ECE 4620/5620 [Q_1; SP 2013]; NO: ☐

Name:

Prob 1 A power system has two fossil-fueled units on economic dispatch loop. The cost functions and the generation limits of the units are given by:

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$$C_1 = 400 + 10 P_{G_1} + 8\times10^{-3} P_{G1}^2 \; ; \; [C] = \$/h$$

$$C_2 = 300 + 8 P_{G2} + 9\times10^{-3} P_{G2}^2 \; ; \; [P_G] = MW$$

$$100^{MW} \leq P_{G1} \leq 600^{MW}$$

$$400^{MW} \leq P_{G2} \leq 1000^{MW}$$

Find the optimum solution $P_{G_1}^*$ and P_{G2}^* for the cases demand power as given:

(4) a) $P_D = 500^{MW}$

(4) b) $P_D = 1600^{MW}$

(4) c) $P_D = 700\,MW$

(4) d) $P_D = 1000\,MW$

(4) e) $P_D = 1300\,MW$

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Solution

$$\begin{cases} C_1 = 400 + 10\,P_{G1} + 8\times10^{-3}\,P_{G1}^2 \\ C_2 = 300 + 8\,P_{G2} + 9\times10^{-3}\,P_{G2}^2 \end{cases}$$

where:
$$\begin{cases} 100^{MW} \leq P_{G1} \leq 600^{MW} \\ 400^{MW} \leq P_{G2} \leq 1000^{MW} \end{cases}$$

a) $P_D = 500^{MW}$ is the minimum power that should be supported by these two units in which case:

$$\begin{cases} P_{G1}^* = P_{G1,min} = 100^{MW} \\ P_{G2}^* = P_{G2,min} = 400^{MW} \end{cases}$$

b) $P_D = 1600^{MW}$ is the Maximum power that can be supported by these two units in which case:

$$P_{G1}^* = P_{G1,Max} = 600^{MW}$$
$$P_{G2}^* = P_{G2,Max} = 1000^{MW}$$

c) Recall the general formula for cost function:

$$C_i = \alpha_i + \beta_i\,P_{Gi} + \gamma_i\,P_{Gi}^2$$

c) [continued]:

You can find $\frac{dC_i}{dP_{Gi}}$ for each machine, then use equal incremental cost criterion and power balance equation:

$$\begin{cases} \frac{dC_1}{dP_{G1}} = 10 + 16\times10^{-3} P_{G1} \\ \frac{dC_2}{dP_{G2}} = 8 + 18\times10^{-3} P_{G2} \end{cases}$$

$\Longrightarrow$

$$\begin{cases} 10 + 16\times10^{-3} P_{G1} = 8 + 18\times10^{-3} P_{G2} \\ P_{G1} + P_{G2} = P_D = 700 \end{cases}$$

Then solve the two equations for P_{G1}^* and P_{G2}^*.

Or, use the following:

$$\begin{cases} \lambda = \frac{P_D + \sum \beta_i/2\gamma_i}{\sum 1/2\gamma_i} \; ; \quad 1 \\ P_{Gi} = \frac{\lambda - \beta_i}{2\gamma_i} \; ; \quad 2 \end{cases}$$

where:
$$\begin{cases} \beta_1 = 10 \; ; \; \gamma_1 = 8\times10^{-3} \\ \beta_2 = 8 \; ; \; \gamma_2 = 9\times10^{-3} \end{cases}$$

c) [continued]:

$$\sum \beta_i / 2\gamma_i = \frac{\beta_1}{2\gamma_1} + \frac{\beta_2}{2\gamma_2}$$

$$= \frac{10}{2 \times 8 \times 10^{-3}} + \frac{8}{2 \times 9 \times 10^{-3}} = 1069.44$$

$$\sum 1/2\gamma_i = \frac{1}{2\gamma_1} + \frac{1}{2\gamma_2} = \frac{1}{2 \times 8 \times 10^{-3}} + \frac{1}{2 \times 9 \times 10^{-3}}$$

$$= 118.06$$

Now,

$$\rightarrow \lambda \overset{1}{=} \frac{700 + 1069.44}{118.06} = 14.99$$

$$\rightarrow P_{G1} \overset{2}{=} \frac{\lambda - \beta_1}{2\gamma_1} = \cdots = 311.76 \text{ MW}$$

$$\rightarrow P_{G2} \overset{2}{=} \frac{\lambda - \beta_2}{2\gamma_2} = \cdots = 388.24 \text{ MW}$$

$$< P_{G2,min}$$

So, the lower limit of P_{G2} is violated; Thus:

$$P_{G2}^* = P_{G2,min} = 400 \text{ MW}$$

$$\Longrightarrow$$

$$P_{G1}^* = P_D - P_{G2}^* = 700 - 400 = 300 \text{ MW}$$

So:
$$\begin{cases} P_{G1}^* = 300 \text{ MW} \\ P_{G2}^* = 400 \text{ MW} \end{cases}$$

d) $P_D = 1000\ MW$

$\rightarrow \lambda \overset{①}{=} \dfrac{1000 + 1069.44}{118.06} = 17.53$

$\rightarrow P_{G1} \overset{②}{=} \dfrac{\lambda - \beta_1}{2\gamma_1} = \dfrac{17.53 - 10}{2 \times 8 \times 10^{-3}} = 470.59\ MW$

$\rightarrow P_{G2} \overset{②}{=} \dfrac{\lambda - \beta_2}{2\gamma_2} = \dfrac{17.53 - \beta_2}{2\gamma_2} = \dots = 529.41$

where both solutions are within the limits:

$$\begin{cases} P_{G1}^{*} = 470.59\ MW \\ P_{G2}^{*} = 529.41\ MW \end{cases}$$

e) $P_D = 1300\ MW$

$\rightarrow \lambda \overset{①}{=} \dfrac{1300 + 1069.44}{118.06} = 20.07$

$\rightarrow P_{G1} \overset{②}{=} \dfrac{\lambda - \beta_1}{2\gamma_1} = \dfrac{20.07 - \beta_1}{2\gamma_1} = 629.41$

$> P_{G1,Max}$

so:

$$\begin{cases} P_{G1}^{*} = P_{G1,max} = 600\ MW \\ P_{G2}^{*} = P_D - P_{G1}^{*} = 1300 - 600 = 700\ MW \end{cases}$$