Deep Learning

Lecture 21

Reinforcement Learning

Part 1

So far… Supervised Learning

Data: (x, y)
x is data, y is label

Goal : Learn a *function* to map x -> y

Examples : Classification, regression, object detection, semantic segmentation, image captioning, etc.

Cat

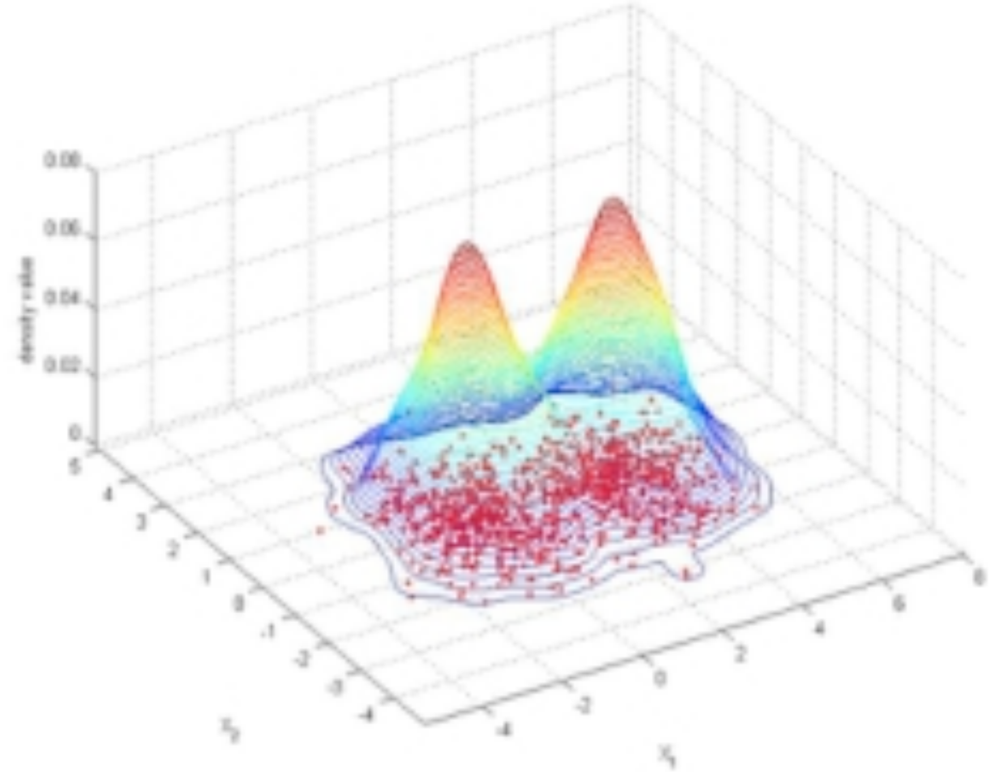
Classification

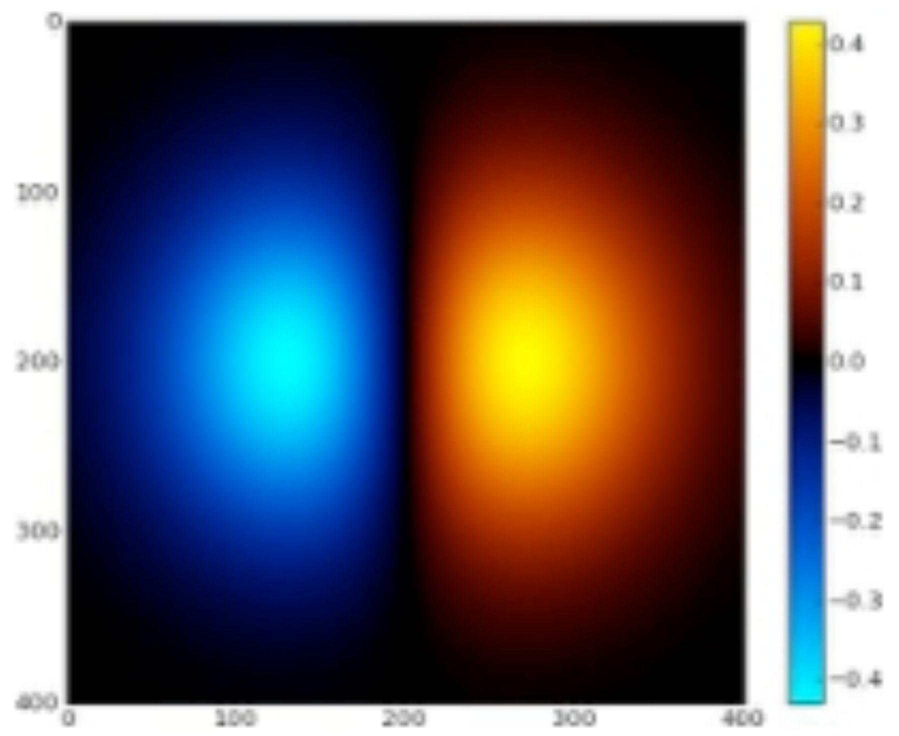
So far… Unsupervised Learning

Data: x
Just data, no labels!

Goal : Learn some underlying hidden *structure* of the data

Examples : Clustering, dimensionality reduction, feature learning, density estimation, etc.



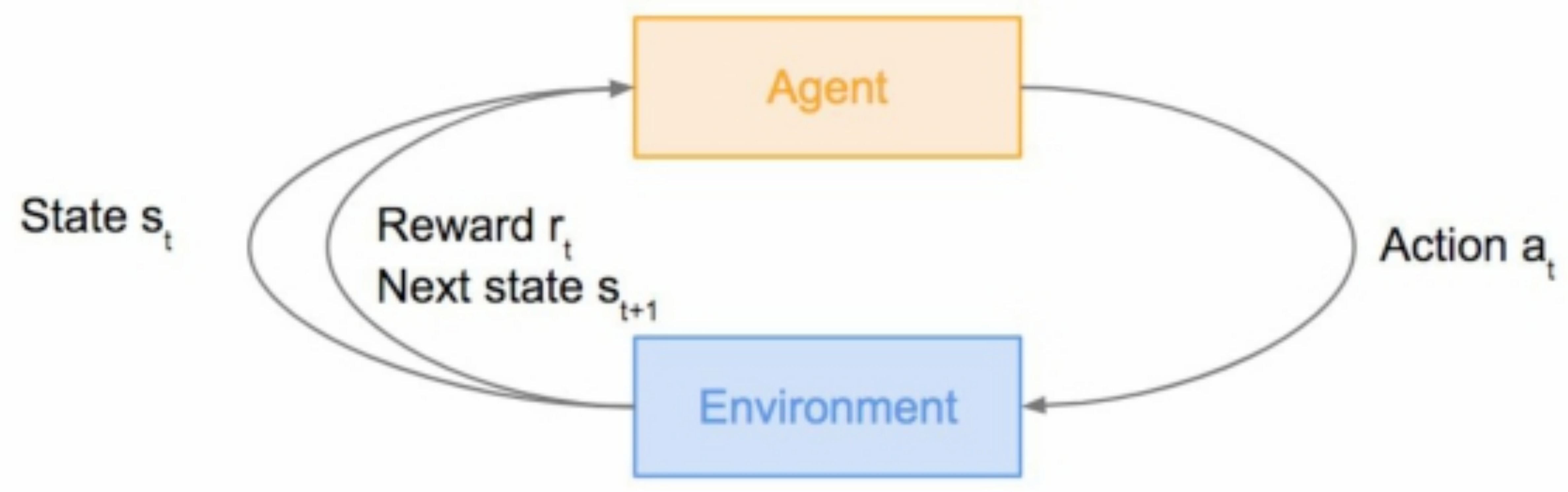


2-d density estimation

Today: Reinforcement Learning

Problems involving an **agent** interacting with an **environment** , which provides numeric **reward** signals

Goal : Learn how to take actions in order to maximize reward

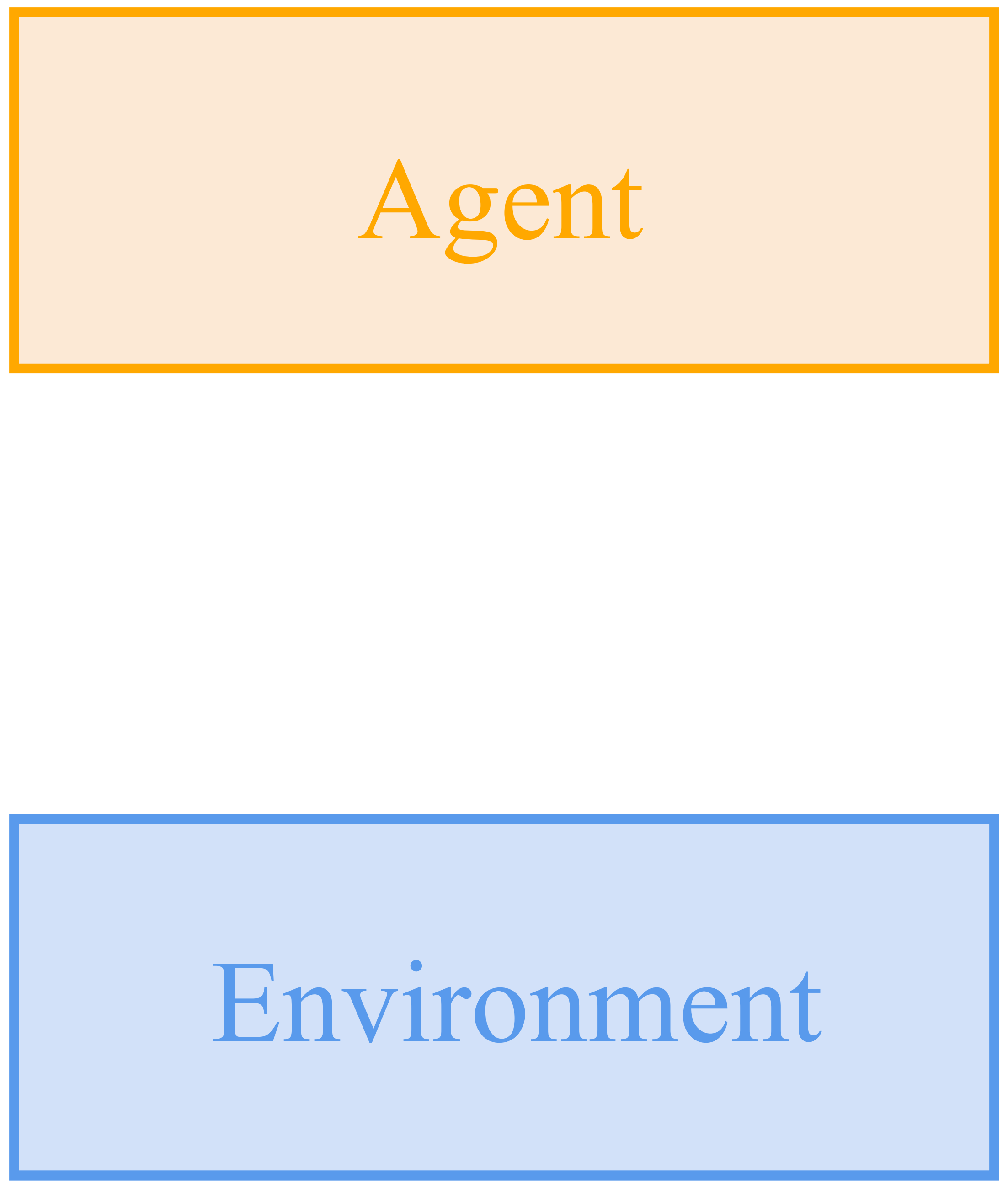


Atari games figure copyright Volodymyr Mnih et al., 2013. Reproduced with permission.

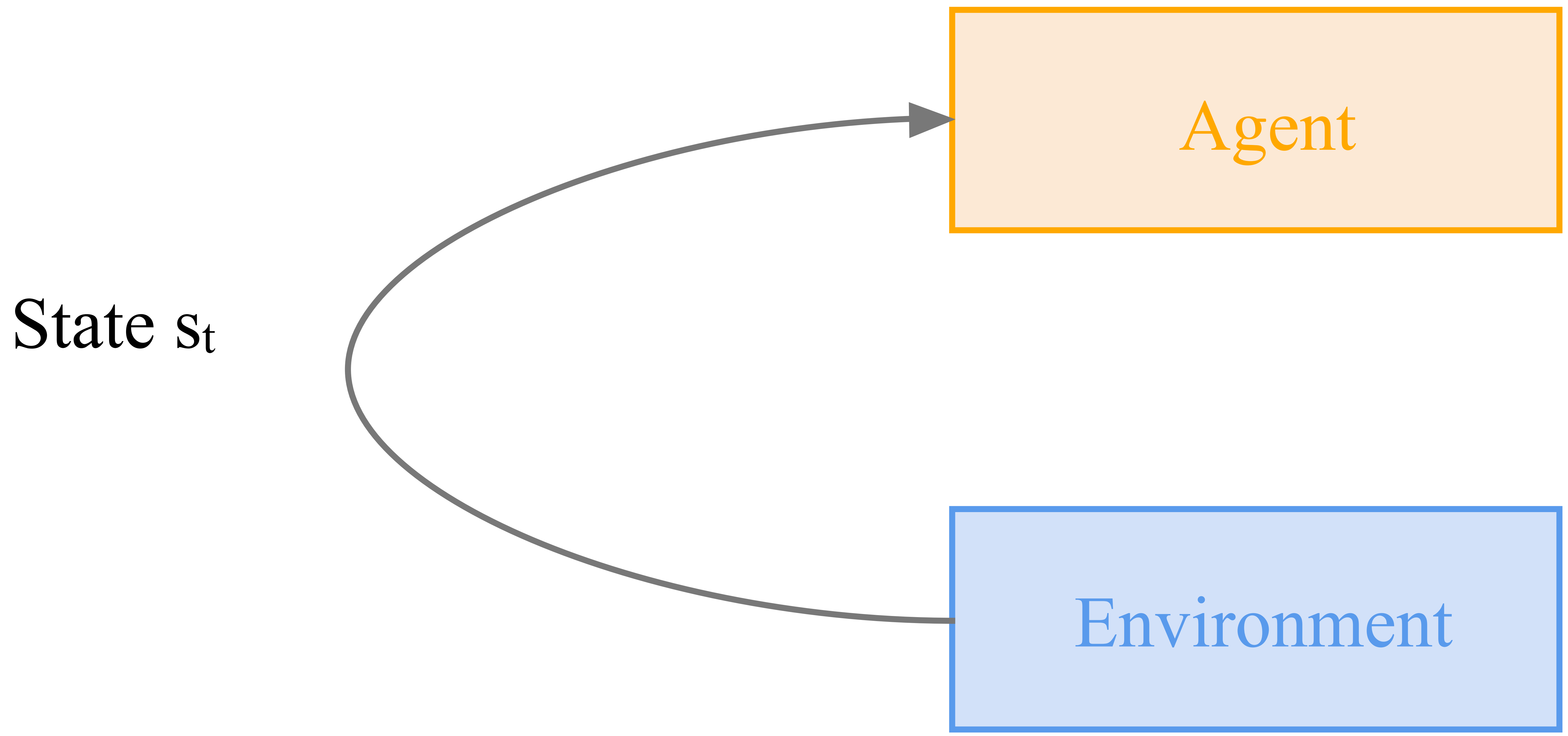
Overview

- What is Reinforcement Learning?
- Markov Decision Processes
- Q-Learning
- Policy Gradients

Reinforcement Learning



Reinforcement Learning



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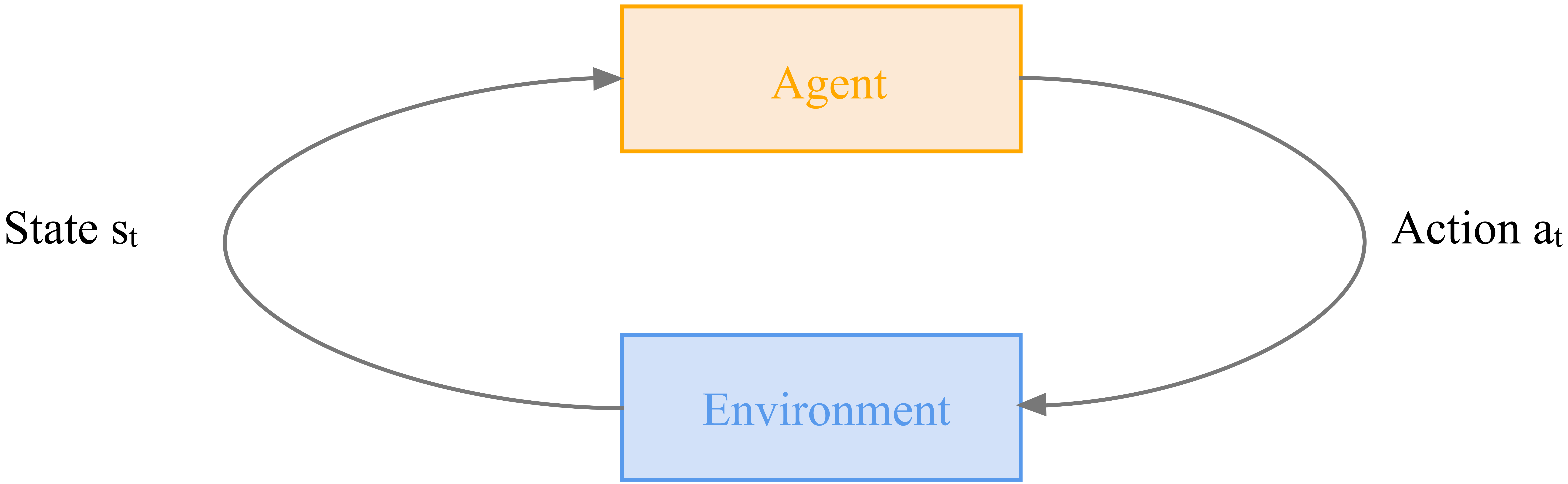

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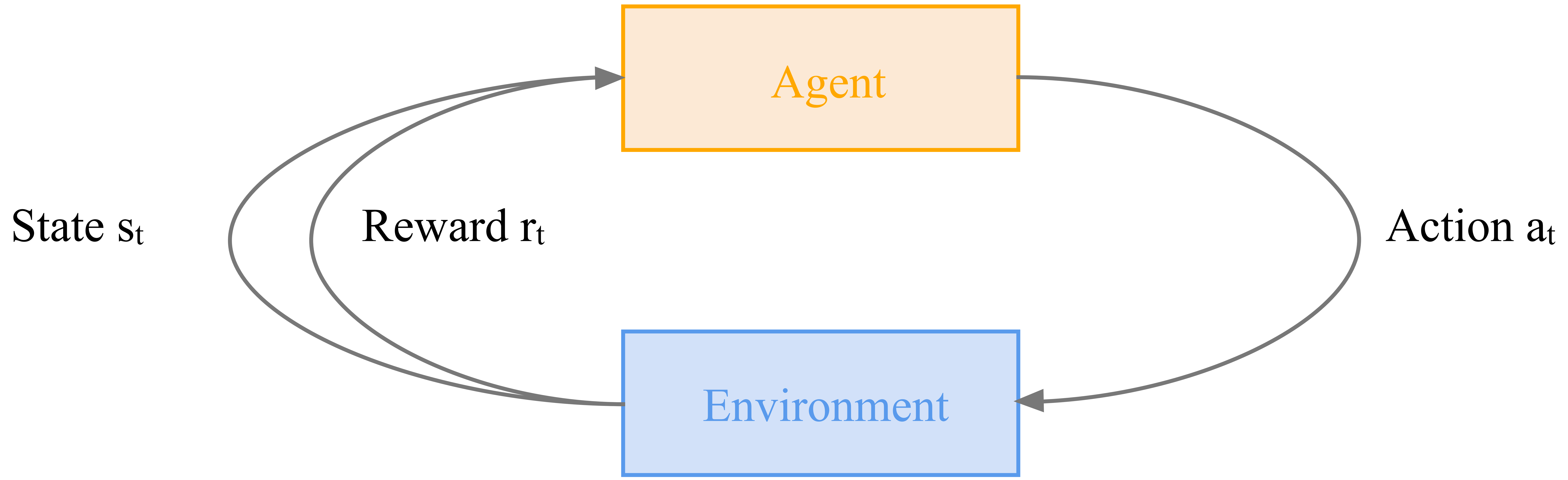
Reinforcement Learning



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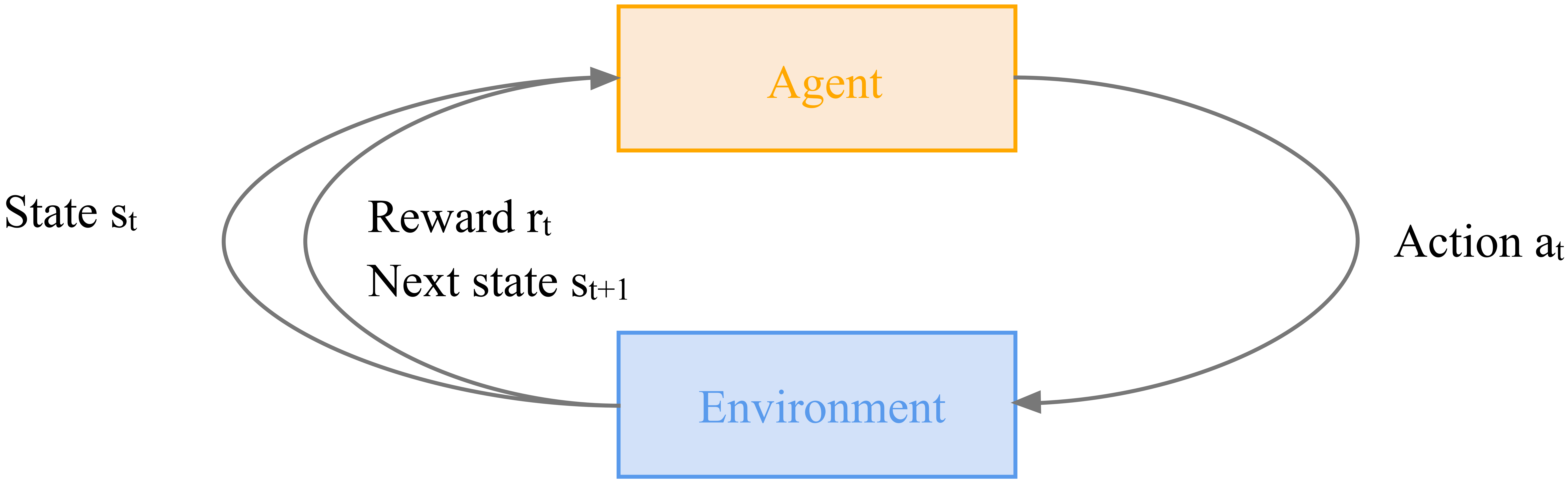

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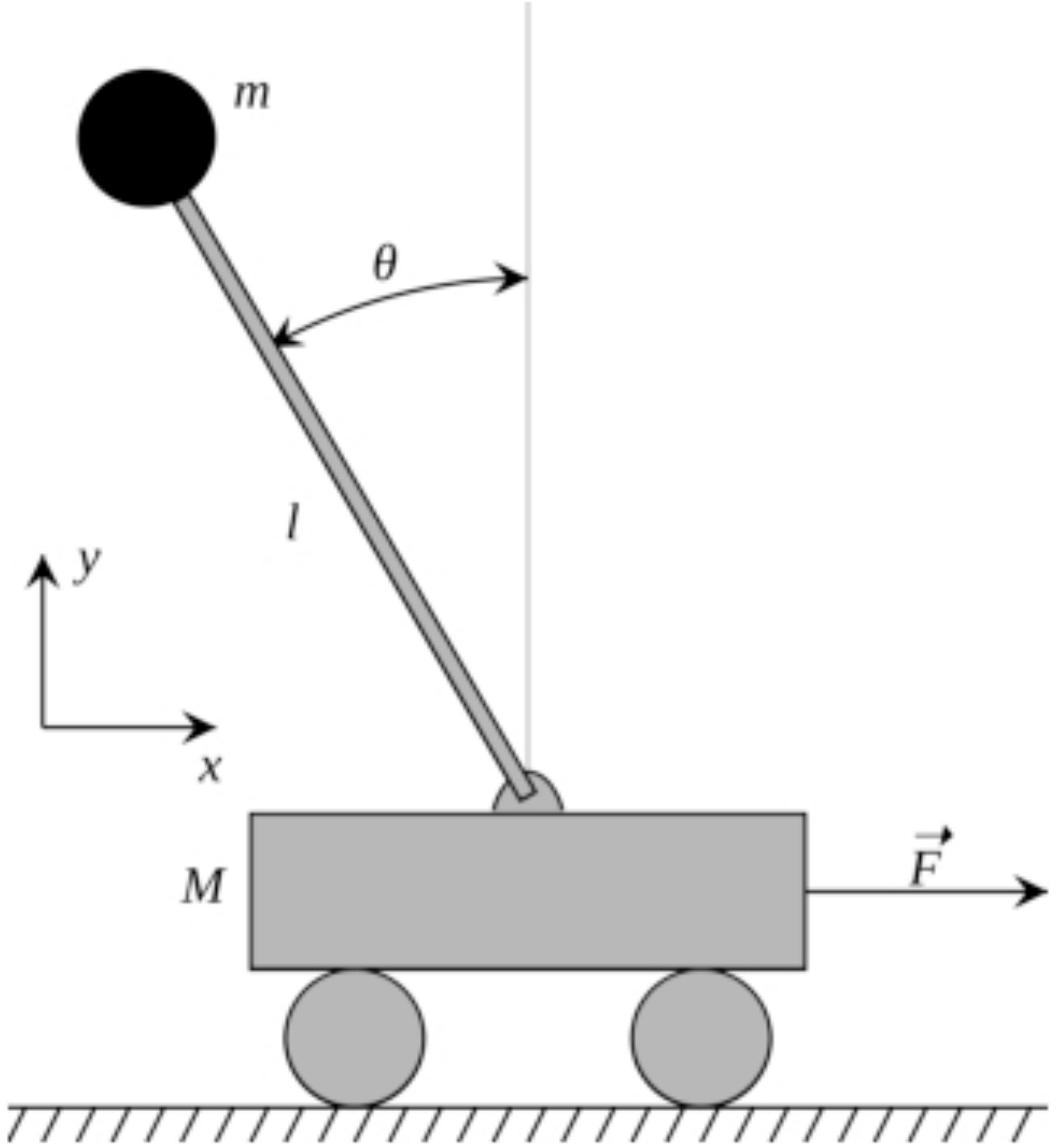
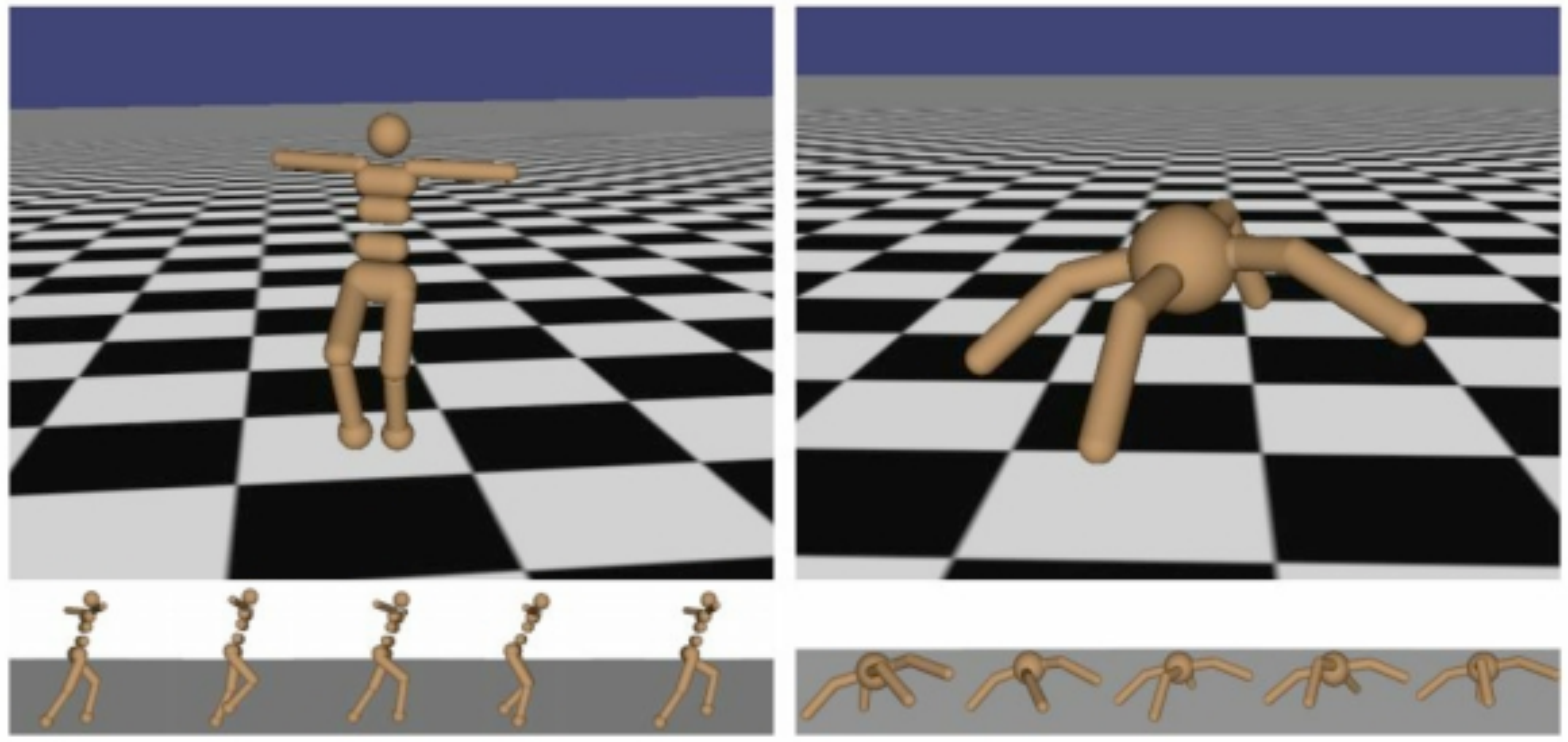
Cart-Pole Problem



Objective : Balance a pole on top of a movable cart

State: angle, angular speed, position, horizontal velocity
Action: horizontal force applied on the cart
Reward: 1 at each time step if the pole is upright

Robot Locomotion



Objective : Make the robot move forward

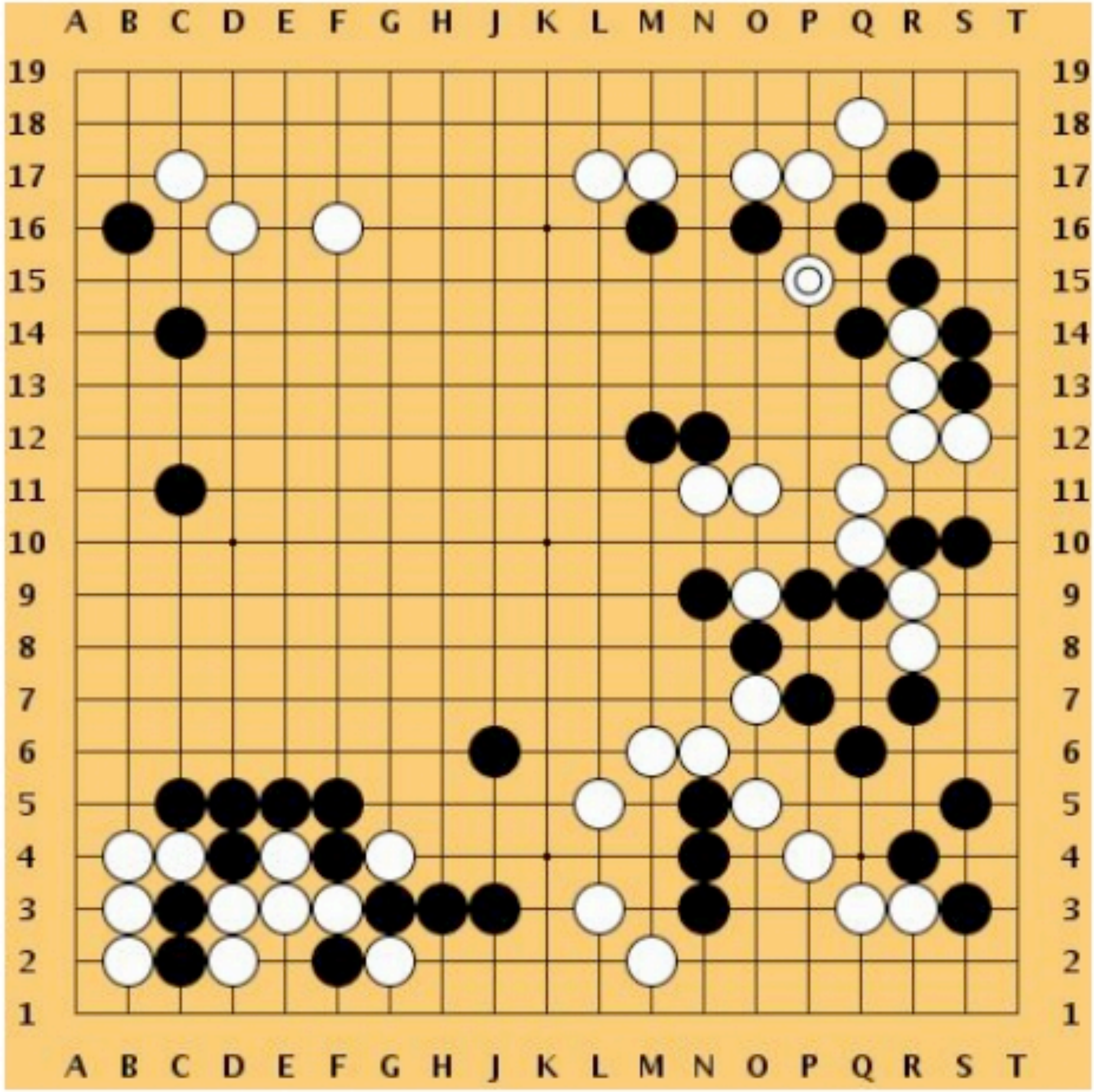
State: Angle and position of the joints
Action: Torques applied on joints
Reward: 1 at each time step upright + forward movement

Atari Games

Objective : Complete the game with the highest score

State: Raw pixel inputs of the game state
Action: Game controls e.g. Left, Right, Up, Down
Reward: Score increase/decrease at each time step

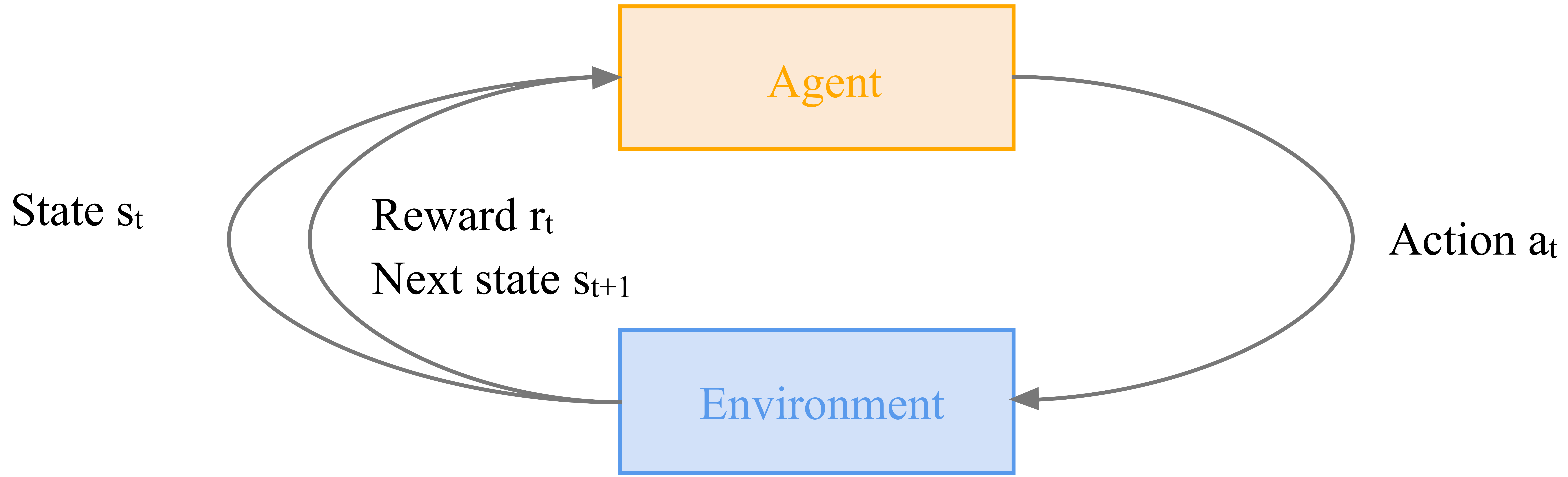
Go



Objective : Win the game!

State: Position of all pieces
Action: Where to put the next piece down
Reward: 1 if win at the end of the game, 0 otherwise

How can we mathematically formalize the RL problem?



Markov Decision Process

- Mathematical formulation of the RL problem
- **Markov property** : Current state completely characterizes the state of the world

Defined by: $(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathbb{P}, \gamma)$

$\mathcal{S}$: set of possible states
$\mathcal{A}$: set of possible actions
$\mathcal{R}$: distribution of reward given (state, action) pair
$\mathbb{P}$: transition probability i.e. distribution over next state given (state, action) pair
γ : discount factor

Markov Decision Process

- At time step t=0, environment samples initial state $s_0 \sim p(s_0)$
- Then, for t=0 until done:
  - Agent selects action a_t
  - Environment samples reward $r_t \sim R(. \mid s_t, a_t)$
  - Environment samples next state $s_{t+1} \sim P(. \mid s_t, a_t)$
  - Agent receives reward r_t and next state s_{t+1}

- A policy **π** is a function from S to A that specifies what action to take in each state
- **Objective** : find policy **π*** that maximizes cumulative discounted reward:

$$\sum_{t \geq 0} \gamma^t r_t$$

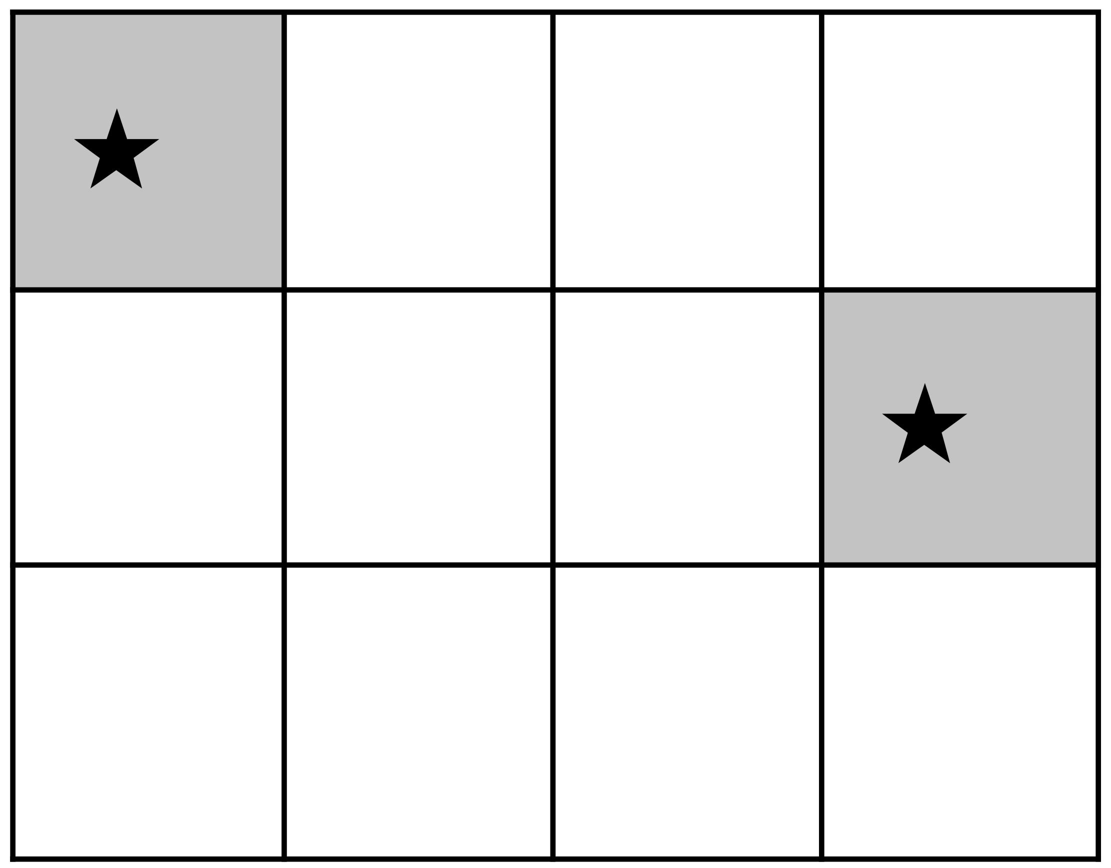
A simple MDP: Grid World

actions = {

1. right
2. left
3. up
4. down

}

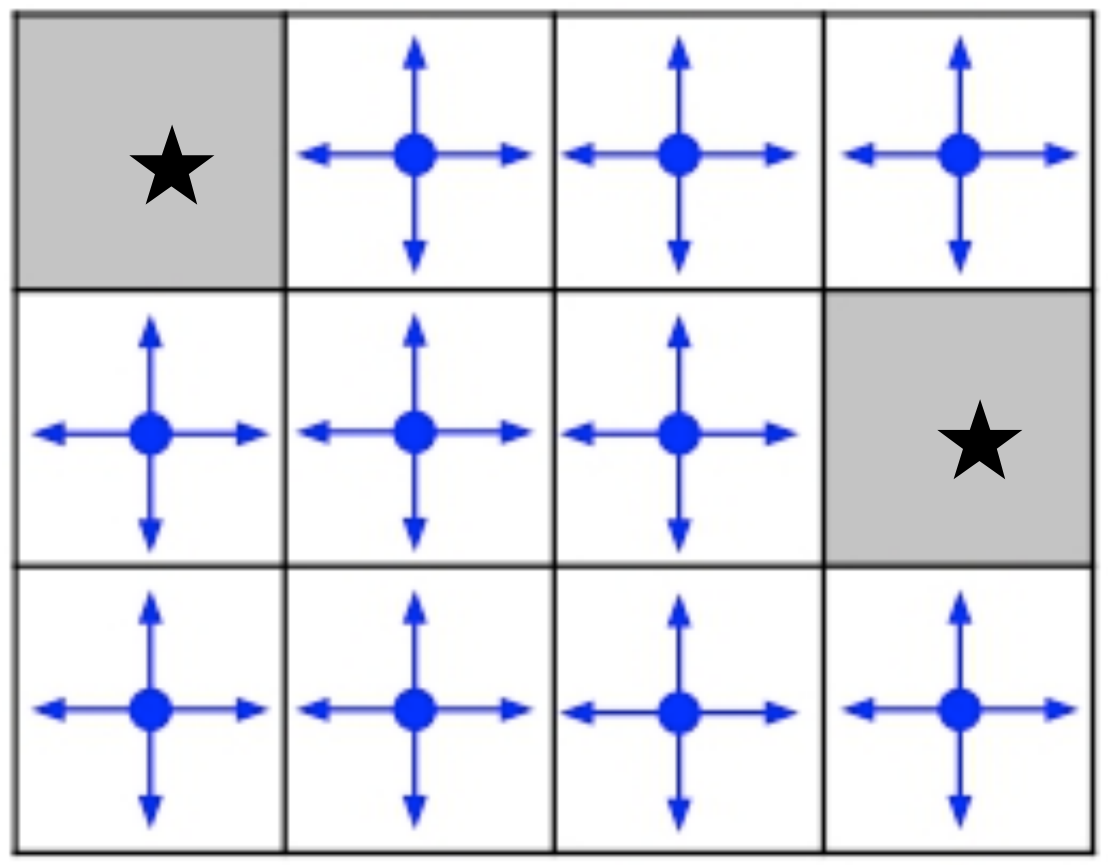
states



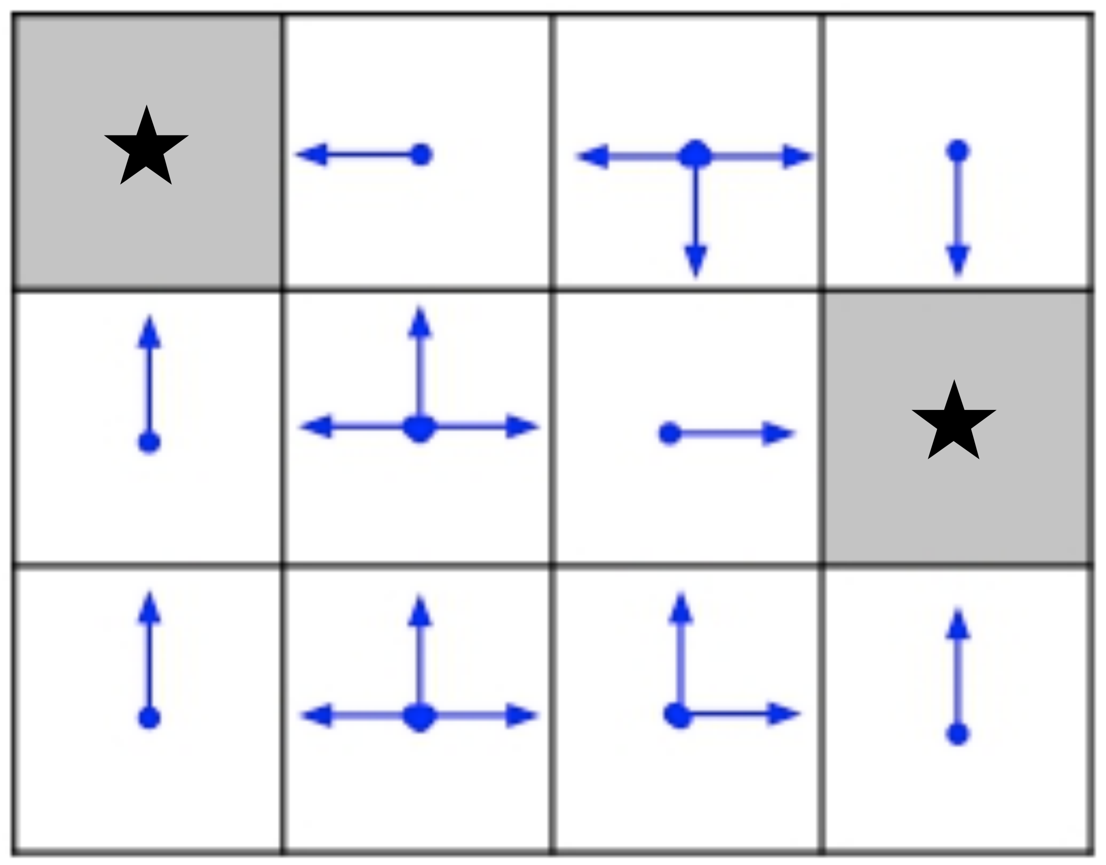
Set a negative “reward” for each transition (e.g. $r = -1$)

Objective: reach one of terminal states (greyed out) in least number of actions

A simple MDP: Grid World



Random Policy



Optimal Policy

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The optimal policy **π***

We want to find optimal policy **π*** that maximizes the sum of rewards.

How do we handle the randomness (initial state, transition probability…)?

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How do we handle the randomness (initial state, transition probability…)?
Maximize the **expected sum of rewards!**

Formally: $\pi^* = \arg\max_{\pi} \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | \pi\right]$ with $s_0 \sim p(s_0), a_t \sim \pi(\cdot|s_t), s_{t+1} \sim p(\cdot|s_t, a_t)$

Definitions: Value function and Q-value function

Following a policy produces sample trajectories (or paths) $s_0, a_0, r_0, s_1, a_1, r_1, \ldots$

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How good is a state?

The **value function** at state s, is the expected cumulative reward from following the policy from state s:

$$V^\pi(s) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, \pi\right]$$

Definitions: Value function and Q-value function

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How good is a state-action pair?

The **Q-value function** at state s and action a, is the expected cumulative reward from taking action a in state s and then following the policy:

$$Q^{\pi}(s, a) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

Bellman equation

The optimal Q-value function Q* is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s,a) = \max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

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Q* satisfies the following **Bellman equation** :

$$Q^*(s,a) = \mathbb{E}_{s' \sim \mathcal{E}}\left[r + \gamma \max_{a'} Q^*(s',a') | s, a\right]$$

Intuition: if the optimal state-action values for the next time-step Q*(s’,a’) are known, then the optimal strategy is to take the action that maximizes the expected value of $r + \gamma Q^*(s', a')$

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The optimal policy **π*** corresponds to taking the best action in any state as specified by Q*

Solving for the optimal policy

Value iteration algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s, a) = \mathbb{E}\left[r + \gamma \max_{a'} Q_i(s', a') | s, a\right]$$

Q_i will converge to Q* as i -> infinity

Solving for the optimal policy

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Not scalable. Must compute Q(s,a) for every state-action pair. If state is e.g. current game state pixels, computationally infeasible to compute for entire state space!

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Not scalable. Must compute Q(s,a) for every state-action pair. If state is e.g. current game state pixels, computationally infeasible to compute for entire state space!

Solution: use a function approximator to estimate Q(s,a). E.g. a **neural network**!

Solving for the optimal policy: Q-learning

Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$

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If the function approximator is a deep neural network => **deep q-learning** !

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$$Q(s, a; \theta) \approx Q^*(s, a)$$

function parameters (weights)

If the function approximator is a deep neural network => **deep q-learning** !

Solving for the optimal policy: Q-learning

Remember: want to find a Q-function that satisfies the Bellman Equation:

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Forward Pass

$$L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot)}\left[(y_i - Q(s,a;\theta_i))^2\right]$$

where $\quad y_i = \mathbb{E}_{s' \sim \mathcal{E}}\left[r + \gamma \max_{a'} Q(s',a';\theta_{i-1}) | s,a\right]$

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Backward Pass

Gradient update (with respect to Q-function parameters θ):

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[Mnih et al. NIPS Workshop 2013; Nature 2015]

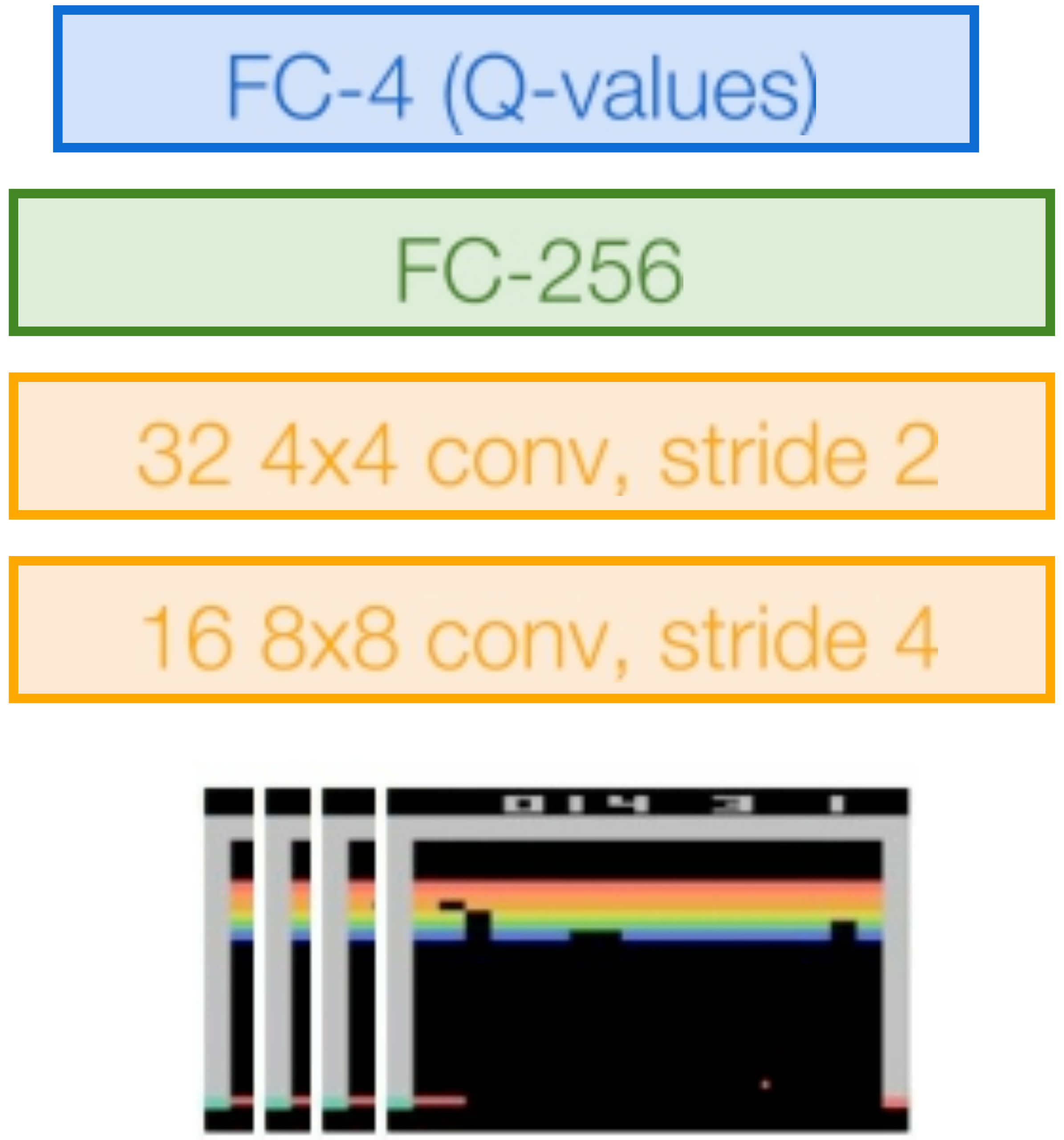
Case Study: Playing Atari Games

Objective : Complete the game with the highest score

State: Raw pixel inputs of the game state
Action: Game controls e.g. Left, Right, Up, Down
Reward: Score increase/decrease at each time step

Q-network Architecture

$Q(s, a; \theta)$:
neural network
with weights θ



Current state s_t: 84x84x4 stack of last 4 frames

(after RGB->grayscale conversion, downsampling, and cropping)

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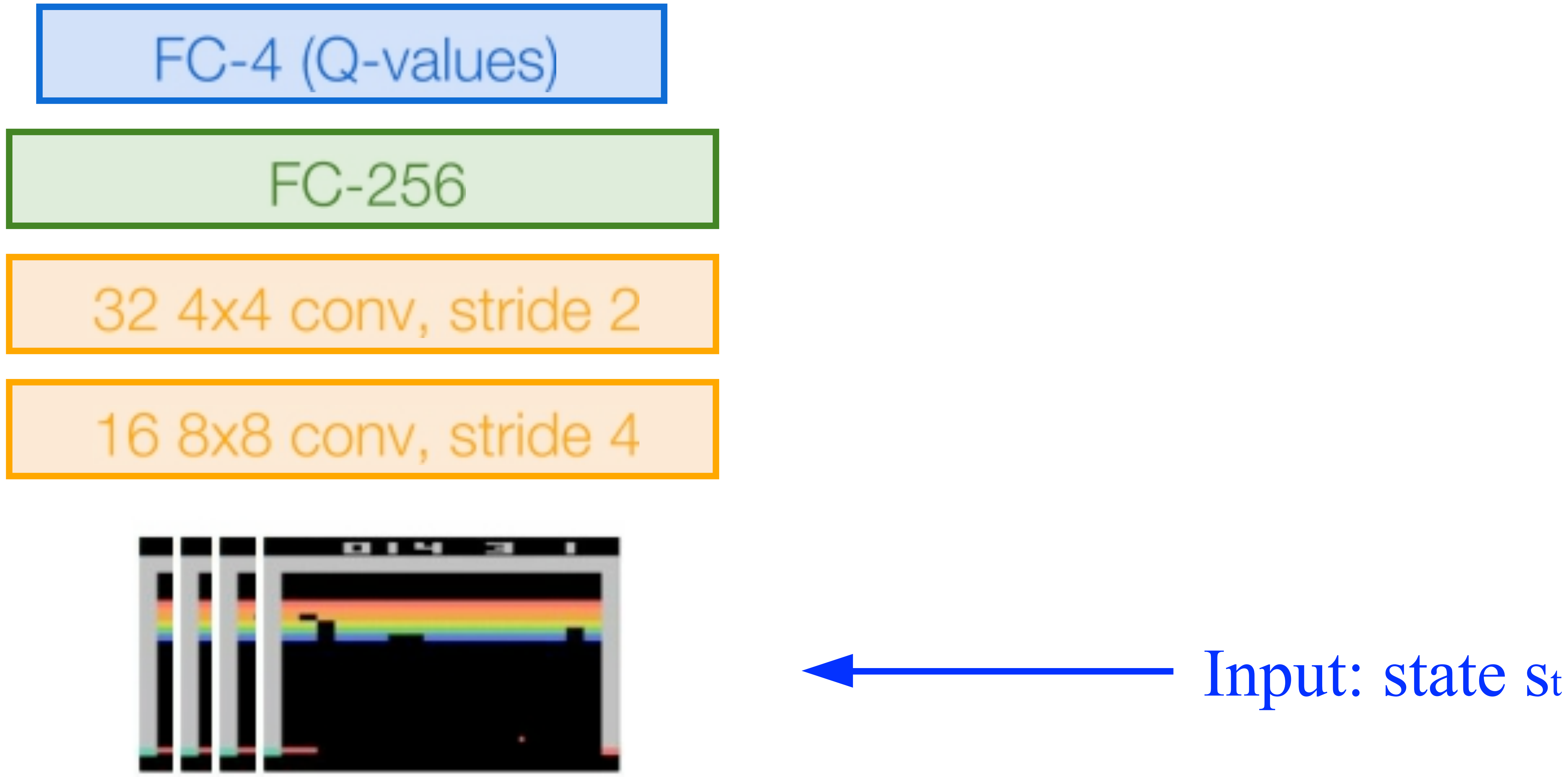

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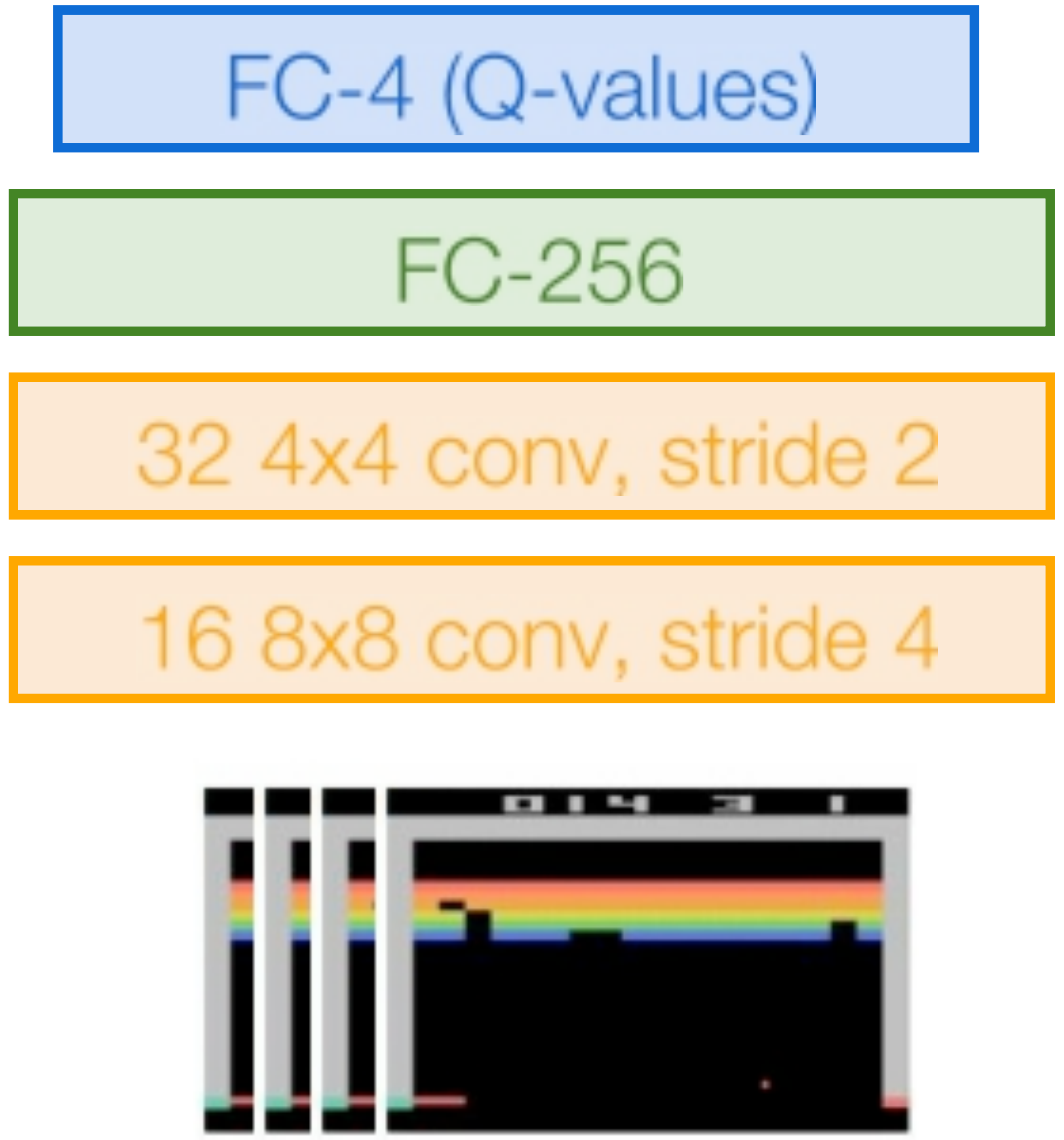

Familiar conv layers,
FC layer

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[Mnih et al. NIPS Workshop 2013; Nature 2015]

Q-network Architecture

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FC-4 (Q-values)

FC-256

32 4x4 conv, stride 2

16 8x8 conv, stride 4

Current state s_t: 84x84x4 stack of last 4 frames

(after RGB->grayscale conversion, downsampling, and cropping)

Last FC layer has 4-d output (if 4 actions), corresponding to

$Q(s_t, a_1)$, $Q(s_t, a_2)$, $Q(s_t, a_3)$, $Q(s_t, a_4)$

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Number of actions between 4-18 depending on Atari game

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Q-network Architecture

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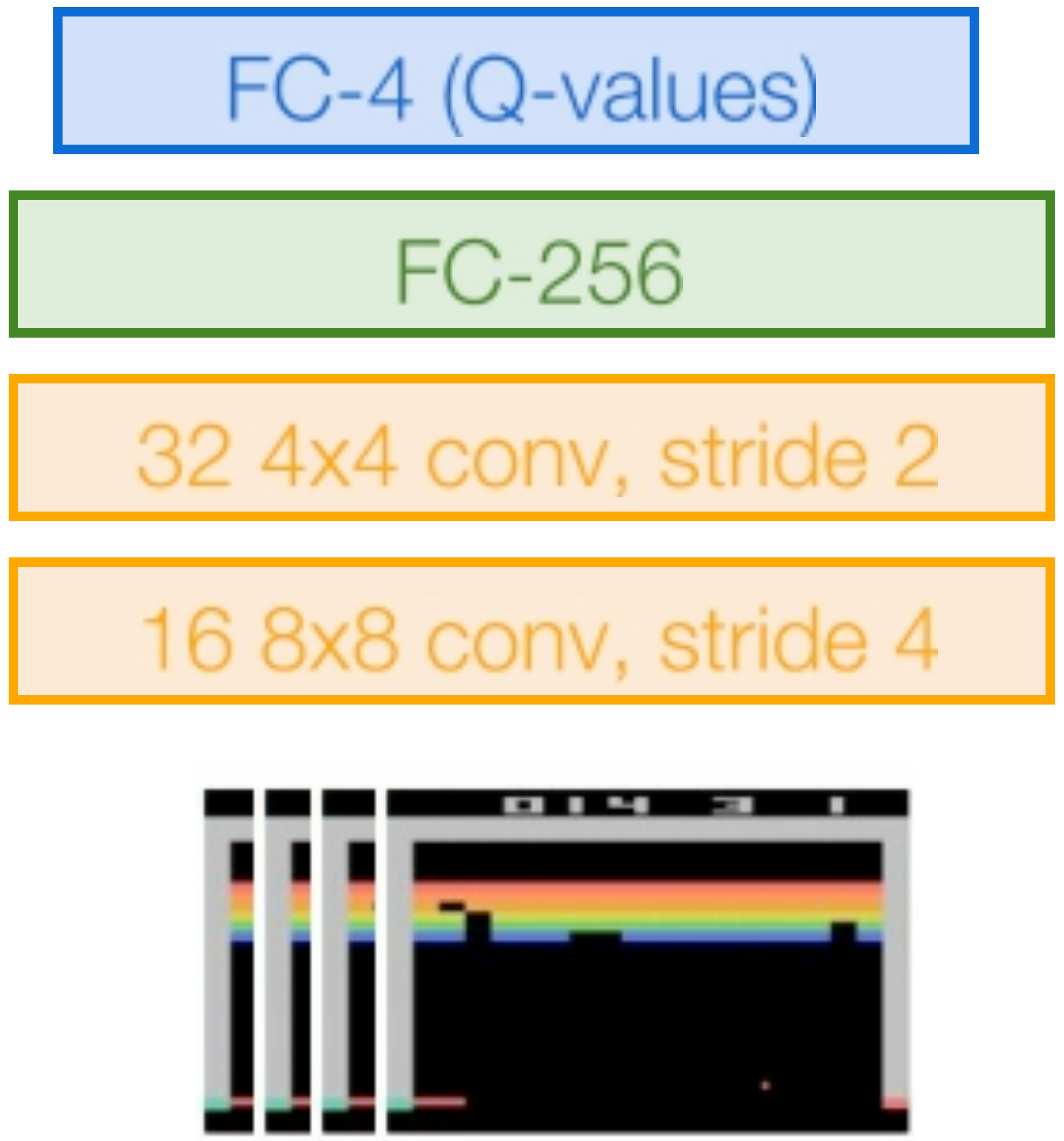
A single feedforward pass to compute Q-values for all actions from the current state => efficient!



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Training the Q-network: Experience Replay

Learning from batches of consecutive samples is problematic:

- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size) => can lead to bad feedback loops

Training the Q-network: Experience Replay

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Address these problems using **experience replay**

- Continually update a **replay memory** table of transitions (s_t, a_t, r_t, s_{t+1}) as game (experience) episodes are played
- Train Q-network on random minibatches of transitions from the replay memory, instead of consecutive samples

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Each transition can also contribute to multiple weight updates => greater data efficiency

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N
Initialize action-value function Q with random weights
for episode $= 1, M$ **do**
  Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
  for $t = 1, T$ **do**
    With probability ϵ select a random action a_t
    otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$
    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
    Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
  end for
end for

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Initialize replay memory, Q-network

[Mnih et al. NIPS Workshop 2013; Nature 2015]

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N
Initialize action-value function Q with random weights
for episode $= 1, M$ **do** ← Play M episodes (full games)
  Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
  for $t = 1, T$ **do**
    With probability ϵ select a random action a_t
    otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
    Execute action a_t in emulator and observe reward r_t and image x_{t+1}
    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
    Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$
    Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
    Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
  end for
end for

Putting it together: Deep Q-Learning with Experience Replay

Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory $\mathcal{D}$ to capacity N
Initialize action-value function Q with random weights
for episode $= 1, M$ **do**
    Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
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        Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
    end for
end for

Initialize state (starting game screen pixels) at the beginning of each episode

Putting it together: Deep Q-Learning with Experience Replay

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    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
  end for
end for

For each timestep t of the game

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        Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
    end for
end for

With small probability, select a random action (explore), otherwise select greedy action from current policy

Putting it together: Deep Q-Learning with Experience Replay

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    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
  end for
end for

Take the action a_t, and observe the reward r_t and next state s_{t+1}

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    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
  end for
end for

Store transition in replay memory

Putting it together: Deep Q-Learning with Experience Replay

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    Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
  end for
end for

Experience Replay: Sample a random minibatch of transitions from replay memory and perform a gradient descent step

https://www.youtube.com/watch?v=V1eYniJ0Rnk

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