

$$\begin{aligned} N &= S + I + R \\ \dot{S} &= -\beta S(1-\theta L)I(1-\theta L) \\ \dot{I} &= \beta S(1-\theta L)I(1-\theta L) - \gamma I \\ -\dot{N} &= D = \phi I \end{aligned}$$

$$\max_L \int_0^\infty e^{-(r+v)t} \left((N - [S + I]L)w + \dot{N}\chi + \frac{v}{r}Nw \right) dt$$

$$V(t_0,x(t_0))=\sup_{x(t),y(t)\in X\times Y}\int_{t_0}^\infty f(t,x,y)dt\;\;s.t.\dot{x}=g(t,x,y)\;\;and\;\lim_t bx\geq x_1$$

$$\text{HJB:}$$

$$f+V_t+V_xg=0$$