9 Relations

§9.1 Relations and their properties

reflexive: if $(a,a) \in R$ for every element $a \in A$

symmetric: if $(b,a) \in R$ whenever $(a,b) \in R$

antisymmetric: for all $a,b \in A$, if $(a,b) \in R$ and $(b,a) \in R$, then $a=b$ is called antisymmetric

transitive: whenever $(a,b) \in R$, $(b,c) \in R$, then $(a,c) \in R$ for all $a,b,c \in A$.

Let R be a relation from a set A to a set B and S a relation from B to a set C. The composite of R and S is the relation consisting of ordered pairs (a,c), where $a \in A$, $c \in C$, and for which there exists an element $b \in B$, such that $(a,b) \in R$ and $(b,c) \in S$. $S \circ R$

We can define 2^{n^2} differene relations on A.

reflexive: if $(A,A) \in R$ for every element $a \in A$

irreflexive: if $(a,a) \notin R$ for every element $a \in A$

symmetric: if $(b,a) \in R$ whenever $(a,b) \in R$ for all $a,b \in A$

antisymmetric: if $a=b$ whenever $(a,b) \in R$ and $(b,a) \in R$.

asymmetric: if $(a,b) \in R$ implies $(b,a) \notin R$.

transitive: if $(a,b) \in R$, $(b,c) \in R$, then $(a,c) \in R$.

there are 2^{n^2-n} elements to generate reflexive relations, $2^{n(n-1)}$ of them

The relation R on a set A is transitive if and only if $R^n \subseteq R$ for all positive integers n.

$A_{m \times k} [a_{ij}]$, $B_{k \times n} [b_{ij}]$

the Boolean product of A and B, ~~$\ast$~~ or $A \odot B$

$$c_{ij} = (a_{i1} \wedge b_{1j}) \vee (a_{i2} \wedge b_{2j}) \vee \cdots \vee (a_{ik} \wedge b_{kj})$$

an edge of the form (b,b) is called a loop.

§9.4 Closures of relations.

R is binary relation on set A

The reflexive closure of R is $R \cup \Delta = R \cup \{(a,a) \mid a \in R\}$

The symmetric closure of R is $R \cup R^{-1} = R \cup \{(a,b) \mid (b,a) \in R\}$

The transitive closure

A path that begins and ends at the same vertex is called a circuit or cycle.

There is a path of length n from a to b if and only if $(a,b) \in R^n$

The transitive closure of a relation R equals the connectivity relation R^* is the union of R^n across all positive integers

$$R^* = \bigcup_{n=1}^{\infty} R^n = R^1 \cup R^2 \cup R^3 \cup \cdots$$

Lemma: if there is a path in R from a to b, then there is such a path with length not exceeding n.

if $a \neq b$ and there is a path in R from a to b, then there is such a path with length not exceeding $(n-1)$.

$$M_{R^*} = M_R \vee M_R^{[2]} \vee M_R^{[3]} \vee \cdots \vee M_R^{[n]}$$

Warshall's Algorithm

$$W_{ij}^{[k]} = w_{ij}^{[k-1]} \vee (w_{ik}^{[k-1]} \wedge w_{kj}^{[k-1]})$$

Equivalence relations: reflexive, symmetric, and transitive

Equivalence classes: the set of ~~A is~~ all elements that are related to an element a of A is called the equivalence class of a.

Set collections of set partition S.

I. no missing

II. all elements is in S

III. $\emptyset$ not allowed

§9.6 Partial Orderings

§ total ordering

partial ordering if it is reflexive, antisymmetric and transsitive

(S, R)
set relation

comparable if either $a \preceq b$ or $b \preceq a$.
neither $a \preceq b$ nor $b \preceq a$, then a and b are called incomparable

Lexicographic Order —字典序

Hasse Diagram

1) start with the directed graph
2) remove all loops
3) remove the transitive edges

the inverse of relation R R^{-1}

the complement of the relation R. $\overline{R}$

如果$(S, \leq)$是偏序集，且S的每对元素都是可比的，则S叫全序集或线序集