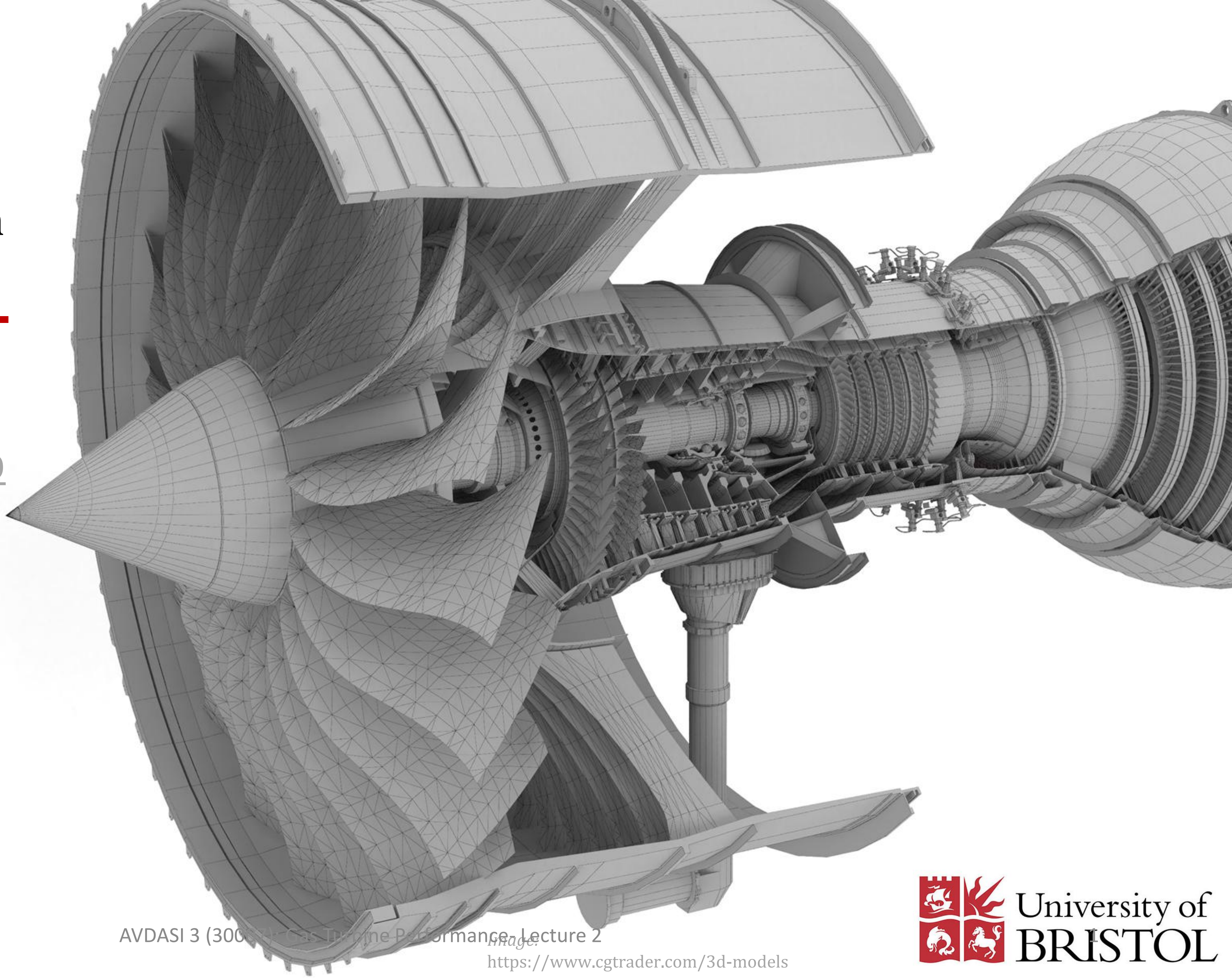
AVDASI 3
(CADE 30007)
Gas Turbine Propulsion

Week 14: Lecture 2

Thermodynamics recap & Practical Gas turbine Cycle



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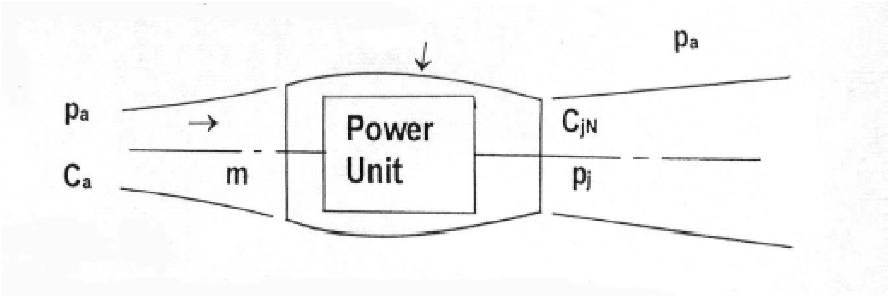
Learning Objectives for Lecture 2

- Recap on basic relationships
- To show the ideal efficiency of the Joule or Brayton Cycle.
- To calculate the main characteristics of a practical turbojet.

Thrust and Propulsive Efficiency: 1



For analysis purposes the Propulsion Unit is stationary in a uniform flow field

THRUST is equal to rate of change of momentum:

If $\dot{m}_f \ll \dot{m}$, $F = \dot{m}(C_{jN} - C_a) + A_j(P_j - Pa)$

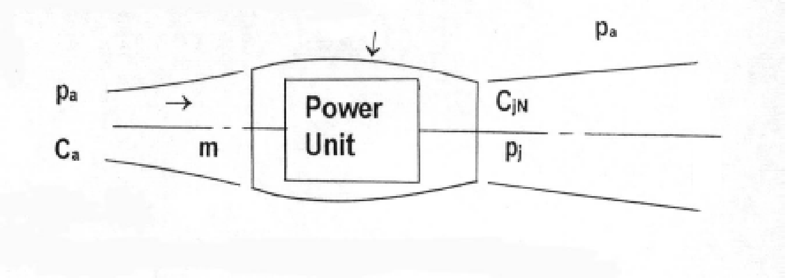
Fuel flow $= \dot{m}_f$ [kg/s]

Net calorific value of fuel $= Q_{net}$

For kerosene:
$Q_{net} \sim 42\ MJ/kg$

Next-gen Batteries:
Energy Density
$\sim 1.5\ MJ/kg$

Thrust and Propulsive Efficiency: 2



Efficiency of Energy Conversion is equal to useful mechanical energy / energy supplied by fuel:

$$\eta_e = \frac{1}{2}\dot{m}\,\frac{(Cj^2 - Ca^2)}{Q_{net}\cdot \dot{m}_f}$$

Propulsive (Froude) Efficiency is equal to useful work / useful work + unused KE in jet:

$$\eta_p = \frac{F\cdot C_a}{\boxed{F\cdot C_a + \frac{1}{2}\dot{m}(C_j - C_a)^2}} = \frac{2}{(1 + C_j/C_a)}$$

$$\frac{1}{2}\dot{m}(C_j^2 - C_a^2) =$$

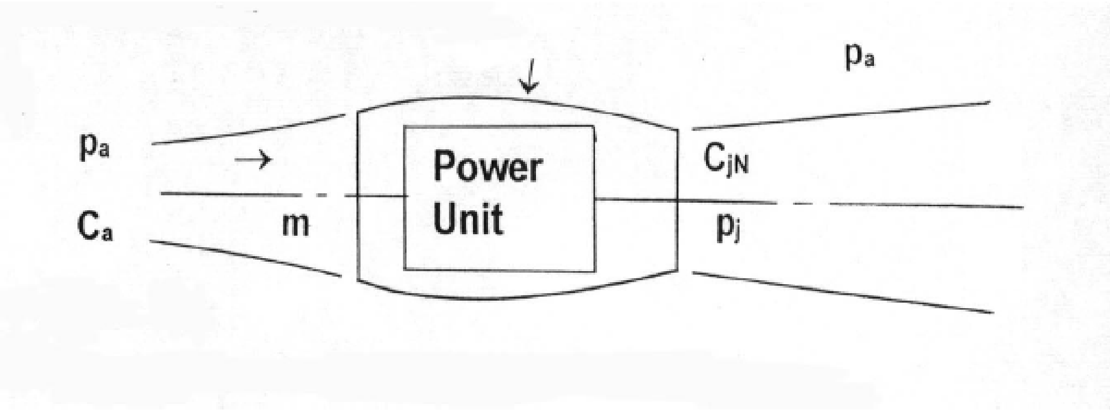
N.B Assuming *fully expanded* jet from propelling nozzle

$$F = \dot{m}(C_j - C_a)$$

OVERALL EFFICIENCY is the product of efficiency of energy conversion and propulsive efficiency:

$$\eta_{overall} = \eta_e \cdot \eta_p$$

Thrust and Propulsive Efficiency: 3



Overall Efficiency: $$\eta_{overall} = \frac{F \cdot C_a}{\dot{m}_f \cdot Q_{net}}$$

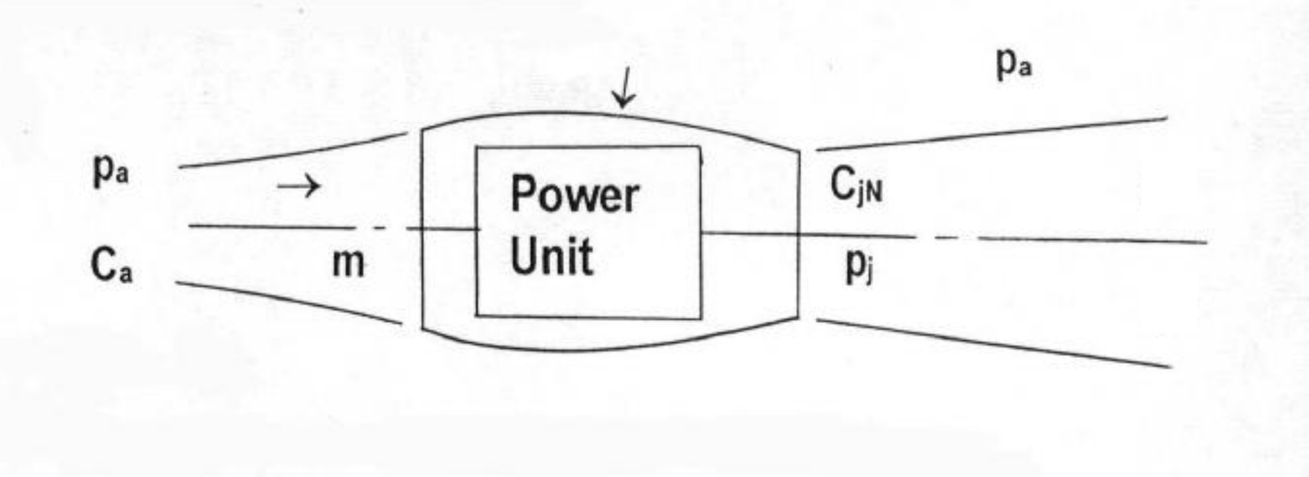
Specific Fuel Consumption: $SFC = \frac{\dot{m}_f}{F}$ [kg/(h N)]

Hence overall efficiency (for a given fuel) ∝ flight speed / SFC:

$$\eta_{overall} = \frac{C_a}{SFC \cdot Q_{net}}$$

Thus SFC is a measure of overall efficiency

Thrust and Propulsive Efficiency: 4



Fuel Air Ratio (FAR):

$$\boldsymbol{FAR} = \frac{fuel\ flow}{air\ flow} = \frac{\dot{m}_f}{\dot{m}}$$

Specific Thrust (ST) is equal to Thrust / unit Mass Flow:

$$\boldsymbol{ST} = \frac{F}{\dot{m}} = (C_j - C_a)$$

That means that:

$$\boldsymbol{SFC} = \frac{FAR}{ST} = \frac{\dot{m}_f}{F}$$

Basic Thermodynamic relationships - 1

Laws of thermodynamics are empirical i.e. based on (many) experimental observations.

First Law of Thermodynamics yields steady flow energy equation , per unit mass flow rate:

$$\boxed{Q = (h_2 - h_1) + \frac{1}{2}(C_2^2 - C_1^2) + W}$$

An expression for the conservation of energy.

Stagnation (total) Enthalpy

$\boxed{h_0 = h + \frac{C^2}{2}}$ and $\boxed{h = C_p T}$ so $\boxed{T_o = T + \frac{C^2}{2.C_p}}$ since $\boxed{h_0 = C_p T_0}$

Second Law of Thermodynamics yields:

$$\boxed{ds = \left(\frac{dQ}{T}\right)_{\text{rev}}}$$

Introduces concept of entropy – a property of a thermodynamic system.

Basic Thermodynamic relationships - 2

Specific Heats

The quantity of heat needed to raise the temperature of unit mass of a substance by 1 K.

Function of temperature only for normal conditions in a gas turbine engine. Often considered constant to simplify preliminary analysis.

Typical Values

For Air:

C_{pa} = 1005 J/kg.K

$\gamma_a = \frac{c_P}{c_V} = 1.4$

$\frac{\gamma}{(\gamma-1)} = 3.5$

For Exhaust Gases:

C_{pg} = 1148 J/kg.K

$\gamma_g = \frac{c_P}{c_V} = 1.333$

$\frac{\gamma}{(\gamma-1)} = 4.0$

$C_p - C_V = R$

For both air and exhaust gases,
the gas constant, R = 287.1 J/kg. K

Basic Thermodynamic relationships - 3

Total temperature:

$$T_0 = T + \frac{C^2}{2C_p} \quad \text{or} \quad T_0 = T\left[1 + \frac{(\gamma - 1)}{2} M^2\right]$$

Total pressure (for isentropic):

$$P_0 = P\left[1 + \frac{(\gamma - 1)}{2} M^2\right]^{\frac{\gamma}{(\gamma-1)}} \quad \text{or} \quad \frac{P_0}{P} = \left[\frac{T_0}{T}\right]^{\frac{\gamma}{(\gamma-1)}}$$

Note also (for isentropic):

$$\frac{P_{02}}{P_{01}} = \left[\frac{T_{02}}{T_{01}}\right]^{\left(\frac{\gamma}{(\gamma-1)}\right)}$$

Speed of sound:

$$a = \sqrt{\gamma R T}$$

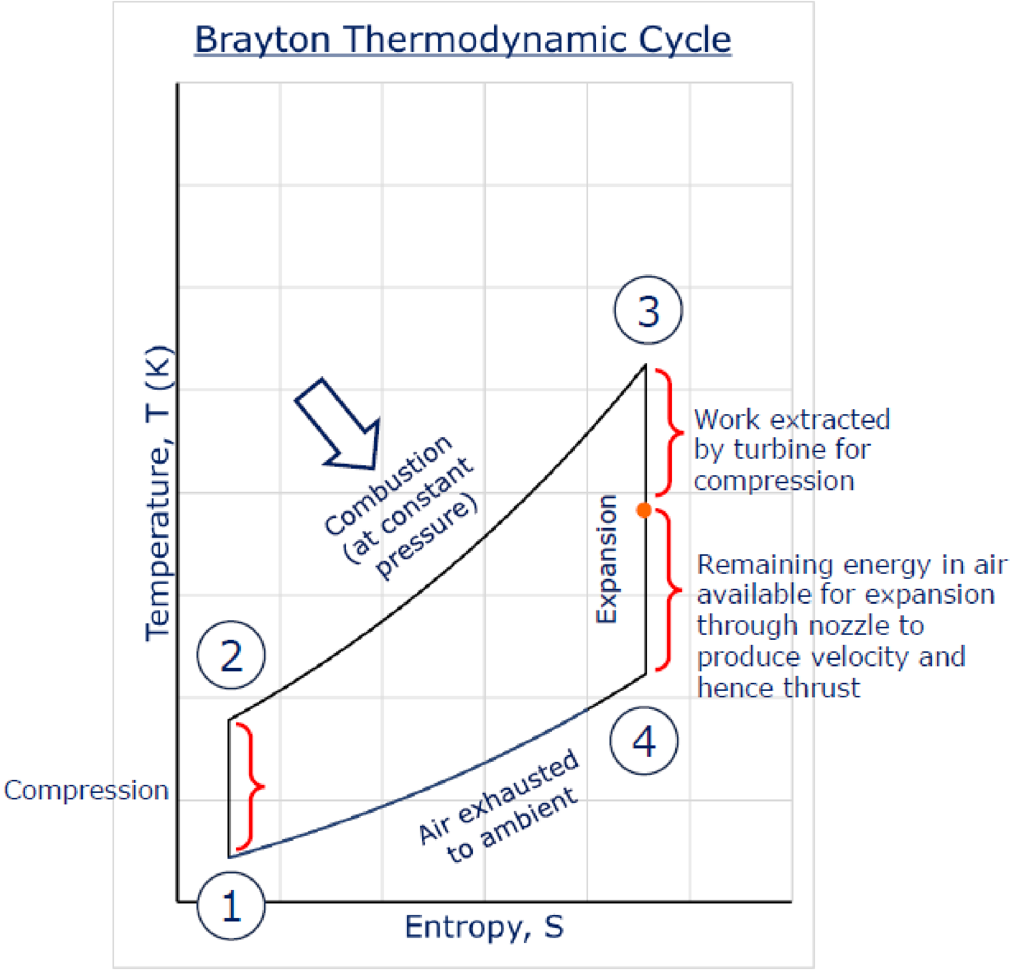
Mach number:

$$M = \frac{C}{\sqrt{\gamma R T}}$$

Ideal Joule-Brayton cycle -1

Assumptions for ideal Brayton cycle:

- The working fluid is air and behaves as a perfect gas
- The mass-flow through the cycle is constant
- The kinetic energy change between inlet/outlet of components is negligible
- No component pressure losses
- Isoentropic compressions and expansions



Ideal Joule-Brayton cycle -2

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$$Q = (h_2 - h_1) + \cancel{\frac{1}{2}(C_2^2 - C_1^2)} + W$$

1»2 Isoentropic compression

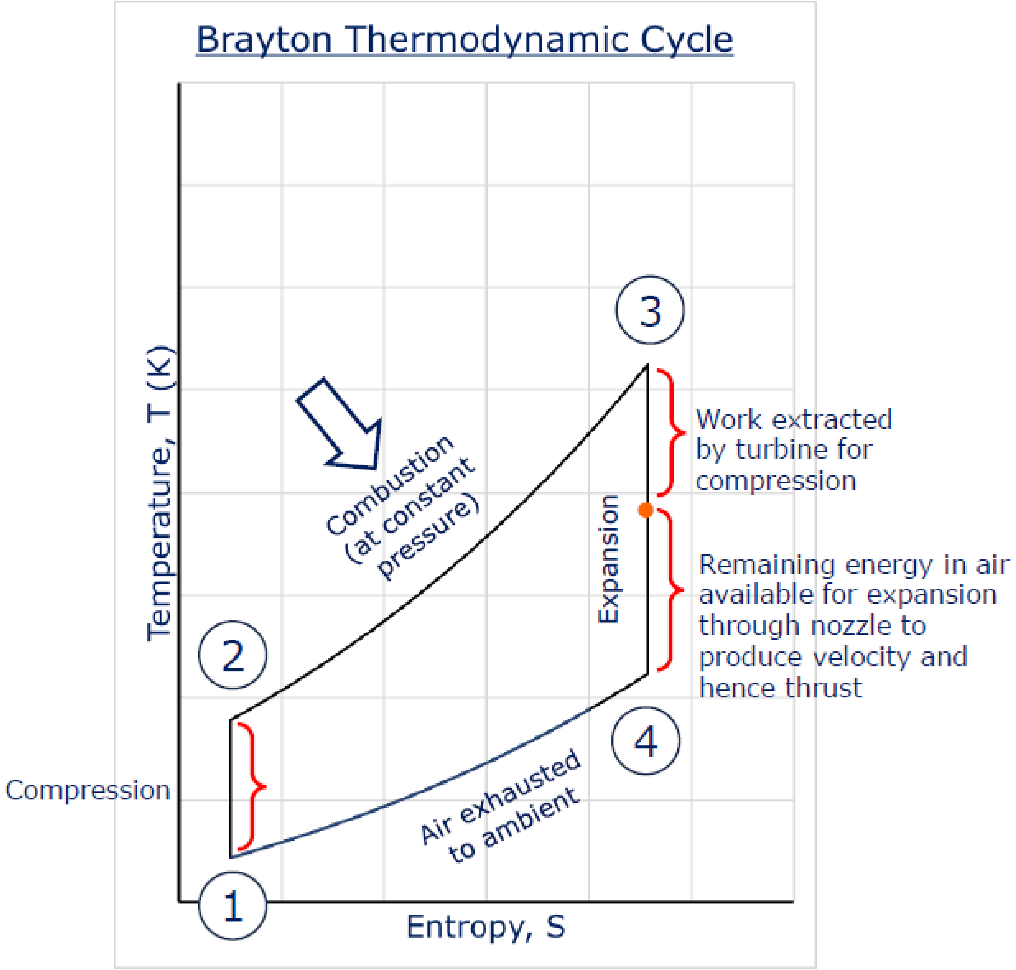
- $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$
- $W_{12} = C_P(T_2 - T_1)$

2»3 Isobaric Combustion

- $P_2 = P_3$
- $Q_{23} = C_P(T_3 - T_2)$

3»4 Isoentropic expansion

- $\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}}$
- $W_{34} = C_P(T_3 - T_4)$



Ideal Joule-Brayton cycle -3

Efficiency is net work produced by the cycle over the heat energy input:

$$\eta_{th} = \frac{W34 - W12}{Q23} = \frac{C_P(T_3 - T_4) - C_P(T_2 - T_1)}{C_P(T_3 - T_2)} =$$

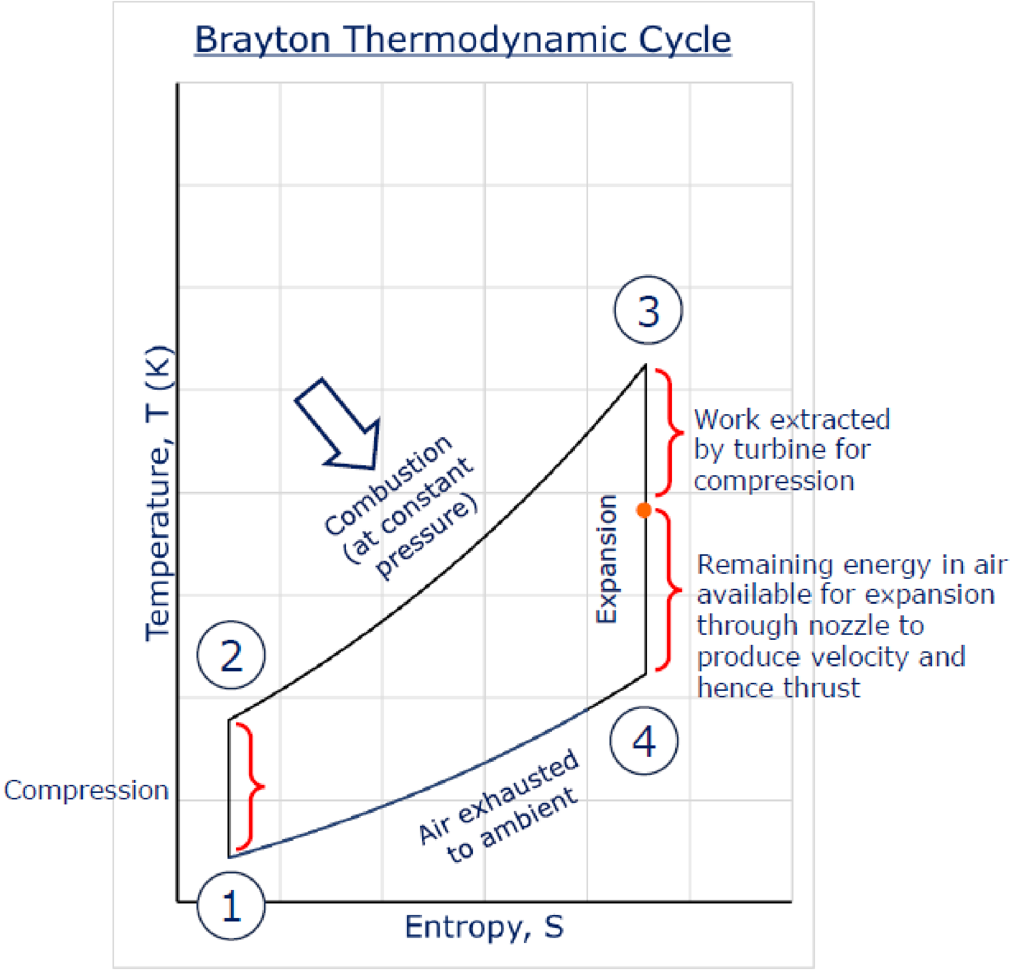
$$= 1 + \frac{T_1 - T_4}{T_3 - T_2} = 1 - \frac{T_1}{T_2}\frac{(1 - T_4/T_1)}{(1 - T_3/T_2)}$$

$P_2 = P_3$ in the comb. chamber

Since $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_3}{T_4}, \rightarrow \frac{T_4}{T_1} = \frac{T_3}{T_2}$

$P_4 = P_1 = P_a$

$$\eta = 1 - \frac{T_1}{T_2}\frac{(1 - T_4/T_1)}{(1 - T_3/T_2)} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$



Ideal Joule-Brayton cycle -4

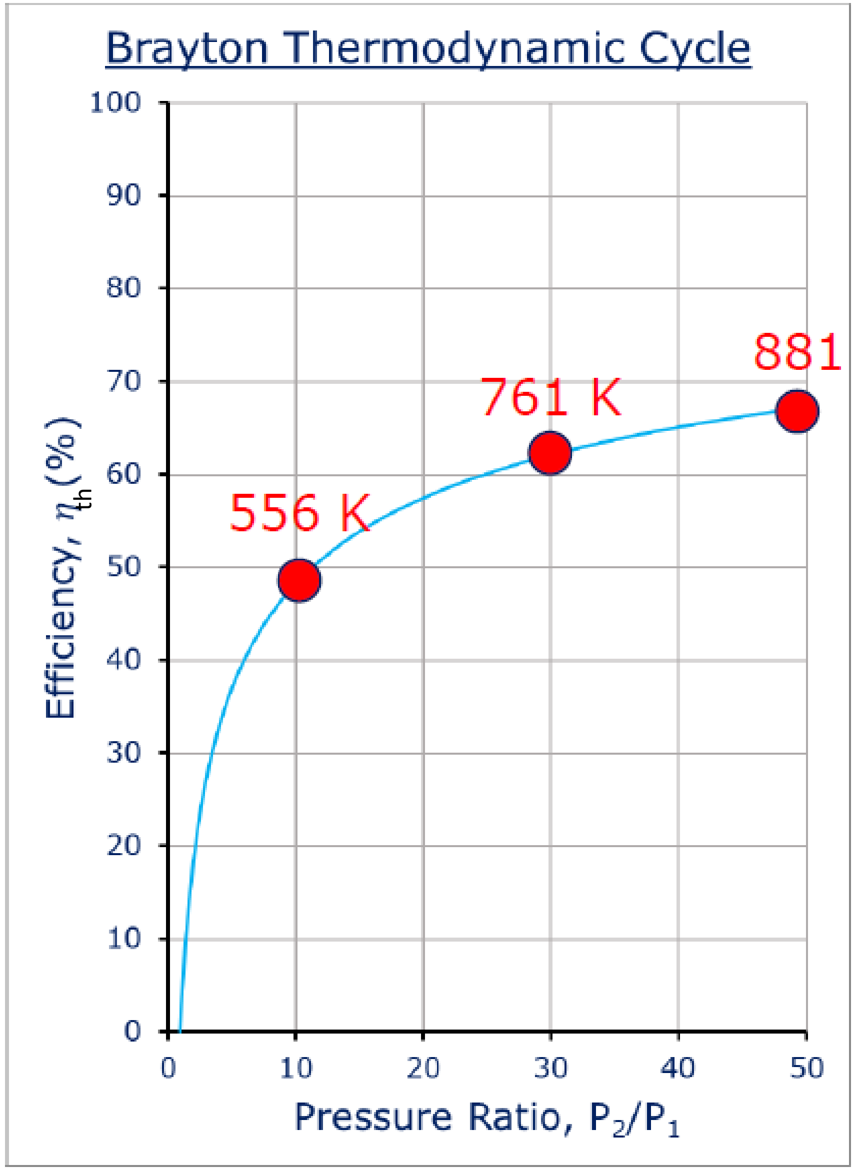
Cycle efficiency:

$$\boxed{\eta_{th} = 1 - \left(\frac{1}{PR}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}}$$

Where $PR = {P_2}/{P_1}$

Cycle efficiency is only a function of pressure ratio.

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}$$



Ideal Joule-Brayton cycle -5

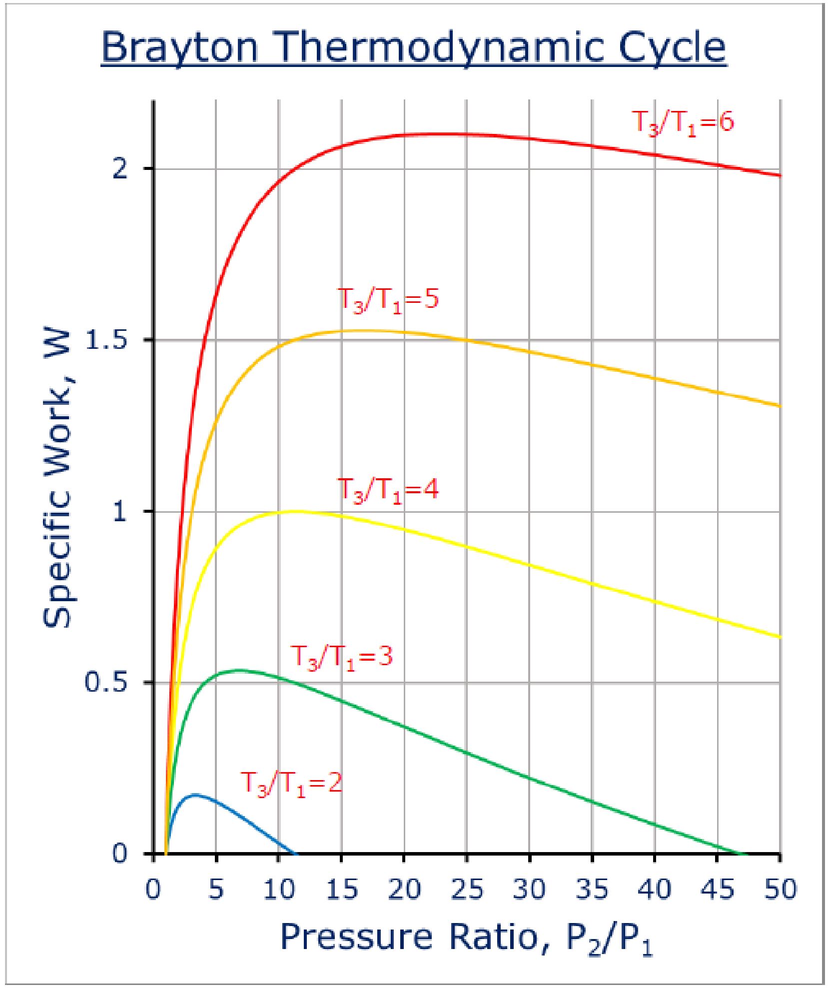
Want an expression for net work in terms of PR and maximum cycle temperature:

$$Net\ Work, W = W_{34} - W_{12} = Cp \times \{(T_3 - T_4) - (T_2 - T_1)\}$$

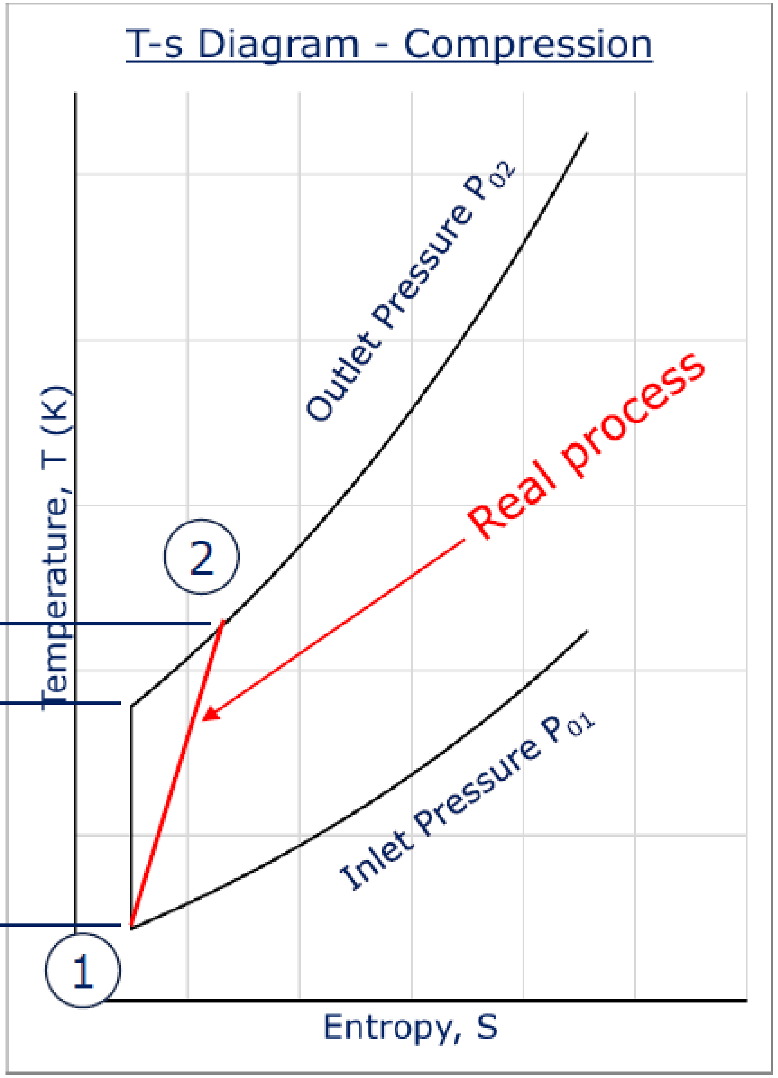
We can non-dimensionalise this:

$$\frac{W}{(Cp \times T_1)} = \left(\frac{T_3}{T_1}\right)\left\{1 - \left(\frac{T_4}{T_3}\right)\right\} - \left\{\left(\frac{T_2}{T_1}\right) - 1\right\}$$

$$\boxed{\frac{W}{(Cp \times T_1)} = \left(\frac{T_3}{T_1}\right)\left\{1 - \left(\frac{1}{PR^{\left(\frac{\gamma-1}{\gamma}\right)}}\right)\right\} - \left\{\left(PR^{\frac{\gamma-1}{\gamma}}\right) - 1\right\}}$$



Practical Joule-Brayton cycle -1



This is amount of extra work needed for real compression to overcome frictional losses for same required compression ratio.

Real T_{02}

Ideal T'_{02}

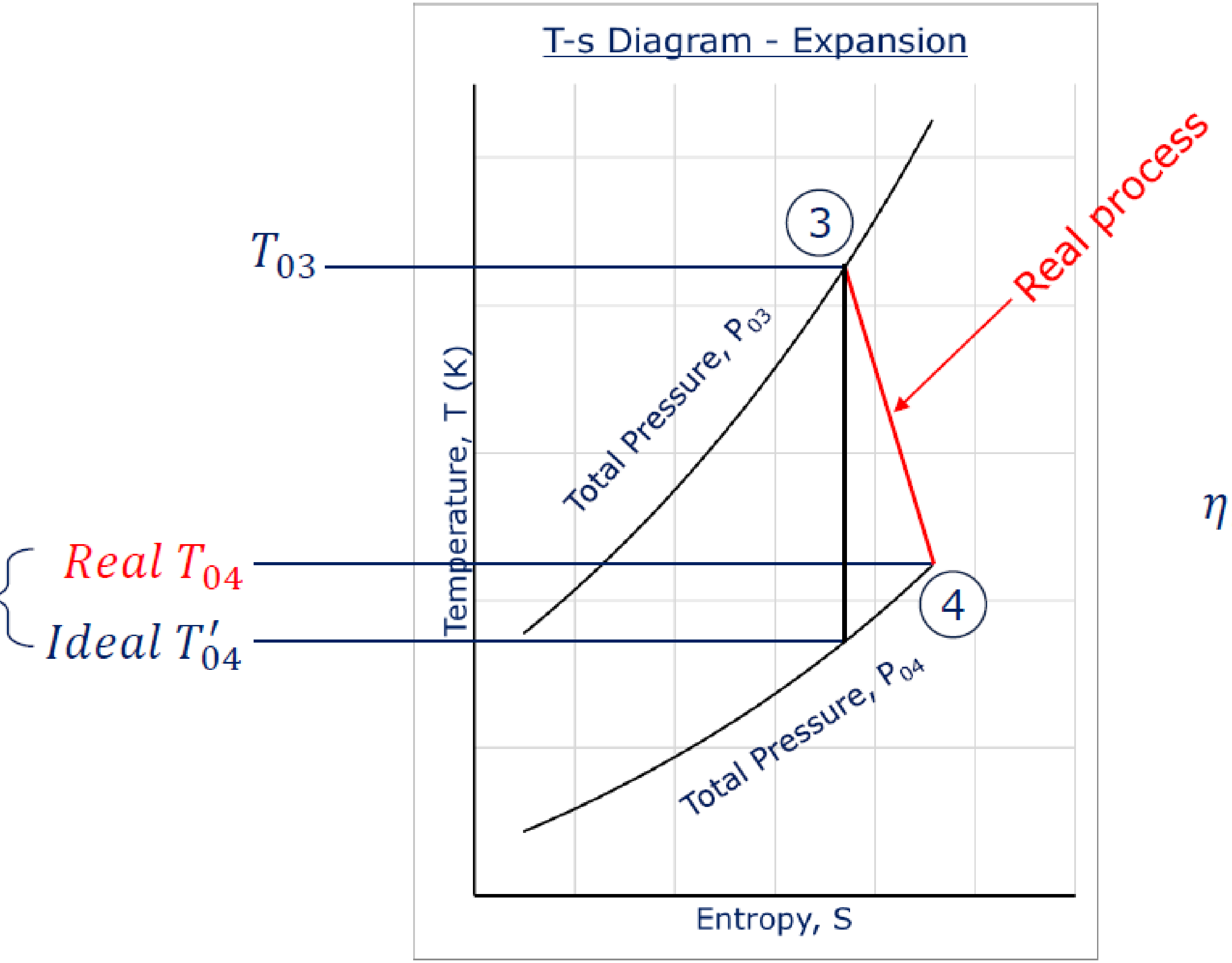
T_{01}

Superscript ‵ often used to denote <u>ideal</u> conditions

$$\eta_{isen} = \frac{(T'_{02} - T_{01})}{(T_{02} - T_{01})}$$

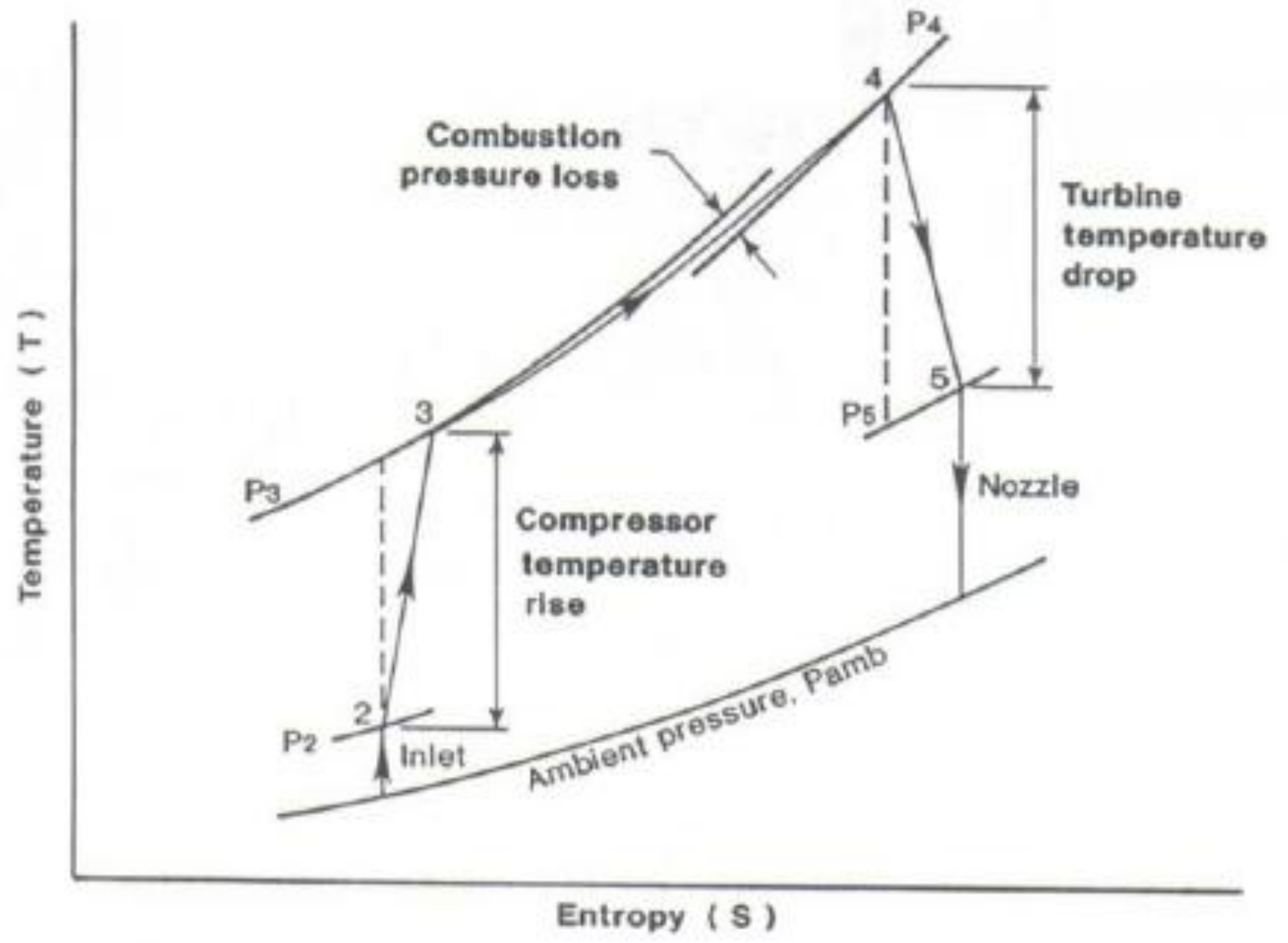
Practical Turbojet cycle -2

This is amount of decreased work produced by the real expansion for the same pressure ratio.



$$\eta_{isen} = \frac{(T_{03} - T_{04})}{(T_{03} - T'_{04})}$$

Practical Turbojet cycle -3



Practical Turbojet cycle -4

How to compute the flow velocity at the nozzle exit?

Adapted nozzle, $P_j = P_a$

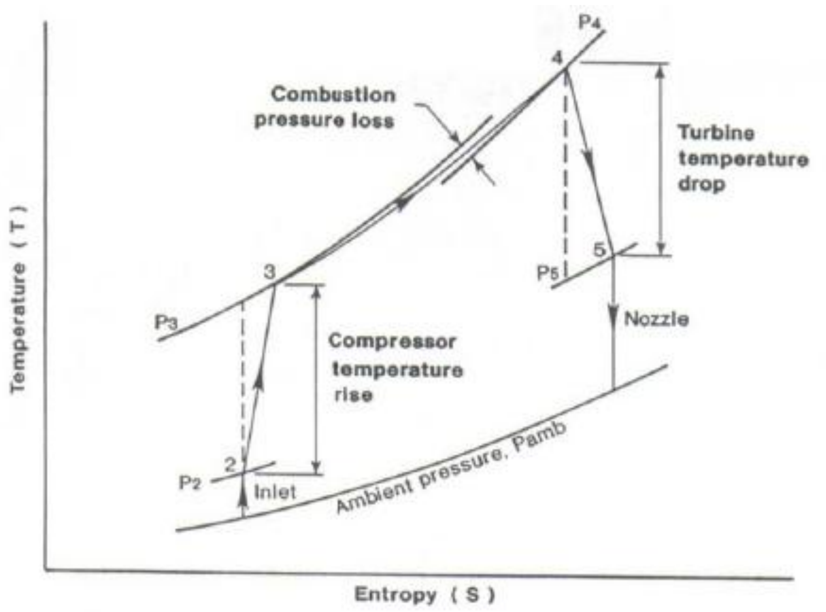
- Jet fully expanded in an ideal De Laval nozzle (convergent-divergent) or subsonic convergent nozzle

- $P_{0,N} = P_{0,5}, \quad T_{0,N} = T_{0,5}$
- $\frac{T_{0,N}}{T_N} = \left(\frac{P_{0,N}}{P_a}\right)^{\frac{\gamma-1}{\gamma}}, \frac{T_{0,5}}{T_N} = \left(\frac{P_{0,5}}{P_a}\right)^{\frac{\gamma-1}{\gamma}}$
- $T_{0,N} = T_{0,5} = T_N + \frac{C_N^2}{2C_P}$

Underexpanded nozzle, $P_j > P_a$

- Usually for choked convergent nozzle that reaches sonic conditions in the throat

- We'll see how to compute the jet speed in the worked example



Worked Example – Turbojet performance 1

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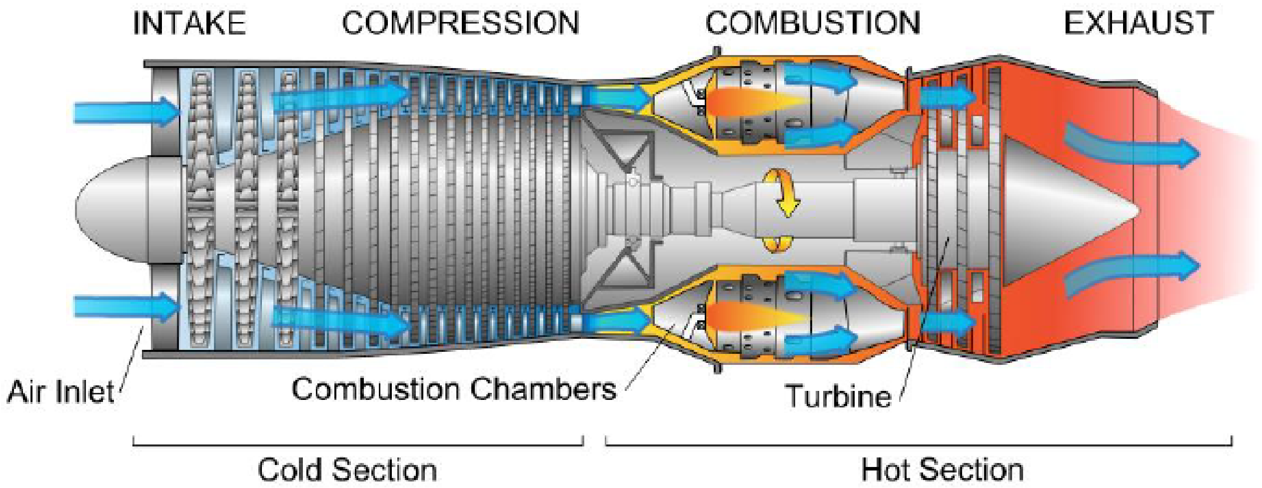


Flight Condition:

$Mach, M = 0.8$

$Altitude, h = 10\ km$

ISA day

Engine Cycle Data:

$Turbojet$

$OPR = 8.0$

$Temperature, T_{04} = 1200K$

Engine Component Efficiencies & Losses:

$Compressor, \eta_{Comp-isen} = 87\%$

$Turbine, \eta_{Turb-isen} = 90\%$

$Transmission, \eta_m = 99\%$

$Combustion\ pressure\ loss = 4\%$

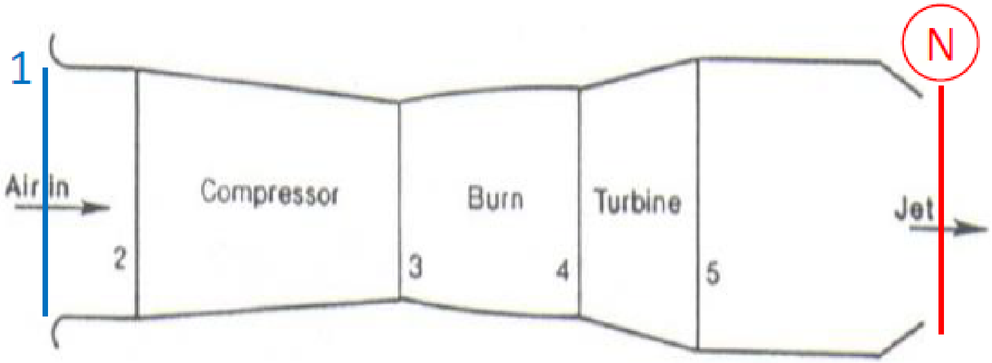
Convergent propelling nozzle
No inlet pressure loss

The question:

What is the specific thrust and SFC of the engine?

(N.B. For this analysis, velocities are relative to the engine i.e. we assume the engine is stationary and the air is entering the engine at the flight velocity)

Worked Example – Turbojet performance 2



Step 0 – Ambient Conditions:

Using tables or formula for ISA:

$$Temperature, T_a = 223K\ (ISA\ day)$$

$$Pressure, P_a = 26.4\ kPa$$

$$Speed\ of\ sound, a = \sqrt{\gamma RT} = 299\ m/s$$

$$Flight\ speed, Ca = 0.8 \times 299 = 240\ m/s$$

Step 1 – Engine Inlet Conditions:

Total temperature:

$$T_{01} = T_a\left[1 + \frac{(\gamma-1)}{2}M^2\right] = 252\text{ K}$$

Total pressure:

$$P_{01} = P_a\left[1 + \frac{(\gamma-1)}{2}M^2\right]^{\frac{\gamma}{(\gamma-1)}} = 40.2\text{ kPa (absolute)}$$

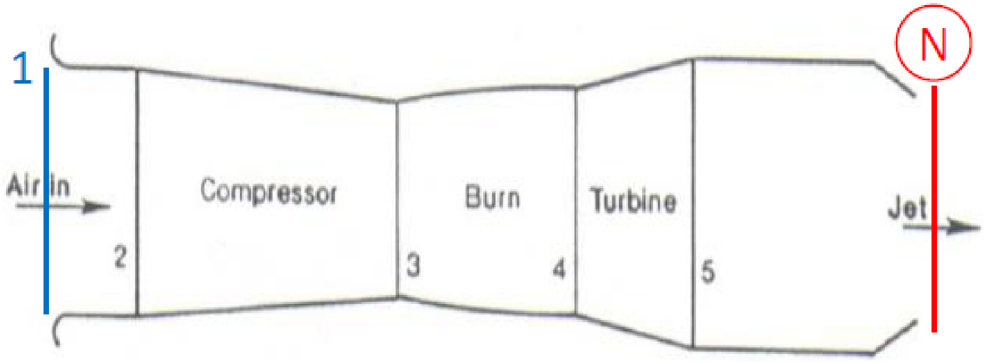
(Remembering that $\frac{\gamma}{(\gamma-1)} = 3.5$ for air)

Step 2 – Compressor Inlet Conditions:

In this case, there is no inlet pressure loss and therefore:

$P_{02} = P_{01}$ and also $T_{02} = T_{01}$

Worked Example – Turbojet performance 3



<u>Step 3 – Compression:</u>

As compressor pressure ratio = 8

$P_{03} = 8 \times 40.2 = 322\ kPa$

If compression was ideal (isentropic):

$\frac{T'_{03}}{T_{02}} = \left[\frac{P_{03}}{P_{02}}\right]^{\left(\frac{\gamma-1}{(\gamma)}\right)} = 1.81$ and therefore T'_{03} = 456 K

By definition $\eta_{comp} = \frac{(T'_{03} - T_{02})}{(T_{03} - T_{02})}$ and therefore $T_{03} = 486\ K$

Specific power <u>required</u> to drive the compressor by turbine:

$$Specific\ Power = \frac{1}{\eta_m} C_{pa}(T_{03} - T_{02})$$

Where this is the transmission efficiency between turbine and compressor – mostly bearing and windage losses.

Worked Example – Turbojet performance 4

Step 4 – Combustion:

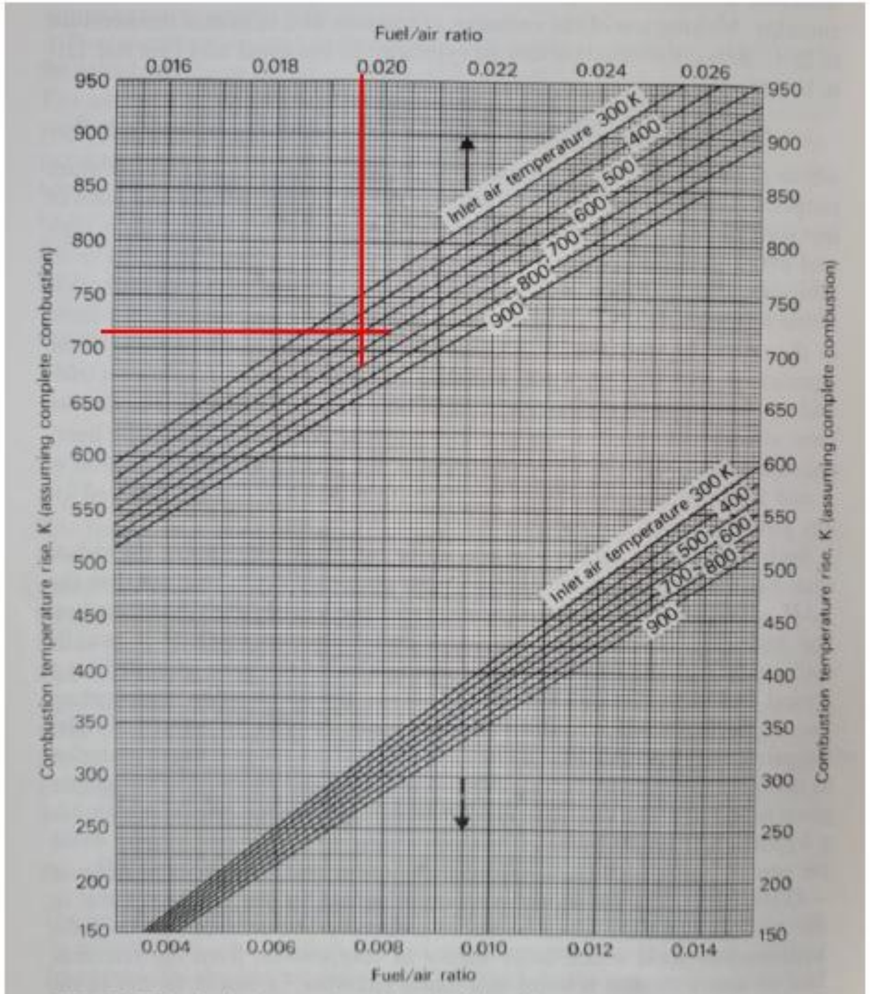
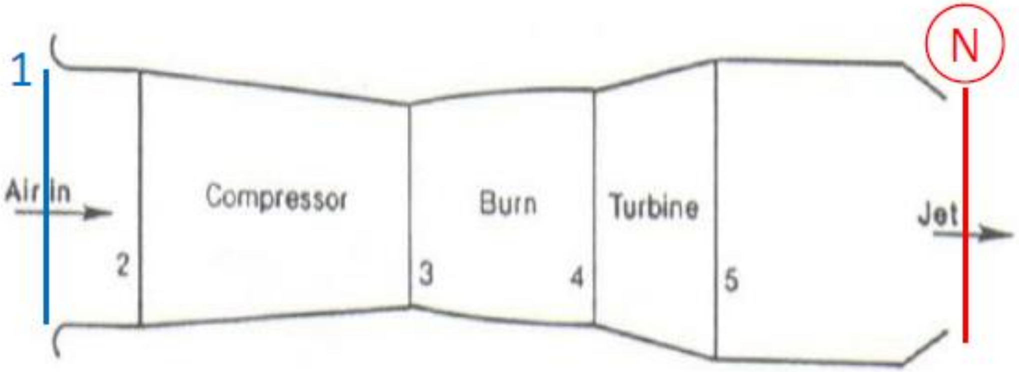


FIG. 2.15 Combustion temperature rise v. fuel/air ratio
(A large-scale version of this diagram is published by Basil Blackwell)



The temperature rise in the combustion chamber is:

$$Temperature\ rise = T_{04} - T_{03} = 1200 - 486 = 714\ K,$$

Using the graph*

$$Fuel/air\ ratio, FAR\ = 0.0195$$

Small 4% pressure loss across combustor, hence:

$$P_{04} = 0.96 \times P_{03} = 309\ \text{kPa}$$

Notes

*For complete combustion of kerosine, 42100 kJ/kg calorific value, stoichiometric FAR 0.068

Worked Example – Turbojet performance 2

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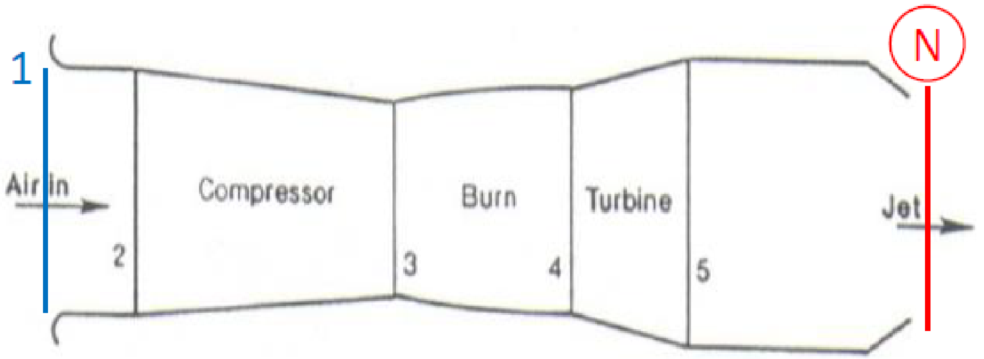
<u>Step 5 – Expansion (Turbine):</u>

$$\underbrace{C_{pg}(T_{04} - T_{05})}_{\text{Specific power delivered by turbine}} = \frac{1}{\eta_m}\underbrace{C_{pa}(T_{03} - T_{02})}_{\text{Specific power needed by compressor}}$$



Hence

$$Turbine\ exit\ temperature, T_{05} = T_{04} - \left\{\frac{1}{\eta_m}\frac{C_{pa}}{C_{pg}}(T_{03} - T_{02})\right\}$$ = 993 K

$$\eta_{turb-isen} = \frac{(T_{04} - T_{05})}{(T_{04} - T'_{05})}$$ and therefore $T'_{05} = 970$ K

$$\frac{P_{05}}{P_{04}} = \left[\frac{T'_{05}}{T_{04}}\right]^{\left(\frac{\gamma}{(\gamma-1)}\right)} = 0.475$$ and therefore $P_{05} = 132$ kPa (Remembering that $\frac{\gamma}{(\gamma-1)}$ = 4 for combustion gases)

Worked Example – Turbojet performance 2

Step 6 – Propelling Nozzle

Assume no losses in nozzle so . . .

$P_{0N} = P_{05}$ and also $T_{0N} = T_{05}$

Available Nozzle pressure ratio is ${}^{P_{0N}}/_{P_a} \approx 5$

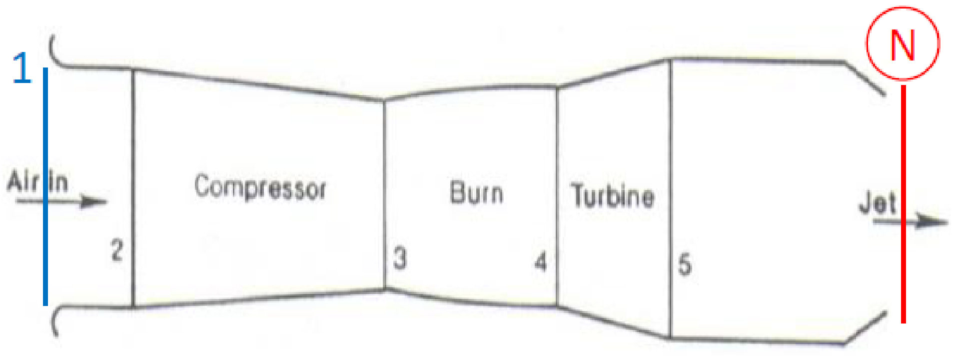
For isentropic flow

$$P_{0N} = P_N \left[1 + \frac{(\gamma - 1)}{2} M^2\right]^{\frac{\gamma}{(\gamma-1)}}$$

So, when M = 1 through convergent nozzle . . .

$$Critical\ pressure\ ratio, \frac{P_{0N}}{P_N} = 1.85$$

Hence nozzle is choked & jet is sonic (Mach 1) as available pressure ratio exceeds this value.



Convergent Nozzle

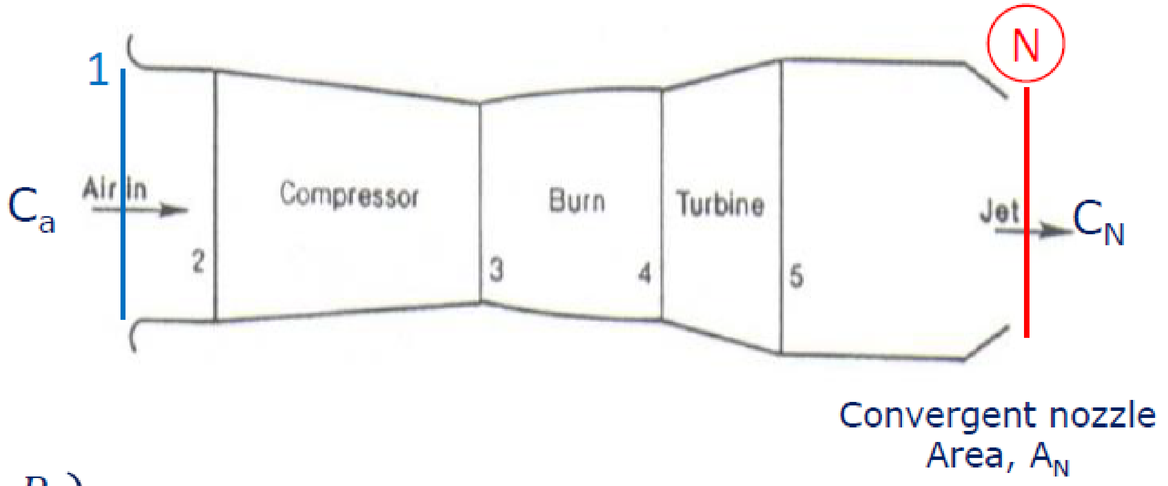
$P_N = 132/1.85 = 71$ kPa (the static pressure at the nozzle exit, which is > Pa)

$T_N = 993/1.167 = 851$ K (the static temperature at the nozzle exit)

$\rho_N = {}^{P_N}/_{(RT_N)} = {}^{71000}/_{(287.1 \times 851)} = 0.29$ kg/m³

$C_N = \sqrt{\gamma R T_N} = 571\ m/s$

Worked Example – Turbojet performance 2



Step 7 – Specific Thrust & SFC

$$Thrust, F = \dot{m}(C_N - C_a) + A_N(P_N - P_a)$$

$$Specific\ Thrust, F_s = \frac{F}{\dot{m}} = (C_N - C_a) + \left(\frac{A_N}{\dot{m}}\right)(P_N - P_a)$$

Mass flow through nozzle is:

$$\dot{m} = \rho_N \times A_N \times C_N$$

Hence $\frac{A_N}{\dot{m}} = \frac{1}{(\rho_5 \times C_5)} = 0.00604$ m²s/kg

And so specific thrust, $F_s = (571 - 240) + 0.00604(71000 - 26400) = 331 + 269 = 600$ N per kg/s

And $SFC = \frac{\dot{m}_f}{F} = \frac{FAR \times m}{F} = \frac{FAR}{F_s} = \frac{0.0195 \times 3600}{600} = 0.117$ kg/h N

Key take-aways of Lecture 2

- Recap on basic relationships

- To show the ideal efficiency of the Joule or Brayton Cycle.

- To calculate the main characteristics of a practical turbojet.

- Worked example

What's in Lectures 3&4?

- Aircraft gas turbine design point and off-design performance
- Performance of
  - Intakes
  - Nozzle types
  - Turbo-prop performance
- Fundamental dimensionless parameters
- Variation of thrust and fuel consumption with inlet conditions