

What is a category ?

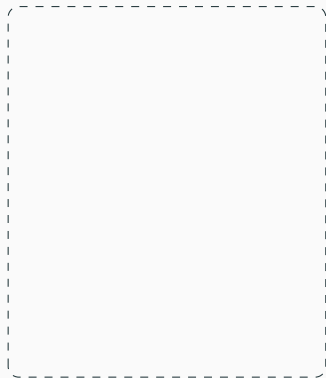
In animated pictures ...

Frédéric Peschanski

Sorbonne University – LIP6

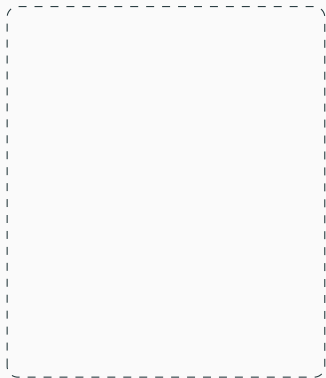
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What is a category ?



What is a category ?

Objects



What is a category ?

Objects

a



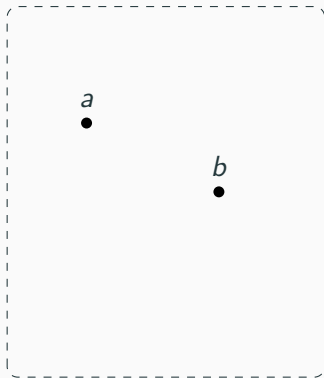
a



What is a category ?

Objects

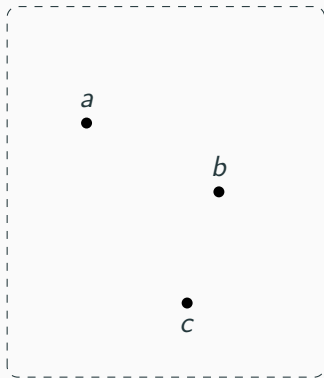
a b
● ●



What is a category ?

Objects

a b c ...
● ● ● ...

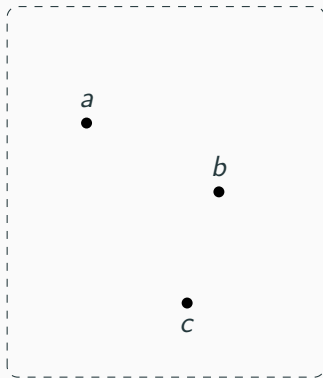


What is a category ?

Objects

a b c ...
• • • ...

Morphisms (arrows)



What is a category ?

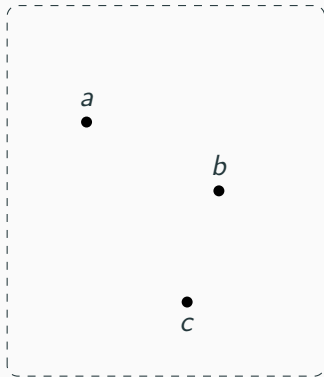
Objects

a b c ...
• • • ...

Morphisms (arrows)

a f b
• \longrightarrow •

$f :: a \longrightarrow b$



What is a category ?

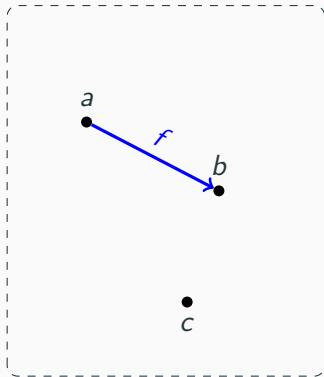
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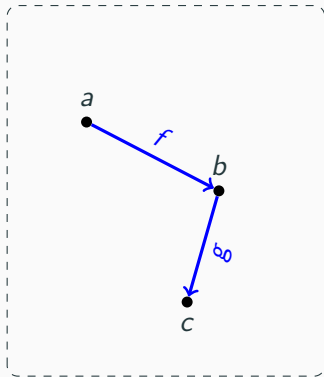
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Morphisms (arrows)



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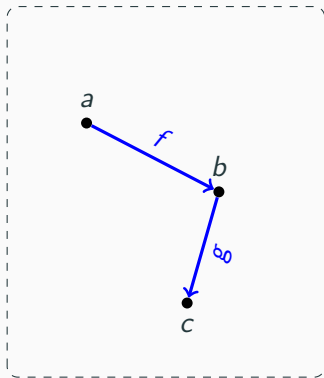
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Identities



What is a category ?

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a b c ...
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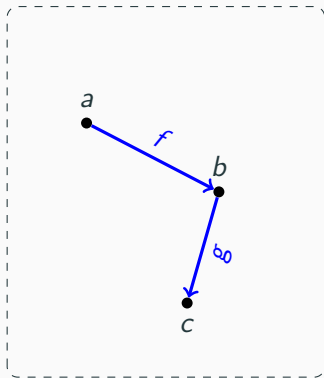
Morphisms (arrows)



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Identities

●
 a



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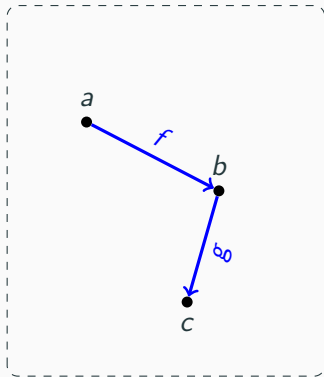
Morphisms (arrows)



$f :: a \longrightarrow b$

Identities

$\bullet \xrightarrow{id_a} \bullet$ $id_a :: a \longrightarrow a$
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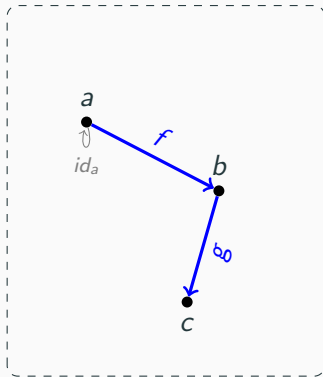
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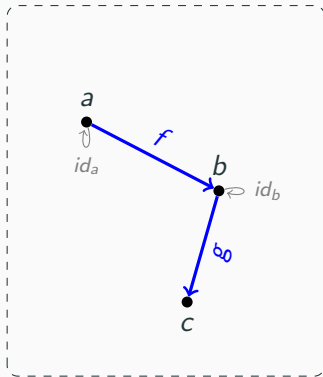
Morphisms (arrows)



$f :: a \longrightarrow b$

Identities

$\bullet \xrightarrow{id_a} a$ $id_a :: a \longrightarrow a$



What is a category ?

Objects

$a \quad b \quad c \quad \dots$

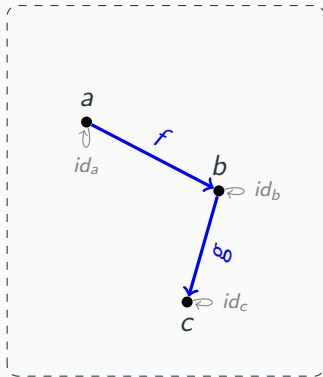
Morphisms (arrows)



$f :: a \longrightarrow b$

Identities

$\bullet \xrightarrow{id_a} a \quad id_a :: a \longrightarrow a$



What is a category ?

Objects

a b c ...
• • • ...

Morphisms (arrows)

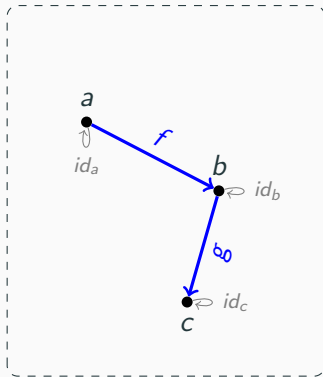


$f :: a \longrightarrow b$

Identities

$\bullet \curvearrowright id_a$ $id_a :: a \longrightarrow a$
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Compositions



What is a category ?

Objects

a b c ...
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Morphisms (arrows)

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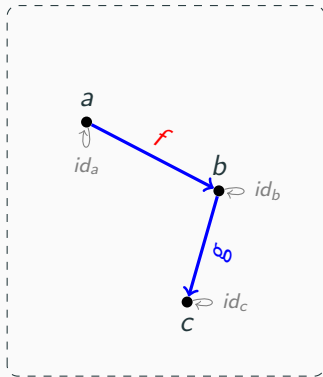
$f :: a \longrightarrow b$

Identities

$\bullet \xrightarrow{id_a} a$ $id_a :: a \longrightarrow a$

Compositions

if $f :: a \longrightarrow b$



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a b c ...
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Morphisms (arrows)

a f b
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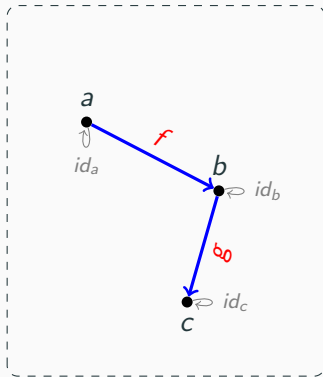
$f :: a \longrightarrow b$

Identities

$\bullet \xrightarrow{id_a} a$ $id_a :: a \longrightarrow a$

Compositions

if $f :: a \longrightarrow b$ and $g :: b \longrightarrow c$



What is a category ?

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a b c ...
• • • ...

Morphisms (arrows)

a \xrightarrow{f} b
• \xrightarrow{f} •

$f :: a \longrightarrow b$

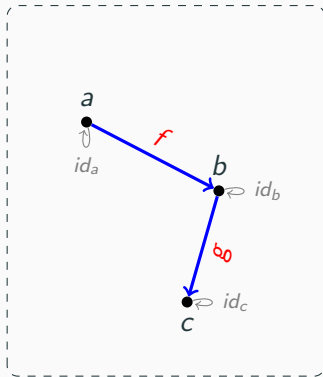
Identities

$\bullet \xrightarrow{id_a} a$ $id_a :: a \longrightarrow a$

Compositions

if $f :: a \longrightarrow b$ and $g :: b \longrightarrow c$

then $g \circ f :: a \longrightarrow c$



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a \xrightarrow{f} b

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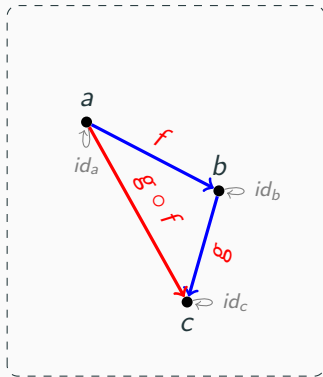
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if $f :: a \longrightarrow b$ and $g :: b \longrightarrow c$

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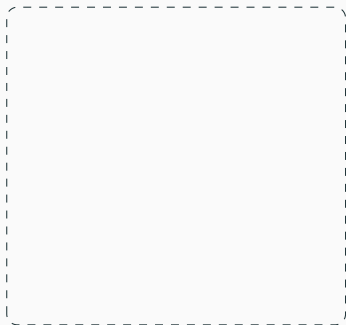


The laws of categories



The laws of categories

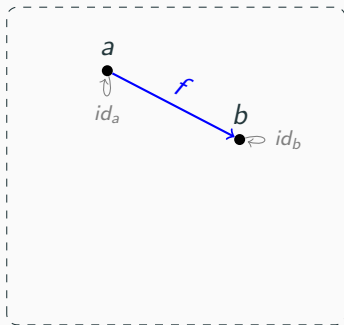
Laws of identity :



The laws of categories

Laws of identity :

For any $f :: a \longrightarrow b$,

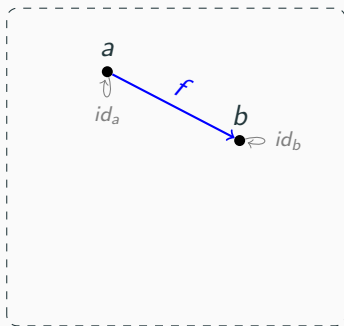


The laws of categories

Laws of identity :

For any $f :: a \longrightarrow b$,

- $f \circ Id_a = f$

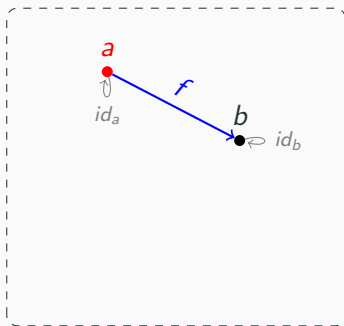


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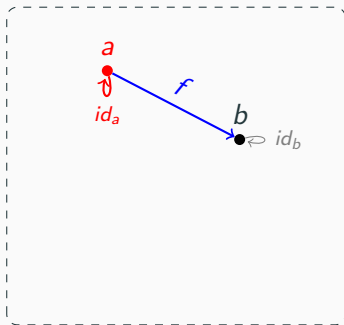


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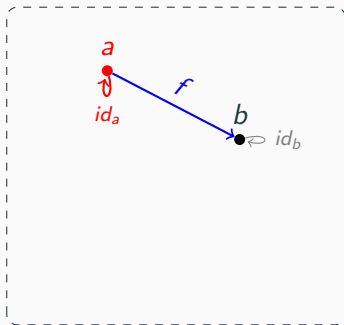


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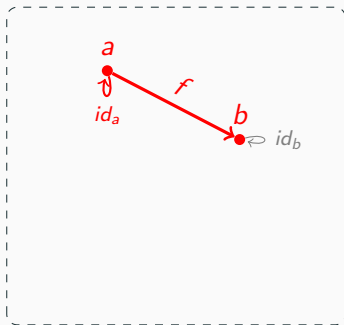


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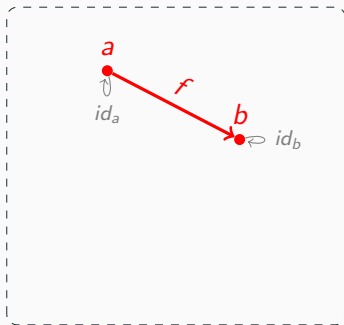


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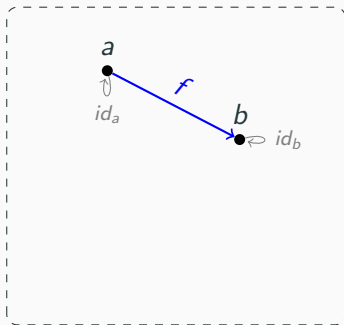


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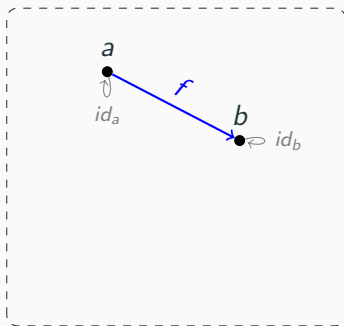


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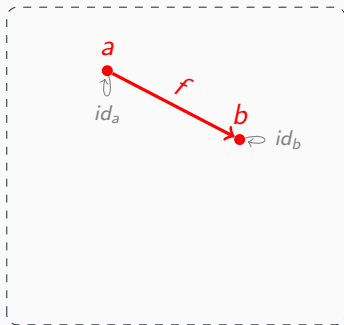


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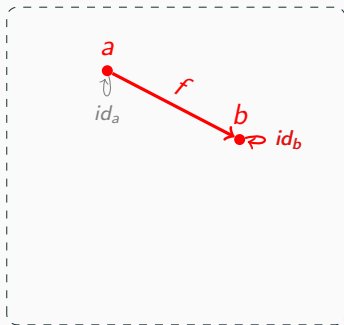


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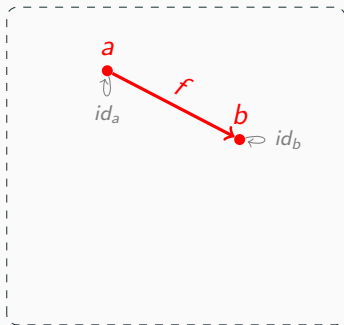


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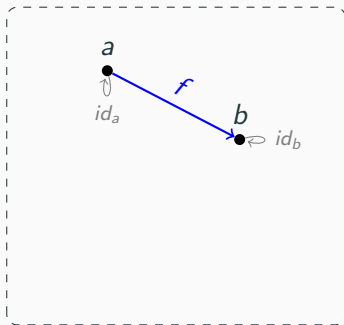
The laws of categories

Laws of identity :

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Law of associativity :



The laws of categories

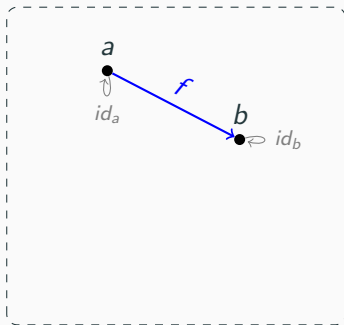
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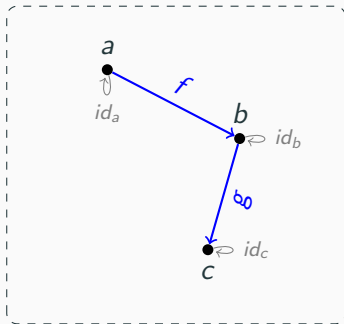
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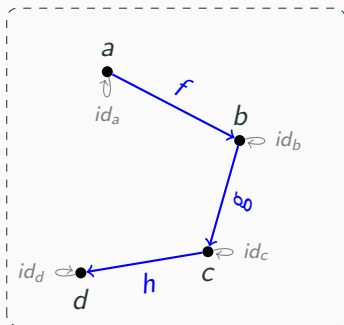
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The laws of categories

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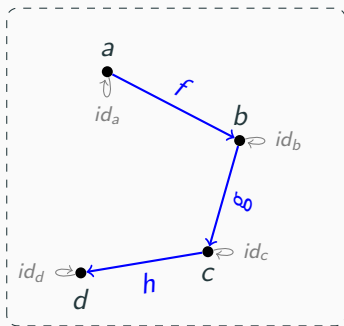
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For any $f :: a \longrightarrow b$,

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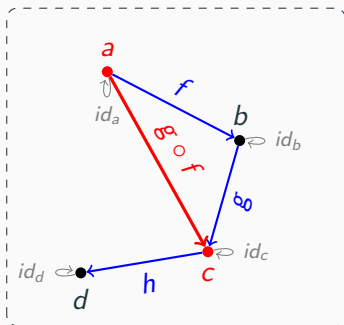
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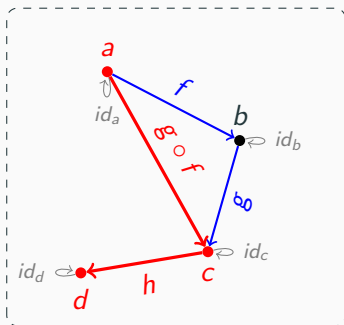
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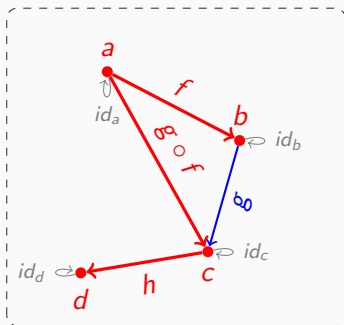
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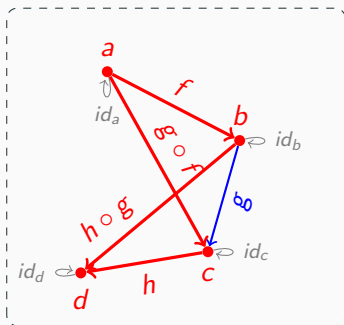
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Law of associativity :

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$h :: c \longrightarrow d$,

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