WORKSHOP Année de Recherche Pré-doctorale à l'Etranger (A.R.P.E.)

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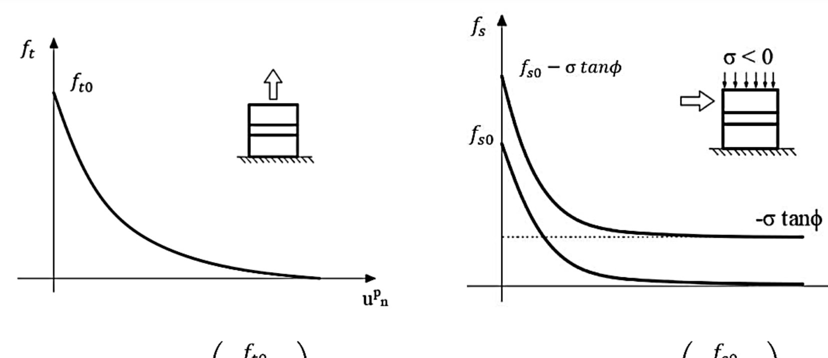
PROPOSAL OF A 3D CONSTITUTIVE MODEL BASED ON ELASTOPLASTICITY WITH DAMAGE FOR THE MODELING OF UNREINFORCED MASONRY INTERFACES

Introduction and objective

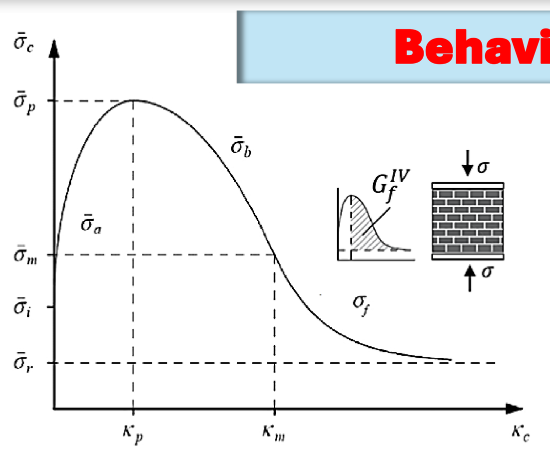
Masonry structures are composite structures made of brick and mortar. They were widely used in antiquity and are still relevant today because of the architectural aesthetics they offer. However, these structures are very vulnerable, especially when they are exposed to castle loads such as earthquakes and winds,...Therefore, understanding the mechanical handling of masonry is essential to prevent accidents but also to build more robust and economical structures. It is in this context that this project is part of the project, the aim of which is to set up a constitutive law based on elastoplasticity and damage to describe the behavior of the interface of these structures when subjected to the combined stresses of tension, shear and compression.



$f_t = f_{t0}\exp\left(-\frac{f_{t0}}{G_f^I}u_n^p\right)$ $\quad f_s = f_{s0}\exp\left(-\frac{f_{s0}}{G_f^{II}}u_s^p\right)$

(a) tensile behaviour (b) shear behaviour

Behavior curve of the model

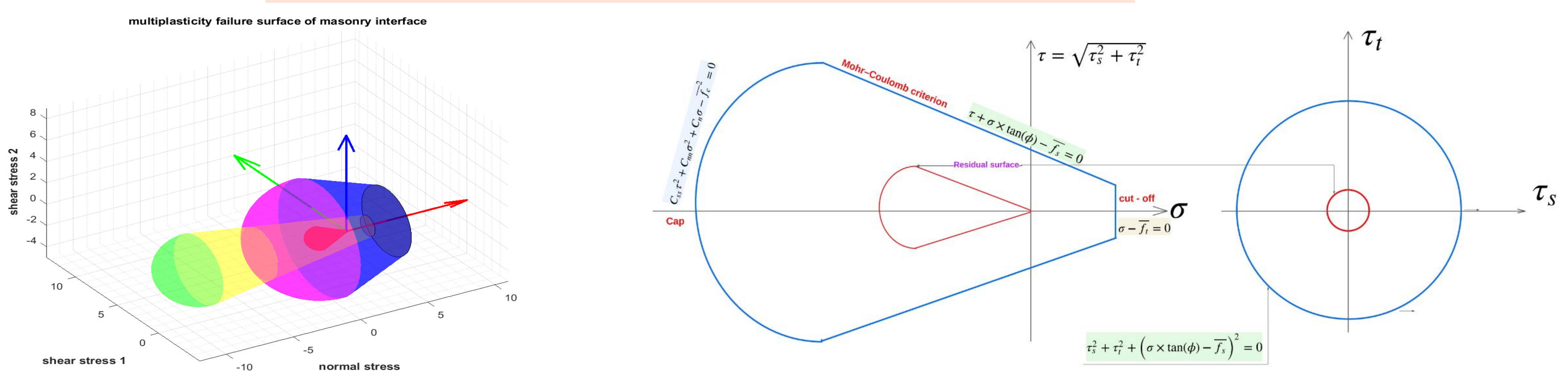


$$\begin{cases} \bar{\sigma}_a(\kappa_3) = \bar{\sigma}_i + (\bar{\sigma}_p - \bar{\sigma}_i)\left[\frac{2\kappa_3}{\kappa_p} - \left(\frac{\kappa_3}{\kappa_p}\right)^2\right]^\beta \\ \bar{\sigma}_b(\kappa_3) = \bar{\sigma}_p + (\bar{\sigma}_m - \bar{\sigma}_p)\left(\frac{\kappa_3 - \kappa_p}{\kappa_m - \kappa_p}\right)^\alpha \\ \bar{\sigma}_c(\kappa_3) = \bar{\sigma}_r + (\bar{\sigma}_m - \bar{\sigma}_r)exp\left(m\frac{\kappa_3 - \kappa_m}{\bar{\sigma}_m - \bar{\sigma}_r}\right) \end{cases}$$

Multiplasticity surface for the model and plastic potential

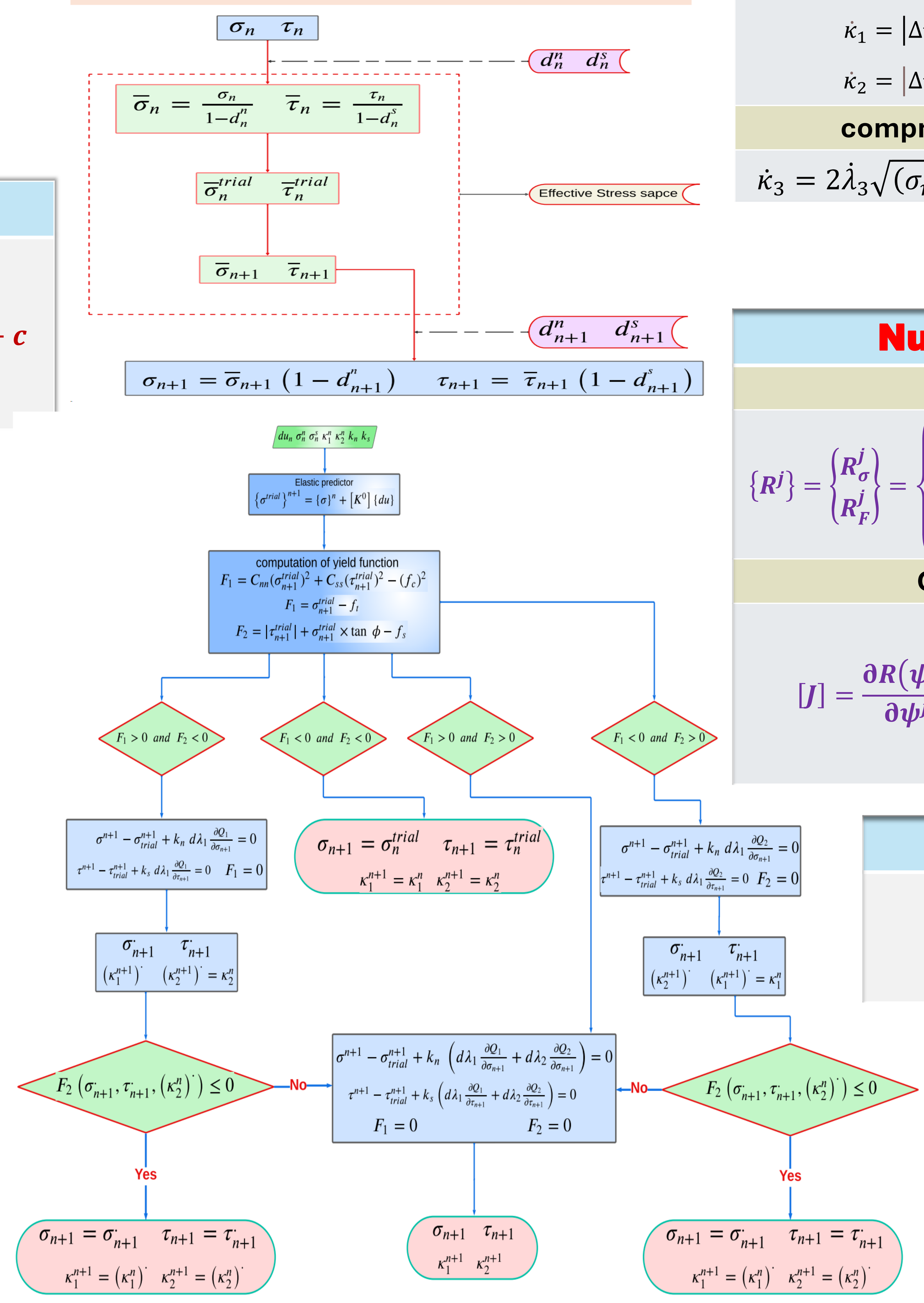
- *A criterion for tension failure (tension Cut – off)* $F_T = Q_T = \sigma_n - \bar{f}_t(\kappa_3)$
- *A Coulomb Criterion for Shear* $F_S = \sqrt{(\sigma_s^2 + \sigma_s^2)} + \sigma_n \tan\phi - \bar{f}_s(\kappa_3)$ $\quad Q_S = \sqrt{(\sigma_s^2 + \sigma_s^2)} + \sigma_n \tan\psi - c$
- *A criterion for compression failure (cap) :* $F_C = Q_C = C_{nn}\sigma_n^2 + C_{ss}(\sigma_s^2 + \sigma_s^2) + C_n\sigma_n - (\bar{\sigma}(\kappa_3))^2$

Multi-plastic threshold surface in 3 and 2 dimensions

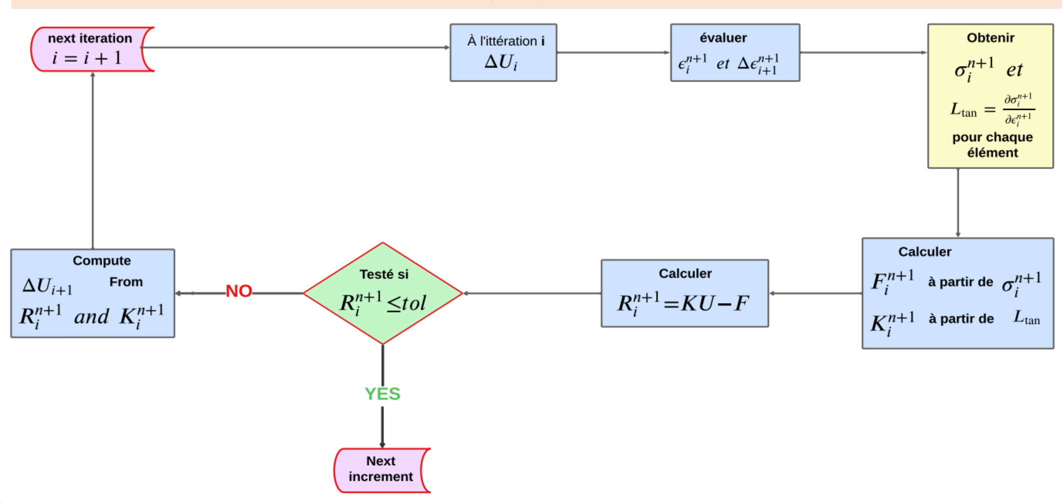


Local and global algorigram

Local algorigram for the model



Global algorigram for FEM



Internal variables

tension and shear	coupling tension and shear
$\dot{\kappa}_1 = \lvert\Delta\dot{u}_n^p\rvert = \dot{\lambda}_1$ $\dot{\kappa}_2 = \lvert\Delta\dot{u}_s^p\rvert = \dot{\lambda}_2$	$d\kappa_1 = \sqrt{\dot{\lambda}_1^{\,2} + \frac{G_f^{I\,2}\,c^2}{G_f^{II\,2}\,ft^2}\dot{\lambda}_2^{\,2}}$
compression	$d\kappa_2 = \sqrt{\dot{\lambda}_2^{\,2} + \frac{G_f^{II\,2}\,ft^2}{G_f^{I\,2}\,c^2}\dot{\lambda}_1^{\,2}}$
$\dot{\kappa}_3 = 2\dot{\lambda}_3\sqrt{(\sigma_n)^2 + (C_{ss}\sigma_s)^2}$	

Numerical implementation

General for of residual vector

$$\{R^j\} = \begin{Bmatrix} R_\sigma^j \\ R_F^j \end{Bmatrix} = \begin{Bmatrix} \{\sigma\}^{n+1} - \{\sigma^{trial}\}^{n+1} + [K_1]\sum_{i\in\{j_{ac}\}} \dot{\lambda}_i^{\,n+1}\left(\frac{\partial Q_i}{\partial\{\sigma\}}\right)^{n+1} \\ F_i^j \end{Bmatrix}$$

General form of Jacobian Matrix

$$[J] = \frac{\partial R(\psi^j)}{\partial\psi^j} = \begin{bmatrix} [I] + \sum_{i\in\{j_{ac}\}} \dot{\lambda}_i^{\,n+1}[K_1]\frac{\partial^2 Q_i}{\partial\{\sigma\}^2} & [K_1]\frac{\partial Q_i}{\partial\{\sigma\}} \\ \left(\frac{\partial F_i}{\partial\{\sigma\}}\right)^T & \left(\frac{\partial F_i}{\partial\dot{\lambda}_i}\right)^T \end{bmatrix}$$

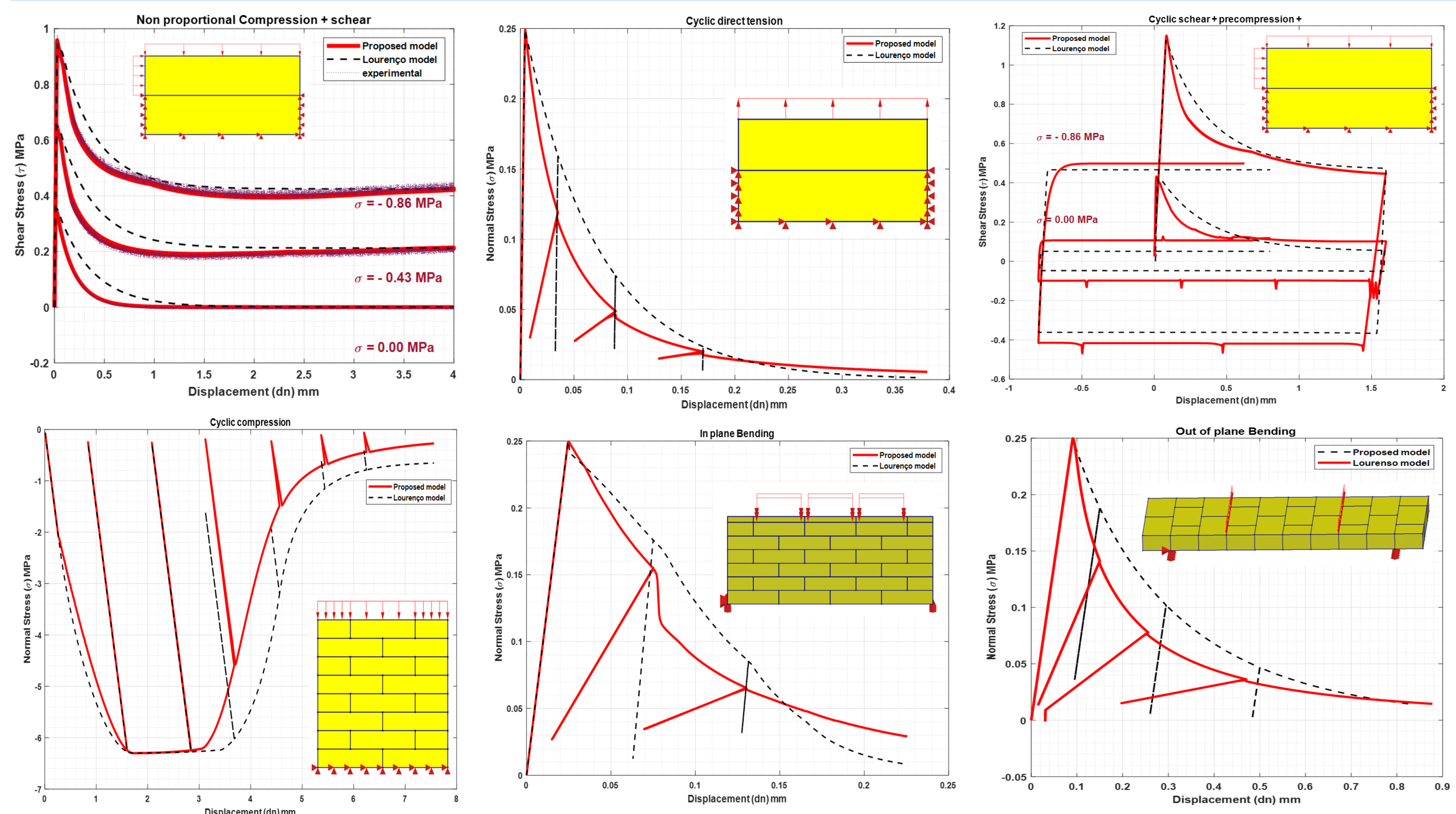
Consistent tangent operator

$$[\mathbb{L}_{n+1}^i] = \frac{\partial\sigma_{n+1}^i}{\partial\varepsilon_{n+1}^i} = H - \frac{H\frac{\partial Q}{\partial\sigma}\gamma^T H}{h + \gamma^T H\frac{\partial Q}{\partial\sigma}}$$

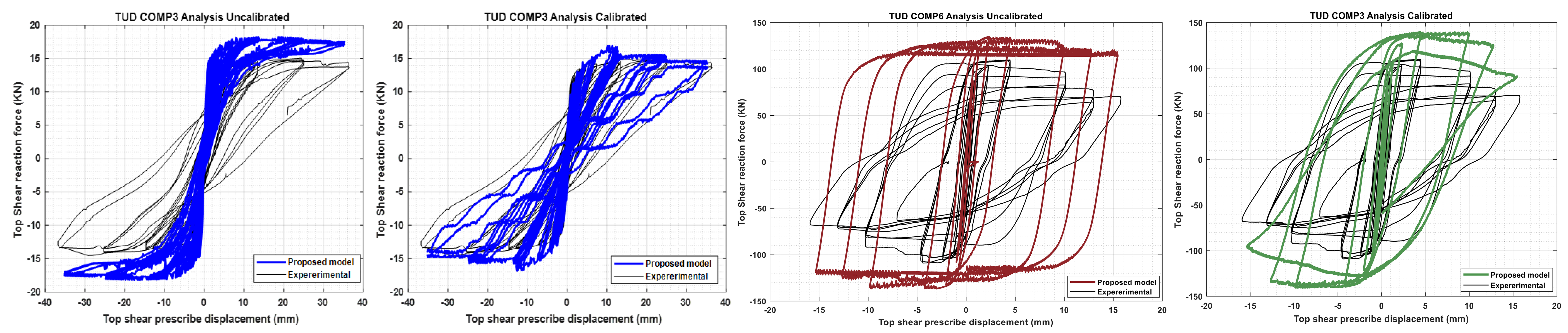
Legend

- $\tan\phi$ *Internal coefficient of friction*
- $\tan\psi$ *dilatancy*
- c *cohesion*
- f_t *tensile strength*
- f_c *compressive strength*
- $[K_1] = (\mathbb{I} - [D])[K]$ *Interface elasticity matrix*
- $\{\sigma\}$ *Vector of interface stress*
- λ_i *plastic multiplier*
- C_{ss}, C_{nn} *and* C_n *paterial parameters of cap*

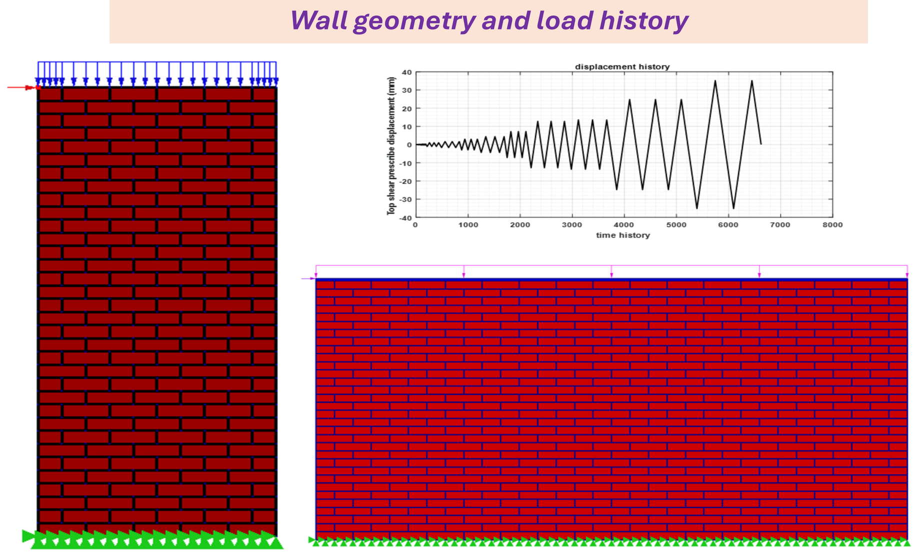
Material level validation of the model



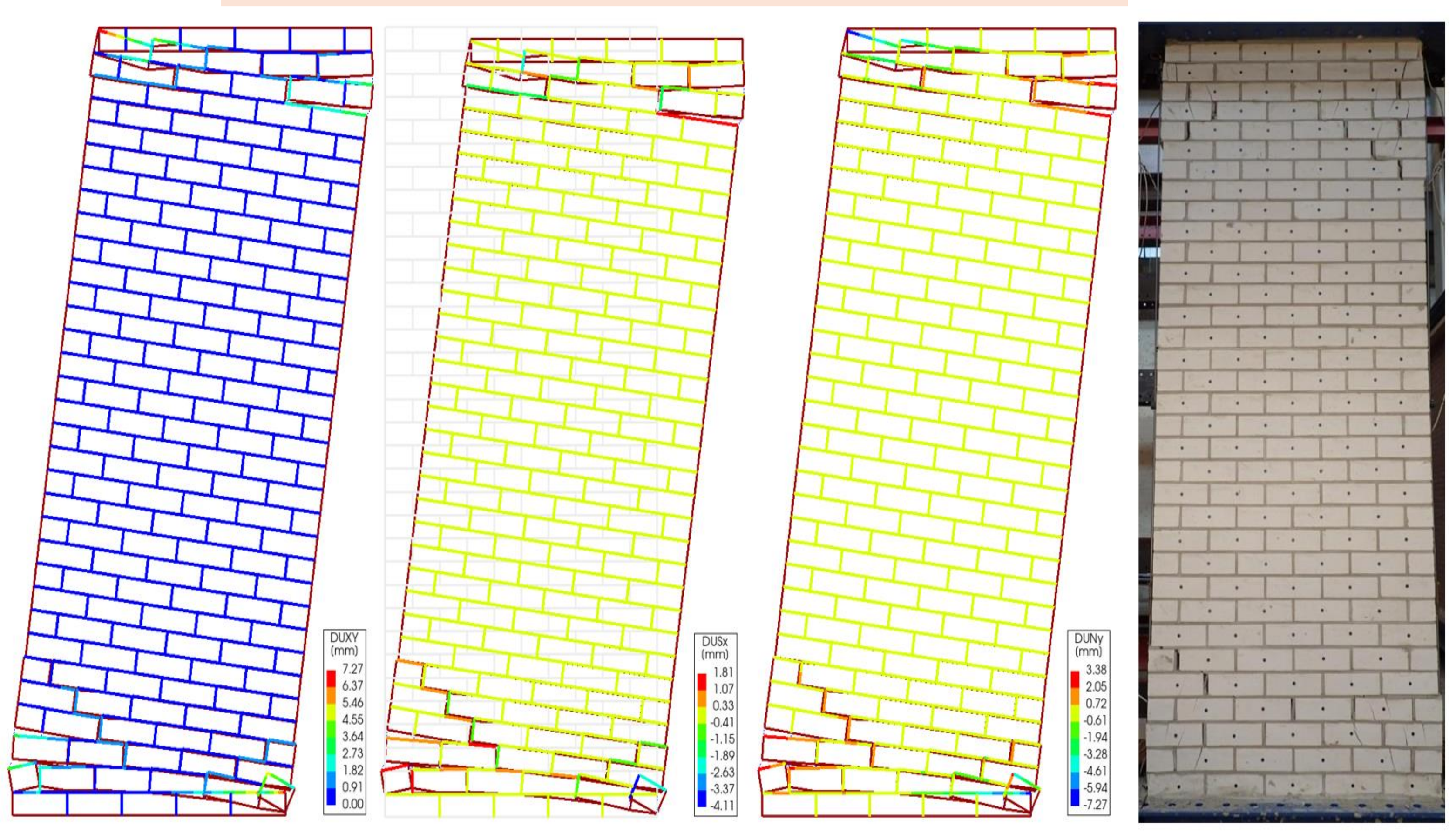
Structural level validation of the model



Wall geometry and load history



Crack pattern of the TUD_COMP3 wall



Crack pattern of the TUD_COMP6 wall