10.7 Planar Graphs

Euler's formula $v-e+r=2$

If G is a connected planar graph with e edges and v vertices where $v \ge 3$, then

$$e \le 3v-6$$

degree of a region : the number of edge on the boundary of the region

KURATOWSKI'S THEOREM

Elementary subdivision: remove $\{u,v\}$, add w, $\{u,w\}$, $\{w,v\}$

$G_1=(V_1,E_1)$ $G_2=(V_2,E_2)$ are homeomorphic ⇔ They can obtained from the same graph by a sequence of elementary ~~sb~~ subdivision

homeomorphic to K_5, $K_{3,3}$

10.8 Graph Coloring

The Four color Theorm

K_n needs n color

$K_{m,n}$ needs 2 color

C_n needs 3 color

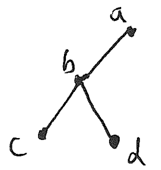
Chap 11 Tree

A tree is a connected undirected graph with no simple circuit. A Forest is a undirected graph with no simple circuit.

An undirected graph is a tree if and only if there is a unique simple path between any two of its vertices

~~Sim~~ specify a vertex as root.

Then, direct each edge away from root.



c's parent b

c's ansestors a,b

a's descendants b c d

level of c , 3

Height 3

leaves c d

The subtree at vertex v

if every internal vertex has more than m. children it is called m-ary tree.

An full m-ary tree is that every internal vertex has exactly m children.

Balanced

if all leaves are at levels H or H-1

A tree with n vertices has n-1 edges

$v-e+r=2$

$n-n+1+r=2$

$r=1$

A full m ary tree with i internal vertex contains $n=mi+1$ vertex

m ary

i internal   n vertices $i=\frac{n-1}{m}$ internal vertices

l leaves $l=\frac{(m-1)n+1}{m}$ leaves

i internal vertices $n=mi+1$

$l=(m-1)i+1$ leaves

$n=i+l$

There are at most m^h leaves in an m-ary tree of height h

$h \ge \lceil \log_m l \rceil$

full and balanced

$h=\lceil \log_m l \rceil$

2^H in a binary tree of height H

binary search tree algorithm

prefix codes

a simple graph is connected if and only if it has a spanning tree