Lecture 4: Discrete Solutions & Taylor Series

Part 1

Numerical and Simulation Methods

- Objective to develop and understand numerical methods to solve (N-S) eqns.
- Considered forms of the equations - PDE's with $\frac{\partial}{\partial t}$, $\frac{\partial}{\partial x,y,z}$, $\frac{\partial^2}{\partial x^2,y^2,z^2}$

- Briefly discussed implications of the form: conservative v non-conservative.

- Introduced a model equation.
- Considered the concept of 'time-marching'

$$\frac{\partial(?)}{\partial t} = -\sum_{spatial}^{dimensions}(spatial\ gradients) = -ERROR$$

TODAY

- Consider discrete solutions.
- Derive simple numerical time-marching method using Taylor series.
- FIRST: recap of Taylor series.

Recap of Taylor Series

The Taylor series defines a polynomial expansion in the neighbourhood of some arbitrary point that approximates the original function close to that point.
Consider a function of a single variable, $f(x)$, the Taylor series polynomial approximation around x reads:

$$f(x+\Delta x) \simeq f(x) + \Delta x \left.\frac{df(x)}{dx}\right|_x + \frac{1}{2}(\Delta x)^2 \left.\frac{d^2 f(x)}{dx^2}\right|_x + \frac{1}{6}(\Delta x)^3 \left.\frac{d^3 f(x)}{dx^3}\right|_x$$
$$+\frac{1}{24}(\Delta x)^4 \left.\frac{d^4 f(x)}{dx^4}\right|_x + \frac{1}{120}(\Delta x)^5 \left.\frac{d^5 f(x)}{dx^5}\right|_x + \frac{1}{720}(\Delta x)^6 \left.\frac{d^6 f(x)}{dx^6}\right|_x + O(\Delta x)^7$$

Where does it come from? A maths textbook. No...

Consider setting $\Delta x = 0$, then is recovers f. Consider differentiating once, then setting $\Delta x = 0$, and it recovers f', or twice and f''. Each time you differentiate the right number of times and evaluate at zero, it recovers the derivative you want, hence the structure of the series.

For a general function of multiple variables (consider coordinates here), we can expand in any variable directions, assuming we expand over a 'small' distance. For example, consider expanding function $F(x, y, z)$ in x:

$$F(x+\Delta x, y, z) \simeq F(x, y, z) + \sum_{n=1}^{n=\infty} \frac{1}{n!}(\Delta x)^n \left.\frac{\partial^n F(x, y, z)}{\partial x^n}\right|_{x,y,z}$$

Expanding out the summation gives:

$$F(x+\Delta x, y, z) \simeq F(x, y, z) + \Delta x \left.\frac{\partial F(x, y, z)}{\partial x}\right|_{x,y,z} + \frac{1}{2}(\Delta x)^2 \left.\frac{\partial^2 F(x, y, z)}{\partial x^2}\right|_{x,y,z} +$$

$$\frac{1}{6}(\Delta x)^3 \left.\frac{\partial^3 F(x, y, z)}{\partial x^3}\right|_{x,y,z} + \frac{1}{24}(\Delta x)^4 \left.\frac{\partial^4 F(x, y, z)}{\partial x^4}\right|_{x,y,z} .. + \frac{1}{\infty!}(\Delta x)^\infty \left.\frac{\partial^\infty F(x, y, z)}{\partial x^\infty}\right|_{x,y,z}$$

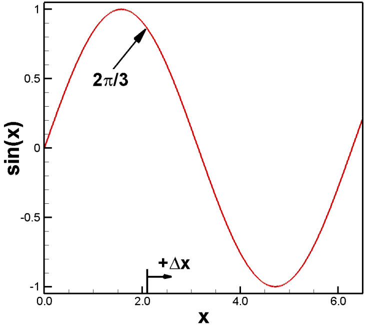
• Clearly we cannot calculate the entire infinite series, so instead we typically truncate the series after a certain number of terms to obtain a *"good enough"* approximation

• The truncated series has an error equal to the truncated terms, this is called the **TRUNCATION ERROR**.

• We describe the truncation error in terms of the lowest power of Δx.

• For example, if you expand in y up to and including $\frac{1}{6}(\Delta y)^3 \frac{\partial^3 F(x,y,z)}{\partial y^3}$, expansion terms of the order of $(\Delta y)^4$ and upward are ignored, so the expansion would be of 4th-order.

Let's consider $f(x) = \sin(x)$:



$$\left.\frac{df(x)}{dx}\right|_x = \cos(x) \qquad \left.\frac{d^2f(x)}{dx^2}\right|_x = -\sin(x) \qquad \left.\frac{d^3f(x)}{dx^3}\right|_x = -\cos(x)$$

$$\left.\frac{d^4f(x)}{dx^4}\right|_x = \sin(x) \qquad \left.\frac{d^5f(x)}{dx^5}\right|_x = \cos(x) \qquad \left.\frac{d^6f(x)}{dx^6}\right|_x = -\sin(x)$$

Evaluate this at $x = 2\pi/3$, and compute with $\Delta x = 0.1, 0.2, 0.4$, i.e. use Taylor expansions to approximate $\sin(2\pi/3 + 0.1), \sin(2\pi/3 + 0.2), \sin(2\pi/3 + 0.4)$, and compare with exact answer at these three points.

Consider variation of approximation error - What variables determine this ?
Only Δx varies here, so we can say, for a general approximation in Δx:

$$ERROR(\Delta x) = ERROR(0) + c(\Delta x)^P$$

where $ERROR(0)$ is the error with zero Δx and c is a scaling. However, for Taylor series, $ERROR(0)$ can only be zero. Hence, we can say:

$$\frac{ERROR(\Delta x_2)}{ERROR(\Delta x_1)} = \left(\frac{\Delta x_2}{\Delta x_1}\right)^P$$

so $\ln \frac{ERROR(\Delta x_2)}{ERROR(\Delta x_1)} = P \ln(2)$ can be used in this example to find P from the error ratio.

	$\sin(\frac{2\pi}{3}) = 0.86550134$	Exact $\sin(\frac{2\pi}{3} + 0.1) = 0.81117022$, $\Delta x = 0.1$		Exact $\sin(\frac{2\pi}{3} + 0.2) = 0.74873415$, $\Delta x = 0.2$			Exact $\sin(\frac{2\pi}{3} + 0.4) = 0.60211730$, $\Delta x = 0.4$		
Order	Terms included	Approxmtn	Error (E1)	Approxmtn	Error (E2)	E2/E1	Approxmtn	Error (E3)	E3/E2
1	$\sin(2\pi/3)$	0.86550134	5.433×10^{-2}	0.86550134	1.168×10^{-1}	2.1	0.86550134	2.634×10^{-1}	2.3
2	$+\Delta x\cos(2\pi/3)$	0.81541068	4.240×10^{-3}	0.76532001	1.659×10^{-2}	3.9	0.66513869	6.302×10^{-2}	3.8
3	$-\frac{1}{2}(\Delta x)^2\sin(2\pi/3)$	0.81108317	-8.705×10^{-5}	0.74800999	-7.242×10^{-4}	8.3	0.59589858	-6.219×10^{-3}	8.6
4	$-\frac{1}{6}(\Delta x)^3\cos(2\pi/3)$	0.81116665	-3.563×10^{-6}	0.74867786	-5.629×10^{-5}	15.8	0.60124158	-8.757×10^{-4}	15.6
5	$+\frac{1}{24}(\Delta x)^4\sin(2\pi/3)$	0.81117026	4.293×10^{-8}	0.74873556	1.411×10^{-6}	32.9	0.60216479	4.749×10^{-5}	33.7
6	$+\frac{1}{120}(\Delta x)^5\cos(2\pi/3)$	0.81117022	1.192×10^{-9}	0.74873423	7.561×10^{-8}	63.4	0.60212205	4.748×10^{-6}	62.8
7	$-\frac{1}{720}(\Delta x)^6\sin(2\pi/3)$	0.81117022	-1.015×10^{-11}	0.74873415	-1.326×10^{-9}	130.6	0.60211712	-1.765×10^{-7}	133.1

This is the standard use of Taylor series: if we can compute approximate values of the gradients of a function we can use the function value at a point in space with the gradients there to approximate the function value at some other point in space.

However, for a discrete solution and numerical method, we have the values of the function at points (hopefully) close to each other in space: to progress the solution in time, it is actually the gradients we need to approximate. Assume we have the solution values at values Δx apart, so at a particular point in space we have $f(x)$ and we also have $f(x+\Delta x), f(x-\Delta x), f(x+2\Delta x), f(x-2\Delta x)$ etc. Then:

$$f(x+\Delta x) \simeq f(x) + \Delta x \left.\frac{df(x)}{dx}\right|_x + \frac{1}{2}(\Delta x)^2 \left.\frac{d^2 f(x)}{dx^2}\right|_x + \frac{1}{6}(\Delta x)^3 \left.\frac{d^3 f(x)}{dx^3}\right|_x + \ldots.$$

$$f(x+\Delta x) - f(x) \simeq \Delta x \left.\frac{df(x)}{dx}\right|_x + \frac{1}{2}(\Delta x)^2 \left.\frac{d^2 f(x)}{dx^2}\right|_x + \frac{1}{6}(\Delta x)^3 \left.\frac{d^3 f(x)}{dx^3}\right|_x + \ldots.$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} \simeq \left.\frac{df(x)}{dx}\right|_x + \frac{1}{2}(\Delta x) \left.\frac{d^2 f(x)}{dx^2}\right|_x + \frac{1}{6}(\Delta x)^2 \left.\frac{d^3 f(x)}{dx^3}\right|_x + \ldots.$$

$$\Rightarrow \left.\frac{df(x)}{dx}\right|_x = \frac{f(x+\Delta x) - f(x)}{\Delta x} + O(\Delta x)$$

We can also say

$$f(x - \Delta x) \simeq f(x) - \Delta x \left. \frac{df(x)}{dx} \right|_x + \frac{1}{2}(\Delta x)^2 \left. \frac{d^2 f(x)}{dx^2} \right|_x - \frac{1}{6}(\Delta x)^3 \left. \frac{d^3 f(x)}{dx^3} \right|_x + \ldots.$$

which also gives a first-order approximation

$$\left. \frac{df(x)}{dx} \right|_x = \frac{f(x) - f(x - \Delta x)}{\Delta x} + O(\Delta x)$$

Alternatively, combining the expansions for $f(x+\Delta x)$ and $f(x-\Delta x)$ we can say

$$f(x+\Delta x) \simeq f(x) + \Delta x \left.\frac{df(x)}{dx}\right|_x + \frac{1}{2}(\Delta x)^2 \left.\frac{d^2 f(x)}{dx^2}\right|_x + \frac{1}{6}(\Delta x)^3 \left.\frac{d^3 f(x)}{dx^3}\right|_x + \ldots.$$

$$f(x-\Delta x) \simeq f(x) - \Delta x \left.\frac{df(x)}{dx}\right|_x + \frac{1}{2}(\Delta x)^2 \left.\frac{d^2 f(x)}{dx^2}\right|_x - \frac{1}{6}(\Delta x)^3 \left.\frac{d^3 f(x)}{dx^3}\right|_x + \ldots.$$

$$f(x+\Delta x) - f(x-\Delta x) \simeq 2\Delta x \left.\frac{df(x)}{dx}\right|_x + 2\frac{1}{6}(\Delta x)^3 \left.\frac{d^3 f(x)}{dx^3}\right|_x + \ldots.$$

and this gives a second-order accurate approximation to the gradient:

$$\Rightarrow \left.\frac{df(x)}{dx}\right|_x = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} + O((\Delta x)^2)$$

$$\left.\frac{d\sin(x)}{dx}\right|_{(\frac{2\pi}{3})} = \cos(x)|_{(\frac{2\pi}{3})} = \text{-0.5000}$$

Order	Gradient Computation	$\Delta x = 0.1$ Approxmtn	Error (E1)	$\Delta x = 0.2$ Approxmtn	Error (E2)	E2/E1	$\Delta x = 0.4$ Approxmtn	Error (E3)	E3/E2
1 Fward	$\frac{\sin(\frac{2\pi}{3}+\Delta x)-\sin(\frac{2\pi}{3})}{\Delta x}$	-0.542432	-4.243×10^{-2}	-0.582988	-8.299×10^{-2}	2.0	-0.657681	-1.577×10^{-1}	1.9
1 Bward	$\frac{\sin(\frac{2\pi}{3})-\sin(\frac{2\pi}{3}-\Delta x)}{\Delta x}$	-0.455902	4.410×10^{-2}	-0.410359	8.964×10^{-2}	2.0	-0.315865	1.841×10^{-1}	2.1
2 Central	$\frac{\sin(\frac{2\pi}{3}+\Delta x)-\sin(\frac{2\pi}{3}-\Delta x)}{2\Delta x}$	-0.499167	8.328×10^{-4}	-0.496673	3.327×10^{-3}	4.0	-0.486773	1.323×10^{-2}	4.0

giving P values of $\ln(2)/\ln(2) = 1$ or $\ln(1.9)/\ln(2) = 0.93$ or Fward, and $\ln(2)/\ln(2) = 1$ or $\ln(2.1)/\ln(2) = 1.07$ for Bward. The central scheme comes out at $P = 2$ for both Δx values.

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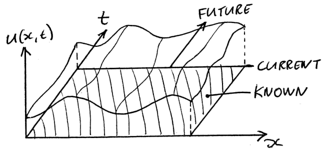
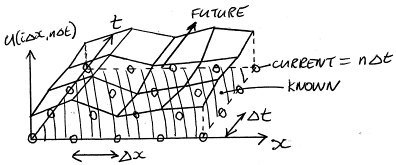
Part 2

Discretisation of Equations

Time-marching is used to predict the evolution of the flow, but at a discrete set of points, and over discrete time periods. This means we can abandon the need to seek continuous functions as a solution to fluid flow problems. Instead we compute numerically the solution at a finite number of points. To do this we introduce a labelling notation



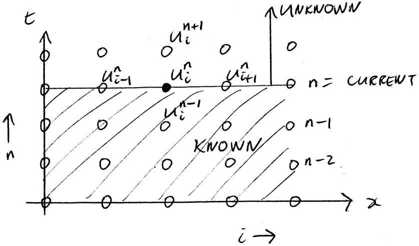


The continuous function $u(x,t)$ is modelled by a finite number of point values

$$u(i\Delta x, n\Delta t) = u_i^n$$

i is the spatial mesh point counter
n is the time level counter

Clearly our time-marching problem is how do we relate u_i^{n+1} to u_i^n for each mesh point (i) ?



Can we define an "equivalent" equation involving the solution values at our discrete mesh points ? This is known as "discretising" the equation. The trick is to use Taylor series expansions. We have the solution u stored at the points on a computational grid.

Consider expressing u_{i+1}^n in terms of quantities at time level n.

$$u_{i+1}^n = u((i+1)\Delta x, n\Delta t) = u(i\Delta x + \Delta x, n\Delta t)$$

$$u_{i+1}^n \simeq u(i\Delta x, n\Delta t) + \Delta x \left.\frac{\partial u}{\partial x}\right|_i^n + \frac{1}{2}(\Delta x)^2 \left.\frac{\partial^2 u}{\partial x^2}\right|_i^n + O(\Delta x^3)$$

Or

$$u_{i+1}^n = u_i^n + \Delta x \left.\frac{\partial u}{\partial x}\right|_i^n + \frac{1}{2}(\Delta x)^2 \left.\frac{\partial^2 u}{\partial x^2}\right|_i^n + O(\Delta x^3)$$

and so

$$\frac{u_{i+1}^n - u_i^n}{\Delta x} = \left.\frac{\partial u}{\partial x}\right|_i^n + \frac{1}{2}(\Delta x)\left.\frac{\partial^2 u}{\partial x^2}\right|_i^n + O(\Delta x^2)$$

If Δx is small we can write

$$\left.\frac{\partial u}{\partial x}\right|_i^n = \frac{u_{i+1}^n - u_i^n}{\Delta x} + O(\Delta x) = \frac{u_{i+1}^n - u_i^n}{\Delta x}$$

to a 'high degree of accuracy'.

But this is not the only choice. Consider

$$u_{i-1}^n = u_i^n - \Delta x\left.\frac{\partial u}{\partial x}\right|_i^n + \frac{1}{2}(\Delta x)^2\left.\frac{\partial^2 u}{\partial x^2}\right|_i^n + O(\Delta x^3)$$

and so

$$\frac{u_i^n - u_{i-1}^n}{\Delta x} = \left.\frac{\partial u}{\partial x}\right|_i^n - \frac{1}{2}(\Delta x)\left.\frac{\partial^2 u}{\partial x^2}\right|_i^n + O(\Delta x^2)$$

$\frac{u_{i+1}^n - u_i^n}{\Delta x}$ is a 'forward' difference approximation to $\left.\frac{\partial u}{\partial x}\right|_i^n$

$\frac{u_i^n - u_{i-1}^n}{\Delta x}$ is a 'backward' difference approximation to $\left.\frac{\partial u}{\partial x}\right|_i^n$

A better approximation is found as follows

$$u_{i+1}^n = u_i^n + \Delta x \left.\frac{\partial u}{\partial x}\right|_i^n + \frac{1}{2}(\Delta x)^2 \left.\frac{\partial^2 u}{\partial x^2}\right|_i^n + \frac{1}{6}(\Delta x)^3 \left.\frac{\partial^3 u}{\partial x^3}\right|_i^n + O(\Delta x^4)$$

$$u_{i-1}^n = u_i^n - \Delta x \left.\frac{\partial u}{\partial x}\right|_i^n + \frac{1}{2}(\Delta x)^2 \left.\frac{\partial^2 u}{\partial x^2}\right|_i^n - \frac{1}{6}(\Delta x)^3 \left.\frac{\partial^3 u}{\partial x^3}\right|_i^n + O(\Delta x^4)$$

So

$$u_{i+1}^n - u_{i-1}^n = 2\Delta x \left.\frac{\partial u}{\partial x}\right|_i^n + \frac{1}{3}(\Delta x)^3 \left.\frac{\partial^3 u}{\partial x^3}\right|_i^n + O(\Delta x^5)$$

or

$$\left.\frac{\partial u}{\partial x}\right|_i^n = \frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x} - \frac{1}{6}(\Delta x)^2 \left.\frac{\partial^3 u}{\partial x^3}\right|_i^n + O(\Delta x^4)$$

This is called a central difference, and is clearly a more accurate approximation for $\left.\frac{\partial u}{\partial x}\right|_i^n$ than the backward or forward, since we neglect terms $O(\Delta x^2)$ instead of $O(\Delta x)$.

So our numerical approximation to our continuous equation is so far

$$\left.\frac{\partial u}{\partial t}\right|_i^n + c \left.\frac{\partial u}{\partial x}\right|_i^n = \left.\frac{\partial u}{\partial t}\right|_i^n + \frac{c}{2\Delta x}(u_{i+1}^n - u_{i-1}^n)$$

Now we must approximate the temporal derivative.
Again expand about i, n

$$u_i^{n+1} = u_i^n + \Delta t \left.\frac{\partial u}{\partial t}\right|_i^n + \frac{1}{2}(\Delta t)^2 \left.\frac{\partial^2 u}{\partial t^2}\right|_i^n + O(\Delta t^3)$$

and so we can say

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \left.\frac{\partial u}{\partial t}\right|_i^n + \frac{1}{2}(\Delta t) \left.\frac{\partial^2 u}{\partial x^2}\right|_i^n + O(\Delta t^2)$$

so we approximate the temporal derivative by

$$\left.\frac{\partial u}{\partial t}\right|_i^n = \frac{u_i^{n+1} - u_i^n}{\Delta t} + O(\Delta t)$$

Hence, we obtain the finite-difference analogue of the original partial differential equation.

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

is approximated by

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + \frac{c}{2\Delta x}(u_{i+1}^n - u_{i-1}^n) + O(\Delta t, (\Delta x)^2) = 0$$

The TRUNCATION ERROR gives useful information about the scheme and will be considered later. Assuming the truncation error is small we can say

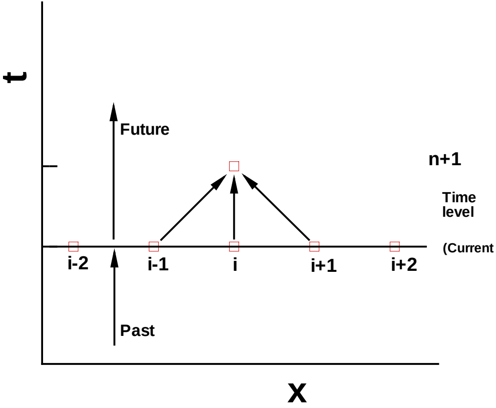
$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + \frac{c}{2\Delta x}(u_{i+1}^n - u_{i-1}^n) = 0$$

or

$$u_i^{n+1} = u_i^n - \frac{c\Delta t}{2\Delta x}(u_{i+1}^n - u_{i-1}^n)$$

This results in a scheme which gives the values of u at the new time level in terms of values of u at the old time level explicitly. Hence, this is known as an explicit scheme.

Back to the wave equation finite-difference stencil... stencil is



The temporal gradient is approximated using a 'forward' difference, and the spatial gradient using a 'central' difference, and so this scheme is called the FTCS scheme (forward time, central space).

Arbitrary stencils

Stencils of arbitrary order can be constructed for any derivative through construction of an appropriate linear system. Let's say we want a second order backward difference approximation to the first derivative. Then say

$$\frac{df}{dx}_i = \alpha f_i + \beta f_{i-1} + \gamma\alpha f_{i-2} \tag{1}$$

while of course

$$f_{i-1} = f_i - \Delta x f_i' + \frac{1}{2}\Delta x^2 f_i'' \tag{2}$$

$$f_{i-2} = f_i - 2\Delta x f_i' + 2\Delta x^2 f_i'' \tag{3}$$

which tells us immediately that (using terms in f) $\alpha + \beta + \gamma = 0$ and (using terms in f') $-\Delta x \beta - 2\Delta x \gamma = 1$ and (using terms in f'') $\frac{\Delta x^2 \beta}{2} + 2\Delta x^2 \gamma = 0$.
Solving this system of three linear equations gives $\alpha = \frac{3}{2\Delta x}$, $\beta = -\frac{2}{\Delta x}$, $\gamma = \frac{1}{2\Delta x}$. It's a second order stencil because the third derivative term in the expansion doesn't vanish, subsequently divided by Δx. So, the linear system more generally was

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & \frac{1}{2} & 2 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\Delta x} \\ \frac{0}{\Delta^2 x} \end{pmatrix} \tag{4}$$

You could of course build similar matrices for forward or central differences for any order of derivative and to any order of accuracy. For example, extending this to third order makes the structure clear, and uses

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & -3 \\ 0 & \frac{1^2}{2!} & \frac{2^2}{2!} & \frac{3^2}{2!} \\ 0 & -\frac{1^3}{3!} & -\frac{2^3}{3!} & -\frac{3^3}{3!} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\Delta x} \\ \frac{0}{\Delta x^2} \\ \frac{0}{\Delta x^3} \end{pmatrix} \tag{5}$$

yielding a third order backward difference of $(1.83/\Delta x, -3/\Delta x, 1.5/\Delta x, -0.333/\Delta x)$.

A central differencing approach for $\frac{d^2 f}{dx^2} \approx \alpha f_{i+2} + \beta f_{i+1} + \gamma f_i + \delta f_{i-1} + \epsilon f_{i-2}$ gives

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & -1 & -2 \\ \frac{2^2}{2!} & \frac{1^2}{2!} & 0 & \frac{1^2}{2!} & \frac{2^2}{2!} \\ \frac{2^3}{3!} & \frac{1^3}{3!} & 0 & -\frac{1^3}{3!} & -\frac{2^3}{3!} \\ \frac{2^4}{4!} & \frac{1^4}{4!} & 0 & \frac{1^4}{4!} & \frac{2^4}{4!} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \\ \epsilon \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{0}{\Delta x} \\ \frac{1}{\Delta x^2} \\ \frac{0}{\Delta x^3} \end{pmatrix} \tag{6}$$

yielding $(-0.083/\Delta x^2, 1.333/\Delta x^2, -2.5/\Delta x^2, 1.333/\Delta x^2, -0.083/\Delta x^2)$
It's worth pointing out that the above matrices can be built with variable Δx values, such that points can be unequally spaced while keeping the order of accuracy unchanged. This only requires inserting the Δx_i increments in the rows of the matrix, but does lead to long expressions for the differencing coefficients. Too long for a lecturer, it seems - refer to textbooks or compute in a symbolic package yourselves!
Don't worry - Taylor series *inversions* beyond a 3x3 would not be expected by hand, but you should understand how to build those matrices for central and one-sided differences.

Summary

• Discretisation is the concept of representing a real continuous funtion, $u(x, t)$, with a discrete solution, i.e. only stored/known at discrete points in time and space, $u(i\Delta x, n\Delta t)$.
• Considered Taylor Series expansions to compute function variation and gradients, and resulting order of accuracy.
• Taylor Series expansions used to derive numerical approximations to spatial and temporal gradients in our discrete solution, leading to a discrete time-marching scheme.
• Difference between our discrete gradients and the real gradients is the truncation error, and this gives us useful information on the accuracy of our scheme.
• Derived central-difference and one-sided difference schemes.
• Simple methods derived use only known data at current time level, so give future time solution EXPLICITLY.

NEXT LECTURE: Now consider properties of the schemes, and particularly stability.