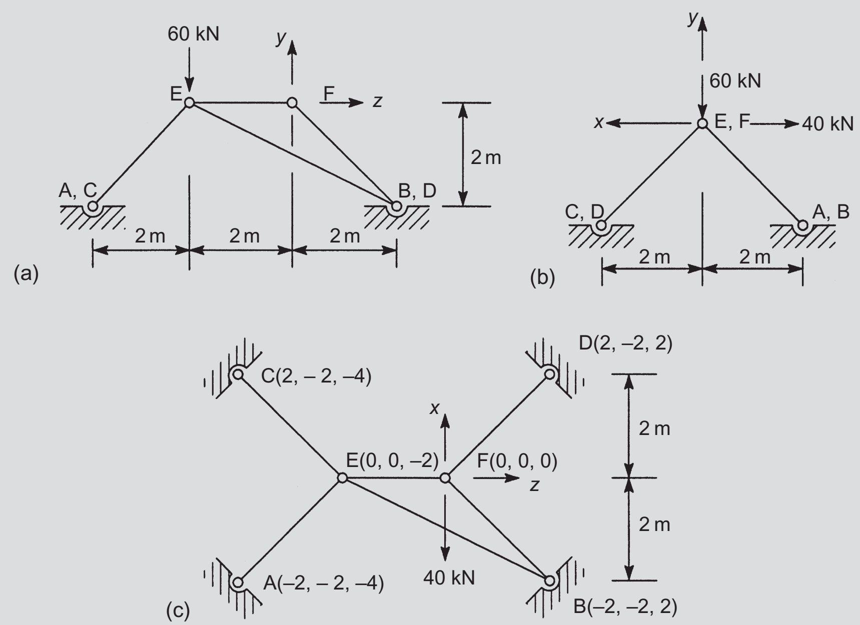

FIGURE 4.25

Elevations and plan of space frame of Ex. 4.8.

The first step is to choose an axis system and an origin of axes. Any system may be chosen so long as care is taken to ensure that there is agreement between the axis directions in each of the three views. Also, any point may be chosen as the origin of axes and need not necessarily coincide with a joint. In this problem it would appear logical to choose F, since the analysis will begin at F. Furthermore, it will be helpful to sketch the axis directions on each of the three views as shown and to insert the joint coordinates on the plan view (Fig. 4.25(c)).

At joint F

$$x\text{ direction}: \; t_{FD}(x_D - x_F) + t_{FB}(x_B - x_F) + t_{FE}(x_E - x_F) - 40 = 0 \tag{i}$$

$$y\text{ direction}: \; t_{FD}(y_D - y_F) + t_{FB}(y_B - y_F) + t_{FE}(y_E - y_F) = 0 \tag{ii}$$

$$z\text{ direction}: \; t_{FD}(z_D - z_F) + t_{FB}(z_B - z_F) + t_{FE}(z_E - z_F) = 0 \tag{iii}$$

Substituting the values of the joint coordinates in Eqs (i)–(iii) in turn we obtain, from Eq. (i)

$$t_{FD}(2-0) + t_{FB}(-2-0) + t_{FE}(0-0) - 40 = 0$$

whence

$$t_{FD} - t_{FB} - 20 = 0 \tag{iv}$$

from Eq. (ii)

$$t_{FD}(-2-0) + t_{FB}(-2-0) + t_{FE}(0-0) = 0$$

which gives

$$t_{FD} + t_{FB} = 0 \tag{v}$$

and from Eq. (iii)

$$t_{FD}(2-0) + t_{FB}(2-0) + t_{FE}(-2-0) = 0$$

so that

$$t_{FD} + t_{FB} - t_{FE} = 0 \tag{vi}$$

From Eqs (v) and (vi) we see by inspection that

$$t_{FE} = 0$$

Now adding Eqs (iv) and (v)

$$2t_{FD} - 20 = 0$$

whence

$$t_{FD} = 10$$

Therefore, from Eq. (v)

$$t_{FB} = -10$$

We now proceed to joint E where, since $t_{EF} = t_{FE}$, there are just three unknown member forces

$$x\text{ direction}: \; t_{EB}(x_B - x_E) + t_{EC}(x_C - x_E) + t_{EA}(x_A - x_E) + t_{EF}(x_F - x_E) = 0 \tag{vii}$$

$$y\text{ direction}: \; t_{EB}(y_B - y_E) + t_{EC}(y_C - y_E) + t_{EA}(y_A - y_E) + t_{EF}(y_F - y_E) - 60 = 0 \tag{viii}$$

$$z\text{ direction}: \; t_{EB}(z_B - z_E) + t_{EC}(z_C - z_E) + t_{EA}(z_A - z_E) + t_{EF}(z_F - z_E) = 0 \tag{ix}$$

Substituting the values of the coordinates and t_{EF} $(=0)$ in Eqs (vii)–(ix) in turn gives, from Eq. (vii)

$$t_{EB}(-2-0)+t_{EC}(2-0)+t_{EA}(-2-0)=0$$

so that

$$t_{EB}-t_{EC}+t_{EA}=0 \tag{x}$$

from Eq. (viii)

$$t_{EB}(-2-0)+t_{EC}(-2-0)+t_{EA}(-2-0)-60=0$$

whence

$$t_{EB}+t_{EC}+t_{EA}+30=0 \tag{xi}$$

and from Eq. (ix)

$$t_{EB}(2+2)+t_{EC}(-4+2)+t_{EA}(-4+2)=0$$

which gives

$$t_{EB}-0.5t_{EC}-0.5t_{EA}=0 \tag{xii}$$

Subtracting Eq. (xi) from Eq. (x) we have

$$-2t_{EC}-30=0$$

so that

$$t_{EC}=-15$$

Now subtracting Eq. (xii) from Eq. (xi) (or Eq. (x)) yields

$$1.5t_{EC}+1.5t_{EA}+30=0$$

which gives

$$t_{EA}=-5$$

Finally, from any of Eqs (x)–(xii),

$$t_{EB}=-10$$

The length of each of the members is now calculated, except that of EF which is given $(=2$ m). Using Pythagoras' theorem

$$L_{FB}=\sqrt{(x_B-x_F)^2+(y_B-y_F)^2+(z_B-z_F)^2}$$

whence

$$L_{FB}=\sqrt{(-2-0)^2+(-2-0)^2+(2-0)^2}=3.46\,\text{m}$$

Similarly

$$L_{FD} = L_{EC} = L_{EA} = 3.46\,\text{m} \quad L_{FB} = 4.90\,\text{m}$$

The forces in the members then follow

$$T_{FB} = t_{FB}\,L_{FB} = -10 \times 3.46\,\text{kN} = -34.6\,\text{kN}(\text{compression})$$

Similarly

$$\begin{aligned} T_{FD} &= +34.6\,\text{kN}\,(\text{tension}) \\ T_{FE} &= 0 \\ T_{EC} &= -51.9\,\text{kN}\,(\text{compression}) \\ T_{EA} &= -17.3\,\text{kN}\,(\text{compression}) \\ T_{EB} &= -49.0\,\text{kN}\,(\text{compression}) \end{aligned}$$

The solution of Eqs (iv)–(vi) and (x)–(xii) in Ex. 4.8 was relatively straightforward in that many of the coefficients of the tension coefficients could be reduced to unity. This is not always the case, so that it is possible that the solution of three simultaneous equations must be carried out. In this situation an elimination method, described in standard mathematical texts, may be used.

4.12 A computer-based approach

The calculation of the member forces in trusses generally involves, as we have seen, in the solution of a number of simultaneous equations; clearly, the greater the number of members the greater the number of equations. For a truss with N members and R reactions $N + R$ equations are required for a solution provided that the truss and the support systems are both statically determinate. However, in some cases such as in Ex. 4.8, it is possible to solve for member forces without first calculating the support reactions. This still could mean that there would be a large number of equations to solve so that a more mechanical approach, such as the use of a computer, would be time and labour saving. For this we need to express the equations in matrix form.

At the joint F in Ex. 4.8 suppose that, instead of the 40 kN load, there are external loads X_F, Y_F and Z_F applied in the positive directions of the respective axes. Eqs (i)–(iii) are then written as

$$\begin{aligned} t_{FD}(x_D - x_F) + t_{FB}(x_B - x_F) + t_{FE}(x_E - x_F) + X_F = 0 \\ t_{FD}(y_D - y_F) + t_{FB}(y_B - y_F) + t_{FE}(y_E - y_F) + Y_F = 0 \\ t_{FD}(z_D - z_F) + t_{FB}(z_B - z_F) + t_{FE}(z_E - z_F) + Z_F = 0 \end{aligned}$$

In matrix form these become

$$\begin{bmatrix} x_D - x_F & x_B - x_F & x_E - x_F \\ y_D - y_F & y_B - y_F & y_E - y_F \\ z_D - z_F & z_B - z_F & z_E - z_F \end{bmatrix} \begin{bmatrix} t_{FD} \\ t_{FB} \\ t_{FE} \end{bmatrix} = \begin{bmatrix} -X_F \\ -Y_F \\ -Z_F \end{bmatrix}$$

or

$$[C][t] = [F]$$

where $[C]$ is the coordinate matrix, $[t]$ the tension coefficient matrix and $[F]$ the force matrix. Then

$$[t] = [C]^{-1}[F]$$

Computer programs exist which will carry out the inversion of [C] so that the tension coefficients are easily obtained.

In the above the matrix equation only represents the equilibrium of joint F. There are, in fact, six members in the truss so that a total of six equations are required. The additional equations are Eqs (vii)–(ix) in Ex. 4.8. Therefore, to obtain a complete solution, these equations would be incorporated giving a 6×6 square matrix for [C].

In practice plane and space frame programs exist which, after the relevant data have been input, give the member forces directly. It is, however, important that the fundamentals on which these programs are based are understood. We shall return to matrix methods later.

PROBLEMS

P.4.1 Investigate the determinacy and stability of the trusses shown in Fig. P.4.1(a)–(d).

Ans.

a. Statically determinate and stable.
b. Statically determinate and stable.
c. Statically in determinate and stable.
d. Statically determinate but unstable unless ABC is in tension.

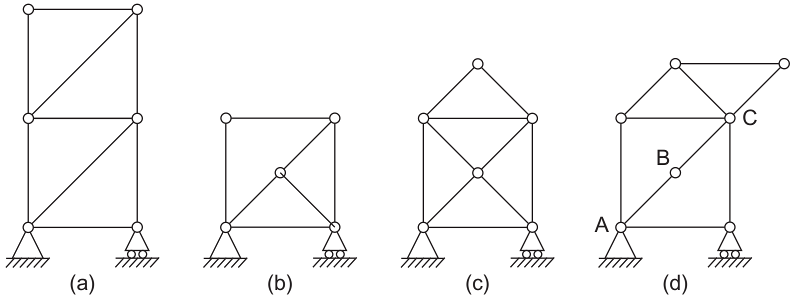

FIGURE P.4.1

P.4.2 Determine the forces in the members of the truss shown in Fig. P.4.2 using the method of joints and check the forces in the members JK, JD and DE by the method of sections.

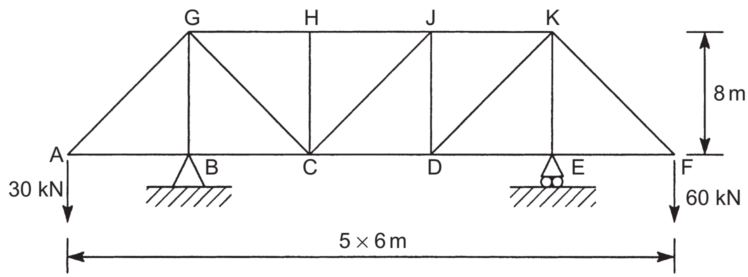

FIGURE P.4.2

Ans. AG = +37.5, AB = −22.5, BG = −20.0, BC = −22.5, GC = −12.5, GH = +30.0, HC = 0, HJ = +30.0, CJ = +12.5, CD = −37.5, JD = −10.0, JK = +37.5, DK = +12.5, DE = −45.0, EK = −70.0, FE = −45.0, FK = +75.0. All in kN.

P.4.3 Calculate the forces in the members of the truss shown in Fig. P.4.3.

Ans. AC = −30.0, AP = +26.0, CP = −8.7, CE = −25.0, PE = +8.7, PF = +17.3, FE = −17.3, GE = −20.0, HE = +8.7, FH = +17.3, GH = −8.7, JG = −15.0, HJ = +26.0, FB = 0, BJ = −15.0. All in kN.

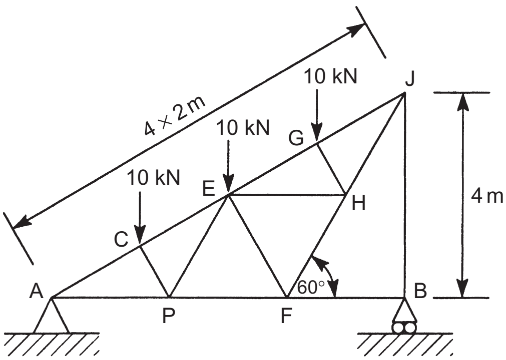

FIGURE P.4.3

P.4.4. A motorway warning display is supported symmetrically by two identical pin-jointed frames as shown in Fig. P.4.4. The frames must be designed to withstand a wind load of 4 kN/m^2. Calculate the forces in the members of a frame and check your solution using a graphical method. Neglect the weight of the display.

Ans. AC = −40.4, AB = BD = 35.1, BC = −20.0, CD = 20.7, CB = −61.5, ED = 30.5. All in kN.

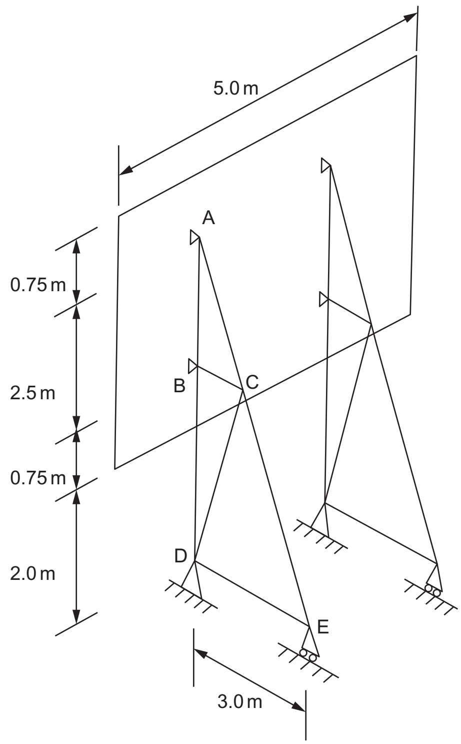

FIGURE P.4.4

P.4.5 If the railway track of P.3.21 is supported at the lower chord joints of the truss, calculate the forces in the members BE, BF, EF, and EG with the head of the train at mid-span.

Ans. BE = 133 (tension), BF = 550.2 (tension), EG = −555.6 (compression), EF = 6.8 (tension). All in kN.

P.4.6 Calculate the forces in the members EF, EG, EH and FH of the truss shown in Fig. P.4.6. Note that the horizontal load of 4 kN is applied at the joint C.

Ans. EF = −20.0, EG = −80.0, EH = −33.3, FH = + 106.6. All in kN.

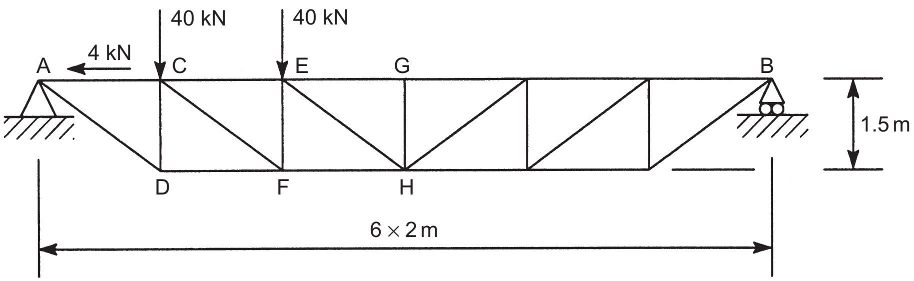

FIGURE P.4.6

P.4.7 The roof truss shown in Fig. P.4.7 is comprised entirely of equilateral triangles; the wind loads of 6 kN at J and B act perpendicularly to the member JB. Calculate the forces in the members DF, EF, EG and EK.

Ans. DF = + 106.4, EF = + 1.7, EG = −107.3, KE = −20.8. All in kN.

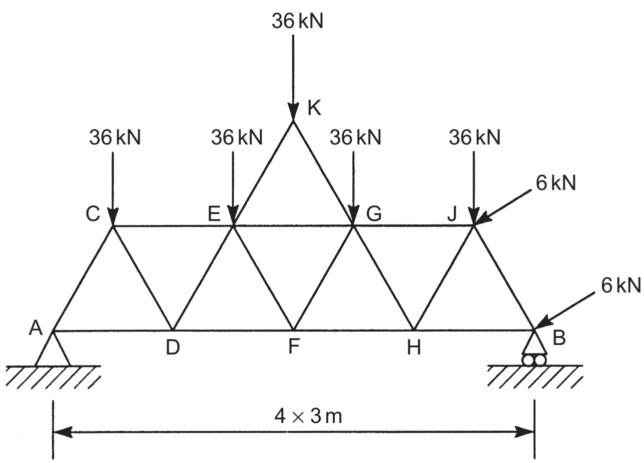

FIGURE P.4.7

P.4.8 The upper chord joints of the bowstring truss shown in Fig. P.4.8 lie on a parabola whose equation has the form $y = kx^2$ referred to axes whose origin coincides with the uppermost joint. Calculate the forces in the members AD, BD and BC.

Ans. DA = −3.1, DB = −0.5, CB = +2.7. All in kN.

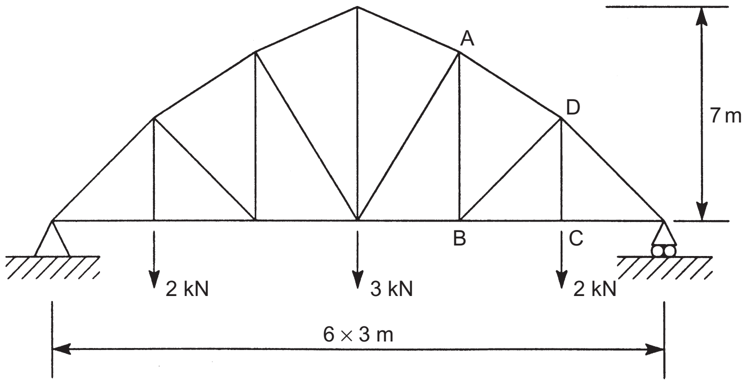

FIGURE P.4.8

P.4.9 The truss shown in Fig. P.4.9 is supported by a hinge at A and a cable at D which is inclined at an angle of 45° to the horizontal members. Calculate the tension, T, in the cable and hence the forces in all the members by the method of tension coefficients.

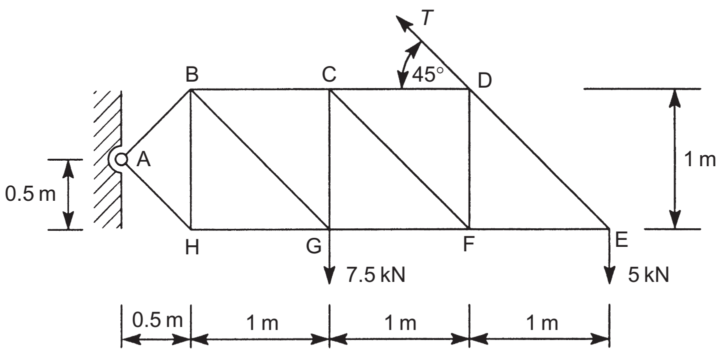

FIGURE P.4.9

Ans. T = 13.6, BA = −8.9, CB = −9.2, DC = −4.6, ED = +7.1, EF = −5.0, FG = −0.4, GH = −3.3, HA = −4.7, BH = +3.4, GB = +4.1, FC = −6.5, CG = +4.6, DF = +4.6. All in kN.

P.4.10 Check your answers to problems P.4.2 and P.4.3 using a graphical method.

P.4.11 Determine the force in the member BC of the crane shown in Fig. P.4.11.

Ans. 707 kN (tension).

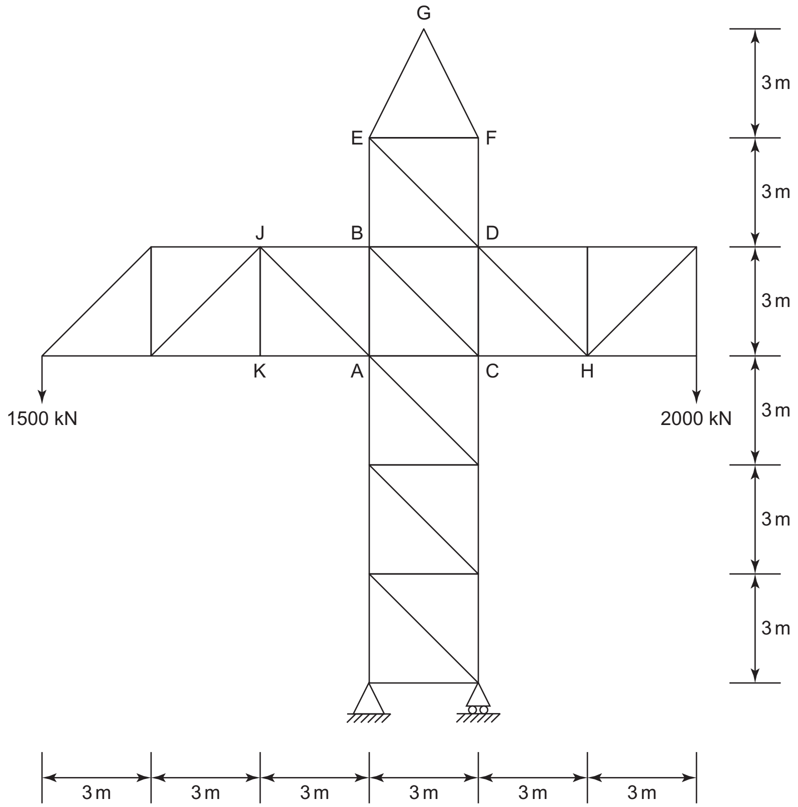

FIGURE P.4.11

P.4.12 The Fink truss shown in Fig. P.4.12 is one of a series of trusses supporting the roof of a building via a system of purlins attached to the truss joints; each truss supports a 2-m length of roof. The members of the truss must be designed to support roof loads as follows: a snow load of 0.5 kN/m^2 on plan, a suction wind load of 0.4 kN/m^2 on the leeward side, and a wind load of 0.6 kN/m^2 on the windward side. If the snow and wind loads may be regarded as being applied symmetrically along the slope of the roof, find the force in the tie member CK.

Ans. 8.1 kN (tension).

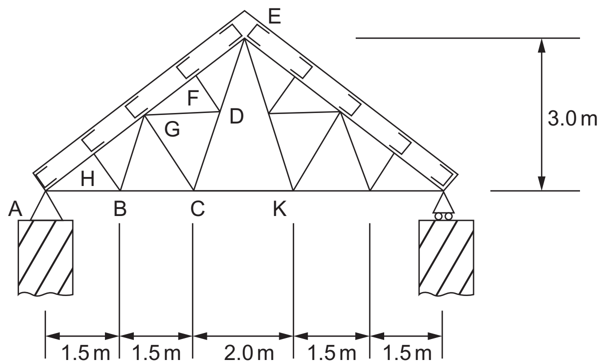

FIGURE P.4.12

P.4.13 Find the forces in the members of the space truss shown in Fig. P.4.13; suggested axes are also shown.

Ans. OA = +24.2, OB = +11.9, OC = −40.2. All in kN.

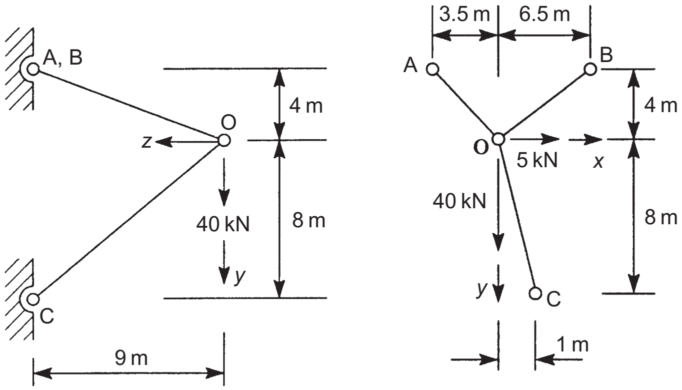

FIGURE P.4.13

P.4.14 Use the method of tension coefficients to calculate the forces in the members of the space truss shown in Fig. P.4.14. Note that the loads P_2 and P_3 act in a horizontal plane and at angles of 45° to the vertical plane BAD.

Ans. AB = +13.1, AD = +13.1, AC = −59.0. All in kN.

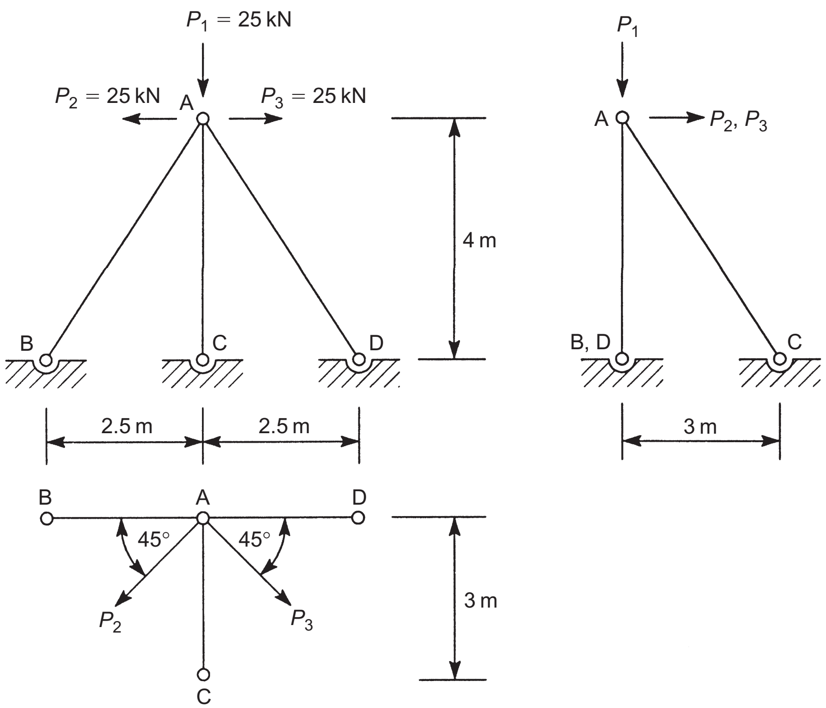

FIGURE P.4.14

P.4.15 The pin-jointed truss shown in Fig. P.4.15 is attached to a vertical wall at the points A, B, C and D; the members BE, BF, EF and AF are in the same horizontal plane. The truss supports vertically downward loads of 9 and 6 kN at E and F, respectively, and a horizontal load of 3 kN at E in the direction EF.

Calculate the forces in the members of the truss using the method of tension coefficients.

Ans. EF = −3.0, EC = −15.0, EB = +12.0, FB = +5.0, FA = +4.0, FD = −10.0. All in kN.

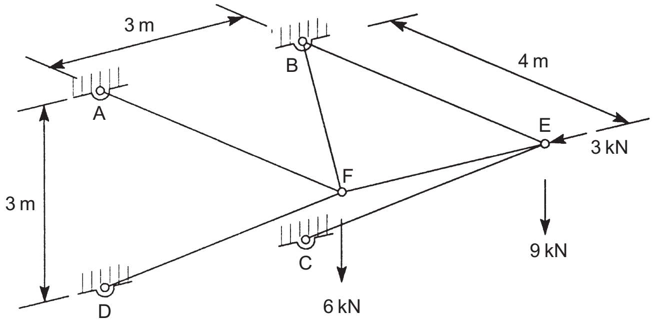

FIGURE P.4.15

P.4.16 Fig. P.4.16 shows the plan of a space truss which consists of six pin-jointed members. The member DE is horizontal and 4 m above the horizontal plane containing A, B and C while the loads applied at D and E act in a horizontal plane. Calculate the forces in the members.

Ans. AD = 0, DC = 0, DE = +40.0, AE = 0, CE = −60.0, BE = +60.0. All in kN.

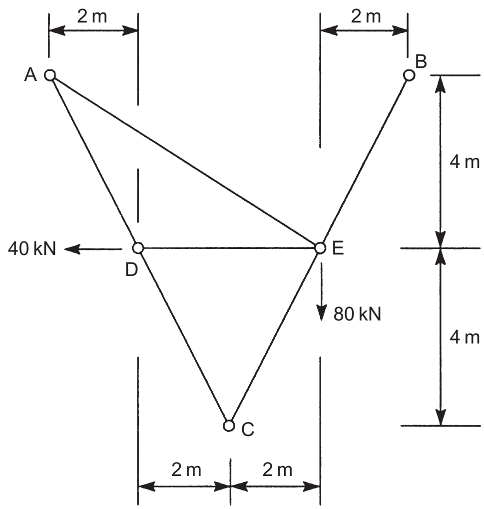

FIGURE P.4.16

P.4.17 Use the method of tension coefficients to calculate the forces in the members of the space truss shown in Fig. P.15.14.

Ans. 27 = 3*P*, 87 = 5*P*/3, 67 = −4*P*/3, 21 = 4*P*, 23 = 38 = 58 = 68 = 13 = 43 = 93 = 03 = 0,
26 = −5*P*, 98 = 5*P*/3, 16 = 3*P*, 56 = −16*P*/3, 15 = −5*P*, 10 = 8*P*.