CHAPTER 9 PROBLEMS

9.1. The system shown in Figure 9.1 is initially on no load with generators operating at their rated voltage with their emfs in phase. The rating of the generators and the transformers and their respective percent reactances are marked on the diagram. All resistances are neglected. The line impedance is $j160\ \Omega$. A three-phase balanced fault occurs at the receiving end of the transmission line. Determine the short-circuit current and the short-circuit MVA.

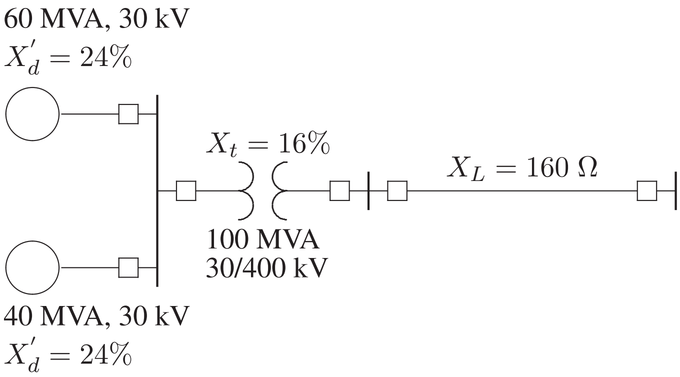

FIGURE 9.1
One-line diagram for Problem 9.1.

The base impedance for line is

$$Z_B = \frac{(400)^2}{100} = 1,600\ \ \Omega$$

and the base current is

$$I_B = \frac{100,000}{\sqrt{3}(400)} = 144.3375\ \ \text{A}$$

The reactances on a common 100 MVA base are

$$
\begin{aligned}
X'_{dg1} &= \frac{100}{60}(0.24) = 0.4 \;\; \text{pu} \\
X'_{dg2} &= \frac{100}{40}(0.24) = 0.6 \;\; \text{pu} \\
X_t &= \frac{100}{100}(0.16) = 0.16 \;\; \text{pu} \\
X_{line} &= \frac{160}{1600} = 0.1 \;\; \text{pu}
\end{aligned}
$$

The impedance diagram is as shown in Figure 9.2.

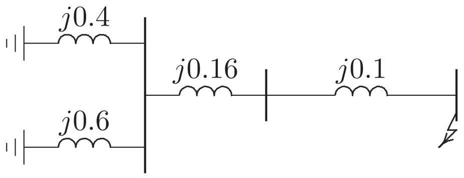

FIGURE 9.2
The impedance diagram for Problem 9.1.

Impedance to the point of fault is

$$X = j\frac{(0.4)(0.6)}{0.4 + 0.6} + j0.16 + j0.1 = j0.5 \;\; \text{pu}$$

The fault current is

$$
\begin{aligned}
I_f &= \frac{1}{j0.5} = 2\angle{-90^\circ} \;\; \text{pu} \\
&= (144.3375)(2\angle{-90^\circ}) = 288.675\angle{-90^\circ} \;\; \text{A}
\end{aligned}
$$

The Short-circuit MVA is

$$\text{SCMVA} = \sqrt{3}(400)(288.675)(10^{-3}) = 200 \;\; \text{MVA}$$

9.2. The system shown in Figure 9.3 shows an existing plant consisting of a generator of 100 MVA, 30 kV, with 20 percent subtransient reactance and a generator of 50 MVA, 30 kV with 15 percent subtransient reactance, connected in parallel to a 30-kV bus bar. The 30-kV bus bar feeds a transmission line via the circuit breaker C which is rated at 1250 MVA. A grid supply is connected to the station bus bar through a 500-MVA, 400/30-kV transformer with 20 percent reactance. Determine the reactance of a current limiting reactor in ohm to be connected between the grid system and the existing bus bar such that the short-circuit MVA of the breaker C does not exceed.

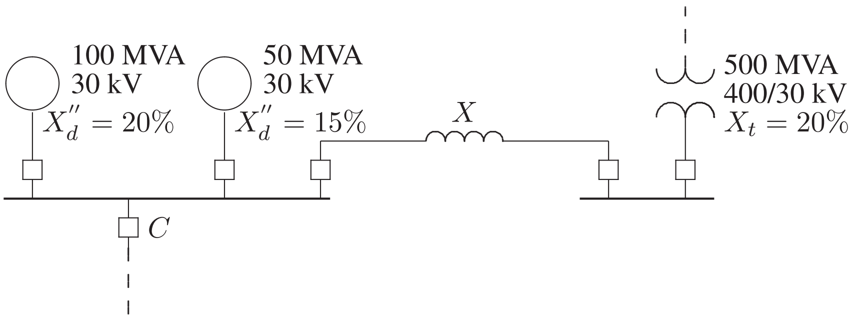

FIGURE 9.3
One-line diagram for Problem 9.2.

The base impedance for line is

$$Z_B = \frac{(30)^2}{100} = 9 \ \ \Omega$$

The reactances on a common 100 MVA base are

$$X''_{d\,g1} = \frac{100}{100}(0.2) = 0.2 \ \ \text{pu}$$
$$X''_{d\,g2} = \frac{100}{50}(0.15) = 0.3 \ \ \text{pu}$$
$$X_t = \frac{100}{500}(0.2) = 0.04 \ \ \text{pu}$$

The impedance diagram is as shown in Figure 9.4.

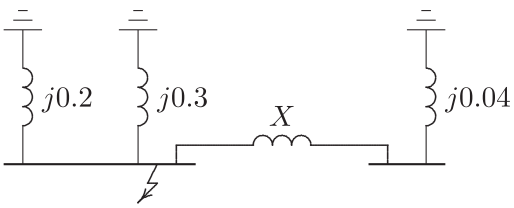

FIGURE 9.4
The impedance diagram for Problem 9.2.

Reactance to the point of fault is

$$X_f = \frac{S_B}{\text{SCMVA}} = \frac{100}{1250} = 0.08 \ \ \text{pu}$$

Parallel reactance of the generators is

$$X_{||} = \frac{(0.2)(0.3)}{0.2 + 0.3} = 0.12 \ \ \text{pu}$$

From Figure 9.4, reactance to the point of fault is

$$\frac{(0.12)(X+0.04)}{0.12+(X+0.04)} = 0.08$$

Solving for X, we get $X = 0.2$ pu., or

$$X_{\Omega} = (X)(Z_B) = (0.2)(9) = 1.8 \ \ \Omega$$

9.3. The one-line diagram of a simple power system is shown in Figure 9.5. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A three-phase fault occurs at bus 1 through a fault impedance of $Z_f = j0.08$ per unit.
(a) Using Thévenin's theorem obtain the impedance to the point of fault and the fault current in per unit.
(b) Determine the bus voltages and line currents during fault.

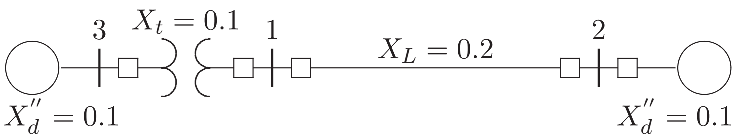

FIGURE 9.5
One-line diagram for Problem 9.3.

The impedance diagram is as shown in Figure 9.6.

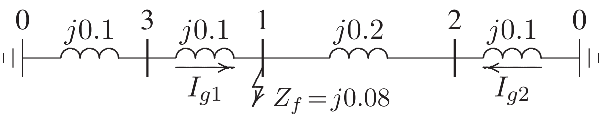

FIGURE 9.6
The impedance diagram for Problem 9.3.

(a) Impedance to the point of fault is

$$X = j\frac{(0.2)(0.3)}{0.2+0.3} = j0.12 \ \ \text{pu}$$

The fault current is

$$I_f = \frac{1}{j0.12 + j0.08} = 5\angle{-90^\circ} \ \ \text{pu}$$

(b)

$$V_1 = (j0.08)(-j5) = 0.4 \ \ \text{pu}$$

$$I_{g1} = \frac{j0.3}{j0.5}(5)\angle{-90^\circ} = 3\angle{-90^\circ} \text{ pu}$$
$$I_{g2} = \frac{j0.2}{j0.5}(5)\angle{-90^\circ} = 2\angle{-90^\circ} \text{ pu}$$
$$V_2 = 0.4 + (j0.2)(-j2) = 0.8 \text{ pu}$$
$$V_3 = 0.4 + (j0.1)(-j3) = 0.7 \text{ pu}$$

9.4. The one-line diagram of a simple three-bus power system is shown in Figure 9.7 Each generator is represented by an emf behind the subtransient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A three-phase fault occurs at bus 3 through a fault impedance of $Z_f = j0.19$ per unit.
(a) Using Thévenin's theorem obtain the impedance to the point of fault and the fault current in per unit.
(b) Determine the bus voltages and line currents during fault.

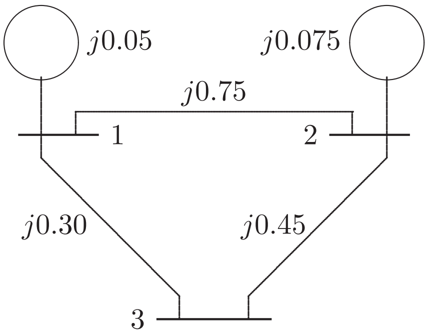

FIGURE 9.7
One-line diagram for Problem 9.4.

Converting the Δ formed by buses 123 to an equivalent Y as shown in Figure 9.8(a), we have

$$Z_{1s} = \frac{(j0.3)(j0.75)}{j1.5} = j0.15 \qquad Z_{2s} = \frac{(j0.75)(j0.45)}{j1.5} = j0.225$$
$$Z_{3s} = \frac{(j0.3)(j0.45)}{j1.5} = j0.09$$

Combining the parallel branches, Thévenin's impedance is

$$Z_{33} = \frac{(j0.2)(j0.3)}{j0.2 + j0.3} + j0.09$$
$$= j0.12 + j0.09 = j0.21$$

From Figure 9.8(c), the fault current is

$$I_3(F) = \frac{V_3(F)}{Z_{33} + Z_f} = \frac{1.0}{j0.21 + j0.19} = -j2.5 \text{ pu}$$

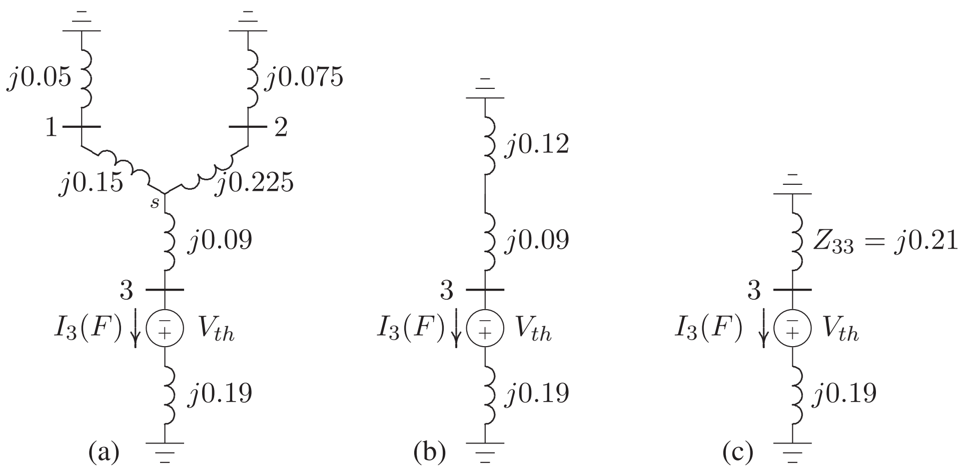

FIGURE 9.8
Reduction of Thévenin's equivalent network.

With reference to Figure 9.8(a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.3}{j0.2 + j0.3} I_3(F) = -j1.5 \quad \text{pu}$$
$$I_{G2} = \frac{j0.2}{j0.2 + j0.3} I_3(F) = -j1.0 \quad \text{pu}$$

For the bus voltage changes from Figure 9.8(a), we get

$$\Delta V_1 = 0 - (j0.05)(-j1.5) \quad = -0.075 \ \text{pu}$$
$$\Delta V_2 = 0 - (j0.075)(-j1) \quad = -0.075 \ \text{pu}$$
$$\Delta V_3 = (j0.19)(-j2.5) - 1.0 = -0.525 \ \text{pu}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, i.e.,

$$V_1(F) = V_1(0) + \Delta V_1 = 1.0 - 0.075 = 0.925 \ \text{pu}$$
$$V_2(F) = V_2(0) + \Delta V_2 = 1.0 - 0.075 = 0.925 \ \text{pu}$$
$$V_3(F) = V_3(0) + \Delta V_3 = 1.0 - 0.525 = 0.475 \ \text{pu}$$

The short circuit-currents in the lines are

$$I_{12}(F) = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.925 - 0.925}{j0.75} = 0 \quad \text{pu}$$
$$I_{13}(F) = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.925 - 0.475}{j0.3} = -j1.5 \quad \text{pu}$$
$$I_{23}(F) = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.925 - 0.475}{j0.45} = -j1.0 \quad \text{pu}$$

9.5. The one-line diagram of a simple four-bus power system is shown in Figure 9.9 Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A bolted three-phase fault occurs at bus 4.

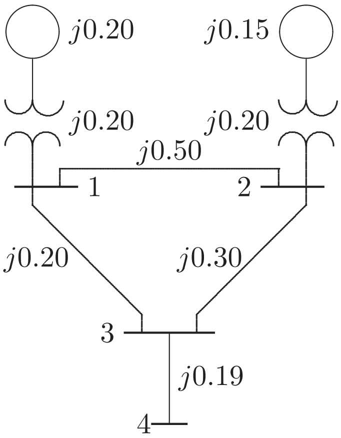

FIGURE 9.9
One-line diagram for Problem 9.5.

(a) Using Thévenin's theorem obtain the impedance to the point of fault and the fault current in per unit.
(b) Determine the bus voltages and line currents during fault.
(c) Repeat (a) and (b) for a fault at bus 2 with a fault impedance of $Z_f = j0.0225$.

(a) Converting the Δ formed by buses 123 to an equivalent Y as shown in Figure 9.10(a), we have

$$Z_{1s} = \frac{(j0.2)(j0.5)}{j1.0} = j0.10 \qquad Z_{2s} = \frac{(j0.5)(j0.3)}{j1.0} = j0.15$$

$$Z_{3s} = \frac{(j0.2)(j0.3)}{j1.0} = j0.06$$

Combining the parallel branches, Thévenin's impedance is

$$Z_{33} = \frac{(j0.5)(j0.5)}{j0.5 + j0.5} + j0.06 + j0.19 = j0.5$$

The fault current is

$$I_4(F) = \frac{V_4(F)}{Z_{44}} = \frac{1.0}{j0.5} = -j2.0 \quad \text{pu}$$

With reference to Figure 9.10(a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.5}{j0.5 + j0.5} I_4(F) = -j1.0 \quad \text{pu}$$

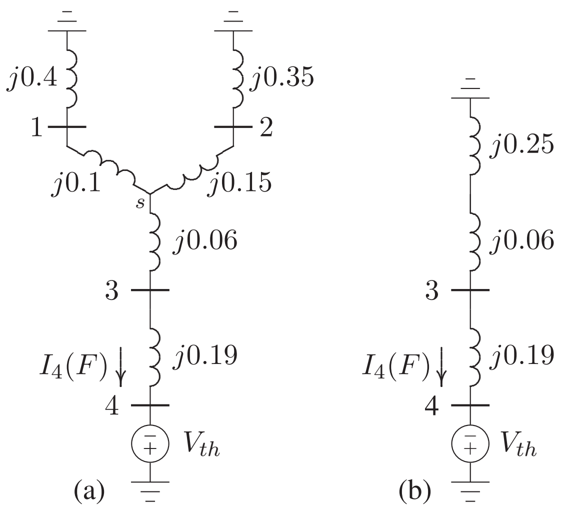

FIGURE 9.10
Reduction of Thévenin's equivalent network.

$$I_{G2} = \frac{j0.5}{j0.5 + j0.5} I_4(F) = -j1.0 \quad \text{pu}$$

(b) For the bus voltage changes from Figure 9.10(a), we get

$$\begin{aligned} \Delta V_1 &= 0 - (j0.4)(-j1.0) &= -0.4 \ \text{pu} \\ \Delta V_2 &= 0 - (j0.35)(-j1) &= -0.35 \ \text{pu} \\ \Delta V_3 &= 1 - (j0.19)(-j2) &= -0.62 \ \text{pu} \end{aligned}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, i.e.,

$$\begin{aligned} V_1(F) &= V_1(0) + \Delta V_1 = 1.0 - 0.4 = 0.60 \ \text{pu} \\ V_2(F) &= V_2(0) + \Delta V_2 = 1.0 - 0.35 = 0.65 \ \text{pu} \\ V_3(F) &= V_3(0) + \Delta V_3 = 1.0 - 0.62 = 0.38 \ \text{pu} \\ V_4(F) &= 0 \ \text{pu} \end{aligned}$$

The short circuit-currents in the lines are

$$\begin{aligned} I_{21}(F) &= \frac{V_2(F) - V_1(F)}{z_{12}} = \frac{0.65 - 0.6}{j0.5} = 0.1\angle{-90^\circ} \ \text{pu} \\ I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.60 - 0.38}{j0.2} = 1.1\angle{-90^\circ} \ \text{pu} \\ I_{23}(F) &= \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.65 - 0.38}{j0.3} = 0.9\angle{-90^\circ} \ \text{pu} \\ I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.38 - 0}{j0.19} = 2.0\angle{-90^\circ} \ \text{pu} \end{aligned}$$

(c) (a) Combining parallel branches between buses 1 and 2 results in the circuit shown in Figure 9.11(a). Combining the parallel branches, Thévenin's impedance is

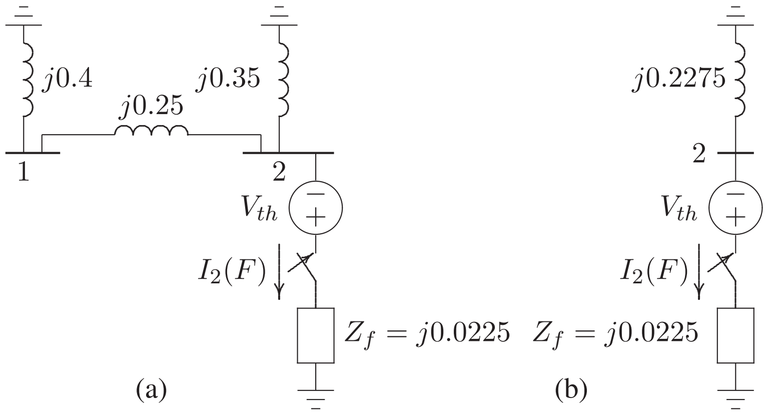

FIGURE 9.11
Reduction of Thévenin's equivalent network.

$$Z_{22} = \frac{(j0.65)(j0.35)}{j0.65 + j0.35} = j0.2275$$

The fault current is

$$I_2(F) = \frac{V_4(F)}{Z_{44} + Z_f} = \frac{1.0}{j0.2275 + j0.0225} = -j4.0 \quad \text{pu}$$

With reference to Figure 9.11(a), the current divisions between the two generators are

$$I_{G1} = \frac{j0.35}{j0.65 + j0.35} I_2(F) = -j1.4 \quad \text{pu}$$
$$I_{G2} = \frac{j0.65}{j0.65 + j0.35} I_2(F) = -j2.6 \quad \text{pu}$$

(c) (b) For the bus voltage changes from Figure 9.10(a), we get

$$\begin{aligned}
\Delta V_1 &= 0 - (j0.4)(-j1.4) \quad = -0.56 \ \text{pu} \\
\Delta V_2 &= 0 - (j0.35)(-j2.6) \quad = -0.91 \ \text{pu} \\
\Delta V_3 &= -j0.56 - (j0.2)(-\frac{j1.4}{2}) \quad = -0.70 \ \text{pu} \\
\Delta V_4 &= \Delta V_3 = -0.70 \ \text{pu}
\end{aligned}$$

The bus voltages during the fault are obtained by superposition of the prefault bus voltages and the changes in the bus voltages caused by the equivalent emf connected to the faulted bus, i.e.,

$$\begin{aligned}
V_1(F) &= V_1(0) + \Delta V_1 = 1.0 - 0.56 = 0.44 \text{ pu}\\
V_2(F) &= V_2(0) + \Delta V_2 = 1.0 - 0.91 = 0.09 \text{ pu}\\
V_3(F) &= V_3(0) + \Delta V_3 = 1.0 - 0.70 = 0.30 \text{ pu}\\
V_4(F) &= V_4(0) + \Delta V_4 = 1.0 - 0.70 = 0.30 \text{ pu}
\end{aligned}$$

The short circuit-currents in the lines are

$$\begin{aligned}
I_{12}(F) &= \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.44 - 0.09}{j0.5} = 0.7\angle{-90^\circ} \text{ pu}\\
I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.44 - 0.30}{j0.2} = 0.7\angle{-90^\circ} \text{ pu}\\
I_{32}(F) &= \frac{V_3(F) - V_2(F)}{z_{23}} = \frac{0.30 - 0.09}{j0.3} = 0.7\angle{-90^\circ} \text{ pu}\\
I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.30 - 0.3}{j0.19} = 0 \text{ pu}
\end{aligned}$$

9.6. Using the method of building algorithm find the bus impedance matrix for the network shown in Figure 9.12.

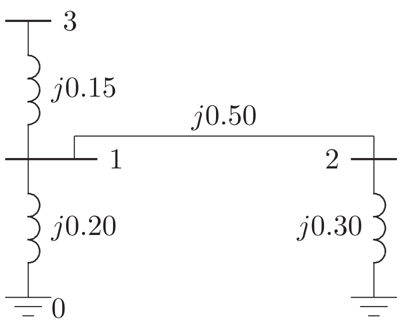

FIGURE 9.12
One-line diagram for Problem 9.6.

Add branch 1, $z_{10} = j0.2$ between node $q = 1$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{11} = z_{10} = j0.20$$

Next, add branch 2, $z_{20} = j0.3$ between node $q = 2$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 \\ 0 & j0.3 \end{bmatrix}$$

Add branch 3, $z_{13} = j0.15$ between the new node $q = 3$ and the existing node $p = 1$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{11} \\ Z_{21} & Z_{22} & Z_{21} \\ Z_{11} & Z_{12} & Z_{11} + z_{13} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 & j0.2 \\ 0 & j0.3 & 0 \\ j0.2 & 0 & j0.35 \end{bmatrix}$$

Add link 4, $z_{12} = j0.5$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\begin{aligned} \mathbf{Z}_{bus}^{(4)} &= \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix} \\ &= \begin{bmatrix} j0.2 & 0 & j0.2 & -j0.2 \\ 0 & j0.3 & 0 & j0.3 \\ j0.2 & 0 & j0.35 & -j0.2 \\ -j0.2 & j0.3 & -j0.2 & Z_{44} \end{bmatrix} \end{aligned}$$

From (9.58)

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = j0.5 + j0.2 + j0.3 - 2(j0) = j1.0$$

and

$$\begin{aligned} \frac{\mathbf{\Delta Z}\,\mathbf{\Delta Z}^T}{Z_{44}} &= \frac{1}{j1.0} \begin{bmatrix} -j0.2 \\ j0.3 \\ -j0.2 \end{bmatrix} \begin{bmatrix} -j0.2 & j0.3 & -j0.2 \end{bmatrix} \\ &= \begin{bmatrix} j0.04 & -j0.06 & j0.04 \\ -j0.06 & j0.09 & -j0.06 \\ j0.04 & -j0.06 & j0.04 \end{bmatrix} \end{aligned}$$

From (9.59), the new bus impedance matrix is

$$\begin{aligned} \mathbf{Z}_{bus} &= \begin{bmatrix} j0.2 & 0 & j0.2 \\ 0 & j0.3 & 0 \\ j0.2 & 0 & j0.35 \end{bmatrix} - \begin{bmatrix} j0.04 & -j0.06 & j0.04 \\ -j0.06 & j0.09 & -j0.06 \\ j0.04 & -j0.06 & j0.04 \end{bmatrix} \\ &= \begin{bmatrix} j0.16 & j0.06 & j0.16 \\ j0.06 & j0.21 & j0.06 \\ j0.16 & j0.06 & j0.31 \end{bmatrix} \end{aligned}$$

9.7. Obtain the bus impedance matrix for the network of Problem 9.3.

Add branch 1, $z_{20} = j0.1$ between node $q = 2$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{22} = z_{20} = j0.10$$

Next, add branch 2, $z_{30} = j0.1$ between node $q = 3$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{22} & 0 \\ 0 & Z_{33} \end{bmatrix} = \begin{bmatrix} j0.1 & 0 \\ 0 & j0.1 \end{bmatrix}$$

Add branch 3, $z_{13} = j0.1$ between the new node $q = 1$ and the existing node $p = 3$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} Z_{33} + z_{13} & 0 & Z_{33} \\ 0 & Z_{22} & 0 \\ Z_{33} & 0 & Z_{33} \end{bmatrix} = \begin{bmatrix} j0.2 & 0 & j0.1 \\ 0 & j0.1 & 0 \\ j0.1 & 0 & j0.1 \end{bmatrix}$$

Add link 4, $z_{12} = j0.2$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\begin{aligned} \mathbf{Z}_{bus}^{(4)} &= \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix} \\ &= \begin{bmatrix} j0.2 & 0 & j0.1 & -j0.2 \\ 0 & j0.1 & 0 & j0.1 \\ j0.1 & 0 & j0.1 & -j0.1 \\ -j0.2 & j0.1 & -j0.1 & Z_{44} \end{bmatrix} \end{aligned}$$

From (9.58)

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = j0.2 + j0.2 + j0.1 - 2(j0) = j0.5$$

and

$$\begin{aligned} \frac{\mathbf{\Delta Z}\,\mathbf{\Delta Z}^T}{Z_{44}} &= \frac{1}{j0.5} \begin{bmatrix} -j0.2 \\ j0.1 \\ -j0.1 \end{bmatrix} \begin{bmatrix} -j0.2 & j0.1 & -j0.1 \end{bmatrix} \\ &= \begin{bmatrix} j0.08 & -j0.04 & j0.04 \\ -j0.04 & j0.02 & -j0.02 \\ j0.04 & -j0.02 & j0.02 \end{bmatrix} \end{aligned}$$

From (9.59), the new bus impedance matrix is

$$\begin{aligned} \mathbf{Z}_{bus}^{(4)} &= \begin{bmatrix} j0.2 & 0 & j0.1 \\ 0 & j0.1 & 0 \\ j0.1 & 0 & j0.1 \end{bmatrix} - \begin{bmatrix} j0.08 & -j0.04 & j0.04 \\ -j0.04 & j0.02 & -j0.02 \\ j0.04 & -j0.02 & j0.02 \end{bmatrix} \\ &= \begin{bmatrix} j0.12 & j0.04 & j0.06 \\ j0.04 & j0.08 & j0.02 \\ j0.06 & j0.02 & j0.08 \end{bmatrix} \end{aligned}$$

9.8. Obtain the bus impedance matrix for the network of Problem 9.4.

Add branch 1, $z_{10} = j0.05$ between node $q = 1$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{11} = z_{10} = j0.05$$

Next, add branch 2, $z_{20} = j0.075$ between node $q = 2$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{bmatrix} = \begin{bmatrix} j0.05 & 0 \\ 0 & j0.075 \end{bmatrix}$$

Add branch 3, $z_{13} = j0.3$ between the new node $q = 3$ and the existing node $p = 1$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{11} \\ Z_{21} & Z_{22} & Z_{21} \\ Z_{11} & Z_{12} & Z_{11} + z_{13} \end{bmatrix} = \begin{bmatrix} j0.05 & 0 & j0.05 \\ 0 & j0.075 & 0 \\ j0.05 & 0 & j0.35 \end{bmatrix}$$

Add link 4, $z_{12} = j0.75$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\begin{aligned} \mathbf{Z}_{bus}^{(4)} &= \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix} \\ &= \begin{bmatrix} j0.05 & 0 & j0.05 & -j0.05 \\ 0 & j0.075 & 0 & j0.075 \\ j0.05 & 0 & j0.35 & -j0.05 \\ -j0.05 & j0.075 & -j0.05 & Z_{44} \end{bmatrix} \end{aligned}$$

From (9.58)

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = j0.75 + j0.05 + j0.075 - 2(j0) = j0.875$$

and

$$\begin{aligned} \frac{\mathbf{\Delta Z}\,\mathbf{\Delta Z}^T}{Z_{44}} &= \frac{1}{j0.875} \begin{bmatrix} -j0.05 \\ j0.075 \\ -j0.05 \end{bmatrix} \begin{bmatrix} -j0.05 & j0.075 & -j0.05 \end{bmatrix} \\ &= \begin{bmatrix} j0.002857 & -j0.004286 & j0.002857 \\ -j0.004286 & j0.006428 & -j0.004286 \\ j0.002857 & -j0.004286 & j0.002857 \end{bmatrix} \end{aligned}$$

From (9.59), the new bus impedance matrix is

$$\begin{aligned} \mathbf{Z}_{bus}^{(4)} &= \begin{bmatrix} j0.05 & 0 & j0.05 \\ 0 & j0.075 & 0 \\ j0.05 & 0 & j0.35 \end{bmatrix} - \begin{bmatrix} j0.002857 & -j0.004286 & j0.002857 \\ -j0.004286 & j0.006428 & -j0.004286 \\ j0.002857 & -j0.004286 & j0.002857 \end{bmatrix} \\ &= \begin{bmatrix} j0.047143 & j0.004286 & j0.047143 \\ j0.004286 & j0.068571 & j0.004286 \\ j0.047143 & j0.004286 & j0.347142 \end{bmatrix} \end{aligned}$$

Add link 5, $z_{23} = j0.45$ between node $q = 3$ and node $p = 2$. From (9.57), we have

$$\mathbf{Z}_{bus}^{(5)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{13} - Z_{12} \\ Z_{21} & Z_{22} & Z_{23} & Z_{23} - Z_{22} \\ Z_{31} & Z_{32} & Z_{33} & Z_{33} - Z_{32} \\ Z_{31} - Z_{21} & Z_{32} - Z_{22} & Z_{33} - Z_{23} & Z_{44} \end{bmatrix}$$

$$= \begin{bmatrix} j0.047143 & j0.004286 & j0.047143 & j0.042857 \\ j0.004286 & j0.068571 & j0.004286 & -j0.064286 \\ j0.047143 & j0.004286 & j0.347142 & j0.342857 \\ j0.042857 & -j0.064286 & j0.342857 & Z_{44} \end{bmatrix}$$

From (9.58)

$$Z_{44} = z_{23} + Z_{22} + Z_{33} - 2Z_{23} = j0.45 + j0.068571 + j0.347142 - 2(j0.004286) = j0.85714$$

and

$$\frac{\mathbf{\Delta Z}\,\mathbf{\Delta Z}^T}{Z_{44}} = \frac{1}{j0.85714} \begin{bmatrix} j0.042857 \\ -j0.064286 \\ j0.342857 \end{bmatrix} \begin{bmatrix} j0.042857 & -j0.064286 & j0.342857 \end{bmatrix}$$

$$= \begin{bmatrix} j0.002143 & -j0.003214 & j0.017143 \\ -j0.003214 & j0.004821 & -j0.025714 \\ j0.017143 & -j0.025714 & j0.137143 \end{bmatrix}$$

From (9.59), the new bus impedance matrix is

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.047143 & j0.004286 & j0.047143 \\ j0.004286 & j0.068571 & j0.004286 \\ j0.047143 & j0.004286 & j0.347142 \end{bmatrix} -$$

$$\begin{bmatrix} j0.002143 & -j0.003214 & j0.017143 \\ -j0.003214 & j0.004821 & -j0.025714 \\ j0.017143 & -j0.025714 & j0.137142 \end{bmatrix} = \begin{bmatrix} j0.0450 & j0.00750 & j0.030 \\ j0.0075 & j0.06375 & j0.030 \\ j0.0300 & j0.03000 & j0.210 \end{bmatrix}$$

9.9. The bus impedance matrix for the network shown in Figure 9.13 is given by

$$Z_{bus} = j \begin{bmatrix} 0.300 & 0.200 & 0.275 \\ 0.200 & 0.400 & 0.250 \\ 0.275 & 0.250 & 0.41875 \end{bmatrix}$$

There is a line outage and the line from bus 1 to 2 is removed. Using the method of building algorithm determine the new bus impedance matrix.

The line between buses 1 and 2 with impedance $Z_{12} = j0.8$ is removed. The removal of this line is equivalent to connecting a link having an impedance equal

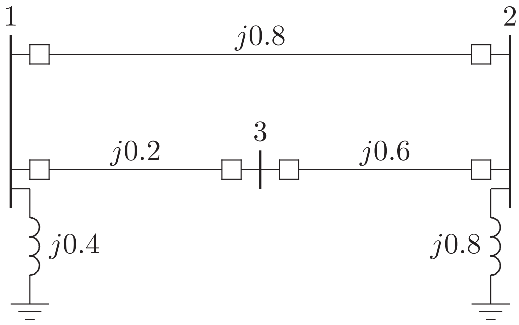

FIGURE 9.13
One-line diagram for Problem 9.9.

to the negated value of the original impedance. Therefore, we add link $z_{12} = -j0.8$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\mathbf{Z}_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix}$$

Thus, we get

$$\mathbf{Z}_{bus}^{(1)} = \begin{bmatrix} j0.300 & j0.200 & j0.27500 & -j0.100 \\ j0.200 & j0.400 & j0.25000 & j0.200 \\ j0.275 & j0.250 & j0.41875 & -j0.025 \\ -j0.100 & j0.200 & -j0.02500 & Z_{44} \end{bmatrix}$$

From (9.58)

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = -j0.8 + j0.3 + j0.4 - 2(j0.2) = -j0.5$$

and

$$\begin{aligned} \frac{\mathbf{\Delta Z}\,\mathbf{\Delta Z}^T}{Z_{44}} &= \frac{1}{-j0.5} \begin{bmatrix} -j0.100 \\ j0.200 \\ -j0.025 \end{bmatrix} \begin{bmatrix} -j0.10 & j0.20 & -j0.025 \end{bmatrix} \\ &= \begin{bmatrix} -j0.020 & j0.040 & -j0.0050 \\ j0.040 & -j0.080 & j0.0100 \\ -j0.005 & j0.010 & -j0.0013 \end{bmatrix} \end{aligned}$$

From (9.59), the new bus impedance matrix is

$$\begin{aligned} \mathbf{Z}_{bus} &= \begin{bmatrix} j0.300 & j0.200 & j0.27500 \\ j0.200 & j0.400 & j0.25000 \\ j0.275 & j0.250 & j0.41875 \end{bmatrix} - \begin{bmatrix} -j0.020 & j0.040 & -j0.00500 \\ j0.040 & -j0.080 & j0.01000 \\ -j0.005 & j0.010 & -j0.00125 \end{bmatrix} \\ &= \begin{bmatrix} j0.320 & j0.160 & j0.280 \\ j0.160 & j0.480 & j0.240 \\ j0.280 & j0.240 & j0.420 \end{bmatrix} \end{aligned}$$

```
Zbus =

      0 + 0.0450i     0 + 0.0075i     0 + 0.0300i
      0 + 0.0075i     0 + 0.0638i     0 + 0.0300i
      0 + 0.0300i     0 + 0.0300i     0 + 0.2100i

Enter Faulted Bus No. -> 3
Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = j*.19

Balanced three-phase fault at bus No. 3
Total fault current =   2.5000 per unit

Bus Voltages during fault in per unit

     Bus     Voltage       Angle
     No.     Magnitude     degrees
      1       0.9250       0.0000
      2       0.9250       0.0000
      3       0.4750       0.0000

Line currents for fault at bus No.  3

     From       To     Current     Angle
     Bus        Bus    Magnitude   degrees
      G          1      1.5000    -90.0000
      1          2      0.0000    -90.0000
      1          3      1.5000    -90.0000
      G          2      1.0000    -90.0000
      2          3      1.0000    -90.0000
      3          F      2.5000    -90.0000
Another fault location? Enter'y' or'n' within single quote->'n'
```

9.11. The per unit bus impedance matrix for the power system of Problem 9.5 is given by

$$Z_{bus} = j\begin{bmatrix} 0.240 & 0.140 & 0.200 & 0.200 \\ 0.140 & 0.2275 & 0.175 & 0.175 \\ 0.200 & 0.175 & 0.310 & 0.310 \\ 0.200 & 0.1750 & 0.310 & 0.500 \end{bmatrix}$$

(a) A bolted three-phase fault occurs at bus 4. Using the bus impedance matrix calculate the fault current, bus voltages, and line currents during fault.
(b) Repeat (a) for a three-phase fault at bus 2 with a fault impedance of $Z_f = j0.0225$.
(c) Check your result using the **Zbuild** and **symfault** programs.

From (9.22), for a solid fault at bus 4 the fault current is

$$I_4(F) = \frac{V_4(0)}{Z_{44}} = \frac{1.0}{j0.5} = -j2 \quad \text{pu}$$

From (9.23), bus voltages during the fault are

$$\begin{aligned}
V_1(F) &= V_1(0) - Z_{14}I_4(F) = 1.0 - (j0.200)(-j2) = 0.60 \ \text{pu} \\
V_2(F) &= V_2(0) - Z_{24}I_4(F) = 1.0 - (j0.175)(-j2) = 0.65 \ \text{pu} \\
V_3(F) &= V_3(0) - Z_{34}I_4(F) = 1.0 - (j0.310)(-j2) = 0.38 \ \text{pu} \\
V_4(F) &= V_4(0) - Z_{44}I_4(F) = 1.0 - (j0.500)(-j2) = 0 \ \text{pu}
\end{aligned}$$

From (9.25), the short circuit currents in the lines are

$$\begin{aligned}
I_{21}(F) &= \frac{V_2(F) - V_1(F)}{z_{12}} = \frac{0.65 - 0.60}{j0.5} = -j0.1 \ \text{pu} \\
I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.60 - 0.38}{j0.2} = -j1.1 \ \text{pu} \\
I_{23}(F) &= \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.65 - 0.38}{j0.3} = -j0.9 \ \text{pu} \\
I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.38 - 0}{j0.19} = -j2 \ \text{pu}
\end{aligned}$$

(c) From (9.22), for a fault at bus 2 with fault impedance $Z_f = j0.0225$ per unit, the fault current is

$$I_2(F) = \frac{V_2(0)}{Z_{22} + Z_f} = \frac{1.0}{j0.2275 + j0.0225} = -j4 \quad \text{pu}$$

From (9.23), bus voltages during the fault are

$$\begin{aligned}
V_1(F) &= V_1(0) - Z_{12}I_2(F) = 1.0 - (j0.140)(-j4) = 0.44 \ \text{pu} \\
V_2(F) &= V_2(0) - Z_{22}I_2(F) = 1.0 - (j0.2275)(-j4) = 0.09 \ \text{pu} \\
V_3(F) &= V_3(0) - Z_{32}I_2(F) = 1.0 - (j0.175)(-j4) = 0.30 \ \text{pu} \\
V_4(F) &= V_4(0) - Z_{42}I_2(F) = 1.0 - (j0.175)(-j4) = 0.30 \ \text{pu}
\end{aligned}$$

From (9.25), the short circuit currents in the lines are

$$\begin{aligned}
I_{12}(F) &= \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.44 - 0.09}{j0.5} = -j0.7 \ \text{pu} \\
I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.44 - 0.30}{j0.2} = -j0.7 \ \text{pu} \\
I_{32}(F) &= \frac{V_3(F) - V_2(F)}{z_{23}} = \frac{0.30 - 0.09}{j0.3} = -j0.7 \ \text{pu} \\
I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.30 - 0.30}{j0.19} = 0 \ \text{pu}
\end{aligned}$$

To check the result, we form the line data for the network of Problem 9.11, and we use the following commands

```
zdata=[0   1    0   0.40
       0   2    0   0.35
       1   2    0   0.50
       1   3    0   0.20
       2   3    0   0.30
       3   4    0   0.19];
Zbus=zbuild(zdata)
symfault(zdata, Zbus)
```

The result is

```
Zbus =
      0 + 0.2400i   0 + 0.1400i   0 + 0.2000i   0 + 0.2000i
      0 + 0.1400i   0 + 0.2275i   0 + 0.1750i   0 + 0.1750i
      0 + 0.2000i   0 + 0.1750i   0 + 0.3100i   0 + 0.3100i
      0 + 0.2000i   0 + 0.1750i   0 + 0.3100i   0 + 0.5000i

Enter Faulted Bus No. -> 4

Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = 0

Balanced three-phase fault at bus No. 4
Total fault current =   2.0000 per unit

Bus Voltages during fault in per unit

     Bus     Voltage       Angle
     No.     Magnitude     degrees
      1       0.6000       0.0000
      2       0.6500       0.0000
      3       0.3800       0.0000
      4       0.0000       0.0000

Line currents for fault at bus No.  4

     From       To     Current     Angle
     Bus        Bus    Magnitude   degrees
      G          1      1.0000    -90.0000
      1          3      1.1000    -90.0000
      G          2      1.0000    -90.0000
      2          1      0.1000    -90.0000
      2          3      0.9000    -90.0000
      3          4      2.0000    -90.0000
      4          F      2.0000    -90.0000
```

```
Another fault location? Enter'y' or'n' within single quote->'y'
Enter Faulted Bus No. -> 2

Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = j*0.0225
Balanced three-phase fault at bus No. 2
Total fault current =   4.0000 per unit

Bus Voltages during fault in per unit

     Bus     Voltage       Angle
     No.     Magnitude     degrees
      1       0.4400       0.0000
      2       0.0900       0.0000
      3       0.3000       0.0000
      4       0.3000       0.0000

Line currents for fault at bus No.  2

     From      To     Current     Angle
     Bus       Bus    Magnitude   degrees
      G         1      1.4000    -90.0000
      1         2      0.7000    -90.0000
      1         3      0.7000    -90.0000
      G         2      2.6000    -90.0000
      2         F      4.0000    -90.0000
      3         2      0.7000    -90.0000
```

9.12. The per unit bus impedance matrix for the power system shown in Figure 9.14 is given by

$$Z_{bus} = j\begin{bmatrix} 0.150 & 0.075 & 0.140 & 0.135 \\ 0.075 & 0.1875 & 0.090 & 0.0975 \\ 0.140 & 0.090 & 0.2533 & 0.210 \\ 0.135 & 0.0975 & 0.210 & 0.2475 \end{bmatrix}$$

A three-phase fault occurs at bus4 through a fault impedance of $Z_f = j0.0025$ per unit. Using the bus impedance matrix calculate the fault current, bus voltages and line currents during fault. Check your result using the **Zbuild** and **symfault** programs.

From (9.22), for a fault at bus 2 with fault impedance $Z_f = j0.0225$ per unit, the fault current is

$$I_4(F) = \frac{V_4(0)}{Z_{44} + Z_f} = \frac{1.0}{j0.2475 + j0.0025} = -j4 \quad \text{pu}$$

From (9.23), bus voltages during the fault are

$$V_1(F) \;=\; V_1(0) - Z_{14}I_4(F) = 1.0 - (j0.135)(-j4) = 0.46 \;\; \text{pu}$$