

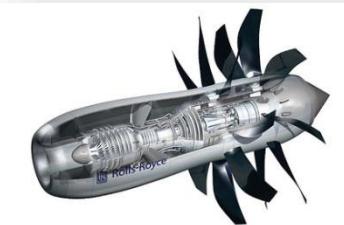
# Propellers and Ducted Fans



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# Analysis of Propeller Aerodynamics

## Lecture 2

Notes in Blackboard: <https://www.ole.bris.ac.uk>

# The Analysis

This is normally achieved through combinations of

Momentum Theory



Blade Element  
Analysis

OR

Vortex Lifting Line Theory



Blade Element  
Analysis

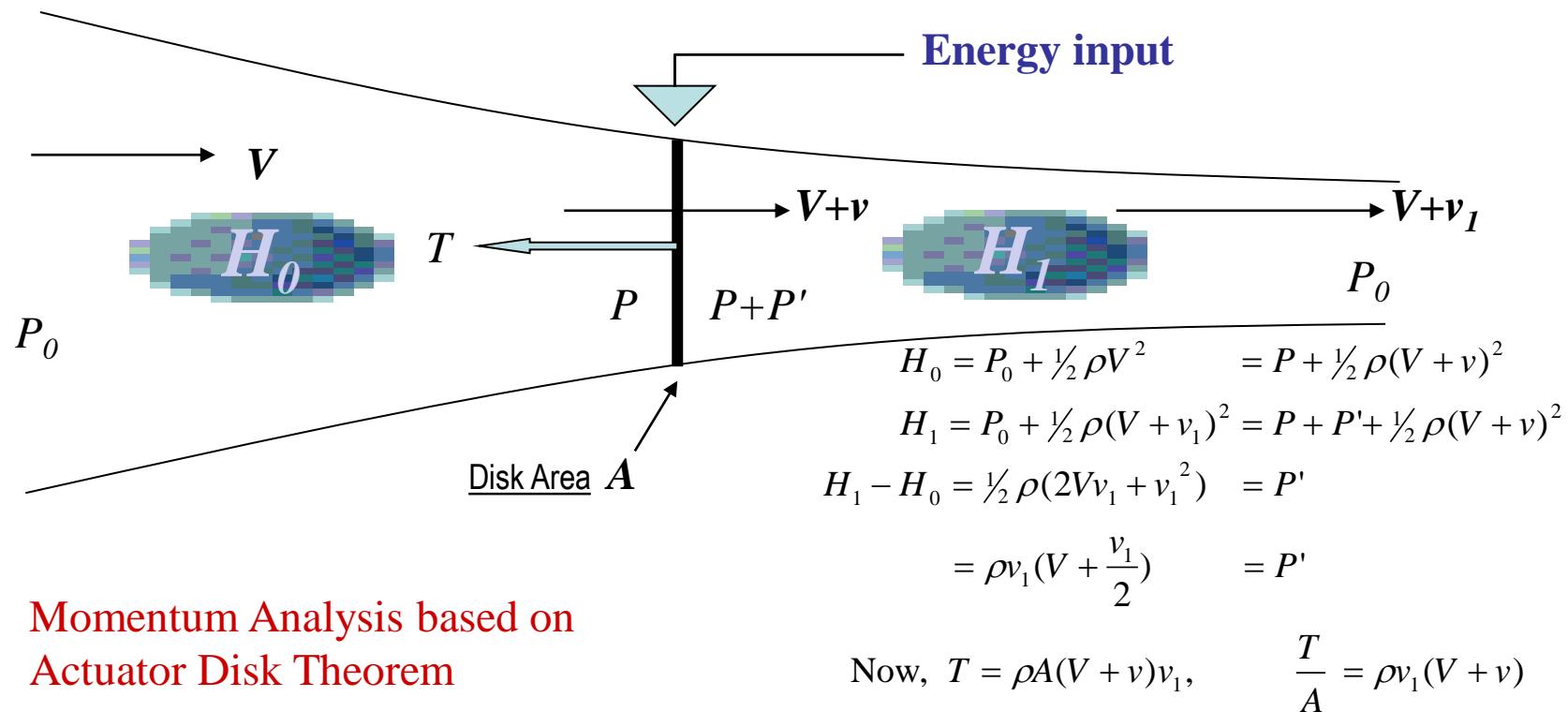
# MOMENTUM THEORY WITH SWIRL



# Actuator Disc Theorem

The Propeller or Axial Fan.

Mechanical energy (in the form of rotating blades) is used to accelerate (***a***) a mass (***m***) of air. Newton's law (every action has a reaction), states ***F = ma***, where ***F***, is the propeller thrust (***T***).



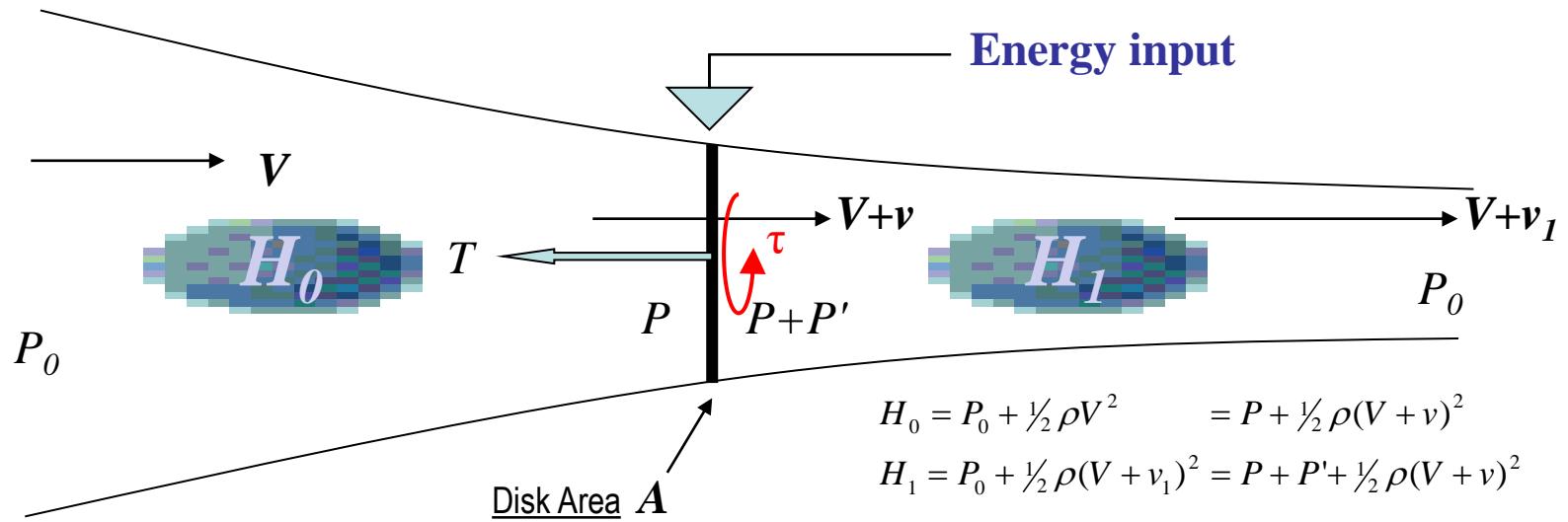
Momentum Analysis based on  
Actuator Disk Theorem

$\rho$  Air density  
 $A$  Disc area

$$so \frac{v_1}{2} = v$$

$$\therefore T = 2\rho A(V + v)v$$

The actuator disk can add only static pressure to the flow and nothing else



Momentum Analysis based on Actuator Disk Theorem can be extended to include swirl, though strictly speaking it can no longer be referred to as an Actuator Disk.

$$H_0 = P_0 + \frac{1}{2} \rho V^2 = P + \frac{1}{2} \rho (V + v)^2$$

$$H_1 = P_0 + \frac{1}{2} \rho (V + v_1)^2 = P + P' + \frac{1}{2} \rho (V + v)^2$$

More generally in both  $H_0$  and  $H_1$  states;

$$\bar{P} + \frac{1}{2} \rho \bar{V}^2 = \text{constant}, \text{ thus } \frac{\bar{P}}{\rho} + \frac{1}{2} \bar{V}^2 = \text{constant}$$

$$\text{or } \frac{\bar{P}}{\rho} + \frac{1}{2} \bar{V}^2 + \frac{K^2}{A} = \text{constant, where } K = \frac{\tau}{\dot{m}}$$

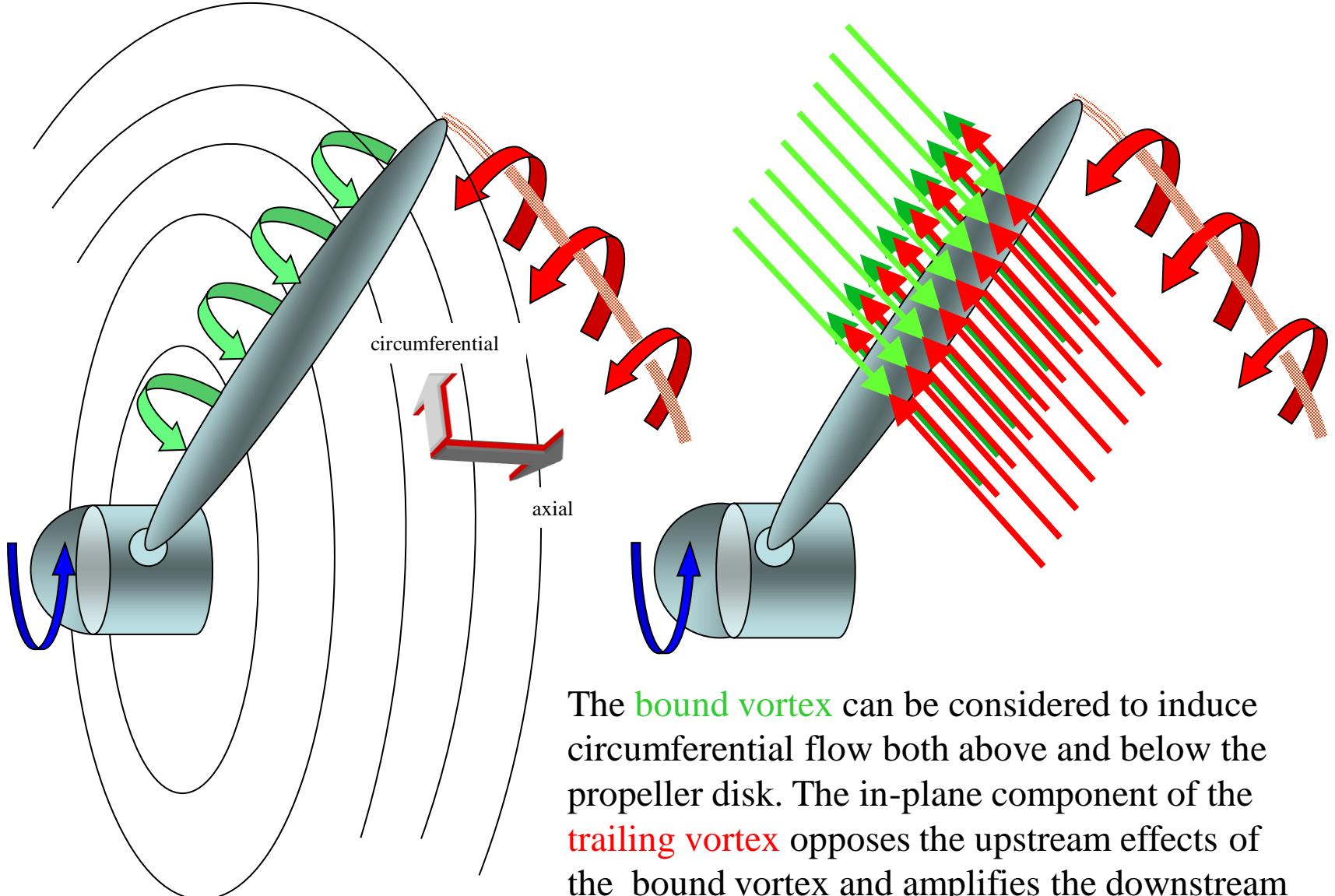
[“ $K$ ” is a “swirl parameter” based on a free vortex and  $\tau$  is fan torque and  $\dot{m}$  is the mass flow rate]

The swirl power =  $\frac{\dot{m}K^2}{A}$  (watts) is supplied directly to the air by the fan.

For the real flow applications that follow in the Blade Element Analysis, the swirl is shown to be the result of combined **blade viscous effects** and in-plane contributions from the **lift induced velocity at the rotor plane**.

It should be noted that the swirl exists only in the **downstream wake**. Upstream of the rotor the flow is irrotational for reasons to be explained later in this lecture.

This is of course why in wind tunnel design the working section is (almost) always upstream of a non-contra-rotating fan.



The **bound vortex** can be considered to induce circumferential flow both above and below the propeller disk. The in-plane component of the **trailing vortex** opposes the upstream effects of the bound vortex and amplifies the downstream effects of the bound vortex.

# BLADE ELEMENT ANALYSIS



The **REAL**ity is that **IDEAL** PROPELLERS don't exist, **REAL** PROPELLERS do!

In analysing and predicting propeller / rotor performance we need to consider:

- Viscosity of the fluid
- Rotor blade tip effects
- Non-uniform inflow
- Blade vortex interaction

$$\eta_p = \frac{TV}{P} \left( = \frac{TV}{T(V + v) + \underline{\text{Losses}}} \right)$$

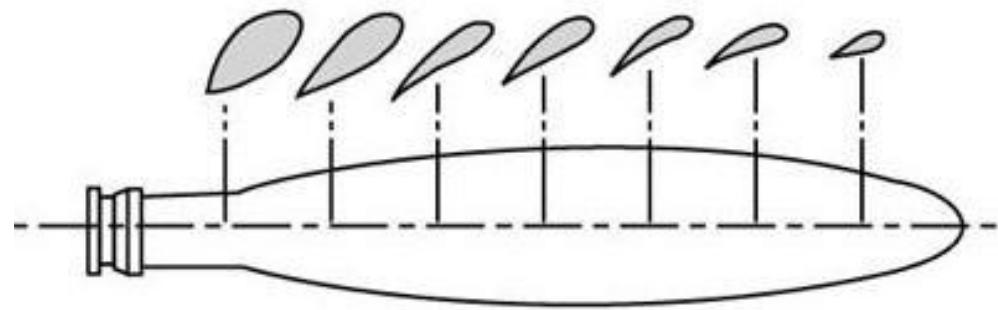
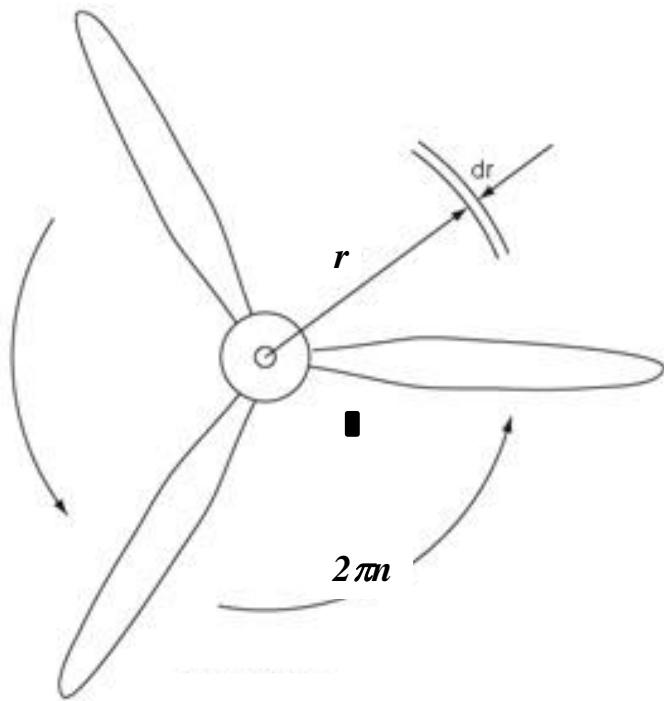
Needless to say, these result in energy loss and therefore reduced efficiency.

In order to obtain a more detailed knowledge of the behaviour of a rotor:

- It is necessary to analyse the forces on the rotor blades.
- The blade has to be considered as a number of separate aerofoil elements.
- Elements are then integrated to represent the characteristics of the whole propeller.

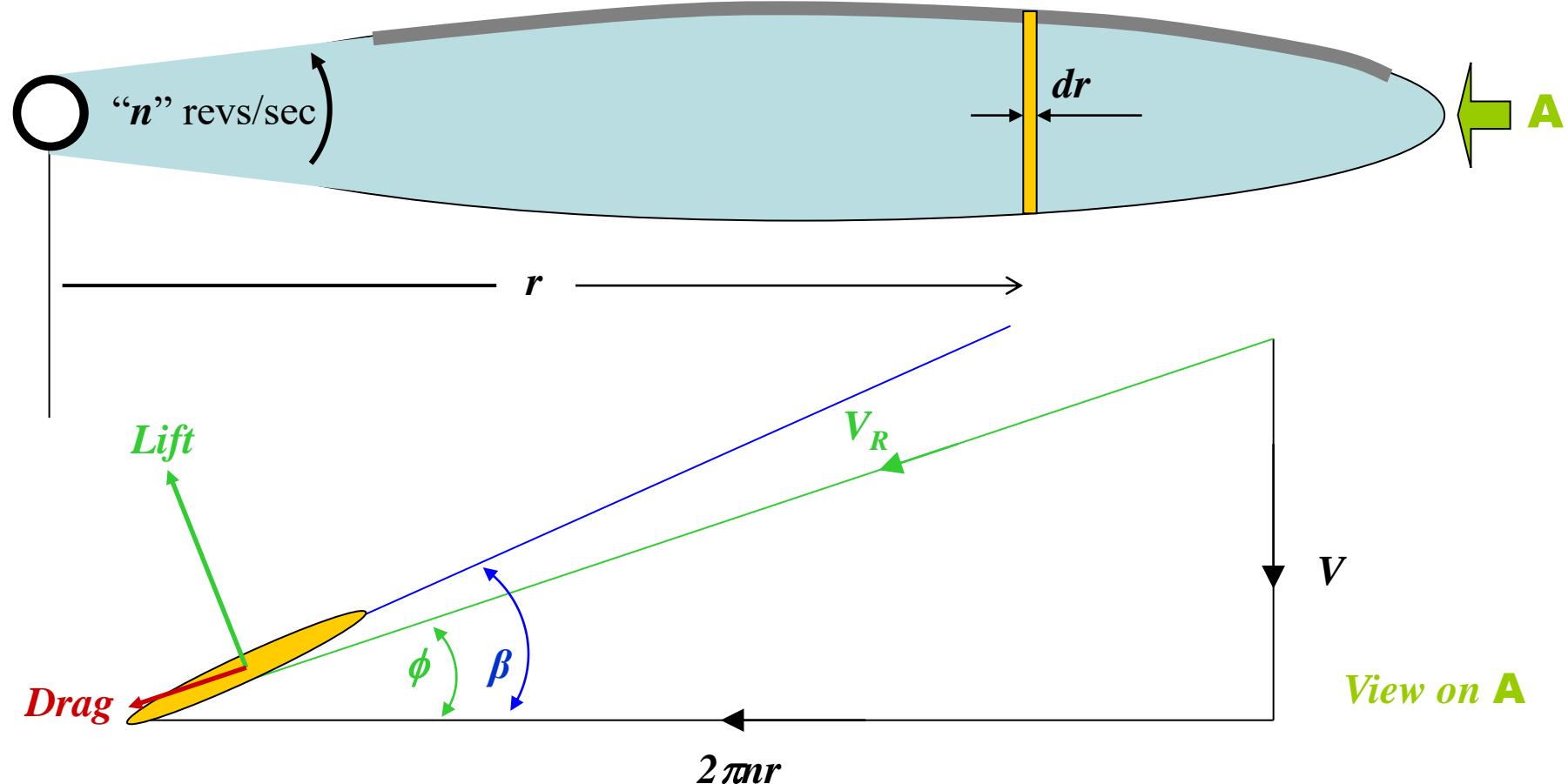
This is achieved by **BLADE ELEMENT** (often called **STRIP ANALYSIS**)

# Blade Element Analysis



# Blade Element Analysis

Considering an element of a propeller blade at a radius  $r$  and of width  $dr$ , the forces on the blade element can be resolved into an axial component  $\delta T$  and an in-plane component  $\delta F$ .

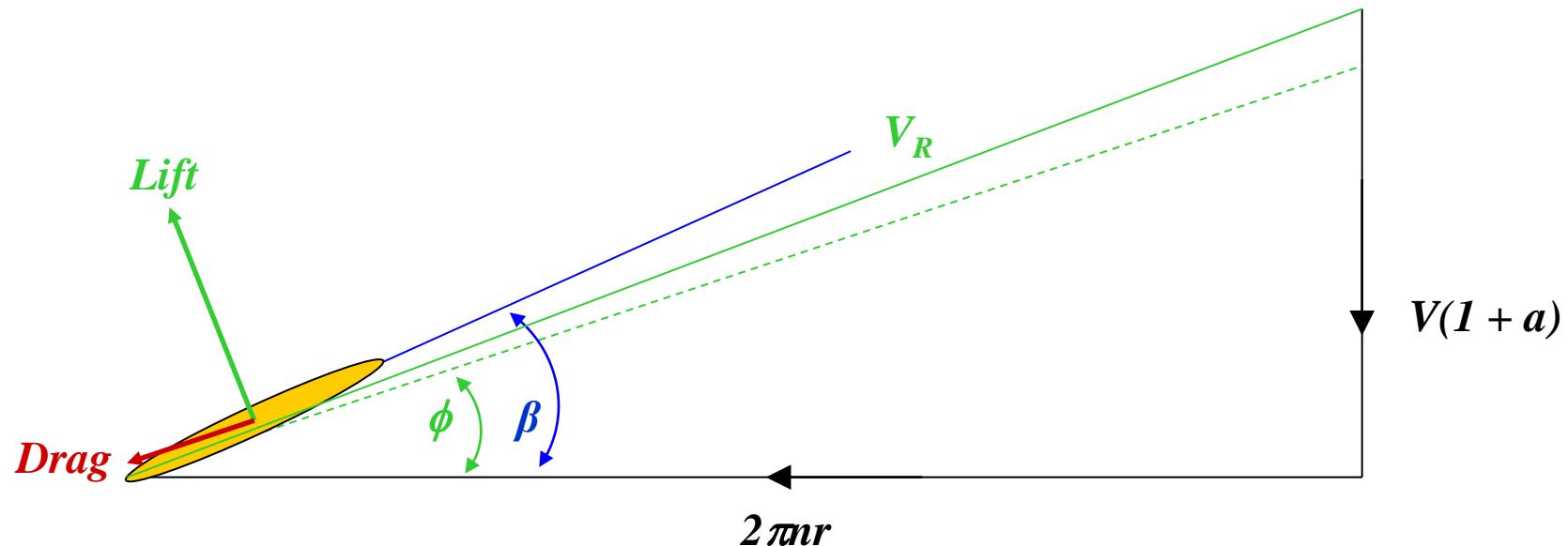


# Interference Flows

These are the axial and rotational flows that are in addition to those of the rotor.

## Axial

The axial interference flow (also referred to as the "induced" velocity  $v$ ) has been determined previously by momentum analysis. It adds to the onset velocity  $V$ . This increases the inflow angle, tilts the lift vector backward resulting in induced drag and reduces the angle of incidence given by  $\alpha = (\beta - \phi)$ .

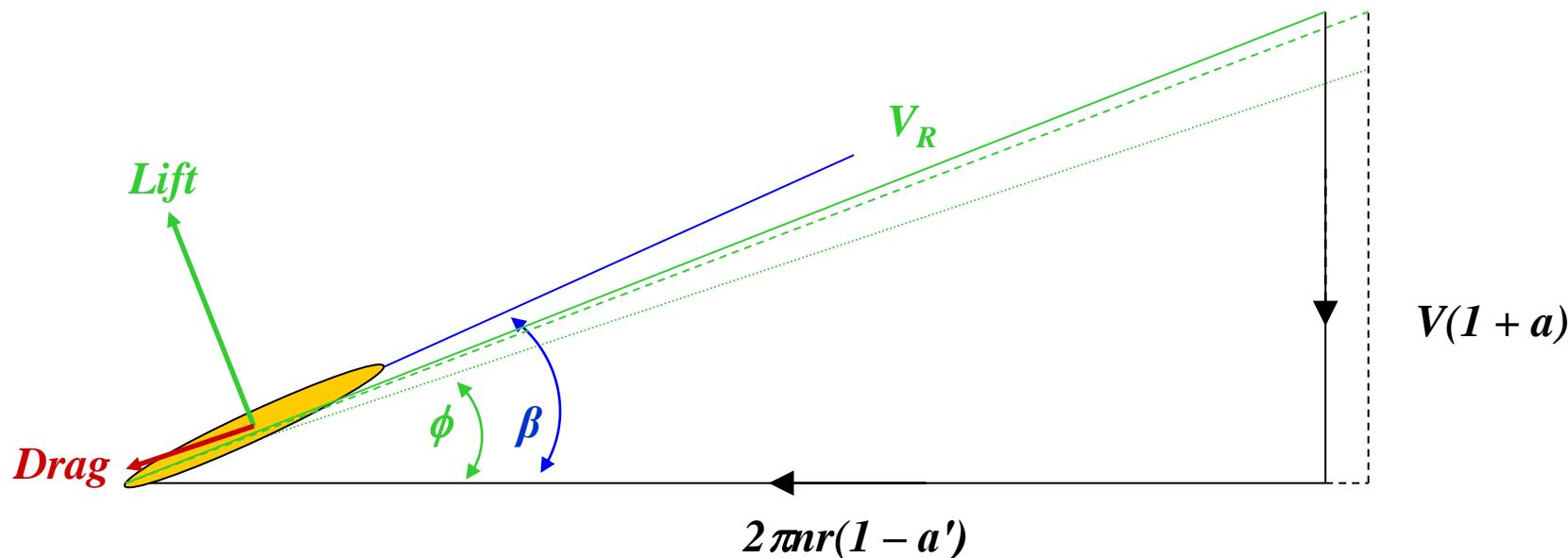


# Interference Flows

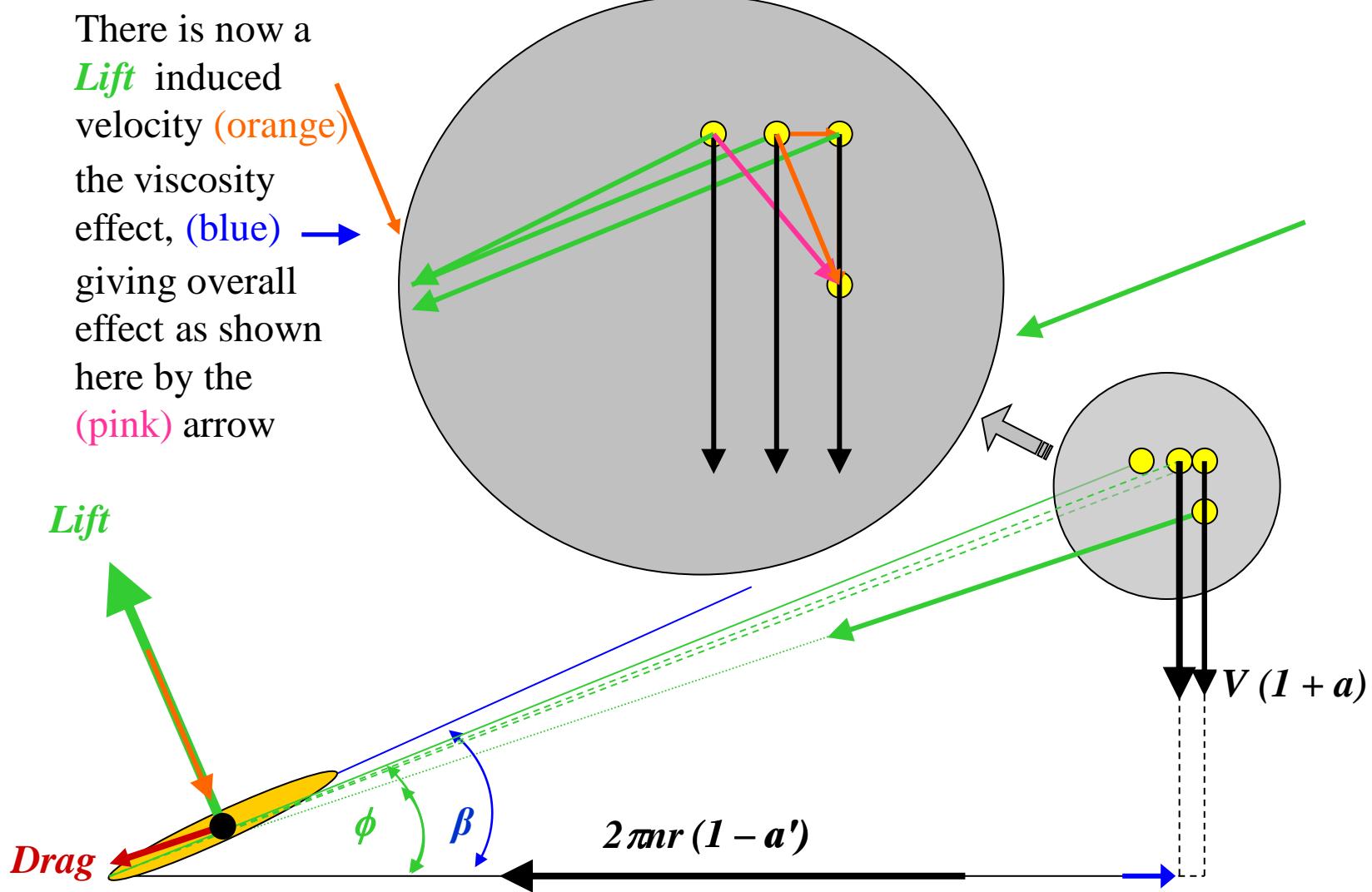
## Radial

The rotational interference flow (which we shall call  $w$ ) can be determined from vortex theory and is physically represented by the swirl of the rotor wake. It is suffice to say that it has a value  $w$  at the rotor and  $2w$  in the far wake. Unlike the axial interference flow, it has no value upstream of the rotor and it is not wholly induced, as it contains a contribution from the viscous drag of the rotor blades.

This also increases the inflow angle, tilting the lift vector backward and further increasing induced drag and reducing the angle of incidence given by  $\alpha = (\beta - \phi)$ .

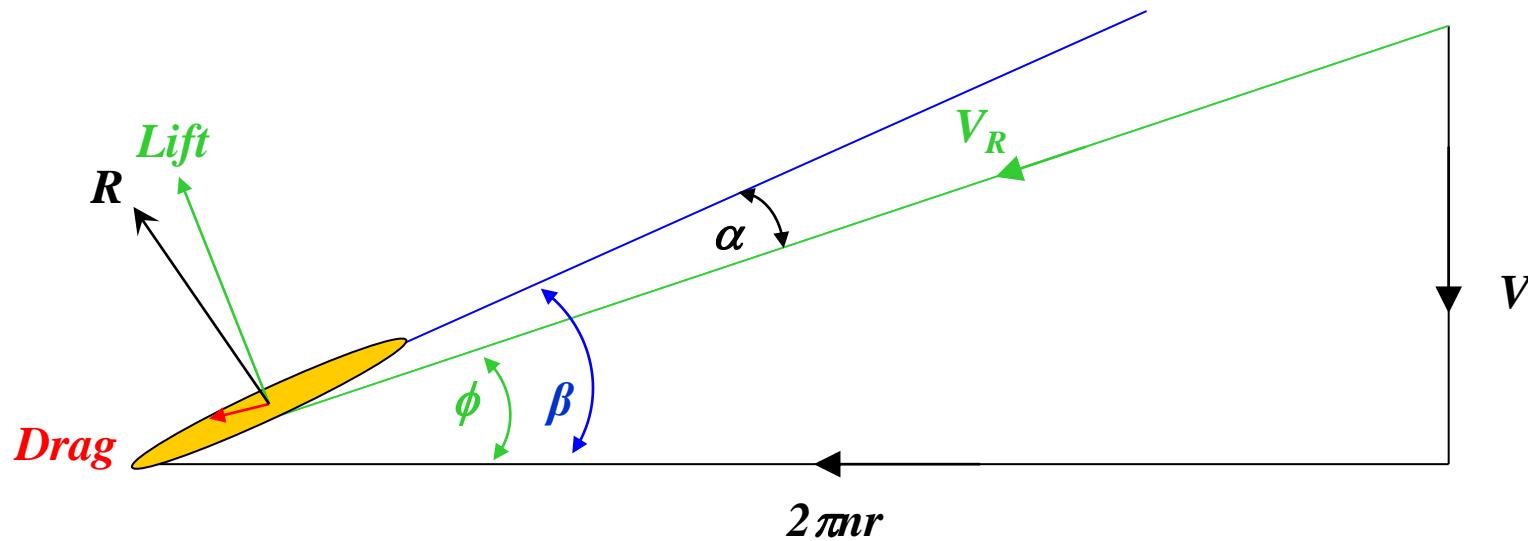


There is now a **Lift** induced velocity (orange) the viscosity effect, (blue) giving overall effect as shown here by the (pink) arrow



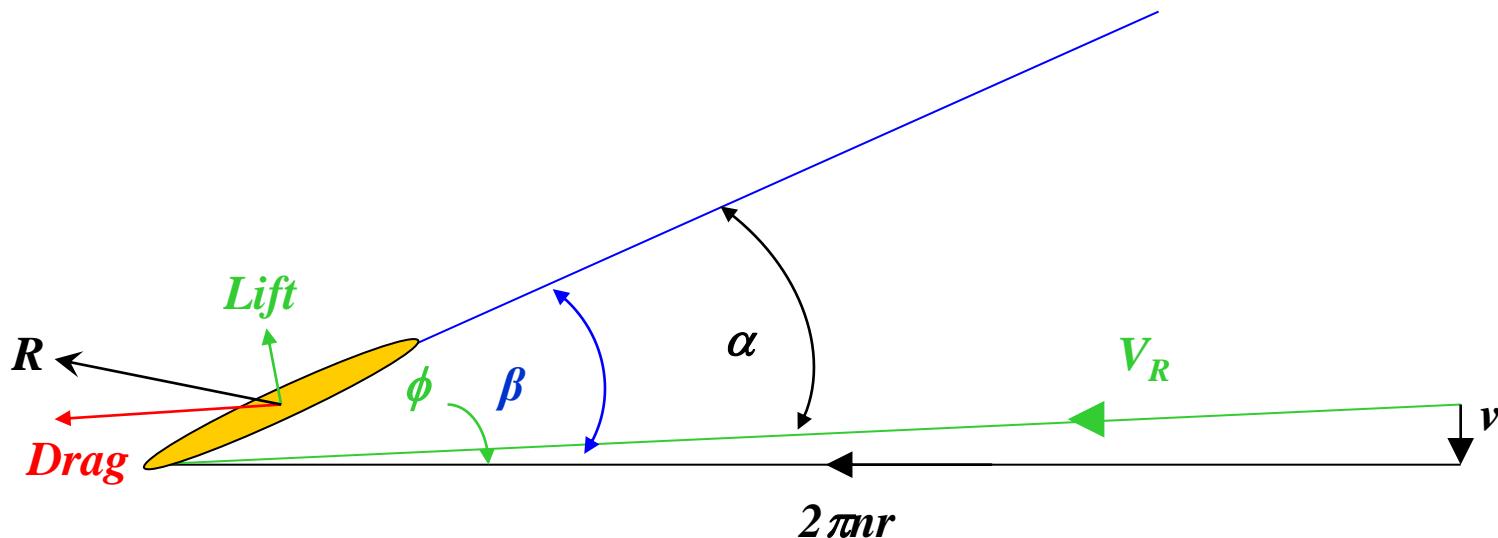
Generally, for propellers in flight,  $V >> v$  and so the axial and rotational interference factors can be neglected. This is not true during the take-off phase of flight and this will be discussed later.....

The velocity diagram below represents propeller inflow for an aircraft in **steady flight** with the angle of incidence “ $\alpha$ ” being a **modest angle**, most probably close to the angle for best lift/drag of the blade aerofoil section.



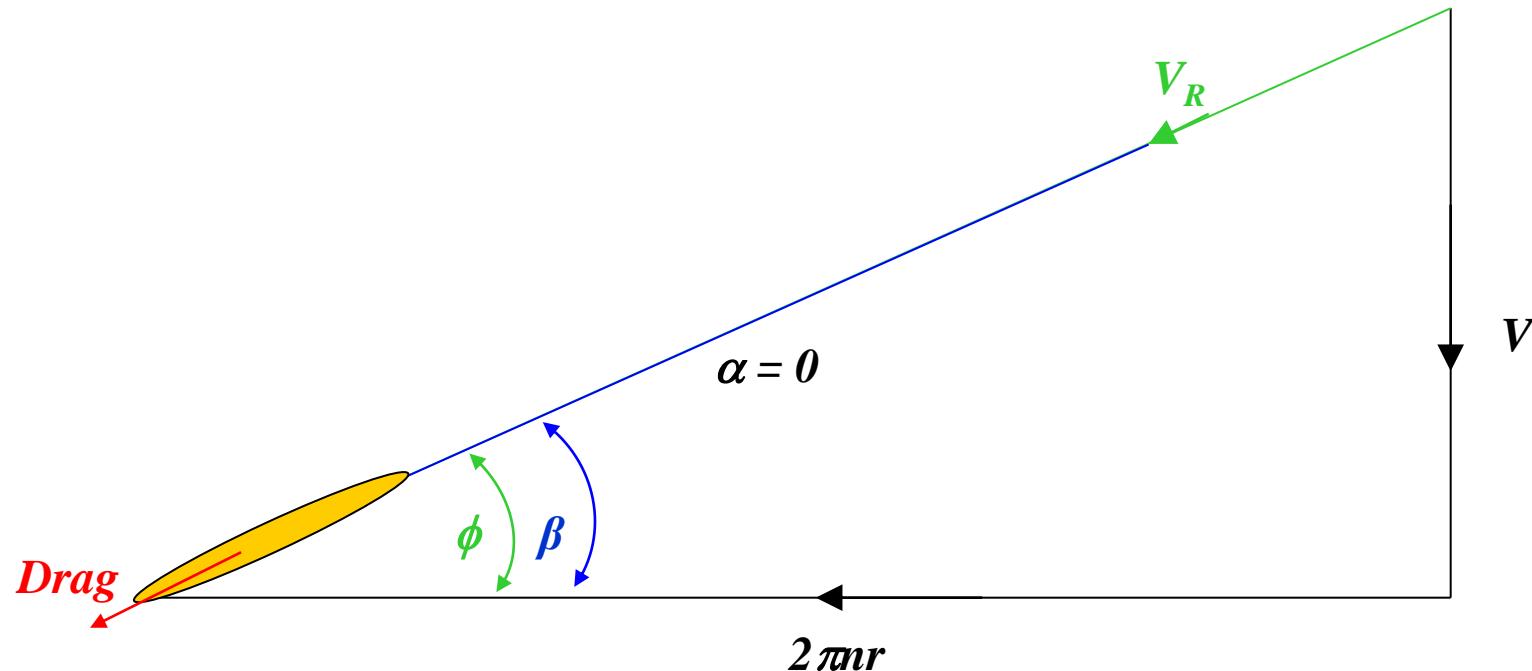
If the blade **Pitch Angle  $\beta$**  is fixed and the propeller rotational speed is constant, which is normally the case, then the optimum angle of incidence (for best lift/drag) occurs at only one aircraft velocity.

The velocity diagram below represents propeller inflow for an aircraft at the start of the **take-off** run with the angle of incidence “ $\alpha$ ” being a **very large angle** and well beyond the point of stall. Whilst some lift will be produced it will be small compared with the drag resulting in low thrust and high torque.



If the blade Pitch Angle ( $\beta$ ) remains fixed and the propeller rotational speed is constant then there is a limit to the aircraft speed if it relies upon the thrust from the propeller.

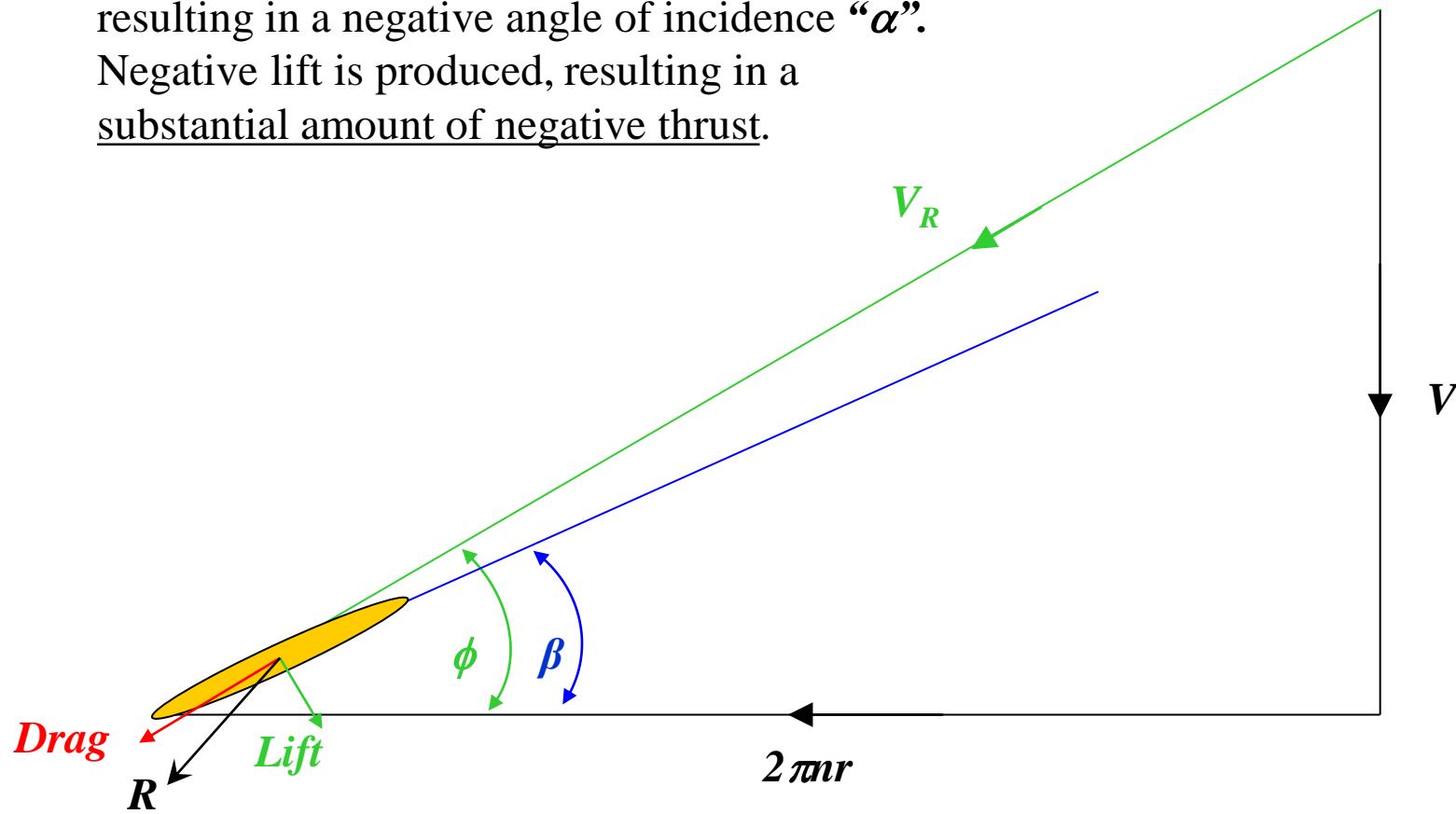
The velocity diagram below represents propeller inflow for an aircraft in a shallow dive with the Inflow Angle equal to the Pitch Angle resulting in zero angle of incidence “ $\alpha$ ”. No lift is produced, resulting in a small amount of negative thrust.



If the blade **Pitch Angle  $\beta$**  remains fixed and the propeller rotational speed is constant then the propeller can provide a limit to the aircraft speed, a useful feature when diving in cloud, akin to speed limiting brakes.

The velocity diagram below represents propeller inflow for an aircraft in a **steep dive** with the **Inflow Angle** exceeding the **Pitch Angle** resulting in a negative angle of incidence “ $\alpha$ ”.

Negative lift is produced, resulting in a substantial amount of negative thrust.

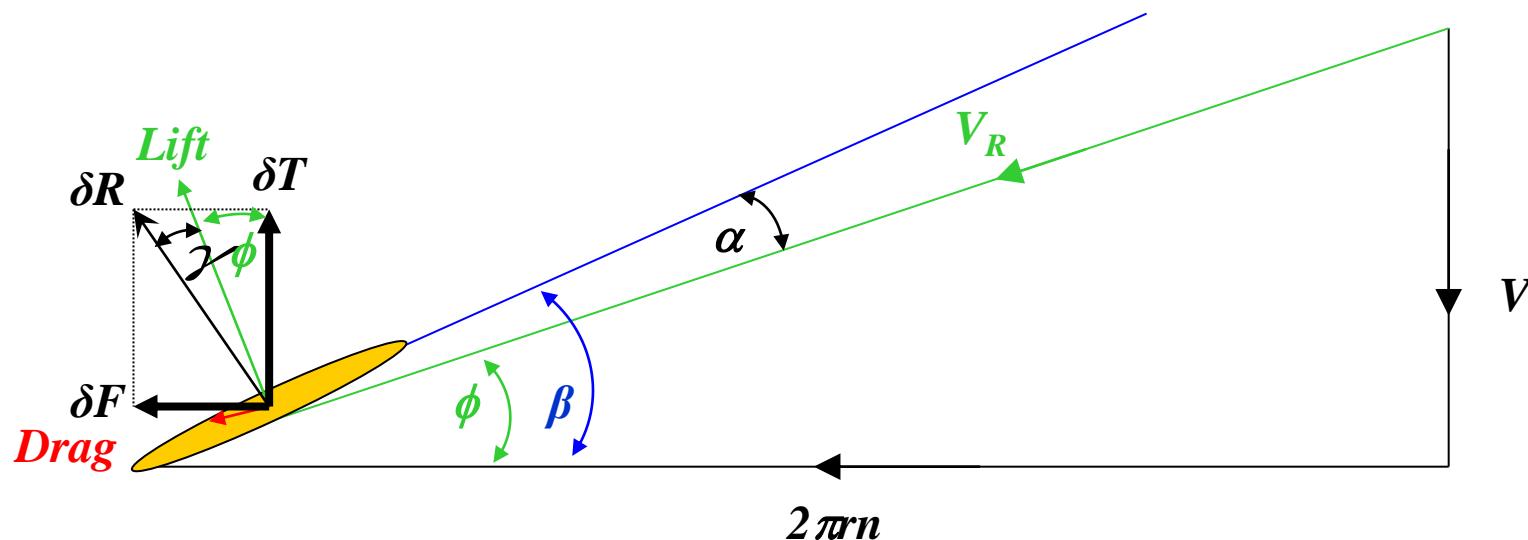


If  $\gamma$  is the angle between the lift component and the resultant force then:

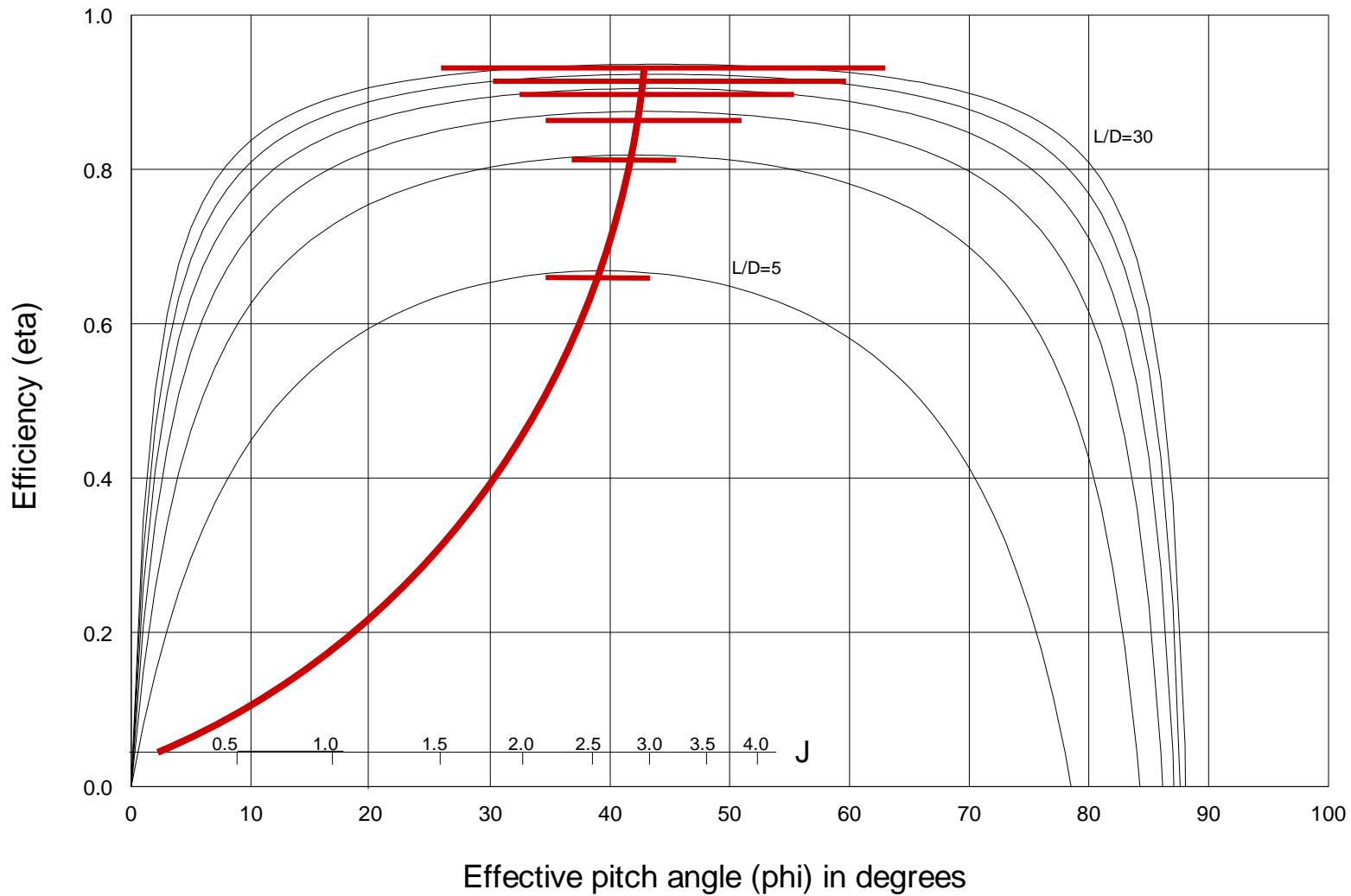
$$\tan \gamma = \frac{D}{L} = \frac{C_D}{C_L}$$

$$\eta' = \frac{\delta T V}{\delta F 2 \pi r n} = \frac{\delta R \cos(\phi + \gamma) V}{\delta R \sin(\phi + \gamma) 2 \pi r n} = \frac{\tan \phi}{\tan(\phi + \gamma)}$$

... using  $\tan \phi = \frac{V}{2 \pi r n}$



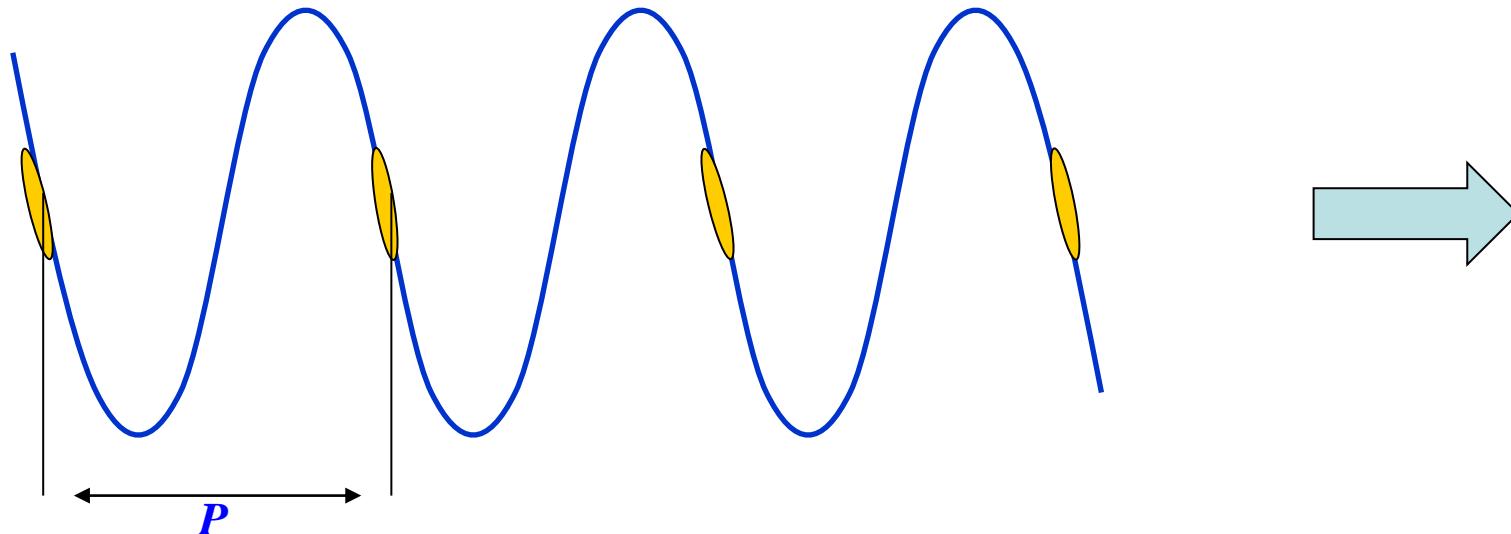
### Blade-element efficiency as a function of effective pitch angle



# Propeller Pitch

Whilst Propeller Pitch Angle ( $\beta$ ) has been referred to in the previous illustrations, Propeller Pitch is a linear distance not an angle and if Propeller Pitch Angle is quoted it should state the radial station at which it is measured (e.g.  $\beta_{0.75R}$ )

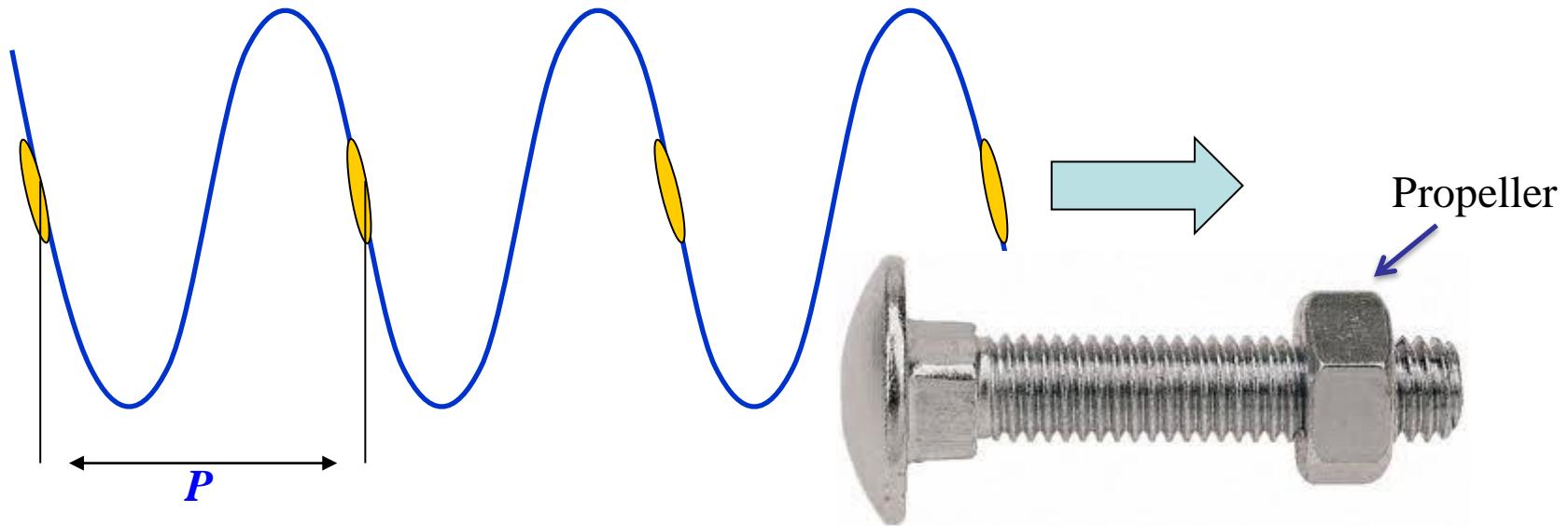
The Geometric Pitch ( $P$ ) of a propeller is the axial displacement of the propeller prescribed by the geometric chord in one revolution. This is analogous with the mechanical screw thread (which is why propellers were originally called "airscrews").



# Propeller Pitch

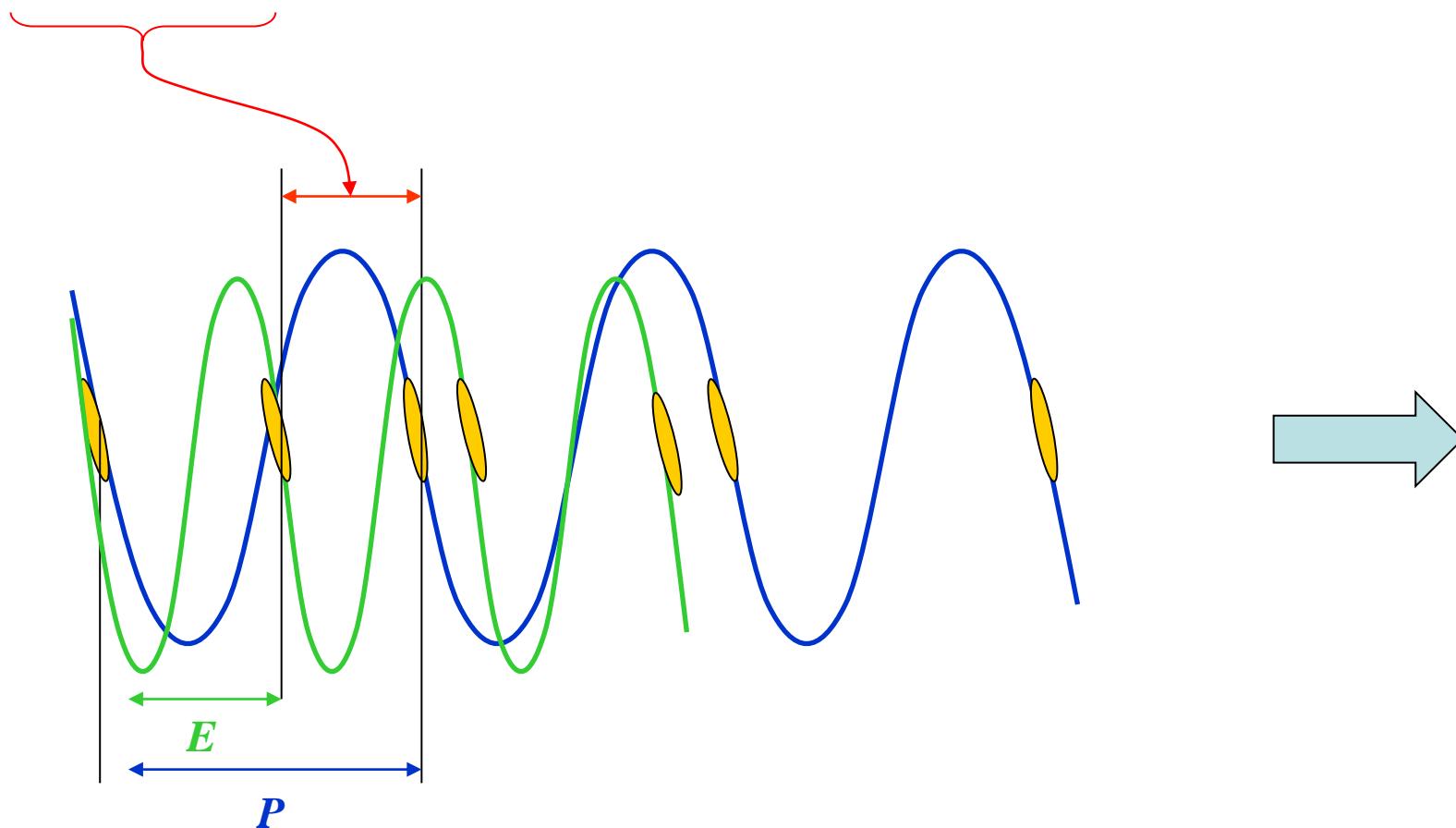
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# Propeller Pitch

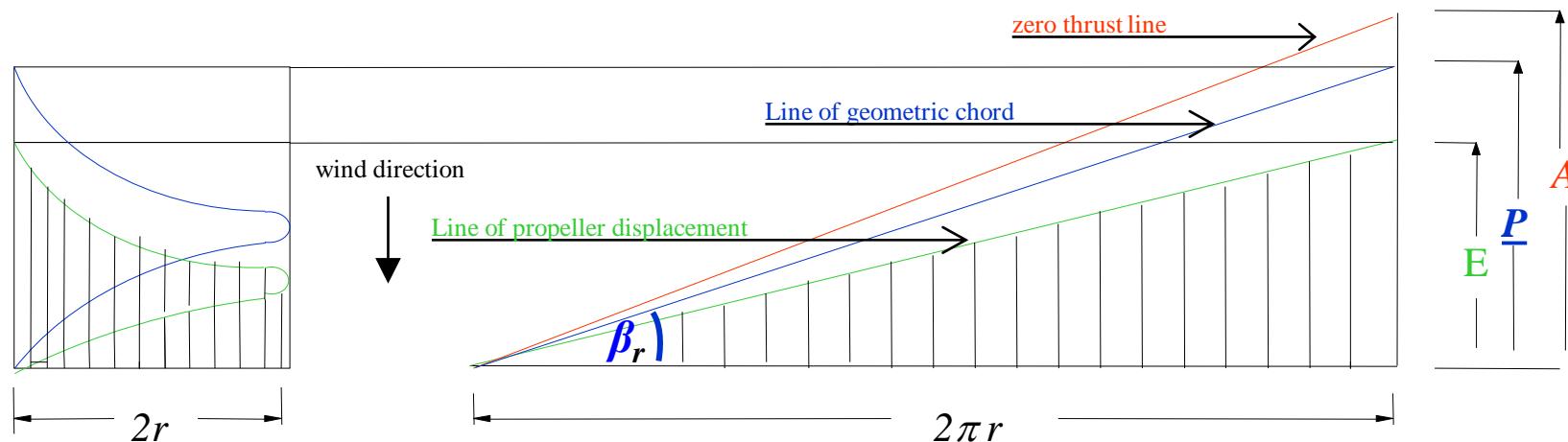
Whilst the **Geometric Pitch ( $P$ )** of a propeller is the **theoretical axial displacement** of the propeller prescribed by the geometric chord in one revolution, the **Effective Pitch ( $E$ )** is the **actual axial displacement** of the propeller. The difference between these two lengths is called the **Propeller Slip**. If thrust is to be produced there must be a finite value of **Propeller Slip**.



# Propeller Pitch

The **Geometric Pitch ( $P$ )** of a propeller cannot be easily measured and this is why use is often made of the **Propeller Pitch Angle ( $\beta$ )** as it can be determined with a protractor. The **Effective Pitch ( $E$ )** is similarly difficult to measure as a length but in unit time it represents the velocity of the aircraft.

Thus the “developed view” of the propeller geometry is analogous of the velocity diagrams previously described.



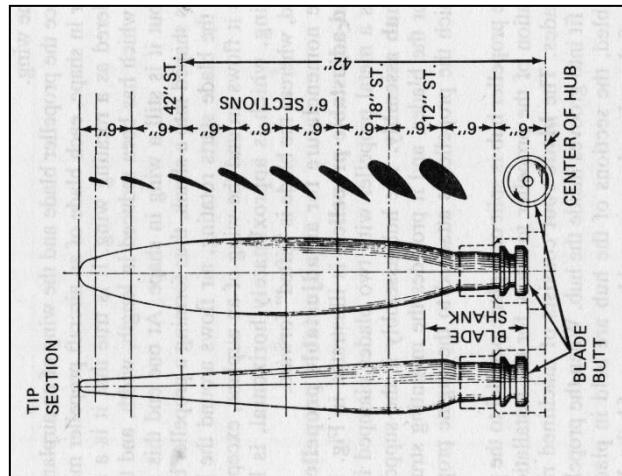
$E$  = Effective Pitch

$P$  = Geometric Pitch

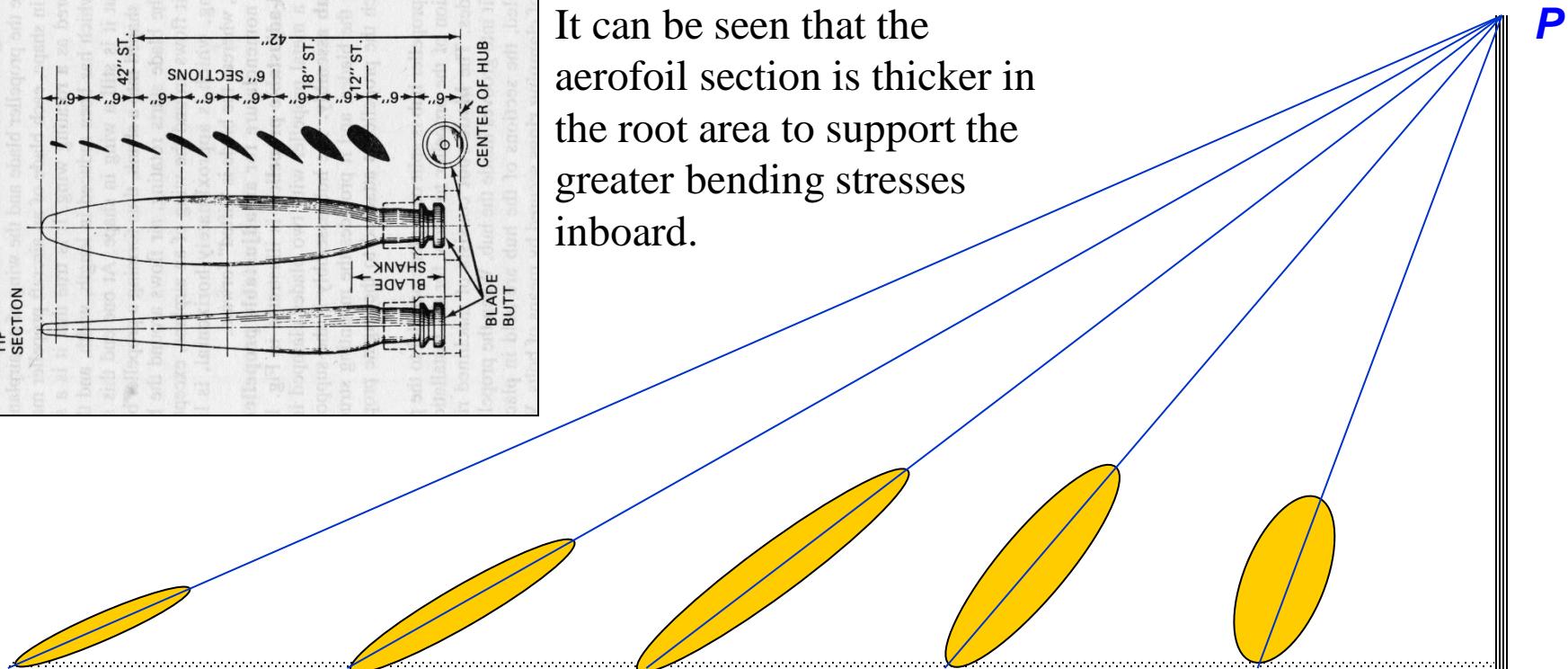
$A$  = Aerodynamic Pitch

# Propeller Pitch

The discussion so far has referred to the **Geometric Pitch ( $P$ )** of an element of the propeller at one radial station. If the propeller blade has a radial twist such that all stations have the **same Geometric Pitch** the propeller is described as a **Constant Pitch Propeller**, as shown below.



It can be seen that the aerofoil section is thicker in the root area to support the greater bending stresses inboard.



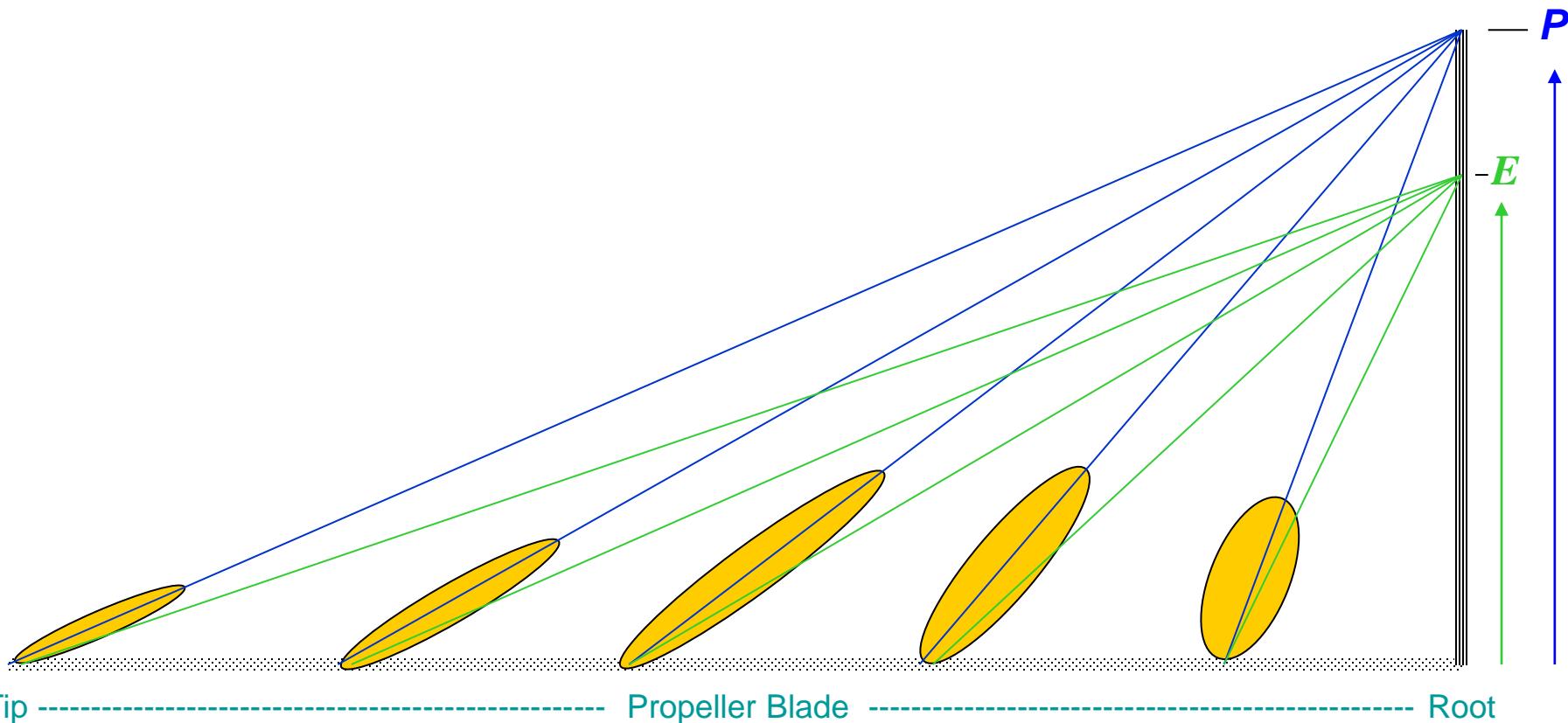
Tip

Propeller Blade

Root

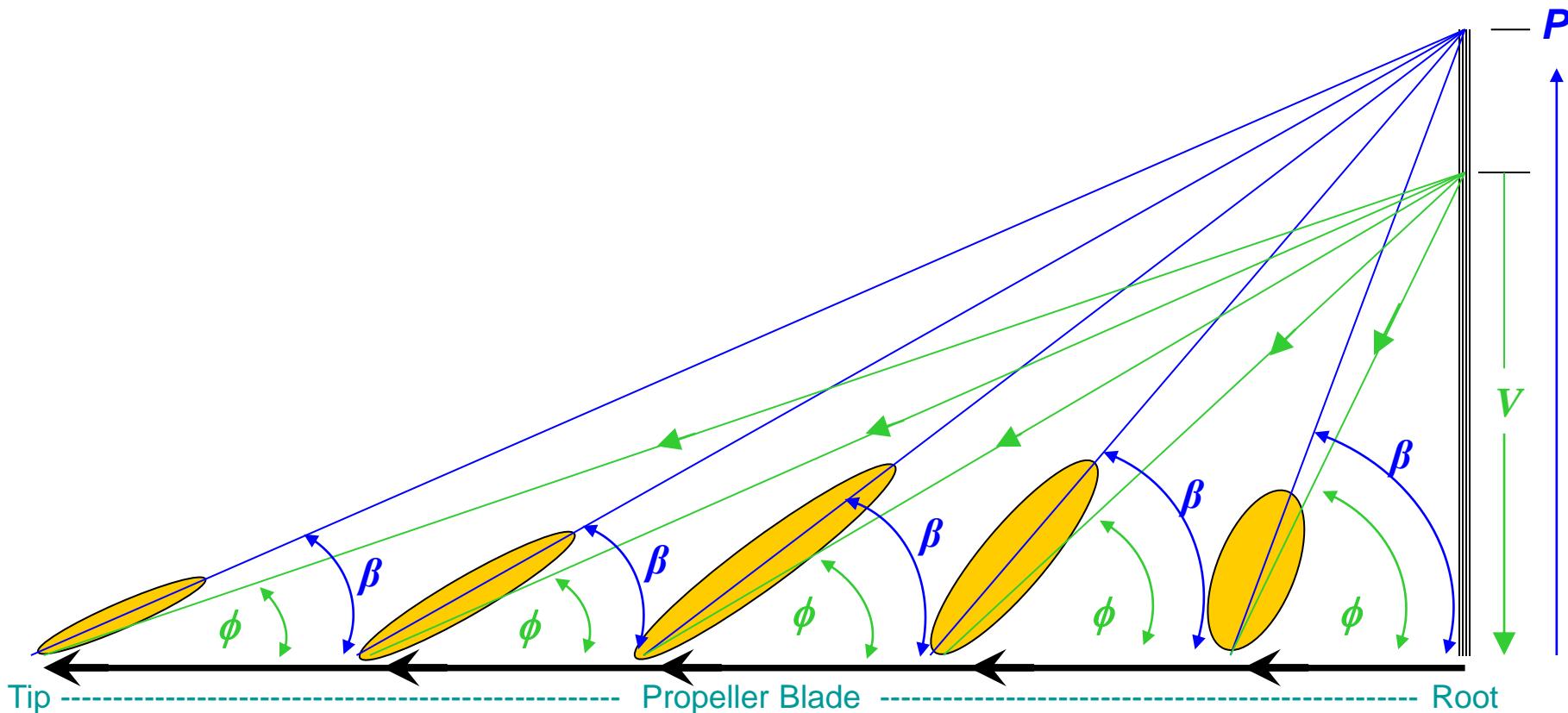
# Propeller Pitch

The **radial twist** shown is normal for propellers. Whilst the rotational speed “ $n$ ” is constant, the **velocity in the plane of rotation increases with radius**. Thus the static analysis that indicates **Geometric Pitch ( $P$ )** and **Effective Pitch ( $E$ )**



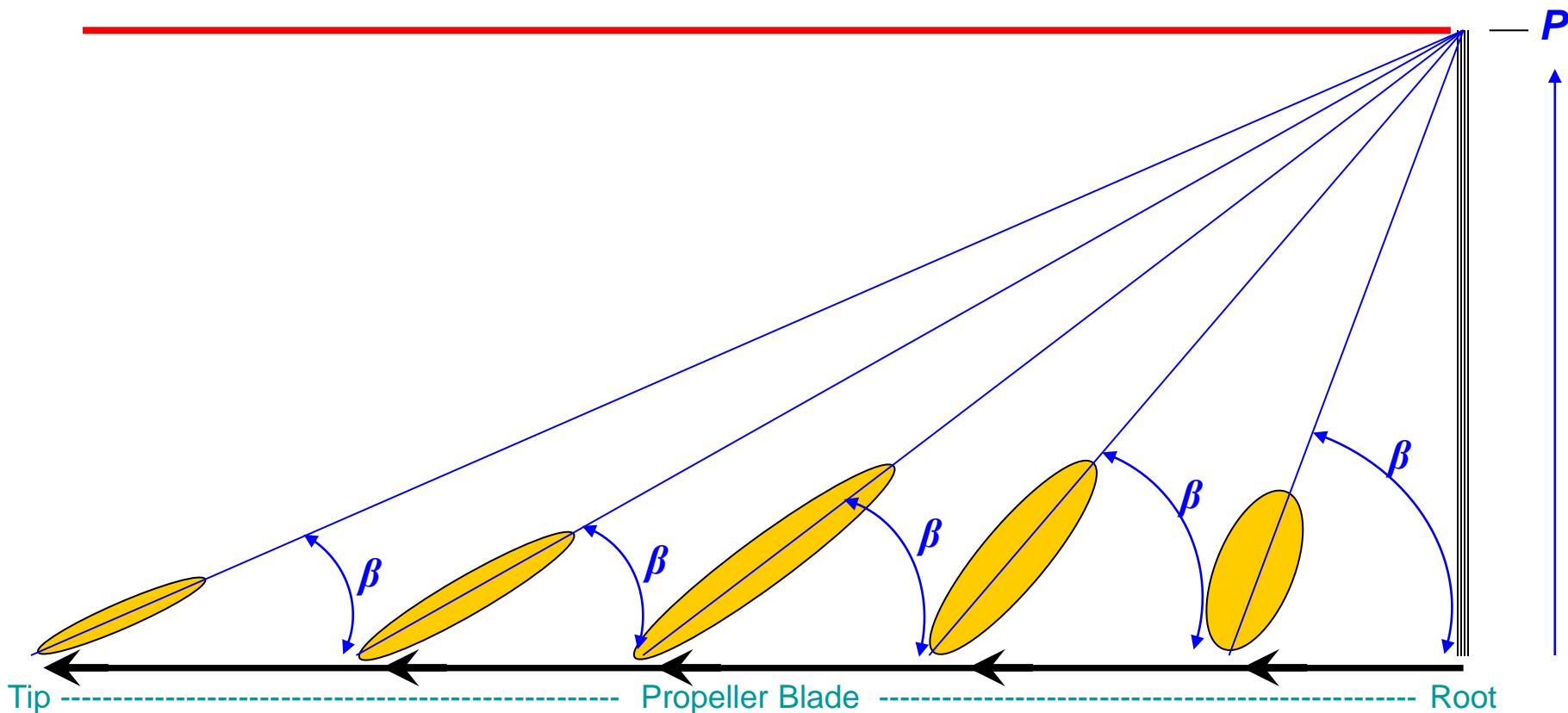
# Propeller Pitch

The **radial twist** shown is normal for propellers. Whilst the rotational speed “ $n$ ” is constant, the **velocity in the plane of rotation increases with radius**. Thus the static analysis that indicates **Geometric Pitch ( $P$ )** and **Effective Pitch ( $E$ )** is analogous to the dynamic analysis of velocities and inflow angles.



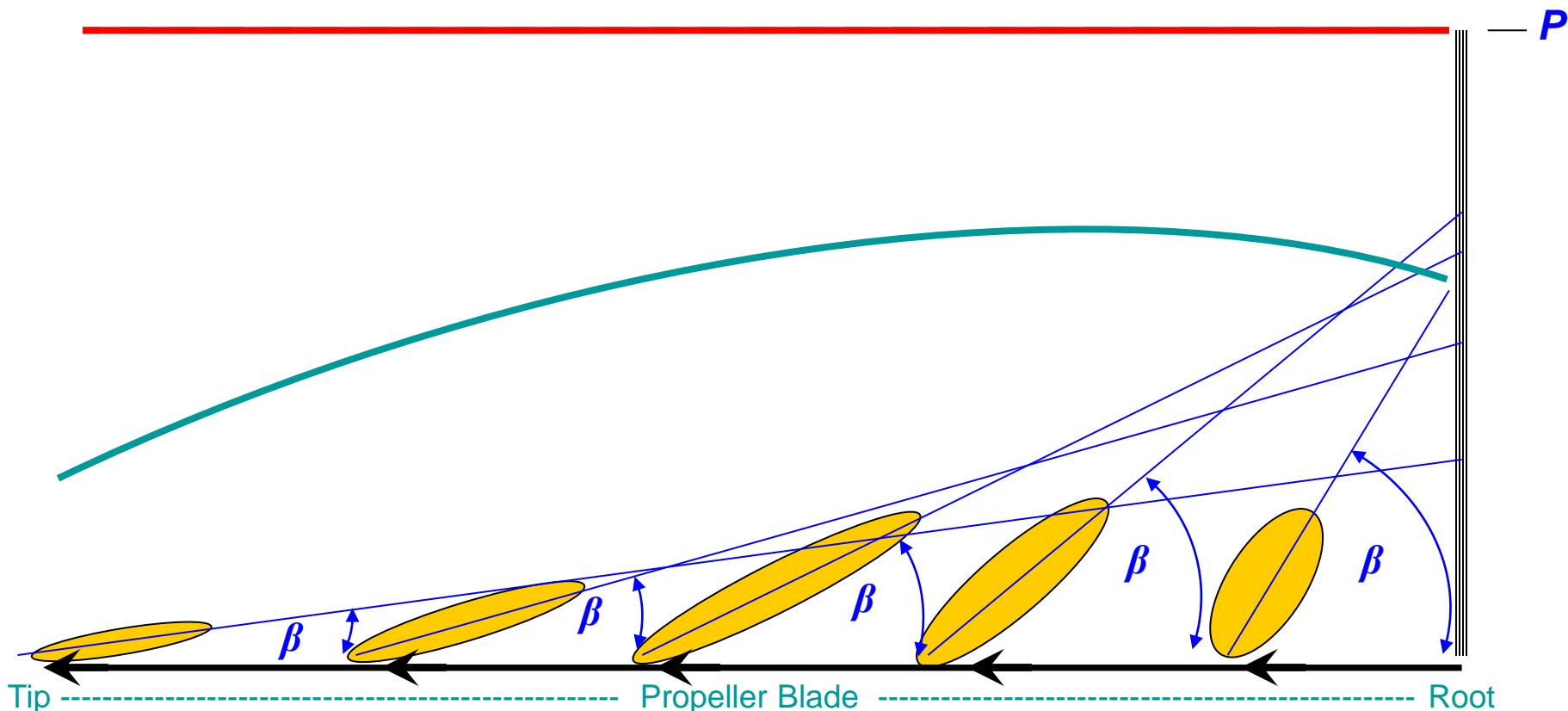
# Propeller Pitch

Considering just the **Blade Pitch (P)** which is **CONSTANT** here,

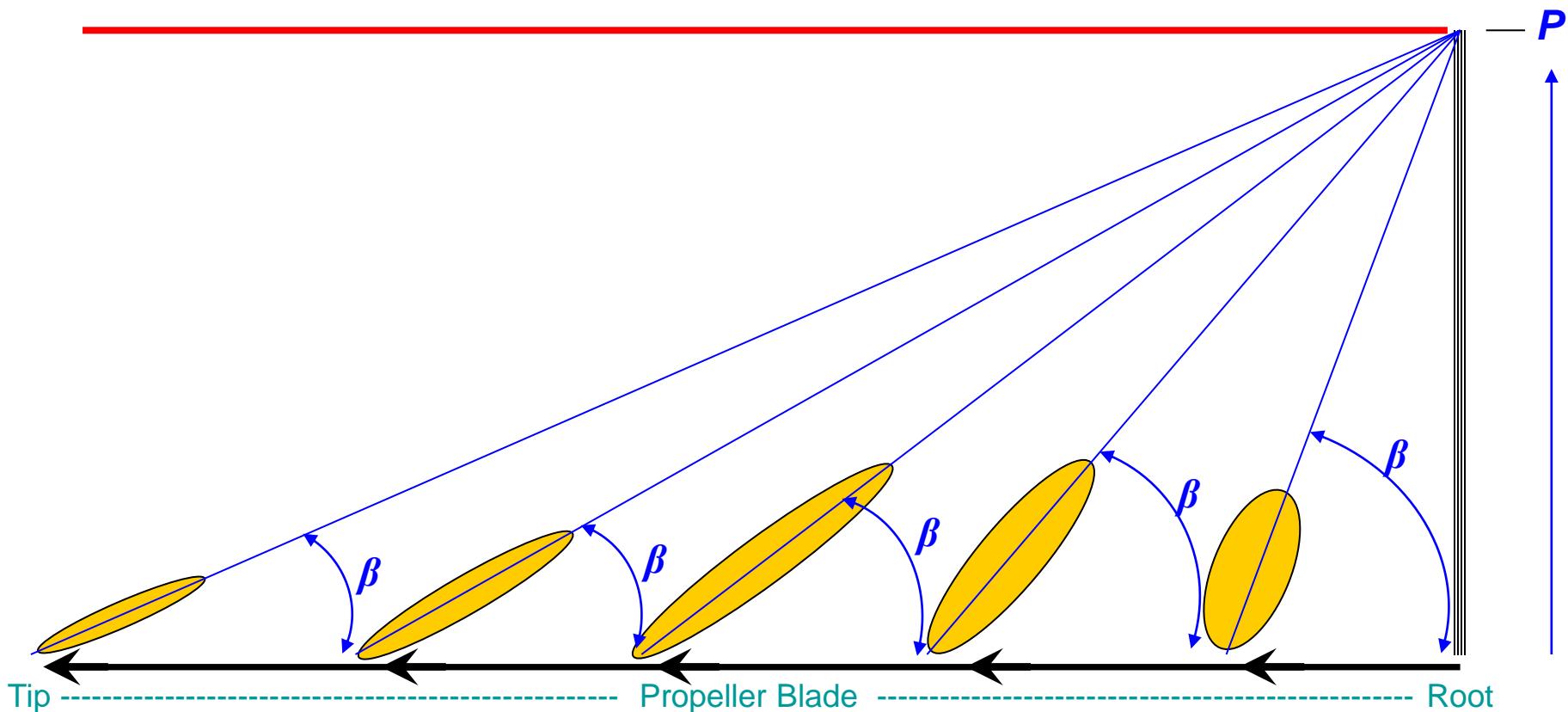


# Propeller Pitch

But for a reduction in Blade Pitch Angle ( $\beta$ ) the Blade Pitch is NOT CONSTANT

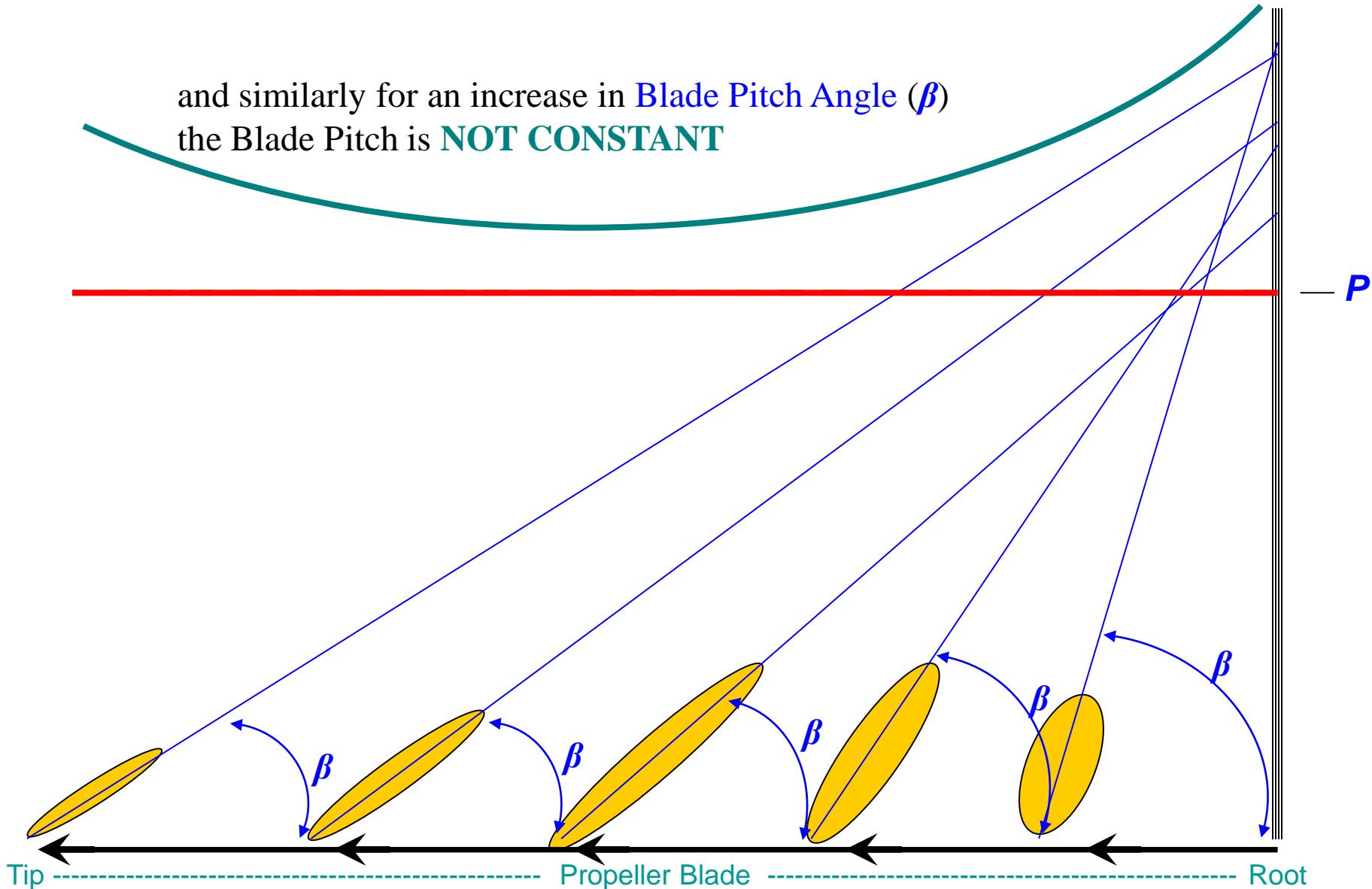


# Propeller Pitch



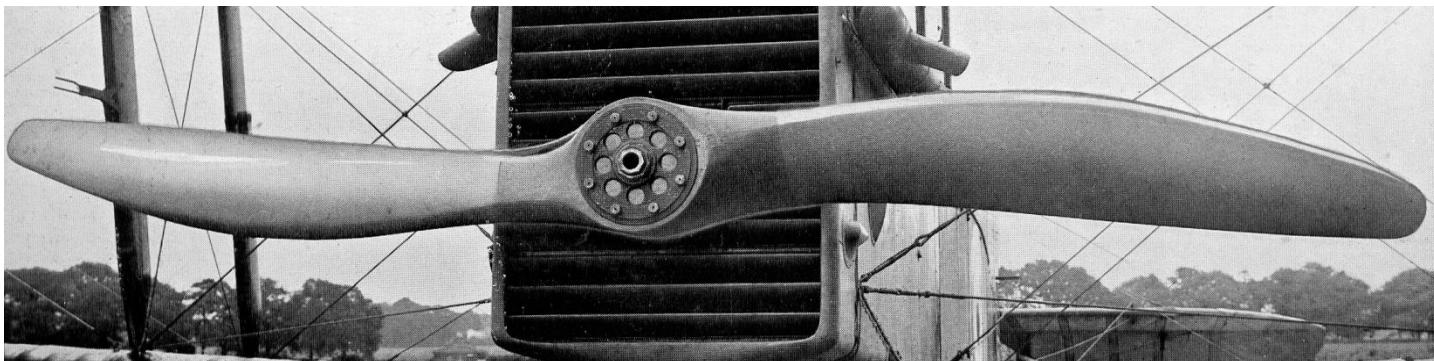
# Propeller Pitch

and similarly for an increase in Blade Pitch Angle ( $\beta$ )  
the Blade Pitch is **NOT CONSTANT**



# Propeller Pitch Control

Clearly a **Fixed Pitch Propeller** has only one design point. It is common place on one-piece propellers (see below) and modern low cost light aircraft.



Most modern propellers have the facility to adjust the geometric pitch angle and these are known as **Adjustable Pitch Propellers**. This can be as simple as setting a pitch angle prior to flight; in order to optimise one part of the flight phase (take-off, climb or cruise).

# Propeller Pitch Control

Propellers with mechanisms that can change the pitch during flight are known as **Controllable (or Variable) Pitch Propellers** and these are well suited to sport aircraft.

A propeller fitted with an autonomous variable pitch mechanism that maintains constant propeller speed (and thus engine speed) irrespective of aircraft speed is known as a **Constant Speed Propeller** and is used primarily on civil transport aircraft.

All the above could also be **Constant Pitch Propellers** – but are unlikely to be !

