Assignment 1

Due on 12:00 a.m. of Sep. 17

1. Please apply the *trapezoidal rule* to compute the integral of the function $4\cos(x) - 2$ over the interval $[0, \pi]$. Compare your result with its analytical answer and discuss the error due to step size.

2. Consider the function $f(x) = 4x^3 + 7x^2 + 2x + 9$. Please apply at least two numerical differentiation schemes to calculate the first derivative of $f(x)$ in the interval [0, 1]. Plot the results and discuss its errors based on the exact $\frac{df(x)}{dx}$ due to step size.

3. $\frac{dy}{dx} = y - 3x^2 + 1,\ y(0) = 0.5$. Please use the *Euler method* to solve the following initial value problem on the interval [0, 0.1] with a step size $h = 0.005$. Then compare your results with the analytical answer.

4. $\frac{dy}{dx} = y - 3x^2 + 1,\ y(0) = 0.5$. Please use *the fourth-order Runge-Kutta method* to solve the following initial value problem on the interval [0, 0.1] with a step size $h = 0.005$. Then compare your results with the ones of Problem 3.

5. Please find the real root of the equation $f(x) = 3\cos(x) + 0.1e^x - 2$ by using one of your favorite root-finding methods to locate the two roots of $f(x)$ that are nearest to $x = 2$.

6. Consider the function $f(x) = 3\sin x + \frac{1}{2}e^x$. Please use polynomial interpolation with proper orders to estimate the values of the function based on points that are evenly spaced within the range of -1 to 1. In addition, analyze how the error is distributed for different numbers of interpolation points with various polynomial orders.