CHAPTER 19

Yield Line Analysis of Slabs

The theory presented in this chapter extends the ultimate load analysis of structures, begun in Chapter 18 for beams and frames, to reinforced concrete slabs.

Structural engineers, before the development of ultimate load analysis, designed reinforced concrete slabs using elastic plate theory. This approach, however, gives no indication of the ultimate load-carrying capacity of a slab and further analysis had to be carried out to determine this condition. Alternatively, designers would use standard tables of bending moment distributions in orthogonal plates with different support conditions. These standard tables were presented, for reinforced concrete slabs, in Codes of Practice but were restricted to rectangular slabs which, fortunately, predominate in reinforced concrete construction. However, for non-rectangular slabs and slabs with openings, these tables cannot be used so that other methods are required. The method presented here, *yield line theory*, was developed in the early 1960s by the Danish engineer, K.W. Johansen.

19.1 Yield line theory

There are two approaches to the calculation of the ultimate load-carrying capacity of a reinforced concrete slab involving yield line theory. One is an energy method which uses the principle of virtual work and the other, an equilibrium method, studies the equilibrium of the various parts of the slab formed by the yield lines; we shall restrict the analysis to the use of the principle of virtual work since this was applied in Chapter 18 to the calculation of collapse loads of beams and frames.

Yield lines

A slab is assumed to collapse at its ultimate load through a system of nearly straight lines which are called *yield lines*. These yield lines divide the slab into a number of *panels* and this pattern of yield lines and panels is termed the *collapse mechanism*; a typical collapse mechanism for a simply supported rectangular slab carrying a uniformly distributed load is shown in Fig. 19.1(a).

The panels formed by the supports and yield lines are assumed to be plane (at fracture elastic deformations are small compared with plastic deformations and are ignored) and therefore must possess a geometric compatibility; the section AA in Fig. 19.1(b) shows a cross section of the collapsed slab. It is further assumed that the bending moment along all yield lines is constant and equal to the value corresponding to the yielding of the steel reinforcement. Also, the panels rotate about axes along the supported edges and, in a slab supported on columns, the axes of rotation pass through the columns, see Fig. 19.2(b). Finally, the yield lines on the sides of two adjacent panels pass through the point of intersection of their axes of rotation. Examples of yield line patterns are shown in Fig. 19.2. Note the conventions for the representation of different support conditions.

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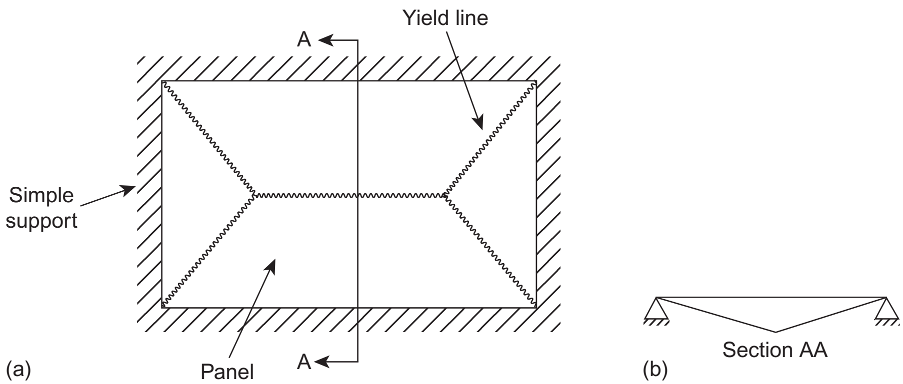

FIGURE 19.1

Collapse mechanism for a rectangular slab.

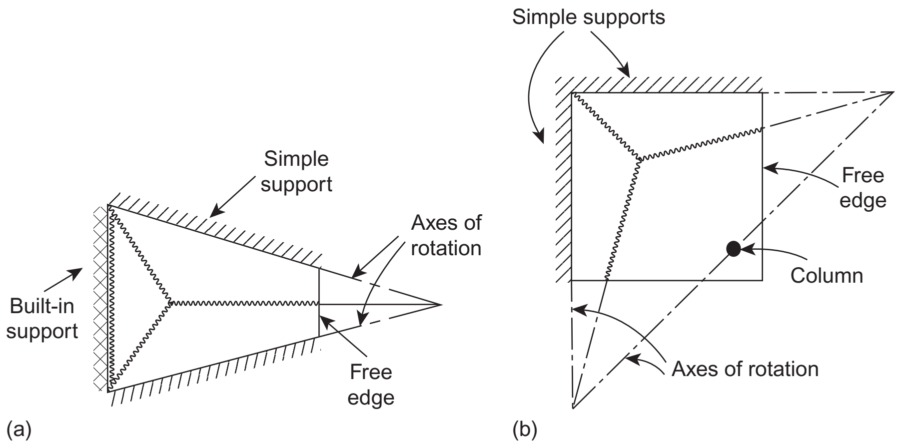

FIGURE 19.2

Collapse mechanisms and diagrammatic representation of support conditions.

In the collapse mechanisms of Figs 19.1(a) and 19.2(b) the supports are simple supports so that the slab is free to rotate along its supported edges. In Fig. 19.2(a) the left-hand edge of the slab is built in and not free to rotate. At collapse, therefore, a yield line will develop along this edge as shown. Along this yield line the bending moment will be hogging, i.e. negative, and the reinforcing steel will be positioned in the upper region of the slab; where the bending moment is sagging the reinforcing steel will be positioned in the lower region.

Ultimate moment along a yield line

Figure 19.3(a) shows a portion of a slab reinforced in two directions at right angles; the ultimate moments of resistance of the reinforcement are m_1 per unit width of slab and m_2 per unit width of slab. Let us suppose that a yield line occurs at an angle α to the reinforcement m_2. Now consider a triangular element formed by a

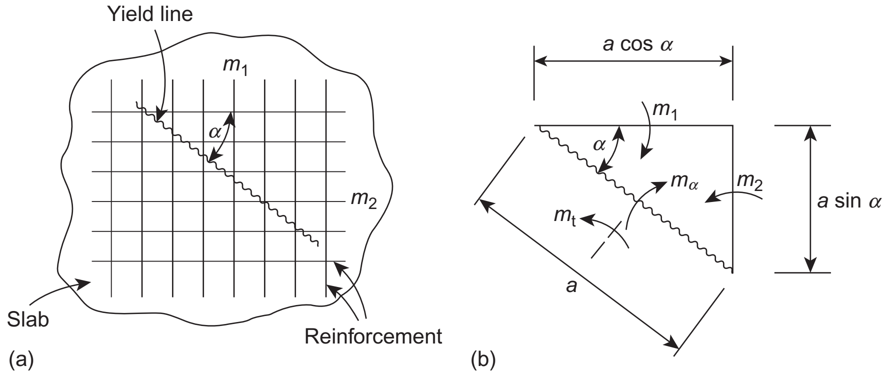

FIGURE 19.3

Determination of the ultimate moment along a yield line.

length a of the yield line and the reinforcement as shown in Fig. 19.3(b). Then, from the moment equilibrium of the element in the direction of m_α, we have

$$m_\alpha a = m_1 a \cos \alpha(\cos \alpha) + m_2 a \sin \alpha(\sin \alpha)$$

i.e.

$$m_\alpha = m_1 \cos^2\alpha + m_2 \sin^2\alpha \tag{19.1}$$

Now, from the moment equilibrium of the element in the direction of m_t

$$m_t a = m_1 a \cos \alpha(\sin \alpha) - m_2 a \sin \alpha(\cos \alpha)$$

so that

$$m_t = \frac{(m_1 - m_2)}{2} \sin 2\alpha \tag{19.2}$$

Note that for an isotropic slab, which is one equally reinforced in two perpendicular directions, $m_1 = m_2 = m$, say, so that

$$m_\alpha = m \quad m_t = 0 \tag{19.3}$$

Internal virtual work due to an ultimate moment

Figure 19.4 shows part of a slab and its axis of rotation. Let us suppose that at some point in the slab there is a known yield line inclined at an angle α to the axis of rotation; the ultimate moment is m per unit length along the yield line. Let us further suppose that the slab is given a small virtual rotation θ. The virtual work done by the ultimate moment is then given by

$$VW(m) = (mL)(\cos \alpha)\theta = m(L \cos \alpha)\theta \tag{19.4}$$

We see, therefore, from Eq. (19.4), that the internal virtual work done by an ultimate moment along a yield line is the value of the moment multiplied by the angle of rotation of the slab and the projection of the yield line on the axis of rotation.

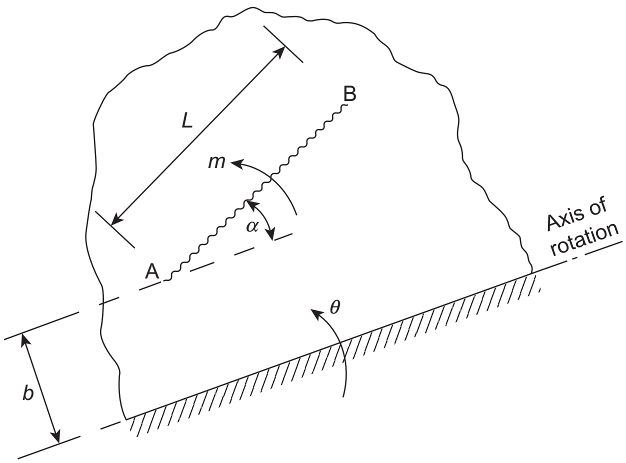

FIGURE 19.4

Determination of the work done by an ultimate moment.

Usually, rather than give a panel of a slab a virtual rotation, it is simpler to give a point on a yield line a unit virtual displacement. If, in Fig. 19.4 for example, the point A is given a unit virtual displacement then

$$\theta = \frac{1}{b}$$

where b is the perpendicular distance of A from the axis of rotation. Clearly the displacement of B due to θ would be greater than unity.

Virtual work due to an applied load

For a slab subjected to a distributed load of intensity $w(x, y)$ the virtual work done by the load corresponding to the virtual rotation of the slab panels is given by

$$VW(w) = \iint wu \, \mathrm{d}x \, \mathrm{d}y \tag{19.5}$$

where u is the virtual displacement at any point (x, y).

Conveniently, many applied loads on slabs are uniformly distributed so that we may calculate the total load on a slab panel and then determine the displacement of its centroid in terms of the given virtual displacement; the virtual work done by the load is then the product of the two and the total virtual work is the sum of the virtual works from each panel.

Having obtained the virtual work corresponding to the internal ultimate moments and the virtual work due to the applied load then the principle of virtual work gives

$$VW(w) = VW(m) \tag{19.6}$$

which gives the ultimate load applied to the slab in terms of its ultimate moment of resistance. This means, in fact, that we can calculate the required moment of resistance for a slab which supports a given load or, alternatively, we can obtain the maximum load that can be applied to a slab having a known moment of resistance. In the former case the given, or working, load is multiplied by a load factor to obtain an ultimate load while in the latter case the ultimate load is divided by the load factor.

The yield line pattern assumed for the collapse mechanism in a slab may not, of course, be the true pattern so that, as for the plastic analysis of beams and frames, the virtual work equation (Eq. (19.6)) gives either the correct ultimate moment or a value smaller than the correct ultimate moment. Therefore, for a given ultimate load (actual load×load factor), the calculated required ultimate moment of resistance is either correct or less than it should be. In other words, the solution is either correct or unsafe so that the virtual work approach gives an upper bound on the carrying capacity of the slab. Generally, in design, two or more yield line patterns are assumed and the maximum value of the ultimate moment of resistance obtained.

EXAMPLE 19.1

The slab shown in Fig. 19.5 is isotropically reinforced and is required to carry an ultimate design load of 12 kN/m^2. If the ultimate moment of resistance of the reinforcement is m per unit width of slab in the direction shown, calculate the value of m for the given yield line pattern.

We note that the slab is simply supported on three sides and is free on the other. Suppose that the junction c of the yield lines is given a unit virtual displacement.

Then

$$\theta_{\mathrm{A}} = \frac{1}{x} \; \theta_{\mathrm{B}} = \theta_{\mathrm{C}} = \frac{1}{2}$$

The internal virtual work is therefore given by

$$VW(m) = m \times 4\frac{1}{x} + 2\,m \times 4\frac{1}{2} \tag{i}$$

The first term on the right-hand side of Eq. (i) is the work done by the ultimate moment on the diagonal yield lines ac and bc on the boundary of panel A and is obtained as follows. We have seen that, for an isotropic slab, the ultimate moment along an inclined yield line is equal to the moment of resistance

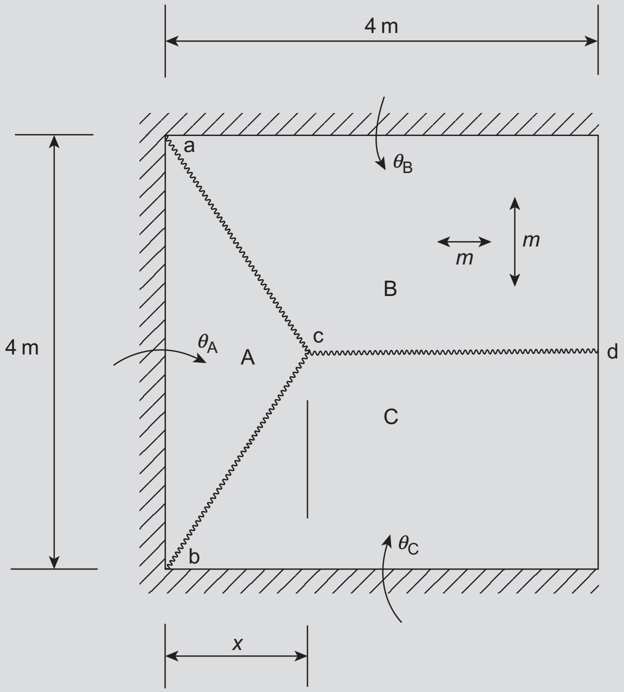

FIGURE 19.5

Slab of Ex. 19.1.

of the reinforcement irrespective of the inclination of the reinforcement to the yield line (Eq. (19.3)). Further, the work done by the ultimate moment on an inclined yield line is the product of the moment, the projection of the yield line on the axis of rotation and the angle of rotation of the panel (Eq. (19.4)). The second term on the right-hand side of Eq. (i) represents the work done by the ultimate moment on the diagonal and horizontal yield lines bordering each of the panels B and C; from symmetry the contribution of both panels will be the same. From the above argument and considering panel B

$$VW(m)_B = mx\left(\frac{1}{2}\right) + m(4-x)\left(\frac{1}{2}\right) = 4\,m\left(\frac{1}{2}\right)$$

Similarly for panel C.

Equation (i) simplifies to

$$VW(m) = 4\,m\left(\frac{1}{x}+1\right) \tag{ii}$$

The work done by the applied load is most easily found by dividing each of the panels B and C into a rectangle and a triangle, panel A is a triangle. Then

$$VW(w) = 12\left\{\frac{1}{2}\times 4x\times\frac{1}{3}+2\left[\frac{1}{2}x\times 2\times\frac{1}{3}+(4-x)\times 2\times\frac{1}{2}\right]\right\} \tag{iii}$$

In Eq. (iii) the displacement of the centroids of the triangles in panels A, B and C is 1/3 while the displacement of the centroids of the rectangular portions of panels B and C is 1/2. Eq. (iii) simplifies to

$$VW(w) = 96 - 8x \tag{iv}$$

Equating Eqs (ii) and (iv)

$$4m\left(\frac{1}{x}+1\right) = 96 - 8x$$

from which

$$m = 2\left(\frac{12x - x^2}{1+x}\right) \tag{v}$$

For a maximum, $(\mathrm{d}m/\mathrm{d}x) = 0$, i.e.

$$0 = \frac{(1+x)(12-2x)-(12x-x^2)}{(1+x)^2}$$

which reduces to

$$x^2 + 2x - 12 = 0$$

from which

$$x = 2.6\,\mathrm{m}\ (\text{the negative root is ignored})$$

Then, from Eq. (v)

$$m = 13.6\,\mathrm{kNm/m}$$

In some cases the relationship between the ultimate moment m and the dimension x is complex so that the determination of the maximum value of m by differentiation is tedious. A simpler approach would be to adopt a trial and error method in which a series of values of x are chosen and then m plotted against x.

In the above we have calculated the internal virtual work produced by an ultimate moment of resistance which acts along a yield line (Fig. 19.4). This situation would occur if the direction of the reinforcement was perpendicular to the direction of the yield line or if the reinforcement was isotropic (see Eq. (19.3)). A more complicated case arises when a band of reinforcement is inclined at an angle to a yield line and the slab is not isotropic.

Consider the part of a slab shown in Fig. 19.6 in which the yield line AB is of length L and is inclined at an angle α to the axis of rotation. Suppose also that the direction of the reinforcement m is at an angle β to the normal to the yield line.

Then, if the point B is given a unit virtual displacement perpendicular to the plane of the slab the angle of rotation θ is given by

$$\theta = \frac{1}{b}$$

where b is the perpendicular distance of B from the axis of rotation. Further, the rotation θ_r of the slab in a plane parallel to the reinforcement is given by

$$\theta_r = \frac{1}{r}$$

where r is the distance of B from the axis of rotation in a direction parallel to the reinforcement.

From the above

$$\theta_r = \theta \frac{b}{r} \tag{19.7}$$

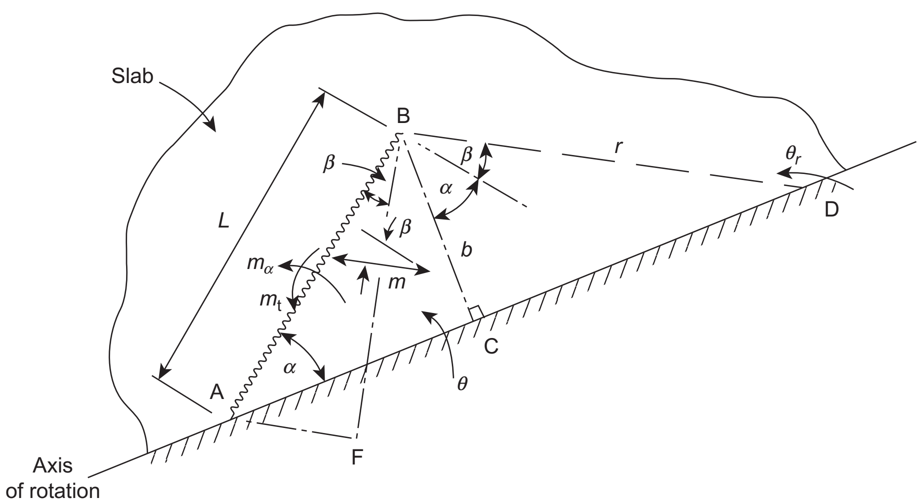

FIGURE 19.6

Reinforcement inclined at an angle to a yield line.

Also, from triangle BCD

$$\frac{b}{r} = \cos(\alpha + \beta)$$

Then, from Eq. (19.7)

$$\theta_r = \theta \cos(\alpha + \beta) \tag{19.8}$$

Now, from Eq. (19.1) in which, in this case, $m_1 = m$, $m_2 = 0$ and $\alpha = \beta$

$$m_\alpha = m \cos^2\beta \tag{19.9}$$

and

$$m_t = \left(\frac{m}{2}\right)\sin 2\beta \tag{19.10}$$

The internal virtual work due to the rotation θ is given by

$$VW(m) = (m_\alpha L)(\cos\alpha)\theta - (m_t L)(\sin\alpha)\theta \tag{19.11}$$

where the component of $(m_t L)$ perpendicular to the axis of rotation opposes the component of $(m_\alpha L)$. Substituting in Eq. (19.11) for m_α and m_t from Eqs (19.9) and (19.10), respectively we have

$$VW(m) = (mL\cos^2\beta)(\cos\alpha)\theta - \left[\left(\frac{m}{2}\right)L\sin 2\beta\right](\sin\alpha)\theta$$

which simplifies to

$$VW(m) = m(L\cos\beta)\theta(\cos\beta\cos\alpha - \sin\beta\sin\alpha)$$

or

$$VW(m) = m(L\cos\beta)\theta\cos(\alpha + \beta) \tag{19.12}$$

Substituting for $\theta \cos(\alpha + \beta)$ from Eq. (19.8) gives

$$VW(m) = m(L\cos\beta)\theta_r \tag{19.13}$$

In Eq. (19.13) the term $L \cos\beta$ is the projection BF of the yield line AB on a line perpendicular to the direction of the reinforcement. Equation (19.13) may be written as

$$VW(m) = m(L\cos\beta)\frac{1}{r} \tag{19.14}$$

where, as we have seen, r is the radius of rotation of the slab in a plane parallel to the direction of the reinforcement.

EXAMPLE 19.2

Determine the required moment parameter m for the slab shown in Fig. 19.7 for an ultimate load of 10 kN/m^2; the relative values of the reinforcement are as shown.

Note that in Fig. 19.7 the reinforcement of 1.2 m resists a hogging bending moment at the built-in edge of the slab and is shown dotted.

The first step is to choose a yield line pattern. We shall assume the collapse mechanism shown in Fig. 19.8; in practice a number of different patterns might be selected and investigated. Note that there

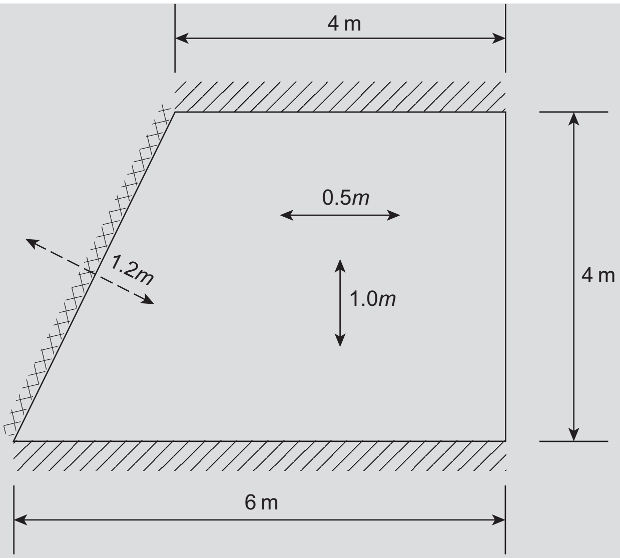

FIGURE 19.7

Slab of Ex. 19.2.

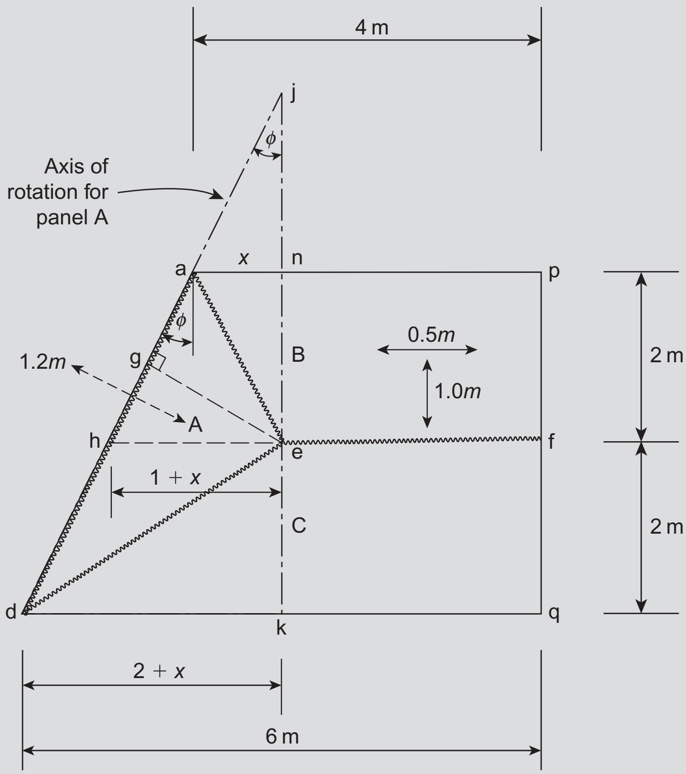

FIGURE 19.8

Yield line pattern for the slab of Ex. 19.2.

will be a yield line ad along the built-in edge. Suppose, now, that we impose a unit virtual displacement on the yield line at f; e will suffer the same virtual displacement since ef and ap are parallel. The angle of rotation of the panel B (and C) is then 1/2. Panel A rotates about the line ad and its angle of rotation is 1/ge where ge is the perpendicular distance of ad from e. From the dimensions given ad = 4.5 m and ge = he cos $\phi = (1+x)(4/4.5) = 0.89(1+x)$.

The slab is not isotropic so that we shall employ the result of Eq. (19.14) to determine the internal virtual work due to the ultimate moments in the different parts of the slab. Therefore, for each yield line we need to determine its projection on a line perpendicular to the reinforcement and the corresponding radius of rotation. We shall adopt a methodical approach.

(1) Panel A

- Reinforcement 1.2 m

  The axis of rotation is the line ad and since the reinforcement is perpendicular to the yield line ad the projected length is ad = 4.5 m. The radius of rotation is ge = $0.89(1+x)$. The virtual work is then

$$1.2\,m \times 4.5\left[\frac{1}{0.89(1+x)}\right] \tag{i}$$

- Reinforcement 0.5 m

  The sum of the projected length of the yield lines de and ea parallel to the reinforcement is 4 m and the radius of rotation is he = $1+x$. The virtual work is then

$$0.5\,m \times 4\left[\frac{1}{(1+x)}\right] \tag{ii}$$

- Reinforcement 1.0 m

  The projection of the yield line de in a direction parallel to the reinforcement is dk = $2+x$ and the corresponding radius of rotation is ej = he/tan $\phi = 2(1+x)$.

  For the yield line ea the projected length is na = x and its radius of rotation is the same as that of the yield line de, i.e. $2(1+x)$. However, since the centre of rotation is at j the displacement of the reinforcement crossing the yield line ea is less than its displacement as it crosses the yield line de. At de, therefore, the reinforcement will be sagging while at ea it will be hogging. The contributions to the virtual work at these two points will therefore be of opposite sign. The virtual work is then

$$\frac{1.0\,m[(2+x)-x]}{2(1+x)} = \frac{1.0\,m}{(1+x)} \tag{iii}$$

(2) Panel B

- Reinforcement 1.0 m

  We note that the 0.5 m reinforcement is parallel to the axis of rotation and does not, therefore, contribute to the virtual work in this panel. The projection of the yield lines ae and ef is 4 m and the radius of rotation is 2 m. The virtual work is then

$$1.0\,m \times \frac{4}{2} = 2.0\,m \tag{iv}$$

(3) Panel C

- Reinforcement 1.0 m

The situation in panel C is identical to that in panel B except that the projection of the yield lines de and ef is 6 m. The virtual work is then

$$3.0\,m \tag{v}$$

Adding the results of Eqs (i)–(v) we obtain the total internal virtual work, i.e.

$$VW(m) = \left(\frac{14.07 + 5x}{1 + x}\right) \tag{vi}$$

The external virtual work may be found by dividing the slab into rectangles enpf and ekqf and triangles ane, ekd and ade. Since the displacement of e is unity the displacement of each of the centroids of the rectangles will be 1/2 and the displacement of each of the centroids of the triangles will be 1/3. The total virtual work due to the applied load is then given by

$$VW(w) = 10\left[2(4-x)\left(\frac{2}{2}\right) + \left(\frac{x}{2}\right)\left(\frac{2}{3}\right) + 2(2+x)\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) + 4.5 \times 0.89(1+x)\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)\right]$$

which simplifies to

$$VW(w) = 10(9.33 - 0.67x) \tag{vii}$$

Equating internal and external virtual works, Eqs (vi) and (vii), we have

$$m = \frac{10(1+x)(9.33 - 0.67x)}{14.07 + 5x} \tag{viii}$$

The value of x corresponding to the maximum value of m may be found by differentiating Eq. (viii) with respect to x and equating to zero. Alternatively, a series of trial values of x may be substituted in Eq. (viii) and the maximum value of m obtained. Using the former approach gives $x = 2.71$ m from which

$$m = 10.9\,\text{kNm/m}$$

EXAMPLE 19.3

The slab shown in Fig. 19.9(a) has an opening at the corner A to allow the passage of a hoist. The slab is built in on the sides AB, BC and AD and is simply supported on the side DC. The relative values of the moments of resistance per unit width for hogging and sagging bending at relevant positions in the slab are shown in Fig. 19.9(b). For the typical yield line pattern shown in Fig. 19.9(a) calculate the value of the moment parameter m if the slab has to carry an ultimate design load of $12\,\text{kN/m}^2$.

The rotations of the different parts of the slab are shown in Fig. 19.10.

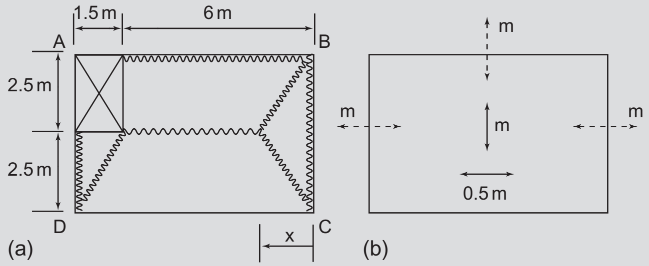

FIGURE 19.9

Slab of Ex. 19.3

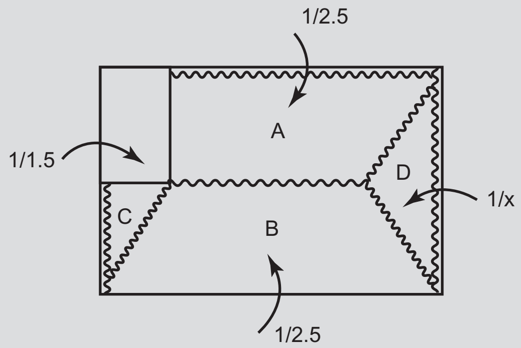

FIGURE 19.10

Rotations for slab of Ex. 19.4

The work absorbed by the different parts of the slab is as follows.

$$\begin{aligned}
&\mathrm{A}: (m+m)\times 6\times(1/2.5) = 4.8\,m \\
&\mathrm{B}: m\times 7.5\times(1/2.5) = 3\,m \\
&\mathrm{C}: (m+0.5\,m)\times 2.5\times(1/1.5) = 2.5\,m \\
&\mathrm{D}: (m+0.5\,m)\times 5\times(1/x) = 7.5\,m/x
\end{aligned}$$

The total work absorbed is therefore $m[10.3 + (7.5/x)]$
The total work done by the load is:

$$12[2.5\times 1.5\times(1/3)+5\times x\times(1/3)+(6-x)\times 5\times(1/2)] = 195-10\,x$$

Equating the work absorbed by the slab to the work done by the load gives

$$m = \frac{195x-10x^2}{10.3x+7.5} \tag{i}$$

Differentiating Eq. (i) and equating to zero gives

$$x^2+1.456\,x-14.199=0$$

the solution of which is

$$x = 3.11\ \mathrm{m}$$

Substituting this value in Eq. (i) gives

$$m = 12.9\ \mathrm{kNm/m}$$

19.2 Discussion

The method presented here for the analysis of reinforced concrete slabs gives, as we have seen, upper bound values for the collapse loads of slabs. However, in relatively simple cases of slab geometry and loading, the yield line method can be used as a design method since the fracture pattern can be obtained with reasonable accuracy. Also, in practice the actual collapse load of a slab may be above the calculated value because of secondary effects such as the kinking of the reinforcing steel in the vicinity of the fracture line and the effect of horizontal edge restraints which induce high compressive forces in the plane of the slab with a consequent increase in load capacity.

An alternative to yield line theory is the *strip method* proposed by A. Hillerborg at Stockholm in 1960. This method is a direct design procedure as opposed to yield line theory which is analytical and therefore will not be investigated here.

PROBLEMS

P.19.1 Determine, for the slab shown in Fig. P.19.1, the required moment parameter m if the design ultimate load is 14 kN/m^2.

Ans. 24.31 kNm/m.

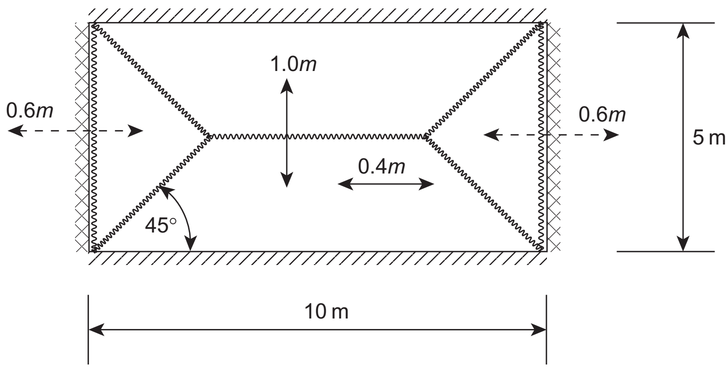

FIGURE P.19.1

P.19.2 The reinforced concrete slab shown in Fig. P.19.2(a) is designed to have an ultimate load capacity of 10 kN/m^2 across its complete area. Determine the required value of the moment parameter m given that the yield line pattern is as shown.

If an opening is introduced as shown in Fig. P.19.2(b) determine the corresponding required value of the moment parameter m.

Ans. 32.37 kNm/m, 35.27 kNm/m.

P.19.3 In the slab shown in Fig. P.19.3 Area 1 carries an ultimate load of intensity 12 kN/m^2 while Area 2 carries an ultimate load of intensity 8 kN/m^2. Determine the value of the moment parameter m assuming the yield line pattern shown.

Ans. 14.73 kNm/m.

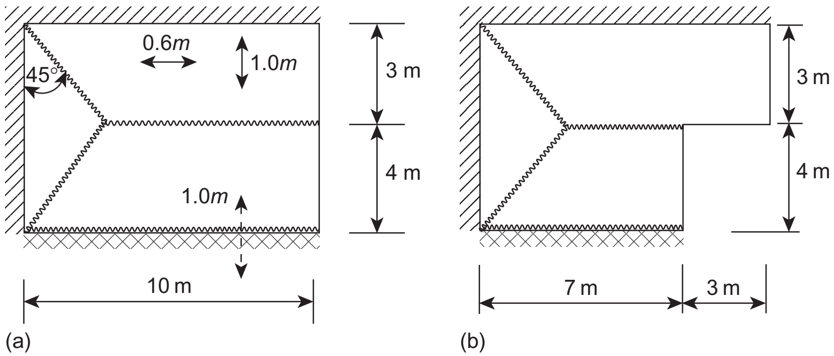

FIGURE P.19.2

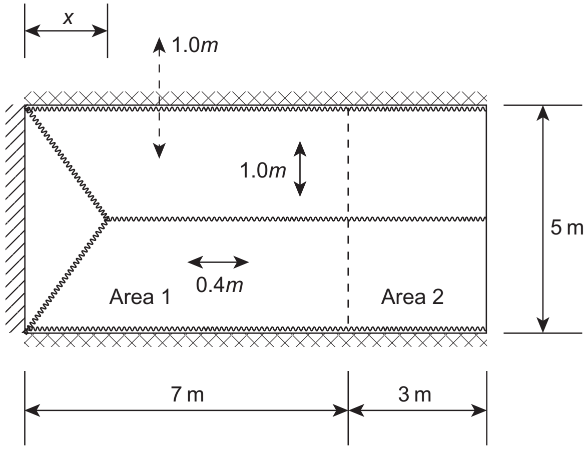

FIGURE P.19.3

P.19.4 Calculate the intensity of uniformly distributed load that would cause the reinforced concrete slab shown in Fig. P.19.4 to collapse given the yield line pattern shown.

Ans. 15.45 kN/m^2.

P.19.5 The reinforced concrete slab shown in Fig. P.19.5 is to be designed to carry an ultimate load of 15 kN/m^2. The distribution of reinforcement is to be such that the ultimate moments of resistance per unit width of slab for sagging bending are isotropic and of value m while the ultimate moment of resistance per unit width at continuous edges is 1.2 m. For the yield line pattern shown derive the general work equation and estimate the value of m by using trial values of $x = 2.0$, 2.5 and 3.0 m.

Ans. 9.70 kNm/m.

P.19.6 The reinforced concrete slab shown in Fig. P.19.6 is reinforced such that the sagging moments of resistance are isotropic and of value 1.0 m while the hogging moment of resistance at all built-in edges is 1.4 m. Estimate the required value of the moment parameter m if the ultimate design load intensity is 20 kN/m^2.

Ans. 15.48 kNm/m for $x = 2.5$ m.

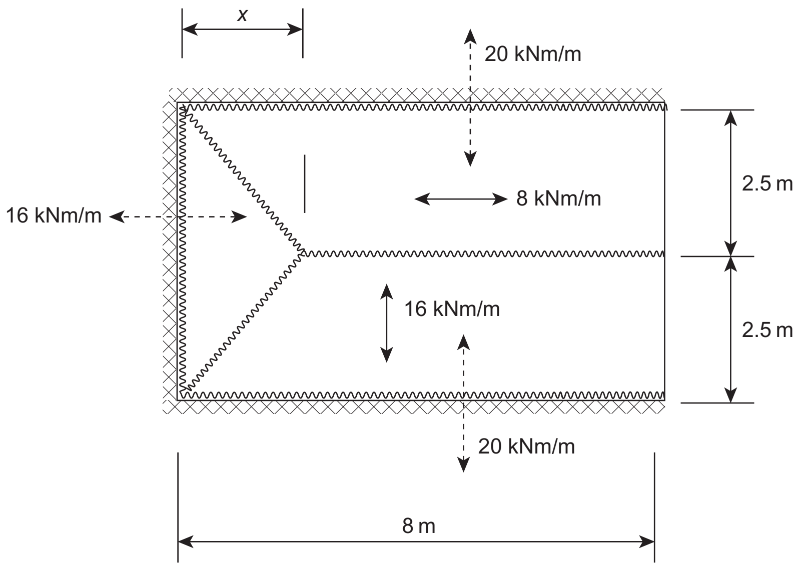

FIGURE P.19.4

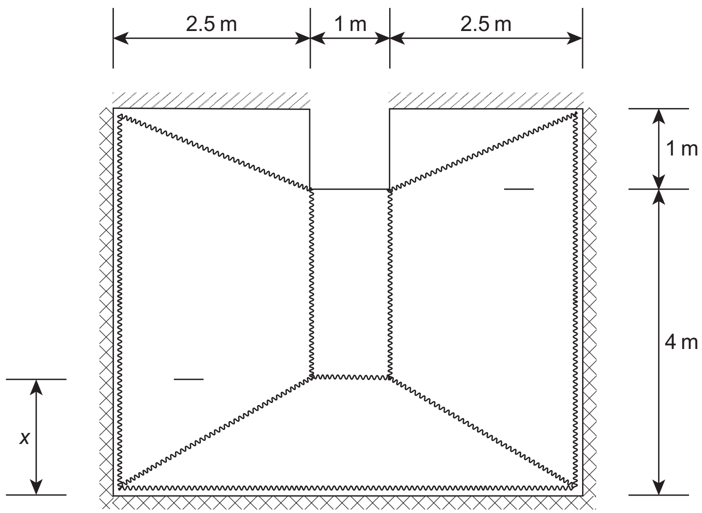

FIGURE P.19.5

P.19.7 The reinforced concrete slab shown in Fig. P.19.7(a) is simply supported on the sides AB and DC and is built in on the sides AD and BC. The layout of reinforcement is such that the sagging moments of resistance about the x and y axes are 1.0 m and 0.4 m per unit width respectively while the hogging moment of resistance about the y direction at the built in edges is 0.6 m per unit width. If the design ultimate load is 14 kN/m^2 determine the required moment parameter m of the resistance of the slab. By consideration of the secondary yield line pattern shown in Fig. P.19.7(b) determine the minimum extent L of the necessary hogging moment of resistance from the built in

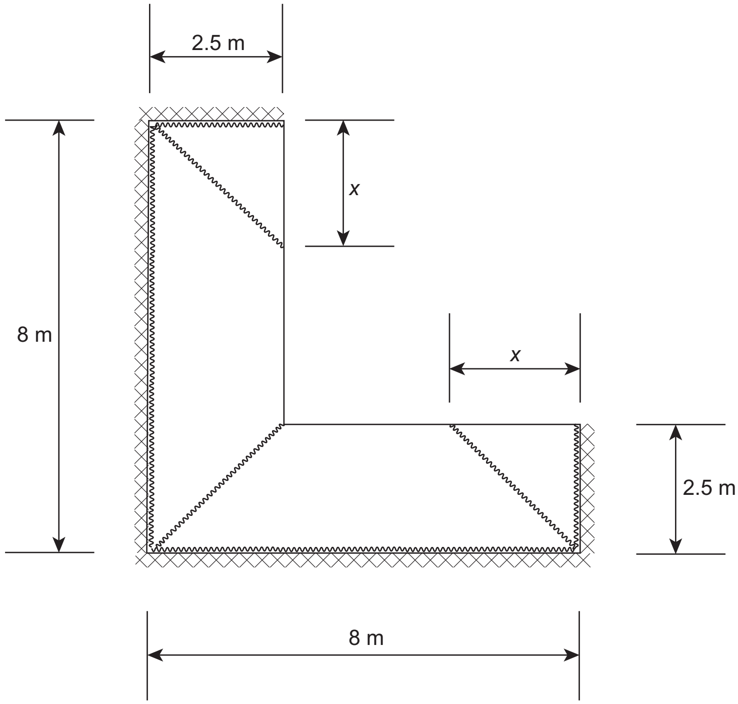

FIGURE P.19.6

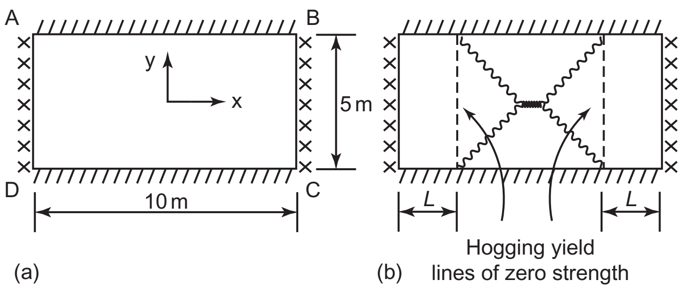

FIGURE P.19.7

edges so that the ultimate load for this pattern is also 14 kN/m^2. Assume that inclined yield lines make an angle of 45° with the x and y axes.

Ans. $m = 24.31$ kNm/m. $L = 1.875$ m.

P.19.8 The reinforced concrete slab shown in Fig. P.19.8 is simply supported on the sides AB, BC, CD and AD and is continuous over the beam EF which is simply supported at E and F. The slab has isotropic sagging moments of resistance m per unit width and a hogging moment of resistance 1.5 m per unit width over the beam EF. If the slab is subjected to an ultimate load of 15 kN/m^2 determine, by consideration of the two yield line patterns shown, the ultimate moment of resistance of the slab and the ultimate moment of resistance of the beam EF.

Ans. m (slab) = 14.83 kNm/m, m (beam) = 452.5 kNm.

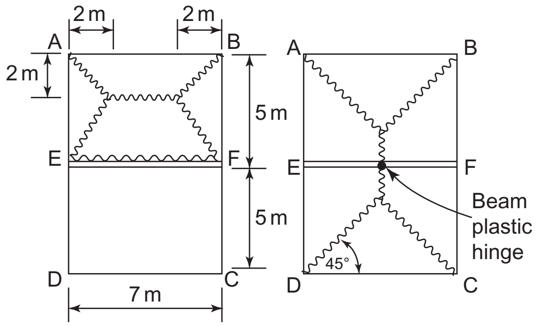

FIGURE P.19.8

P.19.9 The reinforced concrete slab shown in Fig. P.19.9 is simply supported on all its outer edges and has an additional wall support internally along the line aa over which the slab is continuous. The ultimate moments of resistance for sagging bending are isotropic and of value m per unit width while the ultimate moments for hogging bending are also isotropic and of value 1.2 m per unit width. If the design ultimate load is 14 kN/m^2 over the whole slab area and the yield line pattern is as shown find the value of the moment parameter m.

Ans. 22.05 kNm/m.

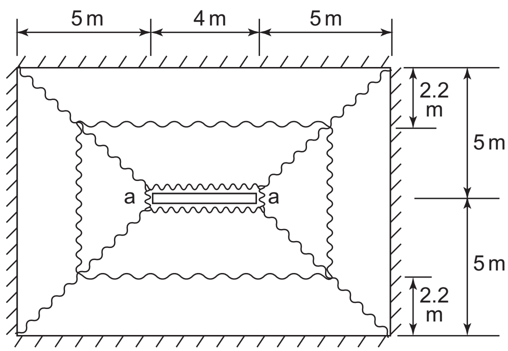

FIGURE P.19.9

P.19.10 In the slab and beam system shown in Fig. P.19.10, the slab is continuous over the beams on the lines A, B, 1, and 2 but has a free edge on the line C. Further, at positions where the slab is continuous, it has a hogging moment of resistance per unit width of 1.5M. If the sagging moments of resistance per unit width are isotropic and of value M in panel I, calculate the maximum uniformly distributed load that panel I can carry. Also, calculate, in terms of M, the isotropic sagging moment the isotropic moments of resistance that panel II must have so that the collapse load intensity is the same as that for panel I.

Ans. 59.12M/L^2, 0.964M.

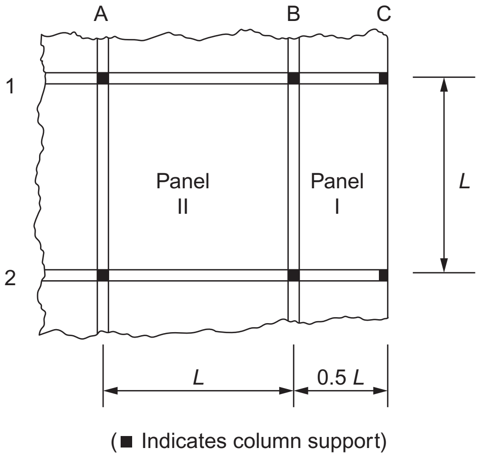

FIGURE P.19.10

P.19.11 Sketch possible yield line patterns for the isotropically reinforced concrete slabs shown in Fig. 19.11(a). The reinforced concrete slab shown in Fig. 19.11(b) is to carry an ultimate design load intensity of 14 kN/m^2. The slab is reinforced such that its moment of resistance per unit width parallel to the x direction is M while that parallel to the y direction is $1.2M$. Assuming the yield line pattern shown calculate the required value of M.

Ans. See Solutions Manual for yield line patterns.
20.19 kNm/m.

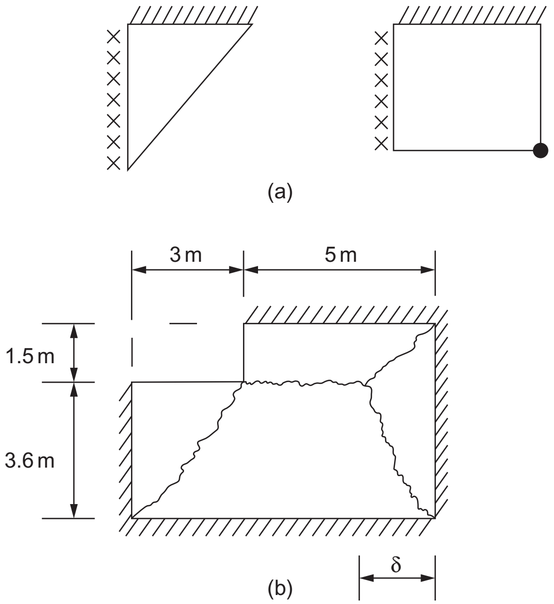

FIGURE P.19.11