1 # Supporting Information

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3 # Unveiling carbon emission flows along sale chains

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18 ## Content

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39   **1. Carbon emission accounts**
40   Since the research focuses on sectoral carbon emissions in domestic production,
41   the import items are removed from the MRIO table to isolate the domestic supply
42   chains in China (Zhang et al., 2017). Then the fundamental equation of input-output
43   table (consisting of n economic sectors) can be expressed as equation

44    $$\mathbf{x}' = \mathbf{v}'(\mathbf{I}-\mathbf{B})^{-1} = \mathbf{v}'\mathbf{G} \tag{1}$$

45    $$\mathbf{x} = (\mathbf{I}-\mathbf{A})^{-1}\mathbf{y} \tag{2}$$

46   where $\mathbf{x}$ represents an $n \times 1$ matrix of total output, $\mathbf{y}$ is an $n \times 1$ matrix of final
47   demand, $'$ represents the inverse of the matrix and $\mathbf{v}$ is an $1 \times n$ matrix of value
48   added. $\mathbf{A}$ is the $n \times n$ direct-input coefficients (technical coefficients) matrix and
49   an element A_{ij} measures direct input from sector i to satisfy unit output of sector
50   j; I is the $n \times n$ identity matrix; $\mathbf{L}$ is the $n \times n$ Leontief inverse and an element
51   L_{ij} of $\mathbf{L}$ measures total input of sector i to satisfy the unit output from sector j.
52    $\mathbf{B}$ is the $n \times n$ direct-output coefficients (allocation coefficients) matrix and an
53   element B_{ij} represents the distribution of sector i's output to sector j that
54   purchase inputs from sector i. $\mathbf{G}$ is the Ghosh inverse and an element G_{ij} measures
55   the total value comes about in sector j per unit of primary input in sector i (Miller
56   and Blair, 2009).
57   The environmental extension of the model introduces an $1 \times n$ intensity vector
58    $\mathbf{f}$ representing direct emissions per unit output. Embodied emission intensity
59    $\mathbf{m} = \mathbf{f}(\mathbf{I}-\mathbf{A})^{-1}$ represents the embodied emissions per unit output and an element m_q
60   measures emissions from all sectors that have become embodied in unit output from
61   sector q. Enabled emission intensity $\boldsymbol{\delta} = (\mathbf{I}-\mathbf{B})^{-1}\mathbf{f}$ measures enabled emissions per
62   unit of input and an element δ_q measures emissions from all sectors induced by per
63   unit input of sector q.

64    $$\mathbf{S} = \mathbf{v}(\mathbf{I}-\mathbf{B})^{-1}\hat{\mathbf{f}}' \tag{3}$$

65    $$\mathbf{E} = \hat{\mathbf{f}}(\mathbf{I}-\mathbf{A})^{-1}\mathbf{y} \tag{4}$$

66   Then the enabled emissions and embodied emissions of various sectors can be
67   given by equation (3)(4). Embodied emissions include direct emissions (emissions
68   during production activities along supply chains) and indirect emissions (emissions
69   induced by intermediate demand along supply chains). Enabled emissions mean the
70   emissions pulled by primary input and intermediate input along the sale chains.
71

72   **2. Attributing emissions to primary input**
73   Ghosh inverse matrix captures the effect of sale chains by describing both direct
74   and indirect outputs from various sectors enabled by primary input of particular
75   sectors. The advantage of Ghosh model is the ability to trace the emission paths
76   through various layers pulled by primary input, which can be expanded based on
77   Taylor series approximation as equation (5)：

78    $$(\mathbf{I}-\mathbf{B})^{-1} = \mathbf{B}^0 + \mathbf{B}^1 + \mathbf{B}^2 + \cdots + \lim_{t \to \infty}\mathbf{B}^t \tag{5}$$

79 Material flows of arbitrary sectors at arbitrary production layers driven by
80 primary or intermediate inputs can be evaluated by referring to the equations of Table
81 1. If the involved production layers are greater than 4, material flows driven by
82 primary or intermediate inputs from sector s_1 at PL^m to sector s_i at PL^n can be
83 assessed by the following equation：

84
$$S^{m \to n}_{s_1,s_2,\cdots s_{i-1},s_i} = v \ \overset{m-1}{\overbrace{\vec{B}}} \ B_{:s_1} B_{s_1 s_1} \cdots B_{s_{i-1},s_i} B_{s_i} \tag{6}$$

85 where $i = n - m + 1$ and $n - m \geq 3$;
86

3. Attributing emissions to final consumption

88 Leontief inverse matrix captures the effect along supply chains by depicting both
89 direct and indirect inputs from various sectors required to satisfy the final demand of
90 products from particular sectors. The advantage of Leontief model is the ability to
91 trace the emission paths through various layers instigated by final demand. The
92 Leontief inverse can be decomposed into contributions from all supply chains, by
93 unravelling the Leontief inverse using its series expansion as equation (7)：

94
$$(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{A}^0 + \mathbf{A}^1 + \mathbf{A}^2 + \cdots + \lim_{t' \to \infty} \mathbf{A}^{t'} \tag{7}$$

95 Material flows of arbitrary sectors at arbitrary production layers pulled by final
96 or intermediate demands can be evaluated by referring to the equations of Table 2. If
97 the involved production layers are greater than 4, material flows pulled by the final or
98 intermediate demands from sector s_1 at $PL^{m'}$ to sector s_i at $PL^{n'}$ can be assessed
99 by the following equation：

100
$$E^{m' \to n'}_{s_1,s_2 \cdots s_{i-1},s_i} = m_{s_1} A_{s_1,s_2} \cdots A_{s_{i-1},s_i} A_{s_{1:}} \ \overset{n'-1}{\overbrace{\vec{A}}} \ y \tag{8}$$

101 where $i = m' - n' + 1$ and $m' - n' \geq 3$.

4. Emission attributions along supply chains

103 Along supply chains, direct emission flows (DE) and embodied emission flows
104 (E) instigated by intermediate or final demands can also be traced at multiple
105 production layers.
106 Figure S1 depicts embodied emission flows and direct emission flows pulled by
107 the intermediate or final demands along the supply chains. P_i represents final
108 production attributions of each sector, $E_i^{0'}$ measures final consumption attributions
109 of each sector and $'$ is the reverse of a matrix. The figure elaborates: direct emissions
110 from each sector at $PL^{0'}$ to $PL^{3'}$ - $DE_i^{0'}$, $DE_j^{1'}$, $DE_k^{2'}$, and $DE_l^{3'}$, respectively;
111 embodied emissions from each sector at $PL^{0'}$ to $PL^{3'}$ - $E_i^{0'}$, $E_j^{1'}$, $E_k^{2'}$ and $E_l^{3'}$,
112 respectively; embodied emission flows between sectors from $PL^{0'}$ to $PL^{1'}$, from $PL^{1'}$
113 to $PL^{2'}$, from $PL^{2'}$ to $PL^{3'}$ are $E_{ij}^{0' \to 1'}$, $E_{jk}^{1' \to 2'}$, and $E_{kl}^{2' \to 3'}$. Similarly, there exist
114 infinite production layers. The sum of final consumption attributions is equal to the
115 sum of final production attributions, i.e.

116
$$\mathbf{fy} + \mathbf{fAy} + \mathbf{fA^2y} + \cdots = \sum_{i=1}^{n} E_i^{0'} = \sum_{i=1}^{n} P_i \tag{9}$$

5. Intermediate, final consumption and production attributions along supply chains

Final consumption attributions of multi-production layers pulled by final demand along supply chains can be expressed as：

$$\mathbf{f}(\mathbf{I}-\mathbf{A})^{-1}\mathbf{y} = \mathbf{f}\mathbf{A}^0\mathbf{y} + \mathbf{f}\mathbf{A}^1\mathbf{y} + \mathbf{f}\mathbf{A}^2\mathbf{y} + \cdots + \lim_{t'\to\infty}\mathbf{f}\mathbf{A}^t\mathbf{y} \tag{10}$$

For sectoral production attributions at arbitrary production layers, an element $DE_i^{t'}$ represents direct emissions released by sector i at $PL^{t'}$. For instance, $DE_i^{2'} = f_i\mathbf{A}_{i:}\mathbf{A}\mathbf{y}$ measures direct emissions released by sector i at $PL^{2'}$ in producing output to meet the intermediate demand $\mathbf{A}_{i:}\mathbf{A}\mathbf{y}$ along the supply chains. For consumption attributions, an element $E_i^{t'}$ represents emissions from all sectors embodied in the output of sector i at $PL^{t'}$. For instance, $E_i^{2'} = m_i\mathbf{A}_{i:}\mathbf{A}\mathbf{y}$ measures emissions from all sectors embodied in sector i at $PL^{2'}$ to meet the intermediate demand $\mathbf{A}_{i:}\mathbf{A}\mathbf{y}$. Embodied emission intensity $\mathbf{m} = \mathbf{f}(\mathbf{I}-\mathbf{A})^{-1}$ represents the embodied emissions per unit output and an element m_q measures emissions from all sectors that have become embodied in unit output from sector q.

Flows of embodied emissions between sectors from arbitrary production layers can also be derived. Equations in Table S5 are displayed to illustrate the emission from all sectors that have become embodied in the output of a given sector to meet intermediate or final demand. Specifically, $E_{ji}^{1'\to 0'}$, $E_{kj}^{2'\to 1'}$, and $E_{lk}^{3'\to 2'}$ measure emissions embodied in the flow of intermediate products between sectors at adjacent layers. For instance, $E_{ji}^{1'\to 0'}$ represents emissions from all sectors embodied in the output of sector j at $PL^{1'}$ purchased by sector i at $PL^{0'}$; $E_{kj}^{2'\to 1'}$ measures emissions from all sectors embodied in the output of sector k at $PL^{2'}$ purchased by sector j at $PL^{1'}$ to meet final demand from all sectors at $PL^{0'}$. The remaining equations provide more detailed extensions. For instance, $E_{kji}^{2'\to 0'}$ measures emission from all sectors embodied in the output of sector k at $PL^{2'}$ purchased by sector j at $PL^{1'}$ to meet the final demand from sector i at $PL^{0'}$. There are n^3 nodes evaluated by $E_{kji}^{2'\to 0'} = m_k A_{kj} A_{ji} y_i$ denoting the path from "$k \to j \to i$".

146 **6. The practical significance of the application**

147   Structure path analysis is an input-output technique to trace and scan entire sale

148 and supply chains in order to extract and rank those that are significant in terms of

149 environmental impacts. For instance, sale chain "labor→ technical service→ coal

150 mining→ electricity-generation" has three sectors and two paths. It can be written as a

151 structural path $v_{LT}B_{TC}B_{CE}f_E$, where B_{TC} is the sales coefficient of the technical

152 services provider to the coal mine sector, B_{CE} is the sales coefficient of technical

153 service provider to coal mine sector, f_E is the emission intensity of electricity-

154 generation sector. Supply chain "technical services→ coal mining→ electricity-

155 generation→ household" also has three intermediate sectors and two paths.[1-3] It can

156 be written as a structural path of $m_T A_{TC} A_{CE} y_{EH}$, where m_T is the emission intensity

157 of technical-service provider, A_{TC} is the requirement coefficient of coal mine for

158 technical service, and A_{CE} is the requirement coefficient of power plant for coal.
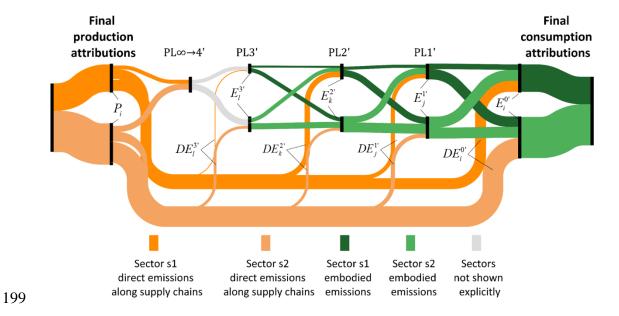
159   In the context of interconnected web of sale and supply chains, everyone is a

160 supplier and a customer at the same time. This is true from the perspective of a

161 corporation (buying primary and intermediate inputs, and selling intermediate and

162 final outputs), as well as from that of a household (buying final outputs and selling

163 primary inputs). Any effort to reduce emissions implies designing policies that

164 allocate responsibilities to actors involved in causing these emissions. There is an

165 implied balance in the relationship between supplier and customer: the supplier has

166 the power to make decisions about to whom it sells (to downstream); the customer has

167 the power to decide from whom it buys (from upstream). Every organisation is both a

168 supplier and a purchaser, and will therefore have both sets of responsibilities.[4,5]

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170 **7. Nomenclature**
171   Definition of the related proper nouns are listed as follows:
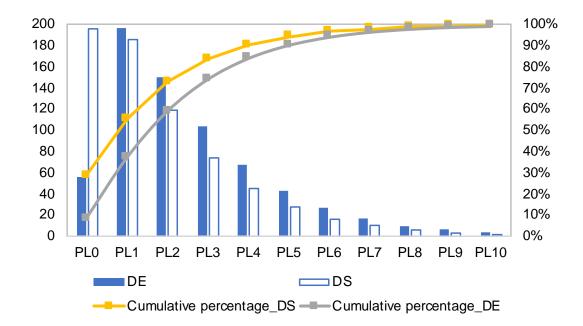172   · *carbon emission attribution* quantifies the emissions virtually embodied in
173   products purchased by final demand, or the emissions driven by the primary
174   input;
175   · *enabled emission* is direct and indirect emissions driven by primary input;
176   · *embodied emission* refers to direct and indirect emissions pulled by final
177   demand;
178   · *primary input* is a commodity that is not produced using something else (for
179   example labor, or capital such as land and resources);
180   · *intermediate inputs and outputs (demand)* are commodities (for example
181   coal) that are traded between companies in order to produce something else
182   (for example electricity);
183   · *final output* is a commodity (for example household electricity) that is not
184   used to produce something else;
185   · *suppliers* can be sellers of primary inputs into production (household
186   electricity), such as households (as workers), or they can be sellers of
187   intermediate inputs (coal), such as companies (power plant);
188   · Customers can be buyers of final outputs of production (household
189   electricity), such as households (as consumers), or they can be buyers of
190   intermediate outputs (coal), such as companies (power plant);
191   · a sale chain is a succession of buyers and sellers, starting with a primary
192   input (for example labor for coal mine), and ending in an emitting
193   intermediate buyer (power plant);
194   · a supply chain is a succession of buyers and sellers, starting with an emitting
195   intermediate seller (coal mines), and ending in a final output (household
196   electricity).
197

198

Figure S1. Illustrative map of direct emission flows (pulled by intermediate or final demand along supply chains) and embodied emission flows for a simple two-sector economy.

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Figure S2. Direct emissions driven by intermediate or final demand along the supply
chains (DE) and direct emissions pulled by primary and intermediate input along the
sale chains (DS) (Unit: Mt). The corresponding cumulative percentage at different
production layers.

210

211   **Table S1.** Abbreviations and classifications of sectors.

212   **Table S2.** Abbreviation of provinces and sectors.

213   **Table S3.** Ranking 1000 top of enabled emission paths.

214   **Table S4.** Ranking 1000 top of embodied emission paths.
215
216   Notably, Table S1-Table S4 could be referred in the document: Supporting
217   Information (.xlsx).

218   **Table S5.** Intermediate and final consumption attributions between adjacent or non-

219   adjacent production layers.

	to sector at $PL^{0'}$	to sector at $PL^{1'}$	to sector at $PL^{2'}$
from sector at $PL^{1'}$	$E_{ji}^{1'\to 0'} = m_j A_{ji} y_i$	-	-
from sector at $PL^{2'}$	$E_{kji}^{2'\to 0'} = m_k A_{kj} A_{ji} y_i$	$E_{kj}^{2'\to 1'} = m_k A_{kj} \mathbf{A_{j:}y}$	-
from sector at $PL^{3'}$	$E_{lkji}^{3'\to 0'} = m_l A_{lk} A_{kj} A_{ji} y_i$	$E_{lkj}^{3'\to 1'} = m_l A_{lk} A_{kj} \mathbf{A_{j:}y}$	$E_{lk}^{3'\to 2'} = m_l A_{lk} \mathbf{A_{k:}Ay}$

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References

(1) Lenzen, M. Structural Path Analysis of Ecosystem Networks. Ecol. Modell. **2007**, 200 (3–4), 334–342. https://doi.org/10.1016/j.ecolmodel.2006.07.041.

(2) Lenzen, M.; Kanemoto, K.; Moran, D.; Geschke, A. Mapping the Structure of the World Economy. Environ. Sci. Technol. **2012**. https://doi.org/10.1021/es300171x.

(3) Liang, S.; Wang, Y.; Zhang, T.; Yang, Z. Structural Analysis of Material Flows in China Based on Physical and Monetary Input-Output Models. J. Clean. Prod. **2017**, 158, 209–217. https://doi.org/10.1016/j.jclepro.**2017**.04.171.

(4) Liang, S.; Qu, S.; Zhu, Z.; Guan, D.; Xu, M. Income-Based Greenhouse Gas Emissions of Nations. Environ. Sci. Technol. **2017**, 51 (1), 346–355. https://doi.org/10.1021/acs.est.6b02510.

(5) Mi, Z.; Zhang, Y.; Guan, D.; Shan, Y.; Liu, Z.; Cong, R.; Yuan, X. C.; Wei, Y. M. Consumption-Based Emission Accounting for Chinese Cities. Appl. Energy **2016**. https://doi.org/10.1016/j.apenergy.2016.06.094.