

## Different Repulsions

Name \_\_\_\_\_

$D$  = diameter of a sphere

$d$  = the distance between sphere centers

$F$  = the magnitude of the force between two spheres

Right now we are using a linear repulsion force.

$$F_1(D,d) = K(D - d)$$

This works really well but as the spheres compress into each other the amount of area that they are in contact with increases. If we think of this like the sphere had pressure we would get.

$A$  = the area of contact between two spheres

$P$  = the pressure in the spheres which we will hold constant

$$F_2(D,d) = P \cdot A$$

Here  $P$  kind of takes the place of  $K$  but we need to know how the area changes as  $d$  goes from  $D$  to zero. Find  $A$  as a function of  $D$  and  $d$ . (attach your work)

$$A(D,d) = \underline{\hspace{4cm}}$$

$$F_2(D,d) = P \cdot A(D,d) = \underline{\hspace{4cm}}$$

If we assume that the spheres are balls of ideal gasses (Which they are not! But let's go with it anyway.) then

$$PV = nRT$$

$V$  = volume of the sphere

$P$  = the pressure in the spheres which will now change

$n$  = how much stuff the spheres are made of

$R$  = ideal gas constant

$T$  = temperature in Kelvin

Let's assume that  $nRT$  is a constant (Which it is not because  $T$  would be changing. But let's go with it.)

$$\text{So } PV = C$$

$C$  = some constant that will kind of be like our  $K$  in the first (linear) force function and  $P$  in the second (quadratic) force function. You should set  $C$  here with an unaffected volume and pressure.  $C = P_0 V_0$

$$P = C/V$$

Assuming that the spheres lose the volume they had past the area of contact, find the volume remaining as a function of  $D$  and  $d$ . (attach your work)

$$V(D,d) = \underline{\hspace{10cm}}$$

Putting this all together we have

$$F_3(D,d) = P \cdot A = [C/V(D,d)] \cdot A(D,d) = \underline{\hspace{10cm}}$$

Now let's compare these functions.

Let  $D = 1$  and  $k = 1$  and graph the linear function below letting  $d$  go from zero to 1.

Let  $D = 1$  and  $P = 4/\pi$  and graph the quadratic function below letting  $d$  go from zero to 1.

Let  $D = 1$  and  $C = 1/6$  and graph the ideal gas function below letting  $d$  go from zero to 1.

What are the differences you see between the functions?

Note: These will be the force function as the spheres are contracting and we will simply multiply them by a number between 0 and 1 to adjust them to force functions going out if we want to make an inelastic collision.