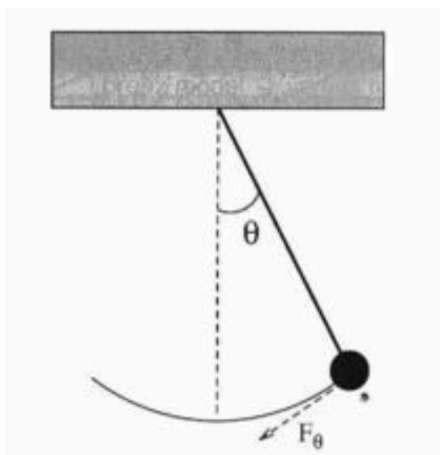
Reading Notes Ch.3 - Oscillatory Motion and Chaos

1. Examples of oscillatory motion: motion of electrons, behaviors of currents and voltages in circuits, planetary motion, pendulums. Ideal versus real pendulum motion differ in that real pendulums have some friction. The motion of this means there is a possibility of chaotic behavior.

- Force perpendicular to the strength is given by the equation:

$$F_\theta = -m\,g\,\sin\theta\ ,$$

Where g = the force of gravity, m = attached mass, theta is the angle the string makes with the vertical axis. Parallel forces add to zero.



Pseudocode for euler method of calculation:

- For each time step i calculate ω and θ at time step $i+1$.
  - ▷ $\omega_{i+1} = \omega_i - (g/\ell)\theta_i\Delta t$
  - ▷ $\theta_{i+1} = \theta_i + \omega_i\Delta t$
  - ▷ $t_{i+1} = t_i + \Delta t$
  - ▷ Repeat for the desired number of time steps.

As seen in the figure in the book, the amplitude of oscillations grows with time, which is contrary to what we would expect. No matter what value of delta t we use, the energy of the pendulum will increase. Because of the instability here, we consider the total energy of the pendulum:

$$E = \frac{1}{2}m\ell^2\omega^2 + mg\ell(1-\cos\theta)\ .$$

Which after substituting becomes:

$$E_{i+1} = E_i + \frac{1}{2} m g \ell \left(\omega_i^2 + \frac{g}{\ell} \theta_i^2 \right) (\Delta t)^2$$

It's clear here that Euler can become unstable, so its usability depends on the context of the question. The modification of the Euler method is referred to as the Euler-Cromer method. Example:

- For each time step i calculate ω and θ at time step $i+1$.
  - ▷ $\omega_{i+1} = \omega_i - (g/\ell)\theta_i \Delta t$
  - ▷ $\theta_{i+1} = \theta_i + \omega_{i+1} \Delta t$ (Note that ω_{i+1} is used to calculate θ_{i+1}.)
  - ▷ $t_{i+1} = t_i + \Delta t$
  - ▷ Repeat for the desired number of time steps.

For problems involving oscillatory motion, the Euler-Cromer method conserves energy over each complete period.

2. To make a pendulum more realistic, friction is added. The method in which it enters the system can vary from the bearing, to air resistance, and so on.
   - The results of the damped and undamped pendulum are slightly different, but nothing too crazy. Because of this, a driving force is added. Since we assume the force is sinusoidal with time, the equation becomes:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{\ell}\theta - q\frac{d\theta}{dt} + F_D \sin(\Omega_D t)$$

*The last term refers to the external driving force. This force pumps energy into or out of the system and completes the frequency, leading to richer behavior.

The new equation of motion becomes:

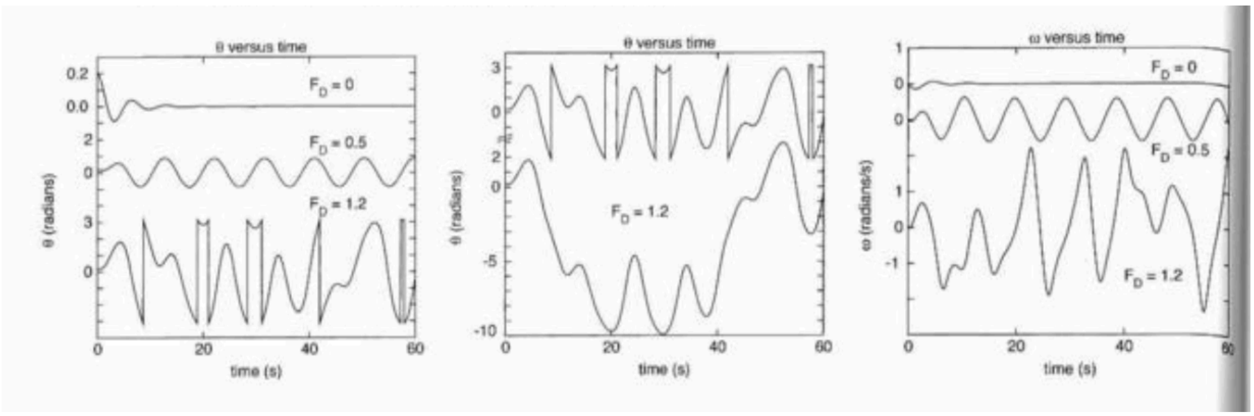
$$\frac{d^2\theta}{dt^2} = -\frac{g}{\ell}\sin\theta$$

*This is because we drop the assumption that the amplitude of the oscillation is small. Without friction and a driving force, total mechanical energy is conserved.

3. ANOTHER equation…

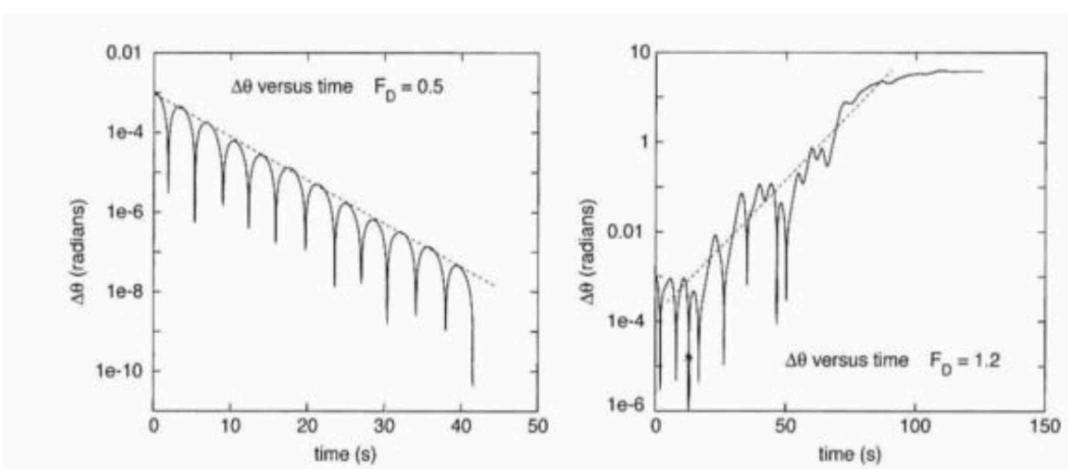
$$\frac{d^2\theta}{dt^2} = -\frac{g}{\ell}\sin\theta - q\frac{d\theta}{dt} + F_D\sin(\Omega_D t)$$

*To reach this equation, we don't assume the small-angle approximation. Then, we include friction. Finally, we add a sinusoidal driving force. We refer to this nonlinear, damped, driven pendulum, as a physical pendulum.



*The textbook includes lengthy explanations in the differences between these graphs. In short, the pendulum is no longer simple. It does not fall into any steady pattern, and so we can call it chaotic behavior.

- At low drive, the motion is simple. On the other hand, high drive leads to chaotic motion.
- The question is put forth of how we can possibly call something unpredictable yet at the same time be able to calculate it. Weirdly, it suggests that it is both deterministic and unpredictable at the same time.



In this experiment, they released two identical pendulums, the only difference being theta, which differed between experiments by 0.001 rad. The left includes results of low drive while the right involves results of high drive.

In the larger view, we can see delta theta increasing rapidly and irregularly with t. The trend is indicated with the dotted line. This is described by saying these trajectories diverge from one another. This divergence is rapid at short times while it stops changing at longer times simply because it can’t get larger. With repeated calculations and averaging of different initial theta values, we get the dotted line shown. It corresponds to the following equation:

$$\Delta\theta \approx e^{\lambda t}$$

The parameter lambda is referred to as the Lyapunov exponent.

Chaotic attractors have a fractal structure and are referred to as strange attractors.
Chaotic Regime - A state of a system where its behavior is highly sensitive to initial conditions, leading to seemingly random outcomes.

Main Ideas:

1. It is possible for a system to be both deterministic and unpredictable.
2. The behavior in the chaotic regime is not completely random, but can be described by a strange attractor in phase space.