Appendix: Kinematics

A state is a tuple $s_i = (p_i, v_i, o_i)$ where $p_i \in \mathbb{R}^2$ is a position (point) denoted $p_i = (p_i^x, p_i^y)$, $v_i \in [v_{\min}, v_{\max}]$ is a velocity with $v_{\min}, v_{\max} \in \mathbb{R}^{\geq 0}$ such that $v_{\min} \leq v_{\max}$, and $o_i \in (-\pi, \pi]$ is an orientation (in radians). Let $b \in \mathbb{R}^{\geq 0}$ be a wheelbase constant. The positions of front and rear wheels $f_i, r_i \in \mathbb{R}^2$ in state $s_i = (p_i, v_i, o_i)$ are:

$$f_i^x = p_i^x + \frac{b}{2} \cos o_i \tag{4}$$

$$f_i^y = p_i^y + \frac{b}{2} \sin o_i \tag{5}$$

$$r_i^x = p_i^x - \frac{b}{2} \cos o_i \tag{6}$$

$$r_i^y = p_i^y - \frac{b}{2} \sin o_i \tag{7}$$

A body has two effectors: throttle and steering. An action is a tuple $a_i = (t_i, e_i)$ where $t_i \in [t_{\min}, t_{\max}]$ is a throttle with $t_{\min}, t_{\max} \in \mathbb{R}$ such that $t_{\min} \leq t_{\max}$, and $e_i \in [e_{\min}, e_{\max}]$ is a steering angle (in radians) with $e_{\min}, e_{\max} \in (-\frac{\pi}{2}, \frac{\pi}{2})$ such that $e_{\min} \leq e_{\max}$. Let $\lambda \in \mathbb{R}^{>0}$ be a time resolution constant. If action $a_i = (t_i, e_i)$ is executed in state $s_i = (p_i, v_i, o_i)$ such that $e_i = 0$, then the successor state is $s_{i+1} = (p_{i+1}, v_{i+1}, o_{i+1})$ where:

$$p_{i+1}^x = p_i^x + v_i \lambda \cos o_i \tag{8}$$

$$p_{i+1}^y = p_i^y + v_i \lambda \sin o_i \tag{9}$$

$$v_{i+1} = \min\{v_{\max}, \max\{v_{\min}, v_i + t_i \lambda\}\} \tag{10}$$

$$o_{i+1} = o_i \tag{11}$$

If action $a_i = (t_i, e_i)$ is executed in state $s_i = (p_i, v_i, o_i)$ such that $e_i \neq 0$, then the successor state is $s_{i+1} = (p_{i+1}, v_{i+1}, o_{i+1})$ where:

$$c_i^x = r_i^x - \frac{b}{\tan e_i} \sin o_i \tag{12}$$

$$c_i^y = r_i^y + \frac{b}{\tan e_i} \cos o_i \tag{13}$$

$$\theta_i = \frac{\operatorname{sgn}(e_i) v_i \lambda}{\sqrt{(c_i^x - p_i^x)^2 + (c_i^y - p_i^y)^2}} \tag{14}$$

$$p_{i+1}^x = c_i^x + (p_i^x - c_i^x) \cos \theta_i - (p_i^y - c_i^y) \sin \theta_i \tag{15}$$

$$p_{i+1}^y = c_i^y + (p_i^x - c_i^x) \sin \theta_i + (p_i^y - c_i^y) \cos \theta_i \tag{16}$$

$$v_{i+1} = \min\{v_{\max}, \max\{v_{\min}, v_i + t_i \lambda\}\} \tag{17}$$

$$o_{i+1} = \operatorname{arctan2}(\sin(o_i + \theta_i), \cos(o_i + \theta_i)) \tag{18}$$

The point c_i is the centre of rotation for the given state-action pair (with non-zero steering action) and θ_i is the corresponding turn angle. The body kinematics specified by Equations 4–18 are illustrated in Figure 6.

$$\theta(o_i, o_{i+1}) = \operatorname{arctan2}\left(\sin\left(o_{i+1} - o_i\right), \cos\left(o_{i+1} - o_i\right)\right) \tag{19}$$

$$e(\theta) = \operatorname{sgn}(\theta) \arctan\left(2b\sqrt{\frac{\theta^2}{4v^2\lambda^2 - b^2\theta^2}}\right) \tag{20}$$

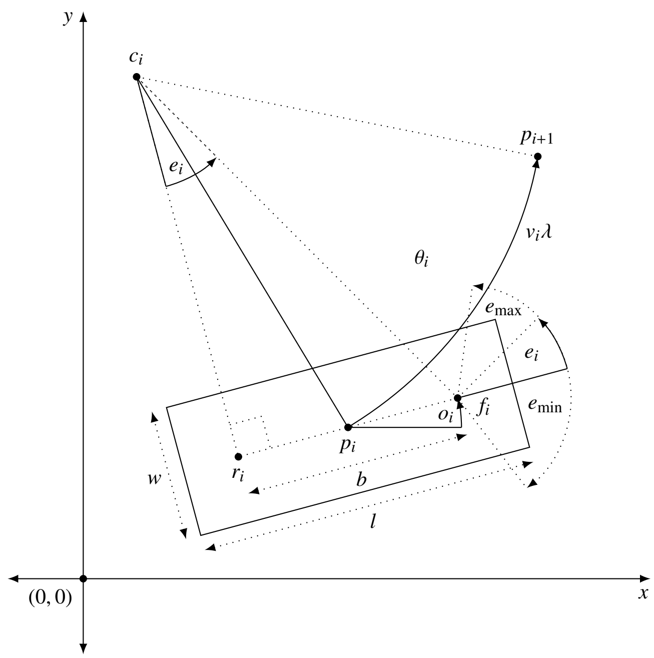

Figure 6: Body kinematics