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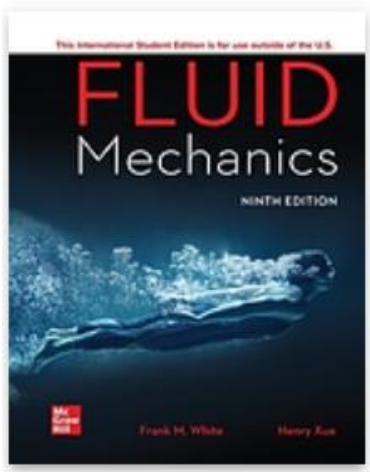
Aerodynamics and Numerical Simulation Methods

Introduction to Laminar Boundary Layers

University of BRISTOL

Course Materials

- Access from: https://www.bristol.ac.uk/library/
- Search for “**Frank White Fluid Mechanics**” in the search tool (eBook available)
- Live chat option available if you are stuck



Fluid mechanics

Authors: Frank M White (Author), Henry Xue (Author)

Print Book 2021, Ninth edition. International student edition.
New York, NY : McGraw Hill LLC, 2021.

Aerodynamics and Numerical Simulation Methods

Introduction to Boundary Layers

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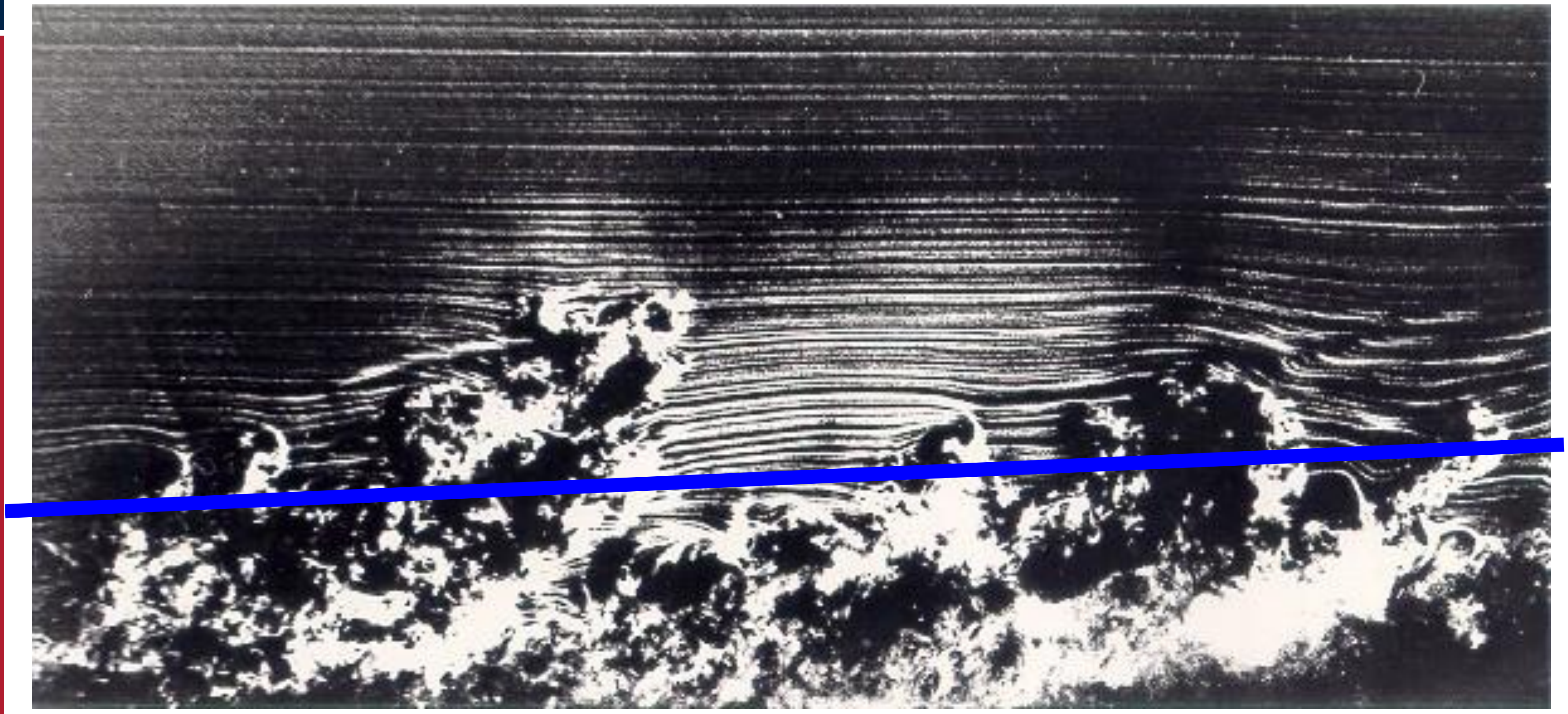
Topics for today

- The concept of boundary layer (BL)
- Why is it relevant?
- How thick is the boundary layer?
- Evaluation of BL thickness in the Hele-Shaw wind tunnel
- Effect of Reynolds number on BL
- Effect of the geometry of the body
- Types of BL & common misconception

What is a Boundary Layer?



What is a Boundary Layer?

- ~1900, aerodynamics consisted of 2 separate fields

  1. hydrodynamics – inviscid fluids, good results far field, but no drag, lift through circulation

  2. Hydraulics – empirical, but gave good results for pipe flows, etc.

- But in 1904 Prandtl (the ‘grandfather’ figure in fluid mechanics) united them through Boundary Layer Theory

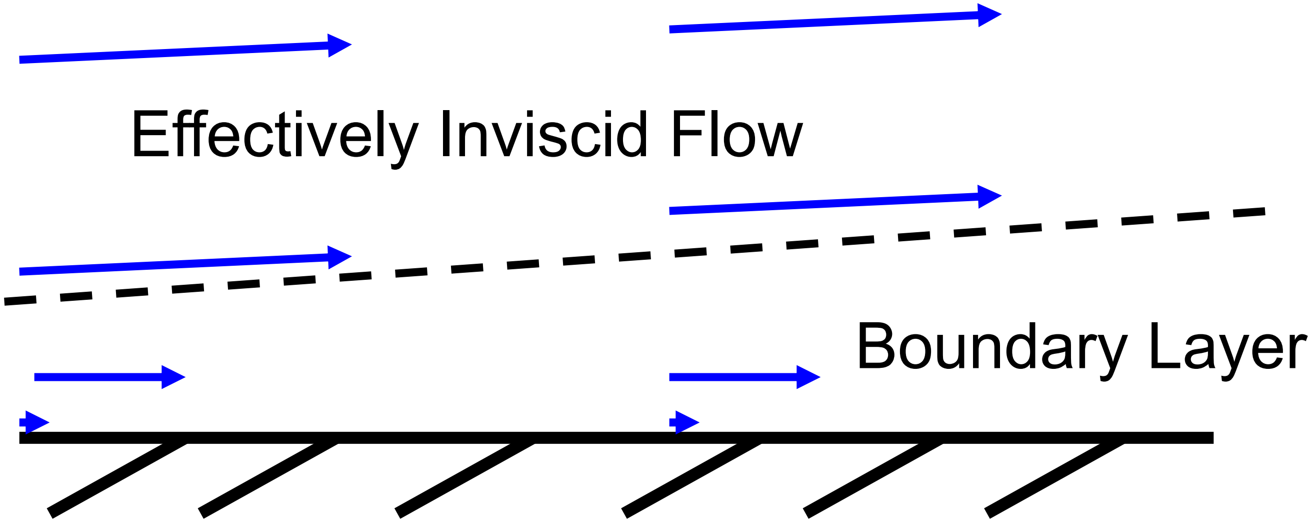
What is a Boundary Layer?

- Even though viscosity is very small, it cannot always be neglected. Specifically, in a Newtonian Fluid, the shear stress is related to the strain rate:

$$\tau = \mu \frac{\partial U}{\partial y}$$

- Wherever velocity changes rapidly in the normal direction, μ can cause significant τ.
- We have a no slip condition at the wall, i.e. U=0. If the flow reaches freestream only a small distance from the wall, then this is the case.

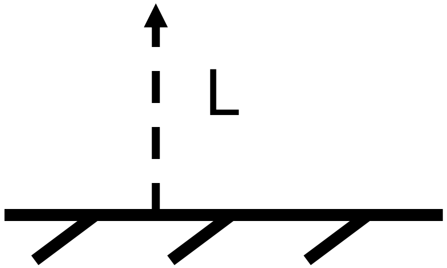
What is a Boundary Layer?



- The boundary layer is *very* thin
- The use of the boundary layer concept depends on large Reynolds number for main flow: >10^4

$$\left(\mathrm{Re} = \frac{\rho U l}{\mu} \right)$$

What is a Boundary Layer?



$$\mathrm{Re} = \frac{\rho U l}{\mu}$$

- Re is the ratio of Inertial to viscous forces
- Usually L is defined in streamwise direction, but imagine for now it is distance normal to surface. Very near surface, L <<1, hence Re is small.
- However, if $\rho U >> \mu$, Re quickly rises with increasing L, and hence viscous forces are only important very near the surface.

Why is Boundary Layer Theory important?

- We can calculate the vast majority of the flow field using inviscid methods – e.g. Joukowski Transformation, full potential, Euler.
- The effects of the boundary layer can be added (i.e. “patched”) afterwards, as a correction.
- The small thickness of the boundary layer means we can make significant reductions in the complexity of the Navier-Stokes Equations in this region.

Definition of Boundary Layer Thickness

- The boundary layer thickness is denoted by δ, and is defined as the distance from the wall where the boundary layer flow is indistinguishable from the external flow, i.e.

$$u = u_e, \ \rho = \rho_{e,} \ \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} = \ldots = 0$$

- Technically this can only happen an infinite distance from the wall, however, δ is usually defined where u = $0.995u_e$, or some similar value

Definition of Boundary Layer Thickness

- Empirical relations can be used to estimate boundary layer thickness

$$\boxed{\frac{\delta}{x} \approx \begin{cases} \dfrac{5.0}{Re^{1/2}}, & laminar \qquad 10^3 < Re_x < 10^6 \\ \dfrac{0.16}{Re^{\frac{1}{7}}}, & turbulent \qquad 10^6 < Re_x \end{cases}}$$

- Question: How do we calculate Re?

$$\mathrm{Re} = \frac{\rho U l}{\mu}$$

So how thick is a boundary layer?

- *Very* thin! For a laminar B.L., δ grows proportional to $Re^{-1/2}$:

$$\frac{\delta}{x} \approx 5\,\mathrm{Re}^{-1/2}$$

- So if on the t.e. of an a/c $Re = 10^6$,

$$\frac{\delta}{c} \approx 0.005$$

- i.e if the chord is say 3 metres, the boundary layer at the t.e. is 1.5 *centimetres* thick

Note – the boundary layer might transition to a turbulent state, which is **thicker**

Worked example: Boundary layer thickness

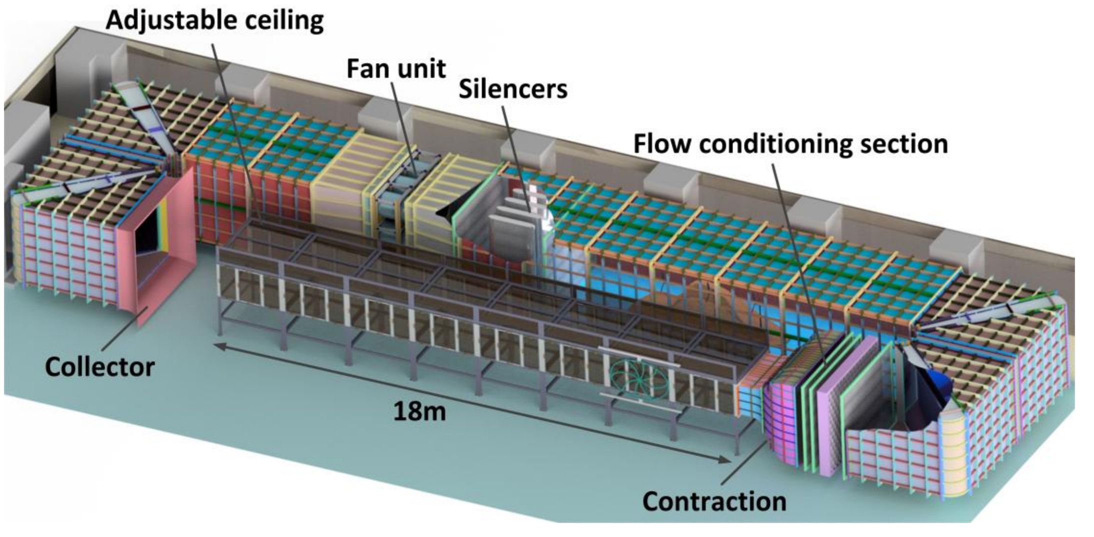
- Estimate the turbulent boundary layer thickness in the new Hele-Shaw lab boundary layer wind tunnel for U=30m/s and x=18m.

$$Re_{\mathrm{x}} = \frac{\mathrm{Ux}}{\nu}$$

$$Re_{\mathrm{x}} = \frac{30(18)}{1.5 \times 10^{-5}}$$

$$Re_{\mathrm{x}} = 3.6 \times 10^{7}$$

$$\delta = \frac{0.16x}{Re^{\frac{1}{7}}} = \sim 24cm$$

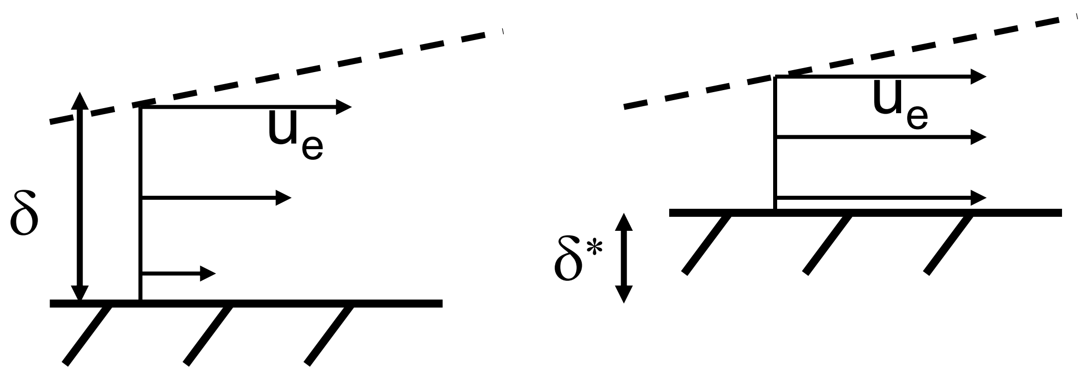


Displacement Thickness, δ^*

- Boundary layer thickness is a bit inexact (depends on how close we set u to u_e). Instead we can use *displacement thickness*, defined as

$$\rho_e u_e \delta^* = \int_0^h (\rho_e u_e - \rho u) dy \qquad \delta^* = \int_0^h \left(1 - \frac{\rho u}{\rho_e u_e}\right) dy$$

- Physically, this is the distance the wall would move to give an equivalent inviscid mass flow:



Momentum Thickness, θ

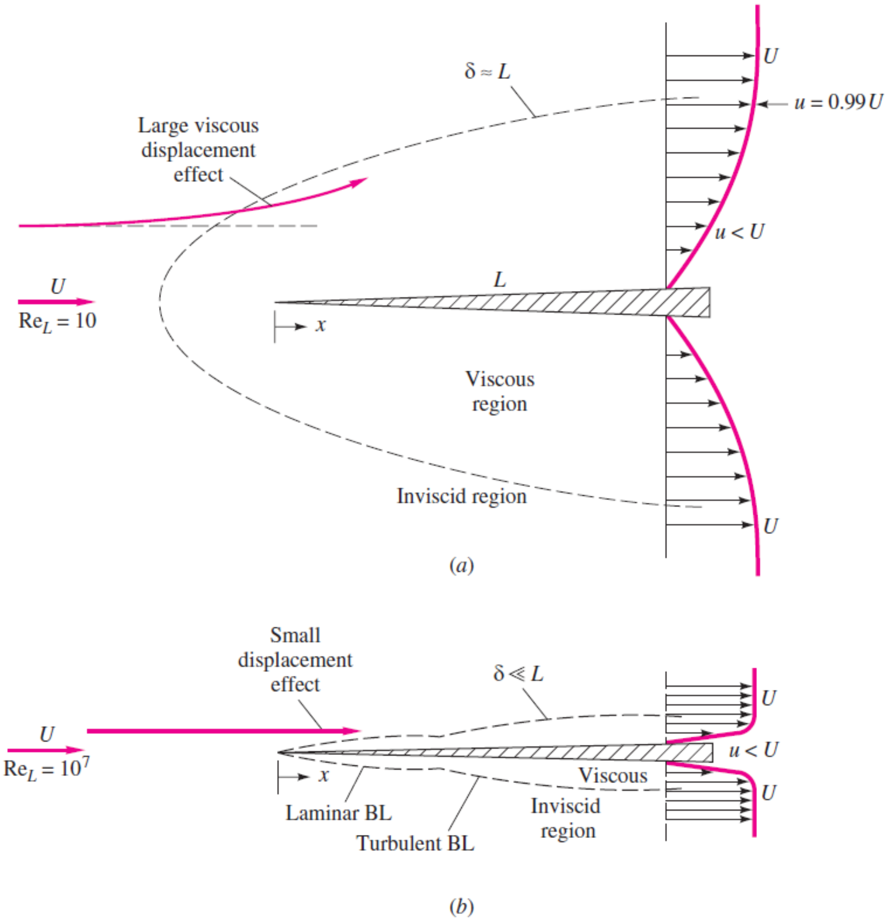
- The *Momentum Thickness* tells us what the loss in the momentum of the flow is, compared to what would be the case were there no boundary layer. It is defined as

$$\rho_e u_e^{\,2}\theta = \int_0^h \rho u(u_e - u)dy \qquad \theta = \int_0^h \frac{\rho u}{\rho_e u_e}\left(1 - \frac{u}{u_e}\right)dy$$

- As a change in momentum is equivalent to a force, this gives us the drag created by the boundary layer, including both *skin friction* and *pressure drags.*

Effects of Reynolds number

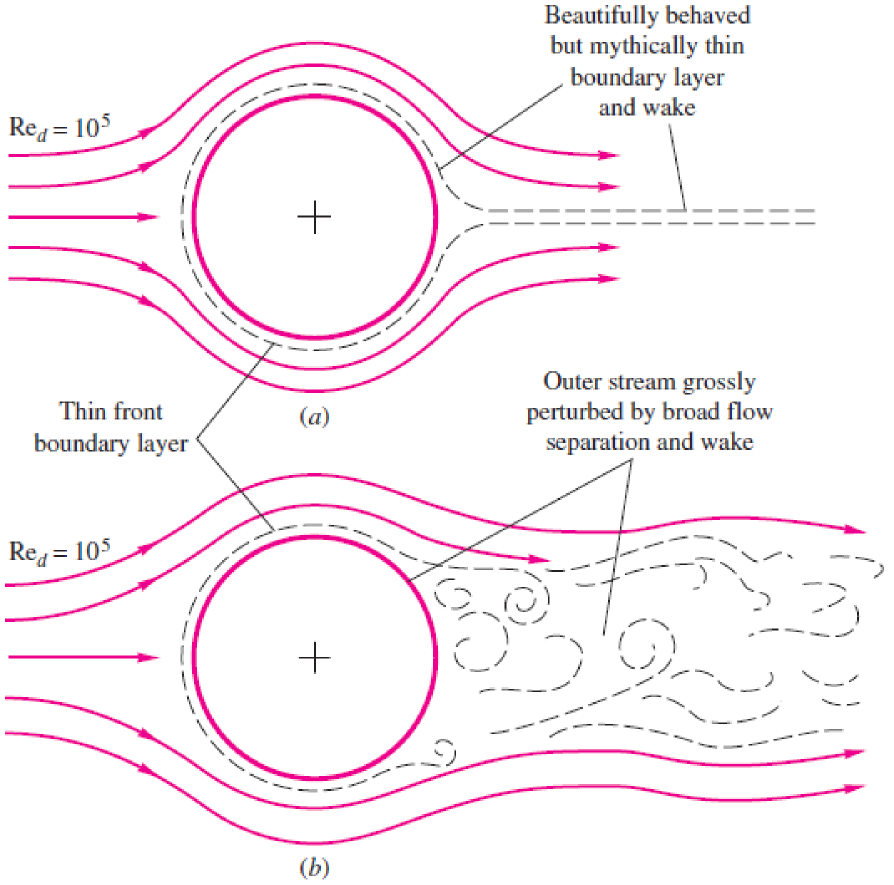
- High Reynolds number flow have very thin viscous layers
- More amenable to boundary layer patching!
- In most aerospace, industry, environmental flow applications, Re is very high



Source: Frank White page 458

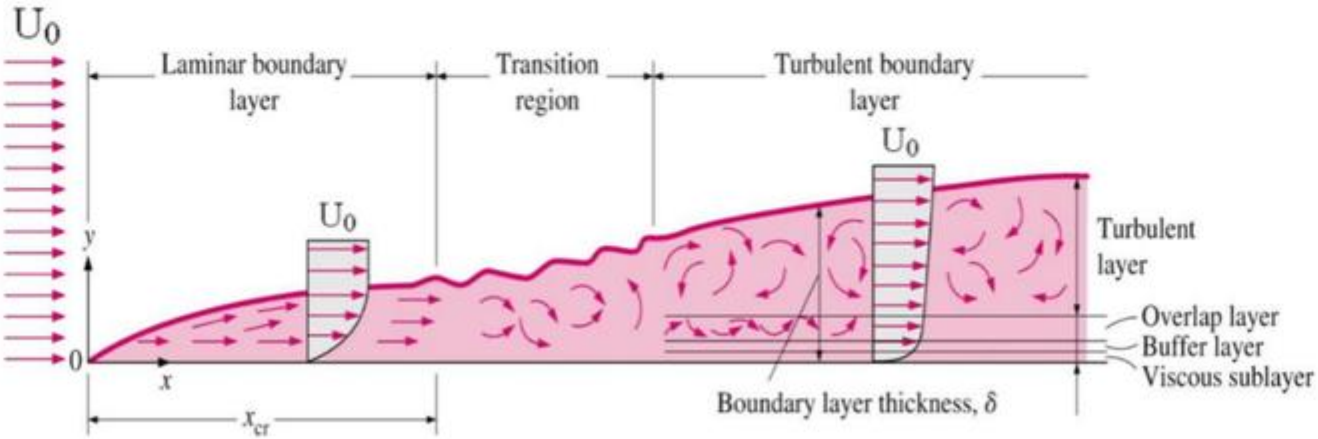
Effects of geometry

- For blunt-body flow, patching concept may not work even at very high Re!
- Favourable pressure gradient on windward side
- Adverse pressure gradient in the rear



Source: Frank White page 458

Types of Boundary Layer

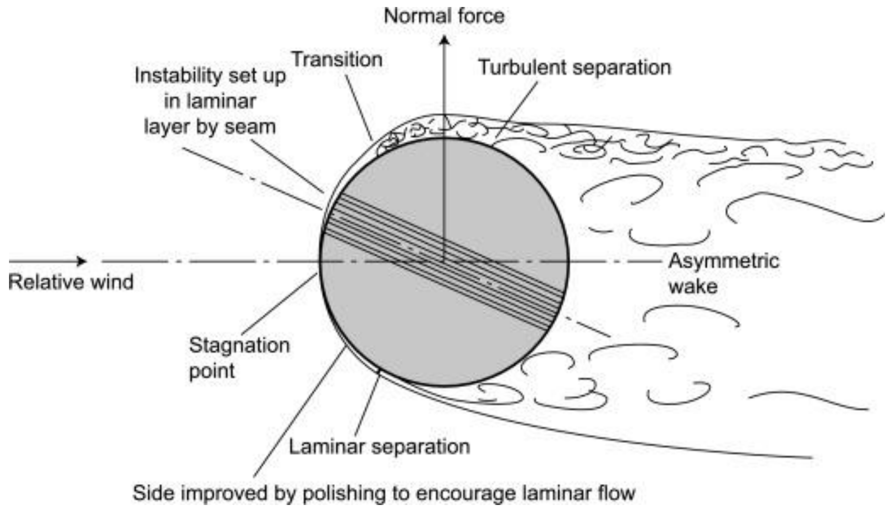


- Laminar BL characterised by smooth layering
- Turbulent BL by eddies and efficient mixing
- Transitional BL by *intermittency*, i.e. the flow is sometimes laminar, sometimes turbulent

Image source: https://www.nuclear-power.com/nuclear-engineering/fluid-dynamics/boundary-layer/

Common misconception

- Consider low Re (<100) flow around a ball. This flow is separated but **still laminar**
- Consider high Re flow around an aerofoil. This is a turbulent boundary layer but it is **still attached**
- **Separation and turbulence are not the same thing**



https://doi.org/10.1016/B978-0-08-100194-3.00003-1