Social Planner, Industrial Structure and Uncertainty for COVID-19

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Abstract

We study the planning problem under the framework of social planner, and explore the dynamic characteristic of one-sector and two-sector model respectively. In order to reflect the risk aversion on the pandemic, we set an expected utility function for agents of the economy. Under certainty scenario, we explore the shock impact on one sector baseline and two-sector model. In light of uncertainty scenario, we construct a stochastic optimal control model to discern these two cases. Under uncertainty setting, the lockdown policy tends to be more rigid.

1 Introduction

COVID-19 pandemic has already cause severe effect to economy all over the world and has taken more than 390000 people(as of June 7, 2020). The shock of pandemic caused global recession and large-scale unemployment of different countries. Recent researchers launched a series of redevelopment of SIR(Susceptible-Infectious-Recovered) model that was promoted by Kermack, McKendrick, and Walker, 1927. The SIR framework and its various extensions model the spread are researched in many inquiries recently (e.g. Berger, Herkenhoff, and Mongey, 2020, Atkeson, 2020). The simplest version of the model that includes three differential equations can provide a good approximation for the dynamic of pandemic. Besides the ability to describe the dynamic nature of the pandemic, it can also provide a basis for the predict the economy and do planning for the economy. Several recent papers have started conducting optimal policy analysis within this framework (e.g. Alvarez, Argente, and Lippi, 2020, Jones, Philippon, and Venkateswaran, 2020, Farboodi, Jarosch, and Shimer, 2020, Rowthorn and Toxvaerd, 2012, Eichenbaum, Rebelo, and Trabandt, 2020.)

Under the framework of SIR, we could describe the dynamic of economy. Another self-evident issue is launched: will industrial structure affect the results? When we consider a lockdown policy related to the specific sector, the results will be different totally. We have to maintain some parts of the society running, such as health care system, power plant system, transportation system for commodity and so forth. Those necessary sectors have to keep operating so that we can meet the basis demands. As Guerrieri, Lorenzoni, Straub, and Werning, 2020, we talk about the problem from one-sector to two-sector. First, in one-sector economies, the shock of pandemic is negative: the drop in supply dominates. The result is well-known in a representative agent economy. And then a less obvious result is proved by them: it holds true in richer incomplete market models that allow for heterogeneous agents, uninsurable income risk and liquidity constraints, creating differences in marginal propensities to consume (MPC). In these models, there is a mechanism from income loss to demand decline, but although it makes the total demand decline larger than the total demand decline in typical agency cases, the decline is still smaller than the output decline caused by supply shock. Intuitively, the largest MPC of people who lose their income may be large, but only one, which means that their consumption decline is always a suppressed version of income loss. Then we turn to economies with multiple sectors. When the impact is concentrated in certain sectors, as in the case of a shutdown in response to an epidemic, there is more room for total spending to contract. Some goods are no longer available, which makes overall consumption less attractive. One explanation is that the suspension of production increases the shadow price of goods in the affected sectors, making current total consumption more expensive, thus inhibiting consumption. On the other hand, due to the unavailability of goods in some sectors, expenditures can be transferred to other sectors through alternative channels. In sectors not directly affected by closures, maintaining full employment depends on the relative intensity of these two impacts.

Therefore, in our model, we present a optimal control model for the COVID-19 epidemic. We adopt the model structure of Alvarez, Argente, and Lippi, 2020, a variation in the SIR epidemiology model reviewed and proposed by Atkeson, 2020 to analyze the optimal lockdown policy. The origin edition of planning problem is promoted by Morton and Wickwire, 1974, they construct a control scheme for the immunisation of susceptibles in the Kermack-McKendrick epidemic model for a closed population is proposed. And they use the approach of dynamic programming and Pontryagin's Maximum Principle and allows one, for certain values of the cost and removal rates, to apply necessary and sufficient conditions for optimality and show that a one-switch candidate is the optimal control. In the remaining cases they show that an optimal control, if it exists, has at most one switch. And then Hansen and Day, 2011 extend the existing work on the time-optimal control of the basic SIR epidemic model with mass action contact rate. They provide analytic solutions for the control that minimizes the outbreak size (or infectious burden) under the assumption that there are limited control resources and optimal control policies for an isolation only model, a vaccination only model and a combined isolation–vaccination model (or mixed model). Several conclusions are given by them: under certain circumstances the optimal isolation only policy is not unique; furthermore the optimal mixed policy is not simply a combination of the optimal isolation only policy and the optimal vaccination only policy.

Some recent papers talk about the planning problem centrally, see Acemoglu, Chernozhukov, Werning, and Whinston, 2020; Barro, Ursua, and Weng, 2020; Eichenbaum, Rebelo, and Trabandt, 2020; Berger, Herkenhoff, and Mongey, 2020; Hall, Jones, and Klenow, 2020; Chari, Kirpalani, and Phelan, 2020; Jones, Philippon, and Venkateswaran, 2020 and Richard Baldwin, 2020. The typical approach in the epidemiology literature is to study the dynamics of the pandemic, for infected, deaths, recovered, as functions of some exogenous chosen diffusion parameters, which are in turn related to various policies, such as the partial lockdown of schools, universities, businesses, and other measures of diffusion mitigation, and where the diffusion parameters are stratified by age and individual covariates.

Differentiating from these research, we put the planning problem in the framework of utility and social planner problem. In order to include the impact of pandemic and make it affect individuals decision, we construct a expectation utility function, which contain parameter to manifest the risk aversion on the choose of labor supply and consumption. For the production part of the economy, we set a linear technology production function. Therefore, we can get the wage under the equilibrium of the economy. In terms of government, we conclude a policy of unemployment insurance for people who are infected. The source of the financing of government is from deficit. So we can explore the deficit in our model. The novel aspect of our analysis is to explicitly formulate and solve a social planner problem; to choose lockdown policy to maximize a given social objective and taking into account the dynamic nature of the problem. At the second part of the model, we tackle with the problem same as the baseline model, but add a new sector into the economy. We want to explore how a specific parameter of industry structure will impact on the economy. Lockdown policy is also specific, we only limit the unnecessary part of the economy. It's self-evident because we have to maintain the flexibility of the necessary part to product for the economy. At this part we analysis the problem from two sectors respectively. At the third part, in light of uncertainty scenario, we introduce a stochastic optimal control problem promoted by Barnett, Brock, and Hansen, 2020a to discern certainty and uncertainty.

2 Planning Model

2.1 SIR Model

As in Atkeson, 2020 and Alvarez, Argente, and Lippi, 2020. At each moment, the population is divided into three categories, the sum of which is N(t). The population was divided into susceptible S(t), infected I(t) and recovered R(t), i.e.

$$N(t) = S(t) + I(t) + R(t) \quad \forall t > 0 \tag{1}$$

The $R(t)$ here include people who have been infected, survived the disease, and are now recovered and assumed to be immune. We normalize the initial population to $N(0) = 1$. The social planner can control a fraction $L(t)$ of the population, where $L \leq 1$ allows us to meet a more actual situation, i.e. we have to keep some significant departments and industries working on such as power plant, food supply vendors. The lockdown efficiency θ measures the proportion that population cannot contact others freely. If $\theta = 1$, the lockdown policy completely wield its effects on population. But the actual scenario we never harbor the ability to control all people to move in different cities to curb the transmission of virus; so we allow parameter $\theta < 1$.

The definition of the law of dynamic of susceptible, infected, and total population as follows specifications. Where β is the susceptible agents are infected by contacting the infected agents per unit time. We set the probability that people get infected from infected agents is 1. For infected people, they can recover at the rate of γ. The death of disease $D(t)$ is defined as product of rate of death per time $\phi(I(t))$ and infected agents. $\phi(I(t)) \in (0, 1)$ is the rate of fatality of infected people, called as "case fatality rate" (CFR) in Alvarez, Argente, and Lippi, 2020. It appears that the CFR manifest the direct proportion with $I(t)$, which reflects the jam effects in health care system.

$$\dot{S} = -\beta S(t)(1 - \theta L(t))I(t)(1 - \theta L(t)) \tag{2}$$

$$\dot{I} = \beta S(t)(1 - \theta L(t))I(t)(1 - \theta L(t)) - \gamma I(t) \tag{3}$$

$$-\dot{N} = D(t) = \phi(I(t))I(t) \tag{4}$$

$$\phi(I(t)) = [\varphi + \kappa I]\gamma \tag{5}$$

2.2 Agents' Utility, Production and Equilibrium

We define the expectation utility for susceptible agents and fixed utility for infected agents. The utility of susceptible people can be divided into two parts: maintaining the presented scenario, namely not be infected; and the possibility of being tracked into infected people population. The utility formula as follows:

$$E[u_s(c_t^s, n_t^s)] = p(L)u_s(c_t^s, n_t^s) + [1 - p(L)]u_i(c_t^i, n_t^i) \tag{6}$$

$$E[u_i(c_t^i, n_t^i)] = u_i(c_t^i, n_t^i) \tag{7}$$

c_t^s, and n_t^s are the consumption and labor supply of susceptible people; c_t^i, n_t^i is for infected people. The probability function $p(L)$ shows direct proportion with lockdown policy $L(t)$ and parameter α, the coefficient of lockdown to probability of being infected.

$$p(L) = \alpha L(t) \tag{8}$$

Now we assume the consumption is inelastic during the period of pandemic, i.e. agents cannot make the optimal decision for c_t, and n_t. Infected agents accept unemployment insurance b. Apparently, social planned is incentive to avoid the worse situation that become infected people for they will suffer from welfare loss. We constrain the consumption and labor supply are as follows:

$$c_t^s = L(t)w(t) \quad n_t^s = L(t) \tag{9}$$

and

$$c_t^i = b \quad n_t^s = 0 \tag{10}$$

The remedy against the ongoing COVID-19 recession that is currently being debated is fiscal stimulus. To consider the effects of fiscal stimulus in our model, we introduce a stylized government sector. We assume that the government chooses paths of government spending G_t, lump-sum transfers (or taxes) T_t that can be targeted by sector, and government debt B_t, subject to the flow budget constraint. For simplifying the model, we set $G_t = 0$ and $T_t = 0$.

$$(1 + r_{t-1})B_t = B_{t+1} \tag{11}$$

$$B_t = I(t)b \tag{12}$$

As Guerrieri, Lorenzoni, Straub, and Werning, 2020, competitive firms produce the final good only from labor using the linear technology, so the wage can be same as price of product.

$$Y_t = N_t \tag{13}$$

$$p_t = w_t \tag{14}$$

Total demanding comes from two parts: the consumption of susceptible agents $S(t)c_t^s$ and of infected agents $I(b)b$:

$$C_t = S(t)c_t^s + I(t)b \tag{15}$$

Another side total supply, namely total labor supply N_t comes from only the sector of susceptible people.

$$N_t = S(t)L(t) \tag{16}$$

Define market equilibrium is:

$$N_t = C_t \tag{17}$$

Under this scenario, the wage of economy is:

$$w_t = \frac{SL - Ib}{S} \tag{18}$$

2.3 Planning Problem

The planning problem is modified version of Alvarez, Argente, and Lippi, 2020 and Acemoglu, Chernozhukov, Werning, and Whinston, 2020, we use a objective function of social planner. Assume that each agents are assumed to live forever, unless they die from the infection. The planner discounts all values at the rate $r > 0$ and with probability ν per unit of time both a vaccine or a cure appear, so that all infected agents can be cured and get the immunity. So the planning problem can be consist in maximizing the following present value:

$$\max_{L(t)} \int_0^\infty e^{-(r+v)t} \left\{ S(t)u(c_t^s, n_t^s) + I(t)b - \chi_d D(t) \right\} dt \tag{19}$$

We get HJB equation:

$$
\begin{aligned}
(r+v)V(S,I) = \min_{L\in[0,\bar{L}]} \Bigg\{ S(t)\left[(1-\alpha L)\frac{SL-Ib}{S}+\alpha Lb\right] + I(t)b + \chi_d\phi(I(t))I(t) + \\
-\partial_S V(S,I)[-\beta S(t)I(t)(1-\theta L(t))^2] \\
+\partial_I V(S,I)[\beta S(t)I(t)(1-\theta L(t))^2-\gamma I(t)] \Bigg\}
\end{aligned}
\tag{20}
$$

The domain of $V(S,I)$ is $S+I \leq 1$. As Alvarez, Argente, and Lippi, 2020, we use value function iteration method to solve this. Note the boundary of value function has an analytical formula. $V(S,0)=\frac{(1+\alpha bS)^2}{4\alpha(r+v)}$ and $V(0,I)=\frac{a^2b^2}{8\alpha\gamma+4\alpha(r+v)}I^2+\frac{4\alpha\chi_d\phi+2ab}{4\alpha\gamma+4\alpha(r+v)}I+\frac{1}{4\alpha(r+v)}$.

3 Incomplete Two-sector Model

As Guerrieri, Lorenzoni, Straub, and Werning, 2020, we construct a two-sector model to explore the impact of the structure of industry. The economy can be divided into two sectors necessary service and productive and unnecessary sector. Proportion of the two sectors is ϕ. We still use linear technology in this part.

$$
Y_{ti} = N_{ti} \tag{21}
$$

N_{ti} means the labor is supplied by a specific part of total population. People cannot choose the sector they work on due to the skill sets are specific, which means people cannot flow inter-sector untetheredly.

$$
Y_{t1} = \phi N_t \quad Y_{t2} = (1-\phi)N_t \tag{22}
$$

The probability of becoming infected is idiosyncratic for sectors:

$$
p_1(L) = \alpha_1 L(t) \quad p_2(L) = \alpha_2 L(t) \tag{23}
$$

Lockdown policy is also specific, we only limit the unnecessary part of the economy. It's self-evident because we have to maintain the flexibility of the necessary part to product for the economy. Therefore, the labor supply for the two sectors are:

$$n_t^{s1} = L(t) \quad n_t^{s2} = 1 \tag{24}$$

And consumption:

$$c_t^{s1} = L(t)w_t^1 \quad c_t^{s2} = w_t^2 \tag{25}$$

w_t^1 is the wage for the sector one, w_t^2 is for sector two. The partial equilibrium for the two sector is idiosyncratic; we assume the demand can be divide into two parts and are equal to the proportion of the sector ϕ. Total demand is

$$C_t = S_1 c_t^{s1} + S_2 c_t^{s2} + I(t)b \tag{26}$$

The demand of sector 1 is:

$$\phi S_t L(t) = \phi S_t L(t) w_t^1 + \phi I(t)b \tag{27}$$

And the wage of sector 1:

$$w_t^1 = \frac{S_t L(t) - I(t)b}{S_t L(t)} \tag{28}$$

For sector 2, the same as sector 1:

$$(1-\phi)S_t = (1-\phi)S_t w_t^2 + (1-\phi)I(t)b \tag{29}$$

$$\frac{S_t - I(t)b}{S_t} = w_t^2 \tag{30}$$

The two sectors planning problem is

$$\max_{L(t)} \int_0^\infty e^{-(r+v)t} \Big\{ S_1(t)u_1(c_t^{s1}, n_t^{s1}) + S_2(t)u_2(c_t^{s2}, n_t^{s2}) + I(t)b - \chi_d D(t) \Big\} dt \tag{31}$$

4 Parameter Setting

As Alvarez, Argente, and Lippi, 2020, We use a value of statistical life of 20 times annual per capita GDP, which is in line with utilitarian values of life.

Table 1: Parameters

Parameter	Value	Definition
β	0.2	Daily increase of active cases if unchecked
γ	0.055	Daily rate of infected recovery (includes those that die)
φ	0.01γ	IFR: fatality per active case (per day)
κ	0.05γ	Implies a 3 percent fatality rate with 40 percent infected
r	0.05	Annual interest rate 5 percent
ν	0.667	Prob rate vaccine + cure (exp. duration 1.5 years)
$\bar{L}$	0.9	1 - GPD share health, retail, government, utilities, and food mfg
θ	0.5	Effectiveness of lockdown
χ	0	Value of Statistical Life $20 \times w$

5 Results

We show the main quantitative results of planning problem described in the previous section. Throughout our main focus is on the comparison between different efficiency parameter θ, the controlled as well as uncontrolled scenario, different policy tool b and so forth. So the rest of the section investigates how the comparison between these different types of policies changes when we modify some of the key parameters. We emphasis the results, in particular, the balance of fiscal policy, lockdown policy and the yield of economy.

5.1 Baseline Model

Lockdown Policy Presented in Figure 1, the lockdown policy is change in tendency and value in terms of different initial value of $I(0)$. When $I(0)$ is relatively low, the curve will increase first and decrease later along with the velocity of increasing of $I(t)$. That manifest the nature that the optimal lockdown policy is adjusted by the proportion of $I(t)$ in the whole N. Another characteristic is any initial value of $I(0)$ will decrease to a fixed value, which manifest the long-

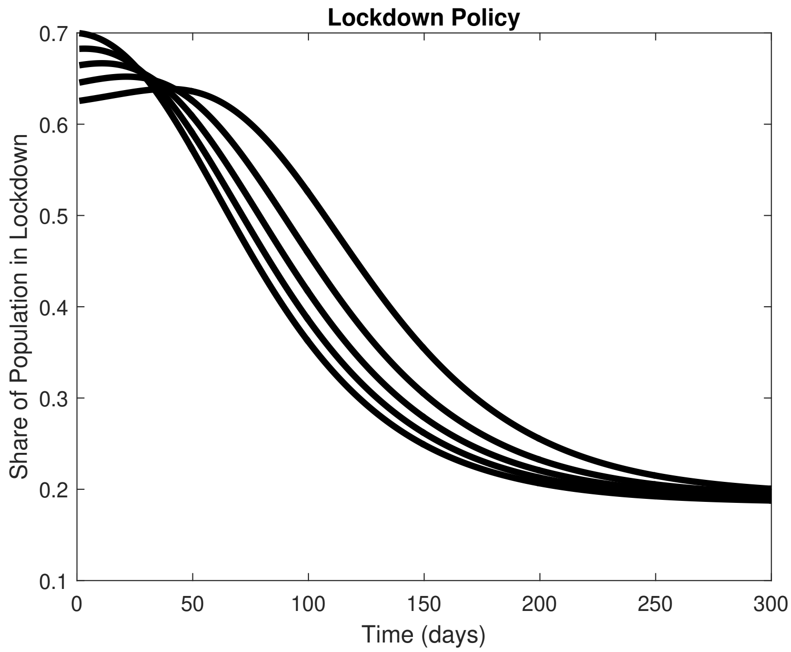


Figure 1: Optimal Policy of Lockdown

lasting policy is a given value if we can describe the specific economy precisely.

Yield For the yield of the economy, the shock of the pandemic is totally negative for the economy. With the proportion $I(t)$ increases the lost of the yield is also enhanced, from 2% to 2.5%.

Price and Wage In our baseline model, the wage is same as price index($w_t = p_t$), so we need only focus on the variation of wage of working population. We can get the conclusion from the numerical solution that the severity of pandemic is negative correlated with the price index(wage), which means the increasing of $I(t)$ means the minimum of price decreases. In our results, it decreases from 0.981 to 0.977. And then, it will increase and converge to a fixed value. The reason for this can be explained by the fiscal policy. At the first stage, there are still many people who work on positions and accept the wage w_t. Pandemic shock decreases the wage at first. And then, fiscal policy exerts its ability to meet the requirement and demanding. Along with people who are infected and given unemployment insurance, the demanding increases again.

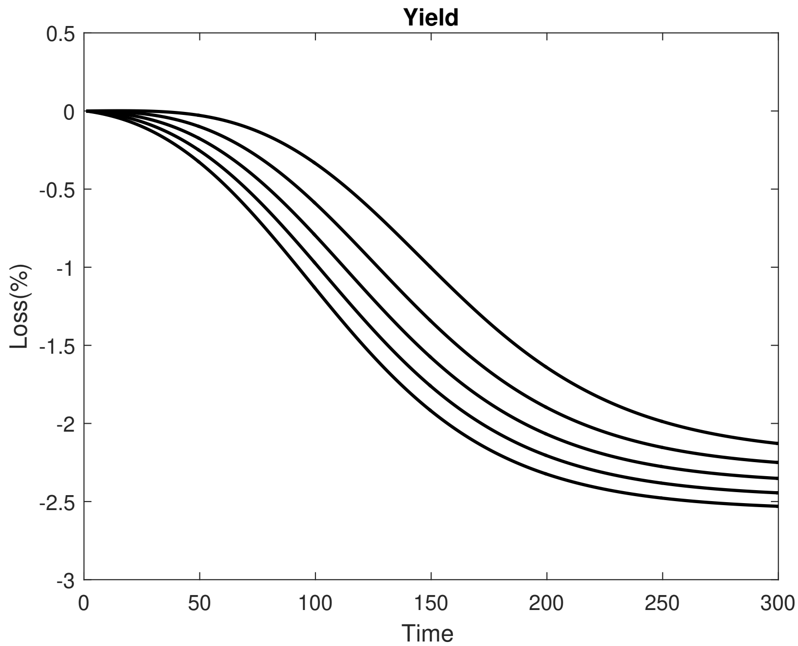


Figure 2: Changes of Yield population with Control

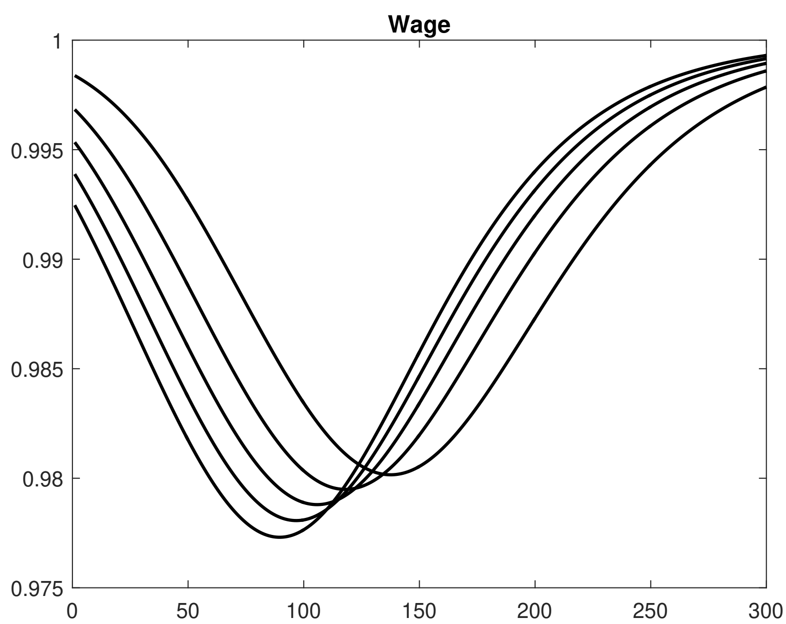


Figure 3: Changes of Wage population without Control

Fiscal Deficit Fiscal deficit comes from the unemployment policy b. According the setting of the model, it should be concert with the trend of infected people. Deficit increases to 2.25%

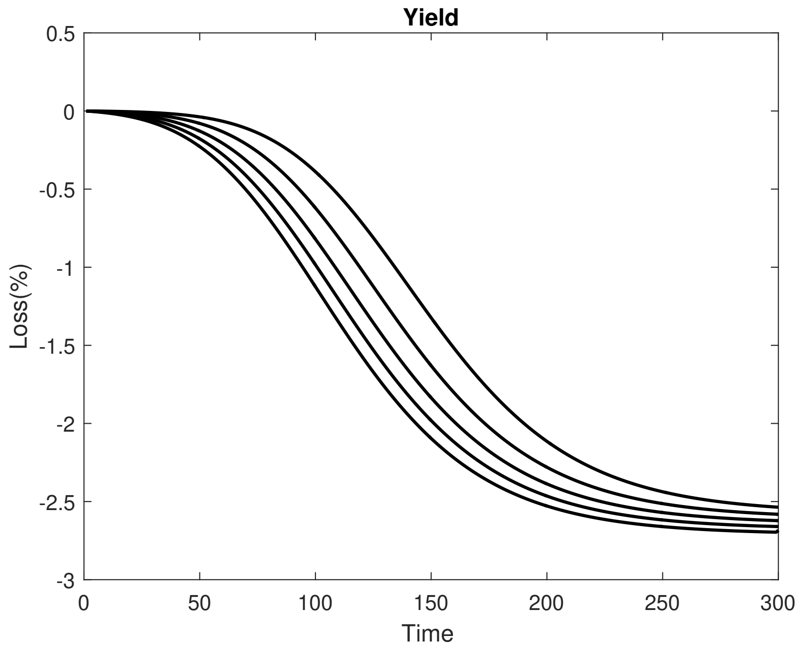


Figure 4: Changes of Yield population with Control, $b = 0.05$

with the proportion $I(t)$ enhances. And then, the same as wage and yield, decrease to a fixed value.

Fiscal Policy change for the baseline model This part we will observe how fiscal policy b will affect the economy of the model. We set $b = 0.05$ and $b = 0.15$ as two comparison. We can get the conclusion that with benefit decreases the yield of economy will also decrease, but fiscal deficit decreases also, as we expected. And the minimum price also increase, which means that a more rigid lockdown policy will be used.

5.2 Two-sector Model

Yield In the new two sector model we can see the difference between two sectors. In the sector 1, the part that is controlled by the government, the loss is less than sector 2. The total loss of the economy depends on the industrial structure; we can see the loss is between the loss of sector 1 and sector 2. When the lockdown policy is less than before, the loss of the two sectors tend to converge to a 1%.

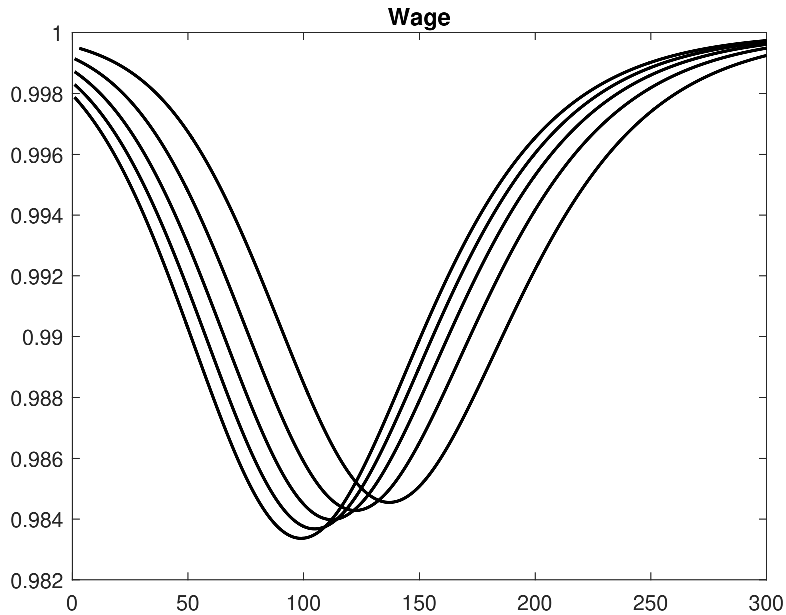


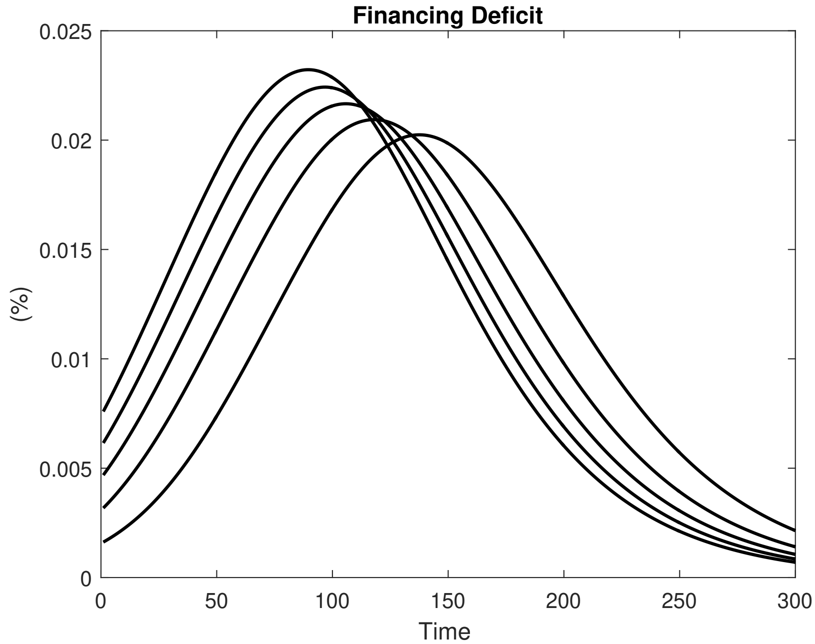
Figure 5: Changes of Wage population without Control, $b = 0.05$

Price and Wage Same as baseline model, price is same as wage. The wage of the sector 1 is less than sector 2 due to the lockdown policy and the degradeing of demands. And then the policy of unemployment insure exerts the efficiency and promote the price of the sector 1 and sector 2.

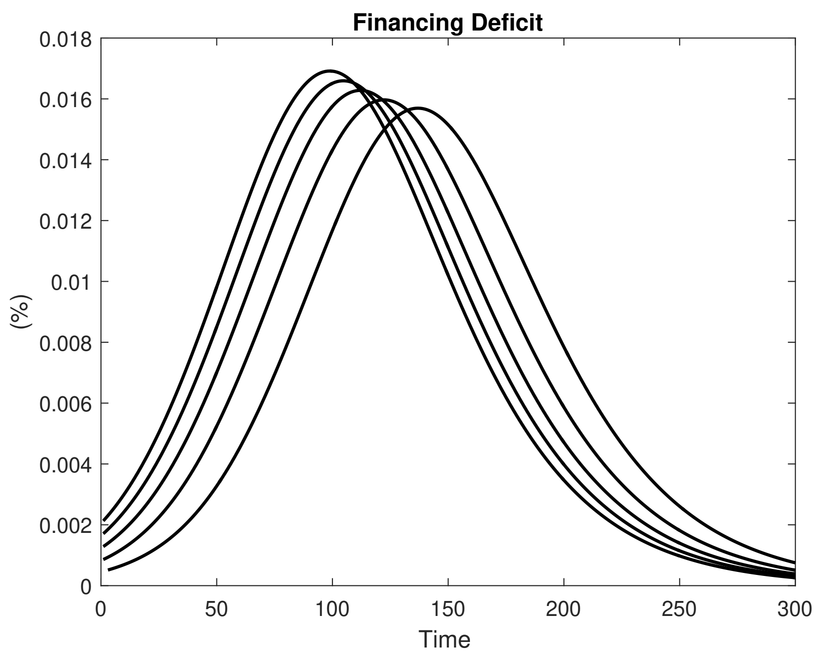
Deficit Compared to the former baseline model, the financial deficit is lower. As the increasing of infected people, the deficit manifests the trending of increasing. From then on, the deficit will decrease to a certain value.

6 Stochastic Scenario

As Barnett, Buchak, and Yannelis, 2020b, we begin by assuming that there is a discrete set Y of possible models v for the pandemic. For each $v \in Y$ there is a set of parameters $\beta(v), \rho(v), \delta(v)$



(a) Changes of Deficit without Control



(b) Changes of Deficit without Control, $b = 0.05$

Figure 6: Deficit

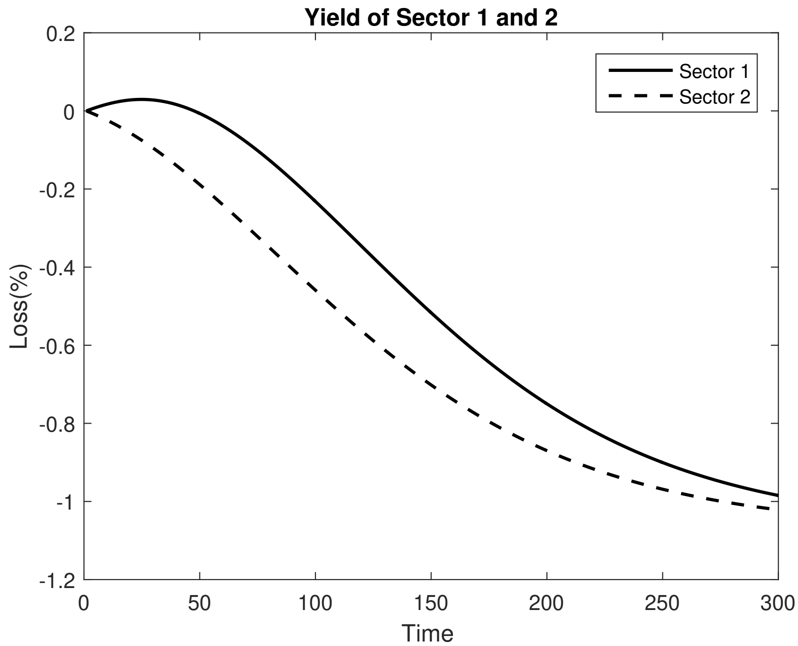


Figure 7: Changes of Yield of the Sectors

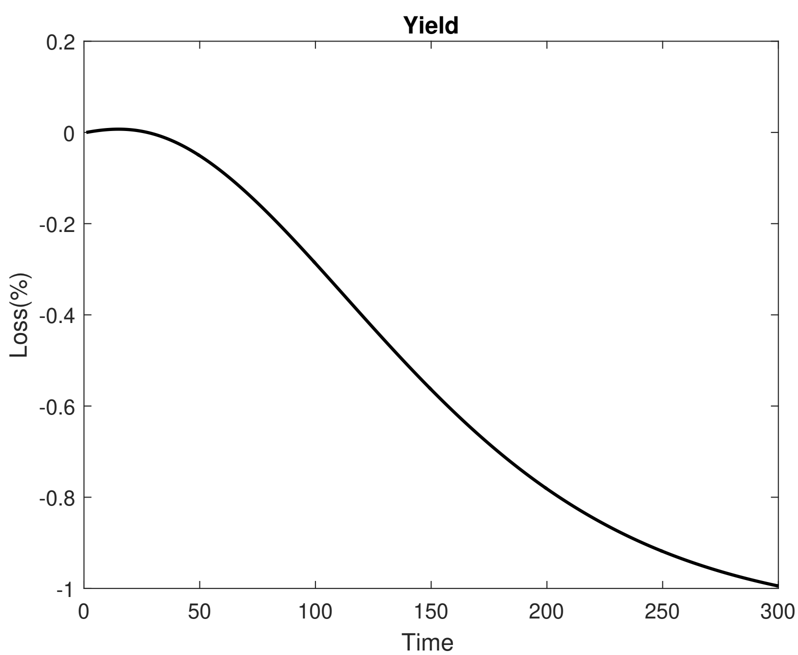


Figure 8: Changes of Total Yield

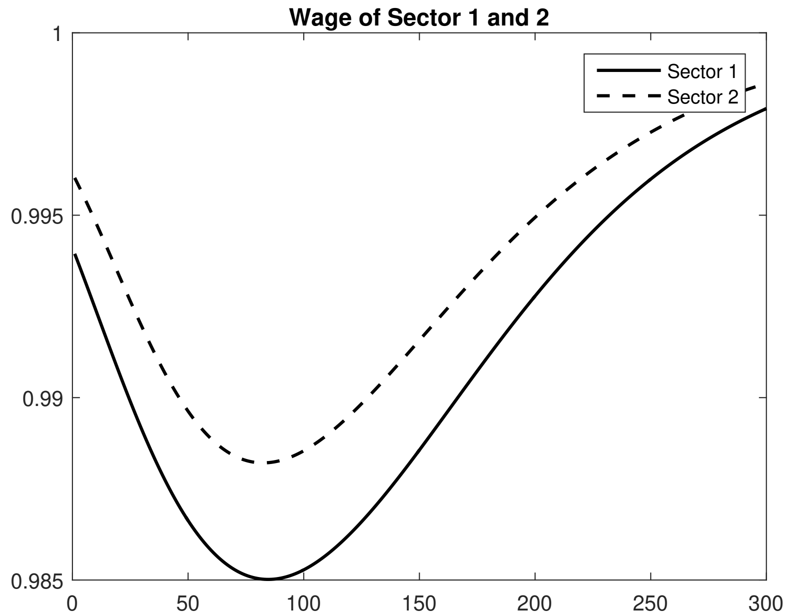


Figure 9: Changes of Wage of the Sectors

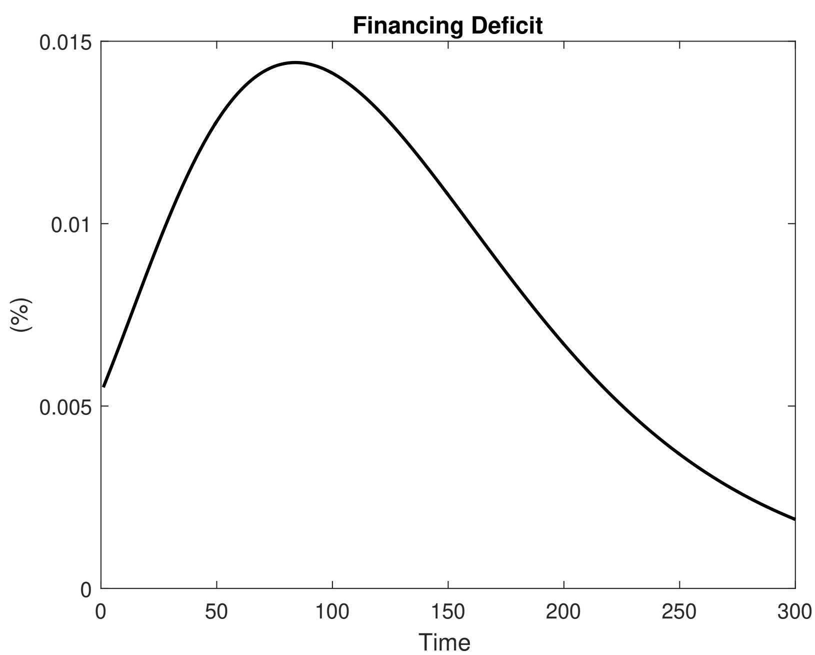


Figure 10: Changes of Deficit with Control

which characterize the state variable evolution equations as follows:

$$
\begin{aligned}
ds_t &= -\beta(v)\left(s_t - q_t\right) i_t dt - \sigma_\beta i_t dW_t \\
di_t &= \beta(v)\left(s_t - q_t\right) i_t dt - (\rho(v) + \delta(v)) i_t dt + \sigma_\beta i_t dW_t - \sigma_\delta i_t dW_t \\
dd_t &= \delta(v) i_t dt + \sigma_\delta i_t dW_t
\end{aligned}
$$

Each of these v conditional models is assumed to come from existing estimates of the model either from historical data of previous viral pandemics or from real-time estimates from different outbreaks and acts as a potential best-guess for what the true pandemic model is for policymakers. The social planner in our model will make optimal policy decisions conditional on each model, and then allow for the fact that the distribution for the set of models is ambiguously specified or unknown and will then adjust their optimal policy decision in response to the pandemic accordingly. And the social planner problem is given by

$$
\max_{L(t)} \int_0^\infty E\left[e^{-(r+v)t}\left\{S(t)u\left(c_t^s, n_t^s\right) + I(t)b - \chi_d D(t)\right\} dt \mid v\right] \tag{32}
$$

We can get HJB equation as follow:

$$
\begin{aligned}
(r+v)V(v) = &\left\{S(t)u\left(c_t^s, n_t^s\right) + I(t)b - \chi_d D(t)\right\} + V_s\left[-\beta(v)\left(S_t - L_t\right) I_t\right] \\
&+ V_i\left[\beta(v)\left(S_t - L_t\right) I_t - (\rho(v) + \delta(v)) I_t\right] + V_d \delta(v) I_t \\
&+ \frac{1}{2}\left\{V_{ss}\sigma_\beta^2 + V_{ii}\left[\sigma_\beta^2 + \sigma_\delta^2\right] + V_{dd}\sigma_\delta^2\right\} I_t^2 - \left\{\sigma_\beta^2 V_{si} + \sigma_\delta^2 V_{id}\right\} I_t^2
\end{aligned} \tag{33}
$$

The numerical method we implement here has been used and developed in other papers, including Barnett, Brock, and Hansen, 2020a and the summary of this algorithm that we provide here closely follows what has been outlined in those papers [15]. We solve the HJB equations that are given by nonlinear partial differential equations using the method of false transient with an implicit finite difference scheme and conjugate gradient solver. The PDEs can be expressed in a conditionally linear form given by

$$
\begin{aligned}
0 = &V_t(x) + A\left(x; V, V_x, V_{xx}\right) V(x) + B\left(x; V, V_x, V_{xx}\right) V_x(x) \\
&+ \frac{1}{2} tr\left[C\left(x; V, V_x, V_{xx}\right) V_{xx}(x) C\left(x; V, V_x, V_{xx}\right)\right] + D\left(x; V, V_x, V_{xx}\right)
\end{aligned}
$$

x is a state variable vector and $V_x(x) = \frac{\partial V}{\partial x}(x), V_{xx}(x) = \frac{\partial^2 V}{\partial x \partial x'}(x)$ are used for notational simplicity. The agent has an infinite horizon and so the problem is time stationary. Thus, $V_t(x) = \frac{\partial V}{\partial t}(x)$ has been added as a "false transient" in order to construct the iterative solution algorithm. In particular, the solution comes by finding a $V(x)$ such that the above equality holds and $V_t(x) = 0$

Therefore, for our social planning problem, we can get following matrix as the same identity as before. We use the same parameters as above. As Figure 11, 13, 12, 15, 14 show, uncertainty make the policy more rigid than before to deal with the severe scenarios.

$$
\begin{aligned}
&A = (r + v) \\
&B = [[-\beta(v)\,(S_t - L^*) \quad I_t]\,[\beta(v)\,(S_t - L^*)\,I_t - (\rho(v) + \delta(v))I_t] \quad \delta(v)I_t] \\
&V_{xx} = \begin{bmatrix} V_{SS} & V_{SI} & V_{SD} \\ V_{SI} & V_{II} & V_{ID} \\ V_{SD} & V_{ID} & V_{DD} \end{bmatrix} \\
&C = \begin{bmatrix} \sigma_\beta I & 0 & 0 \\ 0 & \sqrt{\sigma_\beta^2 + \sigma_\delta^2} I & 0 \\ 0 & 0 & \sigma_\delta I \end{bmatrix}
\end{aligned}
\tag{34}
$$

$$
CV_{xx}C = \begin{bmatrix} \sigma_\beta V_{SS} I & \sigma_\beta V_{SI} I & \sigma_\beta V_{SD} I \\ \sqrt{\sigma_\beta^2 + \sigma_\delta^2} V_{SI} I & \sqrt{\sigma_\beta^2 + \sigma_\delta^2} V_{II} I & \sqrt{\sigma_\beta^2 + \sigma_\delta^2} V_{ID} I \\ \sigma_\delta V_{SS} I & \sigma_\delta V_{SI} I & \sigma_\delta V_{DD} I \end{bmatrix} \begin{bmatrix} \sigma_\beta I & 0 & 0 \\ 0 & \sqrt{\sigma_\beta^2 + \sigma_\delta^2} I & 0 \\ 0 & 0 & \sigma_\delta I \end{bmatrix}
$$

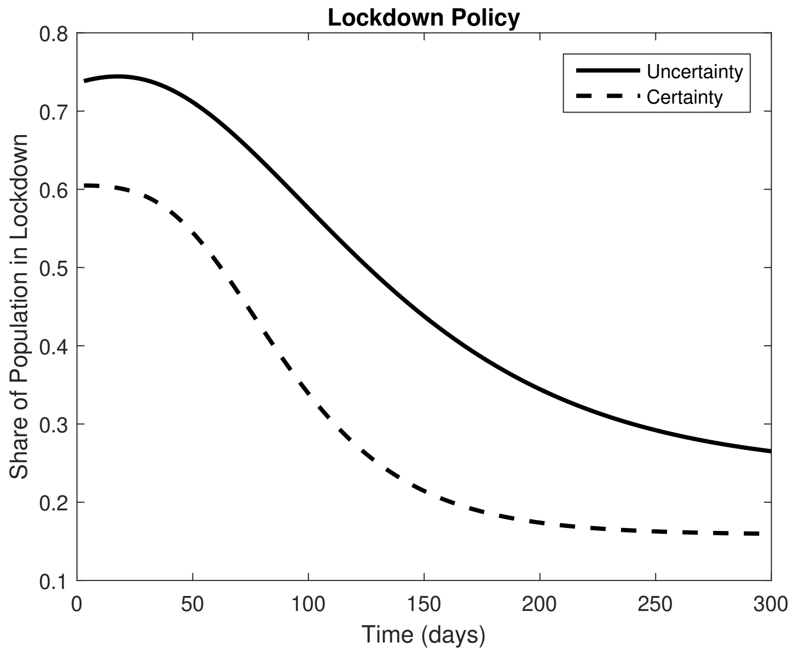


Figure 11: Changes of Lockdown

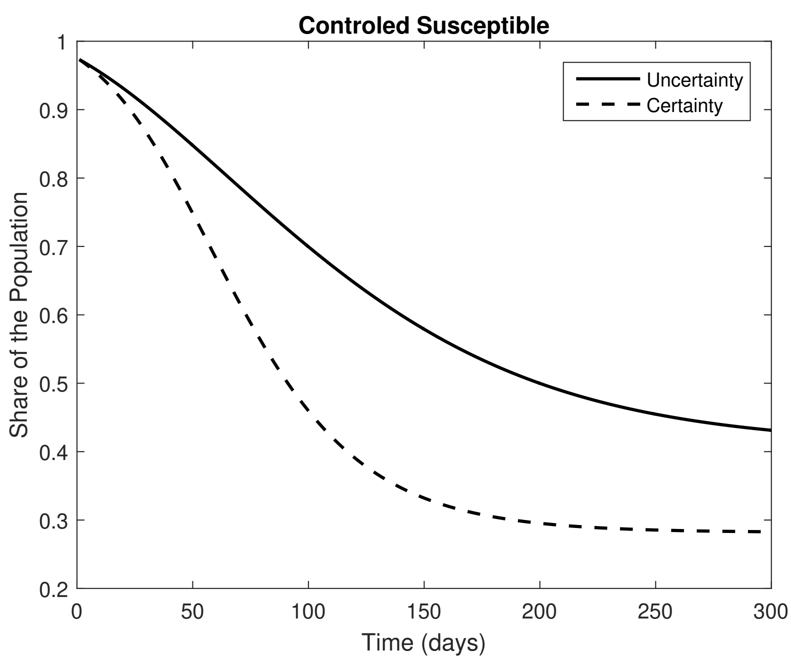


Figure 12: Changes of Susceptible

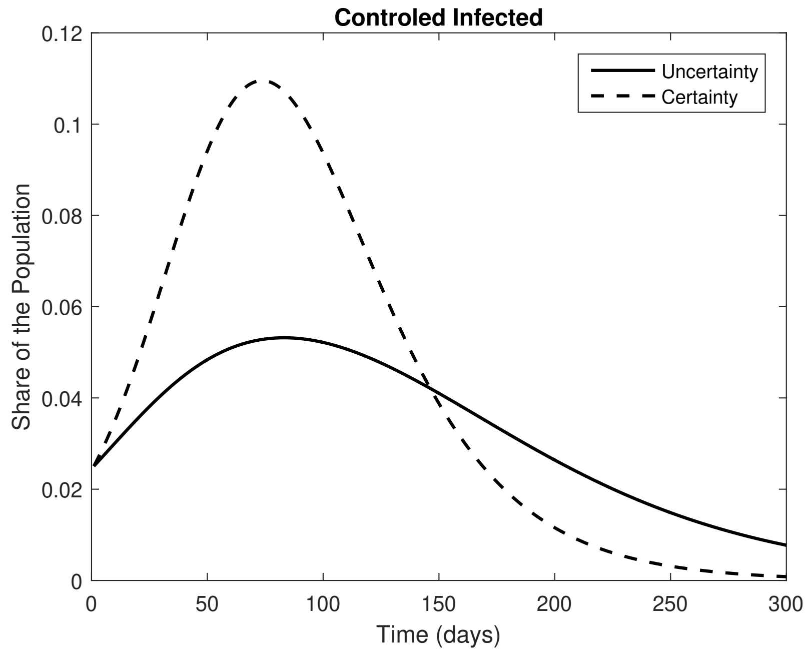


Figure 13: Changes of Infected

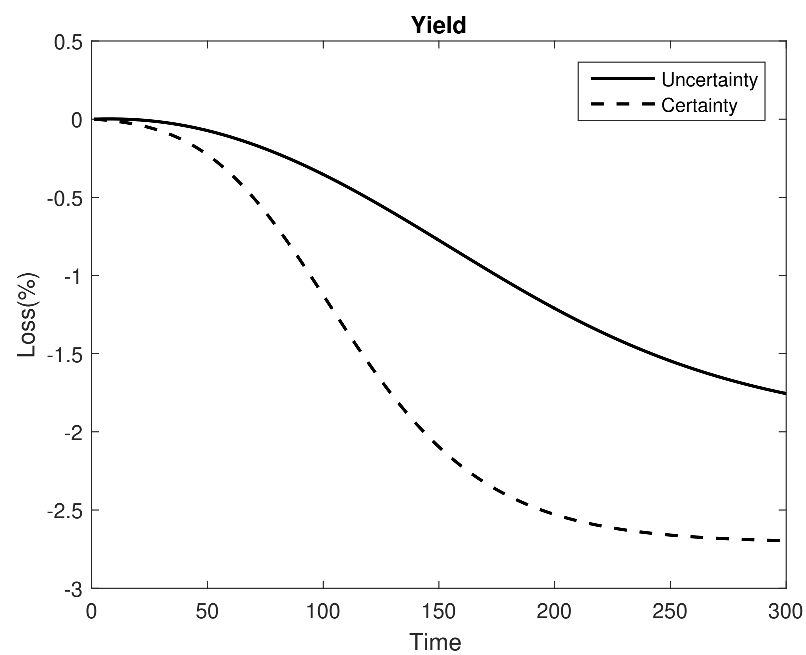


Figure 14: Changes of Yield

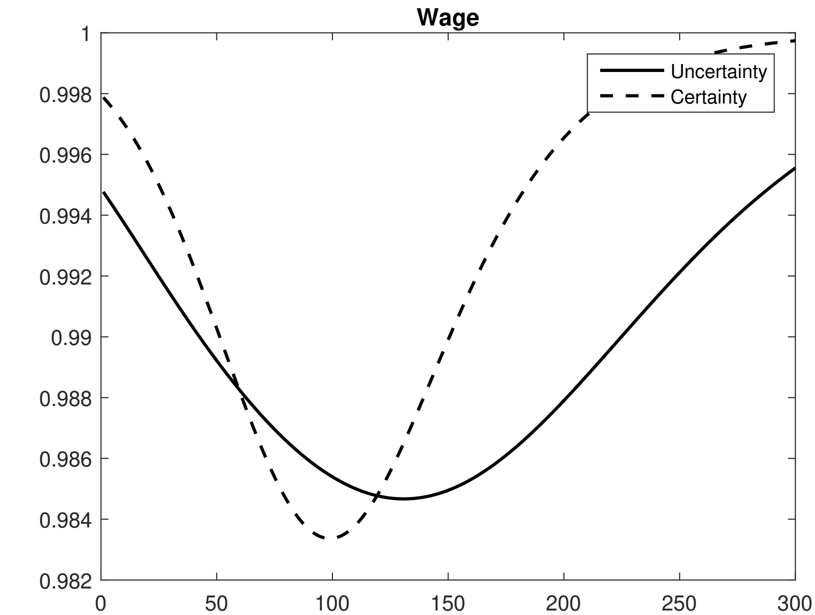


Figure 15: Changes of Wage

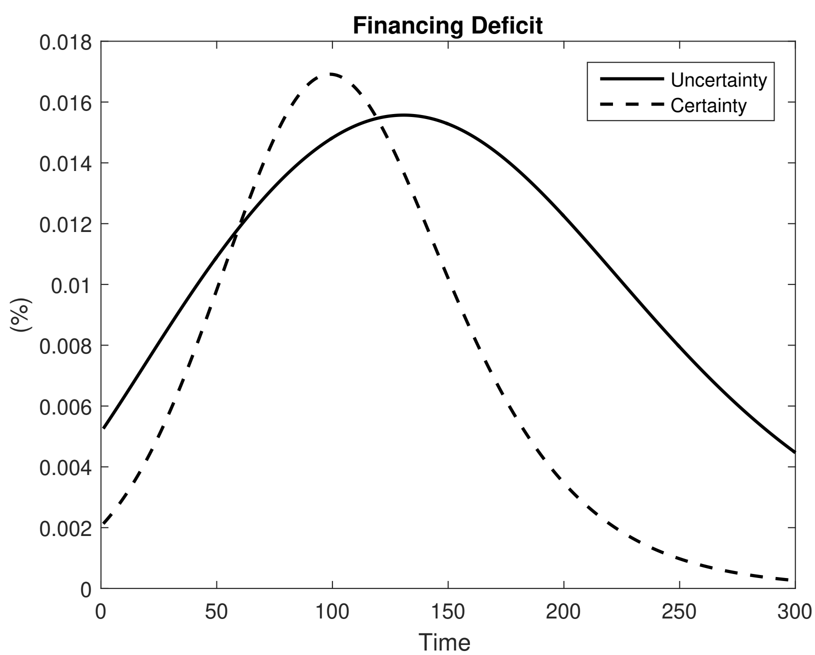


Figure 16: Changes of Deficit

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A Numerical Results

A.1 Baseline Model

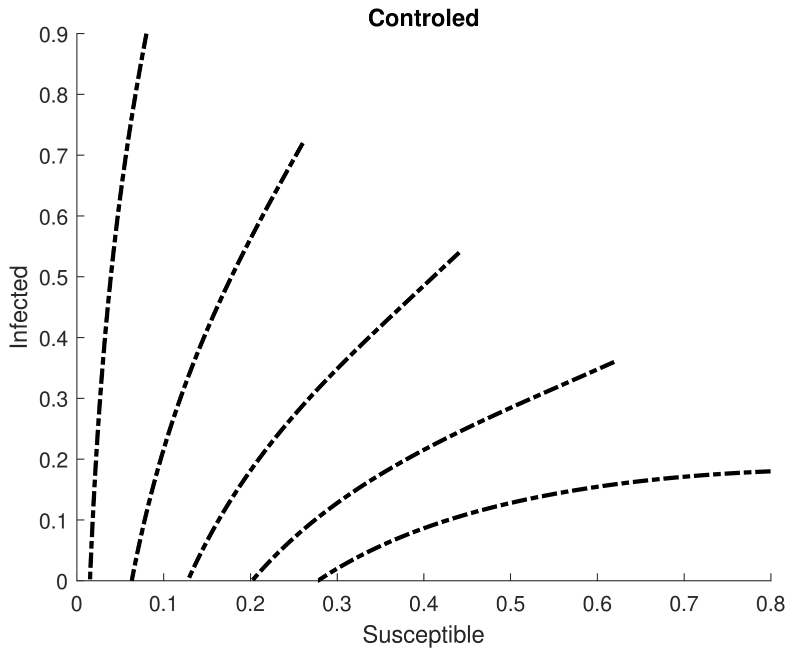

Figure 17: Changes of Susceptible and Infected population under Optimal Control Policy

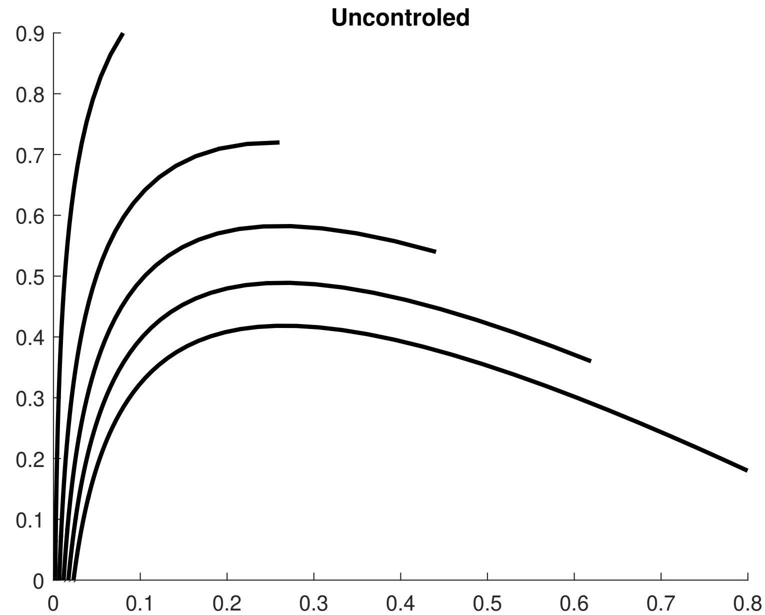

Figure 18: Changes of Susceptible and Infected population without Control

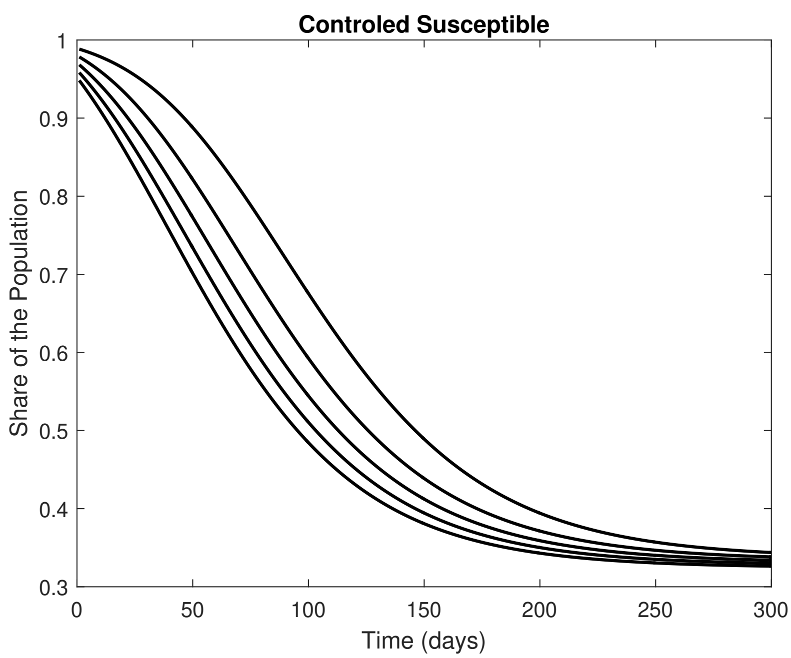

Figure 19: Changes of Susceptible population with Control

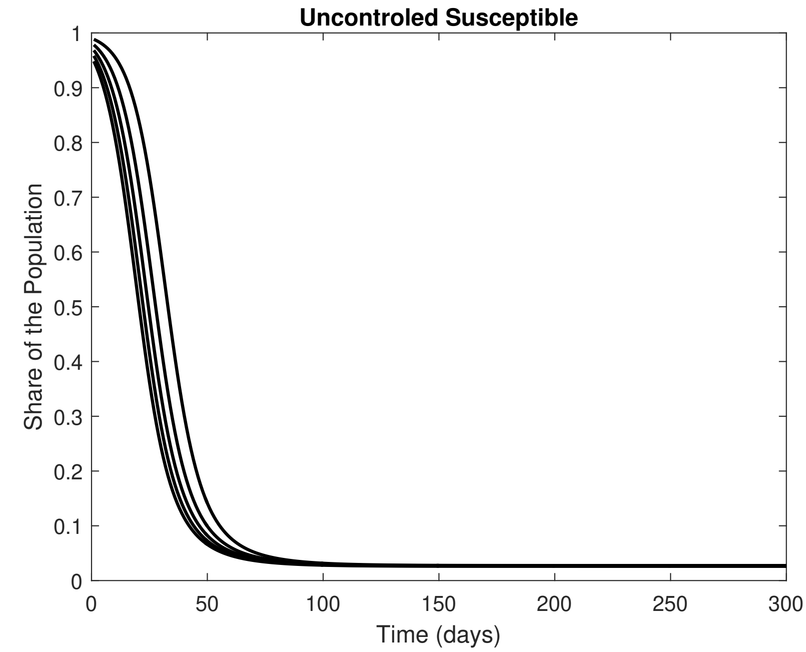

Figure 20: Changes of Infected population without Control

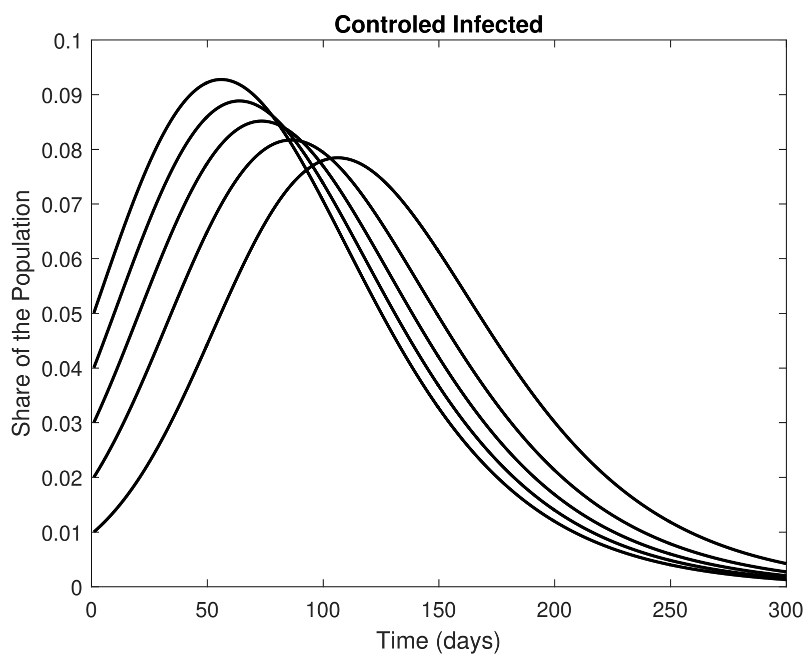

Figure 21: Changes of Infected population with Control

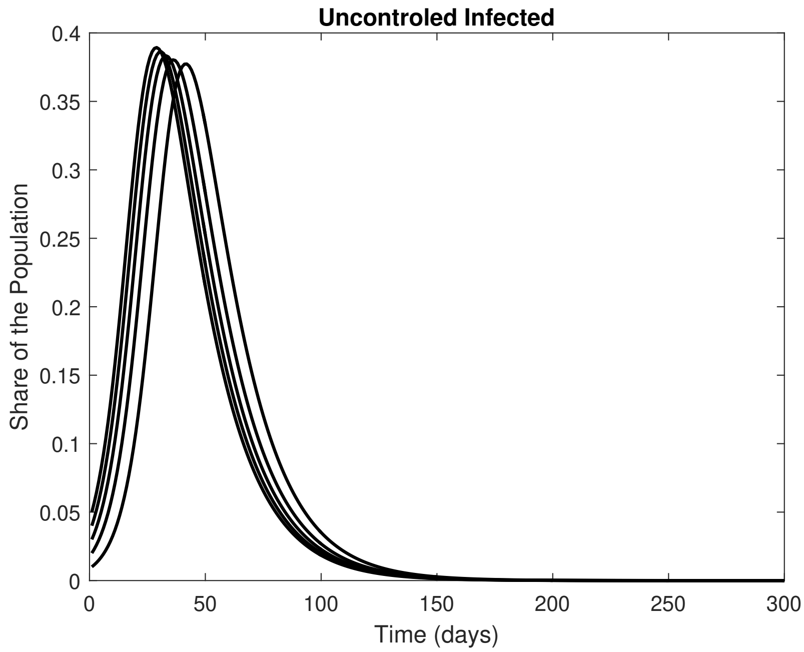

Figure 22: Changes of Infected population without Control

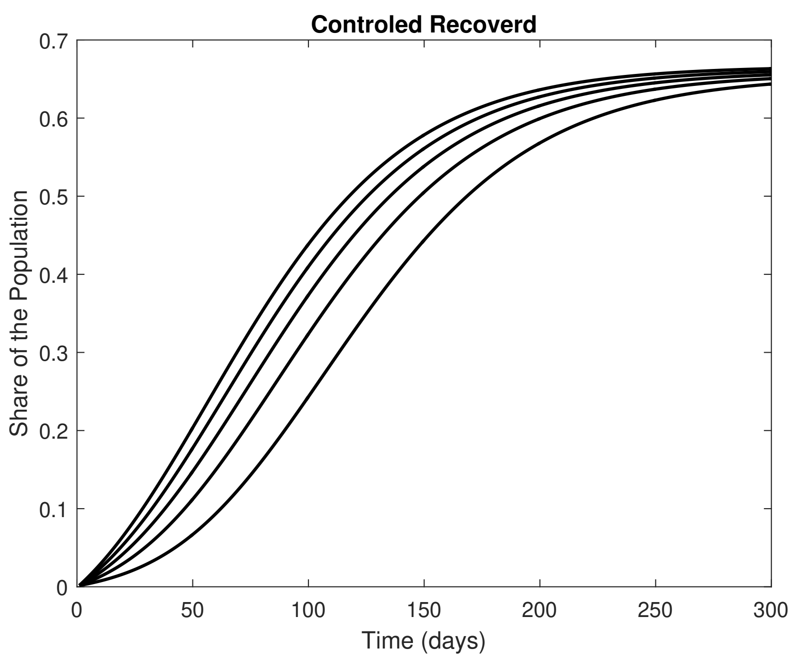

Figure 23: Changes of Recovered population with Control

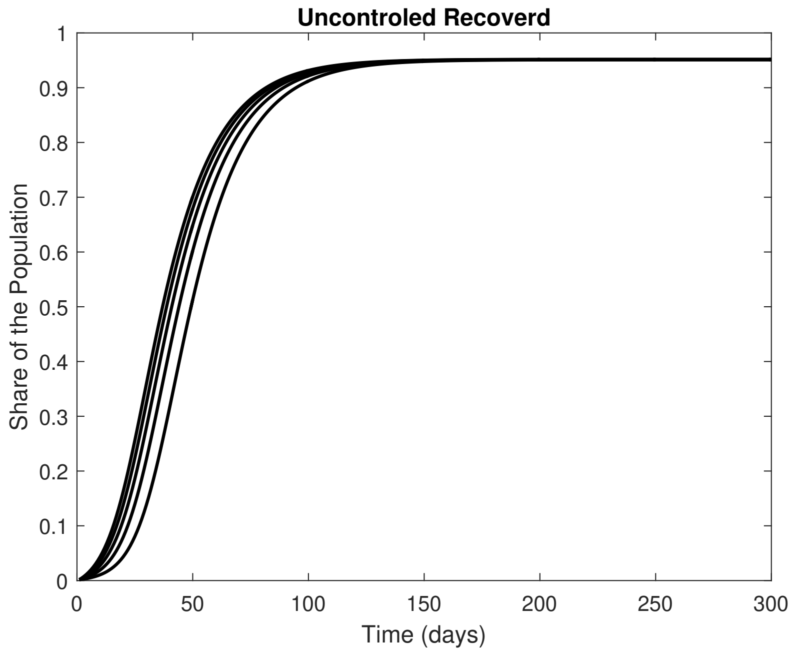

Figure 24: Changes of Recovered population without Control

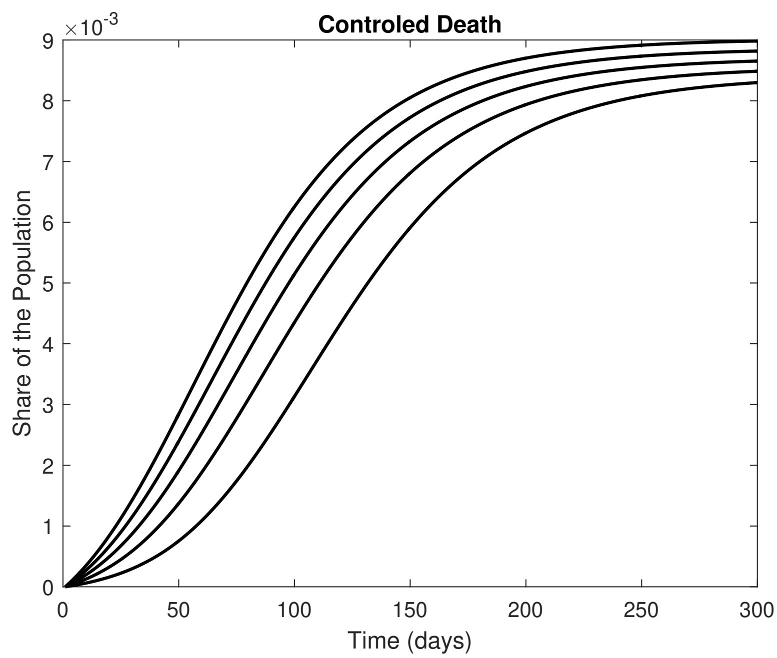

Figure 25: Changes of Dead population with Control

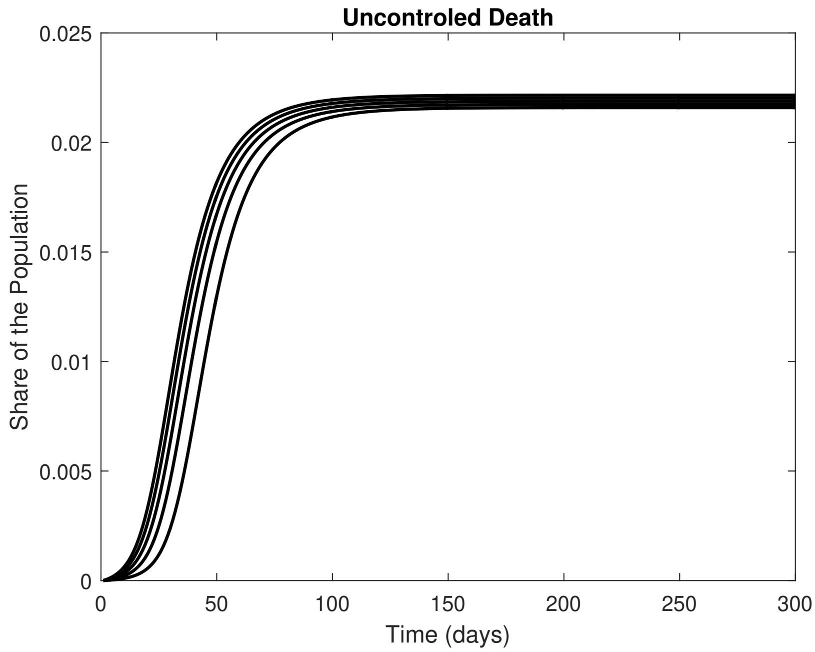

Figure 26: Changes of Dead population without Control

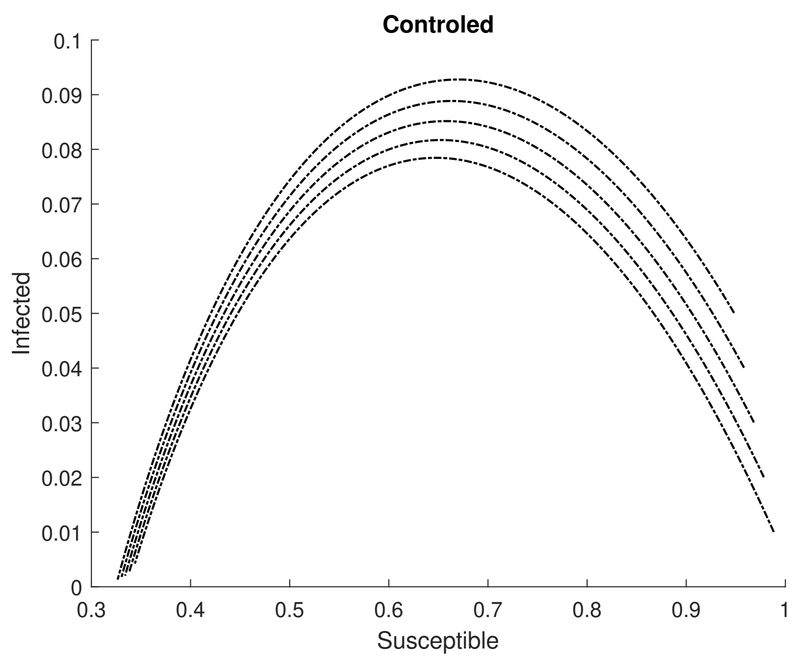

Figure 27: Phase with Control

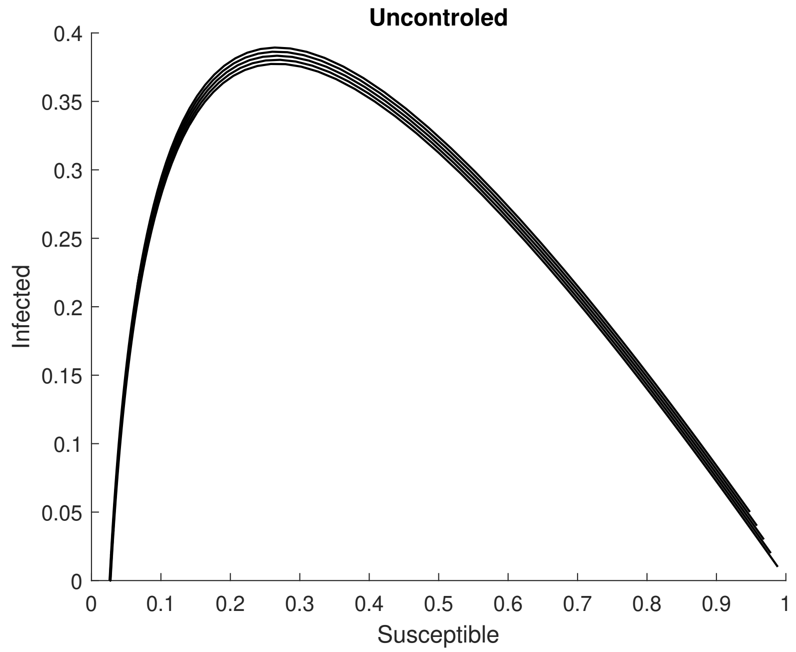

Figure 28: Phase without Control

A.2 Two-sector Model

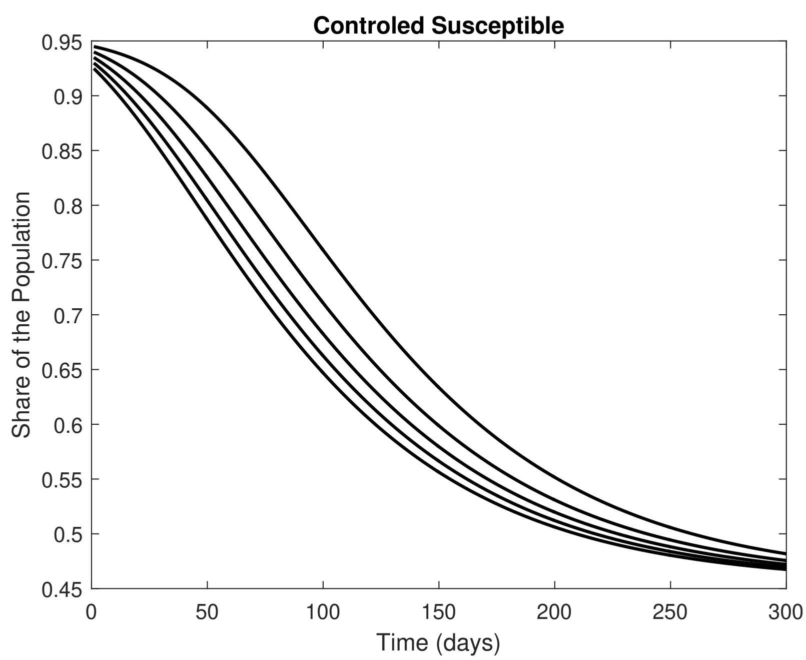

Figure 29: Changes of Susceptible population with Control

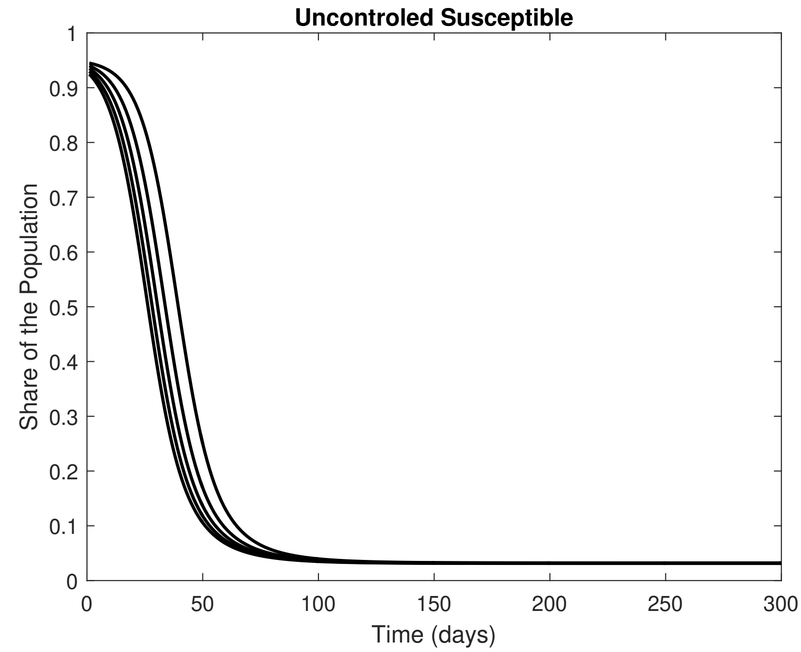

Figure 30: Changes of Infected population without Control

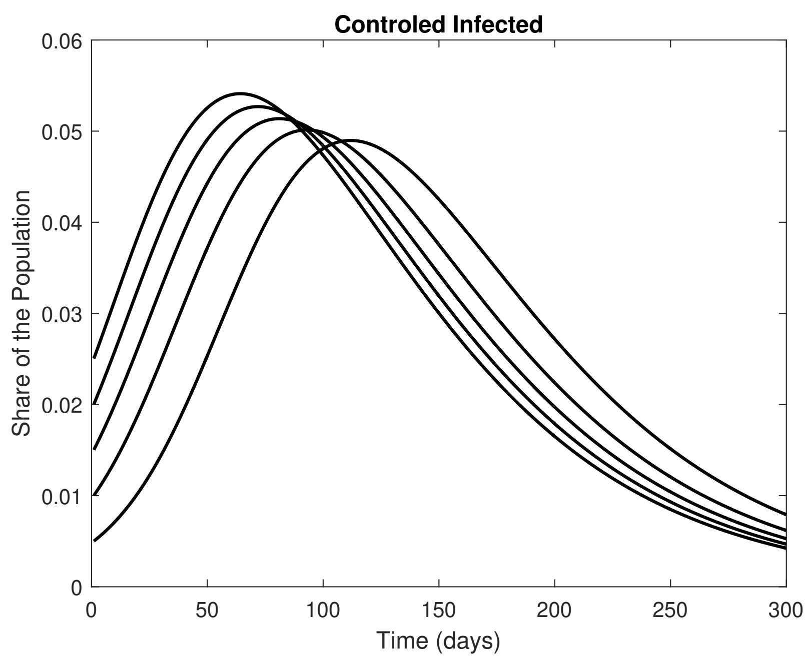

Figure 31: Changes of Infected population with Control

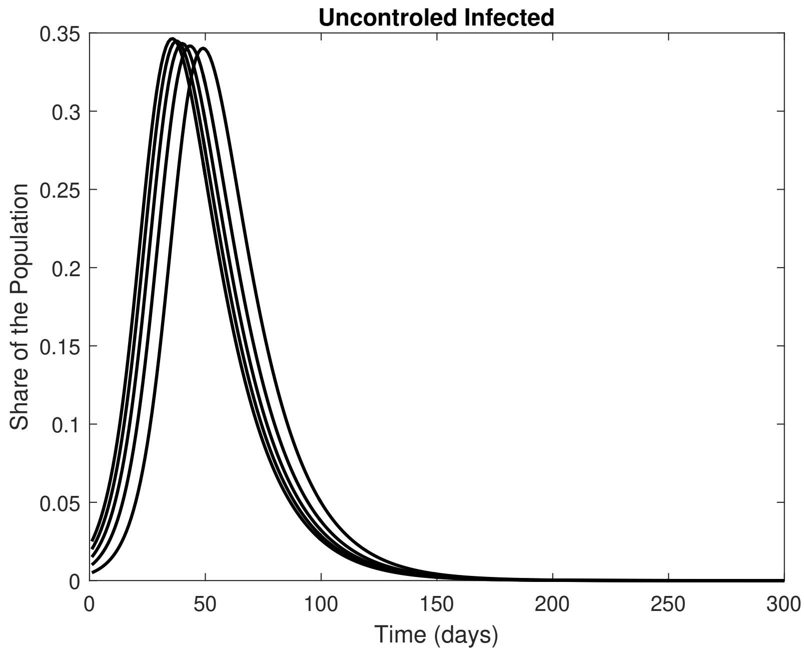

Figure 32: Changes of Infected population without Control

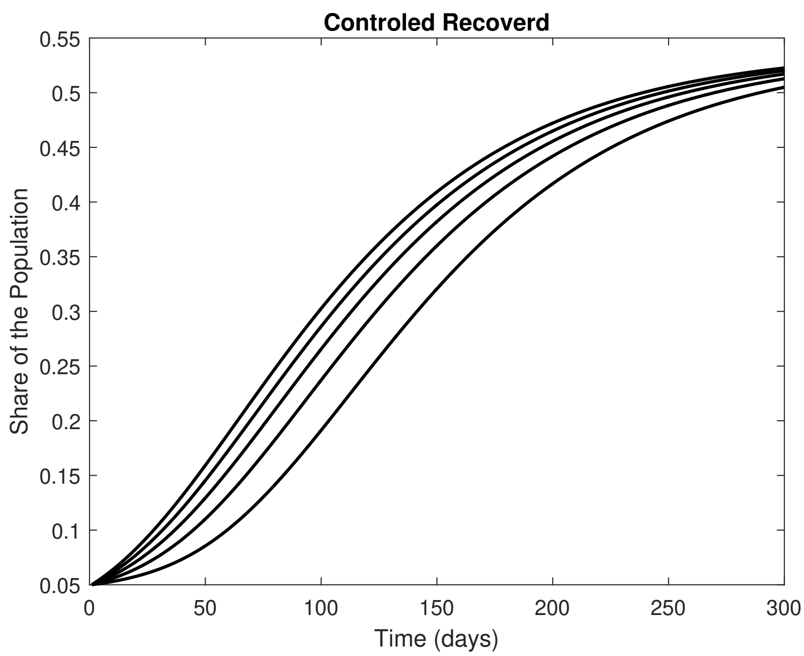

Figure 33: Changes of Recovered population with Control

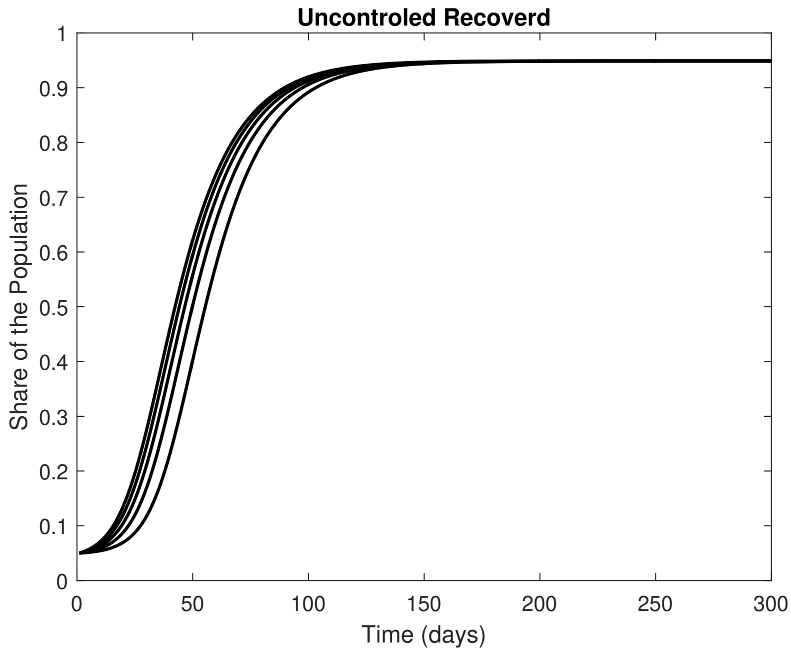

Figure 34: Changes of Recovered population without Control

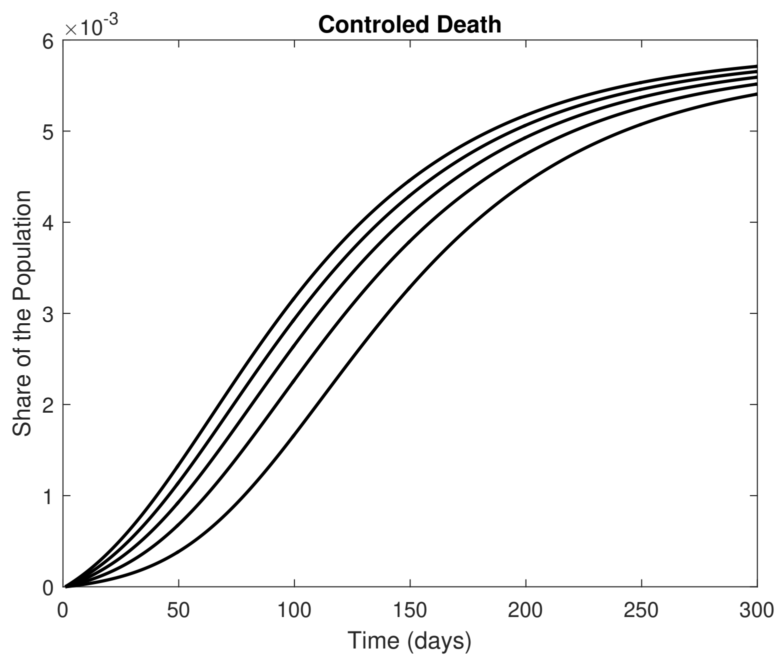

Figure 35: Changes of Dead population with Control

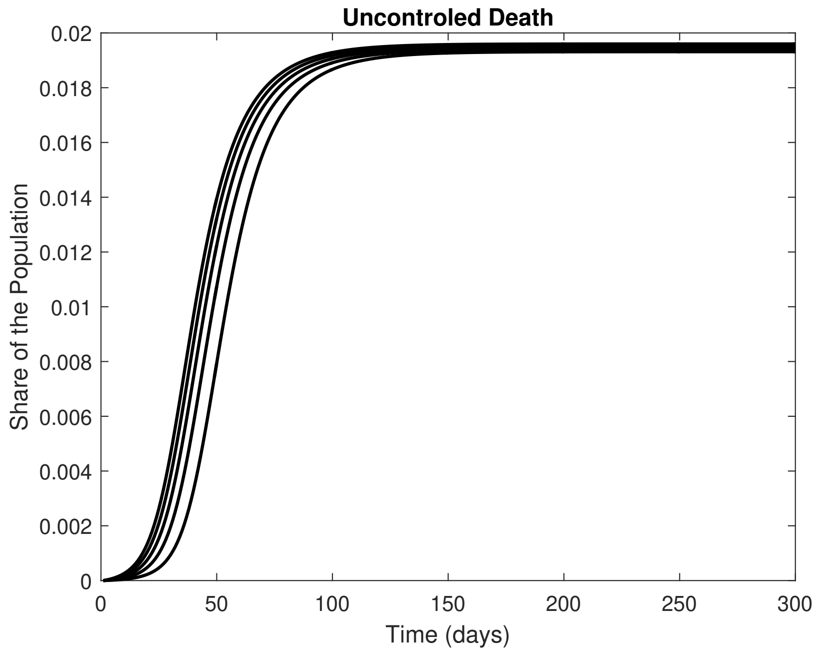

Figure 36: Changes of Dead population without Control

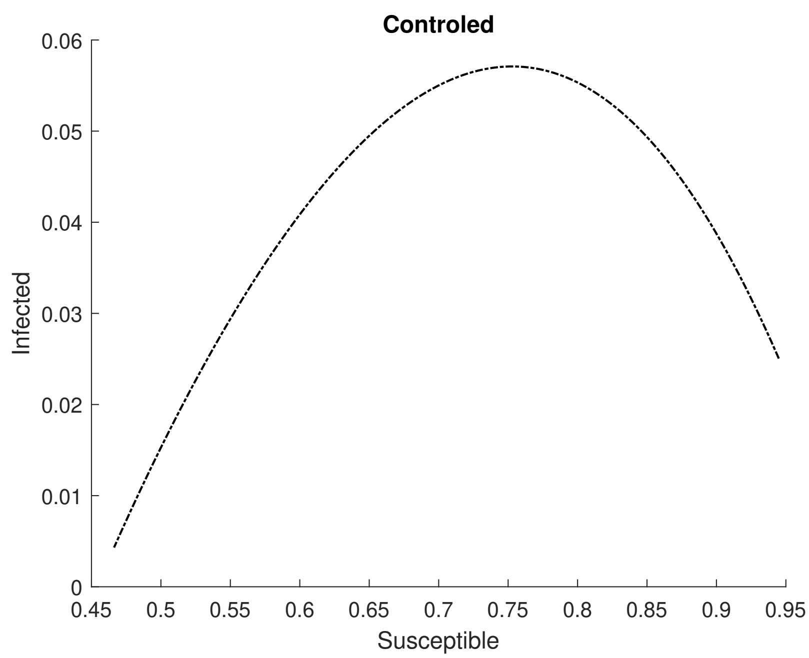

Figure 37: Phase with Control

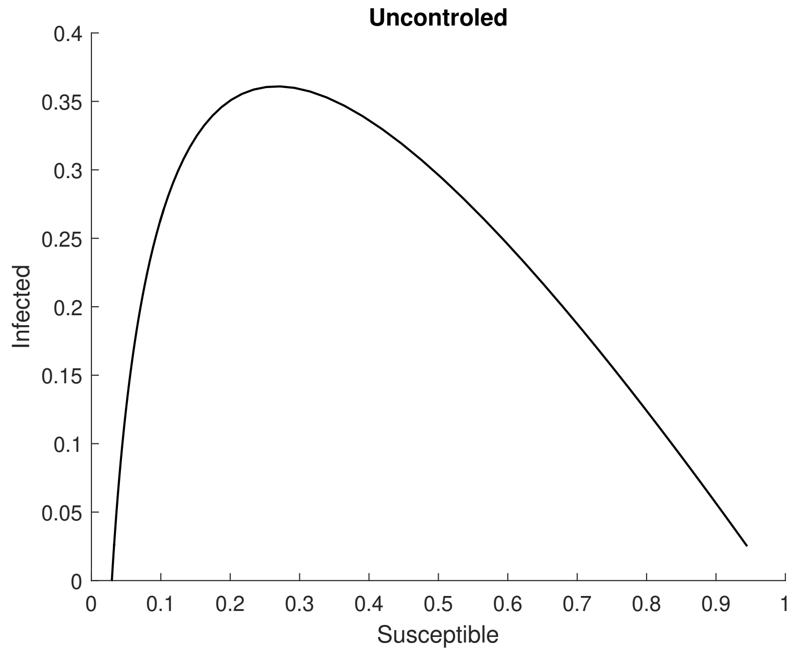

Figure 38: Phase without Control

B Appendix for Numerical Method

B.1 One-sector Model

As Alvarez, Argente, and Lippi, 2020, we use value function iteration algorithm to solve this problem.

The utility of agents is

$$E[u_s(c_t^s, n_t^s)] = (1-\alpha L)Lw + \alpha Lb \tag{35}$$

because we can calculate the equilibrium wage, the utility can be written as:

$$u_t^s = (1-\alpha L)\frac{SL - Ib}{S} + \alpha Lb \tag{36}$$

Solve this social planner's problem:

$$\begin{aligned}
&\max_{L(t)} \int_0^\infty e^{-(r+v)t} \left\{ S(t)\left[(1-\alpha L)\frac{SL-Ib}{S} + \alpha Lb\right] + I(t)b - \chi_d D(t) \right\} dt \\
=&\max_{L(t)} \int_0^\infty e^{-(r+v)t} \left\{ S(t)\left[(1-\alpha L)\frac{SL-Ib}{S} + \alpha Lb\right] + I(t)b + \chi_d \phi(I(t))I(t) \right\} dt
\end{aligned} \tag{37}$$

The HJB equation of this problem is:

$$(r+v)V(S,I) = \min_{L\in[0,\bar{L}]} \left\{ S(t)\left[(1-\alpha L)\frac{SL-Ib}{S} + \alpha Lb\right] + I(t)b + \chi_d\phi(I(t))I(t) + \right.$$
$$- \partial_S V(S,I)[-\beta S(t)I(t)(1-\theta L(t))^2] \tag{38}$$
$$\left. + \partial_I V(S,I)[\beta S(t)I(t)(1-\theta L(t))^2 - \gamma I(t)] \right\}$$

To calculate the partial difference $\partial_S V(S,I)$ and $\partial_I V(S,I)$, we choose to $V_S^-(i,j)$ and $V_I^+(i,j)$

$$V_S^-(i,j) = \frac{V\left(S_i, I_j\right) - V\left(S_{i-1}, I_j\right)}{S_i - S_{i-1}} \tag{39}$$

$$V_I^+(i,j) = \frac{V\left(S_i, I_{j+1}\right) - V\left(S_i, I_j\right)}{I_{j+1} - I_j} \tag{40}$$

$$(r+\nu)V\left(S_i, I_j\right) = \min_{L\in[0,\bar{L}]} \left\{ S_i\left[(1-\alpha L)\frac{SL-Ib}{S} + \alpha Lb\right] + I_j b + \chi_d\phi(I_j)I_j \right.$$
$$\left. + \left[\beta S_i I_j (1-\theta L)^2\right]\left[V_I^+(i,j) - V_S^-(i,j)\right] - \gamma I_j V_I^-(i,j) \right\} \tag{41}$$

We assume that $S_i - S_{i-1} = I_{j+1} - I_j = \Delta$.

$$V_I^+(i,j) - V_S^-(i,j) = \frac{1}{\Delta}\left[V\left(S_i, I_{j+1}\right) - V\left(S_i, I_j\right) - V\left(S_i, I_j\right) + V\left(S_{i-1}, I_j\right)\right] \tag{42}$$

$$[1+(r+\nu)dt]V\left(S_i, I_j\right) = \min_{L\in[0,\bar{L}]} \left\{ S_i\left[(1-\alpha L)\frac{SL-Ib}{S} + \alpha Lb\right]dt + \chi_d\phi(I_j)I_j dt \right.$$
$$\left. + \left[\beta S_i I_j (1-\theta L)^2\right] dt \left[V_I^+(i,j) - V_S^-(i,j)\right] - \gamma I_j dt V_I^-(i,j) + V(S_i, I_j) \right\} \tag{43}$$

$$[1+(r+\nu)dt]V\left(S_i, I_j\right) = \min_{L\in[0,\bar{L}]} \left\{ S_i\left[(1-\alpha L)\frac{SL-Ib}{S} + \alpha Lb\right]dt + \chi_d\phi(I_j)I_j dt \right.$$
$$+ \left[\beta S_i I_j (1-\theta L)^2\right] \frac{dt}{\Delta}\left[V\left(S_i, I_{j+1}\right) - 2V\left(S_i, I_j\right) + V\left(S_{i-1}, I_j\right)\right] \tag{44}$$
$$\left. - \gamma I_j \frac{dt}{\Delta}\left[V\left(S_i, I_j\right) - V\left(S_i, I_{j-1}\right)\right] + V\left(S_i, I_j\right) \right\}$$

$$
\begin{aligned}
&[1+(r+\nu)dt]V\left(S_i,I_j\right) = \min_{L\in[0,\bar{L}]}\left\{S_i\left[(1-\alpha L)\frac{SL-Ib}{S}+\alpha Lb\right]dt+\chi_d\phi(I_j)I_jdt\right.\\
&+[1-(r+\nu)dt]\left\{\frac{\beta S_iI_j(1-\theta L)^2}{1-(r+\nu)dt}\frac{dt}{\Delta}\left[V\left(S_i,I_{j+1}\right)-2V\left(S_i,I_j\right)+V\left(S_{i-1},I_j\right)\right]\right.\\
&\left.\left.-\frac{\gamma I_j}{1-(r+\nu)dt}\frac{dt}{\Delta}\left[V\left(S_i,I_j\right)-V\left(S_i,I_{j-1}\right)\right]+V\left(S_i,I_j\right)\right\}\right\}
\end{aligned}
\tag{45}
$$

$$
\begin{aligned}
V\left(S_i,I_j\right) = &\min_{L\in[0,\bar{L}]}\left\{S_i\left[(1-\alpha L)\frac{SL-Ib}{S}+\alpha Lb\right]dt+\chi_d\phi(I_j)I_jdt\right.\\
&+[1-(r+\nu)dt]\left[1-2\frac{\beta S_iI_j(1-\theta L)^2}{1-(r+\nu)dt}\frac{dt}{\Delta}-\frac{\gamma I_j}{1-(r+\nu)dt}\frac{dt}{\Delta}\right]V\left(S_i,I_i\right)\\
&+[1-(r+\nu)dt]\left[\frac{\beta S_iI_j(1-\theta L)^2}{1-(r+\nu)dt}\frac{dt}{\Delta}\right]V\left(S_i,I_{j+1}\right)\\
&+[1-(r+\nu)dt]\left[\frac{\beta S_iI_j(1-\theta L)^2}{1-(r+\nu)dt}\frac{dt}{\Delta}\right]V\left(S_{i-1},I_j\right)\\
&\left.+[1-(r+\nu)dt]\left[\frac{\gamma I_j}{1-(r+\nu)dt}\frac{dt}{\Delta}\right]V\left(S_i,I_{j-1}\right)\right\}
\end{aligned}
\tag{46}
$$

With respect the situation that we set the interval of different of direction on the discrete space.

$$
\begin{aligned}
&V\left(S_i,I_j\right) = \min_{L\in[0,\bar{L}]}\left\{S_i\left[(1-\alpha L)\frac{SL-Ib}{S}+\alpha Lb\right]dt+\chi_d\phi(I_j)I_jdt\right.\\
&+[1-(r+\nu)dt]\left\{1-\frac{\left[\beta S_iI_j(1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I}-\frac{\left[\beta S_iI_j(1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_S}-\frac{\gamma I_j}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I}\right\}V\left(S_i,I_j\right)\\
&+[1-(r+\nu)dt]\frac{\left[\beta S_iI_j(1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I}V\left(S_i,I_{j+1}\right)\\
&+[1-(r+\nu)dt]\frac{\left[\beta S_iI_j(1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_S}V\left(S_{i-1},I_j\right)\\
&\left.+[1-(r+\nu)dt]\frac{\gamma I_j}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I}V\left(S_i,I_{j-1}\right)\right\}
\end{aligned}
\tag{47}
$$

On the edge of the space:

$$
\begin{aligned}
V\left(S_i, I_j\right) = & \min_{L\in[0,\bar{L}]} \left\{ S_i\left[(1-\alpha L)\frac{SL-Ib}{S} + \alpha Lb\right] dt + \chi_d \phi(I_j) I_j dt \right. \\
& + [1-(r+\nu)dt]\left\{1 - \frac{\left[\beta S_i I_j (1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_S} - \frac{\gamma I_j}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I}\right\} V\left(S_i, I_j\right) \\
& + [1-(r+\nu)dt]\frac{\left[\beta S_i I_j (1-\theta L)^2\right]}{[1-(r+\nu)dt]}\frac{dt}{\Delta_S} V\left(S_{i-1}, I_{j+k}\right) \\
& \left. + [1-(r+\nu)dt]\frac{\gamma I_j}{[1-(r+\nu)dt]}\frac{dt}{\Delta_I} V\left(S_i, I_{j-1}\right) \right\}
\end{aligned} \tag{48}
$$

Before value function iteration, we have to determine the initial value of discrete space:

$$
V(0,I) = \frac{a^2b^2}{8\alpha\gamma + 4\alpha(r+v)} I^2 + \frac{4\alpha\chi_d\phi + 2ab}{4\alpha\gamma + 4\alpha(r+v)} I + \frac{1}{4\alpha(r+v)} \tag{49}
$$

$$
V(S,0) = \frac{\left(\frac{(1+\alpha bS)^2}{4\alpha}\right)}{r+v}; \tag{50}
$$

And then we use FOC to find optimal policy $L(t)$

$$
L = \frac{(1+\alpha bI + abS) - 2\theta[\beta SI]\left[V_I^+ - V_S^-\right]}{2\alpha - 2\theta^2[\beta SI]\left[V_I^+ - V_S^-\right]} \tag{51}
$$

B.2 Two-sector Model

Same as baseline model, we use value function iteration to find the solution of HJB equation:

The HJB equation in thie part is:

$$
\begin{aligned}
\max_{L(t)} \int_0^\infty e^{-(r+v)t} & \left\{ \phi S_t \left[\frac{SL-Ib}{S}(1-\alpha_1 L) + \alpha_1 Lb\right] + (1-\phi) S_t \right. \\
& \left. \left[\frac{S-Ib}{S}(1-\alpha_2 L) + \alpha_2 Lb\right] + I(t)b - \chi_d D(t) \right\} dt
\end{aligned} \tag{52}
$$

$$
\begin{gathered}
V\left(S_i, I_j\right) = \min_{L\in[0,L]} \left\{ \left[(1-\alpha_1 L\left[S_t L(t) - I(t)b\right] + \alpha_1 Lb + (1-\phi)\left[(1-\alpha_2 L)\left[S_t - I(t)b\right] + \alpha_2 Lb\right]\right.\right. \\
+ I(t)b + \chi_d \phi\left(I_j\right) I_j dt \\
+ [1-(r+v)dt]\left\{1 - \frac{\left[\beta S_i I_j(1-\theta L)^2\right]}{[1-(r+v)dt]}\frac{dt}{\Delta_S} - \frac{\gamma I_j}{[1-(r+v)dt]}\frac{dt}{\Delta_I}\right\} V\left(S_i, I_j\right) \\
+ [1-(r+v)dt]\frac{\left[\beta S_i I_j(1-\theta L)^2\right]}{[1-(r+v)dt]}\frac{dt}{\Delta_S} V\left(S_{i-1}, I_{j+k}\right) \\
\left. + [1-(r+v)dt]\frac{\gamma I_j}{[1-(r+v)dt]}\frac{dt}{\Delta_I} V\left(S_i, I_{j-1}\right) \right\}
\end{gathered} \tag{53}
$$

The FOC of optimal policy L is:

$$L = \frac{\alpha_1 \phi I(t) b - \alpha_2(1-\phi)S_t + (1-\phi)I(t)b\alpha_2 + \phi\alpha_1 b + (1-\phi)\alpha_2 b + \phi S_t - 2\theta\beta SI\left[V^+ - V^-\right]}{2\alpha_1\phi S_t - 2\theta^2\beta SI\left[V^+ - V^-\right]} \tag{54}$$

C Stochastic HJB

In this part we will solve the HJB equation of stochastic model:

$$\begin{aligned}(r+v)V(v) = \min_{L\in[0,L]} \{ & [(1-\alpha L)(SL - Ib) + \alpha Lb] + I(t)b - \chi_d D(t) + V_s\left[-\beta(v)\left(S_t - L_t\right)I_t\right] \\ & + V_i\left[\beta(v)\left(S_t - L_t\right)I_t - (\rho(v)+\delta(v))I_t\right] + V_d\delta(v)I_t \\ & + \frac{1}{2}\left\{V_{ss}\sigma_\beta^2 + V_{ii}\left[\sigma_\beta^2 + \sigma_\delta^2\right] + V_{dd}\sigma_\delta^2\right\}I_t^2 - \left\{\sigma_\beta^2 V_{si} + \sigma_\delta^2 V_{id}\right\}I_t^2 \Bigg\}\end{aligned} \tag{55}$$

$$\begin{aligned}= \min_{\mathrm{L}\in[0,\bar{\mathrm{l}}]} & \left\{\left[-\alpha SL^2 + \alpha bIL + SL + \alpha Lb\right] - \chi_d D(t) + V_s\left[-\beta(v)\left(S_t - L_t\right)I_t\right]\right. \\ & + V_i\left[\beta(v)\left(S_t - L_t\right)I_t - (\rho(v)+\delta(v))I_t\right] + V_d\delta(v)I_t \\ & \left. + \frac{1}{2}\left\{V_{ss}\sigma_\beta^2 + V_{ii}\left[\sigma_\beta^2 + \sigma_\delta^2\right] + V_{dd}\sigma_\delta^2\right\}I_t^2 - \left\{\sigma_\beta^2 V_{si} + \sigma_\delta^2 V_{id}\right\}I_t^2\right\}\end{aligned} \tag{56}$$

$$-2\alpha SL + \alpha bI + S + \alpha b + V_s\beta(v)I_t - V_i\beta(v)I_t = 0 \tag{57}$$

$$L = \frac{\alpha bI + S + \alpha b + V_s\beta(v)I_t - V_i\beta(v)I_t}{2\alpha S} \tag{58}$$

$$\begin{aligned}(r+v)V(v) = & \left[-\alpha SL^{*2} + \alpha bIL^* + SL^* + \alpha L^* b\right] - \chi_d D(t) + V_s\left[-\beta(v)\left(S_t - L^*\right)I_t\right] \\ & + V_i\left[\beta(v)\left(S_t - L^*\right)I_t - (\rho(v)+\delta(v))I_t\right] + V_d\delta(v)I_t \\ & + \frac{1}{2}\left\{V_{ss}\sigma_\beta^2 + V_{ii}\left[\sigma_\beta^2 + \sigma_\delta^2\right] + V_{dd}\sigma_\delta^2\right\}I_t^2 - \left\{\sigma_\beta^2 V_{si} + \sigma_\delta^2 V_{id}\right\}I_t^2\end{aligned} \tag{59}$$