数字逻辑

Chap 1 Digital Systems and Information

1.1 signal — Binary values

Analog (模拟信号) continuous: value & time

Time sequence signal:

用物理属性代表二进制例子：(见右侧列表)

CPU — Voltage
Disk — Magnetic Field Direction
CD — Suface Pits/Light
Dynamic RAM — Electrical Charge

1.2 Digital System

Discrete inputs → 各类离散系统状态 → Discrete output

Type: ① Combination Logic system
(1) No state represent   用于: multivariate multi-output F(x)
(2) $f_{output} = Func(Input)$
② Sequential System
(1) state present; state update at discrete times Synchronous | any time Asynchronous
(2) $f_{state} = Func(State, Input)$
$f_{output} = Func(State)$ or $Func(State, Input)$

1.3 Computer Architecture

① memory ② Datapath (BUS) ③ control unit ④ CPU ⑤ I/O device

1.4 进制相关计算:

① BCD adder (当每个BCD和超过9时，该BCD位置加6修正)

例: $448+489 = (0100\ 0100\ 1000)_{BCD} + (0100\ 1000\ 1001)_{BCD}$

1000 1101 0001
+ 0110 0110
1001 0011 0111 = 937

② 二、八、十、十六进制相互转化

2→10 权展开+进制相加

$(110\ 0101.101)_2 = 2^6+2^5+2^2+2^0+2^{-1}+2^{-3} = (813.625)_{10}$

10→2 整数除2取余，小数乘2取整

例: $(725.678)_{10}$

$2\times0.678 = 1.356$ 1
$2\times0.356 = 0.712$ 0
$2\times0.712 = 1.424$ 1
$2\times0.424 = 0.848$ 0
$2\times0.848 = 1.696$ 1

2|725 1
2|362 0
2|181 1
2|90 0
2|45 1
2|22 0
2|11 1
2|5 1
2|2 0
1

$(725)_{10} = (10\ 1101\ 0101)_2$

$(0.678)_{10} = (0.1010\ 1101\ 1001)_2$ can not be accurately converted

1.5 coding

Given n digits in radix r, there are r^n distinct elements that can be represented

几大有用编码: (不在表中的还有 Unicode & ASCII)

Decimal	8421	Excess 3	8,4,-2,-1	Gray
0	0000	0011	0000	0000
1	0001	0100	0111	0100
2	0010	0101	0110	0101
3	0011	0110	0101	0111
4	0100	0111	0100	0110
5	0101	1000	1011	0010
6	0110	1001	1010	0011
7	0111	1010	1001	0001
8	1000	1011	1000	1001
9	1001	1100	1111	1000

偶校验: 补1bit 使全bit中1的个数为偶数，否则补0

odd—奇 even—偶 parity → an extra bit appended onto the code word to make the number of 1's even

Chap 2 Boolean algebra

2.1 Binary Logic and Gates

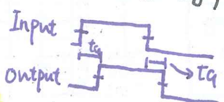
And: $Z = X\cdot Y = XY = X\wedge Y$

Or: $Z = X+Y = X\vee Y$

Not: $Z = \bar{X}$

xy	m_0	m_1	m_2	m_3	M_0	M_1	M_2	M_3
00	1	0	0	0	0	1	1	1
01	0	1	0	0	1	0	1	1
10	0	0	1	0	1	1	0	1
11	0	0	0	1	1	1	1	0

Gate Delay: the output change doesn't occur instantaneously, The delay is denoted by t_G

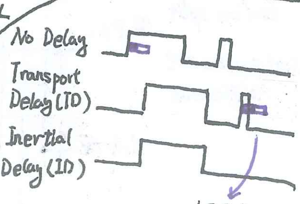


Propagation delay: time for a change Input → Output

low-to-high t_{PLH} high-to-low t_{PHL} 在高低间50%处设立时间点，测量

Transport delay: 输出在经过一个固定特定时间后响应输入的改变.

Inertial delay: 与 transport delay 相同，除了 input 两次变化时间小于 rejection time 的情况(此时不变)



Truth table 独一无二 但 expression 与 logic digrams 则非如此

2.2 Basic concepts of Boolean algebra

$F = f(X_1, X_2, \cdots X_n)$ single output / multi-output

基本等式:

德摩根: $\overline{X+Y} = \bar{X}\cdot\bar{Y}$ $\overline{X\cdot Y} = \bar{X}+\bar{Y}$

$A(A+B) = A$ $A+AB = A$

$A(\bar{A}+B) = AB$ $A+\bar{A}B = A+B$

$(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$

$AB+\bar{A}C+BC = AB+\bar{A}C$

$F_1 = F_2 \Leftrightarrow$ Truth table is completely identical

Complementing funtion (反函数): $\bar{F}$

例: $F = \bar{A}B + C\bar{D} \Rightarrow \bar{F} = (A+\bar{B})(\bar{C}+D)$

Duality rules (对偶规则): AND、OR swap; 0、1 swap; Variable unchanged

Shannon formula

$x f(x,\bar{x},y,\cdots,z_n) = x f(1,0,y,\cdots z)$

$\bar{x} f(x,\bar{x},y,\cdots,z_n) = \bar{x} f(0,1,y,\cdots z)$

$x + f(x,\bar{x},y,\cdots,z_n) = x + f(0,1,y,\cdots z)$

$\bar{x} + f(x,\bar{x},y,\cdots z_n) = \bar{x} + f(1,0,y,\cdots z)$

Shannon expansion

$F(x,\bar{x},Y\ldots z) = x f(1,0,y,\cdots z) + \bar{x} f(0,1,y,\cdots z)$
$= [x+f(0,1,y,\cdots z)][\bar{x}+f(1,0,y,\cdots z)]$

Uses for the XOR: 1. The controllable source / inverted output $A\oplus 0 = A$ $A\oplus 1 = \bar{A}$
2. Digital comparator $A\oplus A = 0$ $A\oplus\bar{A} = 1$
3. half adder Truth table without Carry

NOR:

$F\odot F = T$
$T\odot F = F$
$F\odot T = F$
$T\odot T = T$

2.3 Logic Functions Simplification

2.3.1 An application of Boolean algebra

Contain the smallest number of literals

The most simple expression is not unique

① AND-OR style

"AND-item" 尽可能少，前提是每个item的变量越少越好 $F = A + CD$ (or item, and item)

② OR-AND style

"OR-item" 尽可能少 OR-AND ←duality→ AND-OR

2.4 Standard forms / Canonical Forms (范式)

① Sum of Minterms (SOM) ② Product of Maxterms (POM)

Minterms (最小项): 所有变量都以原变量或反变量形式出现(仅只出现一次)

例: 对于X, Y: XY, $X\bar{Y}$, $\bar{X}Y$, $\bar{X}\bar{Y}$ 为最小项 对n个变量有2^n个最小项

性质: (1) 所有最小项同时只有一个值为1 (2) 两最小项之积为0 $m_i\cdot m_j = 0$
(3) 所有最小项之和 SOM = 1 (4) 任何不在F(x)中的最小项均可现于反函数$\bar{F}$中

Maxterms (最大项): 所有变量都以原变量或反变量形式出现(仅只出现一次)

性质: (1) 所有最大项同时只有一个值为1 (2) 两最大项之和为1 $M_i + M_j = 1$
(3) 所有最大项之积 POM = 0

两者关系: (1) M_i 与 m_i 互反 $m_3 = \bar{A}BC$ $M_3 = A+\bar{B}+\bar{C}$ $m_3 = \overline{M_3}$
(2) Σm_i and ΠM_i is each other Duality
(3) $F = \Sigma m_i = \overline{\Pi M_i}$

记号 $F_{min}(X,Y,Z) = m_0+m_2+m_5+m_7 = \Sigma(0,2,5,7)$

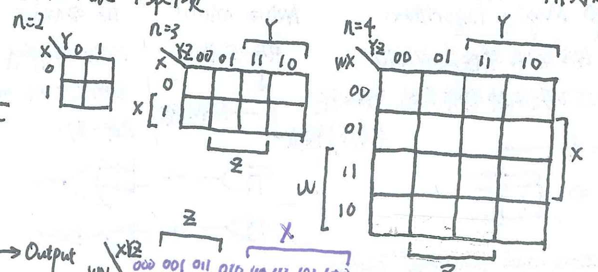
$F_{max}(X,Y,Z) = M_1\cdot M_3\cdot M_4\cdot M_6 = \Pi(1,3,4,6)$

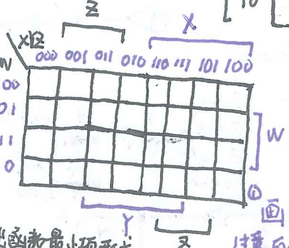
SOP: $ABC+\bar{A}\bar{B}C+B$ POS: $(A+B)(A+\bar{B}+\bar{C})C$

This form often can be simplified (so corresponding circuit is simpler)

2.5 Karnaugh map of Function (卡诺图函数)

每一格代表一个最小项 排列试类似格雷码





1-dimensional block can eliminate one variable, and N-dimensional elimination N variable

① 画K-map时尤其注意X在3列 注意反一下，容易错 从上到下3行

② 画圈时注意4角

画圈也可以圈0，这样得到的是$\bar{F}$ 取反即可

(1) 写出函数最小项形式
(2) 画出K-map
(3) 画圈，尽可能多包含最小项 不要遗漏，尽可能避免重复
(4) 化简

Incompletely Specified Function

Don't care term: (1) not appear in the combination input (2) for some combination of inputs, there is an uncertain output

K-map上有些格会为X，可解释为1 也可解释为0