Class 11 - Mathematics
Sample Paper - 01 (2023-24)

Maximum Marks: 80
Time Allowed: : 3 hours

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. If cos x = $\frac{1}{2}$, then cos 3x = ?
    a) -1
    b) $\frac{2}{3}$
    c) $\frac{3}{2}$
    d) $\frac{1}{6}$
2. If R is a relation on the set A = {1, 2, 3, 4, 5, 6, 7, 8, 9} given by x R y $\Leftrightarrow$ y = 3x, then R =
    a) {(3, 1), (2, 6), (3, 9)}
    b) {(3, 1), (6, 2), (9, 3)}
    c) {(3, 1), (6, 2), (8, 2), (9, 3)}
    d) none of these
3. A dice is tossed once and even number has come up. The chance that it is either 2 or 4 is
    a) none of these
    b) $\frac{2}{3}$
    c) $\frac{1}{3}$
    d) $\frac{5}{6}$
4. $\lim_{x \to 0} \frac{(1+x)^n - 1}{x}$ is equal to
    a) -n
    b) 1
    c) 0
    d) n
5. The inclination of the straight line passing through the point (-3, 6) and the mid-point of the line joining the point (4, -5) and (-2 ,9) is
    a) $\frac{3\pi}{4}$

b) $\frac{\pi}{3}$

c) $\frac{\pi}{4}$

d) $\frac{\pi}{6}$

6. If A $\subset$ B, then

a) $A^c \subset B^c$

b) $B^c \not\subset A^c$

c) $A^c = B^c$

d) $B^c \subset A^c$

7. Mark the correct answer for $(1 + i)^{-1}$ =?

a) $\left(\frac{-1}{2} + \frac{1}{2}i\right)$

b) None of these

c) $\left(\frac{1}{2} - \frac{1}{2}i\right)$

d) (2 - i)

8. The range of the function f(x) = $\frac{x^2-x+1}{x^2+x+1}$ is

a) none of these

b) [3, ∞)

c) R

d) $\left[\frac{1}{3}, 3\right]$

9. What is the solution set for $\frac{|x-2|}{x-2} > 0$?

a) (2, ∞)

b) none of these

c) (-∞, -2)

d) (0, 2)

10. A horse is tied to a post by a rope. If the horse moves along a circular path, always keeping the rope tight and describes 88 m when it traces 72° at the centre, then the length of the rope is

a) 22 m

b) 70 m

c) 35 m

d) 17.5 m

11. If A = {1, 2, 3}, and B = {1, 3, 5, 7}, then A $\cup$ B =

a) none of these

b) {1, 3, 5, 7}

c) {1, 2, 3, 7}

d) {1, 2, 3, 5, 7}

12. The 17th term of the GP 2, $\sqrt{8}$, 4, $\sqrt{32}$... is

a) 256

b) $256\sqrt{2}$

c) $128\sqrt{2}$

d) 512

13. $\left\{\frac{c_1}{c_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \ldots + n \cdot \frac{C_n}{C_{n-1}}\right\} = ?$

a) $\frac{1}{2}n(n+1)$

b) 2n

c) 2^{n-1}

d) 2^n

14. If a, b, c are real numbers such that a $\leq$ b, c < 0, then

a) ac $\leq$ bc

b) ac > bc

c) ac $\geq$ bc

d) none of these

15. If A and B are two given sets , then A $\cap$ $(A \cap B)^c$ is equal to

a) B

b) A

c) $A \cap B^c$

d) ϕ

16. The value of sin 20° sin 40° sin 60° sin 80° is

a) $\frac{-3}{16}$

b) $\frac{3}{16}$

c) $\frac{1}{16}$

d) $\frac{5}{16}$

17. If z = 1 - cos θ + i sin θ, then |z| =

a) $2\left|\sin\frac{\theta}{2}\right|$

b) $2\left|\cos\frac{\theta}{2}\right|$

c) $2\cos\frac{\theta}{2}$

d) $2\sin\frac{\theta}{2}$

18. If in a group of n distinct objects, the number of arrangements of 4 objects is 12 times the number of arrangements of 2 objects, then the number of objects is

a) 8

b) 6

c) None of these

d) 10

19. **Assertion (A):** Number of terms in the expansion of $\left(2x - \frac{4}{x^2}\right)^{10}$ is 11.

Reason (R): Number of terms in the expansion of, $(x + a)^n$ is n + 1.
a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. Consider the following data

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Assertion (A): The variance of the data is 45.8.
Reason (R): The standard deviation of the data is 6.77.
a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Let A = {1, 2}, B = {2, 3, 4}, C = {4, 5}. Find A × (B ∪ C)

OR

Let A and B be two sets. Show that the sets A × B and B × A have an element in common if the sets A and B have an element in common.

22. If f(t) $= \frac{2}{9}t^4 - \frac{5}{3}t^3 + 2t - 1$, then find f'(-3).

23. Find the centre and radius of the circle: $(x + 5)^2 + (y - 3)^2 = 20$

OR

On the line joining (1, 0) and (3, 0) an equilateral triangle is drawn, having its vertex in the first quadrant. Find the equation to the circles described on its sides as diameter.

24. Is the pair of set A = {2, 3} and B = {x : x is solution of $x^2 + 5x + 6 = 0$} equal? Give reason.

25. If A(2, -5), B(-2, 5), C(x, 3) and D(1, 1) be four points such that AB and CD are perpendicular to each other, find the value of x.

Section C

26. Draw the graph of the function f(x) = $\begin{cases} 1 + 2x & x < 0 \\ 3 + 5x, & x \geq 0 \end{cases}$. Also, find its range.

27. To receive Grade **A**, in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Sunita must obtain in fifth examination to get Grade **A** in the course.

28. Three vertices of a parallelogram ABCD are A(3, -1, 2), B(1, 2, -4) and C(-1, 1, 2). Find the coordinates of the fourth vertex.

OR

Show that the points P(2, 3, 5), Q(-4, 7, -7), R (-2, 1, -10) and S(4, -3, 2) are the vertices of a rectangle.

29. Using binomial theorem, expand: $(\sqrt[3]{x} - \sqrt[3]{y})^6$

OR

Show that the coefficient of the middle term in the expansion of$(1 + x)^{2n}$ is equal to the sum of the coefficients of middle terms in the expansion of $(1 + x)^{2n-1}$.

30. Find the square root of -8 - 6i

OR

Find the real values of x and y for which: (x + iy)(3 - 2i) = (12 + 5i)

31. Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that B = C.

Section D

32. One urn contains two black balls (labelled B1 and B2) and one white ball. A second urn contains one black ball and two white balls (labelled W1 and W_2). Suppose the following experiment is performed. One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then a second ball is chosen at random from the same urn without replacing the first ball.
    i. Write the sample space showing all possible outcomes
    ii. What is the probability that two black balls are chosen?
    iii. What is the probability that two balls of opposite colour are chosen?

33. Evaluate: $\lim_{x \to 0} \frac{1 - \cos x \cos 2x \cos 3x}{\sin^2 2x}$

OR

i. If $f(x) = \begin{cases} |x| + 1, & x < 0 \\ 0, & x = 0 \\ |x| - 1, & x > 0 \end{cases}$, for what values (s) of a does $\lim_{x \to a} f(x)$ exist?

ii. Find the derivative of the function $\cos\left(x - \frac{\pi}{8}\right)$ from the first principle.

34. The lengths of three unequal edges of a rectangular solid block are in GP. The volume of the block is 216 cm^3 and the total surface area is 252 cm^2. Find the length of the longest edge.

35. At the foot of a mountain the elevation of its summit is 45^o; after ascending 1000 m towards the mountain up a slope of 30^o inclination, the elevation is found to be 60^o. Find the height of the mountain.

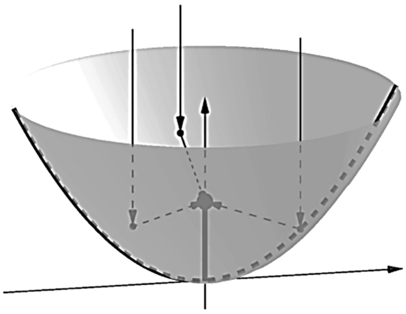
OR

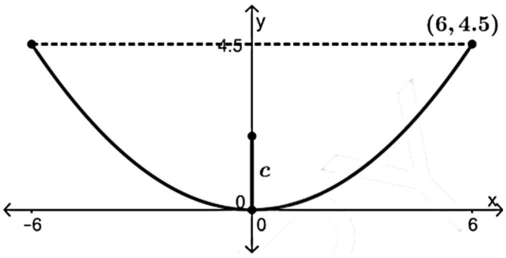
Prove: $\left(\frac{1}{\sec^2 x - \cos^2 x} + \frac{1}{\csc^2 x - \sin^2 x}\right) \sin^2 x \cos^2 x = \frac{1 - \sin^2 x \cos^2 x}{2 + \sin^2 x \cos^2 x}$

Section E

36. **Read the text carefully and answer the questions:**
A satellite dish has a shape called a paraboloid, where each cross section is parabola. Since radio signals (parallel to axis) will bounce off the surface of the dish to the focus, the receiver should be placed at the focus. The dish is 12 ft across,

and 4.5 ft deep at the vertex.





i. Name the type of curve given in the above paragraph and find the equation of curve?
ii. Find the equation of parabola whose vertex is (3, 4) and focus is (5, 4).
iii. Find the equation of parabola Vertex (0, 0) passing through (2, 3) and axis is along x-axis. and also find the length of latus rectum.

OR

Find focus, length of latus rectum and equation of directrix of the parabola $x^2 = 8y$.

37. **Read the text carefully and answer the questions:**

Consider the data

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

i. Find the standard deviation.
ii. Find the variance.
iii. Find the mean.

OR

Write the formula of variance?

38. **Read the text carefully and answer the questions:**

Ashish is writing examination. He is reading question paper during reading time. He reads instructions carefully. While reading instructions, he observed that the question paper consists of 15 questions divided in to two parts - part I containing 8 questions and part II containing 7 questions.

i. If Ashish is required to attempt 8 questions in all selecting at least 3 from each part, then in how many ways can he select these questions
ii. If Ashish is required to attempt 8 questions in all selecting 3 from I part, then in how many ways can he select these questions

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Solution

Section A

1. (a) -1

   Explanation: cos 3x = (4 $\cos^3$x - 3 cos x) $= \left(4\times\frac{1}{8}-3\times\frac{1}{2}\right)=\left(\frac{1}{2}-\frac{3}{2}\right)$ = -1

2. (d) none of these

   Explanation: $\therefore$ For A = {1, 2, 3, 4, 5, 6, 7, 8, 9} the satisfying complete relation is:

   R = {(1, 3), (2, 6), (3, 9)}

3. (b) $\frac{2}{3}$

   Explanation: Total number of ways of getting even number is 3

   Out of these 3 even numbers we have to get either 2 or 4 which can be done in 2 ways.

   So the required probability is $\frac{2}{3}$

4. (d) n

   Explanation: $\lim\limits_{x\to 0}\frac{(1+x)^n-1^n}{(1+x)-(1)}=\lim\limits_{x\to 0} n(1+x)^{n-1}=n$

5. (a) $\frac{3\pi}{4}$

   Explanation: The midpoint of the line joining the points (4, -5) and (-2, 9) is (1, 2)

   Let θ be the inclination of the straight line passing through the points (-3, 6) and (1,2).

   Then , using slope formula of tan θ we get ,

   tan θ = $\frac{2-6}{1+3}$ = -1

   $\Rightarrow \theta=\frac{3\pi}{4}$

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6. (d) $B^c\subset A^c$

   Explanation: Let A $\subset$ B

   To prove $B^c \subset A^c$, it is enough to show that x $\in B^c \Rightarrow$ x $\in A^c$

   Let x $\in B^c$

   $\Rightarrow$ x $\notin$ B

   $\Rightarrow$ x $\notin$ A since A $\subset$ B

   $\Rightarrow$ x $\in A^c$

   Hence $B^c \subset A^c$

7. (c) $\left(\frac{1}{2}-\frac{1}{2}i\right)$

   Explanation: $(1 + i)^{-1}=\frac{1}{(1+i)}=\frac{1}{(1+i)}\times\frac{(1-i)}{(1-i)}=\frac{(1-i)}{\left(1^2-i^2\right)}=\frac{(1-i)}{2}=\left(\frac{1}{2}-\frac{1}{2}i\right)$

8. (d) $\left[\frac{1}{3}, 3\right]$

Explanation: Let y = $\frac{x^2-x+1}{x^2+x+1}$

$\Rightarrow$ y (x^2 + x + 1) = x^2 - x + 1

$\Rightarrow$ (y - 1)x^2 + (y + 1)x + y - 1 = 0

For x to be real, we must have

$(y + 1)^2 - 4(y - 1)^2 \geq 0$ (Discriminant = b^2 - 4ac)

$\Rightarrow -3y^2 + 10y - 3 \geq 0 \Rightarrow 3y^2 - 10y + 3 \leq 0$

$\Rightarrow (3y - 1)(y - 3) \leq 0$

$\Rightarrow \frac{1}{3} \leq y \leq 3 \Rightarrow y \in \left[\frac{1}{3}, 3\right]$

$\therefore$ Range $= \left[\frac{1}{3}, 3\right]$

9. (a) $(2, \infty)$

Explanation: We have $|x - 2| \geq 0$ [$\because$ |x| or absolute value of x is always positive or zero but never negative]

Now $\frac{|x-2|}{x-2} > 0 \Rightarrow x - 2 > 0$ [$\because \frac{a}{b} > 0, a \geq 0 \Rightarrow b > 0$]

$\Rightarrow x > 2$

$\Rightarrow x \epsilon (2, \infty)$

10. (b) 70 m

Explanation: $\theta = 42^\circ = \left(42 \times \frac{\pi}{180}\right)^c = \left(\frac{2\pi}{5}\right)^c$ and l = 88 m.

$\therefore r = \frac{l}{\theta} = \left(88 \times \frac{5}{2\pi}\right)\text{m} = \left(88 \times \frac{5}{2} \times \frac{7}{22}\right)\text{m} = 70$ m.

11. (d) {1, 2, 3, 5, 7}

Explanation: Given A = {1, 2, 3} and B = {1, 3, 5, 7}

$(A \cup B)$ = {1, 2, 3, 5, 7}

12. (d) 512

Explanation: Given GP is $2, 2\sqrt{2}, 4, 4\sqrt{2}, \ldots$

Here, a = 2 and $= \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\therefore T_{17} = ar^{16} = 2 \times (\sqrt{2})^{16} = 2 \times 2^8 = 2^9 = 512$

Therefore, the required 17th term is 512.

13. (a) $\frac{1}{2}n(n + 1)$

Explanation: We know that $\frac{C_r}{C_{r1}} = \frac{n-r+1}{r}$,

Substituting r = 1,2,3,...,n, we obtain

$\frac{C_1}{C_0} + 2 \cdot \frac{C_2}{C_1} + 3 \cdot \frac{C_3}{C_2} + \ldots + n \cdot \frac{C_n}{C_{n-1}}$ = n + (n -1) + (n - 2) + ... + 1= $\frac{1}{2}n(n + 1)$.

14. (c) ac $\geq$ bc

Explanation: The sign of the inequality is to be reversed ($\leq$ to $\geq$ or $\geq$ to $\leq$) if both sides of an inequality are multiplied by the same negative real number.

15. (c) $A \cap B^c$

Explanation: $A \cap B^c$

A and B are two sets.

$A \cap B$ is the common region in both the sets.

$(A \cap B^c)$ is all the region in the universal set except $A \cap B$

Now, $A \cap (A \cap B)^c = A \cap B^c$

16. (b) $\frac{3}{16}$

Explanation: sin 20° sin 40° sin 60° sin 80°.

$= \frac{\sqrt{3}}{2}$ sin 20° sin (60° - 20°) sin (60° + 20°) (since sin 60° = $\frac{\sqrt{3}}{2}$)

$= \frac{\sqrt{3}}{2}$ sin 20° [$\sin^2 60° - \sin^2 20°$] { sin A + sin B = 2sin$\frac{A+B}{2}$ cos $\frac{A-B}{2}$ and sin A - Sin B = 2sin$\frac{A+B}{2}$ cos$\frac{A-B}{2}$

= $\sin^2 60° - \sin^2 20°$ = (sin 60° + sin 20°)(sin 60° - sin 20°)

= (2sin40°cos20°)(2sin20°cos40°)

= (2sin40°cos40°) (2sin20°cos 20°) = sin80° sin 40°}

$= \frac{\sqrt{3}}{2}$ sin 20° [$\frac{3}{4}$ - $\sin^2 20°$]

$= \frac{\sqrt{3}}{2} \times \frac{1}{4}$ [3sin 20° - 4 $\sin^3 20°$]

$= \frac{\sqrt{3}}{2} \times \frac{1}{4}$ (sin 60^o) (since $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$)

$= \frac{\sqrt{3}}{2} \times \frac{1}{4} \times \frac{\sqrt{3}}{2} = \frac{3}{16}$

17. (a) $2\left|\sin\frac{\theta}{2}\right|$

Explanation: $2\left|\sin\frac{\theta}{2}\right|$

$\because z = 1 - \cos\theta + i\sin\theta$

$\Rightarrow |z| = \sqrt{(1-\cos\theta)^2 + \sin^2\theta}$

$\Rightarrow |z| = \sqrt{1 + \cos^2\theta - 2\cos\theta + \sin^2\theta}$

$\Rightarrow |z| = \sqrt{1 + 1 - 2\cos\theta}$

$\Rightarrow |z| = \sqrt{2(1-\cos\theta)}$

$\Rightarrow |z| = \sqrt{4\sin^2\frac{\theta}{2}}$

$\Rightarrow |z| = 2\left|\sin\frac{\theta}{2}\right|$

18. (b) 6

Explanation: According to the question:

${}^nP_4 = 12 \times {}^nP_2$

$\Rightarrow \frac{n!}{(n-4)!} = 12 \times \frac{n!}{(n-2)!}$

$\Rightarrow \frac{(n-2)!}{(n-4)!} = 12$

$\Rightarrow$ (n - 2) (n - 3) = 4 × 3

$\Rightarrow$ n - 2 = 4

$\Rightarrow$ n = 6

19. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Assertion:

no. of terms = 10 + 1

= 11, True

Reason:

no. of term = n + 1, True

A Reason is correct explanation of Assertion.

20. (b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion: Presenting the data in tabular form, we get

x_i	f_i	f_ix_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
4	3	12	-10	100	300
8	5	40	-6	36	180
11	9	99	-3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
	30	**420**			**1374**

N = 30, $\sum_{i=1}^{7} f_ix_i$ = 420, $\sum_{i=1}^{7} f_i\left(x_i - \bar{x}\right)^2$ = 1374

Therefore, $\bar{x} = \frac{\sum_{i=1}^{7} f_ix_i}{N} = \frac{1}{30} \times 420 = 14$

$\therefore$ Variance $\left(\sigma^2\right) = \frac{1}{N}\sum_{i=1}^{7} f_i\left(x_i - \bar{x}\right)^2$

$\frac{1}{30} \times 1374 = 458$

Reason: Standard deviation (σ) = $\sqrt{45.8} = 6.77$

Section B

21. Given, A = {1, 2}, B = {2, 3, 4} and C = {4, 5}

B $\cup$ C = {2, 3, 4} $\cup$ {4, 5}

= {2, 3, 4, 5}

$\therefore$ A $\times$ (B $\cup$ C) = {1, 2} $\times$ {2, 3, 4, 5}

= {(1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5)}

OR

Let (a, b) be an arbitrary element of (A $\times$ B) $\cap$ (B $\times$ A). Then,

(a, b) $\in$ (A $\times$ B) $\cap$ (B $\times$ A)

= (a, b) $\in$ A $\times$ B and (a, b) $\in$ B $\times$ A

= (a $\in$ A and b $\in$ B) and (a $\in$ B and b $\in$ A)

= (a $\in$ A and a $\in$ B) and (b $\in$ A and b $\in$ B)

= a $\in$ A $\cap$ B and b $\in$ A $\cap$ B

Hence, the sets A $\times$ B and B $\times$ A have an element in common if the sets A and B have an element in common.

22. We have, $f(t) = \frac{2}{9}t^4 - \frac{5}{3}t^3 + 2t - 1$

On differentiating both sides w.r.t. t, we get

$$f'(t) = \frac{2}{9}\left(4t^3\right) - \frac{5}{3}\left(3t^2\right) + 2(1) - 0\left[\because \frac{d}{dt}\left(t^n\right) = nt^{n-1}\right]$$

$$= \frac{8}{9}t^3 - 5t^2 + 2$$

$$\therefore f'(-3) = \frac{8}{9}(-3)^3 - 5(-3)^2 + 2[\text{put t = -3}]$$

$= 8 \times \frac{(-27)}{9} - 5(9) + 2$= 8(-3) - 45 + 2

= -24 - 45 + 2 = -67

23. Given, $(x + 5)^2 + (y - 3)^2 = 20$

The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Comparing the given equation of circle with general form we get:

h = - 5 , k = 3, $r^2 = 20$

$\Rightarrow$ Centre = (-5, 3) and radius = $\sqrt{20} = 2\sqrt{5}$ units.

OR

Let (1, 0) and (3, 0) be the coordinate of the points A and B respectively Then

$$AB = \sqrt{(1-3)^2 + (0-0)^2} = 2$$

let Coordinates of C is C(x_1,y_1)

Now, $AC = \sqrt{\left(x_1 - 1\right)^2 + \left(y_1 - 0\right)^2}, \quad BC = \sqrt{\left(x_1 - 3\right)^2 + \left(y_1 - 0\right)^2}$

AC = BC

$AC^2 = BC^2$

$$\left(x_1 - 1\right)^2 + y_1^2 = \left(x_1 - 3\right)^2 + y_1^2 \Rightarrow 4x_1 = 8 \Rightarrow x_1 = 2$$

Again AC = 2

$$\sqrt{\left(x_1 - 1\right)^2 + y_1^2} = 2$$

$$\left(x_1 - 1\right)^2 + y_1^2 = 4$$

$(2-1)^2 + y_1^2 = 4 \Rightarrow y_1 = \pm\sqrt{3} \Rightarrow y_1 = \sqrt{3}$

So, the coordinated of are $(2, \sqrt{3})$

Similarly, the equations of circles with AB and BC as diameters are

(x -1) (x - 3) + (y - 0) (y - 0) = 0 and, (x - 3) (x - 2) + (y - 0) $(y - \sqrt{3}) = 0$

or $x^2 + y^2 - 4x + 3 = 0$ and, $x^2 + y^2 - 5x - \sqrt{3}y + 6 = 0$ respectively,

24. A = {2, 3} and B = {x : x is solution of $x^2 + 5x + 6 = 0$}

Now $x^2 + 5x + 6 = 0 \Rightarrow x^2 + 3x + 2x + 6 = 0$

$\Rightarrow$ (x + 3)(x + 2) = 0 $\Rightarrow$ x = -3, -2

$\therefore$ B = {-2, -3}

Hence A and B are not equal sets.

25. For two lines to be perpendicular, their product of slope must be equal to -1.

Given points are A(2, -5), B(-2, 5) and C(x, 3), D(1, 1)

Therefore, slope $=\left(\frac{y_2-y_1}{x_2-x_1}\right)$

$\Rightarrow$ Slope of line AB is equal to

$\left(\frac{5+5}{-2-2}\right)$

$=\left(\frac{10}{-4}\right)$

$=\left(\frac{-5}{2}\right)$ = -2.5

And the slope of line CD is equal to

$\left(\frac{1-3}{1-x}\right)=\left(\frac{-2}{1-x}\right)$

Their product must be equal to -1

the slope of line AB $\times$ Slope of line CD = -1

$\Rightarrow -2.5\times\left(\frac{-2}{1-x}\right)$ = -1

$\Rightarrow$ 5 = x -1

$\Rightarrow$ x = 6 is the required value.

Section C

26. Given, f(x) = $\begin{cases}1+2x & x<0\\ 3+5x, & x\geq 0\end{cases}$

Here, f(x) = 1 + 2x, x < 0, this gives

f(-4) =1 + 2(-4) = -7

f(-3) = 1 + 2(-3) = -5

f(-2) = 1 + 2(-2) = -3

f(-1) = 1 + 2(-1) = -1

f(x) = 3 + 5x, x $\geq$ 0

f(0) = 3 + 5(0) = 3

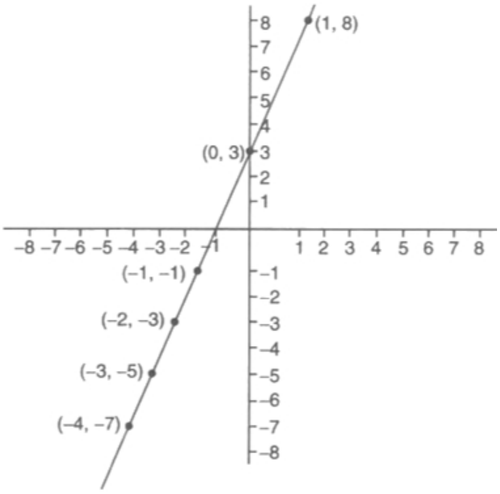
f(1) = 3 + 5(1) = 8

f(2) = 3 + 5(2) = 13

f(3) = 3 + 5(3) = 18

f(4) = 3 + 5(4) = 23

Now the graph of f is as shown in following figure



Range: Let y_1 = f(x), x < 0

$\therefore$ $y_1 = 1 + 2x, x < 0$

$\therefore$ $x = \frac{y_1 - 1}{2}, x < 0$

$\because x < 0 \Rightarrow y_1 - 1 < 0 \Rightarrow y_1 < 1$

Let $y_2 = f(x), x \geq 0$

$\Rightarrow$ $y_2 = 3 + 5x, x \geq 0$

$\Rightarrow x = \frac{y_2 - 3}{5}, x \geq 0$

$\because x \geq 0 \Rightarrow y_2 - 3 \geq 0 \Rightarrow y_2 \geq 3$

Therefore, range of $f(-\infty, 1), \cup [3, \infty)$

27. Let the marks obtained by Sunita in fifth examination be x.

 Then average of five examinations $= \frac{87+92+94+95+x}{5}$

 Now $\frac{87+92+94+95+x}{5} \geq 90 \Rightarrow \frac{368+x}{5} \geq 90$

 Multiplying both sides by 5, we have

 $368 + x \geq 450$

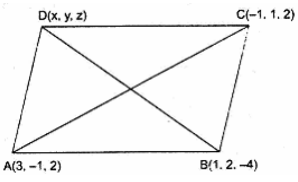
 $\Rightarrow x \geq 450 - 368$

 $\Rightarrow x \geq 82$

 Thus the minimum marks needed to be obtained by Sunita = 82.

28. Let D (x, y, z) be the fourth vertex of parallelogram ABCD.

 We know that diagonals of a parallelogram bisect each other. So the mid points of AC and BD coincide.



 $\therefore$ Coordinates of mid point of AC $\left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right)$

 = (1, 0, 2)

 Also coordinates of mid point of BD $\left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$

 $\therefore \frac{x+1}{2} = 1 \Rightarrow x + 1 = 2 \Rightarrow x = 1$

$\frac{y+2}{2} = 0 \Rightarrow y + 2 = 0 \Rightarrow y = -2$

$\frac{z-4}{2} = 2 \Rightarrow z - 4 = 4 \Rightarrow z = 8$

Thus the coordinates of point D are (1, -2, 8)

OR

To prove: Points P, Q, R, S forms a rectangle.

Formula: The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by

$D = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2}$

Here,

$(x_1, y_1, z_1) = (2, 3, 5)$

$(x_2, y_2, z_2) = (-4, 7, -7)$

$(x_3, y_3, z_3) = (-2, 1, -10)$

$(x_4, y_4, z_4) = (4, -3, 2)$

Length PQ $= \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2}$

$= \sqrt{(-4-2)^2 + (7-3)^2 + (-7-5)^2}$

$= \sqrt{(-6)^2 + (4)^2 + (-12)^2}$

$= \sqrt{36 + 16 + 144}$

Length PQ $= \sqrt{196}$

Length QR $= \sqrt{\left(x_3 - x_2\right)^2 + \left(y_3 - y_2\right)^2 + \left(z_3 - z_2\right)^2}$

$= \sqrt{(-2+4)^2 + (1-7)^2 + (-10+7)^2}$

$= \sqrt{(2)^2 + (-6)^2 + (-3)^2}$

$= \sqrt{4 + 36 + 9}$

Length QR $= \sqrt{49}$

Length RS $= \sqrt{\left(x_4 - x_3\right)^2 + \left(y_4 - y_3\right)^2 + \left(z_4 - z_3\right)^2}$

$= \sqrt{(4+2)^2 + (-3-1)^2 + (2+10)^2}$

$= \sqrt{(6)^2 + (-4)^2 + (12)^2}$

$= \sqrt{36 + 16 + 144}$

Length RS $= \sqrt{196}$

Length PS $= \sqrt{\left(x_4 - x_1\right)^2 + \left(y_4 - y_1\right)^2 + \left(z_4 - z_1\right)^2}$

$= \sqrt{(4-2)^2 + (-3-3)^2 + (2-5)^2}$

$= \sqrt{(2)^2 + (-6)^2 + (-3)^2}$

$= \sqrt{4 + 36 + 9}$

Length PS $= \sqrt{49}$

Length PR $= \sqrt{\left(x_3 - x_1\right)^2 + \left(y_3 - y_1\right)^2 + \left(z_3 - z_1\right)^2}$

$= \sqrt{(-2-2)^2 + (1-3)^2 + (-10-5)^2}$

$= \sqrt{(-4)^2 + (-2)^2 + (-15)^2}$

$= \sqrt{16 + 4 + 225}$

Length PR $= \sqrt{245}$

Length QS $= \sqrt{\left(x_4 - x_2\right)^2 + \left(y_4 - y_2\right)^2 + \left(z_4 - z_2\right)^2}$

$= \sqrt{(4+4)^2 + (-3-7)^2 + (2+7)^2}$

$= \sqrt{(8)^2 + (-10)^2 + (9)^2}$

$= \sqrt{64 + 100 + 81}$

Length QS $= \sqrt{245}$

Here, PQ = RS which are opposite sides of the polygon.

QR = PS which are opposite sides of the polygon.

Also the diagonals PR = QS.

Hence, the polygon is a rectangle.

29. To find: Expension of $(\sqrt[3]{x} - \sqrt[3]{y})^6$ by means of binomial theorem..

Formula used: ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

$(a + b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + nC_nb^n$

We have, $(\sqrt[3]{x} - \sqrt[3]{y})^6$

We can write $\sqrt[3]{x}$, as $x^{\frac{1}{3}}$, and $\sqrt[3]{y}$, as $y^{\frac{1}{3}}$,

Now, we have to solve for $\left(x^{\frac{1}{3}} - y^{\frac{1}{3}}\right)^6$

$$\Rightarrow \left[6C_0\left(x^{\frac{1}{3}}\right)^{6-0}\right] + \left[6C_1\left(x^{\frac{1}{3}}\right)^{6-1}\left(-y^{\frac{1}{3}}\right)^1\right] + \left[6C_2\left(x^{\frac{1}{3}}\right)^{6-2}\left(-y^{\frac{1}{3}}\right)^2\right] + \left[6C_3\left(x^{\frac{1}{3}}\right)^{6-3}\left(-y^{\frac{1}{3}}\right)^3\right]$$

$$+ \left[6C_4\left(x^{\frac{1}{3}}\right)^{6-4}\left(-y^{\frac{1}{3}}\right)^4\right] + \left[6C_5\left(x^{\frac{1}{3}}\right)^{6-5}\left(-y^{\frac{1}{3}}\right)^5\right] + \left[6C_6\left(-y^{\frac{1}{3}}\right)^6\right]$$

$$\Rightarrow \left[{}^6C_0\left(\frac{6}{x^3}\right)\right] - \left[{}^6C_1\left(x^{\frac{5}{3}}\right)\left(y^{\frac{1}{3}}\right)\right] + \left[6C_2\left(x^{\frac{4}{3}}\right)\left(y^{\frac{2}{3}}\right)\right] - \left[6C_3\left(x^{\frac{3}{3}}\right)\left(y^{\frac{3}{3}}\right)\right]$$

$$+ \left[{}^6C_4\left(x^{\frac{2}{3}}\right)\left(y^{\frac{4}{3}}\right)\right] - \left[6C_5\left(x^{\frac{1}{3}}\right)\left(y^{\frac{5}{3}}\right)\right] + \left[6C_6\left(\frac{6}{y^3}\right)\right]$$

$$\Rightarrow \left[\frac{6!}{0!(6-0)!}\left(x^2\right)\right] - \left[\frac{6!}{1!(6-1)!}\left(x^{\frac{5}{3}}\right)\left(y^{\frac{2}{3}}\right)\right] + \left[\frac{6!}{2!(6-2)!}\left(x^{\frac{4}{2}}\right)\left(x^{\frac{2}{3}}\right)\right]$$

$$-\left[\frac{6!}{3!(6-3)!}(x)(y)\right] + \left[\frac{6!}{4!(6-4)!}\left(x^{\frac{2}{3}}\right)\left(y^{\frac{4}{5}}\right)\right] - \left[\frac{6!}{5!(6-5)!}\left(x^{\frac{1}{3}}\right)\left(y^{\frac{5}{3}}\right)\right] + \left[\frac{6!}{6!(6-6)!}\left(y^2\right)\right]$$

$$\Rightarrow \left[1\left(x^2\right)\right] - \left[6\left(x^{\frac{5}{3}}\right)\left(y^{\frac{1}{3}}\right)\right] + \left[15\left(x^{\frac{4}{3}}\right)\left(y^{\frac{2}{3}}\right)\right] - [20(x)(y)] + \left[15\left(x^{\frac{2}{3}}\right)\left(\frac{4}{y^3}\right)\right]$$

$-\left[6\left(x^{\frac{1}{3}}\right)\left(y^{\frac{5}{3}}\right)\right]+\left[1\left(y^{2}\right)\right]$

$\Rightarrow x^2 - 6x^{\frac{5}{2}}y^{\frac{1}{3}} + 15x^{\frac{4}{3}}y^{\frac{2}{3}} - 20xy + 15x^{\frac{2}{3}}y^{\frac{4}{3}} - 6x^{\frac{1}{3}}y^{\frac{5}{3}} + y^2$

Hence the result.

OR

As discussed in the previous example, the middle term in the expansion of $(1 + x)^{2n}$ is given by $T_{n+1} = {}^{2n}C_n x^n$

So, the coefficient of the middle term in the expansion of $(1 + x)^{2n}$ is ${}^{2n}C_n$.

Now, consider the expansion of $(1 + x)^{2n-1}$ Here, the index (2n-1) is odd.

So, $\left(\frac{(2n-1)+1}{2}\right)^{th}$ and $\left(\frac{(2n-1)+1}{2}+1\right)^{\text{th}}$ i.e., n^{th} and $(n + 1)^{\text{th}}$ terms are middle terms.

Now, $T_n = T_{(n-1)+1},= {}^{2n-1}C_{n-1}(1)^{(2n-1)-(n-1)}x^{n-1} = {}^{2n-1}C_{n-1}x^{n-1}$

and, $T_{n+1} = {}^{2n-1}C_n(1)^{(2n-1)-n}x^n = {}^{2n-1}C_n x^n$

So, the coefficients of two middle terms in the expansion of $(1 + x)^{2n-1}$ are ${}^{2n-1}C_{n-1}$ and ${}^{2n-1}C_n$.

$\therefore$ Sum of these coefficients = ${}^{2n-1}C_{n-1} + {}^{2n-1}C_n$

$= {}^{(2n-1)+1}C_n\ [\because\ {}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r]$

$= {}^{2n}C_n$

= Coefficient of middle term in the expansion of $(1 + x)^{2n}$.

30. Let x + yi $= \sqrt{-8-6i}$

Squaring both sides, we get

$x^2 - y^2 + 2x\,yi = -8 - 6i$

Equating the real and imaginary parts

$x^2 - y^2 = -8$.....(i)

and 2xy = -6

$\therefore xy = -3$

Now using the identity

$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$

$= (-8^2 + 4(-3)^2$

= 64 + 36 = 100

$\therefore x^2 + y^2 = 10$ ..... (ii) [Neglecting (-) sign as $x^2 + y^2 > 0$]

Solving (i) and (ii) we get

$x^2 = 1$ and $y^2 = 9$

$\therefore x = \pm 1$ and $y = \pm 3$

Since the sign of xy is negative

$\therefore$ if x = 1, y = -3

and if x = -1, y = 3 $\therefore \sqrt{-8-6i} = \pm(1-3i)$

OR

(x + iy)(3 - 2i) = (12 + 5i)

==> x(3 - 2i) + iy(3 - 2i) = 12 + 5i

$\Rightarrow$ $3x - 2ix + 3iy - 2i^2y = 12 + 5i$

$\Rightarrow$ $3x + i(-2x + 3y) - 2(-1)y = 12 + 5i$ [$\because$ $i^2 = -1$]

$\Rightarrow$ $3x + i(-2x + 3y) + 2y = 12 + 5i$

$\Rightarrow$ $(3x + 2y) + i(-2x + 3y) = 12 + 5i$

Comparing the real parts and the imaginary parts, we get

$3x + 2y = 12$ ...(i)

$-2x + 3y = 5$ ...(ii)

Solving eq. (i) and (ii) to find the value of x and y

Multiply eq. (i) by 2 and eq. (ii) by 3, we get

$6x + 4y = 24$ ...(iii)

$-6x + 9y = 15$ ...(iv)

Adding eq. (iii) and (iv), we get

$6x + 4y - 6x + 9y = 24 + 15$

$\Rightarrow$ $13y = 39$

$\Rightarrow$ $y = 3$

Putting the value of y = 3 in eq. (i), we get

$3x + 2(3) = 12$

$\Rightarrow$ $3x + 6 = 12$

$\Rightarrow$ $3x = 12 - 6$

$\Rightarrow$ $3x = 6$

$\Rightarrow$ $x = 2$

Hence, the value of x = 2 and y = 3

31. We know that $A = A \cap (A \cup B)$ and $A = A \cup (A \cap B)$

Now $A \cap B = A \cap C$ and $A \cup B = A \cup C$

$\therefore B = B \cup (B \cap A) = B \cup (A \cap B) = B \cup (A \cap C)$ [$\because$ $A \cap B = A \cap C$]

$= (B \cup A) \cap (B \cup C)$ (By distributive law)

$= (A \cup C) \cap (B \cup C)$

$= (A \cup C) \cap (B \cup C)$ [$\because$ $A \cup B = A \cup C$]

$= (C \cup A) \cap (C \cup B)$

$= C \cup (A \cap B)$ (by distributive law)

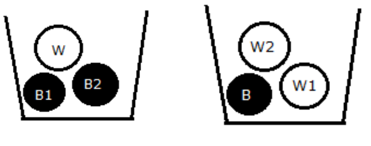
$= C \cup (A \cap C)$ [$\because$ $A \cap B = A \cap C$]

$= C \cup (C \cap A) = C$

Hence B = C.

Section D

32. Given that one urn contains two black balls and one white ball and second urn contains one black ball and two white balls as expressed in the figure below



It is also given that one of the two urns is chosen, then a ball is randomly chosen from the urn, then second ball is chosen at random from the same urn without replacing the first ball and this condition can also be treated as taking out two balls at a time from one of the two urns. So,

i. Sample Space S = $\{B_1B_2, B_1W, B_2W, B_2B_1, WB_1, WB_2, W_1W_2, W_1B, W_2B, W_2W_1, BW_1, BW_2\}$

Total number of sample space = 12

ii. If two black balls are chosen

Total outcomes = 12

Favourable outcomes are B_1B_2, B_2B_1

$\therefore$ Total favourable outcomes = 2

We know that,

Probabiltiy $= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

$\therefore$ Required probability $= \frac{2}{12} = \frac{1}{6}$

iii. If two balls of opposite colours are chosen i.e. one black and one white

Favourable outcomes are B_1W, B_2W, WB_1, WB_2, W_1B, W_2B, BW_1, BW_2

$\therefore$ Total favourable outcomes = 8 and Total outcomes = 12

We know that,

Probabiltiy $= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

$\therefore$ Required probability $= \frac{8}{12} = \frac{2}{3}$

33. Clearly,

$\cos x \cos 2x \cos 3x = \frac{1}{2}\{2\cos x \cos 2x \cos 3x\}$

$= \frac{1}{2}\{(2\cos x \cos 2x)\cos 3x\}$

$= \frac{1}{2}\{(\cos 3x + \cos x)\cos 3x\}$

$= \frac{1}{2}\{\cos^2 3x + \cos 3x \cos x\}$

$= \frac{1}{4}\{2\cos^2 3x + 2\cos 3x \cos x\}$

$= \frac{1}{4}\{1 + \cos 6x + \cos 4x + \cos 2x\}$

$$\therefore \lim_{x \to 0} \frac{1 - \cos x \cos 2x \cos 3x}{\sin^2 2x}$$

$$= \lim_{x \to 0} \frac{1 - \frac{1}{4}(1 + \cos 6x + \cos 4x + \cos 2x)}{\sin^2 2x}$$

$$= \lim_{x \to 0} \frac{4 - 1 - \cos 6x - \cos 4x - \cos 2x}{4\sin^2 2x}$$

$$= \lim_{x \to 0} \frac{(1 - \cos 6x) + (1 - \cos 4x) + (1 - \cos 2x)}{4\sin^2 2x}$$

$$= \lim_{x \to 0} \frac{2\sin^2 3x + 2\sin^2 2x + 2\sin^2 x}{4\sin^2 2x}$$

$$= \lim_{x \to 0} \frac{\frac{\sin^2 3x}{x^2} + \frac{\sin^2 2x}{x^2} + \frac{\sin^2 x}{x^2}}{2\left(\frac{\sin^2 2x}{x^2}\right)}$$

$$= \lim_{x \to 0} \frac{\left(\frac{\sin 3x}{x}\right)^2 + \left(\frac{\sin 2x}{x}\right)^2 + \left(\frac{\sin x}{x}\right)^2}{2\left(\frac{\sin 2x}{x}\right)^2}$$

$$= \lim_{x \to 0} \frac{9 \times \left(\frac{\sin 3x}{3x}\right)^2 + 4 \times \left(\frac{\sin 2x}{2x}\right)^2 + \left(\frac{\sin x}{x}\right)^2}{2 \times 4\left(\frac{\sin 2x}{2x}\right)^2}$$

$$= \frac{9 \times 1 + 4 \times 1 + 1}{8} = \frac{14}{8} = \frac{7}{4}$$

OR

i. $f(x) = \begin{cases} |x| + 1, & x < 0 \\ 0, & x = 0 \\ |x| - 1, & x > 0 \end{cases}$

At x = 0,

RHL $= \lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0 + h)$

$= \lim\limits_{h \to 0} |0 + h| - 1$

= - 1

LHL $= \lim\limits_{h \to 0^-} f(x) = \lim\limits_{h \to 0} f|0 - h|$

$= \lim\limits_{h \to 0} |0 - h| + 1$

$= \lim\limits_{h \to 0} -(0 - h) + 1$

$= \lim\limits_{h \to 0} h + 1$

= 0 + 1 = 1

$\Rightarrow$ RHL $\neq$ LHL

$\Rightarrow$ At x = 0, limi does not exist.

Hence, $\lim\limits_{x \to a} f(x)$ exists for all a $\neq$ 0.

ii. Let $f(x) = \cos\left(x - \frac{\pi}{8}\right)$

By using first principle of derivative

We have,

$f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{\cos\left(x + h - \frac{\pi}{8}\right) - \cos\left(x - \frac{\pi}{8}\right)}{b} \left[\because f(x) = \cos\left(x - \frac{\pi}{8}\right)\right]$

$= \lim\limits_{h \to 0} \frac{-2\sin\left(\frac{x + h - \frac{\pi}{8} + x - \frac{\pi}{8})}{2}\right)\sin\left(\frac{x + h - \frac{\pi}{8} - x + \frac{\pi}{8}}{2}\right)}{h}$

$\left[\because \cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$

$$= \lim_{h \to 0} \frac{-2\sin \frac{2x - 2\left(\frac{\pi}{8}\right) + h}{2}}{2 \times \frac{h}{2}}$$

$$= -\sin \frac{2x - 2\left(\frac{\pi}{8}\right) + 0}{2} \times 1 \left[\therefore \lim_{x \to 0} \frac{\sin \frac{h}{2}}{\frac{h}{2}} = 1 \right]$$

$$= -\sin \frac{2\left(x - \frac{\pi}{8}\right)}{2}$$

$$\Rightarrow f'(x) = -\sin\left(x - \frac{\pi}{8}\right)$$

34. Let the edges of rectangular block in GP be a, ar and ar^2, respectively.... (i)

Now, Volume =216 cm^3

$\Rightarrow$ a(ar) (ar^2) = 216 [$\because$ volume of cuboid = l $\times$ b $\times$ h]

$\Rightarrow$ $(ar)^3 = (6)^3$

$\Rightarrow$ ar = 6 cm [taking cube root] ...(ii)

and total surface area = 252 cm

$\Rightarrow$ 2[a (ar) + ar (ar^2) + a (ar^2)] = 252 [$\because$ sufrace area of cuboid = 2 (lb + bh + hl)]

From Eq. (ii), we get

2 (6a +36r +36) = 252

$\Rightarrow$ 12 (a + 6r + 6) = 252

$\Rightarrow$ a + 6r = 15 [divide both sides by 12] ...(iii)

$\Rightarrow$ a + 6 $\times$ $\left(\frac{6}{a}\right)$ = 15 [from Eq. (ii)]

$\Rightarrow$ $a^2 - 15a + 36 = 0$ $\Rightarrow$ (a - 12) (a - 3) = 0

$\Rightarrow$ a = 3, 12

From Eq. (iii), we get

When a = 3, then 3 + 6r = 15 $\Rightarrow$ r = 2

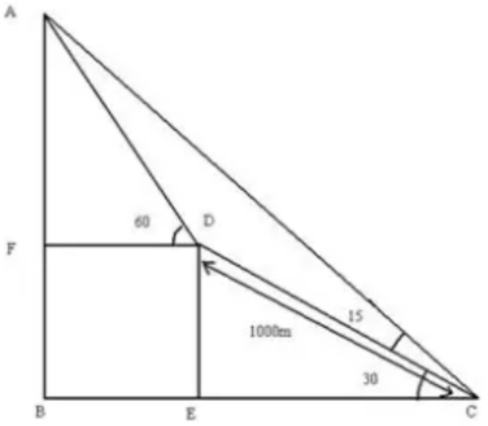
and when a = 12, then 12 + 6r = 15 $\Rightarrow$ r = $\frac{1}{2}$

On putting above values in Eq. (i),

edges are 3, 3 $\times$ 2, 3 $\times$ $(2)^2$ or 12, 12 $\times$ $\left(\frac{1}{2}\right)$, 12 $\times$ $\left(\frac{1}{2}\right)^2$

i.e., 3, 6, 12 or 12, 6, 3.

Hence, the length of the longest edge is 12 cm.

35. 

DE = 1000 sin 30 = 1000 $\times \frac{1}{2}$ = 500 m = FB

EC = 1000 cos 30 = 1000 $\times \frac{\sqrt{3}}{2} = 500\sqrt{3}$ m

Let A F = x m

D F = $\frac{x}{\sqrt{3}}$ m = B E

We know,

From $\triangle$ABC

tan 45 = $\frac{AB}{BC}$

$\Rightarrow 1 = \frac{AF+FB}{BE+EC}$

$\Rightarrow 1 = \frac{x+500}{\frac{x}{\sqrt{3}}+500\sqrt{3}}$

$\Rightarrow \frac{x}{\sqrt{3}} + 500\sqrt{3} = x + 500$

$\Rightarrow x + 1500 = x\sqrt{3} + 500\sqrt{3}$

$\Rightarrow 1500 - 500\sqrt{3} = x\sqrt{3} - x$

$\Rightarrow 500\sqrt{3}(\sqrt{3} - 1) = x(\sqrt{3} - 1)$

$\therefore$ x = $500\sqrt{3}$ m

The height of the triangle is AB = AF + FB = $500(\sqrt{3} + 1)$ m

OR

LHS = $\left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\csc^2\theta - \sin^2\theta}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\left(\frac{1}{\sin^2\theta} - \sin^2\theta\right)}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{1}{\frac{1-\cos^4\theta}{\cos^2\theta}} + \frac{1}{\frac{1-\sin^4\theta}{\sin^2\theta}} \right) \sin^2\theta \cos^2\theta$

$= \left(\frac{\cos^2\theta}{\left(1-\cos^2\theta\right)\left(1+\cos^2\theta\right)} + \frac{\sin^2\theta}{\left(1-\sin^2\theta\right)\left(1+\sin^2\theta\right)} \right) \sin^2\theta \cos^2\theta$ ($\because$ 1 − a4 = 1 − $(a^2)^2 = (1 - a^2)(1 + a^2)$)

$= \left(\frac{\cos^2\theta}{\sin^2\theta\left(1+\cos^2\theta\right)} + \frac{\sin^2\theta}{\cos^2\theta\left(1+\sin^2\right)} \right) \sin^2\theta \cos^2\theta$ ($\because$ $1 - \cos^2\theta = \sin^2\theta$ and $1 - \sin^2\theta = \cos^2\theta$)

$= \left(\frac{\cos^4\theta\left(1+\sin^2\theta\right)+\sin^4\theta\left(1+\cos^2\theta\right)}{\sin^2\theta\cos^2\theta\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)} \right) \sin^2\theta \cos^2\theta$

$= \frac{\cos^4\theta+\sin^2\theta\cos^4\theta+\sin^4\theta+\cos^2\theta\sin^4\theta}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$

$= \frac{\left(\cos^2\theta\right)^2+\left(\sin^2\theta\right)^2+2\cos^2\theta\sin^2\theta-2\cos^2\theta\sin^2\theta+\sin^2\theta\cos^4\theta+\cos^2\theta\sin^4\theta}{\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}$ (adding and subtracting 2 $\cos^2\theta \sin^2\theta$)

$= \frac{\left(\cos^2\theta+\sin^2\theta\right)^2-2\cos^2\theta\sin^2\theta+\sin^2\theta\cos^2\theta\left(\cos^2\theta+\sin^2\theta\right)}{1+\sin^2\theta+\cos^2\theta+\sin^2\theta\cos^2\theta}$

$= \frac{1^2-2\cos^2\theta\sin^2\theta+\sin^2\theta\cos^2\theta\cdot 1}{1+1+\sin^2\theta\cos^2\theta}$

$= \frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$

= RHS

Hence proved.

Section E

36. i. Given curve is a parabola

Equation of parabola is $x^2 = 4ay$

It passes through the point (6, 4.5)

$\Rightarrow$ 36 = 4 × a × 4.5

$\Rightarrow$ 36 = 18a

$\Rightarrow$ a = 2

Equation of parabola is $x^2 = 8y$

ii. Distance between focus and vertex is = a = $\sqrt{(4-4)^2 + (5-3)^2} = 2$

Equation of parabola is $(y - k)^2 = 4a(x - h)$

where (h, k) is vertex

$\Rightarrow$ Equation of parabola with vertex (3, 4) & a = 2

$\Rightarrow$ $(y - 4)^2 = 8(x - 3)$

iii. Equation of parabola with axis along x - axis

$y^2 = 4ax$

which passes through (2, 3)

$\Rightarrow$ 9 = 4a $\times$ 2

$\Rightarrow$ 4a = $\frac{9}{2}$

hence required equation of parabola is

$y^2 = \frac{9}{2}x$

$\Rightarrow$ 2y^2 = 9x

Hence length of latus rectum = 4a = 4.5

OR

x^2 = 8y

a = 2

Focus of parabola is (0, 2)

length of latus rectum is 4a = 4 $\times$ 2 =8

Equation of directrix y + 2 =0

37. i. By using formula, $\sigma^2 = \frac{1}{N}\left[\sum_{i=1}^{n} f_i\left(x_i - \bar{x}\right)^2\right]$

x_i	f_i	f_ix_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
4	3	12	-10	100	300
8	5	40	-6	36	180
11	9	99	-3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
Total	30	420			1374

Given, N = $\sum f_i = 30$, $\sum f_ix_i = 420$ and $\sum f_i\left(x_i - \bar{x}\right)^2 = 1374$

$\therefore \bar{x} = \frac{\sum_{i=1}^{7} f_ix_i}{N} = \frac{420}{30} = 14$

Variance $(\sigma^2) = \frac{1}{N}\sum_{i=1}^{7} f_i\left(x_i - \bar{x}\right)^2 = \frac{1}{30} \times 1374 = 45.8$

Standard deviation, $\sigma = \sqrt{\sigma^2} = \sqrt{45.8} = 6.77$

ii. Variance $(\sigma^2) = \frac{1}{N}\sum_{i=1}^{7} f_i\left(x_i - \bar{x}\right)^2 = \frac{1}{30} \times 1374 = 45.8$

iii. Given, N = $\sum f_i = 30$, $\sum f_ix_i = 420$ and $\sum f_i\left(x_i - \bar{x}\right)^2 = 1374$

$\therefore \bar{x} = \frac{\sum_{i=1}^{7} f_ix_i}{N} = \frac{420}{30} = 14$

OR

$\sigma^2 = \frac{1}{N}\Sigma\left(x_i - \bar{x}\right)$

38. i. Since, at least 3 questions from each part have to be selected

Part I	Part II
3	5
4	4
3	5

So number of ways are

3 questions from part I and 5 questions from part II can be selected in $n^8C_3 \times {}^7C_5$ ways

4 questions from part I and 4 questions from part II can be selected in ${}^8C_4 \times {}^7C_4$ ways

5 questions from part I and 3 questions from part II can be selected in ${}^8C_5 \times {}^7C_3$ ways

So required number of ways are

${}^8C_3 \times {}^7C_5 + {}^8C_4 \times {}^7C_4 + {}^8C_5 \times {}^7C_3$

$\Rightarrow \frac{8!}{5! \times 3!} \times \frac{7!}{5! \times 2!} + \frac{8!}{4! \times 4!} \times \frac{7!}{4! \times 3!} + \frac{8!}{5! \times 3!} \times \frac{7!}{4! \times 3!}$

$\Rightarrow \frac{8 \times 7 \times 6}{3 \times 2 \times 1} \times \frac{7 \times 6}{2 \times 1} + \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} \times \frac{7 \times 6 \times 5}{3 \times 2 \times 1} + \frac{8 \times 7 \times 6}{3 \times 2 \times 1} \times \frac{7 \times 6 \times 5 \times 4}{4 \times 3 \times 2 \times 1}$

$\Rightarrow 56 \times 21 + 70 \times 35 + 56 \times 35$

$\Rightarrow 1344 + 2450 + 1960$

$\Rightarrow 5754$

ii. Ashish is selecting 3 questions from part I so he has to select remaining 5 questions from part II

The number of ways of selection is

3 questions from part I and 5 questions from part II can be selected in ${}^8C_3 \times {}^7C_5$ ways

$\Rightarrow {}^8C_3 \times {}^7C_5$

$\Rightarrow \frac{8!}{5! \times 3!} \times \frac{7!}{5! \times 2!}$

$\Rightarrow \frac{8 \times 7 \times 6}{3 \times 2 \times 1} \times \frac{7 \times 6}{2 \times 1}$

$\Rightarrow 56 \times 21$

$\Rightarrow 1344$