Deep Learning

Lecture 22

Reinforcement Learning

Part 2

Policy Gradients

What is a problem with Q-learning?

Policy Gradients

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The Q-function can be very complicated!

Example: a robot grasping an object has a very high-dimensional state => hard to learn exact value of every (state, action) pair

Policy Gradients

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Example: a robot grasping an object has a very high-dimensional state => hard to learn exact value of every (state, action) pair

But the policy can be much simpler: just close your hand
Can we learn a policy directly, e.g. finding the best policy from a collection of policies?

Policy Gradients

Formally, let's define a class of parameterized policies: $\Pi = \{\pi_\theta, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

$$J(\theta) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi_\theta\right]$$

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Gradient ascent on policy parameters!

REINFORCE algorithm

Mathematically, we can write:

$$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)} \left[r(\tau) \right]$$
$$= \int_{\tau} r(\tau) p(\tau;\theta) \mathrm{d}\tau$$

Where r( ) is the reward of a trajectory $\tau = (s_0, a_0, r_0, s_1, \ldots)$

REINFORCE algorithm

Expected reward:

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Now let's differentiate this:

$$\nabla_{\theta} J(\theta) = \int_{\tau} r(\tau) \nabla_{\theta} p(\tau;\theta) \mathrm{d}\tau$$

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$$\nabla_\theta p(\tau;\theta) = p(\tau;\theta) \frac{\nabla_\theta p(\tau;\theta)}{p(\tau;\theta)} = p(\tau;\theta) \nabla_\theta \log p(\tau;\theta)$$

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If we inject this back:
$$\nabla_\theta J(\theta) = \int_{\tau} \left(r(\tau) \nabla_\theta \log p(\tau;\theta)\right) p(\tau;\theta) \mathrm{d}\tau$$
$$= \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau) \nabla_\theta \log p(\tau;\theta)\right]$$

Can estimate with Monte Carlo sampling

REINFORCE algorithm

Can we compute those quantities without knowing the transition probabilities?

We have: $$p(\tau;\theta) = \prod_{t\geq 0} p(s_{t+1}|s_t, a_t)\pi_\theta(a_t|s_t)$$

REINFORCE algorithm

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Thus: $\log p(\tau;\theta) = \sum_{t \geq 0} \log p(s_{t+1}|s_t, a_t) + \log \pi_\theta(a_t|s_t)$

REINFORCE algorithm

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REINFORCE algorithm

$$\nabla_\theta J(\theta) = \int_\tau \left(r(\tau)\nabla_\theta \log p(\tau;\theta)\right) p(\tau;\theta)\mathrm{d}\tau$$
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Doesn’t depend on transition probabilities!

Therefore when sampling a trajectory , we can estimate J( ) with

$$\nabla_\theta J(\theta) \approx \sum_{t\geq 0} r(\tau)\nabla_\theta \log \pi_\theta(a_t|s_t)$$

Intuition

Gradient estimator: $$\nabla_\theta J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_\theta \log \pi_\theta(a_t | s_t)$$

Interpretation:

- If r( ) is high, push up the probabilities of the actions seen
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However, this also suffers from high variance because credit assignment is really hard. Can we help the estimator?

Variance reduction

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First idea: Push up probabilities of an action seen, only by the cumulative future reward from that state

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Second idea: Use discount factor **γ** to ignore delayed effects

$$\nabla_\theta J(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} \right) \nabla_\theta \log \pi_\theta(a_t|s_t)$$

Variance reduction: Baseline

Problem: The raw value of a trajectory isn't necessarily meaningful. For example, if rewards are all positive, you keep pushing up probabilities of actions.

What is important then? Whether a reward is better or worse than what you expect to get

Idea: Introduce a baseline function dependent on the state.
Concretely, estimator is now:

$$\nabla_\theta J(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} - b(s_t) \right) \nabla_\theta \log \pi_\theta(a_t | s_t)$$

How to choose the baseline?

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A simple baseline: constant moving average of rewards experienced so far from all trajectories

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A simple baseline: constant moving average of rewards experienced so far from all trajectories

Variance reduction techniques seen so far are typically used in "Vanilla REINFORCE"

How to choose the baseline?

A better baseline: Want to push up the probability of an action from a state, if this action was better than the **expected value of what we should get from that state** .

Q: What does this remind you of?

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A: Q-function and value function!

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Intuitively, we are happy with an action a_t in a state s_t if $Q^\pi(s_t, a_t) - V^\pi(s_t)$ is large. On the contrary, we are unhappy with an action if it's small.

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Intuitively, we are happy with an action a_t in a state s_t if $Q^{\pi}(s_t, a_t) - V^{\pi}(s_t)$ is large. On the contrary, we are unhappy with an action if it's small.

Using this, we get the estimator: $\nabla_\theta J(\theta) \approx \sum_{t \geq 0} (Q^{\pi_\theta}(s_t, a_t) - V^{\pi_\theta}(s_t)) \nabla_\theta \log \pi_\theta(a_t|s_t)$

Actor-Critic Algorithm

Problem: we don't know Q and V. Can we learn them?

Yes, using Q-learning! We can combine Policy Gradients and Q-learning by training both an **actor** (the policy) and a **critic** (the Q-function).

- The actor decides which action to take, and the critic tells the actor how good its action was and how it should adjust
- Also alleviates the task of the critic as it only has to learn the values of (state, action) pairs generated by the policy
- Can also incorporate Q-learning tricks e.g. experience replay
- **Remark:** we can define by the **advantage function** how much an action was better than expected

$$A^\pi(s,a) = Q^\pi(s,a) - V^\pi(s)$$

Actor-Critic Algorithm

Initialize policy parameters , critic parameters
For iteration=1, 2 … **do**
Sample m trajectories under the current policy
$$\Delta\theta \leftarrow 0$$
For i=1, …, m **do**
For t=1, ... , T **do**
$$A_t = \sum_{t' \geq t} \gamma^{t'-t} r_t^i - V_\phi(s_t^i)$$
$$\Delta\theta \leftarrow \Delta\theta + A_t \nabla_\theta \log(a_t^i | s_t^i)$$
$$\Delta\phi \leftarrow \sum_i \sum_t \nabla_\phi ||A_t^i||^2$$
$$\theta \leftarrow \alpha\Delta\theta$$
$$\phi \leftarrow \beta\Delta\phi$$

End for

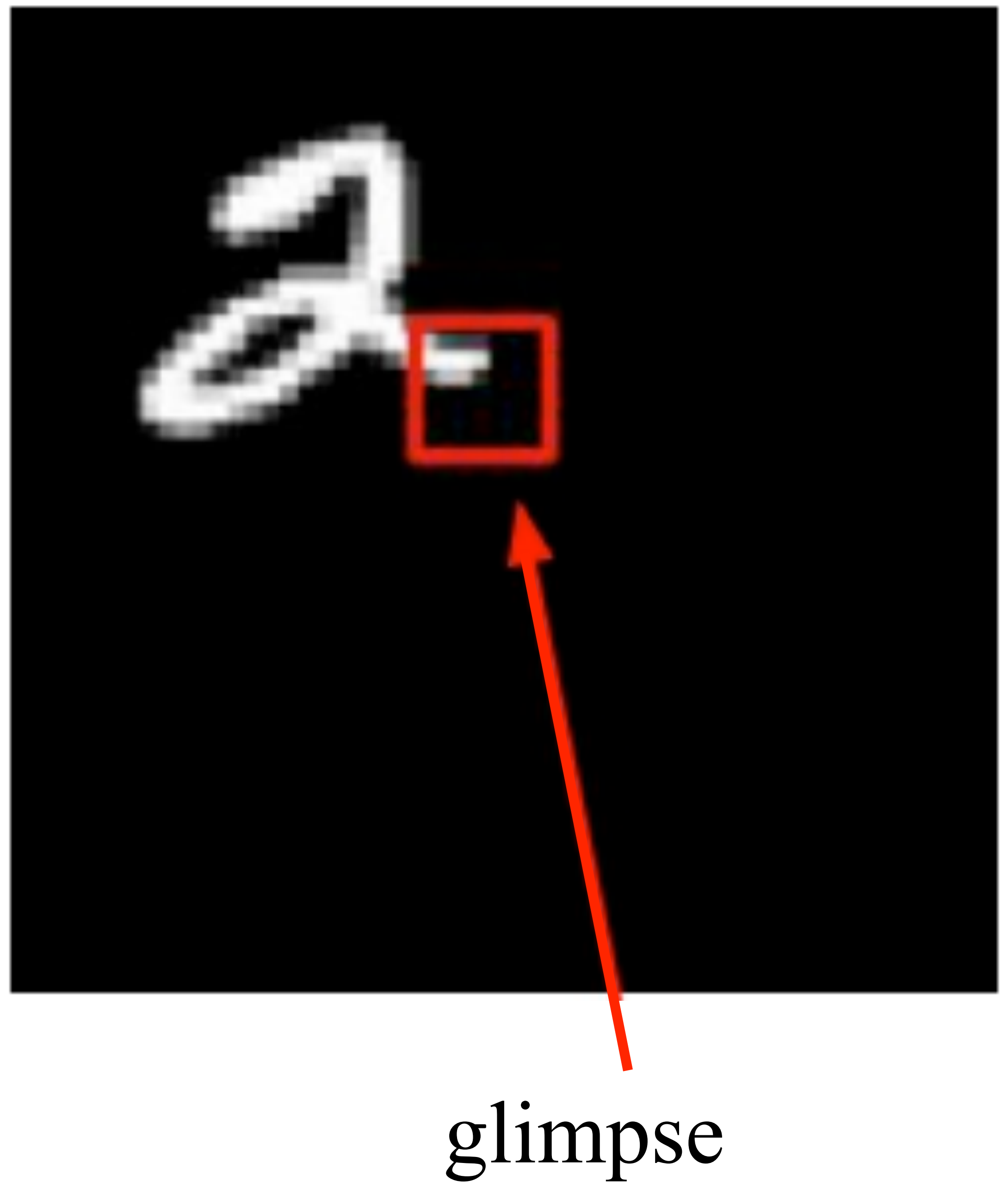
REINFORCE in action: Recurrent Attention Model (RAM)

Objective: Image Classification

Take a sequence of "glimpses" selectively focusing on regions of the image, to predict class

- Inspiration from human perception and eye movements
- Saves computational resources => scalability
- Able to ignore clutter / irrelevant parts of image

State: Glimpses seen so far
Action: (x,y) coordinates (center of glimpse) of where to look next in image
Reward: 1 at the final timestep if image correctly classified, 0 otherwise



[Mnih et al. 2014]

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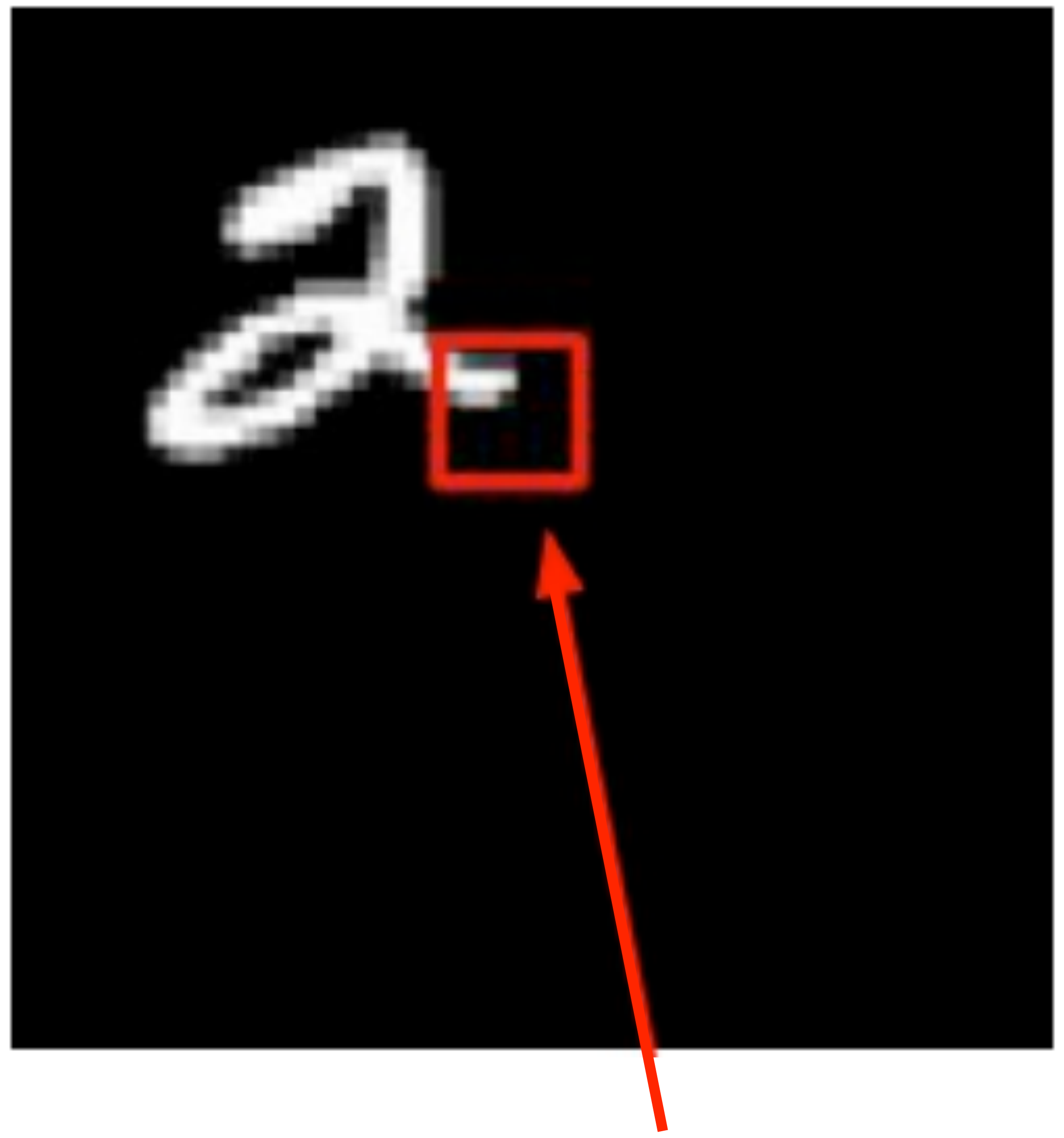
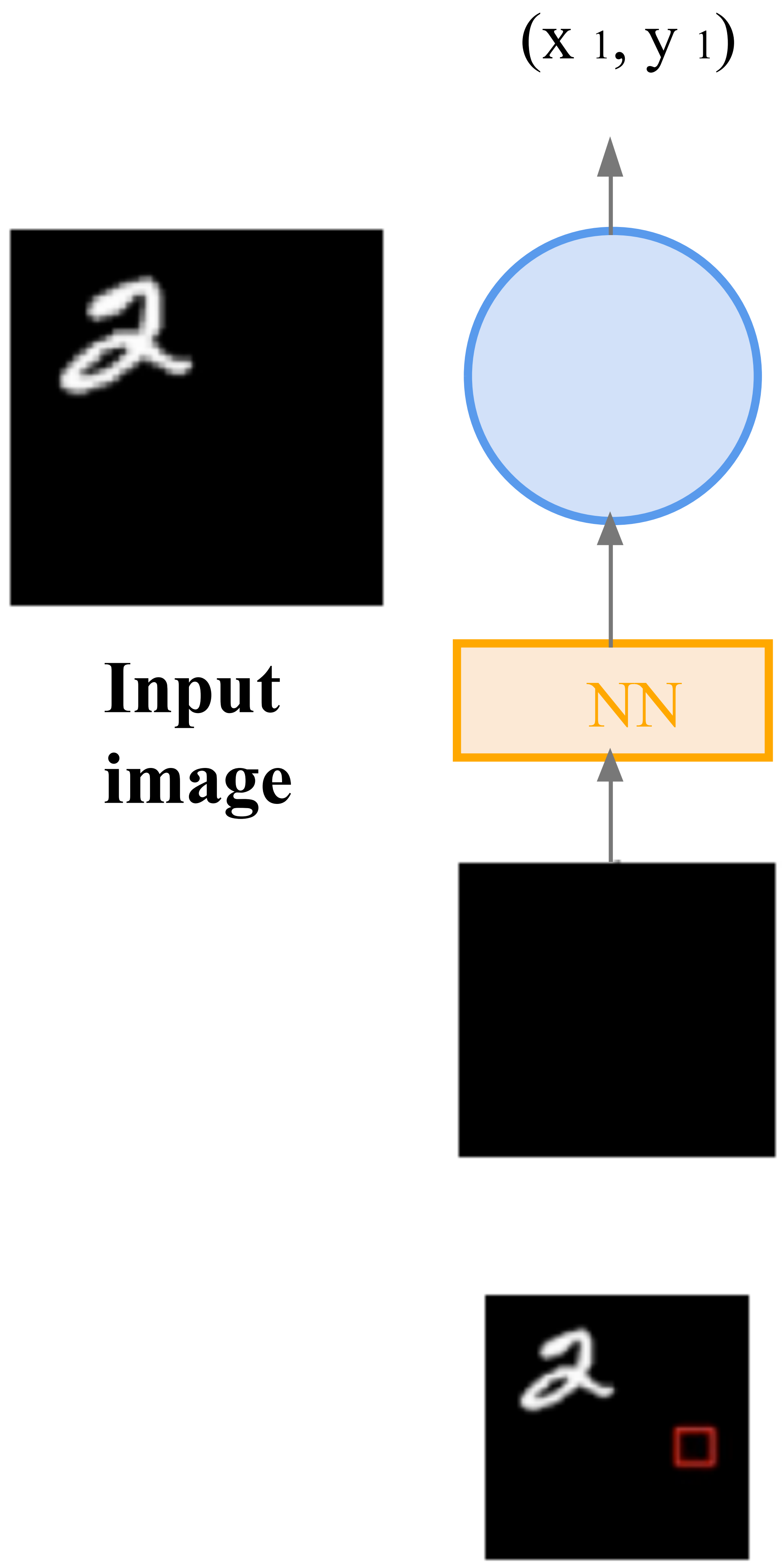


Glimpsing is a non-differentiable operation => learn policy for how to take glimpse actions using REINFORCE
Given state of glimpses seen so far, use RNN to model the state and output next action

[Mnih et al. 2014]

REINFORCE in action: Recurrent Attention Model (RAM)



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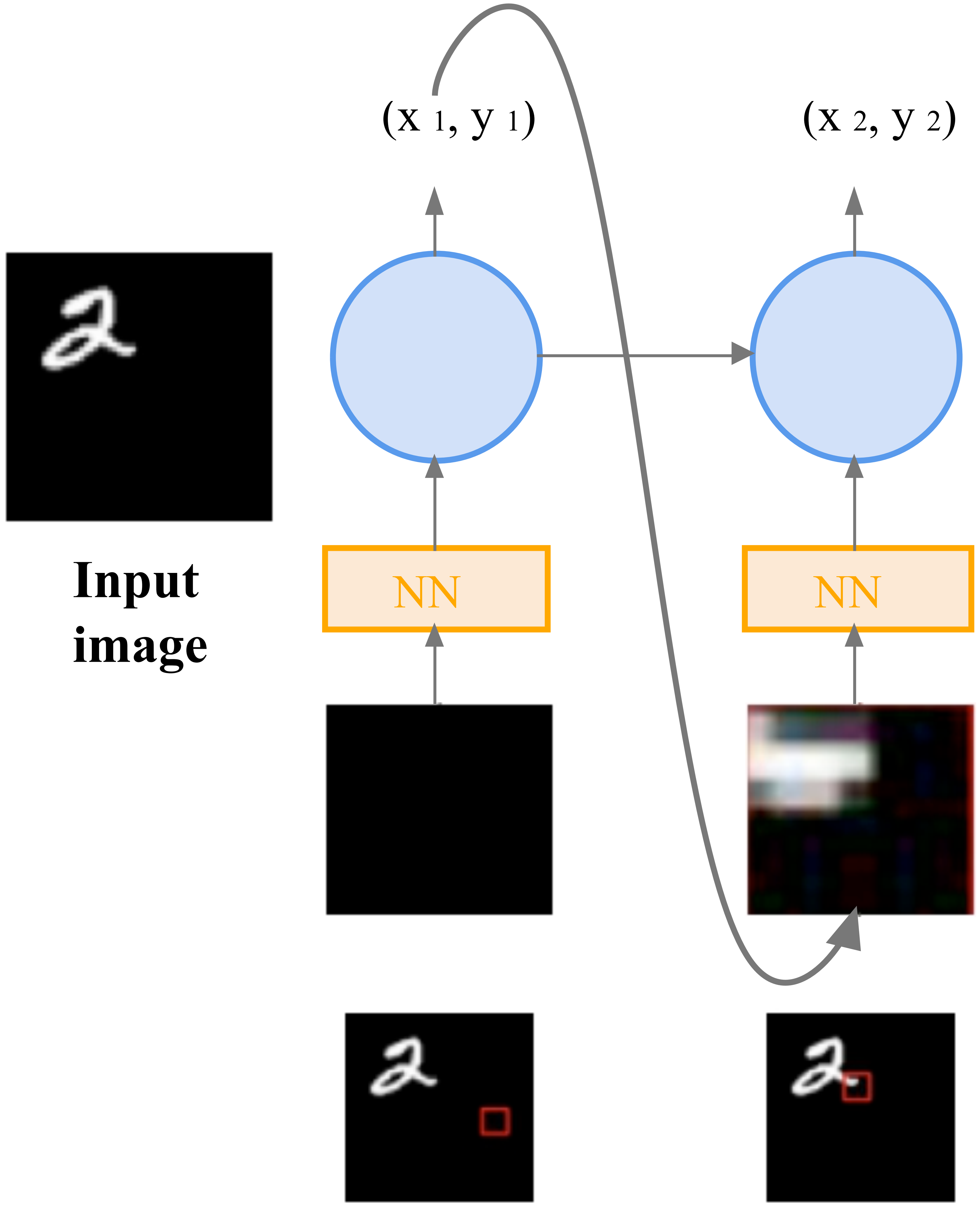

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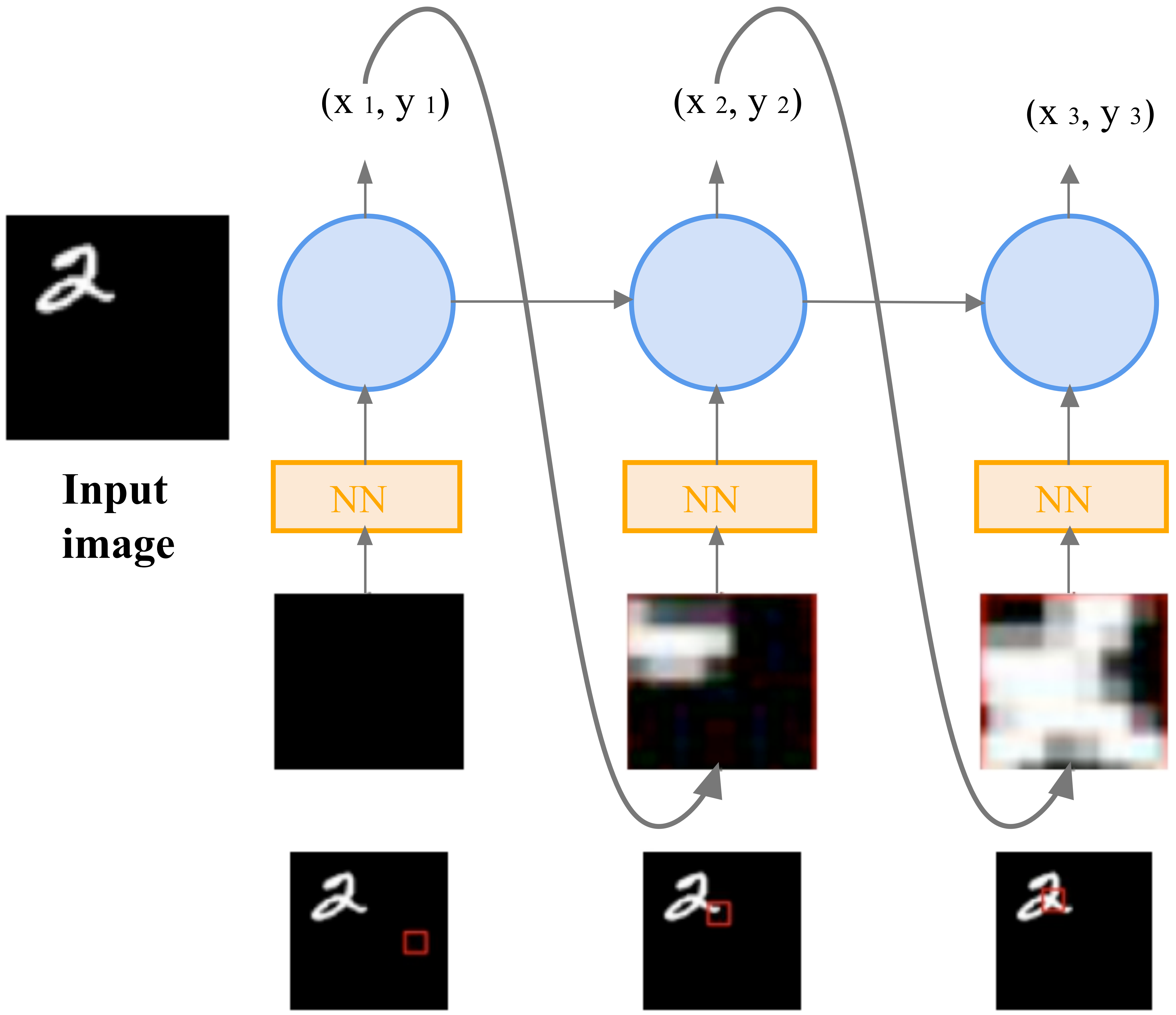

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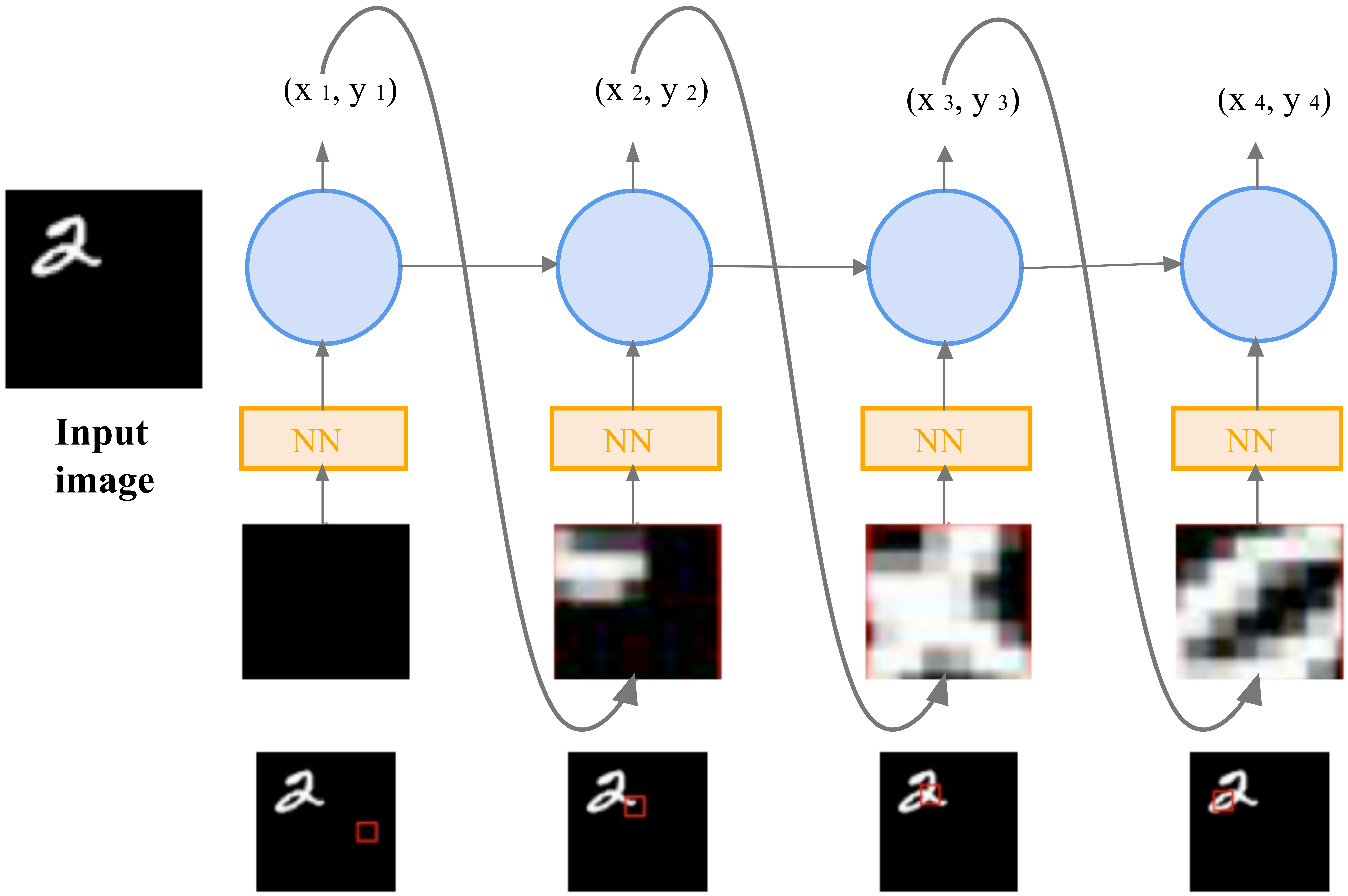

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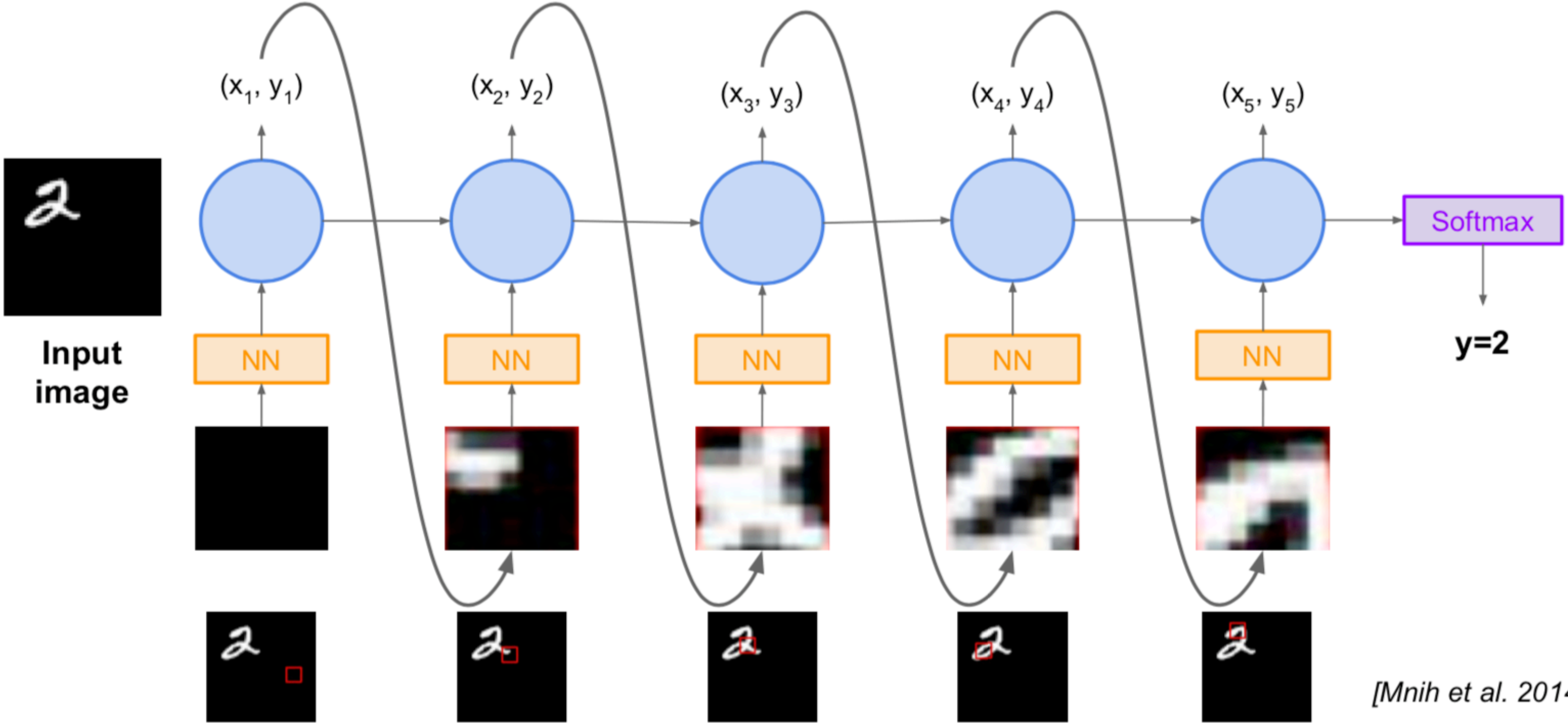

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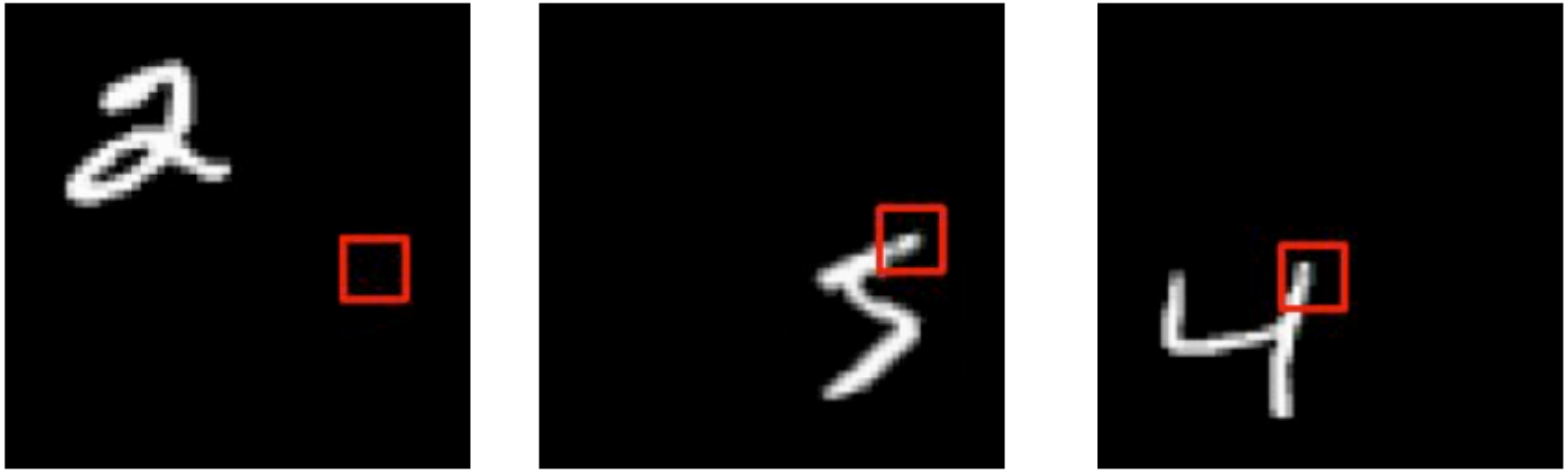


Has also been used in many other tasks including fine-grained image recognition, image captioning, and visual question-answering!

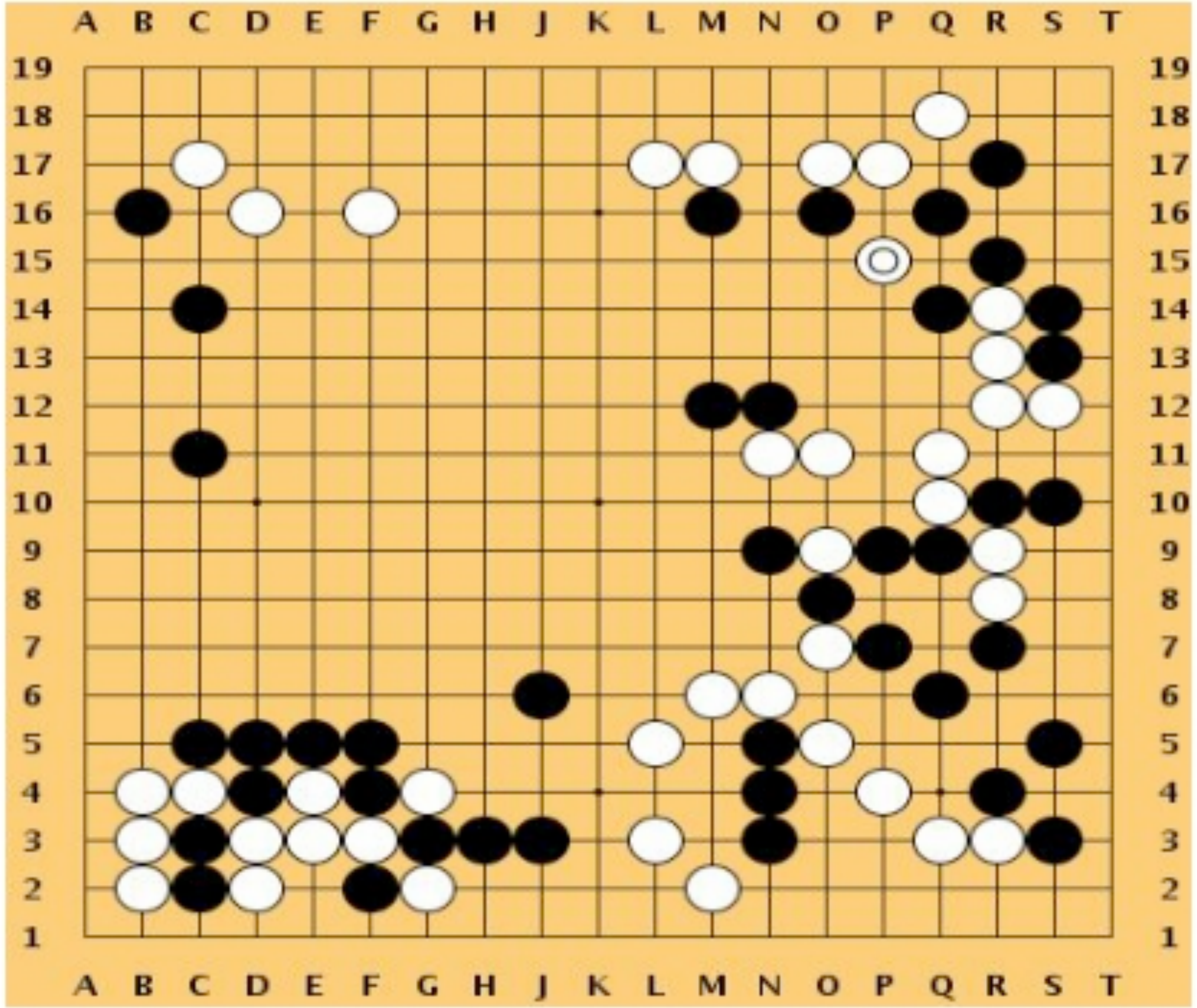
[Mnih et al. 2014]

More policy gradients: AlphaGo

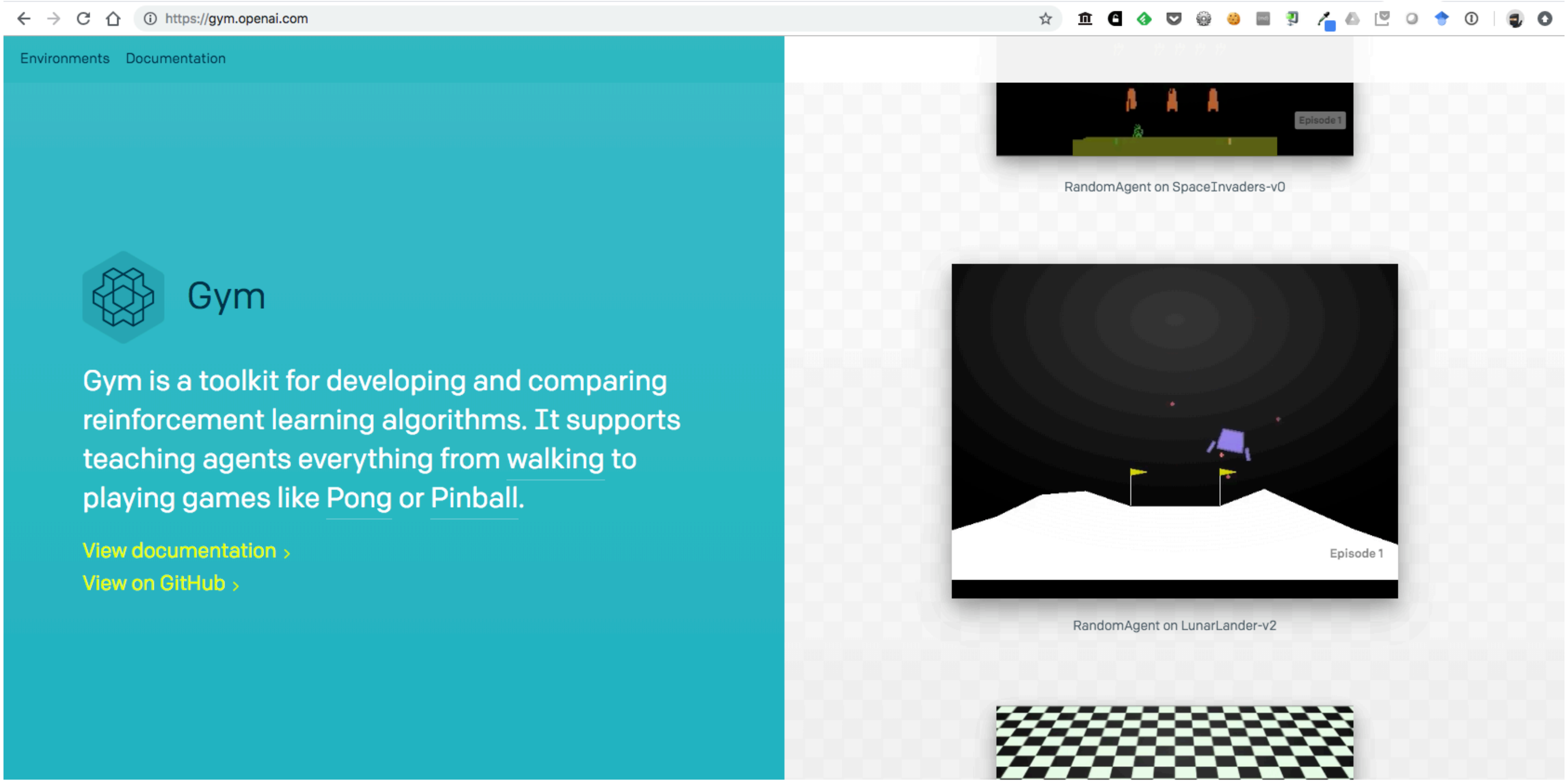
AlphaGo [Nature 2016]:
- Required many engineering tricks
- Bootstrapped from human play
- Beat 18-time world champion Lee Sedol

AlphaGo Zero [Nature 2017]:
- Simplified and elegant version of AlphaGo
- No longer bootstrapped from human play
- Beat (at the time) #1 world ranked Ke Jie

Alpha Zero: Dec. 2017
- Generalized to beat world champion programs on chess and shogi as well



OpenAI Gym


https://gym.openai.com
Environments
Documentation
Gym
Gym is a toolkit for developing and comparing reinforcement learning algorithms. It supports teaching agents everything from walking to playing games like Pong or Pinball.
View documentation ›
View on GitHub ›
Episode 1
RandomAgent on SpaceInvaders-v0
Episode 1
RandomAgent on LunarLander-v2

Summary

- **Policy gradients** : very general but suffer from high variance so requires a lot of samples. **Challenge** : sample-efficiency
- **Q-learning** : does not always work but when it works, usually more sample-efficient. **Challenge** : exploration

- Guarantees:
    - **Policy Gradients** : Converges to a local maximum of J, often good enough!
    - **Q-learning** : Zero guarantees since you are approximating Bellman equation with a complicated function approximator