№1.

$F(x,y) = x^2 + 9y^2 - 6xy - 8x\sqrt{10} + 4y\sqrt{10} + 50 = 0$

1) $A = \begin{pmatrix} 1 & -3 \\ -3 & 9 \end{pmatrix}$

$\chi_A(\lambda) = \begin{vmatrix} 1-\lambda & -3 \\ -3 & 9-\lambda \end{vmatrix} = (1-\lambda)(9-\lambda) - 9 =$

$= 9 - 10\lambda + \lambda^2 - 9 =$

$= \lambda^2 - 10\lambda =$

$= \lambda(\lambda - 10) \Rightarrow \left[ \begin{array}{l} \lambda = 0 \\ \lambda_2 = 10 \end{array} \right.$ - с.з.

① $\lambda_1 = 0 \Rightarrow$

$A - \lambda_1 E \sim (1 \;\; -3) \Rightarrow$ с.в.: $\left\{ \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right\} \sim \frac{1}{\sqrt{10}} \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

② $\lambda_2 = 10$

$A - \lambda_2 E = \begin{pmatrix} -9 & -3 \\ -3 & -1 \end{pmatrix} \sim (3 \;\; 1) \Rightarrow$ с.в.: $\left\{ \begin{pmatrix} 1 \\ -3 \end{pmatrix} \right\} \sim \frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$\Rightarrow U = T_{e \to e'} = \begin{pmatrix} 3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & -3/\sqrt{10} \end{pmatrix}$ — ортог. преобразование в котором $A' = \begin{pmatrix} 0 & 0 \\ 0 & 10 \end{pmatrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = U \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & -3/\sqrt{10} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\Rightarrow \begin{cases} x = \frac{3}{\sqrt{10}} x' + \frac{1}{\sqrt{10}} y' \\ y = \frac{1}{\sqrt{10}} x' - \frac{3}{\sqrt{10}} y' \end{cases}$$ – поворот

$$\Rightarrow F(x,y) \sim F(x',y') = 10(y')^2 - 8\left(\frac{3}{\sqrt{10}} x' + \frac{1}{\sqrt{10}} y'\right)\sqrt{10} + 4\left(\frac{1}{\sqrt{10}} x' - \frac{3}{\sqrt{10}} y'\right)\sqrt{10} + 50 = 0$$

$$10y'^2 - 24x' - 8y' + 4x' - 12y' + 50 = 0$$

$$10y'^2 - 20y' - 20x' + 50 = 0$$

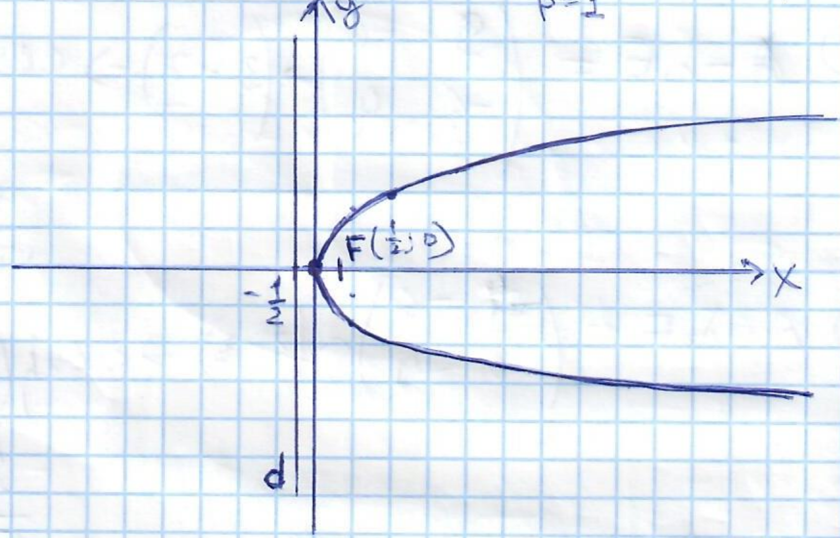
$$10(y'^2 - 2y' + 1) - 20x' + 40 = 0 \quad | :10$$

$$(y'-1)^2 - 2(x'-2) = 0$$

Пусть $\begin{cases} y'' = y' - 1 \\ x'' = x' - 2 \end{cases}$ – сдвиг.

$\Rightarrow F(x,y) \sim F(x'', y'') = y''^2 - 2x'' = 0$, то

есть это кривая $y''^2 = 2 \cdot \underbrace{1}_{p=1} \cdot x''$ – парабола




$p = 1 \Rightarrow F\left(\frac{1}{2}; 0\right)$

$d: x = -\frac{1}{2}$

$\varepsilon = 1$

№2.

$5x^2 - 12xy + 32\sqrt{13}\,x - 24\sqrt{13}\,y + 536 = 0$

$A = \begin{pmatrix} 5 & -6 \\ -6 & 0 \end{pmatrix}$

$\chi_A(\lambda) = \begin{vmatrix} 5-\lambda & -6 \\ -6 & -\lambda \end{vmatrix} = -5\lambda + \lambda^2 - 36 = (\lambda+4)(\lambda-9)$

$$\Rightarrow \left[\begin{array}{l} \lambda_1 = -4 \\ \lambda_2 = 9 \end{array}\right. - \text{С}3.$$

$$① \; A - \lambda_1 E = \begin{pmatrix} 9 & -6 \\ -6 & 4 \end{pmatrix} \to \begin{pmatrix} 3 & -2 \end{pmatrix} \Rightarrow \text{с.в.: } \left\{ \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\} \sim \frac{1}{\sqrt{13}} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$② \; A - \lambda_2 E = \begin{pmatrix} -4 & -6 \\ -6 & -9 \end{pmatrix} \sim \begin{pmatrix} 2 & 3 \end{pmatrix} \Rightarrow \text{с.в.: } \left\{ \begin{pmatrix} -3 \\ 2 \end{pmatrix} \right\} \sim \frac{1}{\sqrt{13}} \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

$$\Rightarrow U = \frac{1}{\sqrt{13}} \begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix}$$ – ортогональное преобразование.

Тогда

$$A' = \begin{pmatrix} -4 & 0 \\ 0 & 9 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = U \begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{\sqrt{13}} \begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\begin{cases} x = \frac{2}{\sqrt{13}} x' - \frac{3}{\sqrt{13}} y' \\ y = \frac{3}{\sqrt{13}} x' + \frac{2}{\sqrt{13}} y' \end{cases}$$ – поворот

$$\Rightarrow F(x,y) \sim F(x',y') = -4x'^2 + 9y'^2 + 32\sqrt{13}\left(\frac{2}{\sqrt{13}}x' - \frac{3}{\sqrt{13}}y'\right) - 24\sqrt{13}\left(\frac{3}{\sqrt{13}}x' + \frac{2}{\sqrt{13}}y'\right) + 536 = 0$$

$$-4x'^2 + 9y'^2 - 8x' - 144y' + 536 = 0$$

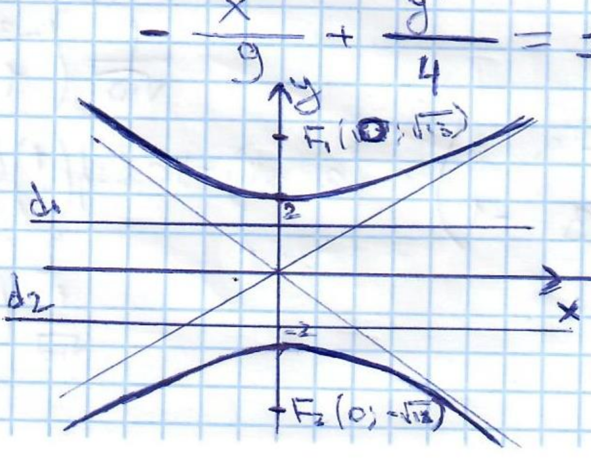
$$-4(x'^2 + 2x' + 1) + 4 + 9(y'^2 - 16y' + 64) - 9\cdot 64 + 536 = 0$$

$$-4(x'+1)^2 + 9(y'-8)^2 - 36 = 0$$

Пусть $\begin{cases} x'' = x' + 1 \\ y'' = y' - 8 \end{cases}$ – сдвиг

$$\Rightarrow -4x''^2 + 9y''^2 = 36 \quad \Big| :36$$

$$-\frac{x''^2}{9} + \frac{y''^2}{4} = 1$$ – сопряжённая гипербола




$a = 3;\ b = 2$

асимптоты: $y = \pm\frac{2}{3}x$

$c = \sqrt{13}$ – фокусное расстояние

$$\varepsilon = \frac{c}{b} = \frac{\sqrt{13}}{2}$$

$d: y = \pm \frac{4}{\sqrt{13}}$

$F: (0; \sqrt{13})$ и $(0; -\sqrt{13})$

№3.

$3x^2 + 11y^2 + 6xy + 6\sqrt{10}\,x - 2\sqrt{10}\,y - 22 = 0$

$$A = \begin{pmatrix} 3 & 3 \\ 3 & 11 \end{pmatrix}$$

$$\chi_A(\lambda) = \begin{vmatrix} 3-\lambda & 3 \\ 3 & 11-\lambda \end{vmatrix} = (3-\lambda)(11-\lambda) - 9 = \lambda^2 - 14\lambda + 24 = 0$$

$$\left[\begin{array}{l} \lambda_1 = 2 \\ \lambda_2 = 12 \end{array}\right. - \text{c.з.}$$

① $A - \lambda_1 E = \begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix} \sim (1 \;\; 3) \Rightarrow$ с.в. $\left\{ \begin{pmatrix} -3 \\ 1 \end{pmatrix} \right\}$

$$\frac{1}{\sqrt{10}} \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

② $A - \lambda_2 E = \begin{pmatrix} -9 & 3 \\ 3 & -1 \end{pmatrix} \sim (3 \;\; -1) \Rightarrow$ с.в. $\left\{ \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\}$

$$\frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$\Rightarrow U = \frac{1}{\sqrt{10}}\begin{pmatrix} -3 & 1 \\ 1 & 3 \end{pmatrix}$ — ортог. преобразование

$$A' = \begin{pmatrix} 2 & 0 \\ 0 & 12 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = U\begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{1}{\sqrt{10}}\begin{pmatrix} -3 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$\Rightarrow \begin{cases} x = -\frac{3}{\sqrt{10}}x' + \frac{1}{\sqrt{10}}y' \\ y = \frac{1}{\sqrt{10}}x' + \frac{3}{\sqrt{10}}y' \end{cases}$$

$\Rightarrow F(x,y) \sim F(x',y') = 2x'^2 + 12y'^2 +$

$+ 6\sqrt{10}\left(-\frac{3}{\sqrt{10}}x' + \frac{1}{\sqrt{10}}y'\right) - 2\sqrt{10}\left(\frac{1}{\sqrt{10}}x' + \frac{3}{\sqrt{10}}y'\right) - 22 = 0$

$$2x'^2 + 12y'^2 - 20x' - 22 = 0$$

$$\underline{x'^2} + 6y'^2 \underline{- 10x'} - 11 = 0$$

$$(x' - 5)^2 + 6y'^2 = 36$$

$$\begin{cases} x'' = x' - 5 \\ y'' = y' \end{cases} \text{ — сдвиг}$$

$$x''^2 + 6y''^2 = 36$$

$$\frac{x''^2}{6^2} + \frac{y''^2}{(\sqrt{6})^2} = 1$$ – эллипс

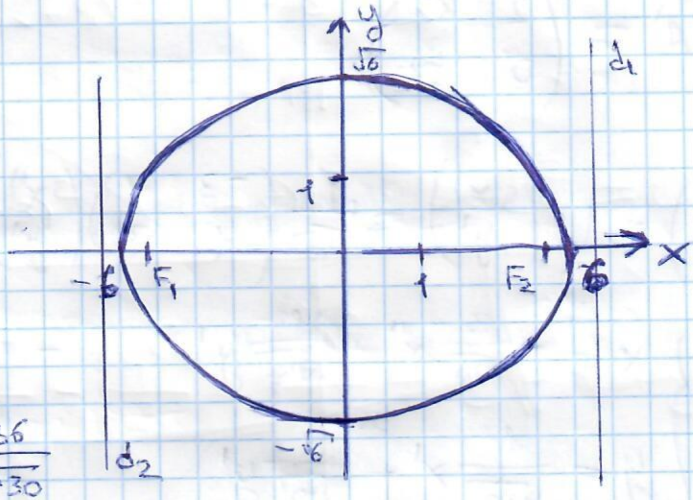
$a = 6$

$b = \sqrt{6}$

$c = \sqrt{30}$

$\varepsilon = \frac{\sqrt{30}}{6}$

$d: \; x = \pm \frac{36}{\sqrt{30}}$




$F: \; (\sqrt{30}; 0)$ и $(-\sqrt{30}; 0)$

№38.10

2) $4x^2 - y^2 - z^2 + 32x - 12z + 44 = 0$

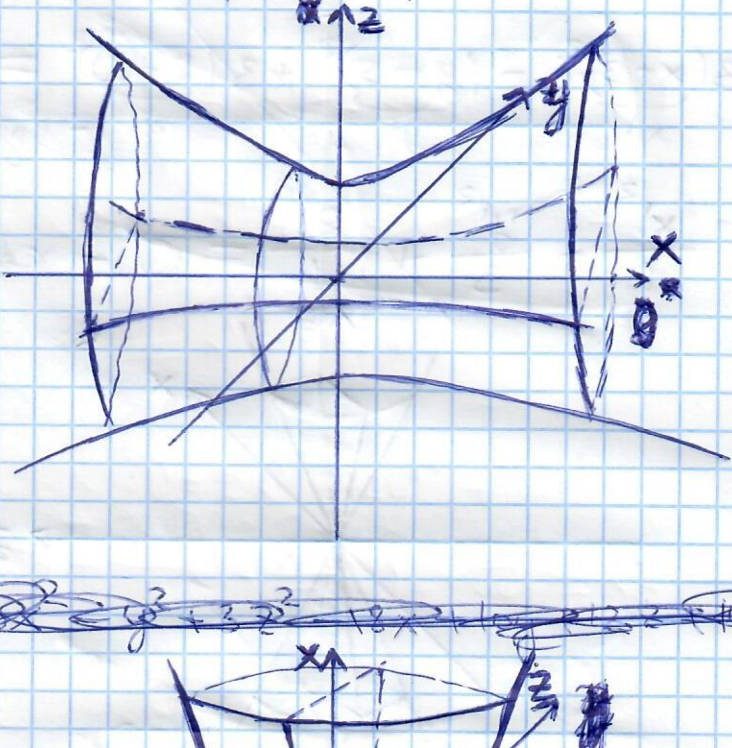
$$4(x^2 + 8x + 16) - (y^2) - (z^2 + 12z + 36) + 44 - 4\cdot 16 + 36 = 0$$

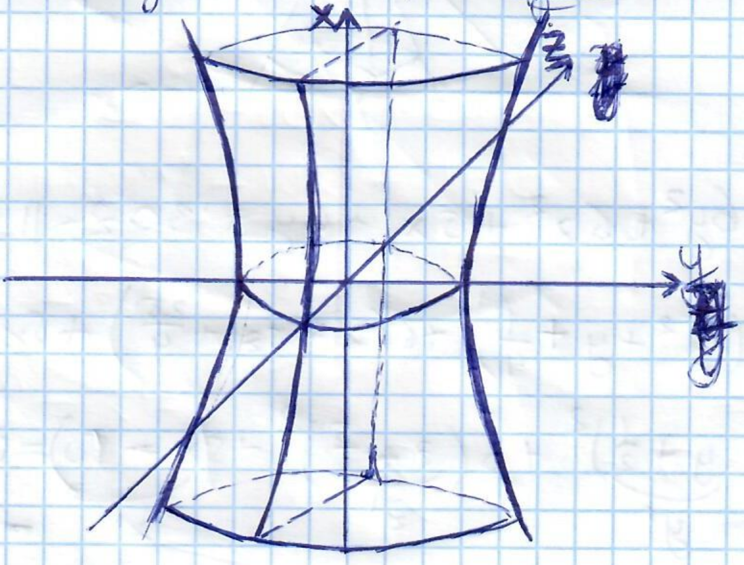
$$4(\underbrace{x+4}_{x'})^2 - \underbrace{y}_{y'}^2 - (\underbrace{z+6}_{z'})^2 + 16 = 0$$ – сдвиг

$$4x'^2 - y'^2 - z'^2 = -16 \quad | : -16$$

$$-\frac{x'^2}{4}+\frac{y'^2}{16}+\frac{z'^2}{16}=1$$

$$-\frac{x'^2}{2^2}+\frac{y'^2}{4^2}+\frac{z'^2}{4^2}=1$$ – однополостный гиперболоид



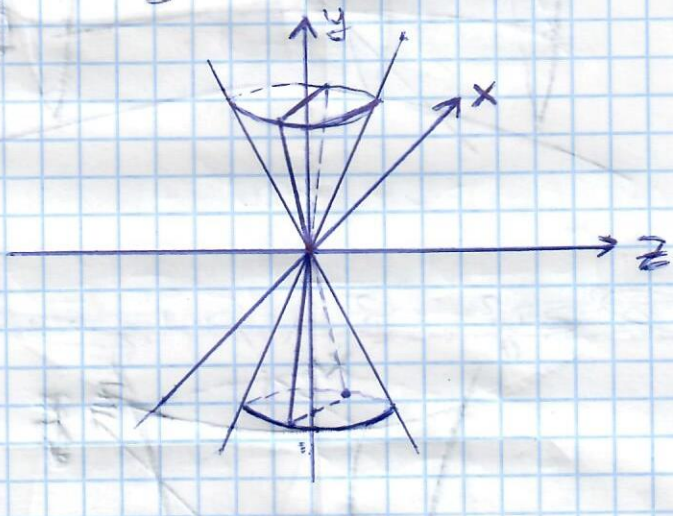
(3) $3x^2 - y^2 + 3z^2 - 18x + 10y + 12z + 14 = 0$

$3(x^2 - 6x + 9) - (y^2 - 10y + 25) + 3(z^2 + 4z + 4) +$

$+ 14 - 27 + 25 - 12 = 0$

$3\underbrace{(x-3)}_{x'}^2 - \underbrace{(y-5)}_{y'}^2 + 3\underbrace{(z+2)}_{z'}^2 = 0$ – сдвиг

$\frac{x'^2}{1} - \frac{y'^2}{3} + \frac{z'^2}{1} = 0$ – конус



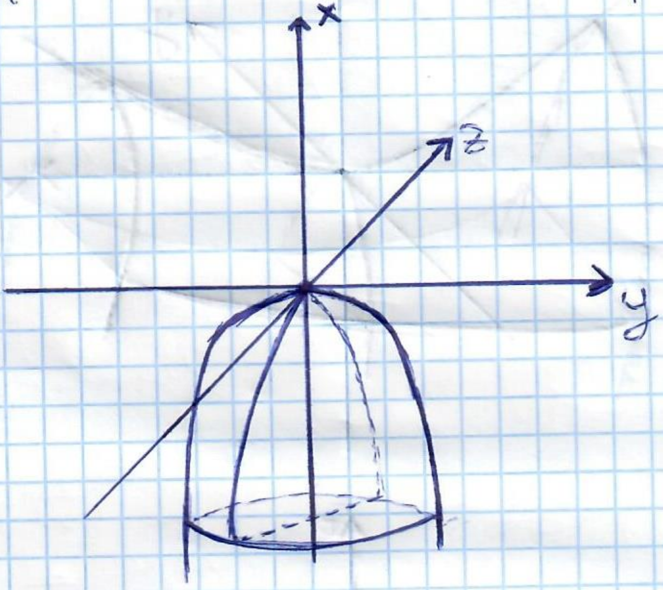

(4) $6y^2 + 6z^2 + 5x + 6y + 30z - 11 = 0$

$6\left(y^2 + y + \frac{1}{4}\right) + 6\left(z^2 + 5z + \frac{25}{4}\right) + 5x - 11 - \frac{6}{4} - \frac{25 \cdot 6}{4} = 0$

$6\underbrace{\left(y + \frac{1}{2}\right)}_{y'}^2 + 6\underbrace{\left(z + \frac{5}{2}\right)}_{z'}^2 + 5\underbrace{(x - 10)}_{x'} = 0$ – сдвиг

$6y'^2 + 6z'^2 + 5x' = 0$

$\frac{y'^2}{1} + \frac{z'^2}{1} = 2\cdot\left(-\frac{5}{12}\right)x$ – эллиптический параболоид.




(5) $z = 2x^2 - 4y^2 - 6x + 8y + 1$

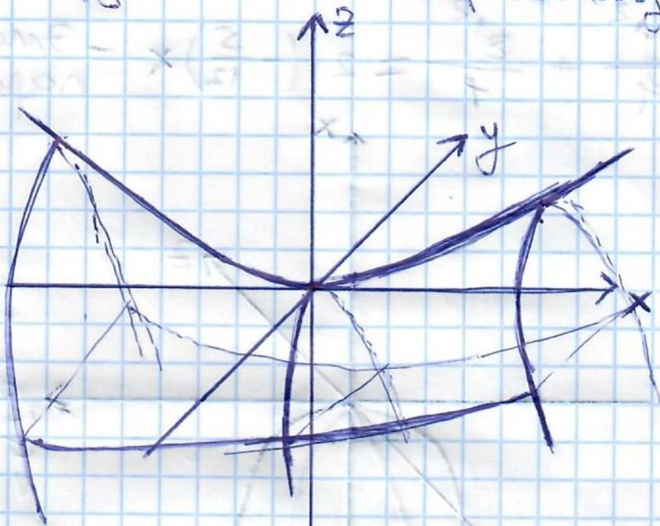
$2(x^2 - 3x + 9) - 4(y^2 - 2y + 4) - 18 + 16 + 1 - z = 0$

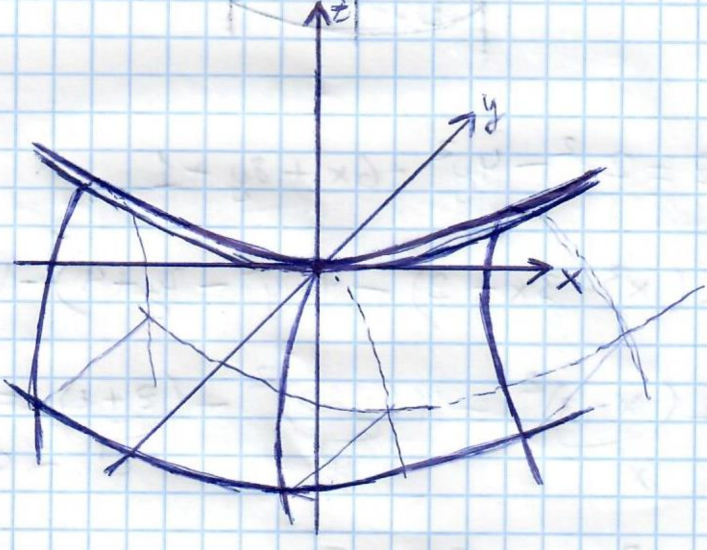
$2\underbrace{(x-3)^2}_{x'} - 4\underbrace{(y-2)^2}_{y'} - \underbrace{(z+1)}_{z'} = 0$ – сдвиг.

$2x'^2 - 4y'^2 - z' = 0$

$\frac{x'^2}{2} - \frac{y'^2}{1} = 2\cdot\frac{z'}{8}$

$$\frac{x'^2}{2/8} - \frac{y'^2}{1/8} = 2z' \text{ — гиперболический параболоид}$$



(10) $3x^2+3y^2-6x+4y-1=0$

$$3(x^2-2x+1)+3\left(y^2+\frac{4}{3}y+\frac{4}{9}\right)-3-\frac{12}{9}-1=0$$

$$3\underbrace{(x-1)^2}_{x'}+3\underbrace{\left(y+{}^2/_3\right)^2}_{y'}={}^{16}/_3$$ — сдвиг

$$\frac{x'^2}{16/9}+\frac{y'^2}{16/9}=1$$ — эллиптический цилиндр.