# Discrete Mathematics and Its Applications

## Chapter 1

### 1.1 propositional Logic

| | | | |
|---|---|---|---|
| Negation | (NOT) | $\neg$ | |
| Conjunction | (AND) | $\wedge$ | |
| Disjunction | (OR) | $\vee$ | |
| Exclusive or | (XOR) | $\oplus$ | (when exactly one of p and q is true) |
| Implication | (if-then) | $p \to q$ | (q unless p, p only if q) |
| Biconditional | (if and only if) | $p \leftrightarrow q$ | |

$p \to q$

$q \to p$ converse

$\neg q \to \neg p$ contrapositive

$\neg p \to \neg q$ inverse

precedence of logical operators

| Operator | precedence |
|---|---|
| $\neg$ | 1 |
| $\wedge$ | 2 |
| $\vee$ | 3 |
| $\to$ | 4 |
| $\leftrightarrow$ | 5 |

bit operations

### 1.3 propositional Equivalences

tautology $p \vee \neg p$ always true

contradiction $p \wedge \neg p$ always false

De Morgan's Laws

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Distributive laws

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Absorption Laws

$p \vee (p \wedge q) \equiv p \quad p \wedge (p \vee q) \equiv p$

Exportation Law

$(p \wedge q) \to r \equiv p \to (q \to r)$

Absurdity Law

$(p \to q) \wedge (p \to \neg q) \equiv \neg p$

$p \wedge q \equiv \neg(p \to \neg q)$

$\neg(p \to q) \equiv p \wedge \neg q$

$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

Propositional Normal Forms

two types:

disjunctive normal form (DNF)

conjunctive normal form (CNF)

如果公式A与一个由基本和之积组成的公式等价 $A \Leftrightarrow A_1 \wedge A_2 \wedge A_3 \cdots A_n$ 合取范式

$A \Leftrightarrow A_1 \vee A_2 \vee \cdots \vee A_n$ 为析取范式.

(1) $\leftrightarrow$ $\to$ 化为 $\wedge, \vee, \neg$

(2) De Morgan's Law

(3) 化简

极小项, 命题变元出现且均只出现一次, 基本积为极小项 主析取范式

| 极小项 | | | 基本和~~最~~极大项 极大项 主合取范式 | | |
|---|---|---|---|---|---|
| $\neg P \wedge \neg Q \wedge \neg R$ | 000 | 0 | $P \vee Q \vee R$ | 000 | 0 |
| $\neg P \wedge \neg Q \wedge R$ | 001 | 1 | $P \vee Q \vee \neg R$ | 001 | 1 |

$(P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R) \vee (\neg P \wedge Q \wedge R)$

记成 $\Leftrightarrow \Sigma 1,3,6,7 \Leftrightarrow \prod 0,2,4,5 \Leftrightarrow (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$

(1) 化归为析取范式、合取范式

(2) 除去 $P \wedge \neg P$, $P \vee \neg P$

(3) 幂等律 $P \wedge P$, $P \vee P$ 增减变元

### 1.4. Predicates and Quantifiers

$\forall x P(x)$

$\exists x Q(x)$

precedence of Quantifiers

$\forall x P(x) \vee Q(x) \equiv (\forall x P(x)) \vee Q(x)$

$\forall x (A(x) \wedge B(x)) \equiv \forall x A(x) \wedge \forall x B(x)$

(1) $\forall x P(x) \vee A \equiv \forall x (P(x) \vee A)$

(2) $\forall x P(x) \wedge A \equiv \forall x (P(x) \wedge A)$

(3) $\exists x P(x) \vee A \equiv \exists x (P(x) \vee A)$

(4) $\exists x P(x) \wedge A \equiv \exists x (P(x) \wedge A)$

(5) $\forall x (A \to P(x)) \equiv A \to \forall x P(x)$