CHAPTER

Manoeuvrability

8

8.1 Introduction

8.1.1 Manoeuvring flight

What is a manoeuvre? An aeroplane executing aerobatics in a vast blue sky or aeroplanes engaged in aerial combat are the images associated with manoeuvring flight. By their very nature, such manoeuvres are difficult to quantify, especially when it is required to describe manoeuvrability in an analytical framework. In reality, most manoeuvres are comparatively mundane and simply involve changing from one trimmed flight condition to another. When a pilot wishes to manoeuvre away from the current flight condition, he applies control inputs which upset the equilibrium trim state by producing forces and moments to manoeuvre the aeroplane toward the desired flight condition. The temporary out-of-trim forces and moments cause the aeroplane to *accelerate* in a sense determined by the combined action of the control inputs. Thus manoeuvring flight is sometimes called *accelerated flight* and is defined as the condition in which the airframe is subject to temporary, or transient, out-of-trim linear and angular accelerations resulting from the displacement of the controls relative to their trim settings. In analytical terms, the manoeuvre is regarded as an *increment* in steady motion, over and above the initial trim state, in response to an increment in control angle.

The main aerodynamic force producing device in an aeroplane is the wing, and wing lift acts normal to the direction of flight in the plane of symmetry. Normal manoeuvring involves rotating the airframe in roll, pitch, and yaw to point the lift vector in the desired direction, and the simultaneous adjustment of both angle of attack and speed enables the lift force to generate the acceleration to manoeuvre. For example, in turning flight the aeroplane is rolled to the desired bank angle when the horizontal component of lift causes it to turn in the desired direction. Simultaneous aft displacement of the pitch stick is required to generate pitch rate, which in turn generates an increase in angle of attack to produce more lift such that the vertical component is sufficient to balance the weight of the aeroplane and hence to maintain level flight in the turn. (The requirements for simple turning flight are illustrated in Example 2.3.) Thus manoeuvrability is mainly concerned with the ability to rotate about aircraft axes, the modulation of the normal or lift force, and the modulation of the axial or thrust force. The use of lateral sideforce to manoeuvre is not common in conventional aeroplanes since it is aerodynamically inefficient and is both unnatural and uncomfortable for the pilot. The principal aerodynamic manoeuvring force is therefore lift, which acts in the plane of symmetry of the aeroplane; this is controlled by operating the control column in the pitch sense. When the pilot pulls back on the pitch stick the aeroplane pitches up to generate an increased lift force; since this results in out-of-trim normal acceleration, the pilot senses, and is very sensitive to, the change in acceleration. He senses what appears to be an increase in the earth's gravitational acceleration g and so is said to be *pulling g*.

8.1.2 Stability

Aircraft stability is generally concerned with the requirement that trimmed equilibrium flight can be achieved and that small transient upsets from equilibrium decay to zero. However, in manoeuvring flight the *transient upset* is the deliberate result following a control input; it may not be small and it may well be prolonged. In the manoeuvre the aerodynamic forces and moments may be significantly different from the steady trim values, and it is essential that the changes do not impair the stability of the aeroplane. In other words, there must be no tendency for the aeroplane to diverge in manoeuvring flight.

The classical theory of manoeuvrability is generally attributed to Gates and Lyon (1944), and various interpretations of that original work may be found in most books on aircraft stability and control. Perhaps one of the most comprehensive and accessible summaries of the theory is included in Babister (1961). In this chapter the subject is introduced at the most basic level in order to provide an understanding of the concepts involved, since they are critically important in the broader considerations of flying and handling qualities. The original work makes provision for the effects of compressibility. In the following analysis subsonic flight only is considered in the interest of simplicity and hence understanding.

The traditional analysis of *manoeuvre stability* is based on the concept of the steady manoeuvre in which the aeroplane is subject to a steady normal acceleration in response to a pitch control input. Although rather contrived, this approach does enable the manoeuvre stability of an aeroplane to be explained analytically. The only realistic manoeuvres which can be flown at constant normal acceleration are the inside or outside loop and the steady banked turn. For the purpose of analysis, the loop is simplified to a pull-up, or push-over, which is just a small segment of the circular flight path. Whichever manoeuvre is analysed, the resulting conditions for stability are the same.

Since the steady acceleration is constrained to the plane of symmetry, the problem simplifies to the analysis of *longitudinal manoeuvre stability* and, since the motion is steady, this analysis is a simple extension of that applied to *longitudinal static stability* as described in Chapter 3. Consequently, it leads to the concept of the *longitudinal manoeuvre margin*, the stability margin in manoeuvring flight, which in turn gives rise to the corresponding control parameters *stick displacement per g* and *stick force per g*.

8.1.3 Aircraft handling

It is not difficult to appreciate that the manoeuvrability of an airframe is a critical factor in its overall flying and handling qualities. Too much manoeuvre stability means that large control displacements and forces are needed to encourage the development of the normal acceleration vital to effective manoeuvring. On the other hand, too little manoeuvre stability implies that an enthusiastic pilot could overstress the airframe by applying excessive levels of normal acceleration. Clearly, the difficult balance between control power, manoeuvre stability, static stability, and dynamic stability must be correctly controlled over the entire flight envelope of the aeroplane.

Today, considerations of manoeuvrability in the context of aircraft handling have moved on from the simple analysis of normal acceleration response to controls alone. Important additional considerations concern accompanying roll, pitch, and yaw rates and accelerations that may be achieved from control inputs, since these determine how quickly a manoeuvre can become established. Manoeuvre entry is also *coloured* by transients associated with the short-term dynamic stability modes. The aggressiveness with which a pilot may fly a manoeuvre and the motion cues available to him also contribute to his perception of the over-all handling characteristics of the aeroplane. The "picture" therefore becomes very complex, and it is further complicated by the introduction of flight control systems to the aeroplane. The subject of *aircraft agility* is a relatively new and exciting topic of research which embraces the ideas just mentioned and which is, unfortunately, beyond the scope of the present book.

8.1.4 The steady symmetric manoeuvre

The analysis of longitudinal manoeuvre stability is based on steady motion which results in constant additional, or incremental, normal acceleration; as mentioned previously, the simplest such manoeuvre to analyse is the pull-up.

In symmetric flight, inertial normal acceleration, referred to the *cg*, is given by equation (5.39):

$$a_z = \dot{w} - qU_e \tag{8.1}$$

Since the manoeuvre is steady, $\dot{w} = 0$ and the aeroplane must fly a steady pitch rate in order to generate the normal acceleration required to manoeuvre. A steady turn enables this condition to be maintained ad infinitum in flight but is less straightforward to analyse. In symmetric flight, a short-duration pull-up can be used to represent the lower segment of a continuous circular flight path in the vertical plane because a continuous loop is not practical for many aeroplanes.

It is worth noting that many modern combat aeroplanes and some advanced civil transport aeroplanes have flight control systems which feature *direct lift control* (DLC). In such aeroplanes pitch rate is not an essential prerequisite for the generation of normal acceleration because the wing is fitted with a system of flaps for producing lift directly. However, in some applications it is common to mix DLC flap control with conventional elevator control to improve manoeuvrability—manoeuvre entry in particular. The manoeuvrability of aeroplanes fitted with DLC systems may be significantly enhanced, although the analysis may become rather more complex.

8.2 The steady pull-up manoeuvre

An aeroplane flying initially in steady level flight at speed V_0 is subject to a small elevator input $\delta\eta$ which causes it to pull up with steady pitch rate q. Consider the situation when the aircraft is at the lowest point of the vertical circle flight path as shown in Fig. 8.1.

To sustain flight in the vertical circle it is necessary that the lift L balances not only the weight mg but also the centrifugal force; thus the lift is greater than the weight and

$$L = nmg \tag{8.2}$$

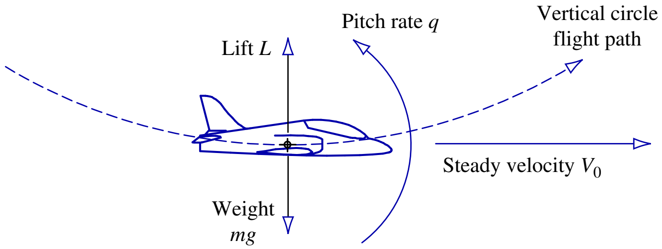

FIGURE 8.1 Symmetric pull-up manoeuvre.

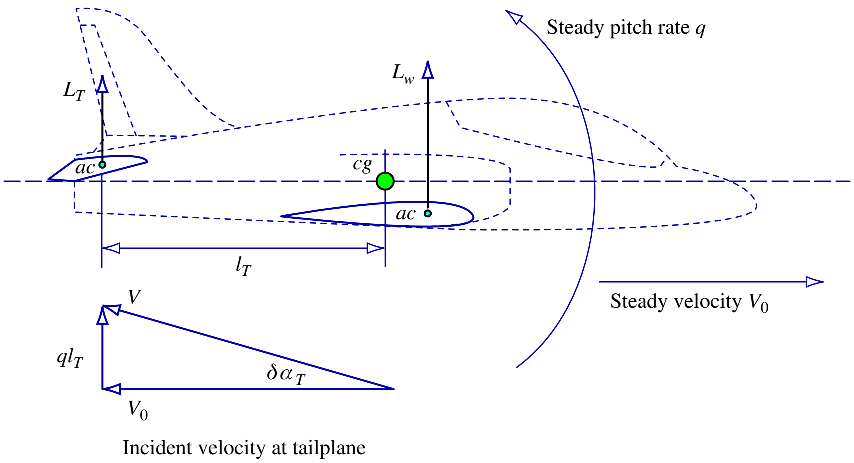

FIGURE 8.2 Incremental tailplane incidence in pull-up manoeuvre.

where n is the italic for emphasis. The normal load factor therefore quantifies the total lift necessary to maintain the manoeuvre and in steady level flight $n = 1$. The centrifugal force balance is therefore given by

$$L - mg = mV_0 q \tag{8.3}$$

and the incremental normal load factor may be derived directly:

$$\delta n = (n - 1) = \frac{V_0 q}{g} \tag{8.4}$$

Now, as the aircraft is pitching up steadily, the tailplane experiences an increase in incidence $\delta\alpha_T$ from the pitch manoeuvre as indicated in Fig. 8.2.

Since small-perturbation motion is assumed, the increase in tailplane incidence is given by

$$\delta\alpha_T \cong \tan\delta\alpha_T = \frac{q l_T}{V_0} \tag{8.5}$$

where l_T is the moment arm of the aerodynamic centre of the tailplane with respect to the centre of rotation in pitch, the cg. Eliminating pitch rate q from equations (8.4) and (8.5),

$$\delta\alpha_T = \frac{(n-1)g l_T}{V_0^2} \tag{8.6}$$

In the steady level flight condition about which the manoeuvre is executed, the lift and weight are now equal; whence

$$V_0^2 = \frac{2mg}{\rho S C_{L_w}} \tag{8.7}$$

where C_{L_w} is the steady level flight value of the wing-body lift coefficient. Thus, from equations (8.6) and (8.7),

$$\delta\alpha_T = \frac{(n-1)\rho S C_{L_w} l_T}{2m} = \frac{(n-1)C_{L_w} l_T}{\mu_1 \bar{\bar{c}}} \equiv \frac{\delta C_{L_w} l_T}{\mu_1 \bar{\bar{c}}} \tag{8.8}$$

where μ_1 is the *longitudinal relative density parameter* and is defined

$$\mu_1 = \frac{m}{\frac{1}{2}\rho S \bar{\bar{c}}} \tag{8.9}$$

and the increment in lift coefficient, alternatively referred to as incremental g, necessary to sustain the steady manoeuvre is given by

$$\delta C_{L_w} = (n-1)C_{L_w} = \delta n C_{L_w} \tag{8.10}$$

Care should be exercised when using the longitudinal relative density parameter since various definitions are in common use.

8.3 The pitching moment equation

Subject to the same assumptions about thrust, drag, speed effects, and so on, in the steady symmetric manoeuvre the pitching moment equation in coefficient form given by equation (3.7) applies and may be written as

$$C'_m = C_{m_0} + C'_{L_w}(h - h_0) - C'_{L_T}\overline{V}_T \tag{8.11}$$

where a dash indicates the manoeuvring value of the coefficient and

$$\begin{aligned} C'_m &= C_m + \delta C_m \\ C'_{L_w} &= C_{L_w} + \delta C_{L_w} \equiv n C_{L_w} \\ C'_{L_T} &= C_{L_T} + \delta C_{L_T} \end{aligned}$$

where C_m, C_{L_w}, and C_{L_T} denote the steady trim values of the coefficients and δC_m, δC_{L_w}, and δC_{L_T} denote the increments in the coefficients required to manoeuvre.

The corresponding expression for the tailplane lift coefficient is given by equation (3.8), which for manoeuvring flight may be written as

$$C'_{L_T} = a_1\alpha'_T + a_2\eta' + a_3\beta_\eta \tag{8.12}$$

It is assumed that the tailplane has a symmetric aerofoil section, $a_0 = 0$, and that the tab angle β_η is held at the constant steady trim value throughout the manoeuvre. In other words, the manoeuvre is the result of elevator input only. Thus, using the above notation,

$$\begin{aligned}\alpha'_T &= \alpha_T + \delta\alpha_T \\ \eta' &= \eta + \delta\eta\end{aligned}$$

Tailplane incidence is given by equation (3.11); in the manoeuvre this may be written as

$$\alpha_T = \frac{C'_{L_w}}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \eta_T \tag{8.13}$$

Total tailplane incidence in the manoeuvre is therefore given by the sum of equations (8.8) and (8.13):

$$\alpha'_T = \frac{C'_{L_w}}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \eta_T + \frac{\delta C_{L_w} l_T}{\mu_1 \bar{\bar{c}}} \tag{8.14}$$

Substituting for α'_T in equation (8.12), the expression for tailplane lift coefficient in the manoeuvre may be written as

$$C'_{L_T} = \frac{C'_{L_w} a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + a_1\eta_T + \frac{\delta C_{L_w} a_1 l_T}{\mu_1 \bar{\bar{c}}} + a_2\eta' + a_3\beta_\eta \tag{8.15}$$

Substitute the expression for tailplane lift coefficient, equation (8.15), into equation (8.11) and, after some rearrangement, the pitching moment equation may be written as

$$C'_m = C_{m_0} + C'_{L_w}(h - h_0) - \overline{V}_T\left(\frac{C'_{L_w} a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + a_1\eta_T + \frac{\delta C_{L_w} a_1 l_T}{\mu_1 \bar{\bar{c}}} + a_2\eta' + a_3\beta_\eta\right) \tag{8.16}$$

Equation (8.16) describes the total pitching moment in the manoeuvre. To obtain the incremental pitching moment equation which describes the manoeuvre effects only, it is first necessary to replace the "dashed'' variables and coefficients in equation (8.16) with their equivalent expressions. Then, after some rearrangement, equation (8.16) may be written as

$$\begin{aligned}C_m + \delta C_m \quad &= \left\{C_{m_0} + C_{L_w}(h - h_0) - \overline{V}_T\left(\frac{C_{L_w} a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + a_1\eta_T + a_2\eta + a_3\beta_\eta\right)\right\} \\ &+ \left\{\delta C_{L_w}(h - h_0) - \overline{V}_T\left(\frac{\delta C_{L_w} a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{\delta C_{L_w} a_1 l_T}{\mu_1 \bar{\bar{c}}} + a_2\delta\eta\right)\right\}\end{aligned} \tag{8.17}$$

Now, in the steady equilibrium flight condition about which the manoeuvre is executed, the pitching moment is zero; therefore,

$$C_m = C_{m_0} + C_{L_w}(h - h_0) - \overline{V}_T\left(\frac{C_{L_w}a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + a_1\eta_T + a_2\eta + a_3\beta_\eta\right) = 0 \tag{8.18}$$

and equation (8.17) simplifies to that describing the incremental pitching moment coefficient,

$$\delta C_m = \delta C_{L_w}(h - h_0) - \overline{V}_T\left(\frac{\delta C_{L_w}a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{\delta C_{L_w}a_1 l_T}{\mu_1\bar{\bar{c}}} + a_2\delta\eta\right) \tag{8.19}$$

8.4 Longitudinal manoeuvre stability

As for longitudinal static stability, discussed in Chapter 3, to achieve a stable manoeuvre the following condition must be satisfied:

$$\frac{dC'_m}{dC'_{L_w}} < 0 \tag{8.20}$$

and for the manoeuvre to remain steady,

$$C'_m = 0 \tag{8.21}$$

Analysis and interpretation of these conditions leads to the definition of *controls-fixed manoeuvre stability* and *controls-free manoeuvre stability*, which correspond with the parallel concepts derived in the analysis of longitudinal static stability.

8.4.1 Controls-fixed stability

The total pitching moment equation (8.16) may be written as

$$C'_m = C_{m_0} + C'_{L_w}(h - h_0) - \overline{V}_T\left(\frac{C'_{L_w}a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + a_1\eta_T + \frac{(C'_{L_w} - C_{L_w})a_1 l_T}{\mu_1\bar{\bar{c}}} + a_2\eta' + a_3\beta_\eta\right) \tag{8.22}$$

and since, by definition, the controls are held fixed in the manoeuvre,

$$\frac{d\eta'}{dC'_{L_w}} = 0$$

Applying the condition for stability, equation (8.20), to equation (8.22) and noting that C_{L_w} and β_η are constant at their steady level flight values and that η_T is also a constant of the aircraft configuration, then

$$\frac{dC'_m}{dC'_{L_w}} = (h - h_0) - \overline{V}_T\left(\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{a_1 l_T}{\mu_1\bar{\bar{c}}}\right) \tag{8.23}$$

or

$$H_m = -\frac{\mathrm{d}C'_m}{\mathrm{d}C'_{L_w}} = h_m - h \tag{8.24}$$

where H_m is the *controls-fixed manoeuvre margin*; the location of the *controls-fixed manoeuvre point* h_m on the mean aerodynamic chord $\bar{\bar{c}}$ is given by

$$h_m = h_0 + \overline{V}_T\left(\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right) + \frac{a_1 l_T}{\mu_1\bar{\bar{c}}}\right) = h_n + \frac{\overline{V}_T a_1 l_T}{\mu_1\bar{\bar{c}}} \tag{8.25}$$

Clearly, for controls-fixed manoeuvre stability, the manoeuvre margin H_m must be positive, which, with reference to equation (8.24), implies that the *cg* must be ahead of the manoeuvre point. Equation (8.25) indicates that the controls-fixed manoeuvre point is aft of the corresponding neutral point by an amount depending on the aerodynamic properties of the tailplane. It therefore follows that

$$H_m = K_n + \frac{\overline{V}_T a_1 l_T}{\mu_1\bar{\bar{c}}} \tag{8.26}$$

which indicates that the controls-fixed manoeuvre stability is greater than the controls-fixed static stability. With reference to Appendix 8, equation (8.26) may be restated in terms of aerodynamic stability derivatives:

$$H_m = -\frac{M_w}{a} - \frac{M_q}{\mu_1} \tag{8.27}$$

A most important conclusion is, then, that additional stability in manoeuvring flight is provided by the aerodynamic pitch damping properties of the tailplane. However, caution is advised because this conclusion may not apply to all aeroplanes in large-amplitude manoeuvring or to manoeuvring in conditions where the assumptions do not apply.

As for controls-fixed static stability, the meaning of controls-fixed manoeuvre stability is easily interpreted by considering the pilot action required to establish a steady symmetric manoeuvre from an initial trimmed level flight condition. Since the steady (fixed) incremental elevator angle needed to induce the manoeuvre is of interest, the incremental pitching moment equation (8.19) is applicable. In a stable steady—and hence by definition—non-divergent manoeuvre, the incremental pitching moment δC_m is zero. Equation (8.19) may therefore be rearranged to give

$$\frac{\delta\eta}{\delta C_{L_w}} = \frac{1}{\overline{V}_T a_2}\left\{(h - h_0) - \overline{V}_T\left(\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right) + \frac{a_1 l_T}{\mu_1\bar{\bar{c}}}\right)\right\} = \frac{-H_m}{\overline{V}_T a_2} \tag{8.28}$$

Or, in terms of aerodynamic stability derivatives,

$$\frac{\delta\eta}{\delta C_{L_w}} = \frac{-H_m}{M_\eta} = \frac{1}{M_\eta}\left(\frac{M_w}{a} + \frac{M_q}{\mu_1}\right) \tag{8.29}$$

Referring to equation (8.10),

$$\delta C_{L_w} = (n-1)C_{L_w}$$

which describes the incremental aerodynamic load acting on the aeroplane causing it to execute the manoeuvre, expressed in coefficient form and measured in units of "g." Thus, both equation (8.28) and equation (8.29) express the *elevator displacement per g* capability of the aeroplane, which is proportional to the controls-fixed manoeuvre margin and inversely proportional to the elevator control power, quantified by the aerodynamic control derivative M_η.

8.4.2 Normal acceleration response to elevator

Since elevator angle and pitch control stick angle are directly related by the control gearing, the very important *stick displacement per g* control characteristic follows directly and is also proportional to the controls-fixed manoeuvre margin. This latter control characteristic is critically important in the determination of longitudinal handling qualities. Measurements of elevator angle and normal acceleration in steady manoeuvres for a range of values of normal load factor provide an effective means for determining controls-fixed manoeuvre stability from flight experiments. However, in such experiments it is not always possible to ensure that all of the underlying assumptions can be adhered to.

Alternatively, and in the context of *small-perturbation* normal acceleration response to controls, it is instructive to use transfer function descriptions in the analysis. Because short-term steady-state response to controls is the principal interest, it is convenient to utilise the longitudinal reduced-order transfer functions for the mathematical analysis. In longitudinal motion the normal acceleration a_z referred to the centre of gravity is given by equation (8.1) and for symmetric manoeuvring is

$$a_z = \dot{w} - qU_e \tag{8.30}$$

where U_e is the steady axial component of velocity. The Laplace transform of equation (8.30), assuming zero initial conditions, may be written as

$$a_z(s) = sw(s) - q(s)U_e \tag{8.31}$$

Or, expressing equation (8.31) in terms of elevator response transfer functions,

$$a_z(s) = s\frac{N_\eta^w(s)}{\Delta(s)}\eta(s) - U_e\frac{N_\eta^q(s)}{\Delta(s)}\eta(s) = \frac{(sN_\eta^w(s) - U_eN_\eta^q(s))\eta(s)}{\Delta(s)} \tag{8.32}$$

where

$$\frac{N_\eta^w(s)}{\Delta(s)} = \frac{k_w(s + 1/T_\alpha)}{(s^2 + 2\zeta_s\omega_s s + \omega_s^2)} \qquad \frac{N_\eta^q(s)}{\Delta(s)} = \frac{k_q(s + 1/T_{\theta_2})}{(s^2 + 2\zeta_s\omega_s s + \omega_s^2)} \tag{8.33}$$

Defining, the incremental normal load factor δn due to manoeuvring as

$$\delta n = -\frac{a_z}{g} \tag{8.34}$$

and since normal acceleration response to elevator is largely determined by the dynamic characteristics of the short-period pitching oscillation, the reduced-order longitudinal transfer functions $w(s)/\eta(s)$ and $q(s)/\eta(s)$ given in equations (8.33) may be substituted in equation (8.32) to give the transfer function for normal load factor response to an increment of elevator displacement as follows:

$$\frac{\delta n(s)}{\delta\eta(s)} = -\frac{\left(k_w\left(s+1/T_\alpha\right)s - k_q\left(s+1/T_{\theta_2}\right)U_e\right)}{g\Delta(s)} \tag{8.35}$$

Application of the final value theorem to equation (8.35), assuming a unit elevator input, leads to an expression for the steady-state gain:

$$\left.\frac{\delta n}{\delta\eta}\right|_{\substack{steady\\state}} \equiv \frac{k_q U_e}{g\omega_s^2 T_{\theta_2}} \tag{8.36}$$

where $k_q = m_\eta$, the concise elevator control derivative. Note the dependence of normal acceleration response to elevator on the dynamic stability parameters ω_s and T_{θ_2}. This is entirely consistent with the understanding that longitudinal dynamic handling is significantly related to the aircraft acceleration motion cue in response to control. Alternatively, equation (8.36) may be expressed entirely in terms of concise aerodynamic stability derivatives:

$$\left.\frac{\delta n}{\delta\eta}\right|_{\substack{steady\\state}} = -\frac{m_\eta z_w U_e}{g(m_q z_w - m_w U_e)} \tag{8.37}$$

With reference to Appendices 2 and 8, the concise derivatives in equation (8.37) may be written in terms of the dimensionless derivatives, which in turn may be substituted with their basic aerodynamic definitions to give, after a substantial algebraic manipulation,

$$\left.\frac{\delta n}{\delta\eta}\right|_{\substack{steady\\state}} = -\frac{\overline{V}_T a_2 U_e^2\left(1 - \frac{\overline{V}_T a_1}{\mu_1}\frac{d\varepsilon}{d\alpha}\right)}{g\mu_1\overline{\overline{c}}\left(H_m - \left(\frac{\overline{V}_T a_1}{\mu_1}\right)^2 \frac{l_T}{\overline{\overline{c}}}\frac{d\varepsilon}{d\alpha}\right)} \tag{8.38}$$

Alternatively, and omitting insignificantly small terms in equation (8.38),

$$\left.\frac{\delta n}{\delta\eta}\right|_{\substack{steady\\state}} \cong -\frac{g\mu_1\overline{\overline{c}}H_m}{\overline{V}_T a_2 U_e^2} \tag{8.39}$$

Equation (8.39) is the equivalent to equation (8.28) and confirms that elevator angle η per g to manoeuvre is proportional to the controls-fixed manoeuvre margin H_m.

8.4.3 Controls-free stability

The controls-free manoeuvre is not a practical way of controlling an aeroplane. It does, of course, imply that the elevator angle required to achieve the manoeuvre is obtained by adjustment of the tab angle. As in the case of controls-free static stability, this equates to the control force required to achieve the manoeuvre, a most important control characteristic. Control force derives from elevator hinge moment in a conventional aeroplane; the elevator hinge moment coefficient in manoeuvring flight is given by equation (3.21) and may be restated as

$$C'_H = C_H + \delta C_H = b_1\alpha'_T + b_2\eta' + b_3\beta_\eta \tag{8.40}$$

Since the elevator angle in a controls-free manoeuvre is indeterminate, it is convenient to express η' in terms of hinge moment coefficient by rearranging equation (8.40):

$$\eta' = \frac{1}{b_2}C'_H - \frac{b_1}{b_2}\alpha'_T - \frac{b_3}{b_2}\beta_\eta \tag{8.41}$$

Substitute the expression for α'_T, equation (8.14), into equation (8.41) to obtain

$$\eta' = \frac{1}{b_2}C'_H - \frac{b_1}{ab_2}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)C'_{L_w} - \frac{b_1}{b_2}\eta_T - \frac{b_1 l_T}{b_2\mu_1\bar{\bar{c}}}\delta C_{L_w} - \frac{b_3}{b_2}\beta_\eta \tag{8.42}$$

Equation (8.42) may be substituted into the manoeuvring pitching moment equation (8.16) to replace the indeterminate elevator angle by hinge moment coefficient. After some algebraic rearrangement, the manoeuvring pitching moment may be expressed in the same format as equation (8.22):

$$C'_m = C_{m_0} + C'_{L_w}(h - h_0) - \overline{V}_T\left(\begin{array}{c} C'_{L_w}\dfrac{a_1}{a}\left(1 - \dfrac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right)\left(1 - \dfrac{a_2 b_1}{a_1 b_2}\right) + a_1\eta_T + C'_H\dfrac{a_2}{b_2} \\ +(C'_{L_w} - C_{L_w})\dfrac{a_1 l_T}{\mu_1\bar{\bar{c}}}\left(1 - \dfrac{a_2 b_1}{a_1 b_2}\right) + \beta_\eta\left(1 - \dfrac{a_2 b_3}{a_3 b_2}\right)\end{array}\right) \tag{8.43}$$

and since, by definition, the controls are free in the manoeuvre,

$$C'_H = 0$$

Applying the condition for stability, equation (8.20), to equation (8.43) and noting that, as before, C_{L_w} and β_η are constant at their steady level flight values and that η_T is also a constant of the aircraft configuration, then

$$\frac{\mathrm{d}C'_m}{\mathrm{d}C'_{L_w}} = (h - h_0) - \overline{V}_T\left(\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right) + \frac{a_1 l_T}{\mu_1\bar{\bar{c}}}\right)\left(1 - \frac{a_2 b_1}{a_1 b_2}\right) \tag{8.44}$$

or

$$H'_m = -\frac{\mathrm{d}C'_m}{\mathrm{d}C'_{L_w}} = h'_m - h \tag{8.45}$$

where H'_m is the *controls-free manoeuvre margin* and the location of the *controls-free manoeuvre point* h'_m on the mean aerodynamic chord $\bar{\bar{c}}$ is given by

$$\begin{aligned} h'_m &= h_0 + \overline{V}_T\left(\frac{a_1}{a}\left(1-\frac{d\varepsilon}{d\alpha}\right)+\frac{a_1 l_T}{\mu_1\bar{\bar{c}}}\right)\left(1-\frac{a_2 b_1}{a_1 b_2}\right) \\ &= h'_n + \overline{V}_T\frac{a_1 l_T}{\mu_1\bar{\bar{c}}}\left(1-\frac{a_2 b_1}{a_1 b_2}\right) \end{aligned} \tag{8.46}$$

Clearly, for controls-free manoeuvre stability, the manoeuvre margin H'_m must be positive. With reference to equation (8.45), this implies that the *cg* must be ahead of the manoeuvre point. Equation (8.46) indicates that the controls-free manoeuvre point is aft of the corresponding neutral point by an amount again dependent on the aerodynamic damping properties of the tailplane. It therefore follows that

$$H'_m = K'_n + \overline{V}_T\frac{a_1 l_T}{\mu_1\bar{\bar{c}}}\left(1-\frac{a_2 b_1}{a_1 b_2}\right) \equiv K'_n + \frac{M_q}{\mu_1}\left(1-\frac{a_2 b_1}{a_1 b_2}\right) \tag{8.47}$$

which indicates that the controls-free manoeuvre stability is greater than the controls-free static stability when

$$\left(1-\frac{a_2 b_1}{a_1 b_2}\right) > 0 \tag{8.48}$$

Since a_1 and a_2 are both positive, the degree of controls-free manoeuvre stability, over and above controls-free static stability, is controlled by the signs of the hinge moment parameters b_1 and b_2. This, in turn, depends on the aerodynamic design of the elevator control surface.

As for controls-free static stability, the meaning of controls-free manoeuvre stability is easily interpreted by considering the pilot action required to establish a steady symmetric manoeuvre from an initial trimmed level flight condition. Since the controls are "free," this equates to a steady tab angle increment or, more appropriately, a steady control force increment that causes the aeroplane to manoeuvre. Equation (8.43) may be rewritten in terms of the steady and incremental contributions to the total controls-free manoeuvring pitching moment in the same way as equation (8.17):

$$\begin{aligned} C_m + \delta C_m = &\left\{ C_{m_0} + C_{L_w}(h-h_0) - \overline{V}_T\left(\begin{array}{l} C_{L_w}\dfrac{a_1}{a}\left(1-\dfrac{d\varepsilon}{d\alpha}\right)\left(1-\dfrac{a_2 b_1}{a_1 b_2}\right) \\ + a_1\eta_T + C_H\dfrac{a_2}{b_2} + \beta_\eta\left(1-\dfrac{a_2 b_3}{a_3 b_2}\right) \end{array}\right)\right\} \\ &+ \left\{ \delta C_{L_w}(h-h_0) - \overline{V}_T\left(\begin{array}{l} \delta C_{L_w}\dfrac{a_1}{a}\left(1-\dfrac{d\varepsilon}{d\alpha}\right)\left(1-\dfrac{a_2 b_1}{a_1 b_2}\right) \\ + \delta C_H\dfrac{a_2}{b_2} + \delta C_{L_w}\dfrac{a_1 l_T}{\mu_1\bar{\bar{c}}}\left(1-\dfrac{a_2 b_1}{a_1 b_2}\right) \end{array}\right)\right\} \end{aligned} \tag{8.49}$$

Now, in the steady equilibrium flight condition about which the manoeuvre is executed, the pitching moment is zero; thus

$$C_m = C_{m_0} + C_{L_w}(h - h_0) - \overline{V}_T \begin{pmatrix} C_{L_w} \dfrac{a_1}{a}\left(1 - \dfrac{d\varepsilon}{d\alpha}\right)\left(1 - \dfrac{a_2 b_1}{a_1 b_2}\right) \\ + a_1 \eta_T + C_H \dfrac{a_2}{b_2} + \beta_\eta \left(1 - \dfrac{a_2 b_3}{a_3 b_2}\right) \end{pmatrix} = 0 \tag{8.50}$$

and equation (8.49) simplifies to that describing the incremental controls-free pitching moment coefficient:

$$\delta C_m = \delta C_{L_w}(h - h_0) - \overline{V}_T \begin{pmatrix} \delta C_{L_w} \dfrac{a_1}{a}\left(1 - \dfrac{d\varepsilon}{d\alpha}\right)\left(1 - \dfrac{a_2 b_1}{a_1 b_2}\right) \\ + \delta C_H \dfrac{a_2}{b_2} + \delta C_{L_w} \dfrac{a_1 l_T}{\mu_1 \overline{\overline{c}}}\left(1 - \dfrac{a_2 b_1}{a_1 b_2}\right) \end{pmatrix} \tag{8.51}$$

In the steady manoeuvre the incremental pitching moment δC_m is zero and equation (8.51) may be rearranged to give

$$\frac{\delta C_H}{\delta C_{L_w}} = \frac{b_2}{a_2 \overline{V}_T}\left\{(h - h_0) - \overline{V}_T\left(\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{a_1 l_T}{\mu_1 \overline{\overline{c}}}\right)\left(1 - \frac{a_2 b_1}{a_1 b_2}\right)\right\} = -\frac{b_2 H'_m}{a_2 \overline{V}_T} \tag{8.52}$$

In a conventional aeroplane the hinge moment coefficient relates directly to the control stick force, as shown in equation (3.32). Equation (8.52) therefore indicates the very important result that the *stick force per g* control characteristic is proportional to the controls-free manoeuvre margin. This control characteristic is critically important in the determination of longitudinal handling qualities, and it must have the correct value. In other words, the controls-free manoeuvre margin must lie between precisely defined upper and lower bounds. As stated above, in an aerodynamically controlled aeroplane this control characteristic can be adjusted independently of the other stability characteristics by selective design of the values of the hinge moment parameters b_1 and b_2. The controls-free manoeuvre stability is critically dependent on the ratio b_1/b_2, which controls the magnitude and sign of the expression (8.48).

For conventional aeroplanes fitted with a plain flap-type elevator control, both b_1 and b_2 are usually negative and, as shown in equation (8.47), the controls-free manoeuvre stability is greater than the controls-free static stability. Adjustment of b_1 and b_2 is normally achieved by aeromechanical means which are designed to modify the elevator hinge moment characteristics. Typically, this involves carefully tailoring the aerodynamic balance of the elevator by means such as set back hinge line, horn balances, spring tabs, servo tabs, and so on. Excellent descriptions of these devices may be found in Dickinson (1968) and Babister (1961).

8.4.4 Elevator deflection and stick force

The elevator hinge moment is denoted H, where

$$H = \frac{1}{2}\rho U_e^2 S_\eta \bar{\bar{c}}_\eta C_H \tag{8.53}$$

Whence, stick force F_η is given by

$$F_\eta = g_\eta H = \frac{1}{2}\rho U_e^2 S_\eta \bar{\bar{c}}_\eta g_\eta C_H \tag{8.54}$$

where g_η is the stick-to-elevator gearing constant, which has a negative value to be consistent with the controls notation. Now, in order to utilise equation (8.54) to relate normal acceleration to stick force, it is first necessary to express stick force F_η in terms of elevator angle η.

The *controls-fixed* manoeuvre margin H_m defines the relationship between the elevator angle and the increment in lift to manoeuvre about trim, as given by equation (8.29):

$$\frac{\delta\eta}{\delta C_{Lw}} = -\frac{H_m}{\overline{V}_T a_2} \tag{8.55}$$

Similarly, the controls-free manoeuvre margin H'_m defines the relationship between the control hinge moment and the increment in lift to manoeuvre about trim as given by equation (8.52):

$$\frac{\delta C_H}{\delta C_{Lw}} = -\frac{b_2 H'_m}{\overline{V}_T a_2} \tag{8.56}$$

Thus from equations (8.55) and (8.56),

$$\frac{\delta C_H}{\delta\eta} = \frac{b_2 H'_m}{H_m} \tag{8.57}$$

From equations (8.54) and (8.57), the stick force per unit of elevator required to manoeuvre about trim is easily derived:

$$\frac{\delta F_\eta}{\delta\eta} = \frac{g_\eta \delta H}{\delta\eta} = \frac{1}{2}\rho U_e^2 S_\eta \bar{\bar{c}}_\eta g_\eta \frac{\delta C_H}{\delta\eta} = \frac{\frac{1}{2}\rho U_e^2 S_\eta \bar{\bar{c}}_\eta g_\eta b_2 H'_m}{H_m} \tag{8.58}$$

The *incremental stick force per g* then follows by multiplying equation (8.58) by equation (8.39):

$$\frac{\delta F_\eta}{\delta n} = \frac{\delta F_\eta}{\delta\eta}\frac{\delta\eta}{\delta n} = -\frac{b_2}{a_2}\frac{S_\eta c_\eta}{\overline{V}_T S} g_\eta m g H'_m \tag{8.59}$$

Referring to equation (8.59), it is clear that adjustment to stick force per g, without otherwise disturbing the aerodynamic design, can only be effected by modifying the control heaviness parameter b_2 or the mechanical gearing g_η. However, the mechanical gearing g_η is chosen to give the correct control sensitivity, rather than optimising the control force. Thus the only effective means for adjusting control force in a mechanical flying control system remains the surface hinge moment parameters b_1 and b_2. In the event that this solution is inadequate, some form of artificial spring feel system is required. Additionally a servo, geared or spring tab, may be used such that the tab angle β_η becomes a function of elevator angle η. In many instances it is also necessary to employ non-linear gearing in place of linear gearing g_η to ensure adequate handling qualities at all flight conditions.

The measurement of stick force per g is easily undertaken in flight. The aeroplane is flown in steady manoeuvring flight, the turn probably being the simplest way of achieving a steady incremental normal acceleration for a period long enough to enable good-quality measurements to be made. Measurements of stick force and normal acceleration permit estimates of the controls-free manoeuvre margin and the location of the controls-free manoeuvre point. With greater experimental difficulty, stick force per g can also be measured in steady pull-ups and steady push-overs. However the experiment is done, it must be remembered that it is not always possible to ensure that all of the assumptions can be adhered to.

8.5 Aircraft dynamics and manoeuvrability

The preceding analysis shows how the stability of an aeroplane in manoeuvring flight is dependent on the manoeuvre margins and, further, that the magnitude of the manoeuvre margins determines the critical handling characteristics, stick displacement per g, and stick force per g. However, the manoeuvre margins are also instrumental in determining some of the dynamic response characteristics of the aeroplane. This fact further reinforces the statement made elsewhere that static, manoeuvre, and dynamic stability and control characteristics are very much inter-related and should not be treated entirely as isolated topics.

In Chapter 6, reduced-order models of an aircraft are discussed. Also, from the longitudinal model representing short-term dynamic stability and response, an approximate expression for the short-period-mode undamped natural frequency is derived, equation (6.21), in terms of dimensional aerodynamic stability derivatives. With reference to Appendix 2, this expression may be restated in terms of dimensionless derivatives:

$$\omega_s^2 = \frac{\frac{1}{2}\rho V_0^2 S\bar{\bar{c}}}{I_y}\left(\frac{\frac{1}{2}\rho S\bar{\bar{c}}}{m}M_q Z_w + M_w\right) = \frac{\frac{1}{2}\rho V_0^2 S\bar{\bar{c}}}{I_y}\left(\frac{M_q Z_w}{\mu_1} + M_w\right) \tag{8.60}$$

where μ_1 is the longitudinal relative density factor defined in equation (8.9).

Now, with reference to Appendix 8, an approximate expression for Z_w is given as

$$Z_w \cong -C_D - \frac{\partial C_L}{\partial \alpha} = -C_D - a \tag{8.61}$$

for small-perturbation motion in subsonic flight. Since $a >> C_D$, equation (8.61) may be approximated further, and substituting for Z_w in equation (8.60) to obtain

$$\omega_s^2 = \frac{\frac{1}{2}\rho V_0^2 S\bar{\bar{c}}a}{I_y}\left(-\frac{M_q}{\mu_1} - \frac{M_w}{a}\right) = kH_m \equiv k\left(K_n - \frac{M_q}{\mu_1}\right) \tag{8.62}$$

where k is a constant at the given flight condition. Equation (8.62) therefore shows that the undamped natural frequency of the longitudinal short-period mode is directly dependent on the controls-fixed manoeuvre margin. Alternatively, this may be interpreted as a dependency on the controls-fixed static margin and pitch damping. Clearly, since the controls-fixed manoeuvre margin must lie between carefully defined boundaries if satisfactory handling is to be ensured, this implies that the longitudinal short-period mode must also be constrained to a corresponding frequency band. Requirements for flying qualities have been developed from this kind of understanding and are discussed in Chapter 10.

In many modern aeroplanes the link between the aerodynamic properties of the control surface and the stick force is broken by a servo-actuator and other components of the flight control system. In this case, the control forces are provided artificially and may not inter-relate with other stability and control characteristics in the classical way. However, it is obviously important that the pilot's perception of the handling qualities of his aeroplane look like those of an aeroplane with acceptable aerodynamic manoeuvre margins. Since many of the subtle aerodynamic inter-relationships do not exist in aeroplanes employing sophisticated flight control systems, it is critically important to be fully aware of the handling qualities implications at all stages of a flight control system design.

8.6 Aircraft with stability augmentation

All aircraft with a flight control system for enhancing flying qualities require a power actuator to drive the control surfaces, and this breaks, or modifies, the mechanical force feedback path between the surface and the control stick. Consequently, some form of artificial feel system is usually required to restore the control force gradient to an acceptable value. In this illustration it is assumed that all of the stick force is provided by an artificial feel unit whereas in practice some of the feel force might be provided by a feel unit and some may be derived from the surface hinge moment via the mechanical feedback linkage around the surface actuator and/or via the addition of a bob-weight to the mechanical stick installation. The feel unit can take many different forms—for example, a simple linear or non-linear spring or an electric or hydraulic device with characteristics scheduled with appropriate flight parameters. A functional block diagram of a typical simple control system is shown in Fig. 8.3.

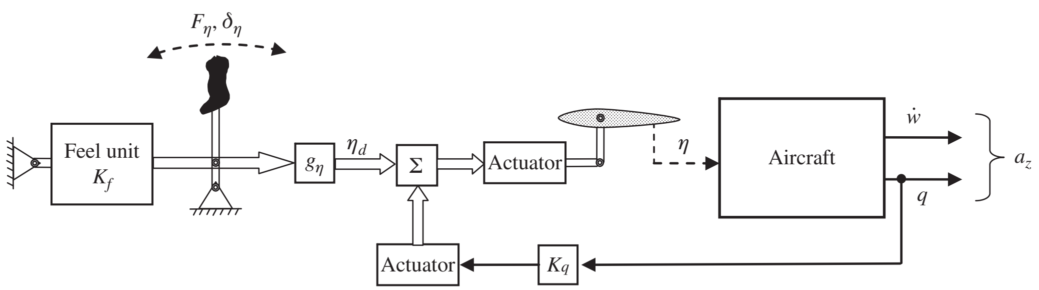

FIGURE 8.3 Simple augmented aircraft control system.

8.6.1 Stick force

The stick force F_η is provided entirely by a "spring" feel unit such that

$$F_\eta = K_f \delta_\eta \tag{8.63}$$

where K_f, the spring constant, may be non-linear and scheduled with flight condition.

8.6.2 Stick force per *g*

With reference to Fig. 8.3, the closed-loop reduced-order transfer functions relating the response variables w and q to the elevator demand variable η_d are given by equations (8.64):

$$\frac{w(s)}{\eta_d(s)} = \frac{k_w\left(s + 1/T_\alpha\right)}{\left(\Delta(s) + K_q k_q\left(s + 1/T_{\theta_2}\right)\right)} \qquad \frac{q(s)}{\eta_d(s)} = \frac{k_q\left(s + 1/T_{\theta_2}\right)}{\left(\Delta(s) + K_q k_q\left(s + 1/T_{\theta_2}\right)\right)} \tag{8.64}$$

Note that K_q is the feedback gain and that the actuator transfer functions are not included in this simple model. Similar to the derivation of equation (8.35), the closed-loop normal load factor response to η_d may be derived using transfer functions (8.64):

$$\frac{\delta n(s)}{\eta_d(s)} = -\frac{\left(k_w\left(s + 1/T_\alpha\right)s - k_q\left(s + 1/T_{\theta_2}\right)U_e\right)}{g\left(\Delta(s) + K_q k_q\left(s + 1/T_{\theta_2}\right)\right)} \tag{8.65}$$

Application of the final value theorem to equation (8.65) leads to an expression for the steady-state gain characteristic,

$$\left.\frac{\delta n}{\eta_d}\right|_{\substack{steady\\ state}} = \frac{k_q U_e}{g(\omega_s^2 T_{\theta_2} + K_q k_q)} \tag{8.66}$$

From equation (8.63) it is easily shown that

$$F_\eta = \frac{K_f}{g_\eta}\eta_d \tag{8.67}$$

Substituting for η_d from equation (8.67) into equation (8.66), the stick force per g characteristic for the augmented aircraft is revealed:

$$\frac{F_\eta}{\delta n} = \frac{K_f g}{g_\eta U_e}\left(\frac{\omega_s^2 T_{\theta_2}}{k_q} + K_q\right) \tag{8.68}$$

Note that the computation of acceleration response to control command, as given by equation (8.66), can normally be obtained as a response transfer function in the solution of the reduced-order equations of motion. Further, acceleration response referred to the *cg*, the pilot's seat, or both can be obtained by this means as required for analysis. Application of equation (8.68) then enables assessment of the stick force per g control characteristic and the design of the feel unit spring constant K_f to achieve the desired manoeuvre handling qualities.

EXAMPLE 8.1

To illustrate the use of transfer functions to evaluate stick force per g, consider the Lockheed F-104A Starfighter aircraft data, which were obtained from Heffley and Jewell (1972). The equations of motion and the data for limited basic handling qualities are given for 10 flight conditions covering the aircraft flight envelope.

For the present example the longitudinal equations of motion for the unaugmented aircraft corresponding to Mach 0.9 at an altitude of 15,000 ft are given by

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.0167 & 0.089 & -79.66 & -32.1 \\ -0.0199 & -1.22 & 948.66 & -1.546 \\ 6.1095e-3 & -0.01942 & -1.4095 & 7.3899e-4 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 17.6 \\ -209 \\ -33.5 \\ 0 \end{bmatrix} \eta \tag{8.69}$$

Note that American imperial units are retained to be consistent with the source data. Since only short-term response to control is of interest, it is appropriate to use the reduced-order model, extracted from the state equation (8.69), as follows:

$$\begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -1.22 & 948.66 \\ -0.01942 & -1.4095 \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix} + \begin{bmatrix} -209 \\ -33.5 \end{bmatrix} \eta \tag{8.70}$$

The normal acceleration response of interest is measured at the pilot's seat and is given by

$$a_{z_p} = a_z - l_x \dot{q} = \dot{w} - U_e q - l_x \dot{q} \tag{8.71}$$

where a_z is the normal acceleration at the aircraft cg and l_x is the distance of the pilot's seat forward of the cg. At the flight condition in question, $l_x = 18.1$ ft and $U_e = 948.66$ ft/s, and expressions for $\dot{w}$ and $\dot{q}$ may be obtained from equation (8.70). Thus, from equation (8.71), expressions for the normal acceleration at the cg and the normal acceleration at the cockpit may be written as

$$\begin{aligned} a_z &= -1.22w - 209\eta \\ a_{z_p} &= -0.8686w + 25.5112q + 397.35\eta \end{aligned} \tag{8.72}$$

Equations (8.72) may be included in the system output equation to provide the acceleration response transfer functions in the solution of the equations of motion. The system state description may then be written as

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{Bu} \\ \mathbf{y} &= \mathbf{Cx} + \mathbf{Du} \end{aligned}$$

where

$$\begin{aligned} \mathbf{x}^{\mathrm{T}} &= [w \quad q] \\ \mathbf{u} &= \eta \\ \mathbf{y}^{\mathrm{T}} &= [w \quad q \quad a_z \quad a_{z_p}] \end{aligned}$$

and

$$\dot{\mathbf{x}} = \begin{bmatrix} -1.22 & 948.66 \\ -0.01942 & -1.4095 \end{bmatrix}\mathbf{x} + \begin{bmatrix} -209 \\ -33.5 \end{bmatrix}\mathbf{u}$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1.22 & 0 \\ -0.8686 & 25.511 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ -209 \\ 397.35 \end{bmatrix}\mathbf{u} \tag{8.73}$$

Solution of equations (8.73) gives the reduced-order response transfer functions:

$$\begin{aligned}
\frac{w(s)}{\eta(s)} &= \frac{-209(s+153.5)}{(s^2+2.629s+20.14)}\frac{\text{ft/s}}{\text{rad}} \\
\frac{q(s)}{\eta(s)} &= \frac{-33.5(s+1.099)}{(s^2+2.629s+20.14)}\frac{\text{rad/s}}{\text{rad}} \\
\frac{a_z(s)}{\eta(s)} &= \frac{-209(s-12.24)(s+13.65)}{(s^2+2.629s+20.14)}\frac{\text{ft/s}^2}{\text{rad}} \\
\frac{a_{z_p}(s)}{\eta(s)} &= \frac{397.4(s^2+0.9353s+87.871)}{(s^2+2.629s+20.14)}\frac{\text{ft/s}^2}{\text{rad}}
\end{aligned} \tag{8.74}$$

Applying the final value theorem to the acceleration response transfer functions, assuming a unit input, values for the steady-state incremental normal acceleration at the *cg* and at the cockpit are easily determined:

$$\left.\frac{a_z(s)}{\eta(s)}\right|_{\substack{steady\\state}} \equiv \left.\frac{a_{z_p}(s)}{\eta(s)}\right|_{\substack{steady\\state}} = 1733.863\frac{\text{ft/s}^2}{\text{rad}} \equiv -0.9397\frac{\text{g}}{\text{deg}} \tag{8.75}$$

It is important to recognise that the quasi-steady manoeuvre value given in equation (8.75) is the same when measured at both the *cg* and the cockpit at the flight condition of interest. However, the dynamic transient response shapes are not the same since the numerators of the acceleration transfer functions in equation (8.74) are clearly different. The difference is shown in Fig. 8.4, where the acceleration response to a unit step input is plotted for the reduced-order transfer functions given in equation (8.74).

It is clear from the figure that the peak transient values of normal acceleration experienced by the pilot at the cockpit are approximately 0.1 *g* less than at the *cg*. The dynamic response "shapes" are otherwise very similar. It is also clear that, by placing the pilot a distance ahead of the *cg*, the instantaneous response to control at $t = 0$ is in the same sense as the command input, whereas at the *cg* the characteristic non-minimum phase effect causes the adverse value of the instantaneous acceleration response. These characteristic differences in normal acceleration contribute positively to the handling qualities of the aircraft, which are based on motion cues measured at the cockpit. To relate the normal acceleration response properties of the airframe to the

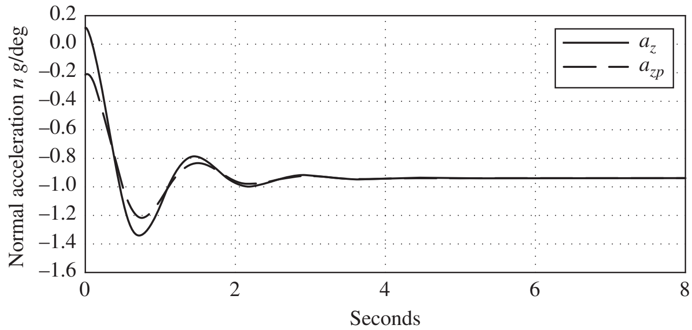

FIGURE 8.4 Normal acceleration response to a unit step input.

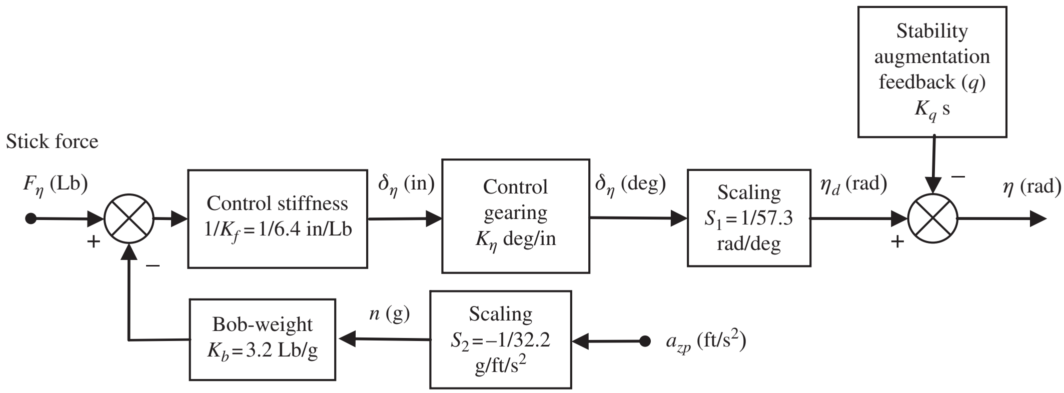

FIGURE 8.5 F-104A longitudinal flying controls.

longitudinal control characteristics requires additional knowledge of the longitudinal flying control system. To facilitate interpretation of the acceleration response characteristics in terms of flying qualities requirements, as discussed in Chapter 10, it is useful to calculate the derivative n_α at this stage:

$$n_\alpha = \frac{U_e}{gT_{\theta_2}} = \frac{948.66}{32.2}1.099 = 32.378\frac{\text{g}}{\text{rad}} \tag{8.76}$$

The longitudinal flying control arrangement of the F-104A, as given by Heffley and Jewell (1972), is shown in Fig. 8.5 using a consistent notational style. In common with many aeroplanes of the late 1950s, the airframe is stable, the controls are mechanical and incorporate artificial feel, and simple rate feedback is used to augment the stability to a more uniform level over the flight envelope. No information about the stability augmentation system is given, but it is safe to assume that pitch rate feedback is used to augment pitch damping and that the feedback gain values typically lie in the range $-0.1 \leq K_q \leq -0.5$ rad/rad/s. A description of the mechanical layout of the system is not given, so it is not known how the artificial feel is implemented.

The stick force reacts against what appears to be mechanical spring stiffness, 6.4 Lb force being required to move the stick control grip 1 in. The stick displacement is then converted to an angular elevator demand η_d through the gearing K_η. The gearing schedule varies non-linearly with trim elevator angle over the range $-2 \leq K_\eta \leq -1.5$ deg/in. approximately. Its purpose is to harmonise control sensitivity over the flight envelope with available elevator travel. To avoid over space control in manoeuvring flight, the stick force required to manoeuvre is increased with the total normal load factor by means of a mechanical bob-weight. The bob-weight is designed to apply a force of 3.2 Lb/g to oppose the pilot force input F_η. Thus the pilot has to pull/push harder to achieve a given manoeuvre as the normal load factor increases. This arrangement represents a fairly rudimentary implementation of mechanical stick force scheduling with flight condition and is typical of combat aircraft of the period.

With reference to Fig. 8.5, the *control law* may be written as

$$\eta_d = \frac{K_\eta S_1}{K_f} F_\eta - \frac{K_b K_\eta S_1 S_2}{K_f} a_{z_p} \tag{8.77}$$

and

$$\eta = \eta_d - K_q q \tag{8.78}$$

Eliminating η_d between equations (8.77) and (8.78), and after some rearrangement,

$$F_\eta = \frac{K_f}{K_\eta S_1}\eta + \frac{K_f K_q}{K_\eta S_1} q + K_b S_2 a_{z_p} \tag{8.79}$$

Whence stick force per g referred to the cockpit may be written as

$$\begin{aligned} \frac{F_\eta}{\delta n_p} &= \frac{-gF_\eta}{a_{z_p}} = \frac{-gK_f}{K_\eta S_1}\frac{\eta}{a_{z_p}} - \frac{gK_f K_q}{K_\eta S_1}\frac{q}{a_{z_p}} - gK_b S_2 \\ &= \frac{K_f}{K_\eta S_1}\frac{\eta}{\delta n_p}\left(1 + K_q\frac{q}{\eta}\right) + K_b \end{aligned} \tag{8.80}$$

In order to determine the steady stick force per g, it is convenient to write equation (8.80) in terms of steady-state transfer functions:

$$\frac{F_\eta}{\delta n} \equiv \left.\frac{F_\eta(s)}{\delta n(s)}\right|_{\substack{steady\\state}} = \frac{K_f}{K_\eta S_1}\left.\frac{\eta(s)}{\delta n(s)}\right|_{\substack{steady\\state}}\left(1 + K_q\left.\frac{q(s)}{\eta(s)}\right|_{\substack{steady\\state}}\right) + K_b \tag{8.81}$$

From equations (8.33) and (8.36) it is easily established that

$$\left.\frac{\eta(s)}{\delta n(s)}\right|_{\substack{steady\\state}} = \frac{g\omega_s^2 T_{\theta_2}}{k_q U_e} \qquad \left.\frac{q(s)}{\eta(s)}\right|_{\substack{steady\\state}} = \frac{k_q}{\omega_s^2 T_{\theta_2}} \tag{8.82}$$

Whence equation (8.81) may be written as

$$\frac{F_\eta}{\delta n} = \frac{gK_f}{g_\eta U_e}\left(\frac{\omega_s^2 T_{\theta_2}}{k_q} + K_q\right) + K_b \tag{8.83}$$

Note that $g_\eta = K_\eta S_1$. Comparison of equation (8.83) with equation (8.68) shows that the effect of the bob-weight is additive and simply increases the apparent stick force per g control characteristic.

Since the pitch rate feedback gain K_q is not given, it is a simple matter to design a suitable value with the aid of the root locus plot using the procedure described in Example 11.4. Since pitch rate feedback to elevator is required to augment longitudinal stability, the root locus is constructed using the second transfer function in equations (8.74). To raise the short-period mode damping to $\zeta_s = 0.7$ requires a feedback gain of $K_q = -0.13$ rad/rad/s, the gain value used in the following analysis.

The values of the additional parameters in equation (8.83) required to evaluate the stick force per g at the flight condition of interest were obtained from Heffley and Jewell (1972) and from the solution of the equations of motion (8.74). They are given in Table 8.1.

Table 8.1 Stick Force per g Parameter Values

Parameter	Value	Units
g	32.2	ft/s^2
K_f	6.4	Lb/in
K_η	−1.49	deg/in.
U_e	948.66	ft/s
$\omega_s{}^2$	20.14	1/s^2
$T_{\theta 2}$	0.9099	s
k_q	−33.5	1/s
K_q	−0.13	s
K_b	3.2	Lb/g
S_1	1/57.3	rad/deg
S_2	−1/32.2	g/ft/s^2
$g_\eta = K_\eta S_2$	−0.026	rad/in.

Whence

$$\frac{F_\eta}{\delta n} = 8.856\ \mathrm{Lb/g}$$

or

$$\frac{F_\eta}{\delta n} = 7.77\ \mathrm{Lb/g}$$

when $K_q = 0$.

The value of $F_\eta/\delta n$ given by Heffley and Jewell (1972) is 7.76 Lb/g. A value close to this is obtained from equation (8.83) when the pitch rate feedback gain K_q is assumed zero, presumably on the basis that its magnitude is small. Since a value for K_q is not given with the aircraft data, it is assumed that its contribution to the transfer function information given by Heffley and Jewell (1972) is ignored on the basis that its significance is small in the present context.

Equivalently, it may be more convenient to calculate stick force per g using the steady values of transfer functions as given by equation (8.81). From equations (8.74), applying the final value theorem for a unit input, the steady-state value of the pitch rate response transfer function is easily evaluated:

$$\left.\frac{q(s)}{\eta(s)}\right|_{\substack{steady\\state}} = -1.828\ 1\ \mathrm{rad/s/rad}$$

From equation (8.75),

$$\left.\frac{\eta(s)}{\delta n(s)}\right|_{\substack{steady\\state}} = -\frac{1}{53.847} = -0.01857\ \mathrm{rad/g}$$

Thus, evaluating equation (8.81) directly, it is confirmed that stick force per g for the augmented aircraft at the example flight condition is

$$\frac{F_\eta}{\delta n} = 8.856\ \mathrm{Lb/g}$$

References

Babister, A. W. (1961). *Aircraft stability and control.* London: Pergamon Press.

Dickinson, B. (1968). *Aircraft stability and control for pilots and engineers.* London: Pitman.

Gates, S. B., & Lyon, H. M. (1944). *A continuation of longitudinal stability and control analysis; part 1, general theory.* Reports and Memoranda No. 2027. Aeronautical Research Council Her Majesty's Stationery Office, London.

Heffley, R. K., & Jewell, W. F. (1972). *Aircraft handling qualities data.* NASA Contractor Report, NASA CR-2144. Washington, DC: National Aeronautics and Space Administration.