

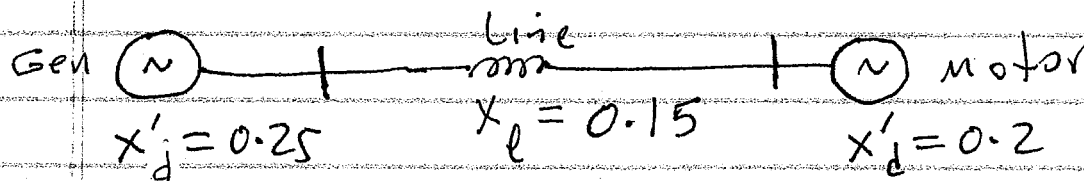
Prob 1: A power system has two fossil-fueled units on economic dispatch loop. The cost functions and the generation limits are given by:

$$\begin{cases} C_1 = 400 + 10 P_{G1} + 8 \times 10^{-3} P_{G1}^2 & [C] = \$/\text{hr} \\ C_2 = 300 + 8 P_{G2} + 9 \times 10^{-3} P_{G2}^2 & [P_G] = \text{MW} \end{cases}$$

$$100 \overset{\text{MW}}{< P_{G1}} \overset{\text{MW}}{\leq} 600 \quad ; \quad 400 \overset{\text{MW}}{< P_{G2}} \overset{\text{MW}}{\leq} 1000$$

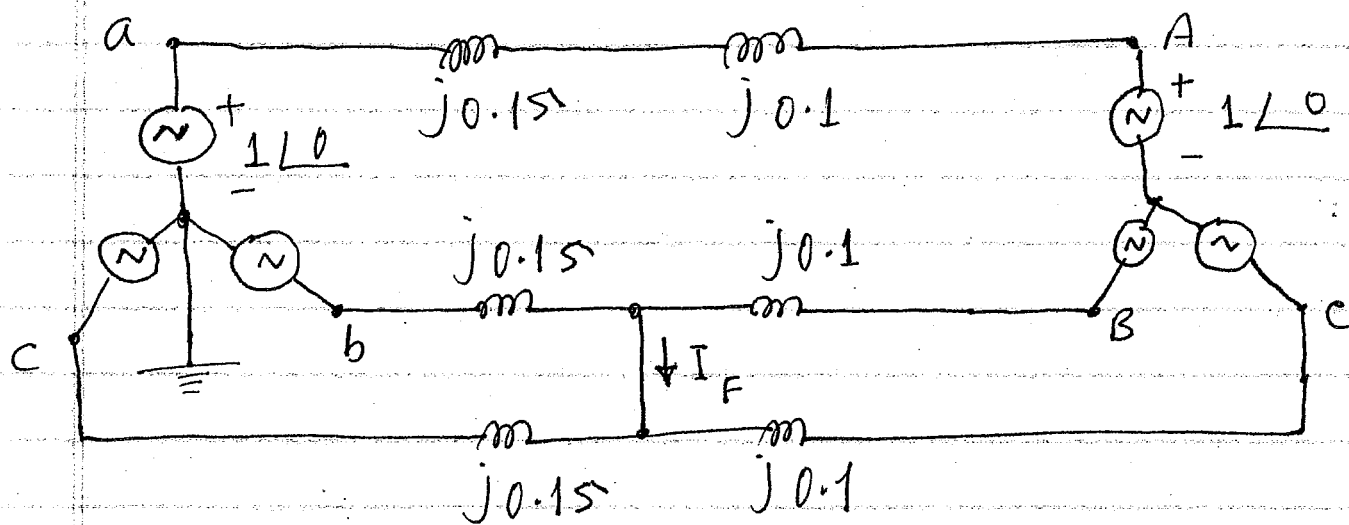
Given  $\lambda = 17.53$ , find the optimal solutions  $P_{G1}^*$  and  $P_{G2}^*$  as well as the total demand  $P_D$  and the total cost.

Prob 2 Assume the power system below where all quantities are given in p.u.

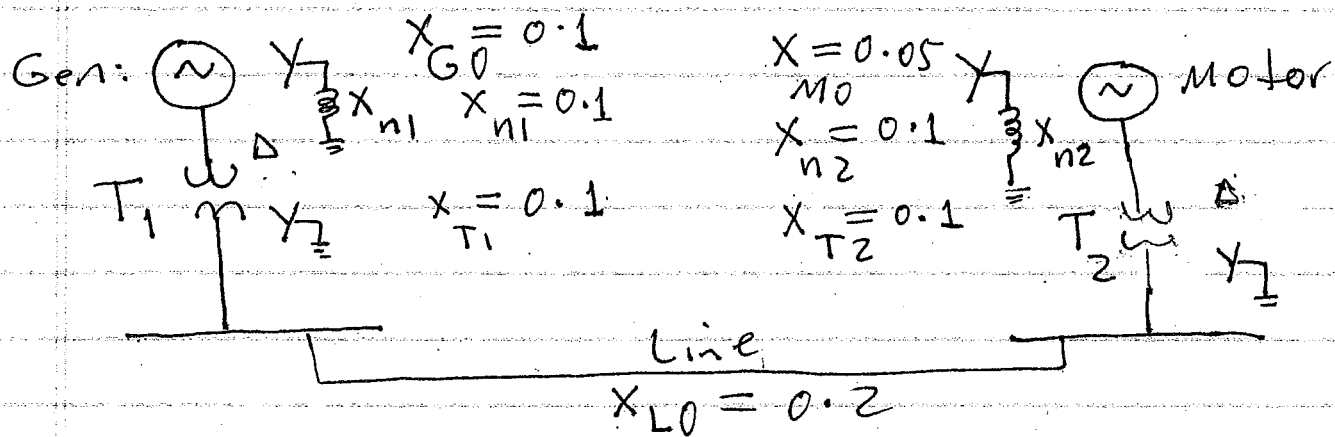


The motor is receiving an apparent power of 1.1 pu at power factor 0.9 (lag) with terminal voltage of 1 p.u. Suddenly a 3-phase fault happens at generator terminal. Find the fault current during transient period.

Prob 3: Given the source voltages are positive sequence sets, find the fault current  $I_F$  as shown in system below.



Prob 4: Consider the power system shown below with all quantities given in p.u.



- Draw the Zero-sequence network for the system.
- Find the Zero-sequence impedance matrix for the system.

ECE 4620/5620 [T<sub>1</sub>; SP 2013]; Key

Prob 1: we are given  $\lambda = 17.53$  and:

$$\begin{cases} C_1 = 400 + 10 P_{G1} + 8 \times 10^{-3} P_{G1}^2 & [C] = \$/\text{hr} \\ C_2 = 300 + 8 P_{G2} + 9 \times 10^{-3} P_{G2}^2 & [P] = \text{MW} \end{cases}$$

$$\begin{aligned} \lambda &= \frac{dC_1}{dP_{G1}} = 10 + 16 \times 10^{-3} P_{G1} = 17.53 \\ \Rightarrow P_{G1}^* &= \frac{17.53 - 10}{16 \times 10^{-3}} = 470.63 \text{ MW} \checkmark \end{aligned}$$

$$\begin{aligned} \lambda &= \frac{dC_2}{dP_{G2}} = 8 + 18 \times 10^{-3} P_{G2} = 17.53 \\ \Rightarrow P_{G2}^* &= \frac{17.53 - 8}{18 \times 10^{-3}} = 529.44 \text{ MW} \checkmark \end{aligned}$$

$$\begin{aligned} P_D &= P_{G1}^* + P_{G2}^* = 470.63 + 529.44 \\ &= 1000 \text{ MW} \checkmark \end{aligned}$$

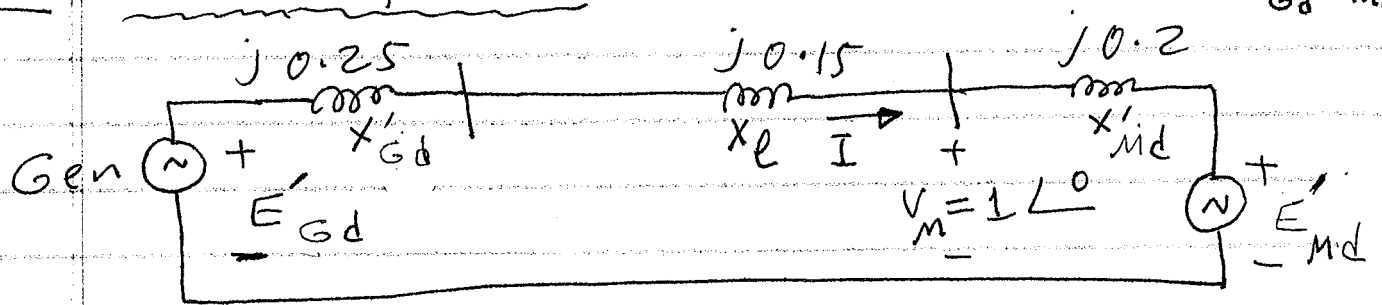
$$C_1 = 400 + 10 P_{G1}^* + 8 \times 10^{-3} P_{G1}^{*2} = \dots = 6,878.24 \text{ } [\$/\text{hr}]$$

$$C_2 = 300 + 8 P_{G2}^* + 9 \times 10^{-3} P_{G2}^{*2} = \dots = 7,058.28 \text{ } [\$/\text{hr}]$$

now,

$$C = C_1 + C_2 = \dots = 13,936.52 \text{ } [\$/\text{hr}] \checkmark$$

Prob2: Before fault: Draw the circuit and find  $E'_{Gd}$ ,  $E'_{Md}$



$$I = \left( \frac{S_m}{V_m} \right)^* = \left( \frac{1.1 \angle \cos^{-1}(0.9)}{1 \angle 0} \right)^* = \frac{1.1 \angle -25.84^\circ}{1 \angle 0}$$

$$= 1.1 \angle -25.84^\circ$$

$$E'_{Gd} = V_m + j(0.25 + 0.15)I$$

$$= 1 \angle 0 + j0.4 (1.1 \angle -25.84^\circ)$$

$$= 1 + j(0.4 - j0.18)$$

$$= 1 + j0.4 + 0.18 = 1.18 + j0.4$$

$$= 1.25 \angle 18.73^\circ \checkmark$$

$$E'_{Md} = V_m - jx'_{Md}I = 1 - j0.2(1.1 \angle -25.84^\circ)$$

$$= 1 - j(0.22 \angle -25.84^\circ)$$

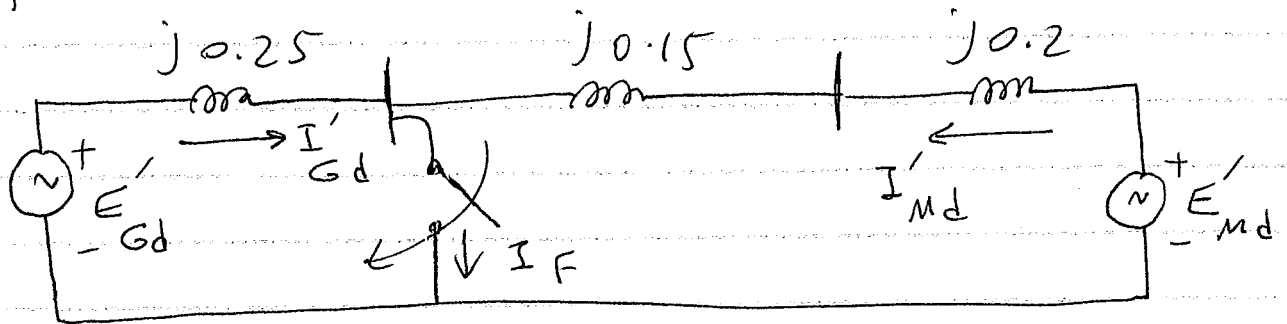
$$= 1 - j(0.2 - j0.1)$$

$$= 1 - j0.2 - 0.1 = 0.9 - j0.2$$

$$= 0.92 \angle -12.53^\circ \checkmark$$

## Prob 2 [Continued]

During the fault the circuit is as follows:



$$I_F = I'_{Gd} + I'_{Md}$$

$$= \frac{E'_{Gd}}{j0.25} + \frac{E'_{Md}}{j(0.15+0.2)}$$

$$= \frac{1.25 \angle 18.73}{j0.25} + \frac{0.92 \angle -12.53}{j0.35}$$

$$= -j \left( \frac{1.25}{0.25} \angle 18.73 \right) - j \left( \frac{0.92}{0.35} \angle -12.53 \right)$$

$$= -j (5 \angle 18.73) - j (2.63 \angle -12.53)$$

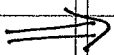
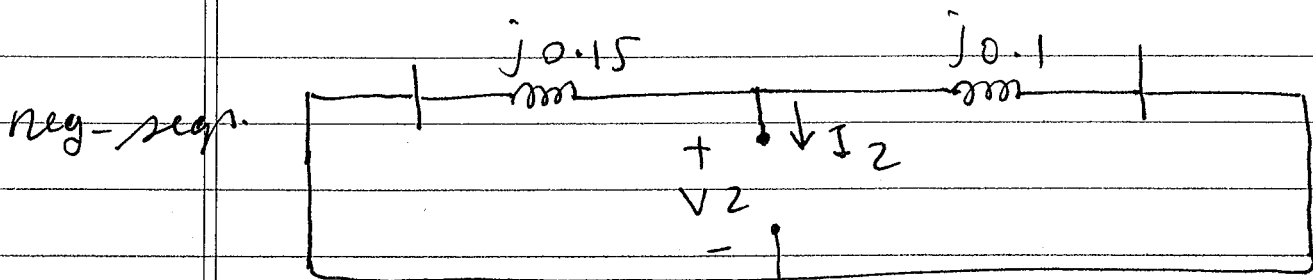
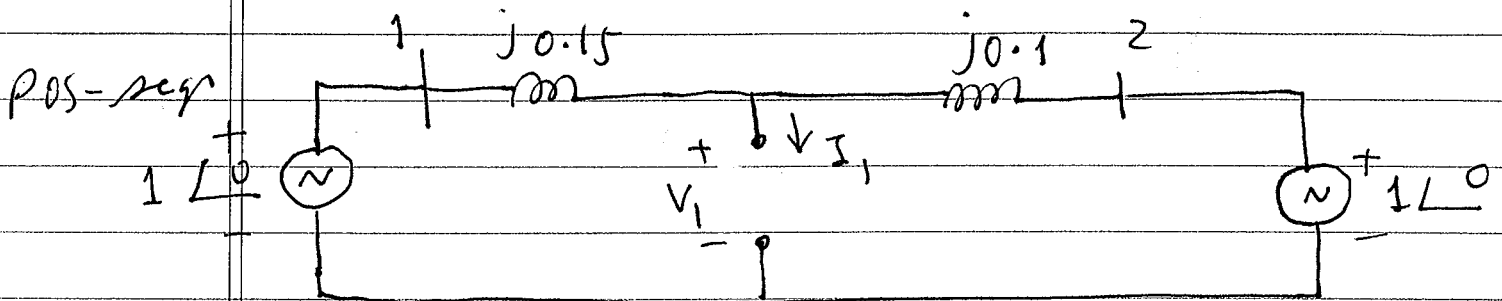
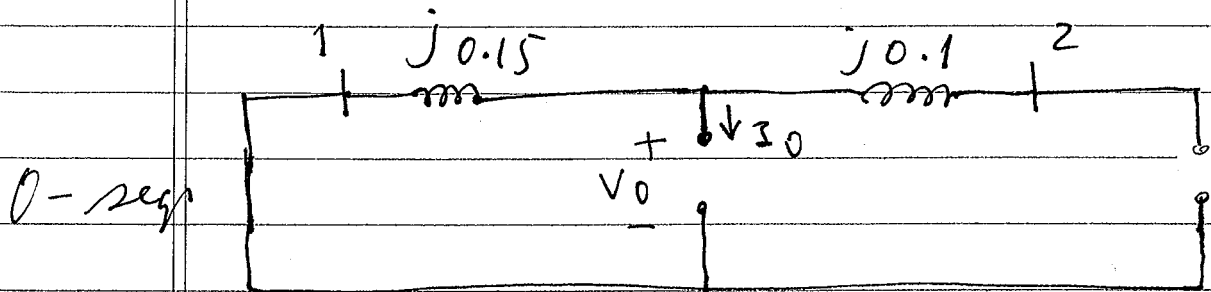
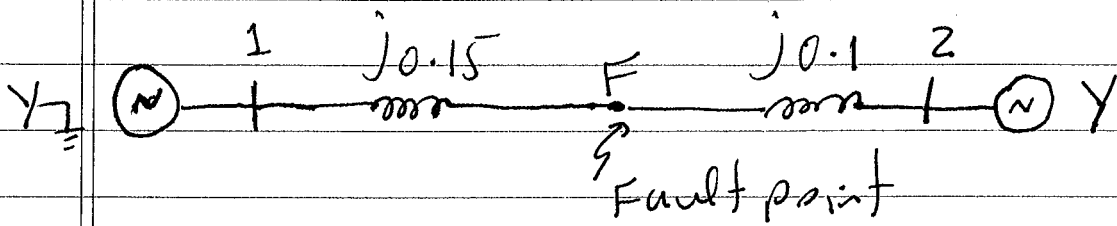
$$= -j (4.74 + j1.61) - j (2.57 - 0.57)$$

$$= -j4.74 + 1.61 - j2.57 + 0.57$$

$$= 2.18 - j7.31$$

$$= 7.63 \angle -73.39^\circ \quad \checkmark$$

Prob 3: First we draw the single-line diagram and the sequence network:



$$Z_0 = j0.15$$

$$\begin{aligned} Z_1 = Z_2 &= (j0.15) \parallel (j0.1) \\ &= j \frac{(0.15)(0.1)}{(0.15) + (0.1)} = j \frac{0.015}{0.25} \\ &= j0.06 \end{aligned}$$

### Prob 3 [continued]

This is a line-to-line fault and in this fault we have:

$$I_0 = 0$$

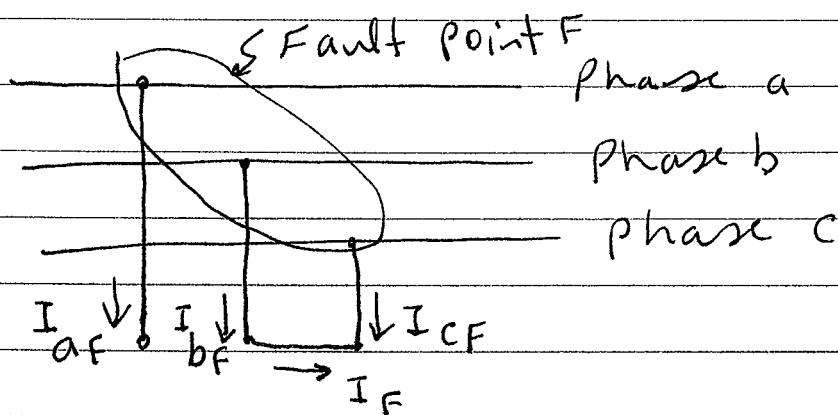
$$I_1 = -I_2 = \frac{V(0)}{Z_1 + Z_2 + Z_F}; \quad V(0) = 1 \angle 0^\circ$$

$$= \frac{1 \angle 0^\circ}{j0.06 + j0.06 + 0} = -j8.33$$

⇒

$$I_0 = 0, \quad I_1 = -j8.33; \quad I_2 = +j8.33$$

These are the sequence values of the fault current as defined below:

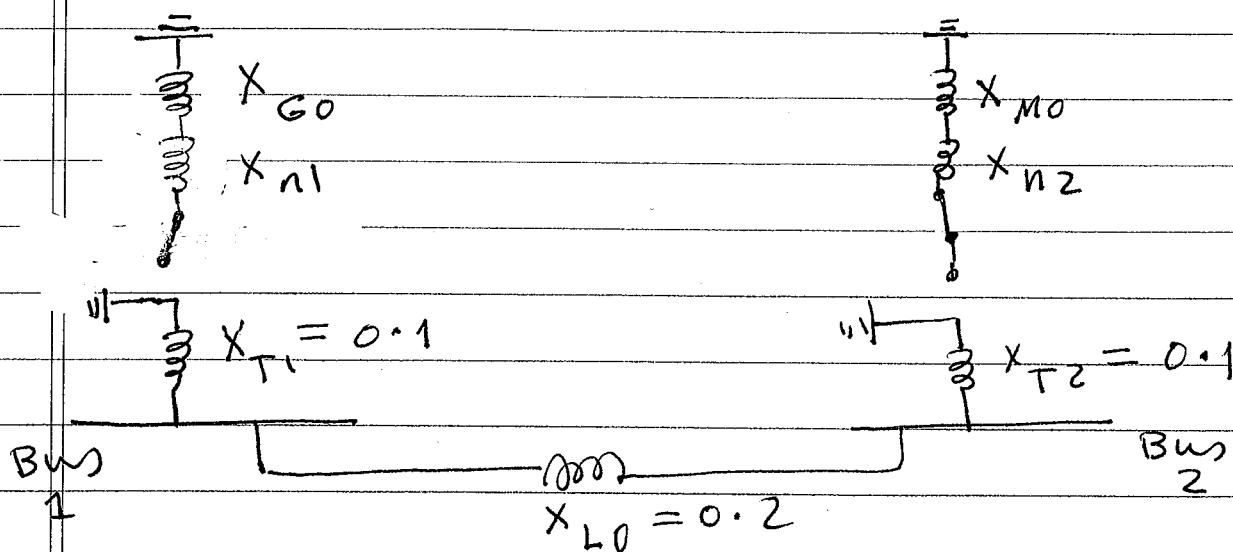


Comparing with the original circuit as given  $I_F = I_{bF}$ ; So using sequence values we find  $I_{bF}$ :

$$\begin{aligned} I_F = I_{bF} &= I_{0F} + a^2 I_{1F} + a I_{2F} \\ &= 0 + (1 \angle 120^\circ)^2 (-j8.33) + (1 \angle 120^\circ) (j8.33) = 14.43 \end{aligned}$$

# Prob 4

a) Let's draw the zero-sequence circuit diagram:



$$b) \quad Y_0 = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

where:

$$Y_{11} = \frac{1}{jX_{T1}} + \frac{1}{jX_{L0}} = \frac{1}{j0.1} + \frac{1}{j0.2} = -j10 - j5 = -j15$$

$$Y_{12} = Y_{21} = \frac{-1}{jX_{L0}} = \frac{-1}{j0.2} = +j5$$

$$Y_{22} = \frac{1}{jX_{T2}} + \frac{1}{jX_{L0}} = \dots = -j15$$

$\Rightarrow$

$$Y_0 = j \begin{bmatrix} -15 & +5 \\ +5 & -15 \end{bmatrix}$$

$$Z_0 = Y_0^{-1} = \left( j \begin{bmatrix} -15 & 5 \\ 5 & -15 \end{bmatrix} \right)^{-1} = \dots = (j \times 10) \begin{bmatrix} 7.5 & 2.5 \\ 2.5 & 7.5 \end{bmatrix}$$