From KF to EKF

Easy and fast way
with concepts

Briefly!

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1},$$

$$z_k = Hx_k + v_k.$$

Time Update ("Predict")

(1) Project the state ahead

$$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_{k-1}$$

(2) Project the error covariance ahead

$$P_k^- = AP_{k-1}A^T + Q$$

Measurement Update ("Correct")

(1) Compute the Kalman gain

$$K_k = P_k^- H^T (HP_k^- H^T + R)^{-1}$$

(2) Update estimate with measurement z_k

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-)$$

(3) Update the error covariance

$$P_k = (I - K_k H)P_k^-$$

Initial estimates for $\hat{x}_{k-1}$ and P_{k-1}

$$x_k = f(x_{k-1}, u_{k-1}, w_{k-1}),$$

$$z_k = h(x_k, v_k),$$

Time Update ("Predict")

(1) Project the state ahead

$$\hat{x}_k^- = f(\hat{x}_{k-1}, u_{k-1}, 0)$$

(2) Project the error covariance ahead

$$P_k^- = A_k P_{k-1} A_k^T + W_k Q_{k-1} W_k^T$$

Measurement Update ("Correct")

(1) Compute the Kalman gain

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + V_k R_k V_k^T)^{-1}$$

(2) Update estimate with measurement z_k

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - h(\hat{x}_k^-, 0))$$

(3) Update the error covariance

$$P_k = (I - K_k H_k)P_k^-$$

Initial estimates for $\hat{x}_{k-1}$ and P_{k-1}

Time Update ("Predict")

(1) Project the state ahead

$$\hat{x}_k^- = f(\hat{x}_{k-1}, u_{k-1}, 0)$$

(2) Project the error covariance ahead

$$P_k^- = A_k P_{k-1} A_k^T + W_k Q_{k-1} W_k^T$$

Measurement Update ("Correct")

(1) Compute the Kalman gain

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + V_k R_k V_k^T)^{-1}$$

(2) Update estimate with measurement z_k

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - h(\hat{x}_k^-, 0))$$

(3) Update the error covariance

$$P_k = (I - K_k H_k) P_k^-$$

Initial estimates for $\hat{x}_{k-1}$ and P_{k-1}

- A is the Jacobian matrix of partial derivatives of f with respect to x, that is

$$A_{[i,j]} = \frac{\partial f_{[i]}}{\partial x_{[j]}}(\hat{x}_{k-1}, u_{k-1}, 0),$$

- W is the Jacobian matrix of partial derivatives of f with respect to w,

$$W_{[i,j]} = \frac{\partial f_{[i]}}{\partial w_{[j]}}(\hat{x}_{k-1}, u_{k-1}, 0),$$

- H is the Jacobian matrix of partial derivatives of h with respect to x,

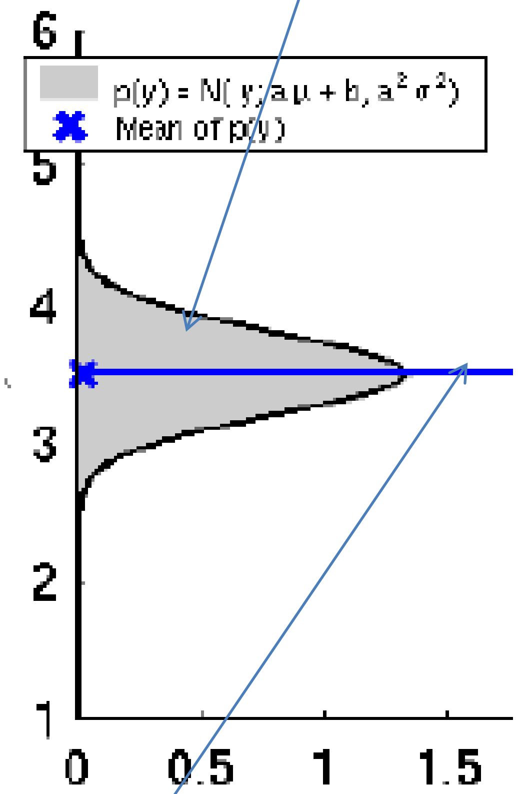
$$H_{[i,j]} = \frac{\partial h_{[i]}}{\partial x_{[j]}}(\tilde{x}_k, 0),$$

- V is the Jacobian matrix of partial derivatives of h with respect to v,

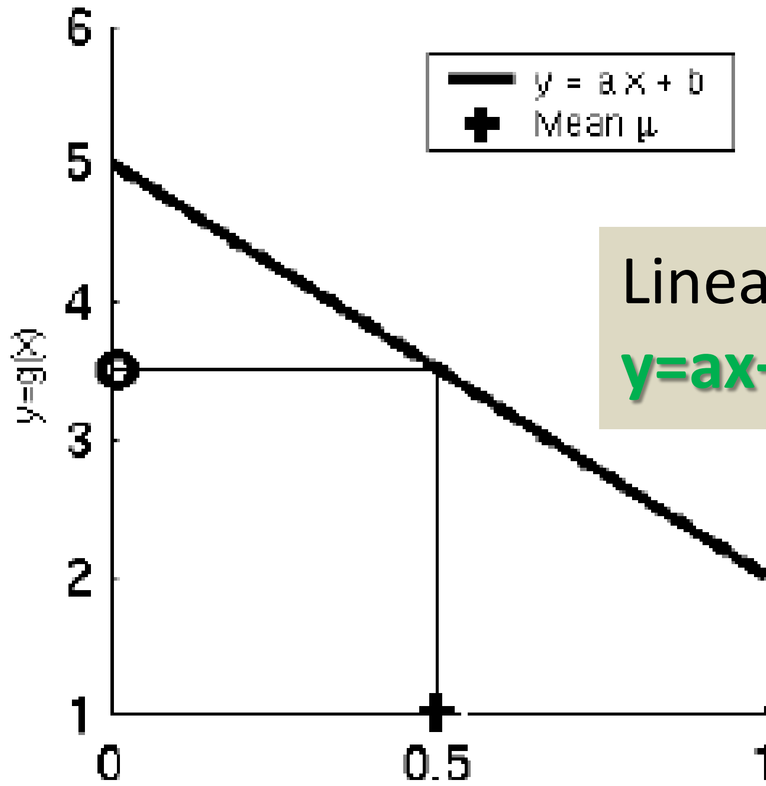
$$V_{[i,j]} = \frac{\partial h_{[i]}}{\partial v_{[j]}}(\tilde{x}_k, 0).$$

Linearity Assumption Revisited

Grey represents true distribution of p(y)

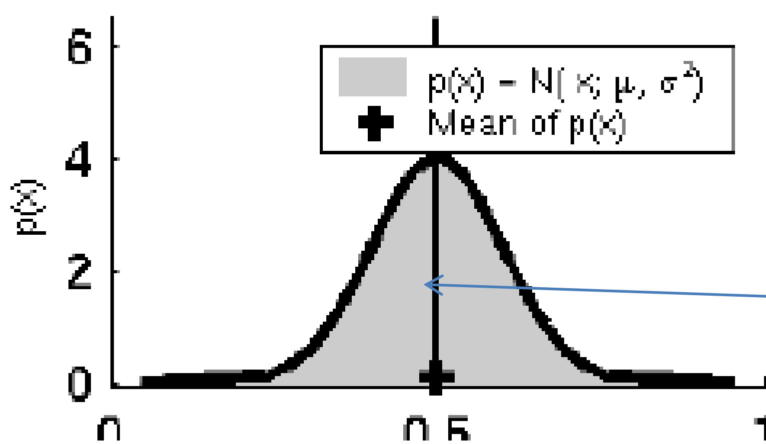


Mean of p(y)



Linear mapping of Mean:

y=ax+b

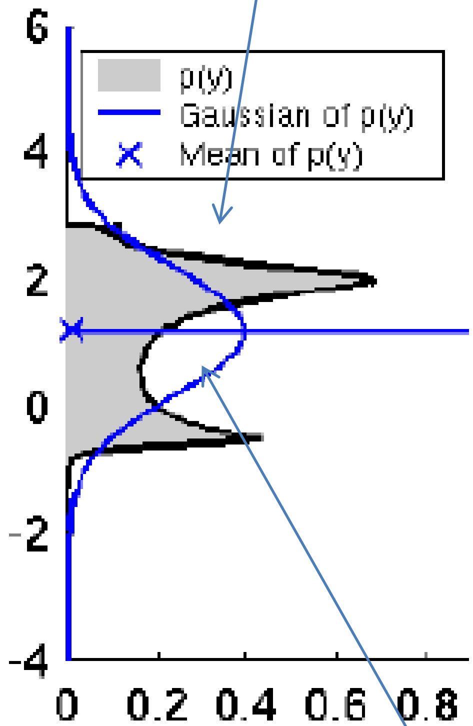


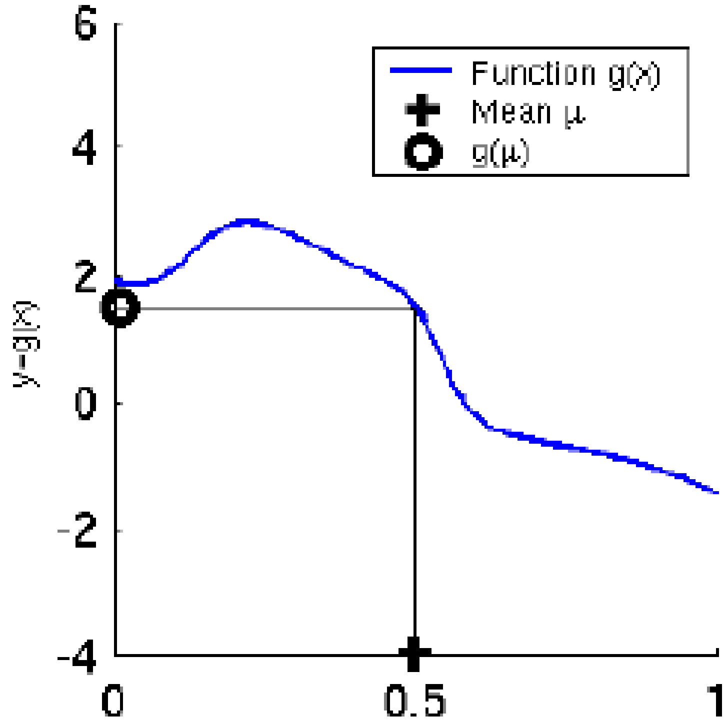
This is ideal case of pushing gaussian through a linear system

Mean of p(x)

Grey represents true distribution of p(y)

Non-linear Function

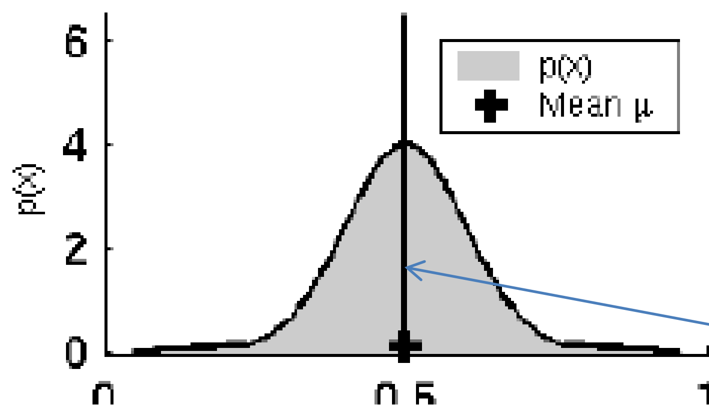




Non-linear function g(x)

Gaussian of p(y)

This is a real case of pushing a gaussian through a non-linear system

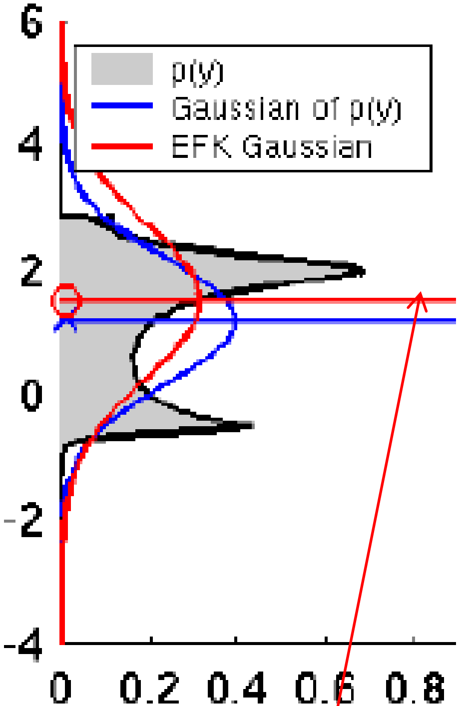


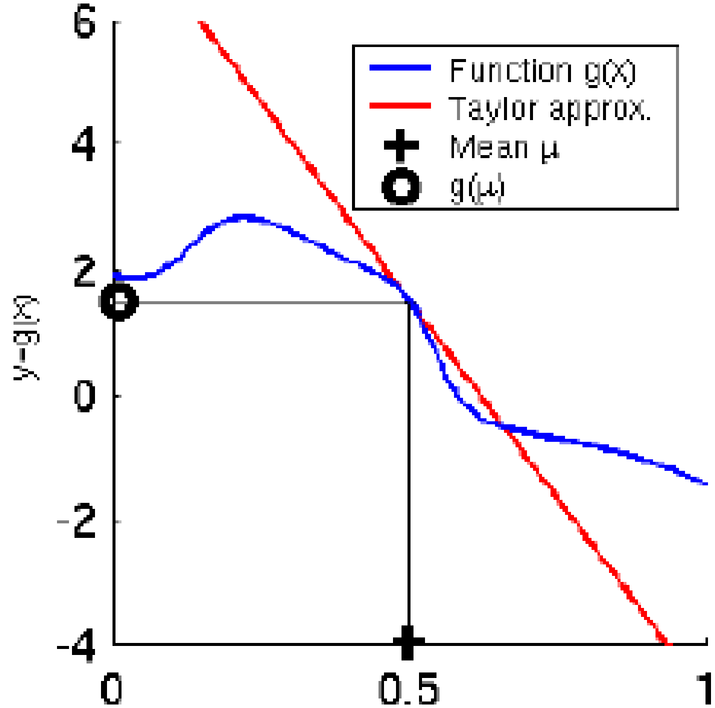
We are approximating grey by blue, not good

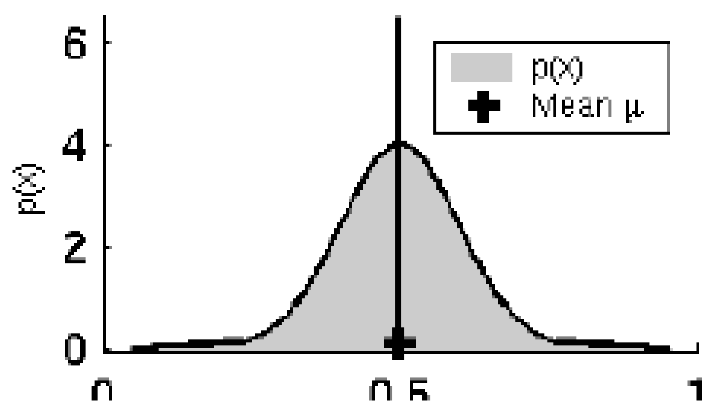
Mean of p(x)

EKF Linearization

Taylor approximation and EKF Gaussian







Better than in last slide.
The mean is closer to grey shape

This example shows tha t Gaussian of EKF better represents estimated va lue than the Gaussian m ean