

Department of Computer Science and Engineering Scilab

LINEAR ALGEBRA AND ITS APPLICATIONS -UE19MA251

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BRANCH : COMPUETR SCIENCE AND ENGINEERING

SEMESTER & SECTION: IV

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8-9	Find the inverse of the following matrix: $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$
10-11	Identify the columns that span the column space of A in the following $A = \begin{pmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{pmatrix}$
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17-18	Apply the Gram – Schmidt process to the following set of vectors and find the orthogonal matrix: (1, 0, 1), (1, 1, 0), (2, 1, 1)
19-20	Find the Eigen values and the corresponding Eigen vectors of the following matrix:
	$\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$

Topic: Gaussian Elimination

Q1) Solve the following system of equations by Gaussian Elimination. Identify the pivots.

$$2x - 3y = 3$$
, $4x - 5y + z = 7$, $2x - y - 3z = 5$

A) Scilab Code :-

```
P1.sce (/Users/trishajain/Desktop/DAA_Assignment/P1.sce) - SciNotes
                  P1.sce (/Users/trishajain/Desktop/DAA_Assignment/P1.sce) - SciNotes
                                             P1.sce 🔀
  1 | clc; clear; close;
  2 A = [2,-3,0;4,-5,1;2,-1,-3], b = [3;7;5];
  3 A_aug = [A - b];
  4 a = A_aug;
  5 \mid n = 3;
   |for-i-=-2:n
      --for-j=2:n+1
            a(i,j) = a(i,j) = -a(1,j)*a(i,1)/a(1,1);
  8
  9
        a(i,1) = 0;
 10
 11 end
 12 for i=3:n
 13 --- for j=3:n+1
            a(i,j) = a(i,j)-a(2,j)*a(i,2)/a(2,2);
 15 ---- end
 16
        a(i,2) -= -0;
 17 end
 18
 19|x(n) = a(n,n+1)/a(n,n);
 20 for i=n-1:-1:1
    - - - sumk - = - 0;
 21
    ----for-k=i+1:n
            sumk = sumk + a(i,k) *x(k);
 24 - - - end
 25 - x(i) = (a(i,n+1) - sumk)/a(i,i);
 26 end
 27
 28 disp("Values-of-x,y,z:")
 29 disp(x(3),x(2),x(1));
 30 disp('Values-of-pivots:');
 31 disp(a(1,1),a(2,2),a(3,3));
 32
```

"Values of x,y,z:"

0.

1.

3.

"Values of pivots:"

2.

1.

-5.

--> |

Topic: LU decomposition of a matrix

Q2) Solve the system of equations by decomposing A as a product A = LU

$$2x - 3y = 3$$
, $4x - 5y + z = 7$, $2x - y - 3z = 5$

A) Scilab Code:-

```
P2.sce (/Users/trishajain/Desktop/DAA_Assignment/P2.sce) - SciNotes
                                                                      P2.sce (/Users/trishajain/Desktop/DAA_Assignment/P2.sce) - SciNotes
                                                P2.sce 🔀
  1 clc; clear; close;
  2 A = [2 - 3 - 0; 4 - 5 - 1; -2 - 1 - 3];
  3 U-=-A;
  4 disp("The given matrix is:", A);
    m - = - det(U(1,1));
    n-=-det(U(2,1));
    a = n/m;
    U(2,:)=-U(2,:)---U(1,:)/(m/n);
  g \mid n = det(U(3,1));
 10 b = n/m;
 11 U(3,:) = U(3,:) - U(1,:)/(m/n);
 12 m = \det(U(2,2));
 13 n = \det(U(3,2));
 14 c = -n/m;
 15 U(3,:) = U(3,:) - U(2,:)/(m/n);
 16 disp("The upper triangular matrix is U:",U);
 17 L = [1,0,0;a,1,0;b,c,1];
 18 disp("The-lower-triangular-matrix-is-L:",L);
 19
```

"The given matrix is:"

- 2. -3. 0.
- 4. -5. 1.
- 2. -1. -3.

"The upper triangular matrix is U:"

- 2. -3. 0.
- 0. 1. 1.
- 0. 0. -5.

"The lower triangular matrix is L:"

- 1. 0. 0.
- 2. 1. 0.
- 1. 2. 1.

Topic: The Gauss - Jordan method of calculating A-1

Q3) Find the inverse of the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

A) Scilab code:-

```
P3.sce (/Users/trishajain/Desktop/DAA_Assignment/P3.sce) - SciNotes
                                          | 🚜 | 🗓 | 🗓 | 🤡 | 쌒 | 💺 | 🛚
P3.sce (/Users/trishajain/Desktop/DAA_Assignment/P3.sce) - SciNotes
                                                  P3.sce 💥
  1 | clc; clear; close;
  2 A = - [1 - 0 - 0; 1 - 1 - 1; 0 - 0 - 1];
  3 n = length(A(1,:));
  4 | Aug = - [A, eye(n, n)];
  5 //Forward Elimination
  6 | for | j=1:n-1
     ----for-i=j+1:n
  7
             Aug(i,j:2*n) = Aug(i,j:2*n)-Aug(i,j)/Aug(j,j)*Aug(j,j:2*n);
 10 end
 11 //Backward-Elimination
 12 for - j - = - n: -1:2
         Aug(1:j-1,:) = Aug(1:j-1,:)-Aug(1:j-1,j)/Aug(j,j)*Aug(j,:);
 13
 14 end
 15 //Diagonal Normalization
 16 for j=1:n
 17
         Aug(j,:) -= Aug(j,:)/Aug(j,j);
 18 end
 19 B = Aug(:, n+1:2*n);
 20 disp("The inverse of A is:");
 21 disp(B);
 22
```

"The inverse of A is:"

- 1. 0. 0.
- -1. 1. -1.
 - 0. 0. 1.

Topic: Span of the Column Space of A

Q4) Identify the columns that span the column space of A in the following case

$$A = \begin{pmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{pmatrix}$$

A) Scilab Code:-

```
P4.sce (/Users/trishajain/Desktop/DAA_Assignment/P4.sce) - SciNotes
P4.sce (/Users/trishajain/Desktop/DAA_Assignment/P4.sce) - SciNotes
                                               P4.sce X
  1 clc; clear; close;
  2 a = [2 - 4 - 6 - 4; 2 - 5 - 7 - 6; 2 - 3 - 5 - 2];
  3 disp("The given matrix is:");
  4 disp(a);
  5|a(2,:)| = a(2,:)-(a(2,1)/a(1,1))*a(1,:);
  6|a(3,:)| = a(3,:)-(a(3,1)/a(1,1))*a(1,:);
  7 disp(a);
  a(3,:) = a(3,:)-(a(3,2)/a(2,2))*a(2,:);
 9 disp(a);
 10 | a(1,:) = a(1,:)/a(1,1);
 11|a(2,:) = a(2,:)/a(2,2);
 12 disp(a);
 13 for i=1:3
     ----for-j=i:4
 14
     -if(a(i,j) \ll 0)
 15
 16 -----disp("column", j, "is-a-pivot-column");
     ----break;
 17
 18 ----- end
 19 --- end
 20 end
 21
```

"The given matrix is:"

- 2. 4. 6. 4.
- 2. 5. 7. 6.
- 2. 3. 5. 2.
- 2. 4. 6. 4.
- 0. 1. 1. 2.
- 0. -1. -1. -2.
- 2. 4. 6. 4.
- 0. 1. 1. 2.
- 0. 0. 0. 0.
- 1. 2. 3. 2.
- 0. 1. 1. 2.
- 0. 0. 0. 0.

"column"

1.

"is a pivot column"

"column"

2.

"is a pivot column"

Topic: The Four Fundamental Subspaces

Q5) Find the four fundamental subspaces of :-

$$A = \begin{pmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{pmatrix}$$

A) Scilab Code :-

```
P4.sce (/Users/trishajain/Desktop/DAA_Assignment/P4.sce) - SciNotes
P4.sce (/Users/trishajain/Desktop/DAA_Assignment/P4.sce) - SciNotes
                                                  P4.sce X
  1 clc; clear; close;
  2 A = [1 - 3 - 3 - 2; 2 - 6 - 9 - 7; -1 - -3 - 3 - 4];
  3 disp("The given matrix is:");
  4 | disp(A);
  5 [m,n] = size(A);
  6 disp("m-=-",m);
  7 | disp("n-=-",n);
  8 [v,pivot] = rref(A);
  g disp("Row-Reduced-Echelon-Form:-", rref(A));
  10 r = length(pivot);
  11 disp(r, "Rank: -");
  12 colspace = A(:,pivot);
  13 disp("Column-Space: -", colspace);
  14 nullspace = kernel(A);
  15 disp("Null-Space: -", nullspace);
  16 rowspace = v(1:r,:)';
  17 disp("Row-Space: - ", rowspace);
  18 leftnullspace = kernel(A');
  19 disp("Left Null Space: ", leftnullspace);
  20
```

```
"The given matrix is:"
           3.
 1.
      3.
                 2.
 2.
                 7.
     6.
           9.
-1. -3.
                 4.
           3.
"m = "
 3.
"n = "
 4.
"Row Reduced Echelon Form: "
 1.
      3.
           0.
                -1.
 0.
      0.
           1.
                 1.
 0.
      0.
           0.
                 0.
 2.
"Rank: "
"Column Space: "
 1.
      3.
 2.
      9.
-1.
      3.
"Null Space: "
-0.2016536
             0.9295686
 0.287881
            -0.2449115
-0.6619896 -0.194834
 0.6619896
             0.194834
"Row Space: "
 1.
      0.
```

3.

0.

-1.

0.

1.

1.

"Left Null Space: "

- 0.9128709
- -0.3651484
 - 0.1825742

Topic: Projections by Least Squares

Q6) Find the line of best fit Ax=b for the following system

$$Ax = \begin{pmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} C \\ D \end{pmatrix}, b = \begin{pmatrix} 4 \\ 5 \\ 9 \end{pmatrix}$$

A) Scilab Code:-

```
P6.sce (/Users/trishajain/Desktop/DAA_Assignment/P6.sce) - SciNotes
                 P6.sce (/Users/trishajain/Desktop/DAA_Assignment/P6.sce) - SciNotes
                                            P6.sce ×
    clc;clear;<u>close</u>;
  2 A = [1 - -1; 1 - 0; 1 - 1];
  3 b = [4;5;9];
  4 disp("The-given-matrix-A-is:")
  5 disp(A);
  6 disp("b:-",b);
  7 x = (A'*A) \setminus (A'*b);
  8 |disp('x=',x);
  9 | C = x(1,1);
 10 D = x(2,1);
 11 disp("C:-",C);
 12 disp("D:-",D);
 13 disp("The-best-fit-line-is-b-=-C+Dt")
 14
```

"The given matrix A is:"

- 1. -1.
- 1. 0.
- 1. 1.
- "b: "
 - 4.
 - 5.
 - 9.
- "x="
 - 6.
- 2.5
- "C: "
- 6.
- "D: "
 - 2.5

"The best fit line is b = C+Dt"

Topic: The Gram- Schmidt Orthogonalization

Q7) Apply the Gram – Schmidt process to the following set of vectors and find the orthogonal matrix: (1, 0, 1), (1, 1, 0), (2, 1, 1)

A) Scilab Code :-

```
P7.sce (/Users/trishajain/Desktop/DAA_Assignment/P7.sce) - SciNotes
                                          X 🗊 🗓 😢 🖢 🖫
P7.sce (/Users/trishajain/Desktop/DAA_Assignment/P7.sce) - SciNotes
                                                 P7.sce 🔀
  1 clc; clear;
  2 A= - [1 - 0 - 1; 1 - 1 - 0; 2 - 1 - 1];
    disp("The-given-matrix-A-is:",A);
    [m, n] - = - size(A);
    for k=1:n
  5
     - - - V(:,k) - = -A(:,k);
     ----for-j=1:k-1
     R(j,k) = V(:,j) *A(:,k);
             V(:,k) = V(:,k)-R(j,k)*V(:,j);
 10 - - - end
     - - - - R(k,k) -= - norm(V(:,k));
 12 - - - V(:,k) = -V(:,k)/R(k,k);
 13 end
 14 disp("Q:-",V);
 15
```

"The given matrix A is:"

- 1. 0. 1.
- 1. 1. 0.
- 2. 1. 1.
- "Q: "
 - 0.4082483 0.7071068 0.842701
 - 0.4082483 0.7071068 0.2407717
 - 0.8164966 -3.140D-16 -0.4815434

Topic: Eigen values and Eigen vectors of a given square matrix

Q8) Find the Eigen values and the corresponding Eigen vectors of the following matrix:

$$\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

A) Scilab Code :-

```
P8.sce (/Users/trishajain/Desktop/DAA_Assignment/P8.sce) - SciNotes
P8.sce (/Users/trishajain/Desktop/DAA_Assignment/P8.sce) - SciNotes
                                                     P8.sce ×
  1 clc; clear; close;
  2 A = [2 \cdot 2 \cdot 1; 1 \cdot 3 \cdot 1; 1 \cdot 2 \cdot 2];
  3 disp("The-given-matrix-A-is:-",A)
  4 | lam = poly(0,"lam");
  5 charMat = A-lam*eye(3,3);
  6 | disp("The - Characteristic - Matrix - is: - ", charMat);
  7 charPoly = poly(A,"lam");
    disp("The Characteristic Polynomial is:", charPoly);
    lam = spec(A);
 10 disp("Eigen - Values: -", lam);
    function[x,lam] = eigenvectors(A)
    --- [n,m] =- size(A);
    ----lam = spec(A);
  3
    \mathbf{x} \cdot \mathbf{x} \cdot \mathbf{x} \cdot \mathbf{x} \cdot \mathbf{x} = \mathbf{x} \cdot []
     ----for-k=1:3
     B = A-lam(k)*eye(3,3);
      C = B(1:n-1,1:n-1);
  7
      b = -B(1:n-1,n);
  8
     y = C b;
     y = [y;1];
 11 - - - - - y = y/norm(y);
 12 - - - - - x - = - [x - y];
 13 --- end
 14 endfunction
 25 [x,lam] = eigenvectors(A);
 26 disp("Eigen-Vectors-of-A:-",x);
```

```
"The given matrix A is: "
```

- 2. 2. 1.
- 1. 3. 1.
- 1. 2. 2.

"The Characteristic Matrix is: "

"The Characteristic Polynomial is:"

$$-5 +11$$
lam -7 lam² $+$ lam³

"Eigen Values: "

- 1. + 0.i
- 5. + 0.i
- 1. + 0.i

"Eigen Vectors of A: "

- 0. 0.5773503 0.
- -0.4472136 0.5773503 -0.4472136
 - 0.8944272 0.5773503 0.8944272