CHAPTER 10 PROBLEMS

10.1. Obtain the symmetrical components for the set of unbalanced voltages $V_a = 300\angle{-120°}$, $V_b = 200\angle 90°$, and $V_c = 100\angle{-30°}$.
The commands

```
Vabc = [300  -120
        200    90
        100   -30];
V012 = abc2sc(Vabc);  % Symmetrical components of phase a
V012p= rec2pol(V012)          % Rectangular to polar form
```

result in

```
V012p =
          42.2650 -120.0000
         193.1852 -135.0000
          86.9473  -84.8961
```

10.2. The symmetrical components of a set of unbalanced three-phase currents are $I_a^0 = 3\angle{-30°}$, $I_a^1 = 5\angle 90°$, and $I_a^2 = 4\angle 30°$. Obtain the original unbalanced phasors.

The commands

```
I012 = [3    -30
        5     90
        4     30];
Iabc = sc2abc(I012);%Unbalanced phasor to symmetrical comp.
Iabcp= rec2pol(Iabc)            % Rectangular to polar form
```

result in

```
Iabcp =
          8.1854    42.2163
          4.0000   -30.0000
          8.1854  -102.2163
```

10.3. The operator a is defined as $a = 1\angle 120°$; show that
(a) $\frac{(1+a)}{(1+a^2)} = 1\angle 120°$
(b) $\frac{(1-a)^2}{(1+a)^2} = 3\angle -180°$
(c) $(a - a^2)(a^2 - a) = 3\angle 0°$
(d) $V_{an}^1 = \frac{1}{\sqrt{3}} V_{bc}^1 \angle 90°$
(e) $V_{an}^2 = \frac{1}{\sqrt{3}} V_{bc}^2 \angle -90°$

(a) Since $1 + a + a^2 = 0$, we have

$$\frac{(1+a)}{(1+a^2)} = \frac{1+a}{-a} = -\frac{1}{a} - 1$$
$$= 0.5 + j0.866 - 1 = 1\angle 120°$$

(b)

$$\frac{(1-a)^2}{(1+a)^2} = \frac{(1-a)^2}{(-a^2)^2} = \frac{1 - 2a + a^2}{a} = \frac{1}{a} - 2 + a =$$
$$= -0.5 - j0.866 - 2 - 0.5 + j0.866 = 3\angle 180°$$

(c)

$$(a - a^2)(a^2 - a) = 2a^3 - a^2 - a^4 = 2a^3 - (a^2 + a)$$
$$= 2(1) - (-1) = 3$$

(d)

$$V_{bc}^1 = V_{bn}^1 - V_{cn}^1 = a^2 V_{an}^1 - a V_{an}^1 = (a^2 - a) V_{an}^1$$
$$= (-0.5 - j0.866 + 0.5 - j0.866) V_{an}^1$$
$$= \sqrt{3}\angle -90° V_{an}^1$$

or

$$V_{an}^1 = \frac{1}{\sqrt{3}} V_{bc}^1 \angle 90°$$

(e)

$$V_{bc}^2 = V_{bn}^2 - V_{cn}^2 = a V_{an}^2 - a^2 V_{an}^2 = (a - a^2) V_{an}^2$$
$$= (-0.5 + j0.866 + 0.5 + j0.866) V_{an}^2$$
$$= \sqrt{3}\angle 90° V_{an}^2$$

or

$$V_{an}^2 = \frac{1}{\sqrt{3}} V_{bc}^2 \angle -90^\circ$$

10.4. The line-to-line voltages in an unbalanced three-phase supply are $V_{ab} = 1000\angle 0^\circ$, $V_{bc} = 866.0254\angle -150^\circ$, and $V_{ca} = 500\angle 120^\circ$. Determine the symmetrical components for line and phase voltages, then find the phase voltages V_{an}, V_{bn}, and V_{cn}.

First find the symmetrical components of line voltages, then find the symmetrical components of phase voltages. Use the inverse symmetrical components transformation to obtain the phase voltages. We use the following commands

```
a = -0.5+j*sqrt(3)/2;
Vabbcca=[1000      0       % Unbalanced line-to-line voltage
         866.0254  -150
         500        120];
VL012=abc2sc(Vabbcca); % Sym. comp. line voltages, rectangular
VL012p=rec2pol(VL012)          % Sym. comp. line voltages, polar
Va012=[ 0               % Sym. comp. phase voltages, rectangular
        VL012(2)/(sqrt(3)*(0.866+j0.5))
        VL012(3)/(sqrt(3)*(0.866-j0.5))];
Va012p=rec2pol(Va012)          % Sym. comp. phase voltage, polar
Vabc=sc2abc(Va012);     % Unbalanced phase voltages, rectangular
Vabcp=rec2pol(Vabc)            % Unbalanced phase voltages, polar
```

The result is

```
  VL012p =
            0.0000   30.0000
          763.7626  -10.8934
          288.6751   30.0000

  Va012p =
            0         0
          440.9586  -40.8934
          166.6667   60.0000

  Vabcp =
          440.9586  -19.1066
          600.9252 -166.1021
          333.3333   60.0000
```

10.5. In the three-phase system shown in Figure 10.1, phase a is on no load and phases b and c are short-circuited to ground.
The following currents are given:
$I_b = 91.65\angle 160.9^\circ$

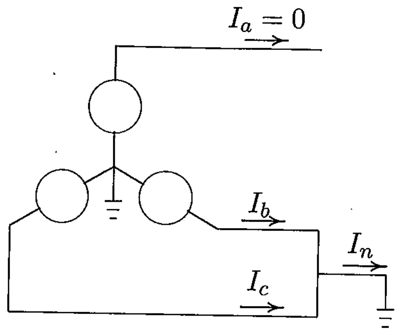

FIGURE 10.1
Circuit for Problem 10.5.

$I_n = 60.00\angle 90°$
Find the symmetrical components of current I_a^0, I_a^1, and I_a^2.

$$I_c = I_n - I_b = 60\angle 90° - 91.65\angle 160.9° = 91.6569\angle 19.1124$$

We use the following commands

```
Iabc = [  0           0
         91.65     160.9
         91.6569    19.1124];
I012=abc2sc(Iabc);  % Sym. components of currents, rectangular
I012p = rec2pol(I012)    % Sym. components of currents , polar
```

The result is

```
I012p =
            20.0000   90.0000
            60.0012  -89.9942
            40.0012   90.0087
```

10.6. A balanced three-phase voltage of 360-V line-to-neutral is applied to a balanced Y-connected load with ungrounded neutral, as shown in Figure 10.2. The three-phase load consists of three mutually-coupled reactances. Each phase has a series reactance of $Z_s = j24\ \Omega$, and the mutual coupling between phases is $Z_m = j6\ \Omega$.
(a) Determine the line currents by mesh analysis without using symmetrical components.
(b) Determine the line currents using symmetrical components.
(a) Applying KVL to the two independent mesh equations, and writing one node equation, results in

$$\begin{bmatrix} (Z_s - Z_m) & -(Z_s - Z_m) & 0 \\ 0 & (Z_s - Z_m) & -(Z_s - Z_m) \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} |V_L|\angle \pi/6 \\ |V_L|\angle -\pi/2 \\ 0 \end{bmatrix}$$

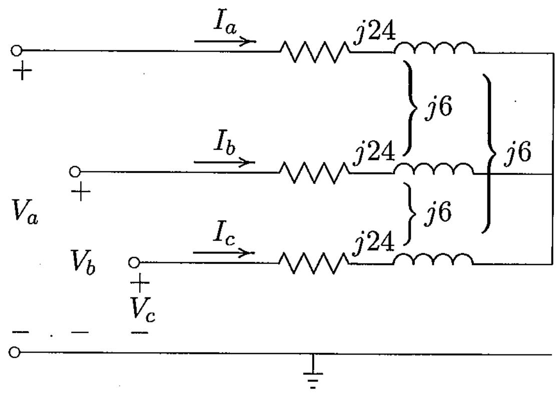

FIGURE 10.2
Circuit for Problem 10.6.

or in compact form

$$\mathbf{Z}_{mesh}\mathbf{I}^{abc} = \mathbf{V}_{mesh}$$

Solving the above equations results in the line currents

$$\mathbf{I}^{abc} = \mathbf{Z}_{mesh}{}^{-1}\mathbf{V}_{mesh}$$

The following commands

```
disp('(a) Solution by mesh analysis')
VL=360*sqrt(3);                              % Line voltage magnitude
Zs=j*24; Zm=j*6;                       % Series and mutual impedances
Z= [(Zs-Zm)  -(Zs-Zm)      0
       0      (Zs-Zm)  -(Zs-Zm) % Matrix from 2 mesh equations
       1         1         1   ];       % and one node equation
V=[VL*cos(pi/6)+j*VL*sin(pi/6)     % RHS of mesh node equations
   VL*cos(-pi/2)+j*VL*sin(-pi/2)
             0                  ];
Iabc=Z\V;                                % Line currents, Rectangular
Iabcp=[abs(Iabc), angle(Iabc)*180/pi]    % Line currents, polar
```

The result is

```
(a) Solution by mesh analysis
    Iabcp =
           20.0000  -90.0000
           20.0000  150.0000
           20.0000   30.0000
```

(b) Using the symmetrical components method, we have

$$\mathbf{V}^{012} = \mathbf{Z}^{012}\mathbf{I}^{012}$$

where

$$\mathbf{V}^{012} = \begin{bmatrix} 0 \\ V_a \\ 0 \end{bmatrix}$$

and from (10.32)

$$\mathbf{Z}^{012} = \begin{bmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix}$$

for the sequence components of currents, we get

$$\mathbf{I}^{012} = [\mathbf{Z}^{012}]^{-1}\mathbf{V}^{012}$$

We write the following commands

```
disp('Solution by Symmetrical components method' )
Z012=[Zs+2*Zm   0      0          % Symmetrical components matrix
        0      Zs-Zm   0
        0        0    Zs-Zm];
V012=[0; VL/sqrt(3); 0];%Symmetrical components phase voltages
I012=inv(Z012)*V012  % Symmetrical components of line currents
a=cos(2*pi/3)+j*sin(2*pi/3);
A=[ 1  1  1; 1 a^2  a; 1 a a^2];          % Transformation matrix
Iabc=A*I012;                            % Line currents, rectangular
Iabcp=[abs(Iabc), angle(Iabc)*180/pi]   % Line currents, Polar
```

which result in

```
(b) Solution by Symmetrical components method
   I012 =
          0
          0 -20.0000i
          0
  Iabcp =
         20.0000  -90.0000
         20.0000  150.0000
         20.0000   30.0000
```

This is the same result as in part (a).

10.7. A three-phase unbalanced source with the following phase-to-neutral voltages

$$\mathbf{V}^{abc} = \begin{bmatrix} 300 & \angle -120^\circ \\ 200 & \angle 90^\circ \\ 100 & \angle -30^\circ \end{bmatrix}$$

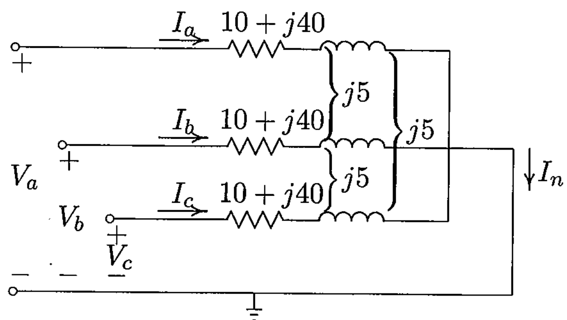

FIGURE 10.3
Circuit for Problem 10.7.

is applied to the circuit in Figure 10.3. The load series impedance per phase is $Z_s = 10 + j40$ and the mutual impedance between phases is $Z_m = j5$. The load and source neutrals are solidly grounded. Determine
(a) The load sequence impedance matrix, $\mathbf{Z}^{012} = \mathbf{A}^{-1}\mathbf{Z}^{abc}\mathbf{A}$.
(b) The symmetrical components of voltage.
(c) The symmetrical components of current.
(d) The load phase currents.
(e) The complex power delivered to the load in terms of symmetrical components, $S_{3\phi} = 3(V_a^0 I_a^{0*} + V_a^1 I_a^{1*} + V_a^2 I_a^{2*})$.
(f) The complex power delivered to the load by summing up the power in each phase, $S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^*$.

We write the following commands

```
Vabc=[300    -120                        % Phase-to-neutral voltages
      200      90
      100     -30];
Zabc=[10+j*40     j*5     j*5 %Self and mutual impedances matrix
          j*5  10+j*40    j*5
          j*5     j*5 10+j*40];
Z012 = zabc2sc(Zabc)      % Symmetrical components of impedance
V012 = abc2sc(Vabc);        % Symmetrical components of voltage
V012p= rec2pol(V012)   % Converts rectangular phasors to polar
I012 = inv(Z012)*V012 ;     % Symmetrical components of current
I012p= rec2pol(I012)   % Converts rectangular phasors to polar
Iabc = sc2abc(I012);                           % Phase currents
Iabcp= rec2pol(Iabc)   % Converts rectangular phasors to polar
S3ph=3*(V012.')*conj(I012) %Power using symmetrical components
Vabcr = Vabc(:,1).*(cos(pi/180*Vabc(:,2))+...
j*sin(pi/180*Vabc(:,2)));
S3ph=(Vabcr.')*conj(Iabc)       % Power using phase quantities
```

The result is

```
Z012 =
        10.0+50.0i     0                 0
        0              10.0 +35.0i       0
        0              0                 10.0+35.0i
V012p =
         42.2650   -120.0000
        193.1852   -135.0000
         86.9473    -84.8961

I012p =
          0.8289    161.3099
          5.3072    150.9454
          2.3886   -158.9507
Iabcp =
          7.9070    165.4600
          5.8190     14.8676
          2.7011    -96.9315
S3ph =
          1036.8+3659.6i
S3ph =
          1036.8+3659.6i
```

10.8. The line-to-line voltages in an unbalanced three-phase supply are $V_{ab} = 600\angle 36.87°$, $V_{bc} = 800\angle 126.87°$, and $V_{ca} = 1000\angle -90°$. A Y-connected load with a resistance of 37 Ω per phase is connected to the supply. Determine
(a) The symmetrical components of voltage.
(b) The phase voltages.
(c) The line currents.

We use the following statements

```
Vabbcca=[600  36.87                    % Unbalanced line voltages
         800  126.87
         1000  -90];
VL012=abc2sc(Vabbcca); % Sym. comp. line voltages, rectangular
VL012p=rec2pol(VL012)        % Sym. comp. line voltages, polar
Va012=[0
       VL012(2)/(sqrt(3)*(0.866+j*.5))
       VL012(3)/(sqrt(3)*(0.866-j*.5))];  % Sym. components of
                                         % phase voltages, rectangular
Va012p=rec2pol(Va012)      % Sym. comp. of phase voltages, polar
Vabc=sc2abc(Va012);                   % Phase voltages, rectangular
Vabcp=rec2pol(Vabc)                        % Phase voltages, polar
Iabc=Vabc/37;                          % Line currents, rectangular
Iabcp=rec2pol(Iabc)                          % Line currents, polar
```

which result in

```
VL012p =
            0.0006    -179.9999
          237.0762     169.9342
          781.3204      24.0621

Va012p =
            0          0
          136.8790     139.9335
          451.1055      54.0628

Vabcp =
          480.7542      70.5606
          333.3386     163.7411
          569.6111     -73.6857

Iabcp =
           12.9934      70.5606
            9.0092     163.7411
           15.3949     -73.6857
```

10.9. A generator having a solidly grounded neutral and rated 50-MVA, 30-kV has positive-, negative-, and zero-sequence reactances of 25, 15, and 5 percent, respectively. What reactance must be placed in the generator neutral to limit the fault current for a bolted line-to-ground fault to that for a bolted three-phase fault?

The generator base impedance is

$$Z_B = \frac{(30)^2}{50} = 18 \ \Omega$$

The three-phase fault current is

$$I_{f\,3\phi} = \frac{1}{0.25} = 4.0 \ \text{pu}$$

The line-to-ground fault current is

$$I_{f\,LG} = \frac{3}{0.25 + 0.15 + 0.05 + 3X_n} = 4.0 \ \text{pu}$$

Solving for X_n, results in

$$\begin{aligned} X_n &= 0.1 \quad \text{pu} \\ &= (0.1)(18) = 1.8 \ \Omega \end{aligned}$$

10.10. What reactance must be placed in the neutral of the generator of Problem 9 to limit the magnitude of the fault current for a bolted double line-to-ground fault to that for a bolted three-phase fault?

The generator base impedance is

$$Z_B = \frac{(30)^2}{50} = 18 \ \Omega$$

From (10.86) the positive sequence component of fault current is

$$I_a^1 = \frac{1}{0.25 + \frac{(0.15)(0.05+3X_n)}{0.2+3X_n}} = \frac{0.2 + 3X_n}{0.0575 + 1.2X_n}$$

and from (10.86) the zero sequence component of fault current is

$$I_a^0 = -\frac{1 - \frac{(0.25)(0.2+3X_n)}{0.0575+1.2X_n}}{0.05 + 3X_n} = -\frac{0.0075 + 0.45X}{3.6X^2 + 0.2325X + 0.002875}$$

The double line-to-ground fault current is

$$I_{f\,DLG} = 3I_a^0$$

Since the magnitude of the fault current is to be equal to 4, we get

$$|I_a^0| = \frac{4}{3}$$

Substituting for I_a^0, we have

$$\frac{4}{3} = \frac{0.0075 + 0.45X}{3.6X^2 + 0.2325X + 0.002875}$$

which results in the following second-order polynomial equation

$$14.4x^2 - 0.42X - 0.011 = 0$$

The positive root of the above polynomial is $X = 0.0458333$ or

$$X_n = (0.0458333)(18) = 0.825 \ \Omega$$

10.11. Three 15-MVA, 30-kV synchronous generators A, B, and C are connected via three reactors to a common bus bar, as shown in Figure 10.4. The neutrals of generators A and B are solidly grounded, and the neutral of generator C is grounded through a reactor of 2.0 Ω. The generator data and the reactance of the reactors are tabulated below. A line-to-ground fault occurs on phase a of the common bus bar. Neglect prefault currents and assume generators are operating at their rated voltage. Determine the fault current in phase a.

Item	X^1	X^2	X^0
G_A	0.25 pu	0.155 pu	0.056 pu
G_B	0.20 pu	0.155 pu	0.056 pu
G_C	0.20 pu	0.155 pu	0.060 pu
Reactor	6.0 Ω	6.0 Ω	6.0 Ω

tance. Therefore, the negative-sequence impedance is

$$\frac{1}{X^2} = \frac{1}{0.255} + \frac{1}{0.255} + \frac{1}{0.255} \quad \text{or} \quad X^2 = 0.085$$

The zero-sequence impedance is

$$\frac{1}{X^0} = \frac{1}{0.156} + \frac{1}{0.156} + \frac{1}{0.26} \quad \text{or} \quad X^0 = 0.06$$

The line-to-ground fault current in phase a is

$$I_a = 3I_a^0 = \frac{3(1)}{j(0.105 + 0.085 + 0.06)} = 12\angle{-90^\circ} \ \text{pu}$$

10.12. Repeat Problem 10.11 for a bolted line-to-line fault between phases b and c.

The positive-sequence fault current in phase a is

$$I_a^1 = \frac{1}{Z^1 + Z^2} = \frac{1}{j(0.105 + 0.085)} = -j5.26316 \ \text{pu}$$

The fault current is

$$I_b = -j\sqrt{3}I_a^1 = -9.116 \ \text{pu}$$

10.13. Repeat Problem 10.11 for a bolted double line-to-ground fault on phases b and c.

The positive- and zero-sequence fault currents in phase a are

$$\begin{aligned} I_a^1 &= \frac{1}{j0.105 + j\left(\frac{(0.085)(0.06)}{0.085+0.06}\right)} = -j7.13407 \ \text{pu} \\ I_a^0 &= -\frac{1 - (j0.105)(-j7.13407)}{j0.06} = j4.182 \ \text{pu} \end{aligned}$$

The fault current is

$$I_f = 3I_a^0 = 12.546\angle 90^\circ$$

10.14. The zero-, positive-, and negative-sequence bus impedance matrices for a three-bus power system are

$$\mathbf{Z}_{bus}^0 = j\begin{bmatrix} 0.20 & 0.05 & 0.12 \\ 0.05 & 0.10 & 0.08 \\ 0.12 & 0.08 & 0.30 \end{bmatrix} \quad \text{pu}$$

$$\mathbf{Z}_{bus}^1 = \mathbf{Z}_{bus}^2 = j\begin{bmatrix} 0.16 & 0.10 & 0.15 \\ 0.10 & 0.20 & 0.12 \\ 0.15 & 0.12 & 0.25 \end{bmatrix} \quad \text{pu}$$

Determine the per unit fault current and the bus voltages during fault for
(a) A bolted three-phase fault at bus 2.
(b) A bolted single line-to-ground fault at bus 2.
(c) A bolted line-to-line fault at bus 2.
(d) A bolted double line-to-ground fault at bus 2.

(a) The symmetrical components of fault current for a bolted balanced three-phase fault at bus 2 is given by

$$I_2^{012}(F) = \begin{bmatrix} 0 \\ \frac{1}{Z_{22}^1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{j0.20} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -j5 \\ 0 \end{bmatrix}$$

The fault current is

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j5 \\ 0 \end{bmatrix} = \begin{bmatrix} 5\angle{-90^\circ} \\ 5\angle 150^\circ \\ 5\angle 30^\circ \end{bmatrix}$$

For balanced fault we only have the positive-sequence component of voltage. Thus, from (10.98), bus voltages during fault for phase a are

$$\begin{aligned} V_1(F) &= 1 - Z_{12}^1 I_2(F) = 1 - j0.10(-j5) = 0.5 \\ V_2(F) &= 1 - Z_{22}^1 I_2(F) = 1 - j0.20(-j5) = 0.0 \\ V_3(F) &= 1 - Z_{32}^1 I_2(F) = 1 - j0.12(-j5) = 0.4 \end{aligned}$$

(b) From (10.90), the symmetrical components of fault current for a single line-to-ground fault at bus 2 is given by

$$\begin{aligned} I_2^0(F) = I_2^1(F) = I_2^2(F) &= \frac{1.0}{Z_{22}^1 + Z_{22}^2 + Z_{22}^0} \\ &= \frac{1.0}{j0.20 + j0.20 + j0.10} = -j2 \end{aligned}$$

The fault current is

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j2 \\ -j2 \\ -j2 \end{bmatrix} = \begin{bmatrix} 6\angle{-90^\circ} \\ 0\angle 0^\circ \\ 0\angle 0^\circ \end{bmatrix}$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 - Z_{12}^0 I_2^0 \\ V_1^1(0) - Z_{12}^1 I_2^1 \\ 0 - Z_{12}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.05(-j2) \\ 1 - j0.10(-j2) \\ 0 - j0.10(-j2) \end{bmatrix} = \begin{bmatrix} -0.10 \\ 0.80 \\ -0.20 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 - Z_{22}^0 I_2^0 \\ V_2^1(0) - Z_{22}^1 I_2^1 \\ 0 - Z_{22}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.10(-j2) \\ 1 - j0.20(-j2) \\ 0 - j0.20(-j2) \end{bmatrix} = \begin{bmatrix} -0.20 \\ 0.60 \\ -0.40 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 - Z_{32}^0 I_2^0 \\ V_3^1(0) - Z_{32}^1 I_2^1 \\ 0 - Z_{32}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.08(-j2) \\ 1 - j0.12(-j2) \\ 0 - j0.12(-j2) \end{bmatrix} = \begin{bmatrix} -0.16 \\ 0.76 \\ -0.24 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.10 \\ 0.80 \\ -0.20 \end{bmatrix} = \begin{bmatrix} 0.50\angle 0^\circ \\ 0.9539\angle -114.79^\circ \\ 0.9539\angle +114.79^\circ \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.20 \\ 0.60 \\ -0.40 \end{bmatrix} = \begin{bmatrix} 0.0\angle 0^\circ \\ 0.9165\angle -109.11^\circ \\ 0.9165\angle +109.11^\circ \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.16 \\ 0.76 \\ -0.24 \end{bmatrix} = \begin{bmatrix} 0.36\angle 0^\circ \\ 0.9625\angle -115.87^\circ \\ 0.9625\angle +115.87^\circ \end{bmatrix}$$

(c) From (10.92) and (10.93), the symmetrical components of fault current for line-to-line fault at bus 2 are

$$I_2^0 = 0$$
$$I_2^1 = -I_2^2 = \frac{V_2(0)}{Z_{22}^1 + Z_{22}^2} = \frac{1}{j0.20 + j0.20} = -j2.5$$

The fault current is

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j2.5 \\ j2.5 \end{bmatrix} = \begin{bmatrix} 0 \\ -4.33 \\ 4.33 \end{bmatrix}$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 \\ V_1^1(0) - Z_{12}^1 I_2^1 \\ 0 - Z_{12}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.10(-j2.5) \\ 0 - j0.10(j2.5) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.75 \\ 0.25 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 \\ V_2^1(0) - Z_{22}^1 I_2^1 \\ 0 - Z_{22}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.20(-j2.50) \\ 0 - j0.20(j2.5) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.50 \\ 0.50 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 \\ V_3^1(0) - Z_{32}^1 I_2^1 \\ 0 - Z_{32}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.12(-j2.5) \\ 0 - j0.12(j2.5) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.70 \\ 0.30 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.75 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.6614\angle -139.11^\circ \\ 0.614\angle +130.11^\circ \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.50 \\ 0.50 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.50\angle 180^\circ \\ 0.50\angle +180^\circ \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.70 \\ 0.30 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.6083\angle -145.285^\circ \\ 0.6083\angle +145.285^\circ \end{bmatrix}$$

(d) From (10.94)–(10.96), the symmetrical components of fault current for a double line-to-ground fault at bus 2 is given by

$$I_2^1 = \frac{V_2(0)}{Z_{22}^1 + \frac{Z_{22}^2(Z_{22}^0)}{Z_{22}^2 + Z_{22}^0}} = -\frac{1}{j0.20 + \frac{j0.20(j0.10)}{j0.20 + j0.10}} = -j3.75$$

$$I_2^2 = -\frac{V_2(0) - Z_{22}^1 I_2^1}{Z_{22}^2} - \frac{1 - j0.20(-j3.75)}{j0.20} = j1.25$$

$$I_2^0 = -\frac{V_2(0) - Z_{22}^1 I_2^1}{Z_{22}^0} = -\frac{1 - j0.20(-j3.75)}{j0.20} = j2.5$$

The phase currents at the faulted bus are

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j2.5 \\ -j3.75 \\ j1.25 \end{bmatrix} = \begin{bmatrix} 0 \\ 5.7282\angle 139.11^\circ \\ 5.7282\angle 40.89^\circ \end{bmatrix}$$

and the total fault current is

$$I_2^b + I_2^c = 5.7282\angle 139.11^\circ + 5.7282\angle 40.89^\circ = 7.5\angle 90^\circ$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 - Z_{12}^0 I_2^0 \\ V_1^1(0) - Z_{12}^1 I_2^1 \\ 0 - Z_{12}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.05(j2.5) \\ 1 - j0.10(-j3.75) \\ 0 - j0.10(j1.25) \end{bmatrix} = \begin{bmatrix} 0.125 \\ 0.625 \\ 0.125 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 - Z_{22}^0 I_2^0 \\ V_2^1(0) - Z_{22}^1 I_2^1 \\ 0 - Z_{22}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.10(j2.5) \\ 1 - j0.20(-j3.75) \\ 0 - j0.20(j1.25) \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 - Z_{32}^0 I_2^0 \\ V_3^1(0) - Z_{32}^1 I_2^1 \\ 0 - Z_{32}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.08(j2.5) \\ 1 - j0.12(-j3.75) \\ 0 - j0.12(j1.25) \end{bmatrix} = \begin{bmatrix} 0.20 \\ 0.55 \\ 0.15 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.125 \\ 0.625 \\ 0.125 \end{bmatrix} = \begin{bmatrix} 0.875\angle 0^\circ \\ 0.50\angle -120^\circ \\ 0.50\angle +120^\circ \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 0.75\angle 0^\circ \\ 0 \\ 0 \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.20 \\ 0.55 \\ 0.15 \end{bmatrix} = \begin{bmatrix} 0.90\angle 0^\circ \\ 0.3775\angle -113.413^\circ \\ 0.3775\angle +113.413^\circ \end{bmatrix}$$

10.15. The reactance data for the power system shown in Figure 10.6 in per unit on a common base is as follows:

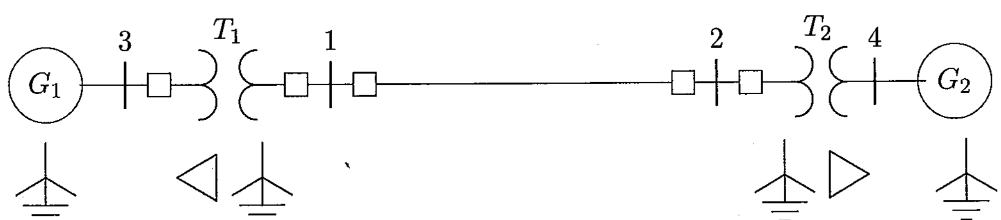

FIGURE 10.6
The impedance diagram for Problem 10.15.

Obtain the Thévenin sequence impedances for the fault at bus 1 and compute the fault current in per unit for the following faults:

(a) A bolted three-phase fault at bus 1.
(b) A bolted single line-to-ground fault at bus 1.
(c) A bolted line-to-line fault at bus 1.
(d) A bolted double line-to-ground fault at bus 1.
The positive-sequence impedance network is shown in Figure 10.7, and impedance to the point of fault is

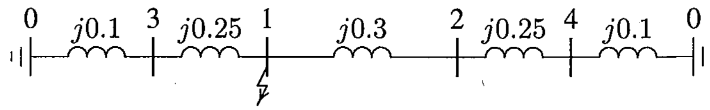

FIGURE 10.7
Positive-sequence impedance network for Problem 10.15.

$$Z^1 = j\frac{(0.35)(0.65)}{0.35+0.65} = j0.2275 \text{ pu}$$

Since negative-sequence reactances are the same as positive-sequence reactances, $X^2 = X^1 = 0.2275$. The zero-sequence impedance network is shown in Figure 10.8, and impedance to the point of fault is

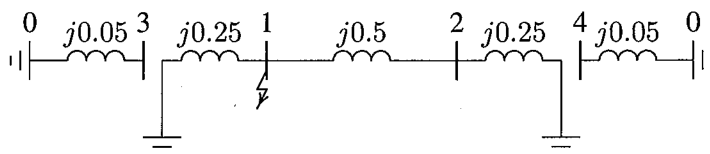

FIGURE 10.8
zero-sequence impedance network for Problem 10.15.

$$Z^0 = j\frac{(0.25)(0.75)}{0.25+0.75} = j0.1875 \text{ pu}$$

(a) For a bolted three-phase fault at bus 1, the fault current is

$$I_f = \frac{1}{j0.2275} = 4.3956\angle{-90^\circ} \text{ pu}$$

(b) For a bolted single-line to ground fault at bus 1, the fault current is

$$I_f = 3I_a^0 = \frac{3}{j(0.2275+0.2275+0.1875)} = 4.669\angle{-90^\circ} \text{ pu}$$

(c) For a bolted line-to-line fault at bus 1, the fault current in phase b is

$$I_a^1 = \frac{1}{j(0.2275+0.2275)} = -j2.1978 \text{ pu}$$

$$I_b(F) = -j\sqrt{3}I_a^1 = -3.8067 \text{ pu}$$

(d) For a bolted double line-to-line fault at bus 1, we have

$$I_a^1 = \frac{1}{j0.2275 + +j\frac{(0.2275)(0.1875)}{0.2275+0.1875}} = -j3.02767 \text{ pu}$$

$$I_a^0 = \frac{1-(0.2275)(-j3.02767)}{j0.1875} = j1.65975 \text{ pu}$$

$$I(F) = 3I_a^0 = 4.979\angle 90^\circ \text{ pu}$$