Chapter 7-- Optimal Dispatch of Generation (*Saadat*)

The formulation of power flow problem and its solution were discussed in Chapter 6. **There,** we assumed we know the active power generations for the different generators. **In practice,** there are many options for scheduling generations to support a set of loads with given constraints. *The objective of this chapter is to schedule the generations of different generators to minimize the cost of the overall generation.* **A more** comprehensive scheduling problem is called Optimal Power Flow (OPF) where generators' voltage and active power are scheduled optimally while satisfying many other requirements. **OPF** is beyond the scope this course.

The following items are covered in this chapter:

- Optimization of non-linear functions
- Operating cost of thermal plants
- Economic dispatch neglecting losses and neglecting generation limits
- Economic dispatch neglecting losses but including generation limits
- Economic dispatch including losses

HW-Assignments: ***Examples*** 7-1, 2, 4, 6, 7 **&** ***Problems*** 7- 1, 2, 7

Non-Linear Optimization **-- Also** called optimization of non-linear function.

Let: $f(x) = f(x_1, x_2, \dots, x_n)$ **Where** $f(\cdot)$ a non-linear scalar function of vector $x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$

Now, we define the following 3 problems:

Unconstrained optimization Problem: **Find** $x = x^*$ such that $f(x^*)$ is minimum. **Or**, in short:

$min: f(x)$

Optimization Problem with equality constrained: **Find** $x = x^*$ such that $f(x^*)$ is minimized and the m equality constraints $h_i(x) = 0$ are satisfied. **Or**, in short:

$$\begin{cases} min: f(x) \\ s.t.: h_i(x) = 0 \;; \quad i = 1, 2, \dots, m \end{cases}$$

Optimization Problem with equality and inequality constraints: **Find** $x = x^*$ such that $f(x^*)$ is minimized and the m equality constraints $h_i(x) = 0$ for $i = 1, 2, \dots, m$ **and** the p inequality constraints $g_j(x) \leq 0$ for $j = 1, 2, \dots, p$ are satisfied. **Or**, in short:

$$\begin{cases} min: f(x) \\ s.t.: h_i(x) = 0 \quad ; \quad i = 1, 2, \dots, m \\ s.t.: g_j(x) \leq 0 \quad ; \quad j = 1, 2, \dots, p \end{cases}$$

Example 1: **Determine** $x^* = x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ that minimizes $f(x) = 2x_1^2 + x_2^2 - 2x_1 - 2x_1x_2$ and the corresponding minimum value of $f(x^*)$

Solution: **This is** an unconstrained optimization problem: $\min f(x)$

At minimum point we must have:

$$\begin{cases} \frac{\partial f(\cdot)}{\partial x_1} = 0 \\ \frac{\partial f(\cdot)}{\partial x_2} = 0 \end{cases} \Rightarrow \begin{cases} 2(2x_1) - 2 - 2x_2 = 0 \\ 2x_2 - 2x_1 = 0 \end{cases} \xRightarrow{solve} \begin{cases} x_1 = 1 \\ x_2 = 1 \end{cases} \Rightarrow x^* = x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

So, the minimum value of $f(\cdot)$ happens at $x^* = x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and for this x the minimum value $f(\cdot)$ is:

$$f_{min}(\cdot) = 2(1)^2 + (1)^2 - 2(1) - 2(1)(2) = -1$$

Example 2: **Determine** the minimum value of $f(x)$ given in Example 1 subject to $x_1 - x_2 + 1 = 0$

Solution: **This is** an optimization problem with one equality constraint, defined as follows:

$$\begin{cases} min: f(x) = 2x_1^2 + x_2^2 - 2x_1 - 2x_1x_2 \\ s.t.: h(x) = x_1 - x_2 - 1 = 0 \end{cases}$$

Method 1: Find x_1 in terms of x_2 using $h(x) = 0$: $h(x) = x_1 - x_2 + 1 = 0 \quad \Rightarrow \quad x_1 = x_2 - 1$

Now, plug $x_1 = x_2 - 1$ into the function $f(x)$:

$$f(x) = 2x_1^2 + x_2^2 - 2x_1 - 2x_1x_2 = 2(x_1 - 1)^2 + x_2^2 - 2(x_2 - 1) - 2(x_2 - 1)x_2$$

$$= 2(x_2^2 - 2x_2 + 1) + x_2^2 - 2x_2 + 2 - 2x_2^2 + 2x_2 = x_2^2 - 4x_2 + 4 \Rightarrow f(x) = x_2^2 - 4x_2 + 4$$

Now, define a new optimization problem as:

$\min: f(x) = x_2^2 - 4x_2 + 4$; This is an unconstrained optimization problem

At the optimum solution point we must have:

$$\frac{\partial f(\cdot)}{\partial x_2} = 2x_2 - 4 = 0 \quad \Rightarrow \quad x_2 = 2$$

Now, using $x_1 = x_2 - 1$, found previously, we have:

$$x_1 = x_2 - 1 = 2 - 1 = 1 \quad \Rightarrow \quad \begin{cases} x_1 = 1 \\ x_2 = 2 \end{cases} \quad \Rightarrow \quad x^* = x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}; \textit{Optimum solution point}$$

At this point we have the minimum value of $f(x)$

$$f_{min}(\cdot) = 2x_1^2 + x_2^2 - 2x_1 - 2x_1x_2 = 2(1)^2 + (2)^2 - 2(1) - 2(1)(2) = 2 + 4 - 2 - 4 = 0$$

Compare the solutions of Examples 1 and 2 for the minimum value **and** the optimum points: In Example 1 we had $x^* = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ with $f_{min}(\cdot) = -1$; but now we have: $x^* = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ with $f_{min}(\cdot) = 0$

Method 2: Method 1 is good if $h(x)$ is linear and can be solved easily. **Otherwise,** we use Lagrangian function (L) as defined below:

$L(\cdot\cdot) = f(x) + \lambda\, h(x)$; Assume λ is a new variable

Now, we solve the unconstrained optimization problem of: $\min: L(x, \lambda)$

At the optimum point we must have: $\begin{cases} \dfrac{\partial L(\cdot\cdot)}{\partial x_1} = 0 \\ \dfrac{\partial L(\cdot\cdot)}{\partial x_2} = 0 \\ \dfrac{\partial L(\cdot\cdot)}{\partial \lambda} = h(x) = 0 \end{cases}$

Now, we solve the above 3 equations simultaneously for the unknown variables of: λ, x_1, x_2 as follows:

$L(\cdot\cdot) = f(x) + \lambda\, h(x) = 2x_1^2 + x_2^2 - 2x_1 - 2x_1x_2 + \lambda\,(x_1 - x_2 + 1)$
$\Rightarrow$

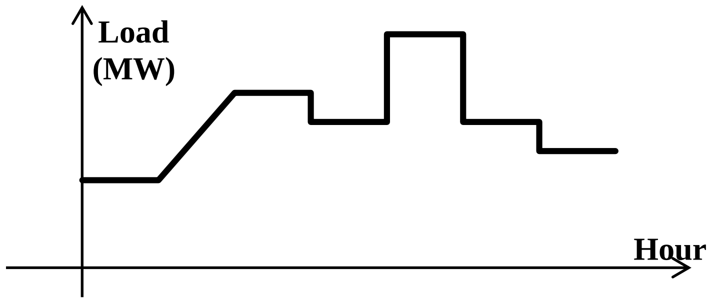
$$\begin{cases} \dfrac{\partial L}{\partial x_1} = 4x_1 - 2 - 2x_2 + \lambda = 0 \\ \dfrac{\partial L}{\partial x_2} = 2x_2 - 2x_1 - \lambda = 0 \\ h(x) = x_1 - x_2 - 1 = 0 \end{cases} \xrightarrow{solve} \begin{bmatrix} x^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

Where $x_1 = 1$ & $x_2 = 2$ are the same as the ones found using Method 1.

***With** inequality constraints the solution becomes more difficult.*

HW: Examples: 7-1, 2 Problems: 7-1, 2

<u>Operating Cost of Thermal Plants</u> **-- The** load in a power system changes during a 24-hour period, as well as different days and different seasons; *a typical average load variation is shown below*:

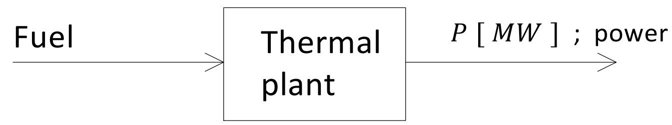


- There is a minimum value (called base load), there is a maximum value (called peak load), and several values in between.
- The generation capacity of power system must be greater than the maximum load.
- For a given amount of load there is infinitely many generation schedule for different power plant to support the load *as long as sum of generation equals to the sum of loads and losses*.

Different types of generation plants have different characteristics:

- Nuclear plants: **are** expensive to build, **but** the fuel is cheap; **so** we use it for base load.
- Hydro-plants: **are** mostly used for frequency control. **If we** have water flowing beyond the reservoir, we should use it; **otherwise** it will be wasted.
- Renewable (solar and wind power): we should use them as they come
- Gas-turbine are used for peak load since their fuel is very expensive
- Thermal plants (burning fossil fuel) vary in efficiency and fuel cost. **Thus,** scheduling of these plants affect the overall cost of the operation.

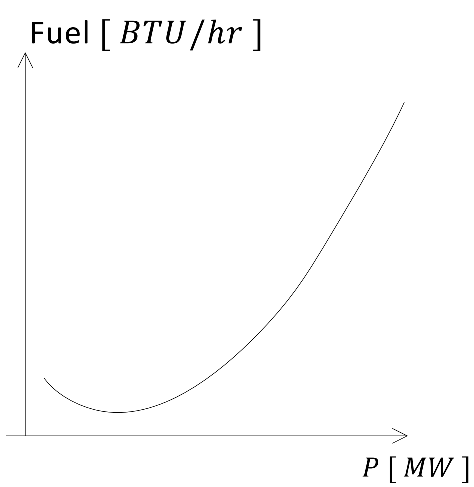
Input-output curves of thermal plants: By burning fossil fuel in a thermal plant we obtain electric power:



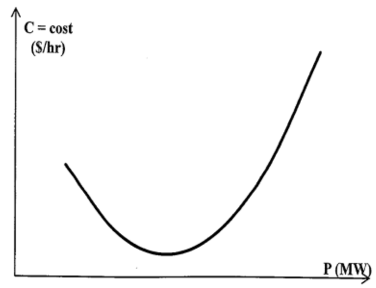
Associated with a thermal plant, we define the following 3 curves:

Heat – Rate curve: A curve that shows the amount consumed fuel in terms of BTU/hour for a given MW-output of the plant. **Where** BTU stands for British thermal unit and is a measure of heat energy obtained by burning fuel.

Heat – Rate curve is typically a quadratic function. It shows the conversion factor from heat energy to electric energy. **The** following figure shows a typical Heat – Rate curve:



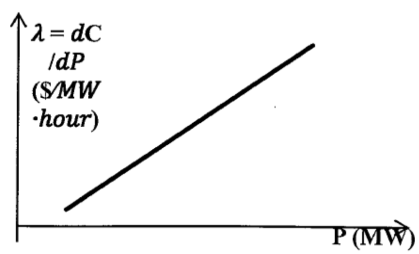
Fuel-cost curve: **This** curve has the same shape as the heat-rate curve. **The** vertical ordinate shows the dollar value of the fuel burned.



Typical function for the Fuel-cost curve is quadratic:

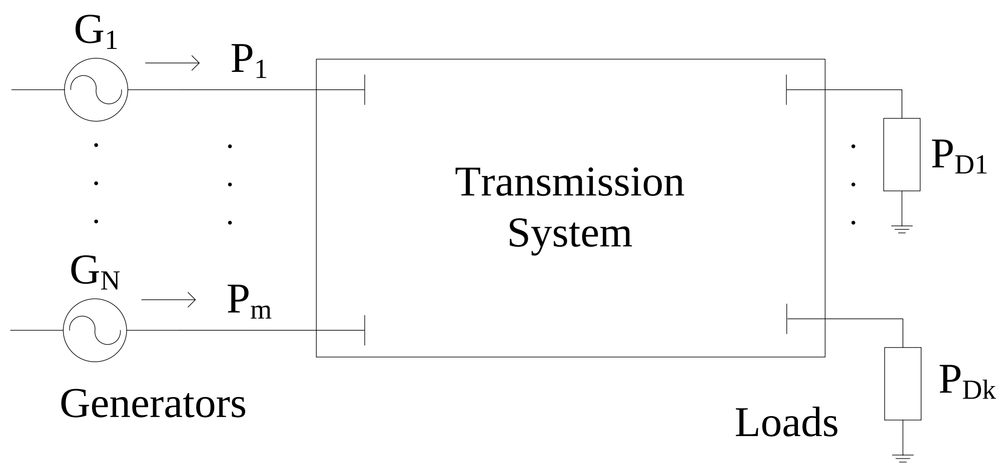
$C = \alpha + \beta P + \gamma P^2$; **Where:** α , β & γ are parameters and P is the plant output power

Incremental fuel cost curve: **This** function is the derivative the Fuel-cost with respect to the power output P as shown below:



$\lambda \triangleq \frac{dC}{dP} = 2\gamma P + \beta$; $\left[\frac{dC}{dP}\right] = \frac{[C]}{[P]} = \frac{\$/\text{hr}}{\text{MW}} = \frac{\$}{\text{MW}\cdot\text{hr}}$ (the unit); *it shows the cost of electrical energy generated by the plant.* ***Different*** *generators have curves that are different from one another*

Economic dispatch neglecting losses and assuming no generation limit -- **Assume** a power system, symbolically shown as follows:



Due to conservation of energy (or conservation of power), we must have:

$\sum_{i=1}^{m} P_i = \sum_{j=1}^{k} P_{Dj} + P_L$ **Where**: P_L represents the power losses in the system

$= \sum_{j=1}^{k} P_{Dj}$; **Ignoring** all losses

Considering only the generators that are on dispatch, we define the following:

$n \triangleq$ The number of generators on dispatch

$P_i \triangleq$ The power delivered by generator i

$C_i \triangleq \alpha_i + \beta_i P_i + \gamma_i P_i^2$; The cost of operation for generator i

$P_D \triangleq$ The balance of the demand that need be supported by the generators on dispatch

Now, the optimization problem is defined as: **Find** P_i's such that the total cost of operation is minimized subject to the satisfaction of power balance equation. **Or,** in short:

$min: C_t = C_1 + C_2 + \cdots + C_n$; The total cost of operation [compare it with $f(x)$]

$s.t.: -P_1 - P_2 - \cdots - P_n + P_D = 0$; The power balance equation [compare it with $h_i(x) = 0$]

To solve this problem we use Lagrange method:

$L(\cdot\cdot) = C_1 + C_2 + \cdots + C_n + \lambda\,(P_D - P_1 - P_2 - \cdots - P_n)$ **Where:** $C_i = \alpha_i + \beta_i P_i + \gamma_i P_i^2$

At the optimum solution we must have:

$$\frac{\partial L}{\partial P_1} = \frac{dC_1}{dP_1} + 0 + 0 + \cdots + \lambda\,(0 - 1 + 0 + \cdots) \quad \Rightarrow \quad \frac{dC_1}{dP_1} - \lambda = 0$$

$$\frac{\partial L}{\partial P_2} = 0 + \frac{dC_2}{dP_2} + 0 + \cdots + \lambda\,(0 + 0 - 1 + 0 + \cdots) \quad \Rightarrow \quad \frac{dC_2}{dP_2} - \lambda = 0$$

Similarly ... ... $\Rightarrow \quad \frac{dC_i}{dP_i} - \lambda = 0$

$\frac{\partial L}{\partial \lambda} = P_D - P_1 - P_2 - \cdots - P_n = 0$; *Power balance equation* $\Rightarrow \quad P_D - P_1 - P_2 - \cdots - P_n = 0$

***Now,* summarizing the above results we have**:

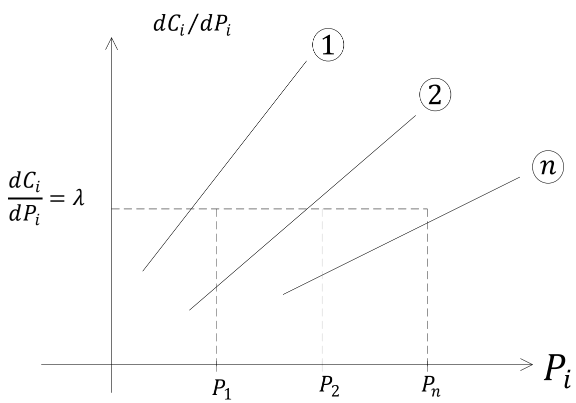
$\lambda = \frac{dC_1}{dP_1} = \cdots = \frac{dC_n}{dP_n}$; This is called the equal incremental cost criterion

$P_D - (P_1 + P_2 + \cdots + P_n) = 0$; This is the power balance equation

So, we have $n + 1$ equations with $n + 1$ unknowns: $P_1, P_2, \cdots, P_n$ & λ to solve for.

$\lambda = \frac{dC_1}{dP_1} = \cdots = \frac{dC_n}{dP_n}$ & $P_D - (P_1 + P_2 + \cdots + P_n) = 0$; (Repeat)

The above set of equations can be shown graphically as:



Now, considering the cost function $C_i = \alpha_i + \beta_i P_i + \gamma_i P_i^2$ at optimum point we have:

$$\frac{dC_i}{dP_i} = \beta_i + 2\gamma_i P_i \quad \Rightarrow \quad P_i = \frac{\lambda - \beta_i}{2\gamma_i}$$

Plugging these values into the power balance equation we obtain:

$$P_D = P_1 + P_2 + \cdots + P_n = \frac{\lambda - \beta_1}{2\gamma_1} + \cdots + \frac{\lambda - \beta_n}{2\gamma_n} \quad \Rightarrow \quad \lambda = \frac{P_D + \sum_{i=1}^{n}\left(\frac{\beta_i}{2\gamma_i}\right)}{\sum_{i=1}^{n}\left(\frac{1}{2\gamma_i}\right)}$$

So, to solve the economic dispatch problem, we first use: $\lambda = \frac{P_D + \sum_{i=1}^{n}\left(\frac{\beta_i}{2\gamma_i}\right)}{\sum_{i=1}^{n}\left(\frac{1}{2\gamma_i}\right)}$ to find λ, and then use $P_i = \frac{\lambda - \beta_i}{2\gamma_i}$ to find P_i.

Example 3 [Problem 11-8 / Bergen] -- **The** fuel cost functions for the 3 generating units on economic dispatch are given below:

$$\begin{cases} C_1 = 300 + 8P_{G1} + 0.0015P_{G1}^2 \\ C_2 = 450 + 8P_{G2} + 0.0005P_{G2}^2 \\ C_3 = 700 + 7.5P_{G3} + 0.001P_{G3}^2 \end{cases} \quad \text{Where } [C_i] = \$/\text{hr} \quad \& \quad [P_{Gi}] = \text{MW}.$$

Now, assuming the total demand is $P_D = 500^{\text{MW}}$, no generation limits for the units, and neglecting line losses find the total system cost C_T in $\$/\text{hr}$.

Solution: **First** find the incremental cost (IC) for each unit:

$$IC_1 = \frac{dC_1}{dP_{G1}} = 8 + 0.003P_{G1} \qquad IC_2 = \frac{dC_2}{dP_{G2}} = 8 + 0.001P_{G2} \qquad IC_3 = \frac{dC_3}{dP_{G3}} = 7.5 + 0.002P_{G3}$$

Now, solve the following:

$$\begin{cases} IC_1 = 8 + 0.003P_{G1} = \lambda \\ IC_2 = 8 + 0.001P_{G2} = \lambda \\ IC_3 = 7.5 + 0.002P_{G3} = \lambda \\ P_D = P_{G1} + P_{G2} + P_{G3} = 500^{\text{M}} \end{cases} \xRightarrow{solve} \begin{cases} \lambda = 8.136\,\$/hr \\ P_{G1} = 45.33^{\text{MW}} \\ P_{G2} = 136^{\text{MW}} \\ P_{G3} = 318^{\text{MW}} \end{cases}$$

Alternatively, we could use $\lambda = \frac{P_D + \sum_{i=1}^{n}\left(\frac{\beta_i}{2\gamma_i}\right)}{\sum_{i=1}^{n}\left(\frac{1}{2\gamma_i}\right)}$ to find λ, then use $P_i = \frac{\lambda - \beta_i}{2\gamma_i}$ to find P_i. **Do it as HW.**

$$\begin{cases} C_1 = 300 + 8P_{G1} + 0.0015P_{G1}^2 \\ C_2 = 450 + 8P_{G2} + 0.0005P_{G2}^2 \\ C_3 = 700 + 7.5P_{G3} + 0.001P_{G3}^2 \end{cases}$$; Repeated

$$\begin{cases} \lambda = 8.136\,\$/hr \\ P_{G1} = 45.33^{\text{MW}} \\ P_{G2} = 136^{\text{MW}} \\ P_{G3} = 318^{\text{MW}} \end{cases}$$; Repeated

Finding the total cost of operation:

The cost of operation for the generators are:

$C_1 = 300 + 8P_{G1} + 0.0015P_{G1}^2$ (Plug in $P_{G1} = 45.33^{\text{MW}}$ into this equation)

$= \cdots = 665.75\,\$/\text{hr}$

Similarly, we have:

$C_2 = \cdots = 1547.25\,\$/\text{hr}$
$C_3 = \cdots = 3186.12\,\$/\text{hr}$

Finally, the total cost of operation is:

$C_T = C_1 + C_2 + C_3 = \cdots = 5399.12\,\$/\text{hr}$

The gradient method-- **If the** cost functions, C_i are not quadratic, and equal incremental cost equations $\frac{dC_i}{dP_i} = \lambda$ cannot be solved easily, we may use the *gradient method*. **This** is an iterative method.

Using equal incremental cost criterion: $\frac{dC_i}{dP_i} = \lambda$ we obtain a function $P_i = f_i(\lambda)$ for each generator.

Using power balance equation: $P_1 + P_2 + \cdots + P_n = P_D$ & $P_i = f_i(\lambda)$ (from above) we obtain:

$f_1(\lambda) + f_2(\lambda) + \cdots + f_n(\lambda) = P_D$ **Or,** $f(\lambda) = P_D$ **Where** $f(\cdot) \triangleq f_1(\cdot) + \cdots + f_n(\cdot)$

Using Taylor Series expansion about a given $\lambda = \lambda_0$ and ignoring the higher order terms, we have:

$$f(\lambda) + \frac{df(\lambda)}{d\lambda}\,\Delta\,\lambda = P_D$$

$\Rightarrow$

$\frac{df(\lambda)}{d\lambda} \cdot \Delta\lambda = P_D - f(\lambda)$ Now, define: $\Delta P \triangleq P_D - f(\lambda)$ for $\lambda = \lambda_0$

$\Rightarrow$

$\Delta\lambda = \frac{\Delta P}{\left(\frac{df(\lambda)}{d\lambda}\right)}$ ***This** equation is used for an iterative solution as described below*: (**Over**)

Iterative procedure: Given the cost function C_i , then: **(a)** -- find the functions $\frac{dC_i}{dP_i} = \lambda$ for each generator **(b)**-- solve the above equations for P_i in terms of λ and call these function $f_i(\lambda)$; this means: $P_i = f_i(\lambda)$ & **(c)**-- find $f(\lambda) = f_1(\lambda) + f_2(\lambda) + \cdots + f_n(\lambda)$.

Now, with $f(\lambda)$ at hand, perform the following iterative process:

Assume $\lambda = \lambda^{(k)}$ is our estimate of λ in iteration k, then:

(1) -- Find $\Delta P^{(k)} = P_D - f\left(\lambda^{(k)}\right) \ = P_D - \sum_{i=1}^{n} P_i^{(k)}$; the mismatch

(2) -- Find $\left(\frac{df(\lambda)}{d\lambda}\right)^{(k)} = \left.\frac{df(\lambda)}{d\lambda}\right|_{\lambda=\lambda^{(k)}}$

(3) -- Find $\Delta\,\lambda^{(k)} = \frac{\Delta P^{(k)}}{\left(\frac{df(\lambda)}{d\lambda}\right)^{(k)}}$

(4) -- Find $\lambda^{(k+1)} = \lambda^{(k)} + \Delta\,\lambda^{(k)}$; the updated value for λ

The process is continued until $\Delta P^{(k)}$ is less than a specified value for acceptable accuracy.

Assuming quadratic cost function $C_i \triangleq \alpha_i + \beta_i P_i + \gamma_i P_i^2$ the equations for update becomes:

$\Delta\,\lambda^{(k)} = \frac{\Delta P^{(k)}}{\sum_{i=1}^{n}(\frac{1}{2\gamma_i})}$ Where: $\Delta P^{(k)} = P_D - \sum_{i=1}^{n}(\frac{\lambda^{(k)} - \beta_i}{2\gamma_i})$

HW: Example 7-4

Economic Dispatch [*Neglecting transmission losses but including generation limits*]

It is important to notice that each generator is designed to operate within its rated values: $P_{i(min)} \leq P_i \leq P_{i(max)}$. **With** this in mind, the *economic dispatch* problem is defined as follows:

Minimize: $C_t = C_1 + C_2 + \cdots + C_n$ The cost function

Subject to: $P_1 + P_2 + \cdots + P_n = P_D$ Equality constraint

Subject to: $P_{i(min)} \leq P_i \leq P_{i(max)}$ For all i The inequality constraint

The solution is achieved as follows:

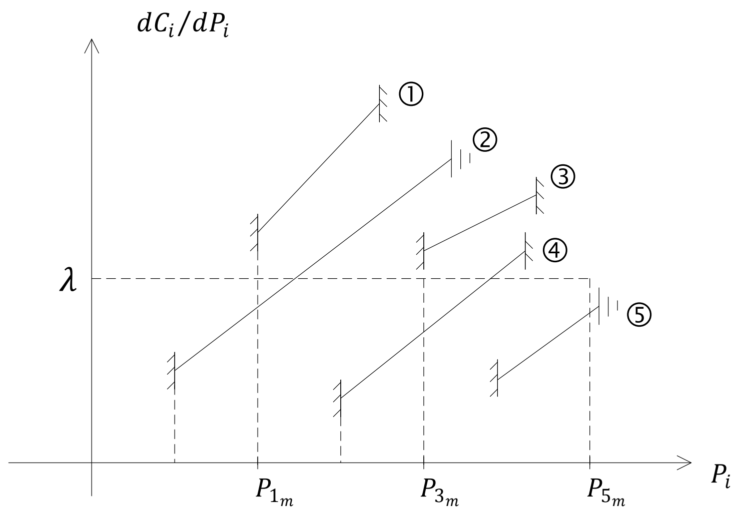
Assume no limits and solve the economic dispatch problem without the inequality constraint.

Check if any limit is violated:

If no: then we have the solution

If yes: Set the dispatch values of the generators whose limits have been violated equal to the violated limits. Then solve the economic dispatch problem for the remaining plants and the balance of power.

As an example assume we found the following λ as the solution of the unconstrained economic dispatch problem:



From the above we make this decision: $P_1 = P_{1m}$, $\quad P_3 = P_{3m} \quad \& \quad P_5 = P_{5M}$

Then we solve another unconstrained dispatch problem for generators **2** and **4** (those generators whose limits have not been violated) for the balance of power:

$$P_{bal} = P_D - (P_{1m} + P_{3m} + P_{5m})$$

<u>HW</u>: Example: 7-6 & Problems: 7-7:10

Example 4 [*example 11.4 / Glover*]

An area has 2 fossil-fuel units operating on economic dispatch loop. **The** variable costs of the units are:

$$\begin{cases} C_1 = 10P_{G1} + 8 \times 10^{-3}\, P_{G1}^2 \\ C_2 = 8P_{G2} + 9 \times 10^{-3}\, P_{G2}^2 \end{cases} \quad ; \quad [C] = \$/\mathrm{hr} \quad \& \quad [P] = \mathrm{MW}$$

The generation limits of the units are:

$$100^{\mathrm{MW}} \le P_{G1} \le 600^{\mathrm{MW}}$$
$$400^{\mathrm{MW}} \le P_{G2} \le 1000^{\mathrm{MW}}$$

(a)-- Find the range of the power demand (P_D) that can be supported by these units.

(b)-- Find the range of P_D for which the Equal Incremental Cost Criterion (EICC) holds.

(c)-- Find the optimal solution for all P_D as P_D varies within the range found in part (a).

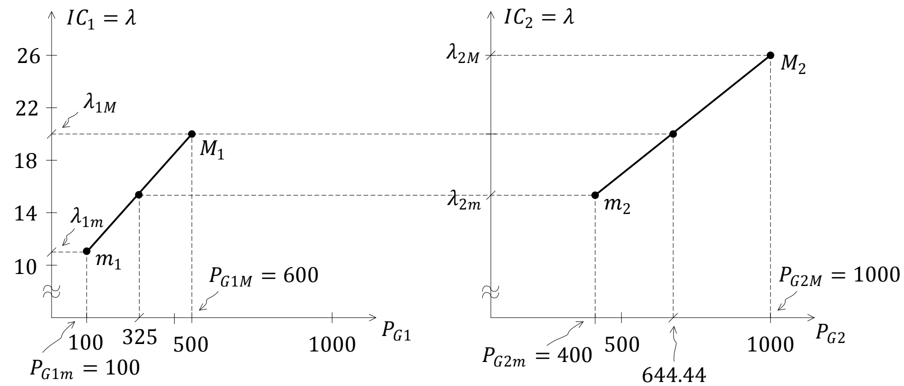
Solution -- Let's plot the Incremental Cost functions:

$$IC_1 = \frac{dC_1}{dP_{G1}} = 10 + 16 \times 10^{-3} P_{G1} \quad ; \quad 100^{\mathrm{MW}} \le P_{G1} \le 600^{\mathrm{MW}}$$
$$IC_2 = \frac{dC_2}{dP_{G2}} = 8 + 18 \times 10^{-3} P_{G2} \quad ; \quad 400^{\mathrm{MW}} \le P_{G2} \le 1000^{\mathrm{MW}}$$

(Over)

$$IC_1 = \frac{dC_1}{dP_{G1}} = 10 + 16 \times 10^{-3} P_{G1} \quad ; \quad 100^{\text{MW}} \leq P_{G1} \leq 600^{\text{MW}} \quad ; \quad \text{Repeated}$$

$$IC_2 = \frac{dC_2}{dP_{G2}} = 8 + 18 \times 10^{-3} P_{G2} \quad ; \quad 400^{\text{MW}} \leq P_{G2} \leq 1000^{\text{MW}} \quad ; \quad \text{Repeated}$$



Next, let us find the values of λ at Maximum and minimum limits of the different generators:

$$P_{G1m} = 100 \quad \Rightarrow \quad IC_1 = 11.6 = \lambda_{1m} \quad \Rightarrow \quad \text{Point } m_1$$

$$P_{G1M} = 600 \quad \Rightarrow \quad IC_1 = 19.6 = \lambda_{1M} \quad \Rightarrow \quad \text{Point } M_1$$

$$P_{G2m} = 400 \quad \Rightarrow \quad IC_2 = 15.2 = \lambda_{2m} \quad \Rightarrow \quad \text{Point } m_2$$

$$P_{G2M} = 1000 \quad \Rightarrow \quad IC_2 = 26 = \lambda_{2M} \quad \Rightarrow \quad \text{Point } M_2$$

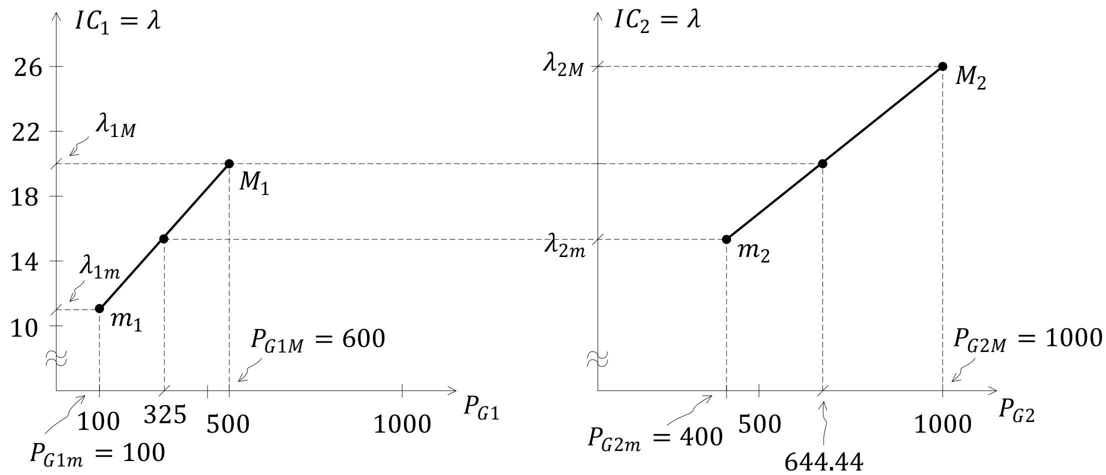
(a)-- The minimum and Maximum values of the demand is as follows:

$$P_{Dm} = P_{G1m} + P_{G2m} = 100 + 400 = 500 \text{ MW} \quad ; \quad \text{The minimum demand}$$

$$P_{DM} = P_{G1M} + P_{G2M} = 600 + 1000 = 1600 \text{ MW} \quad ; \quad \text{The Maximum demand}$$

So, the load range is: $500 \leq P_D \leq 1600$

(b)-- As seen from the figure the EICC can only hold when: $\lambda_{2m} \le \lambda = IC_i \le \lambda_{1M}$



Now, we can find the range of P_D for the above range of λ as follows:

For $\lambda = \lambda_{2m} = 15.2$, we have: $P_{G2} = 400 \text{ MW} = P_{G2m}$

$$\lambda = 15.2 = \ IC_1 = 10 + 16 \times 10^{-3}\, P_{G1} \quad \overset{solve}{\Longrightarrow} \quad P_{G1} = \cdots = 325^{\text{MW}}\ ; \quad \text{Marked on figure}$$

$$P_D = P_{G1} + P_{G2} = 325 + 400 = 725^{\text{MW}}$$

For $\lambda = \lambda_{1M} = 19.6$, we have: $P_{G1} = 600 \text{ MW} = P_{G1M}$

$$\lambda = 19.6 = \ IC_2 = 8 + 18 \times 10^{-3}\, P_{G2} \quad \overset{solve}{\Longrightarrow} \quad P_{G2} = \cdots = 644.44^{\text{MW}}; \quad \text{Marked on figure}$$

$$P_D = P_{G1} + P_{G2} = 600 + 644.44 = 1244.44^{MW}$$

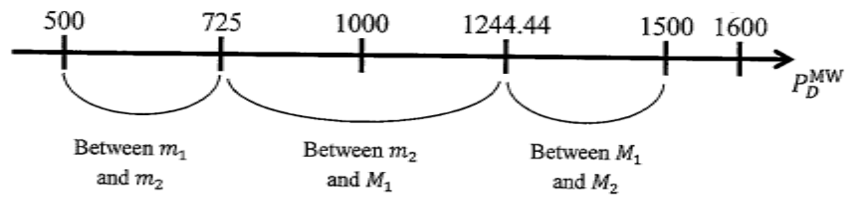
Thus, equivalently, the EICC can apply for the range of demand as: $725^{\text{MW}} \le P_D \le 1244.44^{\text{MW}}$

(c)-- So far, we found the following:

For: $500 \le P_D \le 1600$ We can supply the load without bringing in another unit

For: $725 \le P_D \le 1244.44$ EICC applies

Using above and in conjunction with previous figure we can draw the following:



So, If: $P_D \le 725$ $\Rightarrow$ $\begin{cases} P_{G2} = P_{G2m} = 400 \\ P_{G1} = P_D - 400 \end{cases}$

If: $1244.44 \le P_D \le 1600$ $\Rightarrow$ $\begin{cases} P_{G1} = P_{G1M} = 600 \\ P_{G2} = P_D - 600 \end{cases}$

If: $725 \le P_D \le 1244.44$ $\Rightarrow$ EICC holds, and (P_{G1}, P_{G2}) must be calculated

The calculation for the case of: $725 \le P_D \le 1244.44$ is as follows.

Using the cost functions we have:

$$\begin{cases} C_1 = \alpha_1 + \beta_1 P_1 + \gamma_1 P_1^2 \\ C_2 = \alpha_2 + \beta_2 P_2 + \gamma_2 P_2^2 \end{cases} \Rightarrow \begin{cases} IC_1 = \beta_1 + 2\,\gamma_1 P_1 = \lambda \\ IC_2 = \beta_2 + 2\,\gamma_2 P_2 = \lambda \end{cases} \Rightarrow \beta_1 + 2\,\gamma_1 P_1 = \beta_2 + 2\,\gamma_2 P_2$$

Using the power balance equation we have:

$P_1 + P_2 = P_D \quad \Rightarrow \quad P_2 = P_D - P_1$ now plug this into equation below:

$\beta_1 + 2\,\gamma_1 P_1 = \beta_2 + 2\,\gamma_2 P_2$; Repeated $\Rightarrow \quad \beta_1 + 2\,\gamma_1 P_1 = \beta_2 + 2\,\gamma_2 (P_D - P_1)$

$\Rightarrow$

$(2\gamma_1 + 2\gamma_2)P_1 = \beta_2 - \beta_1 + 2\,\gamma_2 P_D \quad \Rightarrow \quad P_1 = \frac{\beta_2 - \beta_1 + 2\,\gamma_2 P_D}{2\gamma_1 + 2\gamma_2}$ We know $\beta_1, \beta_2, \gamma_1, \gamma_2$

Now,

$P_2 = P_D - P_1 = P_D - \frac{\beta_2 - \beta_1 + 2\,\gamma_2 P_D}{2\gamma_1 + 2\gamma_2} = \frac{2\,\gamma_1 P_D + 2\,\gamma_2 P_D - \beta_2 + \beta_1 - 2\,\gamma_2 P_D}{2\gamma_1 + 2\gamma_2} \quad \Rightarrow \quad P_2 = \frac{2\,\gamma_1 P_D + \beta_1 - \beta_2}{2\gamma_1 + 2\gamma_2}$

Now, you can plot P_{G1}, P_{G2} vs. P_D

Optimal Economic Dispatch Considering Losses -- **Given** a set of loads, then there are many generation patterns that can supply the load. **Different** patterns of generation result in different flows of power through the lines and thus different values of losses in lines. **These** losses have to be supported by generation also. **Therefor** the power balance equation becomes:

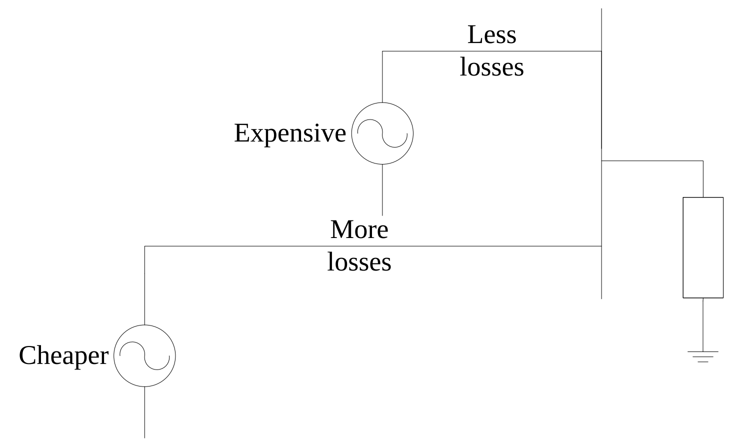
$\sum_{i=1}^{n} P_{Gi} = P_D + P_L \quad ; \quad P_L \triangleq$ The total losses of lines

It should be noted that P_L is a function of the generation pattern:

$P_L = P_L(P_{G1}, \ldots, P_{Gn})$

The further away (in electrical distance) a generator is from the load center, the more would be the associated losses

For example, consider the system shown: **The** power from the generator on the right may be more expensive but getting the power to the load involves less losses. **The** opposite is true for the other generator. **So,** we need to let the economic dispatch to make the decision for the optimum solution.



Statement of the problem:

$min: \; C_T = \sum_{i=1}^{n} C_i\,(P_{Gi})$; The cost function

$s.t.: \; \sum P_{Gi} = P_L\,(P_G) + P_D$; The power balance equation

Using Lagrangian method, the solution to the optimum dispatch problem becomes as follows.

Define the penalty factor as:

$$L_i \triangleq \frac{1}{1-\frac{\partial P_L}{\partial P_{Gi}}}$$ Penalty factor of generator i

At the optimum solution we have (it can be proven):

$$L_1 \frac{dC_1}{dP_{G1}} = L_2 \frac{dC_2}{dP_{G2}} = \cdots\cdot = L_n \frac{dC_n}{dP_{Gn}} = \lambda$$ Or $$L_i \frac{dC_i}{dP_{Gi}} = \lambda$$ For all i

So, at optimum point we have the above equations together with power balance equation as:

$$L_i \frac{dC_i}{dP_{Gi}} = \lambda$$ For all i

$\sum_{i=1}^{n} P_{Gi} = P_L + P_D$; Power balance equation

Now, we can solve the $n+1$ equations above for the $n+1$ unknowns: $P_{G1}, \ldots, P_{Gn}$, and λ.

HW: Example 7-7

Example 5 -- **An** area has 2 units on economic dispatch loop. **The** variable costs of the units are:

$$\begin{cases} C_1 = 10P_{G1} + 8 \times 10^{-3}\ P_{G1}^2 \\ C_2 = 8P_{G2} + 9 \times 10^{-3}\ P_{G2}^2 \end{cases} \quad ; \quad [C] = \$/\text{hr} \quad \& \quad [P] = \text{MW}$$

The total transmission losses for the system is given by:

$$P_L = 1.5 \times 10^{-4}\ P_{G1}^2 + 2 \times 10^{-5}\ P_{G1}P_{G2} + 3 \times 10^{-5}\ P_{G2}^2$$

(a)-- Given $P_D = 679.5$ MW, find the economic dispatch solution and the total fuel cost C_T in \$/hr.

(b)-- The economic solution for P_{G1} is 250 MW for a load of P_D, find P_{G2}, P_L, P_D, λ and C_T.

(c)-- The economic dispatch solution results in $\lambda = 15.75$ \$/MW·hr for a load demand of P_D. Now, find: P_{G1}, P_{G2}, P_L, P_D, λ, and C_T.

Solution -- Find the incremental cost $\frac{dC_i}{dP_{Gi}}$ and the penalty factor $L_i = \frac{1}{1-\frac{\partial P_L}{\partial P_{Gi}}}$ for each unit:

$$\frac{dC_1}{dP_{G1}} = 10 + 16 \times 10^{-3} P_{G1} \quad \& \quad \frac{dC_2}{dP_{G2}} = 8 + 18 \times 10^{-3} P_{G2}$$

$$\frac{\partial P_L}{\partial P_{G1}} = 3.0 \times 10^{-4}\ P_{G1} + 2 \times 10^{-5}\ P_{G2} \quad \Rightarrow \quad L_1 = \frac{1}{1-\frac{\partial P_L}{\partial P_{G1}}} = \frac{1}{1-3.0\times10^{-4}\ P_{G1}+2\times10^{-5}\ P_{G2}}$$

$$\frac{\partial P_L}{\partial P_{G2}} = 6 \times 10^{-5}\ P_{G2} + 2 \times 10^{-5}\ P_{G1} \quad \Rightarrow \quad L_2 = \frac{1}{1-\frac{\partial P_L}{\partial P_{G2}}} = \frac{1}{1-2\times10^{-5}\ P_{G1}-6\times10^{-5}\ P_{G2}}$$

(a)-- Given $P_D = 679.5$ MW, find the economic dispatch solution and the total fuel cost C_T in \$/hr.

At optimum solution we have:

$$\begin{cases} L_1 \dfrac{dC_1}{dP_{G1}} = \lambda \\ L_2 \dfrac{dC_2}{dP_{G2}} = \lambda \\ P_{G1} + P_{G2} = P_D + P_L \end{cases} \Rightarrow \begin{cases} L_1 \dfrac{dC_1}{dP_{G1}} = L_2 \dfrac{dC_2}{dP_{G2}} \\ P_{G1} + P_{G2} = P_D + P_L \end{cases}$$

$\Rightarrow$

$$\begin{cases} \dfrac{10+16\times10^{-3}P_{G1}}{1-3.0\times10^{-4}\,P_{G1}+2\times10^{-5}\,P_{G2}} = \dfrac{8+18\times10^{-3}P_{G2}}{1-2\times10^{-5}\,P_{G1}-6\times10^{-5}\,P_{G2}} \\ P_{G1} + P_{G2} = P_D + 1.5\times10^{-4}\,P_{G1}^2 + 2\times10^{-5}\,P_{G1}P_{G2} + 3\times10^{-5}\,P_{G2}^2 \end{cases}$$

The above is a set of 2 non-linear equations with 2 unknowns. Using numerical solution (you may use MATLAB) we obtain the following:

$P_{G1} = 282^{\text{MW}}$ & $P_{G2} = 417^{\text{MW}}$

Now, plug in these values into the cost functions $\begin{cases} C_1 = 10P_{G1} + 8\times10^{-3}\,P_{G1}^2 \\ C_2 = 8P_{G2} + 9\times10^{-3}\,P_{G2}^2 \end{cases}$ to obtain C_1 & C_2

Then,

$C_T = C_1 + C_2 = \cdots$

(b)-- The economic solution for P_{G1} is 250 MW for a load of P_D, find P_{G2}, P_L, P_D, λ and C_T.

As discussed in part **(a)**, at optimum solution we have:

$$\begin{cases} L_1 \frac{dC_1}{dP_{G1}} = L_2 \frac{dC_2}{dP_{G2}} \\ P_{G1} + P_{G2} = P_D + P_L \end{cases}$$

$\Rightarrow$

$$\begin{cases} \frac{10 + 16 \times 10^{-3} P_{G1}}{1 - 3.0 \times 10^{-4}\ P_{G1} + 2 \times 10^{-5}\ P_{G2}} = \frac{8 + 18 \times 10^{-3} P_{G2}}{1 - 2 \times 10^{-5}\ P_{G1} - 6 \times 10^{-5}\ P_{G2}} \\ P_{G1} + P_{G2} = P_D + 1.5 \times 10^{-4}\ P_{G1}^2 + 2 \times 10^{-5}\ P_{G1} P_{G2} + 3 \times 10^{-5}\ P_{G2}^2 \end{cases}$$

The first equation includes 2 variables P_{G1} & P_{G2}. Given $P_{G1} = 250^{\text{MW,}}$ we can solve this equation for P_{G2}.

Now, plug in P_{G1} & P_{G2} into the second equation to find P_D

Now, plug in P_{G1} & P_{G2} into the Loss equation below to find P_L

$$P_L = 1.5 \times 10^{-4}\ P_{G1}^2 + 2 \times 10^{-5}\ P_{G1} P_{G2} + 3 \times 10^{-5}\ P_{G2}^2$$

Now, $\lambda = L_1 \frac{dC_1}{dP_{G1}} = \frac{10+16\times10^{-3} P_{G1}}{1-3.0\times10^{-4}\ P_{G1}+2\times10^{-5}\ P_{G2}} = \cdots$

Now, find C_T the same way as part **(a)**

(c)-- The economic dispatch solution results in $\lambda = 15.75$ \$/MW·hr for a load demand of P_D. Now, find: P_{G1}, P_{G2}, P_L, P_D, λ, and C_T.

As discussed in part **(a),** at optimum solution we have:

$$\begin{cases} L_1 \dfrac{dC_1}{dP_{G1}} = \lambda \\ L_2 \dfrac{dC_2}{dP_{G2}} = \lambda \\ P_{G1} + P_{G2} = P_D + P_L \end{cases}$$

Now, plug in for L_1 , $\dfrac{dC_1}{dP_{G1}}$ & L_2 , $\dfrac{dC_2}{dP_{G2}}$ from part **(a),** to obtain:

$$\begin{cases} \dfrac{10 + 16 \times 10^{-3} P_{G1}}{1 - 3.0 \times 10^{-4}\, P_{G1} + 2 \times 10^{-5}\, P_{G2}} = \lambda \\ \dfrac{8 + 18 \times 10^{-3} P_{G2}}{1 - 2 \times 10^{-5}\, P_{G1} - 6 \times 10^{-5}\, P_{G2}} = \lambda \\ P_{G1} + P_{G2} = P_D + 1.5 \times 10^{-4}\, P_{G1}^2 + 2 \times 10^{-5}\, P_{G1}P_{G2} + 3 \times 10^{-5}\, P_{G2}^2 \end{cases}$$

Now, given $\lambda = 15.75$ \$/MW·hr, solve the first equation for P_{G1} and the second equation for P_{G2}

Now, plug in P_{G1} & P_{G2} into the third equation to obtain P_D

Now, calculate P_L and C_T the same way done in part **(b).**