№1358

$a_1 = (1, 1, 1, -2)$; $a_2 = (1, 2, 3, -3)$

$(a_1, a_2) = 1 + 2 + 3 + 6 \neq 0 \Rightarrow a_1, a_2$ не ортогональны

$L = \langle a_1, a_2 \rangle \Rightarrow L^{\perp} = \{ x \mid (a_1, x) = (a_2, x) = 0 \} \Rightarrow$

$\Rightarrow L^{\perp} = \{ x \mid (a_1\ a_2)^T x = 0$

$$\begin{pmatrix} 1 & 1 & 1 & -2 \\ 1 & 2 & 3 & -3 \end{pmatrix} X = 0$$

$$\begin{pmatrix} 1 & 1 & 1 & -2 \\ 1 & 2 & 3 & -3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & -2 \\ 0 & 1 & 2 & -1 \end{pmatrix} \rightarrow$$

$$\rightarrow \begin{pmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 2 & -1 \end{pmatrix} \Rightarrow \begin{cases} x_1 = x_3 + x_4 \\ x_2 = -2x_3 + x_4 \\ x_3 = x_3 \\ x_4 = x_4 \end{cases} \Rightarrow$$

$$\Rightarrow L^{\perp} = \left\langle \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

Пусть $a_1' = \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} \Rightarrow$

$\Rightarrow$ по формуле Грама-Шмидта,

$$a_2' = \begin{pmatrix}1\\1\\0\\1\end{pmatrix} - \frac{1-2}{1+4+1}\begin{pmatrix}1\\-2\\1\\0\end{pmatrix} = \begin{pmatrix}1\\1\\0\\1\end{pmatrix} + \frac{1}{6}\begin{pmatrix}1\\-2\\1\\0\end{pmatrix} \sim$$

$$\sim \begin{pmatrix}6\\6\\0\\6\end{pmatrix} + \begin{pmatrix}1\\-2\\1\\0\end{pmatrix} = \begin{pmatrix}7\\4\\1\\0\end{pmatrix}$$

Ответ. $\left\{ \begin{pmatrix}1\\-2\\1\\0\end{pmatrix}, \begin{pmatrix}7\\4\\1\\0\end{pmatrix} \right\}$

№1360.

$$L = \left\langle \underset{a_1}{\begin{pmatrix}1/2\\1/2\\1/2\\1/2\end{pmatrix}}; \underset{a_2}{\begin{pmatrix}1/2\\1/2\\-1/2\\-1/2\end{pmatrix}} \right\rangle$$

$$L^\perp = \{ x \mid (a_1, x) = (a_2, x) = 0 \} \Rightarrow$$

$$= \{ x \mid (a_1\ a_2)^T x = 0 \}$$

$$\begin{pmatrix}1/2 & 1/2 & 1/2 & 1/2\\ 1/2 & +1/2 & -1/2 & -1/2\end{pmatrix} x = 0$$

$$\begin{pmatrix}1 & 1 & 0 & 0\\ 0 & 0 & -1 & -1\end{pmatrix} \Rightarrow \begin{cases} x_1 = -x_2\\ x_2 = x_2\\ x_3 = -x_4\\ x_4 = x_4 \end{cases} \Rightarrow$$

$$\Rightarrow L^\perp = \left\langle \begin{pmatrix}-1\\1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\-1\\1\end{pmatrix} \right\rangle = \left\langle \begin{pmatrix}-1/\sqrt{2}\\1/\sqrt{2}\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\-1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix} \right\rangle$$

ортогональны; $|a_i'| = 1$

Ответ: $\begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$

№43.18

а) $L: \begin{cases} 2x_1 + x_2 + 3x_3 - x_4 = 0 \\ 3x_1 + 2x_2 - 2x_4 = 0 \\ 3x_1 + x_2 + 4x_3 - x_4 = 0 \end{cases}$

$$L: \begin{pmatrix} 2 & 1 & 3 & -1 \\ 3 & 2 & 0 & -2 \\ 3 & 1 & 4 & -1 \end{pmatrix} X = 0$$

$\Downarrow$

$$L^\perp = \left\langle \begin{pmatrix} 2 \\ 1 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 4 \\ -1 \end{pmatrix} \right\rangle$$

$$\begin{pmatrix} 2 & 1 & 3 & -1 \\ 3 & 2 & 0 & -2 \\ 3 & 1 & 4 & -1 \end{pmatrix} \to$$

$$\to \begin{pmatrix} 0 & 1 & 1 & -1 \\ 0 & 1 & -4 & -1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 1 & 1 & -1 \end{pmatrix} \to$$

$$\to \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} \to L^\perp = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \right\rangle$$

$$\Rightarrow \forall x \in L^{\perp} \quad Rg\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix} = 3$$

$$\Rightarrow \begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{vmatrix} = 0$$

$$x_1 \cdot \underbrace{\begin{vmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{vmatrix}}_{0} - \begin{vmatrix} x_2 & x_3 & x_4 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = 0$$

$$+x_2 + x_4 = 0$$

Ответ: $x_2 + x_4 = 0$

б) $$\begin{cases} 2x_1 - 3x_2 + 4x_3 - 3x_4 = 0 \\ 3x_1 - x_2 + 11x_3 - 13x_4 = 0 \\ 4x_1 + x_2 + 18x_3 - 23x_4 = 0 \end{cases}$$

$$L: \begin{pmatrix} 2 & -3 & 4 & -3 \\ 3 & -1 & 11 & -13 \\ 4 & 1 & 18 & -23 \end{pmatrix} X = 0$$

$$\Rightarrow L^{\perp} = \left\langle \begin{pmatrix} 2 \\ -3 \\ 4 \\ -3 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \\ 11 \\ -13 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ 18 \\ -23 \end{pmatrix} \right\rangle$$

$$\begin{pmatrix} 2 & -3 & 4 & -3 \\ 3 & -1 & 11 & -13 \\ 4 & 1 & 18 & -23 \end{pmatrix} \to \begin{pmatrix} 0 & -7 & -10 & 17 \\ 1 & 2 & 7 & -10 \\ 0 & 7 & 10 & -17 \end{pmatrix} \to$$

$$\to \begin{pmatrix}1 & 2 & 7 & -10\\ 0 & 7 & 10 & -17\end{pmatrix} \to \begin{pmatrix}1 & -5 & -3 & 7\\ 0 & 7 & 10 & -17\end{pmatrix}$$

$$\Rightarrow \forall x \in L^{\perp} \; \exists \alpha_1, \dots, \alpha_2 : \alpha_1 \begin{pmatrix}1\\-5\\-3\\7\end{pmatrix} + \alpha_2 \begin{pmatrix}0\\7\\10\\-17\end{pmatrix} = \begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}1 & 0\\-5 & 7\\-3 & 10\\7 & -17\end{pmatrix}\begin{pmatrix}\alpha_1 & \alpha_2\end{pmatrix} = \begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix} \text{ — СЛАУ } (A\underbrace{\alpha^T}_{\binom{\alpha_1}{\alpha_2}} = x)$$

Совместна при $\operatorname{Rg}\begin{pmatrix}1 & 0\\-5 & 7\\-3 & 10\\7 & -17\end{pmatrix} = \operatorname{Rg}\begin{pmatrix}\boxed{\begin{matrix}1 & 0\\-5 & 7\end{matrix}} & \begin{matrix}x_1\\x_2\end{matrix}\\ \begin{matrix}-3 & 10\\7 & -17\end{matrix} & \begin{matrix}x_3\\x_4\end{matrix}\end{pmatrix}$ по т. Кронекера-Капелли

$$\Rightarrow \forall M^{j_1 j_2 j_3}_{i_1 i_2 i_3} = 0 : \begin{cases}\begin{vmatrix}1 & 0 & x_1\\-5 & 7 & x_2\\-3 & 10 & x_3\end{vmatrix} = 0\\[2ex] \begin{vmatrix}1 & 0 & x_1\\-5 & 7 & x_2\\7 & -17 & x_4\end{vmatrix} = 0\end{cases} \quad \begin{cases}\begin{vmatrix}1 & 0 & x_1\\-3 & 10 & x_3\\2 & 3 & x_3 - x_2\end{vmatrix} = 0\\[2ex] \begin{vmatrix}1 & 0 & x_1\\-3 & -3 & x_4 + 2x_2\\2 & -10 & x_4 + x_2\end{vmatrix} = 0\end{cases}$$

$$\Rightarrow \begin{cases}\begin{vmatrix}10 & x_3\\3 & x_3 - x_2\end{vmatrix} + x_1\begin{vmatrix}-3 & 10\\2 & 3\end{vmatrix} = 0\\[2ex] \begin{vmatrix}-3 & x_4 + 2x_2\\-10 & x_4 + x_2\end{vmatrix} + x_1\begin{vmatrix}-3 & -3\\2 & -10\end{vmatrix} = 0\end{cases}$$

$$\begin{cases}10(x_3 - x_2) - 3x_3 - 9x_1 - 20x_1 = 0\\ -3(x_4 + x_2) + 10(x_4 + 2x_2) + 30x_1 + 6x_1 = 0\end{cases}$$

$$\begin{cases} 7x_3 - 10x_2 - 29x_1 = 0 \\ 7x_4 + 36x_1 + 17x_2 = 0 \end{cases}$$

Ответ:

№1358

$a_1 = (1\ 1\ 1\ 2),\ a_2 = (1\ 2\ 3\ -3)$

$(a_1, a_2) = 1 + 2 + 3 - 6 = 0 \Rightarrow a_1 \perp a_2.$

$L = \langle a_1, a_2 \rangle \Rightarrow L^{\perp} = \{x \mid (a_1, x) = (a_2, x) = 2\} \Rightarrow$

$\Rightarrow L^{\perp} = \{x \mid (a_1\ a_2)^T x = 0\}$

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 1 & 2 & 3 & -3 \end{pmatrix} x = 0$$

$$\begin{pmatrix} 1 & 0 & -1 & 7 \\ 0 & 1 & 2 & -5 \end{pmatrix} x = 0 \Rightarrow \begin{cases} x_1 = x_3 - 7x_4 \\ x_2 = -2x_3 + 5x_4 \\ x_3 = x_3 \\ x_4 = x_4 \end{cases}$$

$$\Rightarrow L^{\perp} = \left\langle \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -7 \\ 5 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

Ответ: $\left\{ \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -7 \\ 5 \\ 0 \\ 1 \end{pmatrix} \right\}.$

№43.36

б) $a_1 = (1\ 1\ 1\ 1)$; $a_2 = (1\ -1\ -1\ 1)$;

$a_3 = (2\ 1\ 1\ 3)$; $a_4 = (0\ 1\ -1\ 0)$

$$V(a_1, a_2, a_3, a_4) = |\det(a_1\ a_2\ a_3\ a_4)| =$$

$$= \left| \begin{vmatrix} 1 & 1 & 2 & 0 \\ 1 & -1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 3 & 0 \end{vmatrix} \right| = \left| \begin{vmatrix} 1 & 1 & 2 & 0 \\ 0 & -2 & -1 & 1 \\ 0 & 0 & 0 & -2 \\ 1 & 1 & 3 & 0 \end{vmatrix} \right| =$$

$$= \left| \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{vmatrix} \right| = 4$$

г) $a_1 = (1\ 0\ 0\ 2\ 5)$; $a_2 = (0\ 1\ 0\ 3\ 4)$

$a_3 = (0\ 0\ 1\ 4\ 7)$ $a_4 = (2\ -3\ 4\ 11\ 12)$

$a_5 = (0\ 0\ 0\ 0\ 1)$

$$V(a_1, a_2, a_3, a_4, a_5) = |\det(a_1, a_2, a_3, a_4, a_5)| =$$

$$= \left| \begin{vmatrix} 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 2 & 3 & 4 & 11 & 0 \\ 5 & 4 & 7 & 12 & 1 \end{vmatrix} \right| = \left| \begin{vmatrix} 1 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} \right| = 0$$

Ответ: 0