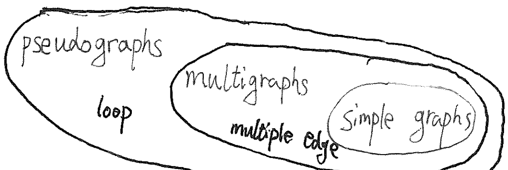
10 Graphs

10.1 Introduction of Graph

TYPE	EDGES	MULTIPLE EDGES ALLOWED	Loops ALLOWED
Simple graph	Undirected	no	no
Multigraph	undirected	yes	no
pseudograph	undirected	yes	yes
directed graph	directed	no	yes
directed multigraph	directed	yes	yes

Undirected Graphs

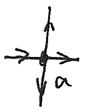


10.2 Graph Terminology

Degree of a vertex   loop contributes twice to the degree of that vertex.

Handshaking Theorem

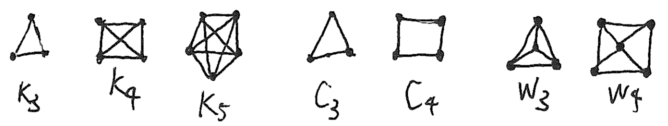
I. $\sum deg(v)=2e$ II. $\sum deg^+(v) = \sum deg^-(v) = e$



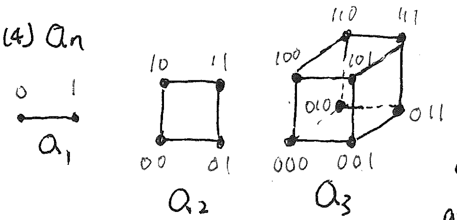
$deg^-(a)=1$

$deg^+(a)=3$

(1) K_n (2) C_n (3) W_n (wheels)



(4) Q_n



Vertex: 2^n

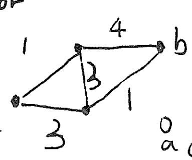
edges: $\sum deg(v) = 2|E| \quad 2^{n-1} n$

$= n 2^{n-1}$

(Shaking Hands Theorem)

If G is a simple graph with n vertices with $n \geq 3$ such that the degree of every vertex in G is at least $\frac{n}{2}$, then G has a Hamilton circuit

If G is a simple graph with n vertices with $n \geq 3$ such that $deg(u)+deg(v) \geq n$ for



Bipartite graphs

C_6, K_3,

Complete Bipartite graphs $K_{n,m}$

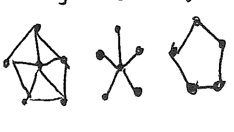
Sub graph

C_5, K_5,

C_5 is a subgraph of K_5

Union

W_5 S_5 C_5



学术单词

Pseudograph	伪图
terminology	用辞
adjacent	邻近的
bipartite	二分图
spanning	生成的
isomorphism	同构
bijection	双射
component	元件

10.3 representing graph and graph isomorphism

Adjacency Matrix

$a_{ij} = \begin{cases} 1 & \text{if } (v_i, v_j) \text{ is an edge in } G \\ 0 & \text{otherwise} \end{cases}$

Isomorphism of graphs.

$\forall a,b \in V_1$, $(a,b) \in E_1$

$\Longleftrightarrow (f(a), f(b)) \in E_2$

The same number of vertices

The same number of edges

The same degree of vertices

Check the compliment of the graph

10.4 connectivity

1. Path of length n.

Circuit, path is simple

strangly connected   a path from a to b and b to a

weakly connected   a path between any two vertices

$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ $A^4 = \begin{bmatrix} 8 & 0 & 0 & 8 \\ 0 & 8 & 8 & 0 \\ 0 & 8 & 8 & 0 \\ 8 & 0 & 0 & 8 \end{bmatrix}$

10.5 Euler and Hamilton Paths

G has Euler Circuit $\Rightarrow$ Every vertex in V has even degree

G has an Euler path but not an Euler circuit

I. G connected

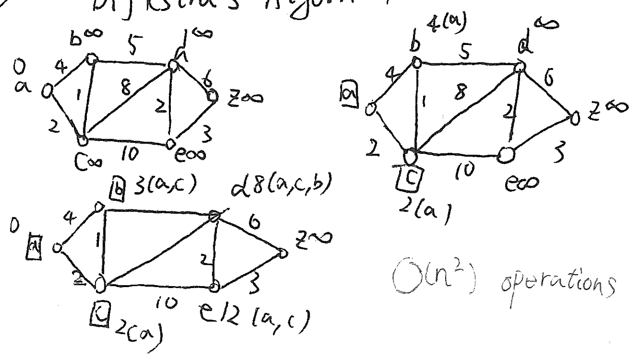
II. has exactly two vertices of odd degree.

K_n has a Hamilton circuit

If G is a connected simple graph with n vertices where $n \geq 3$, then G has a Hamilton circuit if the degree of each vertex is at least $n/2$

10.6 Shortest Path Problem

Dijkstra's Algorithm



$O(n^2)$ operations