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Summary Sheet

Four States in the USA of Various Energy Analysis and Forecast

Summary

The first part, first of all, we use the principal component analysis to analyze the state of energy development in the past 50 years in 2009. Get the most relevant variables associated with energy changes in each state. See the text of the specific content.

Then we use the principal component regression method to analyze the evolution of energy in each state and the factors that cause similarities and differences. See the text for details.

We used AHP to select the nine states that best use clean and renewable energy, based on the nine selected energy indicators, which is Arizona.

Finally, the data from 1960 to 2009 of each state are used to forecast using the gray system, and the predicted values of several key energy indicators of the four states in 2025 and 2050 are obtained. See the text of the specific content.

The second part, for question A, taking into account the two indicators of the best economic indicators and the largest use of renewable energy, we use the target energy use value from 1960 to 2050 and the economic value of GDP over this period of time, and we establish a multi-objective Optimize the model. Finally, 2025 AZ, CA, NM, TX renewable energy accounted for 10.57%, 9.29%, 5.34%, 5.07% of the total energy consumption.

For question B, according to the results of the previous analysis, we put forward several suggestions according to the energy use of each state such as increasing the self-sufficiency rate and utilization rate of energy. See the text of the specific content.

Key words: GM, PCA, PCR, AHP, Multi-target planning

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1. Introduction

1.1 Problem Background

Energy production and use are an important part of any economy and an important material basis for social progress. The consumption structure of energy is not only related to the quality and speed of national growth, but also to the quality of human life and the ecological environment.

Since the 21st century, with the rise of clean energy and renewable energy and the worsening of the international energy situation, all countries in the world have paid more attention to energy issues in order to effectively cope with global climate change and ensure energy security, and have given renewable energy development highly valued. Introducing a series of incentive measures to achieve the purpose of improving energy efficiency, reducing energy consumption and improving energy consumption structure.

In the United States, many energy policies are decentralized to the state. In addition, the geographical regions and industrial levels in different states have affected the use and production of energy. Intercontinental deed is a contractual arrangement between multiple states. Take a set of standards or cooperate with each other in these states on specific issues. CA, AZ, NM and TX, these four states are important U.S. energy producers and are adjacent to each other and are uniquely suited to the development of clean renewable energy through interstate energy contracts.

1.2 Our Work

We first conducted a pre-treatment of the data, delete some of the non-compliance data.

Part I.A: we use principal component analysis to analyze the state of energy development in each state over the past 50 years.

Part I.B: using the principal component regression method to analyze the evolution of energy in each state and the factors that cause similarities and differences

Part I.C: using the analytic hierarchy process and the nine selected energy indicators, the states that best use clean and renewable energy are selected.

Part I D: After the previous data processing and screening, we selected eleven energy indicators and used the gray prediction method to predict the energy development in each state in the next 40 years.

Part II. A: First, the model goal is to determine the highest economic efficiency and increase the production of renewable energy. Then we use the multi-objective optimization model based on the least deviation method to determine the target of using renewable energy in 2025 and 2050.

Part II. B: Based on the previous analysis, taking into account the energy situation in each state, we propose a concrete implementation plan for the energy compact.

2. Nomenclature

We use the Nomenclature in Table.1 to describe our model in this paper. Others that are used only once will be described later.

Table 1. The definition of symbol

Symbol	Definition
F_i	Evaluation function
\hat{y}	the dependent variable of PCR
$A = (a_{ij})_{9 \times 9}$	The paired comparison matrix of the target layer with the guideline layer
$f_1(x)$	energy usage
$f_2(x)$	renewable energy production
$r_{k,\min}(i, j)$	Kth energy in the unit of energy input minimum

3. Simplifying Assumptions

- The data given is true and valid, there is no fake data.
- In the future, energy industry is stable, no major changes, no impact of new energy.
- Federal and state policies have not changed much for all kinds of energy sources.
- Regardless of extreme conditions, such as war broke out, the climate anomalies.

4. Our Solution

4.1 Solution for part one

4.1.1 Data processing

First, based on the data provided, summarize the energy use profile for each state. We have to deal with the data given in the annex, with its 605 different indicators and its data for recent decades. We start from the data. For its 605 kinds of data, by comparing the table seseds and msn codes, we found that 24 indicators msn codes table seseds not in the table, so this data only has 583 indicators. By continuing to analyze the table seseds, we find that two indicators, all of which are zero, are deleted, leaving 581 useful indicators left.

Continuing with the analysis of the seseds, we found that because of the different years recorded by different states for different indicators, the data length varies and can not be quantified, so we interpolated the remaining 581 sets of splines to implement each Indicators have 50 data.

For ease of analysis, we classify seseds into four categories (CA, AZ, NM, TX) according to

their geographical location, with 581 indicators in each region. Because of the number of indicators and the lack of indicators, we cleaned the data and deleted the indicators that have the number of zero more than 20 to improve the accuracy of the data. The CA cleaned up 90 indicators, retaining 491. AZ cleaned 195 indicators, leaving 386. The NM cleaned 167 indicators, retaining 414. TX cleaned 128 indicators, retaining 483. Even if the processed data is still a lot. And there are a lot of energy-independent data that can not be described one by one, so the data needs to be processed again. By looking at the explanations in the attachment, we focus on the unit of BTU in the data, which is a unit of energy.

4.1.2 Solution for question a

4.1.2.1 Problem analysis

Question a is ask us to create an energy for each of four states by using the data provided. By extracting this unit of data, we consider all of its indicators as state energy profiles and then use principal component analysis Dimensional treatment, the number of variables into a few principal components (ie, integrated variables), and then assist with the corresponding description

4.1.2.1 Developing and Solve the model

There are four sets of data since the data has been classified by state. The principal component analysis and treatment method is the same in each state except that the number of indicators varies. Therefore, the establishment of this model we only give a brief description of the demonstration, according to the four states of different results for a detailed description.

Developing the model

Standardize the raw data matrix

Suppose p variables for n times the observation data can be the following matrix(in the title p for the filtered variables, n is 50 years of data)

$$X_{n \times p} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{np} \end{bmatrix} \quad (1)$$

make it standardize:

$$\tilde{x} = \frac{(x_{ij} - \bar{x}_j)}{S_j}, (i = 1, 2, \dots, n; j = 1, 2, \dots, p) \quad (2)$$

(2) Find the covariance matrix Z

$$z_j = \sum_k b_{kj} \tilde{x}_k \quad (3)$$

(3) Eigenvalue decomposition is $Z = U \Lambda U^T$, Get p non-negative eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_p$ of Z, The p eigenvalues are the variance of the principal

components Determine the number of the main components

$$\eta_m = \lambda_1 + \lambda_2 + \dots + \lambda_m / (\lambda_1 + \lambda_2 + \dots + \lambda_p) \quad (4)$$

(4) Write the main component expression

$$Z_{n \times m} = X_{n \times p} U_{p \times m} \quad (5)$$

Solute the model

We conducted a principal component analysis of the data extracted from the four states, of which 132 were AZ, 129 were CA, 135 were NM, and 132 were TX. The length of the observation is 50.

First, the eigenvalues of the correlation matrix of the four sets of data are obtained, as shown in Table 2 and Table 3 below.

Table 2. Eigenvalue of the Correlation Marix

	NM				TX			
	Eigenvalue	Difference	Proportion	Cumulative	Eigenvalue	Difference	Proportion	Cumulative
1	63.2264827	42.816311	0.4683	0.4683	69.116496	48.713975	0.5236	0.5236
2	20.4101717	7.4248248	0.1512	0.6195	20.4025145	7.2687184	0.1546	0.6782
3	12.985347	3.4283479	0.0962	0.7157	13.133796	5.0161776	0.0995	0.7777
4	9.5569991	3.817534	0.0708	0.7865	8.1176184	3.2690815	0.0615	0.8392
5	5.739465	1.6260502	0.0425	0.829	4.8485369	1.4323562	0.0367	0.8759
6	4.1134149	0.8832591	0.0305	0.8595	3.4161806	1.0642294	0.0259	0.9018

We start with NM and analyze the results of principal component analysis. It can be seen from the results that the ratio of the cumulative variance of the first principal component to the sixth principal component has exceeded 85%. So just ask for the $\lambda_1, \lambda_2, \dots, \lambda_6$ corresponding to the normalized eigenvector $\alpha_i (i=1 \dots 6)$. According to the results we can see:

$$Z_1 = \alpha_1 Y^T, Z_2 = \alpha_2 Y^T, \dots, Z_6 = \alpha_6 Y^T \quad (6)$$

Ps. $Y = [y_1, y_2, \dots, y_{135}]$

Feature vector $\alpha_i (i=1 \dots 6)$ is a matrix of 1*135, because of many data, it is not a list.

So the Evaluation function is:

$$F_1 = P_1 \times Z_{NM} \quad (7)$$

Ps. $Z = (Z_1, Z_2, \dots, Z_6)^T$

P is the proportion, specifically $P = (0.4683, 0.1512, 0.0962, 0.0708, 0.0425, 0.0305)$

We analyze the above evaluation function. Since the representative of the components has reached 85.95%, we will analyze the six main components as the characteristics of NM. By sorting the eigenvectors of the first to sixth principal components, we can find:

- 1) The first principal component is electricity and oil
- 2) The second principal component indicates natural gas production and petroleum by-products
- 3) The third principal component indicates the liquid consumption of natural gas and natural gas
- 4) The fourth principal component indicates the consumption of natural gas and its portion for power generation
- 5) The fifth principal component is kerosene and petroleum by-products
- 6) The sixth principal component is renewable energy, wind power generation and so on

In addition, we have selected several sets of representative indicators, the data for nearly 50 years to make a map, as shown in Figure1.

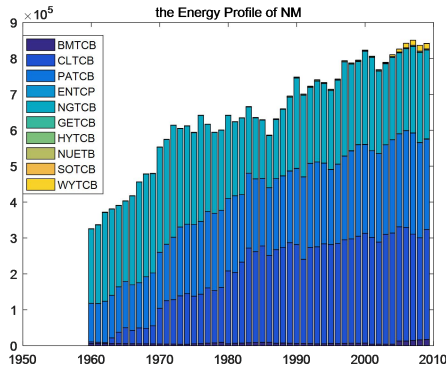


Figure1.the Energy Profile of NM

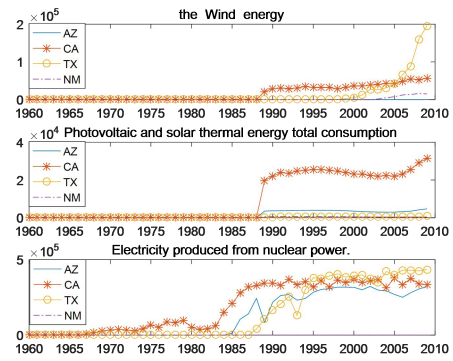


Figure2

Through the analysis of the principal components and images, we found that New Mexico mainly produces oil, natural gas, natural gas liquids and coal. Among them, the utilization rate of petroleum is very high, and the output of gasoline, diesel, propane, butane and heavy fuel oil is very large. Although New Mexico produces a lot of gas, it uses very little gas. Coal production plays an important role but with a downward trend, and the same consumption is small, mainly for power generation. Natural gas is used to generate a lot of electricity. There are a lot of renewable energy, wind energy utilization of a lot, generating capacity is also great.

Next, the Evaluation function of TX is:

$$F_2 = P_2 \times Z_{TX} \quad (8)$$

$$\text{Ps. } Z_{TX} = (Z_1, Z_2, \dots, Z_5)^T$$

P is the proportion, specifically $P_2 = (0.5236, 0.1546, 0.0995, 0.0615, 0.0367)$

We analyze the above evaluation function. Since the representative of the

components has reached 87.59%, we will analyze the six main components as the characteristics of TX. By sorting the eigenvectors of the first to fifth principal components, we can find:

- 1) The first principal component is electricity and oil,
- 2) The second principal component represents oil by-products and natural gas
- 3) The third principal component is kerosene and fuel oil
- 4) The fourth principal component indicates the consumption of natural gas and its portion for power generation
- 5) The fifth principal component is wind power and solar energy

In addition, we have selected several sets of representative indicators, the data for nearly 50 years to make a map, as shown in Figure3.

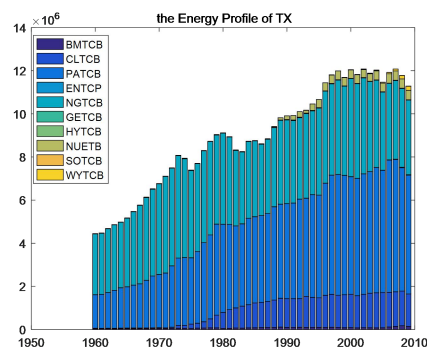


Figure3.the Energy Profile of TX

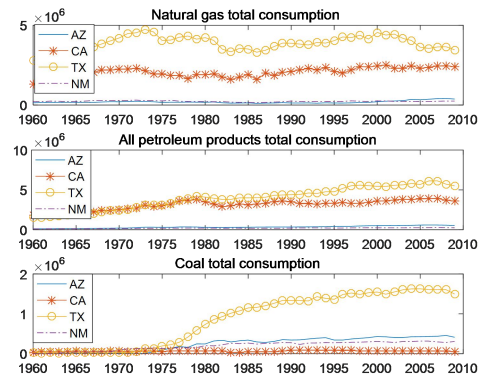


Figure4

Through the principal component analysis and image analysis, we found that Texas is the leader in energy production, mainly from crude oil and natural gas. Texas has a very high total oil consumption and high utilization of petroleum by-products such as distillate fuel oil, residual fuel oil, and liquefied petroleum gas. Texas is also higher than the country in terms of natural gas consumption. The use of natural gas in Texas dominates the industrial and power sectors. The more natural gas recovered, processed and distributed, the natural gas consumption ratio is consumed jointly by the country's residential and commercial sectors. There is a lot of natural gas in Texas for electricity generation. Texas is the largest coal-consuming state, primarily for electricity generation and the largest electricity consumer in the United States, both in terms of both electricity demand and electricity supply. The share of renewable energy is very small, and mainly wind power.

Table 3.Eigenvalue of the Correlation Marix

	CA				AZ			
	Eigenvalue	Difference	Proportion	Cumulative	Eigenvalue	Difference	Proportion	Cumulative
1	60.269662	38.746157	0.4709	0.4709	68.663041	41.152081	0.5202	0.5202
2	21.523504	11.490158	0.1682	0.639	27.51096	13.241922	0.2084	0.7286
3	10.033346	1.6760469	0.0784	0.7174	14.269037	7.1812471	0.1081	0.8367

4	8.3572996	3.2531245	0.0653	0.7827	7.0877906	4.1347618	0.0537	0.8904
5	5.1041751	1.4187808	0.0399	0.8226	2.9530288	0.8016724	0.0224	0.9128
6	3.6853943	0.1476333	0.0288	0.8514	2.1513564	0.4055532	0.0163	0.9291

Continue, the Evaluation function of CA is:

$$F_3 = P_3 \times Z_{CA} \quad (9)$$

Ps. $Z_{TX} = (Z_1, Z_2 \dots, Z_6)^T$

P is the proportion, specifically $P_3 = (0.4709, 0.1682, 0.0784, 0.0653, 0.0399, 0.0288)$

We analyze the above evaluation function. Since the representative of the components has reached 85.14%, we will analyze the six main components as the characteristics of CA. By sorting the eigenvectors of the first to sixth principal components, we can find:

1)The first principal component is the total electricity consumption and electricity generated from nuclear power and geothermal power.

2)The second principal component shows the production of petroleum as well as the production of petroleum by-products and the consumption of geothermal and solar energy for civilians.

3)The third principal component is the production of oil by-products and the consumption of natural gas.

4)The fourth principal component indicates the consumption of biomass energy such as ethanol and some other nuclear power.

5)The fifth principal component indicates the generation and consumption of clean energy such as solar energy, wind energy and nuclear energy.

6)The sixth principal component represents the total production and consumption of hydropower.

In addition, we have selected several sets of representative indicators, the data for nearly 50 years to make a map, as shown in Figure5.

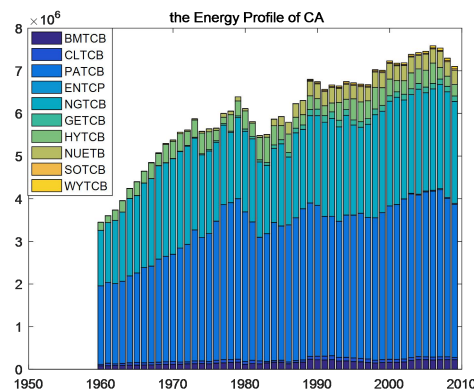


Figure5.the Energy Profile of CA

Through the principal component analysis and image analysis, we found that the

energy demand in California is very large, but the residential area per capita energy is very low. In electricity, the state's traditional hydroelectricity is well developed. At the same time, the state also has abundant clean and renewable energies through solar energy, geothermal energy, biomass energy and nuclear power generation, with a high utilization rate. In terms of oil, California's crude oil output has remained high despite declining levels in recent years, and oil consumption is mainly used in the transport sector. As for natural gas, the output is small, but most of it is for household consumption. It is worth mentioning that the state's coal use is very small, perhaps the use of clean energy more, on the other hand the state has almost no coal production.

Finally, the Evaluation function of AZ is:

$$F_4 = P_4 \times Z_{AZ} \quad (10)$$

$$Ps. Z_{TX} = (Z_1, Z_2, \dots, Z_4)^T$$

P is the proportion, specifically $P_4 = (0.5202, 0.2084, 0.1081, 0.0537)$

We analyze the above evaluation function. Since the representative of the components has reached 89.04%, we will analyze the six main components as the characteristics of AZ. By sorting the eigenvectors of the first to fifth principal components, we can find:

1) The first principal component states that renewable energy, oil consumed by the transport sector.

2) The second principal component is the total consumption of fossil fuels, as well as the consumption of electricity and coal and the production of coal.

3) The third principal component is the total natural gas consumption and the natural gas consumed by the power sector.

4) The fourth principal component indicates the amount of nuclear power generated, the amount of solar power generated, and the consumption of nuclear energy.

In addition, we have selected several sets of representative indicators, the data for nearly 50 years to make a map, as shown in Figure 4.

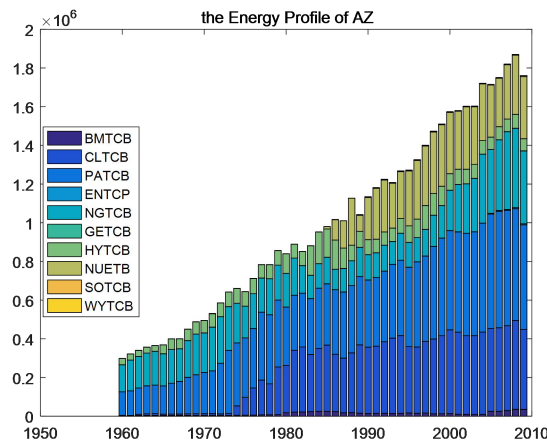


Figure6.the Energy Profile of AZ

Arizona has less fossil fuel resources but abundant solar and geothermal energy. In terms of oil, Arizona produces very little oil and almost no petroleum by-products, and its oil is mainly used for transportation. Natural gas production is also small, mostly for electricity production and residential consumption. Similarly, coal production is also very small, almost all for power generation. However, a large part of Arizona's electricity comes from nuclear power. The sources of power in Arizona are mainly nuclear power, natural gas power generation and coal power generation. It is worth mentioning that the state's solar power generation has a place, but the proportion is not large.

4.1.3 Solution for question b

4.1.3.1 Problem analysis

For problem b, we use principal component regression to describe energy evolution. Continuing the above ideas, we do multiple regression on the result of principal component analysis of problem a. Using this result allows governors to understand the differences in geographical and industrial populations and climate between states

4.1.3.2 Developing and Solve the model

Linear regression requires independent variables are independent of each other, but often encounter problems related to independent variables, a good solution is to use principal component analysis, using principal component regression to find the regression coefficient.

There are p regression variables x_1, x_2, \dots, x_p , Their i th test in the value of

$x_{i1}, x_{i2}, \dots, x_{ip} (i = 1, 2, \dots, n)$, Write in matrix:

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1p} \\ x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{np} \end{pmatrix} \quad (11)$$

(11) is Design matrix, Consider the linear model:

$$Y = \beta_0 1 + X\beta + \varepsilon, \varepsilon \sim N(0, \sigma^2 I) \quad (12)$$

$$\hat{\beta}_0 = \bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i \quad (13)$$

For any one linear combination of independent variables:

$$z = c_1x_1 + c_2x_2 + \cdots + c_px_p, \sum_{j=1}^p c_j^2 = 1 \quad (14)$$

Suppose: $Z = (z_1, z_2, \dots, z_p)$, $Q = (\eta_1, \eta_2, \dots, \eta_p)_{p \times p}$ and $Z = XQ$.

Introduce new parameters: $\alpha = Q^T \beta$ or $\beta = Q\alpha$

$$Y = \beta_0 1 + ZQ^T \beta + \varepsilon = \beta_0 1 + Z\alpha + \varepsilon \quad (15)$$

In this question, we use the total energy consumption as the dependent variable to regress.

First of all, we still analyze NM, there are Analysis of Variance as Table3 and Parameter Estimates as Table4 after Principal component regress.

Table3 NM Analysis of Variance

Source	DF	Sum of Square	Mean Square	F Value	Pr > F
Model	6	5.92E+11	98641693523	955.02	<.0001
Error	43	4441350159	103287213		
Corrected Total	49	5.96E+11			
Root MSE	10163	R-Square	0.9926	Adj R-Sq	0.991
Dependent Mean		526237	Coeff Var		1.93127

By analyzing Table3, we found that the value of $\text{Pr} > F$ less than 0.0001, within a reasonable range, so that the regression equation passed the test.

Table4 NM Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Standardized Estimate
Intercept	1	526237	1437.26972	366.14	<.0001	0
Prin1	1	13505	183.9545	73.42	<.0001	0.96625
Prin2	1	1528.60218	321.39787	4.76	<.0001	0.0626
Prin3	1	7121.47276	403.72447	17.64	<.0001	0.23216
Prin4	1	-290.15763	469.64091	-0.62	0.5399	-0.00813
Prin5	1	1226.61149	606.06026	2.02	0.0492	0.02664
Prin6	1	974.72577	715.88068	1.36	0.1804	0.01792

By analyzing Table4, we find that the values of the fourth to the sixth principal components are both greater than 0.01 and fail to pass the test. Therefore, the regression equation is:

$$\hat{y}_{NM} = 526237 + 13505\text{prin1} + 1528.60218\text{prin2} + 7121.47276\text{prin3} \quad (16)$$

To save space, put the Analysis of Variance and Parameter Estimates in the Appendix. The regression equations for the remaining three states are as follows:

$$\hat{y}_{TX} = 8930452 + 271785\text{prin1} + 64443\text{prin2} - 19310\text{prin3} + 109329\text{prin4} - 44815\text{prin5}$$

$$\hat{y}_{CA} = 655630 + 156729\text{prin1} + 41391\text{prin2} + 34615\text{prin3} + 20004\text{prin4} \quad (18)$$

$$\hat{y}_{AZ} = 880081 + 47398\text{prin1} + 7031.721\text{prin4} \quad (19)$$

After the regression of the principal components of each state is given, in order to better describe the changes in clean energy and non-clean energy in each state, we add a diagram, Figure 7, which, together with Figure 2 and Figure 4, better describes the energy evolution.

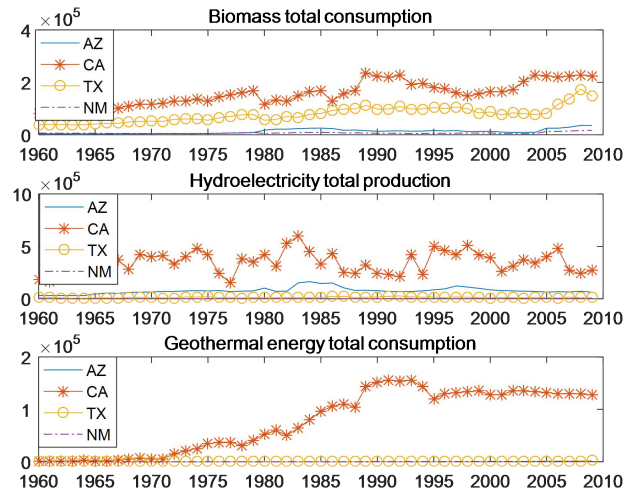


Figure7

Combining the analysis of the principal components of question a, we will only discuss energy evolution here, and we conclude:

- (1) For New Mexico, oil production and natural gas are steadily increasing, coal production is declining and electricity production is growing rapidly. However, the proportion of renewable energy is very small, mainly wind power.
- (2) For Texas, oil production steadily increased and then declined, while oil consumption has been growing and so has natural gas. The state's energy generation is rapidly increasing, with wind power generating particularly rapidly.
- (3) For California, production of oil and gas has been declining over the past few decades, and coal has been declining more rapidly. However, hydropower, wind energy and nuclear energy are developing rapidly and occupy an important position. At the same time the development of solar energy is also more rapid.
- (4) For Arizona, production of oil, gas and coal is small, and natural gas production is declining rapidly. Nuclear power, hydroelectric power, wind power occupy an important position to develop more rapidly.

4.1.4 Solution for question c

4.1.4.1 Problem analysis

Based on the first part of the questions a, b, we select the typical energy situation of BMTCB, CLTCB, GETCB, HYTCB, NGTCB, NUETB, PATCB, SOTCB, WYTCB as indicators to analyze the energy status of the four states.

4.1.3.2 Developing and Solve the model

We use analytic hierarchy process to establish a hierarchical structure model, as shown in the following table5.

Table5.Analytic hierarchy

Target level	Overview of clean and renewable energy		
Guidelines layer	BMTCB(B1)	CLTCB(B2)	GETCB(B3)
	HYTCB(B4)	NGTCB(B5)	NUETB(B6)
	PATCB(B7)	SOTCB(B8)	WYTCB(B9)
Program floor	AZ,CA,TX,NM		

Based on the conclusions of the previous questions and Table 6, constructed to compare the matrix A,B.

Table6.Relative standard reference

	1	2	3	4	5	6	7	8	9
Standard importance	Identical		A little better		better		best		Absolutely best

Program-level factors, due to space limitations, only B1 is displayed here:

$$A = \begin{pmatrix} 1 & 3 & 1/3 & 1/4 & 3/2 & 1/3 & 4 & 1/4 & 1/4 \\ 1/3 & 1 & 1/9 & 1/13 & 1/2 & 1/9 & 4/3 & 1/12 & 1/12 \\ 3 & 9 & 1 & 3/4 & 4 & 1 & 12 & 3/4 & 3/4 \\ 4 & 12 & 4/3 & 1 & 6 & 4/3 & 16 & 1 & 1 \\ 2/3 & 2 & 2/7 & 1/6 & 1 & 2/9 & 8/3 & 1/6 & 1/6 \\ 3 & 9 & 1 & 3/4 & 9/2 & 1 & 12 & 3/4 & 3/4 \\ 1/4 & 3/4 & 1/12 & 1/14 & 3/8 & 1/12 & 1/13 & 1/12 & 9 \\ 4 & 12 & 4/3 & 1 & 6 & 4/3 & 16 & 1 & 1 \\ 4 & 12 & 4/3 & 1 & 6 & 4/3 & 16 & 1 & 1 \end{pmatrix} \quad B_1 = \begin{pmatrix} 1 & 1/3 & 1/2 & 1 \\ 3 & 1 & 3/2 & 3 \\ 2 & 2/3 & 1 & 2 \\ 1 & 1/3 & 1/2 & 1 \end{pmatrix}$$

Then we can get eigenvalue λ and eigenvector ϖ :

$$\lambda = 6.54$$

$$\varpi = (-0.21, 0.03, 0.27, 0.36, 0.06, 0.27, 0.36, 0.36) \quad (20)$$

Table7.Random consistency indicator

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

$n = 9$, according to Table7: $RI = 1.45$

Based on $CI = \frac{\lambda - n}{n - 1}$ and $CR = \frac{CI}{RI}$, we can get the Table8.

Table8.Consistency check

Index	B1	B2	B3	B4	B5	B6	B7	B8	B9
CR	0.085	0.11	0.01	0.03	0.10	0.07	0.08	0.03	0.091

$$CR_i < 0.1 (i = 1 \dots 9)$$

Nine indicators passed the consistency test, can solve most of the situation. Followed by the scores of these four states, as shown in Table9.

Table9.Score in four states

States	AZ	CA	TX	NM
Score	0.589320309	0.254188034	0.27023199	0.426723647

According to the state scores, the higher the score, the more important, the better its clean energy use. So, The use of clean energy is as follows: $AZ > CA > TX > NM$.

4.1.5 Solution for question d

4.1.5.1 Problem analysis

Due to the small time span of basic data, we consider using GM.

4.1.5.1 Developing and Solve the model

we define the model of the differential equation of GM (1,1) as

$$d(k) + az^{(1)}(k) = b \text{ Equal to } x^{(0)}(k) + az^{(1)}(k) = b$$

$$\begin{cases} x^{(0)}(2) + az^{(1)}(2) = b \\ x^{(0)}(3) + az^{(1)}(3) = b \\ \dots \\ x^{(0)}(n) + az^{(1)}(n) = b \end{cases} \quad (21)$$

$$Y = (x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(n))^T, u = (a, b)^T, B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \dots & \dots \\ -z^{(1)}(n) & 1 \end{bmatrix} \quad (22)$$

The least squares method can be obtained as:

$$\hat{u} = (\hat{a}, \hat{b})^T = (B^T B)^{-1} B^T Y \quad (23)$$

So we get the whitening Differential equations of GM(1,1):

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \quad (24)$$

Absolute residual formula:

$$\Delta = \left| X^{(0)}(k) - \hat{x}^{(0)}(k) \right| \quad (25)$$

Relative residual formula:

$$\theta = \frac{\left| X^{(0)}(k) - \hat{x}^{(0)}(k) \right|}{\left| X^{(0)}(k) \right|} \quad (26)$$

In order to make the test results more intuitive, we find the standard deviation of the relative residuals as the test result of the posterior difference.

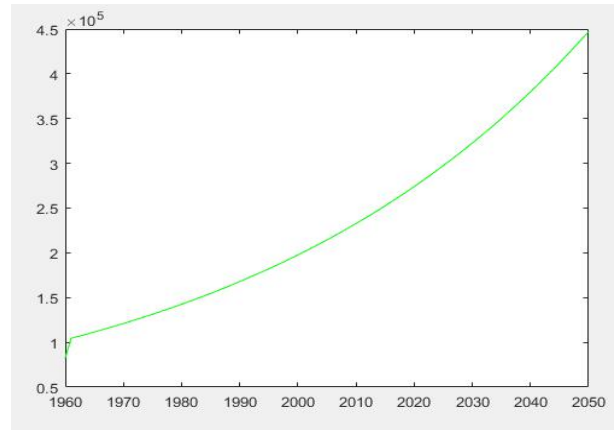


Figure8

Take California as an example, We got more than 100 more reliable predictions from our forecast result, such as the Predictive model of BMTCB, Neglect policies and other mandatory conditions. The estimated total biomass consumption in 2025 and 2050 are respectively 297300 billion btu, 447300 billion btu as is shown in Figure 8.

From our previous calculations, we can calculate the posterior difference matrix C for the main indicators of the four states (Where $i = 1, 2, 3, 4$ respectively represent AZ, CA, NM, TX)

$$C_1 = \{0.44 \ 0.30 \ 582.20 \ 0.13 \ 1.2777 \ 0.68 \ 0.44 \ 0.84 \ 0.16 \ 1.03\}$$

$$C_2 = \{0.35 \ 0.56 \ 36.87 \ 0.16 \ 0.31 \ 0.52 \ 0.44 \ 0.35 \ 0.36 \ 1.03\}$$

$$C_3 = \{0.56 \ 0.30 \ 0.92 \ 0.09 \ 3.49 \ 0.36 \ 0.62 \ 0.0 \ 0.29 \ 0.39\}$$

$$C_4 = \{0.36 \ 0.34 \ 0.16 \ 10.46 \ 0.60 \ 0.50 \ 1.80 \ 0.19 \ 1.58 \ 1.77 \times 10^5\}$$

After excluding a few data of poor prediction results, we obtained the forecast results of the main indicators of the four states through the gray system as shown in Figure 9.

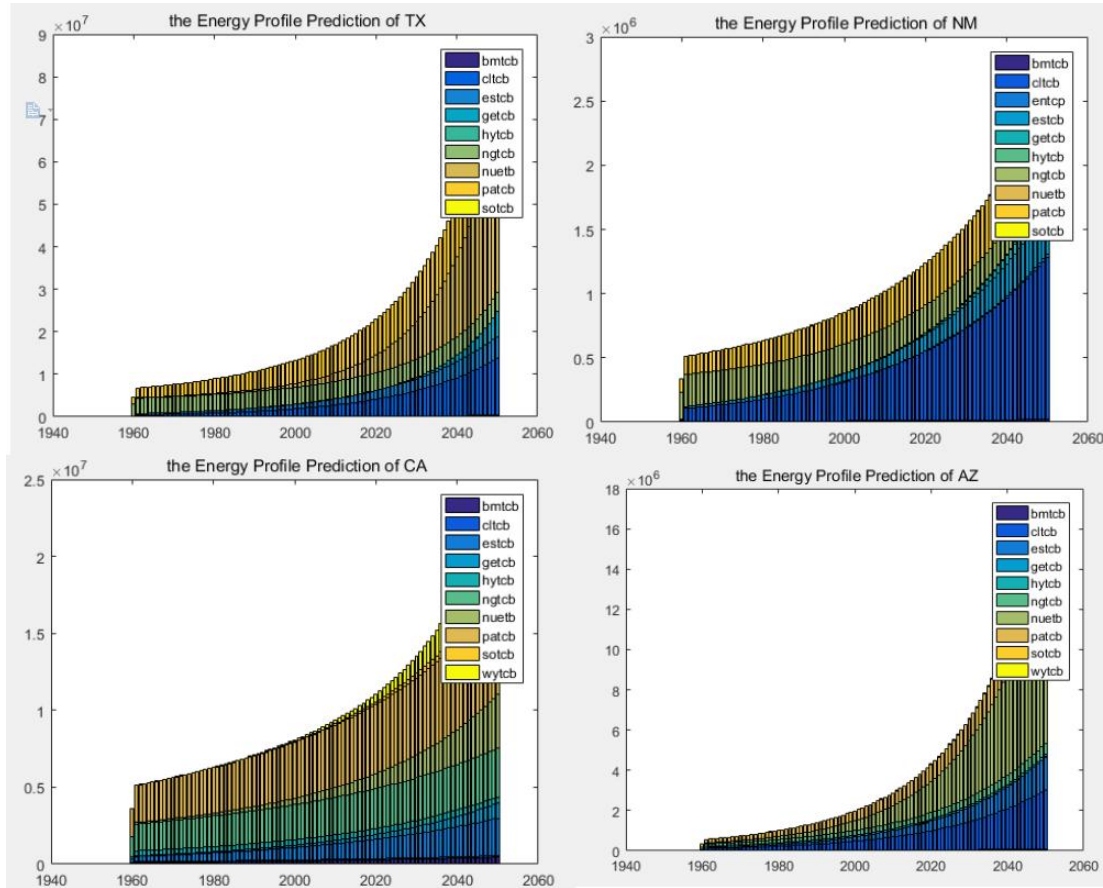


Figure9

4.2 Solution for part two

4.2.1 Solution for question a (Multi-objective optimization model)

1.The objective function

Taking into account the economy and energy are closely related,optimal economic indicators for the optimization of renewable energy use,To maximize the benefits brought by energy and maximize the production of renewable energy as two objective functions.

$$\begin{aligned}
 \max F &= \max(f_1(x), f_2(x)) \\
 f_1(x) &= \sum_k a_k(i, j) * x_k(i, j) \\
 f_2(x) &= \sum_k b_k(i, j) * x_k(i, j)
 \end{aligned} \tag{27}$$

2. Restrictions

(1)Non-renewable energy output constraints

$$r_{k,\min}(i, j) \leq x_k(i, j) \leq r_{k,\max}(i, j) \tag{28}$$

(2)energy supply constraints

$$\sum_j x_k(i, j) \leq p(i) \quad (29)$$

(3)Energy demand constraints

$$\sum_i x_k(i, j) \leq q(j) \quad (30)$$

3. Minimum deviation method

Convert the objective functions $f_1(x)$ and $f_2(x)$ into a single target form:

$$\max F^s = \frac{f_1(x) - f_1^{\min}}{f_1^{\max} - f_1^{\min}} + \frac{f_2(x) - f_2^{\min}}{f_2^{\max} - f_2^{\min}} \quad (31)$$

f_1^{\min} and f_2^{\min} represents the minimum value of the objective function f_1^{\max} and

f_2^{\max} represents the maximum value of the objective function

Considering the constraints, the converted renewable energy allocation model is $\max F^s$.

$$s.t. \begin{cases} \sum_j x_k(i, j) \leq p(i) \\ \sum_i x_k(i, j) \geq q(j) \\ r_{k,\min}(i, j) \leq x_k(i, j) \leq r_{k,\max}(i, j) \end{cases} \quad (32)$$

4. data analysis

Combined with the data, wind energy, hydropower, biomass and other renewable energy data together, calculate the state energy demand, the proportion of renewable energy.

Table10.the proportion of renewable energy

State name	Energy demand /billion btu	Renewable energy supply/billion btu	The share of renewable energy
AZ	1454.3	153.763	10.57%
ca	8005.5	743.392	9.29%
NM	670.1	35.786	5.34%
TX	11297.4	573.323	5.07%

4.2.1 Solution for question b

Based on our data analysis and the information we have reviewed, an analysis of the energy profiles of the four states shows that the four most commonly used energy sources in the four states are oil, gas, coal, nuclear power and renewable energy. Texas holds the largest oil consumption in the country with a significant share of natural gas consumption, and Texas also accounts for a

large proportion of coal consumption; New Mexico's oil demand is the second largest of the four states, Natural gas consumption, coal consumption ranked second only to Texas, relatively speaking, New Mexico energy consumption value is more balanced; California's renewable energy is the largest proportion of the four states, the data we process, California has relatively low coal consumption, low growth rates, a large proportion of oil consumption and production, and relatively low natural gas consumption; Arizona has the largest nuclear power plant in the United States and in our projections, During the year, Arizona enjoyed the fastest growth in nuclear power, with less energy demand in Arizona than in the other three states.

In this regard, I would like to make some immature suggestions in the interest of energy compactness:

Increase energy self-sufficiency rate. Each of the four states exerts its own energy advantages and has its own unique features in energy use. The states should include energy independence in their energy policies for a long time. The government should attach great importance to the issue of energy independence, vigorously develop the energy in the state and minimize the dependence on energy imports.

improve energy efficiency. In our data, we saw the word "waste," which, although not within the scope of the data we analyzed, still accounts for a certain percentage of "waste." Optimize the industrial structure, eliminate high energy-consuming industries. We should adhere to the industrial restructuring as a starting point for energy conservation strategies, strictly control and reduce redundant construction, speed up the transformation or elimination of "high energy consumption and high emission" production capacity, speed up technological transformation and upgrading, and reduce unnecessary energy consumption.

formulate the overall development of energy industry strategy to promote the diversification of energy development pattern

Through successive government efforts, the United States has gradually built a relatively complete energy and energy saving policy system and has ensured the effective implementation of policies through relevant laws and decrees, thus greatly boosting the development of new energy and energy-saving industries in the United States. Drawing on the experiences and lessons learned from overseas renewable energy development, we should make clear the medium- and long-term energy development strategies in each state, set milestones, establish a clear timetable, target value and roadmap scientifically, from extensive and inefficient to intensive and efficient The transformation of modern energy system, and to achieve energy production and consumption patterns of innovation.

Strengthen the protection of the environment, the use of clean energy and renewable energy. Through the data analysis and data query of the four states, we can see that the consumption of coal, petroleum and natural gas account for a large proportion because the pollution caused by environmental pollution also accounts for a large proportion of the pollution and strengthens the energy

management level. Change the structure of unreasonable energy consumption dominated by fossil fuels and promote the development of industries with high added value, low energy consumption and low pollution.

5. Strengths and Weaknesses

5.1 Strengths

1. Analytic Hierarchy Process to the actual measurement is not easy to target, easy measurement objectives, did not weaken the original amount of information. In addition, the weights are determined hierarchically, and the composite index is calculated by combining the weights, thus reducing the deviation of the traditional subjective right.
2. The original variables can be replaced with a few composite variables through dimensionality reduction techniques.

5.2 Weaknesses

1. Matrix order is too large, and we need to calculate a large number of comparison matrix, resulting in poor promotion of the model.
 2. Most of the actual statistics of the numerical value is 0, resulting in larger error prediction model results. The prediction model is relatively ineffective.
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6. Page memo

Dear governor,

We are here to raise some energy contracts. This letter explains our best solution model and makes a series of recommendations in a variety of contexts.

It is well known that energy is the foundation for a country to live in its own right. Without energy, modern civilized society will disappear. However, the use of energy brings us other problems as well as quicker lives.

In terms of energy use, we are based on two basic goals, using as much of the renewable energy as possible to protect the environment and maintaining sufficient economic benefits to maintain social stability.

In the past five decades, our country's energy consumption has been on the rise. In Texas, the level of consumption of all energy values is increasing year by year. Dezhou's demand for petroleum, natural gas and coal is higher than the other three states, and Texas has the most advantageous wind energy. Excluding policy factors and other reasons, Texas, nuclear power and coal consumption will continue to increase in the next 40 years, which is still a great pressure on resources. New Mexico's coal consumption forecast continues to rise, New Mexico also has a lot of wind, water can be used; The level of nuclear power in Arizona is growing fastest among the four states while the growth of other energy sources is relatively stable. California's nuclear growth rate is also fast, while California's oil consumption is maintained at a relatively high level.

Our model is feasible and reasonable and can be adjusted to various kinds of situation in reality. Although we have proposed a better solution, we suggest that the authority should take overall account of the reality according to the factors we listed in our paper. To know how the our model run in details, please check our paper.

Wish our optimal investment strategy can inspire you at the key point of solving the probable solution of energy. We are very eager to hear your opinion on our performance and to have more communication about it. We look forward to hearing from you.

Yours sincerely,

A group of modelers who are enthusiastic about mathematical modeling.

13/2/2018

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8. Appendix

TX Analysis of Variance

Source	DF	Sum of Square	Mean Square	F Value	Pr > F
Mode	5	2.88E+14	5.76E+13	2749.77	<.0001
Error	44	9.22E+11	20946943978		
Corrected Total	49	2.89E+14			
Root MSE	144731	R-Square	0.9968	Adj R-Sq	0.9964
Dependent Mean		8930452	Coeff Var	1.62064	

TX Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Standardized Estimate
Intercept	1	8930452	20468	436.31	<.0001	0
Prin1	1	271785	2360.83201	115.12	<.0001	0.98024
Prin2	1	64443	4385.68018	14.69	<.0001	0.12512
Prin3	1	-19310	5516.49626	-3.5	0.0011	-0.0298
Prin4	1	109329	7004.1745	15.61	<.0001	0.13291
Prin5	1	-44815	9171.37626	-4.89	<.0001	-0.04161

CA Analysis of Variance

Source	DF	Sum of Square	Mean Square	F Value	Pr > F
Mode	6	9.48E+13	1.58E+13	2379.3	<.0001
Error	43	2.86E+11	6643817204		
Corrected Total	49	9.51E+13			
Root MSE	81510	R-Square	0.997	Adj R-Sq	0.9966
Dependent Mean		6556300	Coeff Var		1.24323

CA Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Standardized Estimate
Intercept	1	6556300	11527	568.77	<.0001	0
Prin1	1	156729	1334.48572	117.45	<.0001	0.98148
Prin2	1	41391	2249.56194	18.4	<.0001	0.15376
Prin3	1	34615	3298.94938	10.49	<.0001	0.08769
Prin4	1	20004	3676.44056	5.44	<.0001	0.04547
Prin5	1	8867.77421	4825.08699	1.84	0.073	0.01536
Prin6	1	-5191.4072	5770.53443	-0.9	0.3733	-0.00752

AZ Analysis of Variance

Source	DF	Sum of Square	Mean Square	F Value	Pr > F
Mode	4	6.89E+12	1.72E+12	1574.53	<.0001
Error	45	49200866048	1093352579		
Corrected Total	49	6.94E+12			
Root MSE	33066	R-Square	0.9929	Adj R-Sq	0.9923
Dependent Mean		880081	Coeff Var		3.75714

AZ Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Standardized Estimate
Intercept	1	880081	4676.22193	188.2	<.0001	0
Prin1	1	47398	598.30703	79.22	<.0001	0.99469
Prin2	1	-2645.7179	1161.36986	-2.28	0.0275	-0.0286
Prin3	1	934.79344	1234.33761	0.76	0.4528	0.00951
Prin4	1	7031.72104	1736.3192	4.05	0.0002	0.05085