# **Examples**By Kalman Filter

- 1 KF & EKF
- 2 GPS + IMU + Motion tracker
- 6 DVL + IMU + Camera + Depth sensor

1 KF & EKF



#### Time Update ("Predict")

(1) Project the state ahead

$$\hat{x}_k = A\hat{x}_{k-1} + Bu_{k-1}$$

(2) Project the error covariance ahead

$$P_k = AP_{k-1}A^T + Q$$



(1) Compute the Kalman gain

$$K_k = P_k^{\mathsf{T}} H^T (H P_k^{\mathsf{T}} H^T + R)^{-1}$$

(2) Update estimate with measurement zk

$$\hat{x}_k = \hat{x}_k + K_k(z_k - H\hat{x}_k)$$

(3) Update the error covariance

$$P_k = (I - K_k H) P_k$$



Initial estimates for  $\hat{x}_{k-1}$  and  $P_{k-1}$ 

Linear system



#### Time Update ("Predict")

(1) Project the state ahead

$$\hat{x}_{k} = f(\hat{x}_{k-1}, u_{k-1}, 0)$$

(2) Project the error covariance ahead

$$P_{k} = A_{k}P_{k-1}A_{k}^{T} + W_{k}Q_{k-1}W_{k}^{T}$$



(1) Compute the Kalman gain

$$K_{k} = P_{k}^{T}H_{k}^{T}(H_{k}P_{k}^{T}H_{k}^{T} + V_{k}R_{k}V_{k}^{T})^{-1}$$

(2) Update estimate with measurement  $z_k$ 

$$\hat{x}_k = \hat{x}_k + K_k(z_k - h(\hat{x}_k, 0))$$

(3) Update the error covariance

$$P_k = (I - K_k H_k) P_k$$



Initial estimates for  $\hat{x}_{k-1}$  and  $P_{k-1}$ 

Nonlinear system

#### Time Update ("Predict")

(1) Project the state ahead

$$\hat{x}_k = A\hat{x}_{k-1} + Bu_{k-1}$$

(2) Project the error covariance ahead

$$P_k = AP_{k-1}A^T + Q$$



Initial estimates for  $\hat{x}_{k-1}$  and  $P_{k-1}$ 

#### Measurement Update ("Correct")

(1) Compute the Kalman gain

$$K_k = P_k^{\scriptscriptstyle -} H^T (H P_k^{\scriptscriptstyle -} H^T + R)^{-1}$$

(2) Update estimate with measurement z<sub>k</sub>

$$\hat{x}_k = \hat{x}_k + K_k(z_k - H\hat{x}_k)$$

(3) Update the error covariance

$$P_k = (I - K_k H) P_k$$

Nonlinear system

## Linear system

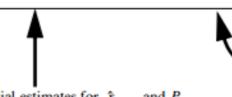
#### Time Update ("Predict")

(1) Project the state ahead

$$\hat{x}_k = f(\hat{x}_{k-1}, u_{k-1}, 0)$$

(2) Project the error covariance ahead

$$P_{k}^{-} = A_{k} P_{k-1} A_{k}^{T} + W_{k} Q_{k-1} W_{k}^{T}$$



Initial estimates for  $\hat{x}_{k-1}$  and  $P_{k-1}$ 

#### Measurement Update ("Correct")

(1) Compute the Kalman gain

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + V_k R_k V_k^T)^{-1}$$

(2) Update estimate with measurement z<sub>k</sub>

$$\hat{x}_k = \hat{x}_k + K_k(z_k - h(\hat{x}_k, 0))$$

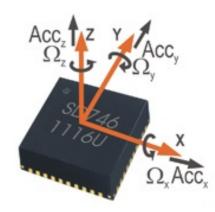
(3) Update the error covariance

$$P_k = (I - K_k H_k) P_k$$

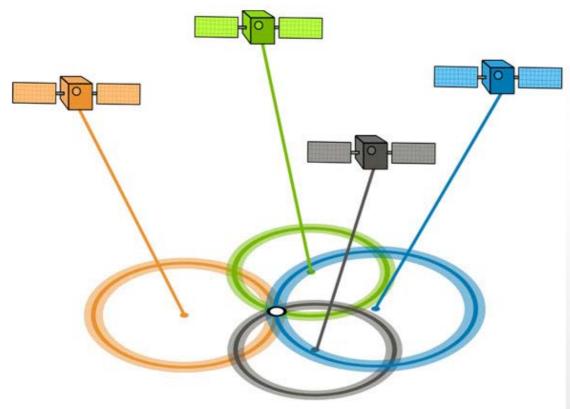
2 IMU + GPS + Motion tracker



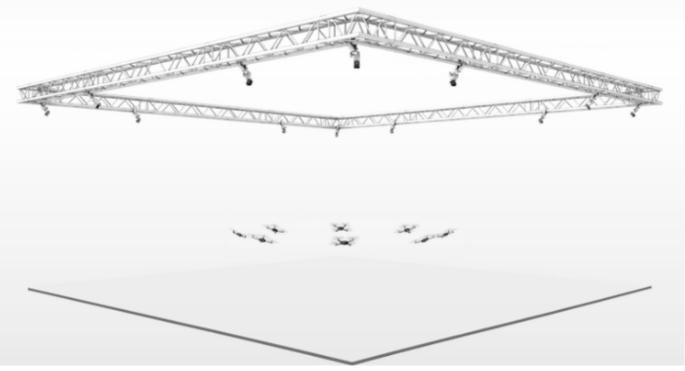




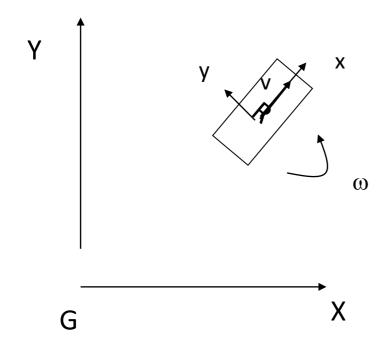
**GPS** 



# **Motion tracker**



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The discrete time state estimate (including noise) looks like this:

From a robot-centric  $x_t = V_t$  perspective, the velocities look like this:  $\dot{\phi}_t = \omega_t$ 

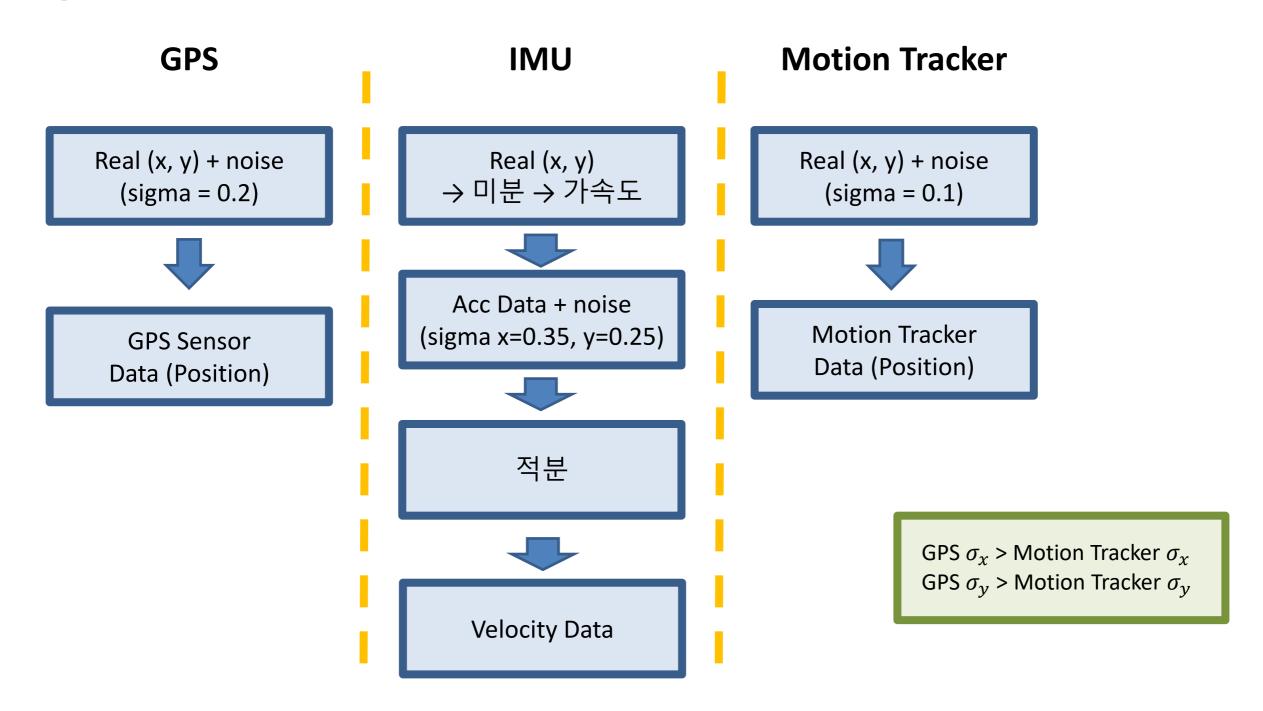
From the global  $\dot{x}_t = V_t \cos \phi_t$  perspective, the  $\dot{y}_t = V_t \sin \phi_t$  velocities look like  $\dot{\phi}_t = \omega_t$  this:

$$\hat{x}_{t+1} = \hat{x}_t + (V_t + w_{V_t}) \delta t \cos \hat{\phi}_t$$

$$\hat{y}_{t+1} = \hat{y}_t + (V_t + w_{V_t}) \delta t \sin \hat{\phi}_t$$

$$\hat{\phi}_{t+1} = \hat{\phi}_t + (\omega_t + w_{\omega_t}) \delta t$$

#### 센서 데이터 생성 방법 조건



# **Geometric Sensor Fusion**

$$\hat{X}_k = \mathbf{w}_{A,k} * \mathbf{Z}_{A,k} + \mathbf{w}_{B,k} * \mathbf{Z}_{B,k}$$

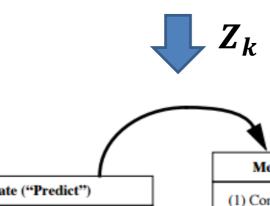
$$\hat{X}_{k} = \left(\frac{\sigma_{B,k}^{2}}{\sigma_{A,k}^{2} + \sigma_{B,k}^{2}}\right) * Z_{A,k} + \left(\frac{\sigma_{A,k}^{2}}{\sigma_{A,k}^{2} + \sigma_{B,k}^{2}}\right) * Z_{B,k}$$

GPS  $\sigma_x$  > Motion Tracker  $\sigma_x$  GPS  $\sigma_y$  > Motion Tracker  $\sigma_y$ 

GPS와 Motion Tracker를 Geometry Sensor Fusion



## Geometric Sensor Fusion



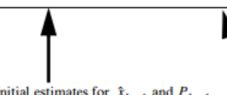
#### Time Update ("Predict")

(1) Project the state ahead

$$\hat{x_k} = f(\hat{x}_{k-1}, u_{k-1}, 0)$$

(2) Project the error covariance ahead

$$P_{k}^{-} = A_{k} P_{k-1} A_{k}^{T} + W_{k} Q_{k-1} W_{k}^{T}$$



Initial estimates for  $\hat{x}_{k-1}$  and  $P_{k-1}$ 

#### Measurement Update ("Correct")

(1) Compute the Kalman gain

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + V_k R_k V_k^T)^{-1}$$

(2) Update estimate with measurement zk

$$\hat{x}_k = \hat{x}_k + K_k(z_k - h(\hat{x}_k, 0))$$

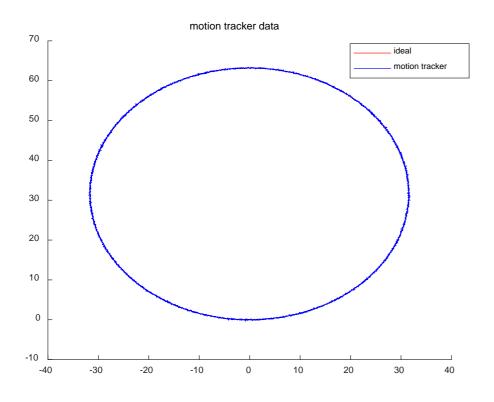
(3) Update the error covariance

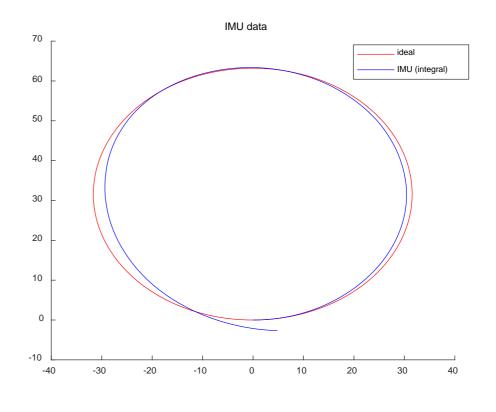
$$P_k = (I - K_k H_k) P_k$$

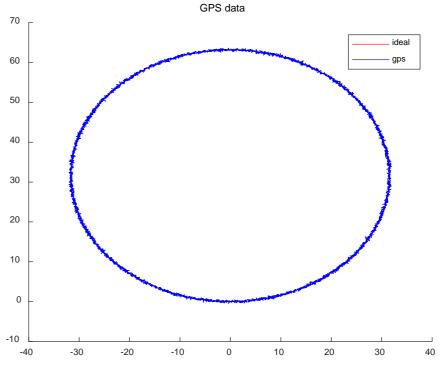
$$Q = diag(0.1, 0.1, 0.1, 0.1)$$

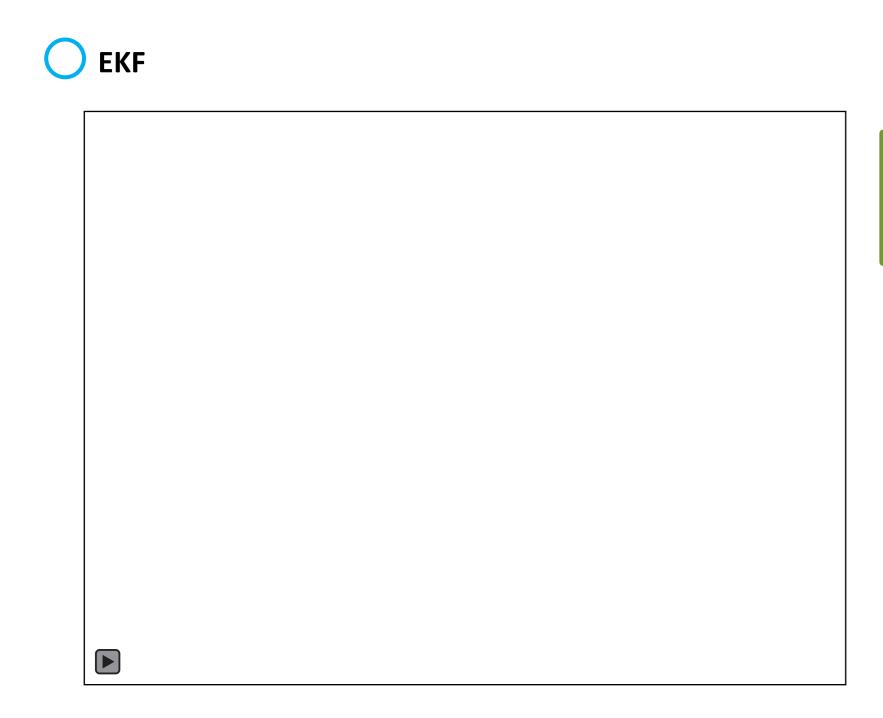
$$P = I$$

R = diag(100,100,100,100)







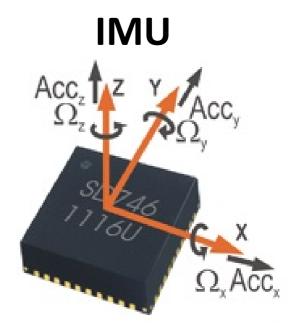


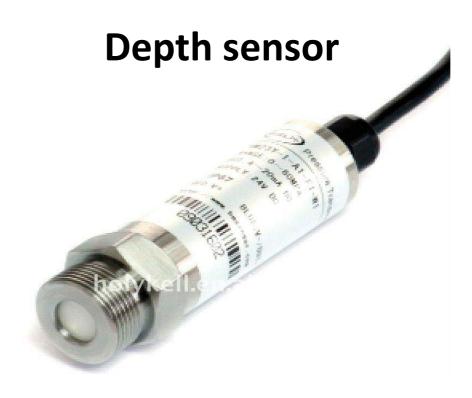


2 DVL + IMU + Depth sensor

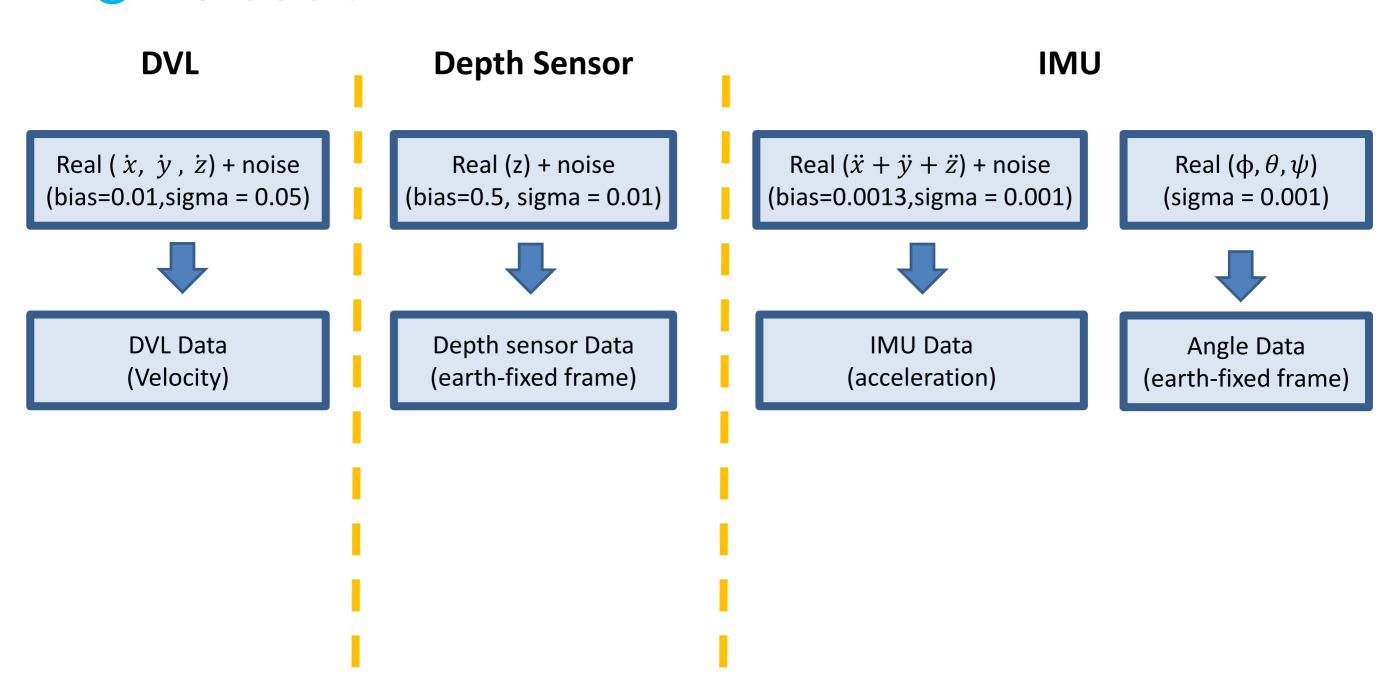


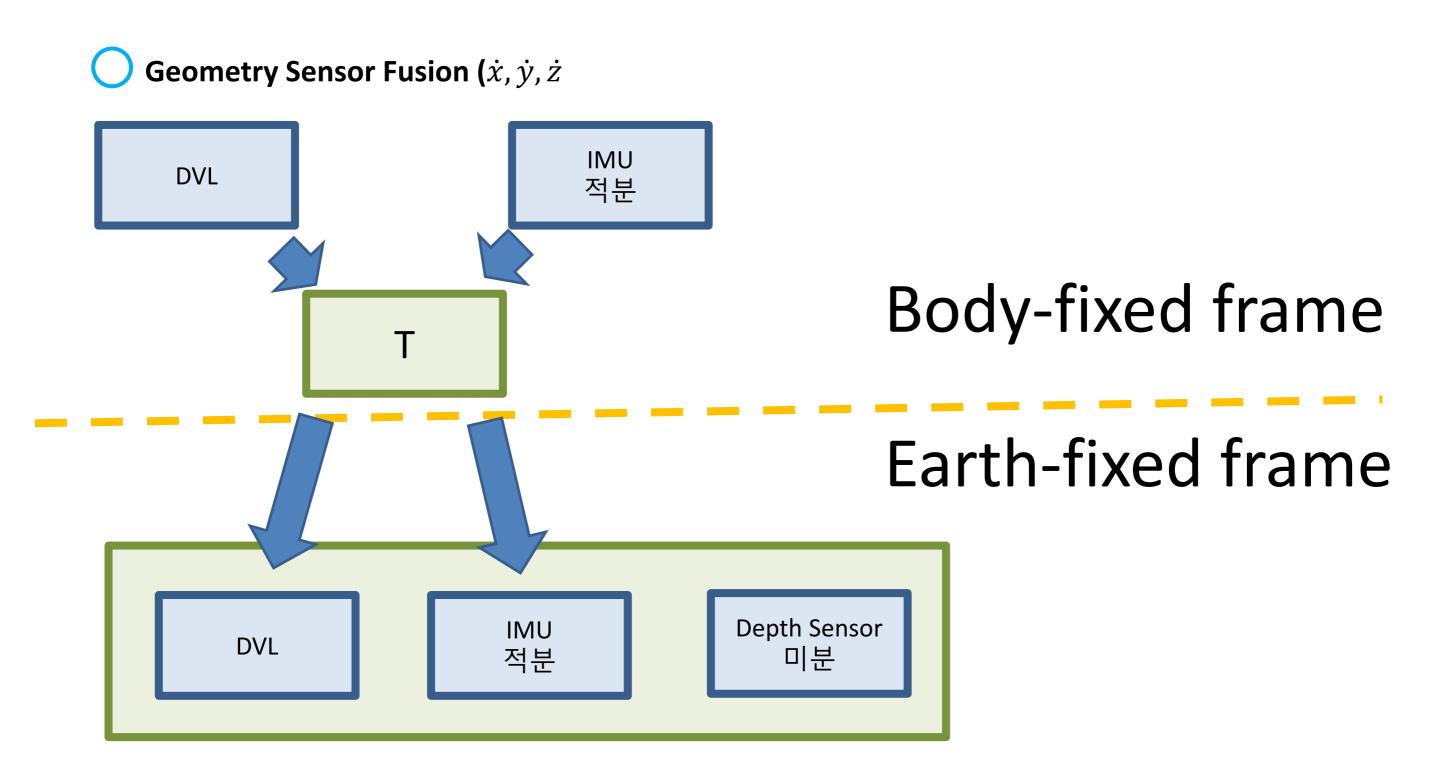






## ◯ 센서 데이터 생성 방법 조건







### Geometric Sensor Fusion



#### Time Update ("Predict")

(1) Project the state ahead

$$\hat{x}_{k} = f(\hat{x}_{k-1}, u_{k-1}, 0)$$

(2) Project the error covariance ahead

$$P_{k}^{-} = A_{k} P_{k-1} A_{k}^{T} + W_{k} Q_{k-1} W_{k}^{T}$$



Initial estimates for  $\hat{x}_{k-1}$  and  $P_{k-1}$ 

#### Measurement Update ("Correct")

(1) Compute the Kalman gain

$$K_k = P_k^T H_k^T (H_k P_k^T H_k^T + V_k R_k V_k^T)^{-1}$$

(2) Update estimate with measurement  $z_k$ 

$$\hat{x}_k = \hat{x}_k + K_k(z_k - h(\hat{x}_k, 0))$$

(3) Update the error covariance

$$P_k = (I - K_k H_k) P_k$$



♦ Ideal position
♠ EKF position