

# ERRATA OF *INTRODUCTION TO TOPOLOGY IN AND VIA LOGIC*

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## CHAPTER 2

- **Exercise 2.7.** The definition of  $T(A)$  is:

$$T(A) = \{s \in \omega^\omega : s \triangleleft a \text{ for some } a \in A\}.$$

## CHAPTER 3

- **Figure 3.1.** We follow the convention of drawing Kripke frames, namely, we omit the relations obtained by reflexivity and transitivity.
- **Exercise 3.4.** The real numbers  $a$  and  $b$  should be required to satisfy  $a < b$ .
- **Exercise 3.7.** An *interior map* is a map that is continuous and open.
- **Exercise 3.9.** In (3), the map  $f$  should be required to respect  $\sim$ , that is, for any  $x, x' \in X$ ,  $x \sim x'$  implies  $f(x) = f(x')$ .

## CHAPTER 4

- **Definition 4.2.6.** We usually allow filters (filter bases) to be a collection of subsets, not necessarily open.
- **Example 4.4.1.** Existential quantifier in the definition of closed set  $U$  “ $\exists S \subseteq \text{Pol}(R)$ ” should not appear inside  $U = \{x \in \mathbb{R} : \dots\}$ ; instead,  $U$  is defined relative to  $S$ :

a set  $U$  is closed iff there is a collection  $S \subseteq \text{Pol}(R)$  of polynomials such that

$$U = \{x \in \mathbb{R} : \forall f \in S, f(x) = 0\}$$

- **Example 4.4.6.**  $\mathfrak{F} = (W, R)$  is reflexive and transitive, as in Example 2.2.3.
- **Definition 4.5.2.** The map  $f$  separating  $E, F$  should be continuous, as is suggested by the name.
- **Example 4.5.7.** Second to last paragraph, second to last line: “given  $\langle x, -x \rangle$ , consider the rectangle  $[x, x + \epsilon) \times [-x, -x + \epsilon)$  . . .”

## CHAPTER 5

- **Theorem 5.2.13.** There’s a gap in the proof. Indeed  $p_{i_0}^{-1}[U_{i_0}]$  only leaves  $(X_{i_0} \setminus U_{i_0}) \times \prod_{j \neq i} X_j$  uncovered in the product space, but the original subbasis cover may not have enough sets on the  $i_0$ -th coordinate, i.e.  $U_{i_k}$  where  $k \neq 0$ , to cover the remaining part of  $X_{i_0}$ . So the line “Then by assumption we can cover  $X_{i_j}$  using some of the sets  $U_k$  occurring in the cover above” is not well-justified. We need to use AC carefully; in fact, Tychonoff’s theorem is equivalent to AC.
- **Definition 5.3.2.**  $(Y, f)$  is a proper extension if  $f$  is a topological embedding and  $X$  is non-compact.

- **Definition 5.3.8.** A Stone-Čech compactification  $(\bar{Y}, i)$  is a compactification as in Definition 5.3.2 in case there's any confusion.