***DSGE Likely-Hood***

***Step 1***

$$Parameters = \left[\tau, \kappa, \psi_1, \psi_2, r_A, \pi_A, \gamma_Q, \rho_R, \rho_g, \rho_z, \sigma_R, \sigma_g, \sigma_z\right]$$

$$\Gamma_0 s_t = \Gamma_1 s_{t-1} + \Psi \epsilon_t + \Pi \eta_t$$

*QZ Decomposition*

$$Q'\Lambda Z' = \Gamma_0,\;\; Q'\Omega Z' = \Gamma_1,\;\; QQ' = ZZ' = I$$

$$QQ'\Lambda Z s_t = QQ'\Omega Z' s_{t-1} + Q\Psi \epsilon_t + Q\Pi \eta_t$$

*Upper-Triangular Matrix* $\Lambda$

$$\Lambda Z s_t = \Omega Z' s_{t-1} + Q(\Psi \epsilon_t + \Pi \eta_t)$$

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{22} \end{bmatrix} \begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{bmatrix} \begin{bmatrix} w_{1,t-1} \\ w_{2,t-1} \end{bmatrix} + \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} (\Psi \epsilon_t + \Pi \eta_t)$$

$$w_{2,t} = \Lambda_{22}^{-1} \Omega_{22} w_{2,t-1} + \Lambda_{22}^{-1} Q_2 (\Psi \epsilon_t + \Pi \eta_t)$$

*Define* $\hat{x} = \ln \frac{x_t}{x}$, *8*

$$x_t = \left[\hat{y}_t, \widehat{\pi_t}, \widehat{R_t}, \hat{y}_{t-1}, \hat{g}_t, \hat{z}_t, \epsilon_y, \epsilon_\pi\right]'$$

Expectation errors for inflation and output

$$\eta_{y,t} = y_t - E_{t-1}[\hat{y}_t],\;\; \eta_{\pi,t} = \pi_t - E_{t-1}[\widehat{\pi_t}]$$

***Step 2: DSGE system***

$$\begin{bmatrix} 1 & 0 & \frac{1}{\tau} & 0 & -(1-\rho_g) & -\frac{\rho_z}{\tau} & -1 & -\frac{1}{\tau} \\ -\kappa & 1 & & & \kappa & & 0 & -\text{ב} \\ 0 & 0 & \dots & & \dots & & 0 & \\ & . & \dots & \dots & & 0 & 0 & \\ & . & & \dots & & 0 & 0 & \\ & & & \dots & & 0 & 0 & \\ & . & & \dots & & 0 & 0 & \\ & . & & \dots & & 0 & 0 & \end{bmatrix} s_t$$

$$= \begin{bmatrix} 1 & 0 & \frac{1}{\tau} & 0 & -(1-\rho_g) & -\frac{\rho_z}{\tau} & -1 & -\frac{1}{\tau} \\ -\kappa & 1 & & & \kappa & & 0 & -\text{ב} \\ 0 & 0 & & & & & 0 & \\ & & & & & 0 & 0 & \\ & & & & & 0 & 0 & \\ & & & & & 0 & 0 & \\ & & & & & 0 & 0 & \\ & & & & & 0 & 0 & \end{bmatrix} s_{t-1} + \begin{bmatrix} 1 & 0 & -\frac{\rho_z}{\tau} \\ -\kappa & 1 & \\ 0 & 0 & \\ & & 0 \\ & & 0 \\ & & 0 \\ & & 0 \\ & & 0 \end{bmatrix} \epsilon_t$$

$$+ \begin{bmatrix} 1 & 0 \\ -\kappa & 1 \\ 0 & 0 \\ & \\ & \\ & \\ & \\ & \end{bmatrix} \eta_t$$

***Step 3: Sims Algorithm: QZ decomposition by GENSYS***

$$\Gamma_0 s_t = \Gamma_1 s_{t-1} + \Psi \epsilon_t + \Pi \eta_t$$

To

$$y(t) = G1y(t-1) + C + impact * z(t) + ywt * inv\big(I - fmat * inv(L)\big) fwtz(t+1)$$

[T1,TC,T0,TY,M,TZ,TETA,GEV,RC] = gensys(GAM0,GAM1,C,PSI,PPI,1+1E-8);

y(t)=G1*y(t-1)+C+impact*z(t)+ywt*inv(I-fmat*inv(L))*fwt*z(t+1) .

***Step 4: State Space Representation***

*Translation Equations:* $[A, B, H, R, Se, \Phi]$

$$s(t) = \Phi_{8*8} s(t-1)_{8*1} + R_{8*3} \times e(t)_{3*1} \quad e(t) \sim i.i.d.N(0, Se)$$

$$y(t) = A_{3*1} + B_{3*8} s(t) + u(t) \;\; u(t) \sim N(0, HH)$$

***Step 5: Kalman Filter***

**y** is vector (80*3) of observable variables and **s** is latent variables.

*T=80, l=3, n=8,* $s_{81*8}$

$$a = [I - \Phi \otimes \Phi] RSeR'$$

$$y' = A + B\Phi s$$

$$v = y_{obs} - \hat{y}$$

$$F = B(\Phi P \Phi' + RSeR')B' + H$$

$$\mathcal{L} = -\frac{1}{2} l \log 2\pi - \frac{1}{2} \log\det(F) - \frac{1}{2} v' F^{-1} v$$