Regelung eines Furuta Pendulums

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September 5, 2015

Mechatronics Lab
Summer 2015
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2.1 Furuata Pendel

Bei dem Furuta Pendel handelt es sich um ein 1992 von Katsuhisa Furuta entwickeltes nichtlineares Pendel...
Test, ob Umlaute untersttzt werden: Strae (Strasse) Huser (Haeuser)

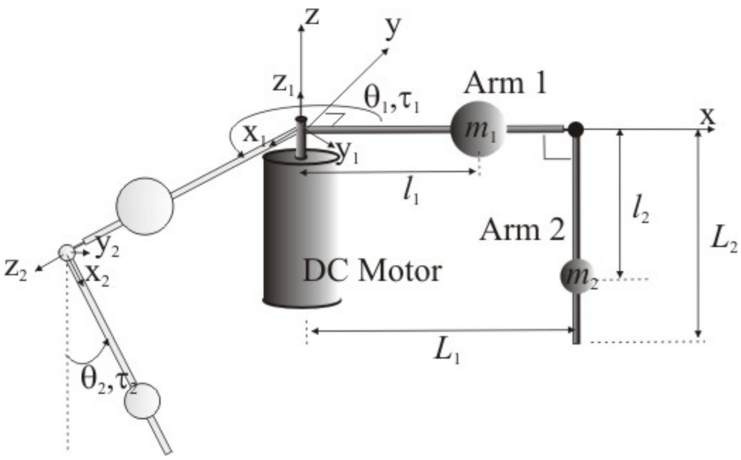

Figure 1: Furuta Pendel Quelle: Wiki

$$E_{p1} = 0 \tag{1}$$

$$E_{k1} = \frac{1}{2}(v_{1c}^T m_1 v_{1c} + \omega_1^T J_1 \omega_1) = \frac{1}{2}\dot{\theta}_1^2 (m_1 l_1 + J_{1zz}) \tag{2}$$

$$E_{p2} = g m_2 l_2 (\cos(\theta_2) - 1) \tag{3}$$

$$\begin{aligned} E_{k2} &= \frac{1}{2}(v_{2c}^T m_2 v_{2c} + \omega_2^T J_2 \omega_2) \\ &= \frac{1}{2}\dot{\theta}_1^2 (m_2 L_2^2 + (m_2 l_2^2 + J_{2yy}) \sin^2(\theta_2) + J_{2xx} \cos^2(\theta_2)) \\ &\quad + \frac{1}{2}\dot{\theta}_2^2 (J_{2zz} + m_2 l_2^2) + m_2 L_1 l_2 \cos(\theta_2) \dot{\theta}_1 \dot{\theta}_2 \end{aligned} \tag{4}$$

$$L = E_k - E_p \tag{5}$$

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_i}) + b_i \dot{q}_i - \frac{\partial L}{\partial q_i} = Q_i \tag{6}$$

$$\begin{bmatrix}\tau_1\\\tau_2\end{bmatrix}=\begin{bmatrix}\begin{pmatrix}\ddot{\theta}_1(J_{1zz}+m_1l_1^2+m_2L_1^2+(J_{2yy}+m_2l_2^2)\\\times\sin^2(\theta_2)+J_{2xx}cos^2(\theta_2))+\ddot{\theta}_2m_2L_1l_2\cos(\theta_2)\\-m_2L_1l_2\sin(\theta_2)\dot{\theta}_2^2+\dot{\theta}_1\dot{\theta}_2\sin(2\theta_2)\\\times(m_2l_2^2+J_{2yy}-J_{2xx})+b_1\dot{\theta}_1\end{pmatrix}\\\begin{pmatrix}\ddot{\theta}_1m_2L_1l_2\cos(\theta_2)+\ddot{\theta}_2(m_2l_2^2+J_{2zz})\\+\frac{1}{2}\dot{\theta}_1\sin(2\theta_2)(-m_2l_2^2-J_{2yy}+J_{2xx})\\+b_2\dot{\theta}_2+gm_2l_2\sin(\theta_2)\end{pmatrix}\end{bmatrix}\tag{7}$$

$$\begin{aligned}J_1&=\begin{bmatrix}J_{1xx}&0&0\\0&J_{1yy}&0\\0&0&J_{1zz}\end{bmatrix}\approx\begin{bmatrix}0&0&0\\0&J_1&0\\0&0&J_1\end{bmatrix}\\J_2&=\begin{bmatrix}J_{2xx}&0&0\\0&J_{2yy}&0\\0&0&J_{2zz}\end{bmatrix}\approx\begin{bmatrix}0&0&0\\0&J_2&0\\0&0&J_2\end{bmatrix}\end{aligned}\tag{8}$$

$$\begin{aligned}\hat{J}_1&=J_1+m_1l_1^2\\\hat{J}_2&=J_2+m_2l_2^2\\\hat{J}_0&=J_1+m_1l_1^2+m_2L_1^2\end{aligned}\tag{9}$$

$$\begin{bmatrix}\tau_1\\\tau_2\end{bmatrix}=\begin{bmatrix}\begin{pmatrix}\ddot{\theta}_1(\hat{J}_0+\hat{J}_2\sin^2(\theta_2))+\ddot{\theta}_2m_2L_1l_2\cos(\theta_2)\\-m_2L_1l_2\sin(\theta_2)\dot{\theta}_2^2+\dot{\theta}_1\dot{\theta}_2\hat{J}_2\sin(2\theta_2)+b_1\dot{\theta}_1\end{pmatrix}\\\begin{pmatrix}\ddot{\theta}_1m_2L_1l_2\cos(\theta_2)+\ddot{\theta}_2\hat{J}_2-\frac{1}{2}\dot{\theta}_1^2\hat{J}_2\sin(2\theta_2)\\+b_2\dot{\theta}_2+gm_2l_2\sin(\theta_2)\end{pmatrix}\end{bmatrix}\tag{10}$$

Die folgenden Gleichungen enthalten noch den Fehler, den wir finden sollten. Ich habe die Lsung gerade nicht parat.

$$\ddot{\theta}_1=\frac{\begin{bmatrix}-\hat{J}_2b_1\\m_2L_1l_2\cos(\theta_2)b_2\\-\hat{J}_2^2\sin(2\theta_2)\\-\frac{1}{2}\hat{J}_2m_2L_1l_2\cos(\theta_2)\sin(2\theta_2)\\\hat{J}_2m_2L_1l_2\sin(\theta_2)\end{bmatrix}^T\begin{bmatrix}\dot{\theta}_1\\\dot{\theta}_2\\\dot{\theta}_1\dot{\theta}_2\\\dot{\theta}_1^2\\\dot{\theta}_2^2\end{bmatrix}+\begin{bmatrix}\hat{J}_2\\-m_2L_1l_2\cos(\theta_2)\\\frac{1}{2}m_2^2l_2^2L_1\sin(2\theta_2)\end{bmatrix}^T\begin{bmatrix}\tau_1\\\tau_2\\g\end{bmatrix}}{\hat{J}_0\hat{J}_2+\hat{J}_2^2\sin^2(\theta_2)-m_2^2L_1^2l_2^2\cos^2(\theta_2)}\tag{11}$$

$$\ddot{\theta}_2=\frac{\begin{bmatrix}m_2L_1l_2\cos(\theta_2)b_1\\-b_2(\hat{J}_0+\hat{J}_2\sin^2(\theta_2))\\m_2L_1l_2\hat{J}_2\cos(\theta_2)\sin(2\theta_2)\\-\frac{1}{2}\sin(2\theta_2)(\hat{J}_0\hat{J}_2+\hat{J}_2^2\sin^2(\theta_2))\\-\frac{1}{2}m_2^2L_1^2l_2^2\sin(2\theta_2)\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\\dot{\theta}_2\\\dot{\theta}_1\dot{\theta}_2\\\dot{\theta}_1^2\\\dot{\theta}_2^2\end{bmatrix}+\begin{bmatrix}-m_2l_2\cos(\theta_2)\\\hat{J}_0+\hat{J}_2\sin^2(\theta_2)\\-m_2l_2\sin(\theta_2)(\hat{J}_0+\hat{J}_2\sin^2(\theta_2))\end{bmatrix}\begin{bmatrix}\tau_1\\\tau_2\\g\end{bmatrix}}{\hat{J}_0\hat{J}_2+\hat{J}_2^2\sin^2(\theta_2)-m_2^2L_1^2l_2^2\cos^2(\theta_2)}\tag{12}$$

$$
\begin{aligned}
\theta_{1e} &= 0 \\
\theta_{2e} &= \pi \\
\dot{\theta}_{1e} &= 0 \\
\dot{\theta}_{2e} &= 0
\end{aligned} \tag{13}
$$

$$
\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ B_{31} & B_{32} \\ B_{41} & B_{42} \end{bmatrix} \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \tag{14}
$$

$$
\begin{aligned}
A_{31} &= 0 \\
A_{32} &= \frac{g m_2^2 l_2^2 L_1}{(\hat{J}_0 \hat{J}_2 - m_2^2 L_1^2 l_2^2)} \\
A_{33} &= \frac{-b_1 \hat{J}_2}{(\hat{J}_0 \hat{J}_2 - m_2^2 L_1^2 l_2^2)} \\
A_{34} &= \frac{-b_2 m_2 l_2 L_1}{(\hat{J}_0 \hat{J}_2 - m_2^2 L_1^2 l_2^2)} \\
A_{41} &= 0 \\
A_{42} &= \frac{g m_2 l_2 \hat{J}_0}{(\hat{J}_0 \hat{J}_2 - m_2^2 L_1^2 l_2^2)} \\
A_{43} &= \frac{-b_1 m_2 l_2 L_1}{(\hat{J}_0 \hat{J}_2 - m_2^2 L_1^2 l_2^2)} \\
A_{44} &= \frac{-b_2 \hat{J}_0}{(\hat{J}_0 \hat{J}_2 - m_2^2 L_1^2 l_2^2)} \\
B_{31} &= \frac{\hat{J}_2}{(\hat{J}_0 \hat{J}_2 - m_2^2 L_1^2 l_2^2)} \\
B_{41} &= \frac{m_2 L_1 l_2}{(\hat{J}_0 \hat{J}_2 - m_2^2 L_1^2 l_2^2)} \\
B_{32} &= \frac{m_2 L_1 l_2}{(\hat{J}_0 \hat{J}_2 - m_2^2 L_1^2 l_2^2)} \\
B_{42} &= \frac{\hat{J}_0}{(\hat{J}_0 \hat{J}_2 - m_2^2 L_1^2 l_2^2)}
\end{aligned} \tag{15}
$$

$$
\tau = K_m i \tag{16}
$$

Herleitung der Parameter (1 zu 1 aus dem Dokument bernommen.. die Herangehensweise muss also noch verndert werden!):

$$\begin{aligned} L_1 &= l_a \\ L_2 &= l_{pm} \end{aligned} \tag{17}$$

$$\begin{aligned} m_1 &= m_b + m_r + m_{sens} \\ m_2 &= m_p + m_{pm} \end{aligned} \tag{18}$$

$$J_{arm} = m_r \frac{l_r^2}{12} + m_r(\frac{1}{2}l_r + l_b - l_a)^2 \tag{19}$$

$$J_{pend1} = m_b l_a^2 \tag{20}$$

$$J_{sens} = m_{sens}(l_a - l_b - l_r - \frac{1}{2}l_c)^2 \tag{21}$$

$$J_{ps} = m_p(r_b + \frac{1}{2}l_p)^2 \tag{22}$$

$$J_{pm} = m_{pm} l_{pm}^2 \tag{23}$$

$$J_{arm} + J_{pend1} + J_{sens} = m_1 l_1^2 J_{pm} + J_{ps} = m_2 l_2^2 \tag{24}$$

$$l_1 = \sqrt{\frac{m_r \frac{l_r^2}{12} + m_r(\frac{1}{2}l_r + l_b - l_a)^2 + m_b l_a^2 + m_{sens}(l_a - l_b - l_r - \frac{1}{2}l_c)^2}{m_1}} \tag{25}$$

$$l_2 = \sqrt{\frac{m_{pm} l_{pm}^2 + m_p(r_b + \frac{1}{2}l_p)^2}{m_2}} \tag{26}$$

... Hier sollten wir gucken, ob wie die Umformungen evtl. anders vornehmen.

$$Listeder Zahlenwerte \tag{27}$$

$$\tag{28}$$

$$\tag{29}$$

Swing-Up

$$U = n \cdot g \cdot sign(E - E_0)\dot{\theta}_2 \cos(\theta_2) \tag{30}$$

$$E = E_{pot} + E_{kin} \tag{31}$$

$$\begin{aligned} E_{pot} &= m_2 g l_2 (\cos(\theta_2) - 1) \\ E_{kin} &= \frac{1}{2} \dot{\theta}_2^2 \hat{J}_2 \end{aligned} \tag{32}$$

$$\tag{33}$$

3 Identifikation der Parameter

4 Design der Controller

5 Ergebnisse