№37.6.

б) $A_e = \begin{pmatrix} 0 & 2 & 1 \\ -2 & 2 & 0 \\ -1 & 0 & 3 \end{pmatrix}; \quad \begin{cases} e_1' = e_1 + 2e_2 - e_3 \\ e_2' = e_2 - e_3 \\ e_3' = -e_1 + e_2 - 3e_3 \end{cases}$

$$T_{e\to e'} = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 1 \\ -1 & -1 & -3 \end{pmatrix}$$

$$A_{e'} = T_{e\to e'}^{T} \cdot A_e \cdot T_{e\to e'} =$$

$$= \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ -1 & 1 & -3 \end{pmatrix} \begin{pmatrix} 0 & 2 & 1 \\ -2 & 2 & 0 \\ -1 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 1 \\ -1 & -1 & -3 \end{pmatrix} =$$

$$= \begin{pmatrix} -3 & 6 & -2 \\ -1 & 2 & -3 \\ 1 & 0 & -10 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 1 \\ -1 & -1 & -3 \end{pmatrix} = \begin{pmatrix} 11 & 8 & 15 \\ 6 & 5 & 12 \\ 11 & 10 & 29 \end{pmatrix}$$

Ответ:

## №38.16

a) $f(x, y) = 2x_1y_1 - 3x_1y_2 - 4x_1y_3 + x_2y_1 - 5x_2y_3 + x_3y_3$

$q(x) = f(x, x) = 2x_1^2 - 3x_1x_2 - 4x_1x_3 + x_1x_2 - 5x_2x_3 + x_3x_3 =$

$= 2x_1^2 - 2x_1x_2 - 4x_1x_3 - 5x_2x_3 + x_3^2$

$$Q = \begin{pmatrix} 2 & -1 & -2 \\ -1 & 0 & -5/2 \\ -2 & -\frac{5}{2} & 1 \end{pmatrix} \Rightarrow$$

$\Rightarrow b(x, y) = 2x_1y_1 - x_1y_2 - 2x_1y_3 - y_1x_2 -$

$- \frac{5}{2}x_2y_3 - 2x_3y_1 - \frac{5}{2}x_3y_2 + x_3y_3$

## №1181

$q(x) = \underline{4x_1^2} + \underline{x_2^2 + x_3^2} - \underline{4x_1x_2 + 4x_1x_3} - 3x_2x_3 =$

$= (2x_1)^2 - 2 \cdot 2x_1 \cdot (x_2 - x_3) + (x_2 - x_3)^2 + 2x_2x_3 - 3x_2x_3 =$

$= (2x_1 + x_2 - x_3)^2 - x_2x_3 =$

$$= y_1^2 - (y_2 - y_3)(y_2 + y_3) =$$

$$\begin{cases} y_1 = 2x_1 - x_2 + x_3 \\ y_2 = \frac{x_2 + x_3}{2} \\ y_3 = \frac{x_3 - x_2}{2} \end{cases} \qquad \begin{cases} x_2 = y_2 - y_3 \\ x_3 = y_2 + y_3 \end{cases}$$

$$= \boxed{y_1^2 - y_2^2 + y_3^2} \qquad \begin{cases} x_1 = \frac{y_1 - 2y_3}{2} \\ x_2 = y_2 - y_3 \\ x_3 = y_2 + y_3 \end{cases}$$

Ответ.

## №1183.

$$q(x) = 2x_1^2 + 18x_2^2 + 8x_3^2 - 12x_1x_2 + 8x_1x_3 - 27x_2x_3 =$$

$$= 2x_1^2 - x_1(12x_2 - 8x_3) + 18x_2^2 + 8x_3^2 - 27x_2x_3 =$$

$$\cancel{= (x_1^2 - 2\cdot x_1 \cdot 6x_2 + 36x_2^2) + (x_1^2 + 2\cdot x_1 \cdot 4x_3 + 16x_3^2) -}$$

$$\cancel{- 36x_2^2 + 18x_2^2 - 16x_3^2 + 8x_3^2 - 27x_2x_3 =}$$

$$(x_1\sqrt{2})^2 - 2\cdot x_1\sqrt{2}\cdot\left(3x_2\sqrt{2} - 2x_3\sqrt{2}\right) + 18x_2^2 + 8x_3^2 - 27x_2x_3 =$$

$$=\left((x_1\sqrt{2})^2 - 2\cdot(x_1\sqrt{2})(3x_2\sqrt{2} - 2x_3\sqrt{2}) + (3x_2\sqrt{2} - 2x_3\sqrt{2})^2\right) -$$

$$-\underbrace{(3x_2\sqrt{2} - 2x_3\sqrt{2})^2}_{18x_2^2 - 24x_2x_3 + 8x_3^2} + 18x_2^2 + 8x_3^2 - 27x_2x_3 =$$

$$= (\underbrace{x_1\sqrt{2} - 3x_2\sqrt{2} + 2x_3\sqrt{2}}_{y_1})^2 - 3x_2x_3 =$$

$$\begin{cases} y_1 = x_1\sqrt{2} - 3x_2\sqrt{2} + 2x_3\sqrt{2} \\ y_2 = \dfrac{x_2 + x_3}{2} \\ y_3 = \dfrac{x_3 - x_2}{2} \end{cases} \qquad \begin{cases} x_2 = y_2 - y_3 \\ x_3 = y_2 + y_3 \end{cases}$$

$$= y_1^2 - 3(y_2 - y_3)(y_2 + y_3) = y_1^2 - 3(y_2^2 - y_3^2) =$$

$$= y_1^2 - 3y_2^2 + 3y_3^2 = z_1^2 - z_2^2 + z_3^2$$

$$\begin{cases} z_1 = y_1 \\ z_2 = y_2\sqrt{3} \\ z_3 = y_3\sqrt{3} \end{cases} \Rightarrow \begin{cases} y_1 = z_1 \\ y_2 = z_2/\sqrt{3} \\ y_3 = z_3/\sqrt{3} \end{cases}$$

$$\Rightarrow \begin{cases} z_1 = x_1\sqrt{2} - 3x_2\sqrt{2} + 2x_3\sqrt{2} \\ x_2 = \dfrac{z_2}{\sqrt{3}} - \dfrac{z_3}{\sqrt{3}} \\ x_3 = \dfrac{z_2}{\sqrt{3}} + \dfrac{z_3}{\sqrt{3}} \end{cases}$$

$$z_1 = x_1\sqrt{2} - 3\left(\frac{z_2 - z_3}{\sqrt{3}}\right)\sqrt{2} + 2\left(\frac{z_2 + z_3}{\sqrt{3}}\right)\sqrt{2}$$

$$z_1 = x_1\sqrt{2} - z_2\sqrt{6} + z_3\sqrt{6} + \frac{2z_2\sqrt{6}}{3} + \frac{2z_3\sqrt{6}}{3}$$

$$z_1 = x_1\sqrt{2} - z_2 \cdot \frac{\sqrt{6}}{3} + \frac{5\sqrt{6}}{3} z_3$$

$$\frac{z_1}{\sqrt{2}} = x_1 - \frac{z_2}{\sqrt{3}} + \frac{5z_3}{\sqrt{3}}$$

$$x_1 = \frac{z_1}{\sqrt{2}} + \frac{z_2}{\sqrt{3}} - \frac{5z_3}{\sqrt{3}}$$

Ответ: $q(x) = z_1^2 - z_2^2 + z_3^2$,

$$\begin{cases} x_1 = z_1 \frac{\sqrt{2}}{2} + z_2 \frac{\sqrt{3}}{3} - z_3 \cdot \frac{5\sqrt{3}}{3} \\ x_2 = z_2 \sqrt{3}/3 - z_3 \sqrt{3}/3 \\ x_3 = z_2 \sqrt{3}/3 + z_3 \sqrt{3}/3 \end{cases}$$

№1185.

$q(x) = \cancel{\ldots} x_1x_2 + x_2x_3 + x_3x_4 + x_4x_1 =$

$= x_1(x_2+x_4) + x_3(x_2+x_4) =$

$= (x_1+x_3)(x_2+x_4) = (y_1-y_2)(y_1+y_2) = \boxed{y_1^2 - y_2^2}$

$\begin{cases} x_1 + x_3 = y_1 - y_2 \\ x_2 + x_4 = y_1 + y_2 \\ y_3 = x_3 \\ y_4 = x_4 \end{cases}$ $\begin{cases} x_1 = y_1 - y_2 - y_3 \\ x_2 = y_1 + y_2 - y_4 \\ x_3 = y_3 \\ x_4 = y_4 \end{cases}$

Ответ: $y_1^2 - y_2^2$;

№1190

$f = 2x_1^2 + 9x_2^2 + 3x_3^2 + 8x_1x_2 - 4x_1x_3 - 10x_2x_3$

$g = 2y_1^2 + 3y_2^2 + 6y_3^2 - 4y_1y_2 - 4y_1y_3 + 8y_2y_3$

$A_f = \begin{pmatrix} 2 & 4 & -2 \\ 4 & 9 & -5 \\ -2 & -5 & 3 \end{pmatrix}; A_g = \begin{pmatrix} 2 & -2 & -2 \\ -2 & 3 & 4 \\ -2 & 4 & 6 \end{pmatrix}$

$$\cancel{A_g = T_{f\to g}^{T} A_f \cdot T_{f\to g}}$$

$$\cancel{A_f = T_{g\to f}^{T} A_g \cdot T_{g\to f}}$$

$$\cancel{\begin{pmatrix} 2 & 4 & -2 \\ 4 & 9 & -5 \\ -2 & -5 & 3 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} 2 & -2 & -2 \\ -2 & 3 & 4 \\ -2 & 4 & 6 \end{pmatrix} \cdot \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}$$

$$f = 2x_1^2 + 9x_2^2 + 3x_3^2 + 8x_1x_2 - 4x_1x_3 - 10x_2x_3 =$$

$$= (x_1\sqrt{2})^2 + 2\cdot x_1\sqrt{2}\cdot(2x_2\sqrt{2}) - 2\cdot x_1\sqrt{2}\cdot(x_3\sqrt{2}) +$$

$$+ 9x_2^2 + 3x_3^2 - 10x_2x_3 =$$

$$= (x_1\sqrt{2})^2 + 2\cdot x_1\sqrt{2}\left(2x_2\sqrt{2} - x_3\sqrt{2}\right) + (2x_2\sqrt{2} - x_3\sqrt{2})^2 -$$

$$- 8x_2^2 + 8x_2x_3 - 2x_3^2 + 9x_2^2 + 3x_3^2 - 10x_2x_3 =$$

$$= \left(x_1\sqrt{2} + 2x_2\sqrt{2} - x_3\sqrt{2}\right)^2 + x_2^2 + x_3^2 - 2x_2x_3 =$$

$$= \left(x_1\sqrt{2} + 2x_2\sqrt{2} - x_3\sqrt{2}\right)^2 + (x_2 - x_3)^2 =$$

$$= q_1^2 + q_2^2$$

$$\begin{cases} q_1 = x_1\sqrt{2} + 2x_2\sqrt{2} - x_3\sqrt{2} \\ q_2 = x_2 - x_3 \\ q_3 = x_3 \end{cases}$$

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$$g = 2y_1^2 + 3y_2^2 + 6y_3^2 - 4y_1y_2 - 4y_1y_3 + 8y_2y_3 =$$

$$= (y_1\sqrt{2})^2 - 2\cdot y_1\cdot(2y_2 + 2y_3) + 3y_2^2 + 6y_3^2 + 8y_2y_3 =$$

$$= (y_1\sqrt{2})^2 - 2\cdot(y_1\sqrt{2})\cdot(y_2\sqrt{2} + y_3\sqrt{2}) + (y_2\sqrt{2} + y_3\sqrt{2})^2 -$$

$$- 2y_2^2 - 4y_2y_3 - 2y_3^2 + 3y_2^2 + 6y_3^2 + 8y_2y_3 =$$

$$= \left(y_1\sqrt{2} \cancel{+} - y_2\sqrt{2} - y_3\sqrt{2}\right)^2 + y_2^2 + 4y_2y_3 + 4y_3^2 =$$

$$= \left(y_1\sqrt{2} - y_2\sqrt{2} - y_3\sqrt{2}\right)^2 + (y_2 + 2y_3)^2 = q_1^{*2} + q_2^{*2}$$

$$\begin{cases} q_1^* = y_1\sqrt{2} - y_2\sqrt{2} - y_3\sqrt{2} \\ q_2^* = y_2 + 2y_3 \\ q_3^* = \cancel{y_2+2y} \; y_3 \end{cases}$$

$$\Rightarrow \begin{cases} x_1\sqrt{2} + 2x_2\sqrt{2} - x_3\sqrt{2} = y_1\sqrt{2} - y_2\sqrt{2} - y_3\sqrt{2} \\ x_2 - x_3 = y_2 + 2y_3 \\ y_3 = x_3 \end{cases}$$

$$\begin{cases} x_1\cancel{\sqrt{2}} + 2y_2\cancel{\sqrt{2}} + 6y_3\cancel{\sqrt{2}} - y_3\cancel{\sqrt{2}} = y_1\cancel{\sqrt{2}} - y_2\cancel{\sqrt{2}} - y_3\cancel{\sqrt{2}} \\ x_2 = y_2 + 3y_3 \\ x_3 = y_3 \end{cases}$$

$$\begin{cases} x_1 = y_1 - 3y_2 - 6y_3 \\ x_2 = y_2 + 3y_3 \\ x_3 = y_3 \end{cases}$$

Ответ: $\nearrow$

№1191

$f = 3x_1^2 + 10x_2^2 + 25x_3^2 - 12x_1x_2 - 18x_1x_3 + 40x_2x_3 =$

$= 3(x_1^2 - 4x_1x_2 - 6x_1x_3) + 10x_2^2 + 25x_3^2 + 40x_2x_3 =$

$= 3\left(x_1^2 - 2\cdot x_1\cdot(2x_2 + 3x_3) + (2x_2+3x_3)^2 - 4x_2^2 - 12x_2x_3 - \right.$

$\left. - 9x_3^2\right) + 10x_2^2 + 25x_3^2 + 40x_2x_3 =$

$$= 3(x_1 - 2x_2 - 3x_3)^2 - 12x_2^2 - 36x_2x_3 - 27x_3^2 +$$
$$+ 10x_2^2 + 25x_3^2 + 40x_2x_3 -$$

$$= 3(x_1 - 2x_2 - 3x_3)^2 - 2x_2^2 + 4x_2x_3 - 2x_3^2 =$$

$$= 3(x_1 - 2x_2 - 3x_3)^2 - \left((x_2\sqrt{2})^2 - 2\cdot x_2\sqrt{2}\cdot x_3\sqrt{2} + (x_3\sqrt{2})^2\right) =$$

$$= 3(x_1 - 2x_2 - 3x_3)^2 - (x_2\sqrt{2} - x_3\sqrt{2})^2$$

---

$$g = 5y_1^2 + 6y_2^2 + 12y_1y_2 =$$

$$\cancel{y_1^2 + 2\cdot y_1 \cdot 6y_2} = 6y_1^2 + 2\cdot\sqrt{6}y_1\cdot y_2\sqrt{6} + 6y_2^2 - y_1^2 =$$

$$= (y_1\sqrt{6} + y_2\sqrt{6})^2 - y_1^2 = 3(y_1\sqrt{2} + y_2\sqrt{2})^2 - y_1^2$$

---

$$f = g \Rightarrow \begin{cases} x_1 - 2x_2 - 3x_3 = y_1\sqrt{2} + y_2\sqrt{2} \\ x_2\sqrt{2} - x_3\sqrt{2} = y_1\frac{\sqrt{2}}{2} \\ x_3 = y_3 \end{cases}$$

$$\cancel{\begin{cases} x_1 - 2(y_1 + y_3\sqrt{2}) - 3y_3 = y_1\sqrt{2} + y_2\sqrt{2} \\ x_2 = y_1 + y_3\sqrt{2} \\ x_3 = y_3 \end{cases}}$$

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$$\begin{cases} x_1 - 2y_1 - 2y_3\sqrt{2} - 3y_3 = y_1\sqrt{2} + y_2\sqrt{2} \\ x_2 = y_1 + y_3\sqrt{2} \\ x_3 = y_3 \end{cases}$$

$$\begin{cases} x_1 = y_1(2+\sqrt{2}) + y_2\sqrt{2} + y_3(2\sqrt{2}+3) \\ x_2 = y_1 + y_2\sqrt{2} \\ x_3 = y_3 \end{cases}$$

Ответ:

$$\begin{cases} x_1 - 2y_3 - y_1\sqrt{2} - 3y_3 = y_1\sqrt{2} + y_2\sqrt{2} \\ x_2 = y_3 + y_1\frac{\sqrt{2}}{2} \\ x_3 = y_3 \end{cases}$$

$$\begin{cases} x_1 = y_1 \cdot 2\sqrt{2} + y_2\sqrt{2} + 5y_3 \\ x_2 = y_1\frac{\sqrt{2}}{2} + y_3 \\ x_3 = y_3 \end{cases}$$

Ответ: ↗

## №1201

$$f_1 = x_1^2 - x_2 x_3 = x_1'^2 - (x_2' - x_3')(x_2' + x_3') = x_1'^2 - x_2'^2 + x_3'^2$$

$$f_2 = y_1 y_2 - y_3^2 = (y_1' - y_2')(y_1' + y_2') - y_3'^2 = y_1'^2 - y_2'^2 - y_3'^2$$

$$f_3 = z_1 z_2 + z_3^2 = (z_1' - z_2')(z_1' + z_2') + z_3^2 = z_1'^2 - z_2'^2 + z_3^2$$

Сигнатуры: $f_1 - (2,1)$

$f_2 - (1,2)$

$f_3 - (2,1)$

$\Rightarrow$ $f_1$ и $f_3$ эквивалентны.

## №1202

1) $f_1 = x_1^2 + 4x_2^2 + x_3^2 + 4x_1x_2 - 2x_1x_3 =$

$= x_1^2 + 2\cdot x_1 \cdot 2x_2 + (2x_2)^2 + x_3^2 - 2x_1x_3 = (x_1 + 2x_2)^2 + x_3^2 - 2x_1x_3 =$

$= (x_1 + 2x_2)^2 + x_3^2 - 2x_1x_3 + x_1^2 - x_1^2 =$

$= (x_1 + 2x_2)^2 + (x_3 - x_1)^2 - x_1^2$

$\Rightarrow$ сигнатура: $(2, 1)$

2) $f_2 = \underline{y_1^2} + 2y_2^2 - y_3^2 \underline{+ 4y_1y_2} \underline{- 2y_1y_3} - 4y_2y_3 =$

$= \cancel{y_1^2 - 2y_1y_3 + y_3^2}$

$$= y_1^2 + 2 \cdot y_1 \cdot (2y_2 - y_3) + 2y_2^2 - y_3^2 - 4y_2y_3 =$$

$$= y_1^2 + 2y_1(2y_2 - y_3) + (2y_2 - y_3)^2 - 4y_2^2 + \cancel{4y_2y_3} - y_3^2 +$$

$$+ 2y_2^2 - y_3^2 - \cancel{4y_2y_3} =$$

$$= (y_1 + 2y_2 - y_3)^2 - 2y_2^2 - 2y_3^2$$

$\Rightarrow$ сигнатура: $(1, 2)$

3) $f_3 = \underline{-4z_1^2} - z_2^2 - z_3^2 \underline{-4z_1z_2} \underline{+4z_1z_3} + 18z_2z_3 =$

$$= -(2z_1)^2 - 2 \cdot 2z_1(z_2 - z_3) - z_2^2 - z_3^2 + 18z_2z_3 =$$

$$= -(2z_1)^2 - 2 \cdot 2z_1(z_2 - z_3) - (z_2 - z_3)^2 + (z_2 - z_3)^2 -$$

$$- z_2^2 - z_3^2 + 18z_2z_3 =$$

$$= -\left((2z_1)^2 + 2 \cdot 2z_1(z_2 - z_3) + (z_2 - z_3)^2\right) +$$

$$+ \cancel{z_2^2} - 2z_2z_3 + \cancel{z_3^2} - \cancel{z_2^2} - \cancel{z_3^2} + 18z_2z_3 =$$

$$= -(2z_1 + z_2 - z_3)^2 + 16z_2z_3 =$$

$$= -(2z_1 + z_2 - z_3)^2 + 16z_2'^2 - 16z_3'^2$$

$\Rightarrow$ сигнатура: $(1, 2)$

$\Rightarrow f_2, f_3$ эквивалентны.