$\neg\forall x P(x) \equiv \exists x \neg P(x)$

$\neg\exists x P(x) \equiv \forall x \neg P(x)$

$\exists! x$ Exactly one student

1.5 Nest Quantifiers

Two quantifiers are nested if one is within the scope of the other quantifiers

1.6 Prenex Normal Form

1. Eliminate all occurrences of $\rightarrow$ and $\leftrightarrow$ from the formula in question;
2. Move all negations inward such that, in the end, negations only appear as part of literals
3. Rename the variables (when necessary)
4. The prenex normal fom can now be obtained by moving all quntifiers to the front of the formula

Modus ponens 假言推理

$$\begin{array}{l} p \\ p\rightarrow q \\ \hline q \end{array}$$

Modus tollens 取拒式

$$\begin{array}{l} \neg q \\ p\rightarrow q \\ \hline \neg p \end{array}$$

Hypothetical Syllogism 假言三段论

$$\begin{array}{l} p\rightarrow q \\ q\rightarrow r \\ \hline p\rightarrow r \end{array}$$

Disjunctive syllogism 析取三段论

$$\begin{array}{l} p\vee q \\ \neg p \\ \hline q \end{array}$$

Addition 附加

$$\begin{array}{l} p \\ \hline p\vee q \end{array}$$

Simplification 化简

$$\begin{array}{l} p\wedge q \\ \hline p \end{array}$$

Conjunction 合取

$$\begin{array}{l} p \\ q \\ \hline p\wedge q \end{array}$$

Resolution 消解

$$\begin{array}{l} p\vee q \\ \neg p\vee r \\ \hline q\vee r \end{array}$$

Chapter 2

The power set

2^A or $P(A)$ power set of A

cartesian product

$A\times B = \{(a,b) \mid a\in A \wedge b\in B\}$

$A\neq B \equiv A\times B \neq B\times A$

$|A\times B| = |A| \times |B|$

元素个数

Boolean Algebra

Union $\cup$

Intersection $\cap$

Difference $-$

Complement $\sim$ (补集)

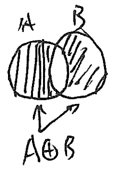
$|A\cup B| = |A| + |B| - |A\cap B|$

Symmetric Difference

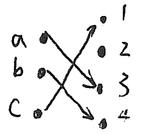
$A\oplus B = (A-B)\cup(B-A)$

$A_1\cup A_2\cup\cdots\cup A_n = \bigcup_{i=1}^{n} A_i$

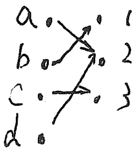
$A_1\cap A_2\cap\cdots\cap A_n = \bigcap_{i=1}^{n} A_i$



one to one functions 一对一



onto function 满射



floor function

$\lfloor \frac{1}{2} \rfloor = 0 \quad \lfloor -\frac{1}{2} \rfloor = -1 \quad \lfloor -4.3 \rfloor = -5$

Ceiling function

$\lceil \frac{1}{2} \rceil = 1 \quad \lceil -\frac{1}{2} \rceil = 0$

Factorial Function

$f(n) = n!$

Partial Function

Example $f: N\rightarrow R$ where $f(n)=\sqrt{n}$ is a partial function from Z to R where the domain of definition is the set of nonegative integers

2.5 Cardinality of Sets

Show that the set of real numbers is uncontable

1. suppose R is countable
2. The real numbers ~~由~~ between 0 and 1 can be listed in order $r_1, r_2 r_3 \cdots$
3. $r_1 = 0.d_{11}d_{12}\cdots$
   $r_2 = 0.d_{21}d_{22}\cdots$
   $r_3 = 0.d_{31}d_{32}\cdots$
4. ~~Fima~~ Form a new real number r
   $d_i = \begin{cases} 3 & \text{if } d_{ii}\neq 3 \\ 4 & \text{if } d_{ii}=3 \end{cases} \qquad r = 0.d_1d_2\cdots$
5. it is different from all the real numbers had been l

2.6 Matrices

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$$A\odot B = \begin{bmatrix} (1\wedge 1)\vee(0\wedge 0) & (1\wedge 1)\vee(0\wedge 1) & (1\wedge 0)\vee(0\wedge 1) \\ (0\wedge 1)\vee(1\wedge 0) & (0\wedge 1)\vee(1\wedge 1) & (0\wedge 0)\vee(1\wedge 1) \\ (1\wedge 1)\vee(0\wedge 0) & (1\wedge 1)\vee(0\wedge 1) & (1\wedge 0)\vee(0\wedge 1) \end{bmatrix}$$