

ERRATA OF *INTRODUCTION TO TOPOLOGY IN AND VIA LOGIC*

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CHAPTER 2

- **Exercise 2.7.** The definition of $T(A)$ is:

$$T(A) = \{s \in \omega^\omega : s \triangleleft a \text{ for some } a \in A\}.$$

CHAPTER 3

- **Figure 3.1.** We follow the convention of drawing Kripke frames, namely, we omit the relations obtained by reflexivity and transitivity.
- **Exercise 3.4.** The real numbers a and b should be required to satisfy $a < b$.
- **Exercise 3.7.** An *interior map* is a map that is continuous and open.
- **Exercise 3.9.** In (3), the map f should be required to respect \sim , that is, for any $x, x' \in X$, $x \sim x'$ implies $f(x) = f(x')$.

CHAPTER 4

- **Definition 4.2.6.** We usually allow filters (filter bases) to be a collection of subsets, not necessarily open.
- **Example 4.4.1.** Existential quantifier in the definition of closed set U “ $\exists S \subseteq Pol(R)$ ” should not appear inside $U = \{x \in \mathbb{R} : \dots\}$; instead, U is defined relative to S :

a set U is closed iff there is a collection $S \subseteq Pol(R)$ of polynomials such that

$$U = \{x \in \mathbb{R} : \forall f \in S, f(x) = 0\}$$

- **Example 4.4.6.** $\mathfrak{J} = (W, R)$ is reflexive and transitive, as in Example 2.2.3.
- **Definition 4.5.2.** The map f separating E, F should be continuous, as is suggested by the name.
- **Example 4.5.7.** Second to last paragraph, second to last line: “given $\langle x, -x \rangle$, consider the rectangle $[x, x + \epsilon] \times [-x, -x + \epsilon]$. . .”

CHAPTER 5

- **Theorem 5.2.13.** There’s a gap in the proof. Indeed $p_{i_0}^{-1}[U_{i_0}]$ only leaves $(X_{i_0} \setminus U_{i_0}) \times \prod_{j \neq i} X_j$ uncovered in the product space, but the original subbasis cover may not have enough sets on the i_0 -th coordinate, i.e. U_{i_k} where $k \neq 0$, to cover the remaining part of X_{i_0} . So the line “Then by assumption we can cover X_{i_j} using some of the sets U_k occurring in the cover above” is not well-justified. We need to use AC carefully; in fact, Tychonoff’s theorem is equivalent to AC.

- **Definition 5.3.2.** (Y, f) is a proper extension if f is a topological embedding and X is non-compact.
- **Definition 5.3.8.** A Stone-Čech compactification (Y, i) is a compactification as in Definition 5.3.2 in case there's any confusion.