Improvement about Re-Tiling Polygonal Surfaces

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This paper presents a method of creating triangle meshes at several levels of detail, based on the parametric manifolds of original mesh.This method is an improvement on the result of Greg Turk[1997].Greg Turk proposed a important method named *Re-Tiling*, to generate different detail meshes from an original mesh.But his method have two defects: (1)His method is suitable only for relatively smooth meshes. The output mesh created by his method,may lose the sharp features of inpute mesh; (2)When we generate high-detail models using Re-tiling,there may be more than four points coplanar on the output meshes.This is undesirable situation.So in this paper we have settled these problems by adjusting the Re-Tiling algorithm and using parametric manifolds.

Additional Key Words and Phrases: Triangle mesh,manifold,Re-Tiling,sharp features.

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1 INTRODUCTION

Representing models at various levels of detail is important for achieving high frame rates in interactive graphics application and also for speeding-up the off-line rendering of complex scenes[Greg Turk,1992].One benefit is that we can use low-detail model during rendering if the object our mesh represented just cover a small portion of the screen.So we can increase the rendering rate,especially important for interactive application.Another benefit of having different detail representation of one mesh is that this is a good way to avoid sampling problems when rendering an image.

The method named *Re-Tiling*,proposed by Greg Turk[1992],settle this kind of problems well. But his method has two defects:(1)His method is suitable only for smooth models,so the output mesh may lose the features when input mesh has sharp features;(2)When we generate high-detail models using Re-tiling,there may be more than four points coplanar on the output meshes.This is undesirable situation.In this paper,we propose our methods to settle the two defects.

The main idea of Re-Tiling is to remove old points on original mesh and use new points placed over mesh to generate a new mesh. The new points are uniformly distributing on original mesh after being moved. To settle the first defect,we adjust the algorithm proposed by Greg Turk. When we remove the old points,we can retain the old points with high curvature, so the output mesh will maintain the features of original mesh. But this way may cause the remanent points are not evenly distributing on mesh,we will get a output mesh with poor quality. We will add a *constrain points set* in this paper to overcome this contradiction. For settling the second defect,we use the *parametric pseudo-manifold*[M.Siqueira et al. 2009].Since pseudo-manifold is smooth surface in three dimensional space, and it has the similar features with original mesh. So it would not appear four-point coplanar situation locally. This defect is settled also.

We make the following contributions in this paper:

- We adjust the Re-tiling algorithm, and make the output mesh maintain similar features with original mesh.
- By using the parametric pseudo-manifold,we improved the Re-tiling algorithm.

2 PREVIOUS WORK

There are many methods to generate new meshes that users need from input mesh.Greg Turk's[1992] method named Re-Tiling, can create different detail meshes for users,but his method can not maintain shape features of original mesh.The quadric error metric(*QEM*), proposed by Garland et al. [1997],is a representative work.This algorithm is efficient and can maintain high fidelity to the original mesh.Liu et al' method [2015] is also based on the QEM framework,but it takes a different strategy for collapse.This method is faster than QEM and maintaining a similar level of accuracy.

The first manifold-based construction for surface modeling was proposed by Grimm et al.[1995]. After some time,Grimm et al.[2006] introduced in detail the manifold-based techniques in his paper. M. Siqueira et al.[2009]introduced a new manifold construction for fitting surfaces of arbitrary smoothness to triangle meshes. In this paper,we will use the M. Siqueira et al.'s work to settle the defect in Re-Tiling.

3 RE-TILING ALGORITHM

3.1 Overview

Greg Turk[1992] proposed an automatic method of creating surface models at several levels of detail from an original polygonal description of a given object,named Re-Tiling.This method include several steps.

The Re-Tiling begins by having a given number of *new points*(specified by the user)placed randomly over the surface of the triangle mesh. Once all the points have been randomly placed on the surface, a relaxation procedure is applied to move each point away from all other nearby points. The basic operation of this relaxation procedure is to fold or project nearby points onto a plane tangent to the surface at one point, to calculate the repelling force that each

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ALGORITHM 1: Re-Tiling algorithm

Input: a triangular mesh,the number of output mesh points m.
Place m new points randomly over the surface of the inpute mesh.
for *nLoop* $\leftarrow 1$ **to** k **do**
  for *each new point q on mesh* **do**
    determine nearby points to q;
    map these nearby points onto the plane containing the triangle of q;
    compute and store the repulsive forces that the mapped points exert on q;
  end
  for *each new point q on mesh* **do**
    compute the new position of q based on the repulsive forces;
  end
end
generate an intermediate triangle surface that incorporates both the old vertices of the original surface and the new points.
remove old vertices that satisfy the removal condition one by one,and triangulate the resulting hole.
Output: a new triangular mesh with m points.

nearby point has on the given point and then to move this point over the triangular mesh based on the force exerted against it. A point that is pushed off one polygon is moved onto an adjacent polygon. In this paper,we use triangle meshes as input meshes,so the polygon means triangle. The final step is to remove old vertices that satisfy the removal condition one by one.

The Re-Tiling algorithm is summarized as follows:

3.2 Improvement 1

As mentioned above, the mesh obtained by Re-Tiling algorithm can not matain sharp features of original mesh(see Fig.1 and Fig.2). We introduce a points set, named constrained points set(*CSP set*),to modify the Re-Tiling algorithm. CSP set consists of the original mesh points with high curvature. And readers can refer to M.Botsch et al's book[2010] for calculating curvature of mesh points.

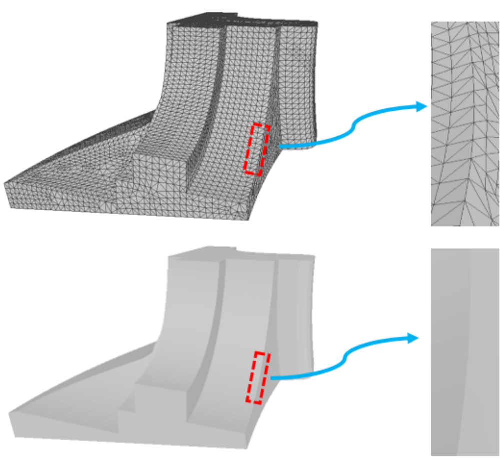

Fig. 1. The input mesh and its local features.

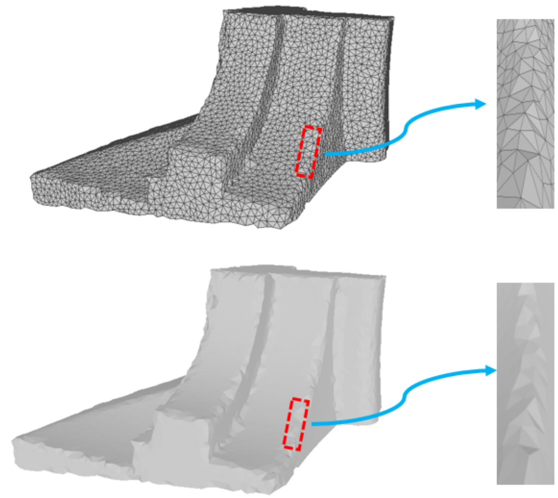

Fig. 2. The Re-Tiling result of input mesh and its local features.

Now we adjust Re-Tiling algorithm. When we do Re-Tiling ,calculating the repulsive force, we not only calculate the force from nearby new points, but also calculate the force coming from the CSP set. In the algorithm, computing the new position, we need not to move the CSP positions. By doing these adjustment, the high curvature points on the inpute mesh are ramained, so the output mesh retain more features of the input mesh than Re-Tiling algorithm result.

Because we also compute the repulsive force of CSP against new points, the points of output mesh with CSP and new points,are still uniformly distributing.

4 PARAMETRIC PSEUDO-MANIFOLDS

4.1 Overview

M.Siqueira et al.[2009] introduced a new manifold-based construction for fitting surfaces of arbitrary smoothness to triangle meshes. And their construction combines, in the same framework, most of the best features of previous constructions.

Let's start with a brief introduction to the Parametric pseudo-manifolds.

Definition 4.1. Let n and d be two integers with $n > d \geqslant 1$ and let k be integer with $k \geqslant 1$ or $k = \infty$. A *parametric C^k pseudo-manifold* of dimension d in $\mathbb{R}^n$, $\mathscr{M}$, is a pair

$$\mathscr{M} = (\mathscr{G}, (\theta_i)_{i\in I})$$

where $\mathscr{G} = \left((\Omega_i)_{i\in I}, (\Omega_{ij})_{(i,j)\in I\times I}, (\varphi_{ji})_{(i,j)\in K}\right)$ is a set of gluing data, for some finite set I, and each θ_i is a C^k function, $\theta_i : \Omega_i \rightarrow \mathbb{R}^n$, called a parametrization such that
$\theta_i = \theta_j \circ \varphi_{ji}$
for all $(i, j) \in K$. The subset, $M \subset \mathbb{R}^n$, given by
$M = \bigcup_{i\in I} \theta_i (\Omega_i)$
is called the image of the parametric pseudo-manifold, $\mathscr{M}$. When

$d = 2$ and $n = 3$ in definition, we call $\mathscr{M}$ a *parametric pseudo-surface*(*PPS*).

Reader can refer to M.Siqueira et al.[2009] paper for what is gluing data.

Our goal is to fit a surface, $S \in \mathbb{R}^3$, to a triangle mesh $\mathscr{T}$. More specifically, we want to build a surface S that approximates the vertices of $\mathscr{T}$. To build S, our construction defines a set of gluing data and a set of parametrizations of a PPS.
The set of gluing data,
$\mathscr{G} = \left((\Omega_i)_{i\in I}, (\Omega_{ij})_{(i,j)\in I\times I}, (\varphi_{ji})_{(i,j)\in K}\right)$,
is defined from the elements of $\mathscr{T}$, while the set of parametrizations, $(\theta_i)_{i\in I}$, where $\theta_i : \Omega_i \rightarrow \theta(\Omega_i) \subset \mathbb{R}^3$, for every $i \in I$, is defined from $|\mathscr{T}|$. The key idea is to define a PPS,

$$\mathscr{M} = (\mathscr{G}, (\theta_i)_{i\in I})$$

such that the image,

$$S = \bigcup_{i\in I} \theta_i(\Omega_i)$$

of $\mathscr{M}$ in $\mathbb{R}^3$ is a surface, $S \subset \mathbb{R}^3$, that approximates $|\mathscr{T}|$(is the point set of $\mathscr{T}$). So if we can build $(\theta_i)_{i\in I}$, we will get the surface S.

From M.Siqueira et al.'s paper we can know that,

$$\theta_u(p) = \sum_{v\in J_u(p)} \omega_{vu}(p) \times (\psi_v \circ \varphi_{vu}(p))$$

where,
$u \in |\mathscr{T}|$,
$p \in \Omega_u$,
$\theta_u : \Omega_u \longrightarrow \theta_u(\Omega_u) \subset \mathbb{R}^2$,
$\omega_{vu}(p) = \frac{\gamma_v \circ \varphi_{vu}(p)}{\sum_{w\in I_u(p)} \gamma_w \circ \varphi_{wu}(p)}$.

For more detail definitions of these symbols,readers can refer to M.Siqueira et al.'s paper.

From the equation about θ_u,we notice that we can get the PPS position of every $p \in \Omega_u$.

4.2 Improvement 2

In this section,we settle the four points coplanar problem. In Re-Tiling algorithm, because the new points lie on the triangle mesh, the new points lying in one triangle containing more than three points are coplanar.we do not want appear this situation. So we improve the Re-Tiling algorithm by computing PPS position of new points.

Fig.4 shows the four points coplanar situation.Red points(a, b, c, d) are new points, blue points(A, B, C)are old points of inpute mesh.

From the PPS costruction in M.Siqueira et al.'s[2009] paper, every triangle of $\mathscr{T}$ can correspond to a part of region Ω_u by barycentric coordinates. So we can compute PPS position of each new point in the following step, such as a new point q,$q \in T$,$T \subset \mathscr{T}$ is a triangle.

(1) we calculate the barycenter coordinates (λ, v, η) of q in T.
(2) according to (λ, v, η), we can find a p in parameter area Ω_u correspond to q,mentioned in M.Siqueira et al.'s paper.

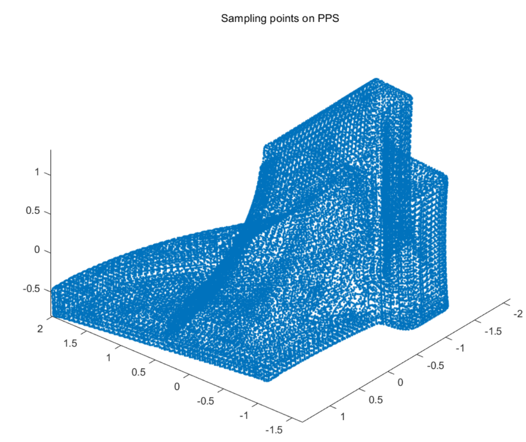

Fig. 3. Sampling points on PPS of the input mesh.

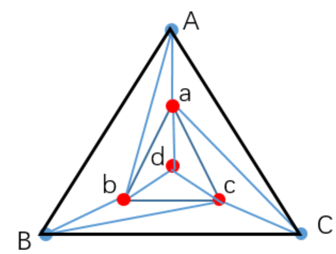

Fig. 4. four points coplanar situation.

(3) using p and the equation of θ_u,we can get the PPS position of new point q.

Because manifolds are smooth and concave-convex,we can generate more reasonable meshes,which would not appear four points coplanar situation.So we can update the new points positions with their PPS positions after the last step of the Re-Tiling algorithm.

We conclude our algorithm in "Re-Tiling and PPS algorithm".

5 RESULT AND CONCLUSIONS

In this section we present the results of some model tests in Fig.5.

From the result in Fig.5,we can know that our method can retain more features of original meshes than Re-Tiling algorithm.

Re-Tiling algorithm is a useful method to generate meshes at several levels of detail,but it doesn't work well with shape features meshes and four points coplanar problem. Our method can make up for its deficiency.

6 FUTURE WORK

In fact,our method is not perfect. For maintaining features of the input mesh,we calculate the curvature of old points, and retain them. If the points with high curvature are very close together in the inpute mesh,and we need retain them in our method.This situation cause some points in the output mesh are close together.We will get a poor quality mesh.

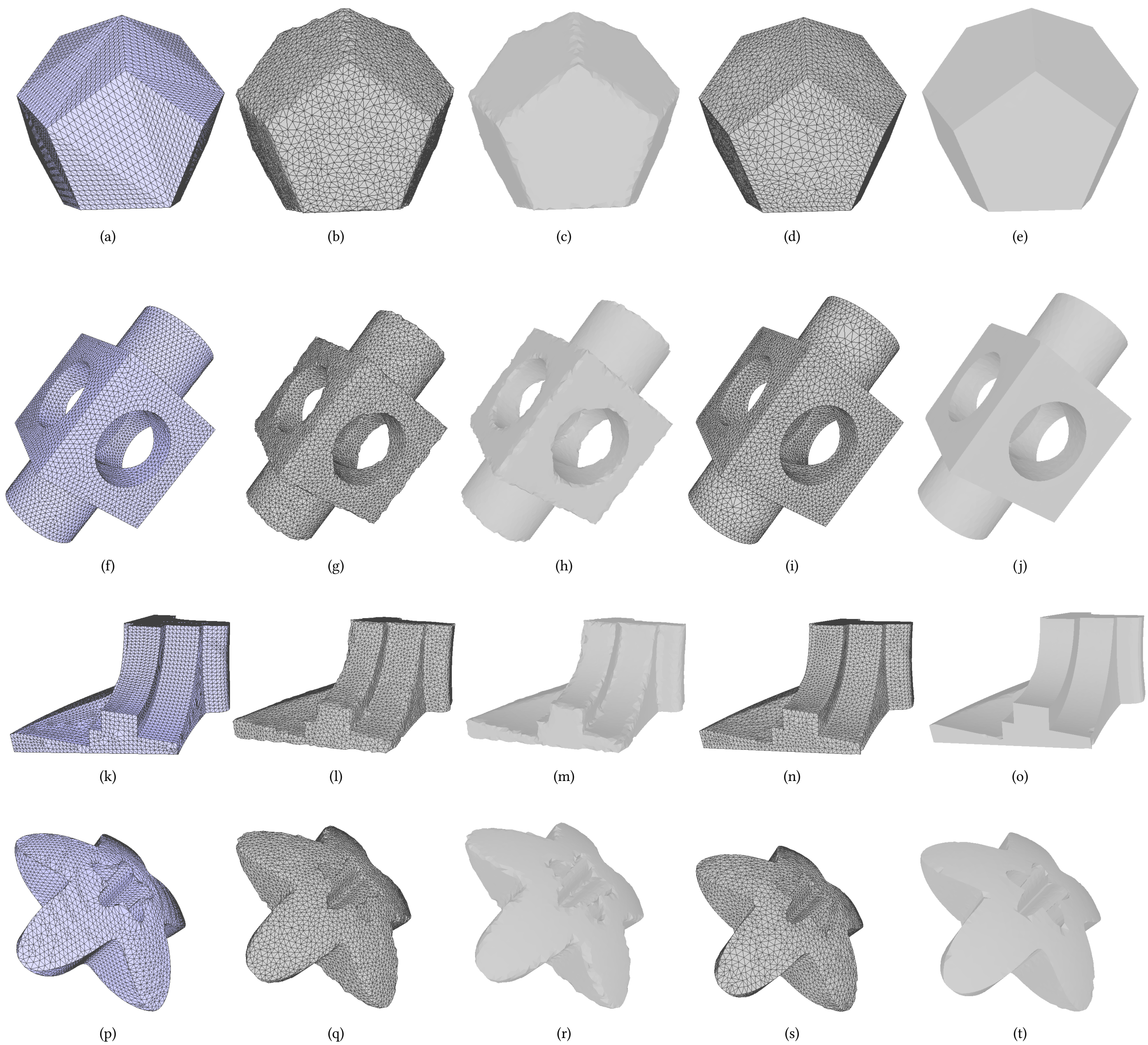

Fig. 5. Results and visual comparison. The first column meshes in the figure are inpute meshes with sharp features. The second and the third columns are the result of Re-Tiling algorithm. The fourth and fifth columns are the result of our algorithm.

So in the future we will try to deal with the special situation. Before doing our method, adjusting the very close vertices as far away as possible in some ways may be a worth try.

REFERENCES

[1] G. Turk, "Re-Tiling Polygonal Surfaces", 1992.
[2] R. YI, Y. LIU, "Delaunay Mesh Simplification with Differential Evolution", 2018.
[3] M. Botsch, L. Kobbelt, "Polygon Mesh Processing", 2010.
[4] M. Siqueira, D. Xu, J. Gallier, "A new construction of smooth surfaces from triangle meshes using parametric pseudo-manifolds", 2009.