Full equation:

$$\frac{d\mathbf{v}}{dt} = \mathbf{F} - 2\vec{\Omega}_0 \times \vec{v} + \Omega_0^2 \vec{\rho}$$

Dropping anything second order or higher in (x_i/a_0). First term:

$$\begin{aligned}
\mathbf{F} &= -\frac{GM_*}{a^3}\vec{a} \\
&= -\frac{GM_*}{\left((a_0+x)^2 + y^2 + z^2\right)^{3/2}} \begin{pmatrix} a_0 + x \\ y \\ z \end{pmatrix} \\
&= -\frac{GM_*}{a_0^3\left(1 + \frac{2x}{a_0} + \frac{x^2}{a_0^2} + \frac{y^2}{a_0^2} + \frac{z^2}{a_0^2}\right)^{3/2}} \begin{pmatrix} a_0 + x \\ y \\ z \end{pmatrix} \\
&\approx -\frac{GM_*}{a_0^3\left(1 + \frac{2x}{a_0}\right)^{3/2}} \begin{pmatrix} a_0 + x \\ y \\ z \end{pmatrix} \\
&\approx -\Omega_0^2\left(1 - \frac{3x}{a_0}\right) \begin{pmatrix} a_0 + x \\ y \\ z \end{pmatrix} \\
&= -\Omega_0^2 a_0 \left(1 - \frac{3x}{a_0}\right) \begin{pmatrix} 1 + \frac{x}{a_0} \\ \frac{y}{a_0} \\ \frac{z}{a_0} \end{pmatrix} \\
&\approx -\Omega_0^2 a_0 \begin{pmatrix} 1 - \frac{2x}{a_0} \\ \frac{y}{a_0} \\ \frac{z}{a_0} \end{pmatrix} = -\Omega_0^2 \begin{pmatrix} a_0 - 2x \\ y \\ z \end{pmatrix}
\end{aligned}$$

So:

$$\begin{aligned}
\frac{d\mathbf{v}}{dt} &= -\Omega_0^2 \begin{pmatrix} a_0 - 2x \\ y \\ z \end{pmatrix} - 2\begin{pmatrix} 0 \\ 0 \\ \Omega_0 \end{pmatrix} \times \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} + \Omega_0^2 \begin{pmatrix} a_0 + x \\ y \\ 0 \end{pmatrix} \\
\frac{d\mathbf{v}}{dt} &= \Omega_0^2 \begin{pmatrix} 3x \\ 0 \\ -z \end{pmatrix} + 2\begin{pmatrix} v_y\Omega_0 \\ -v_x\Omega_0 \\ 0 \end{pmatrix} \\
\frac{d\mathbf{v}}{dt} &= \begin{pmatrix} 2v_y\Omega_0 + 3x\Omega_0^2 \\ -2v_x\Omega_0 \\ -\Omega_0 z \end{pmatrix}
\end{aligned}$$