

# Expected Utility, Industrial Structure and Social Planner's Optimal Lockdown for COVID-19

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## Abstract

In order to flatten the curve of COVID-19 and reduce potential deaths, most countries have adopted a policy of blocking non-essential sectors. However, such policies are controversial, especially when it comes to economic losses and the inequalities incurred by sectoral differences. In this paper, we try to show the nexus between economic loss, social inequality, and loss of value of life under optimal lockdown. The single sector model is first introduced as a benchmark model to study the impact of lockdown on the whole economy. The two-sector models, dispersed and concentrated sector relations, respectively, are established to study the optimal lockdown rate for non-essential sectors. Rather than the loss of their wages under lockdown, which is the setting of most others, we employ the individuals' expected utility to reflect the risk aversion in the pandemic and model the impact of the deaths. The insights are two-fold: first, in a country with a higher proportion of the tertiary industry, it is possible to ease the lockdown to reduce economic losses; second, the fiscal policy, especially unemployment insurance, even if it is effective, can lead to big output losses.

**Keywords:** SIR Model; Risk Aversion; Sectoral Income Inequality; Optimal Lockdown; COVID-19

# 1 Introduction

COVID-19 pandemic has already caused severe effects on the economy worldwide and has taken more than 1 million people as of October 2020. The shock of the pandemic caused a global recession and large-scale unemployment in different countries. Recent researchers launched a series of the redevelopment of SIR (Susceptible-Infectious-Recovered) model (Shown in Figure 1, one result of our planning model) that was promoted by Kermack et al. [1927]. The SIR framework and its various extensions model the spread are researched in many inquiries recently (e.g. Berger et al. [2020], Atkeson [2020]). The simplest version of the model that includes three differential equations can provide a good approximation for the pandemic's dynamic. Besides the ability to describe the pandemic's dynamic nature, it can also provide a basis for predicting the economy and planning for the economy. Several recent papers have started conducting optimal policy analysis within this framework (e.g. Alvarez et al. [2020], Jones et al. [2020], Farboodi et al. [2020], Rowthorn and Toxvaerd [2012], Eichenbaum et al. [2020].) But they only use the amount of people represent the value of the welfare. Differentiating from them, we build the model based on the classic theory of utility.

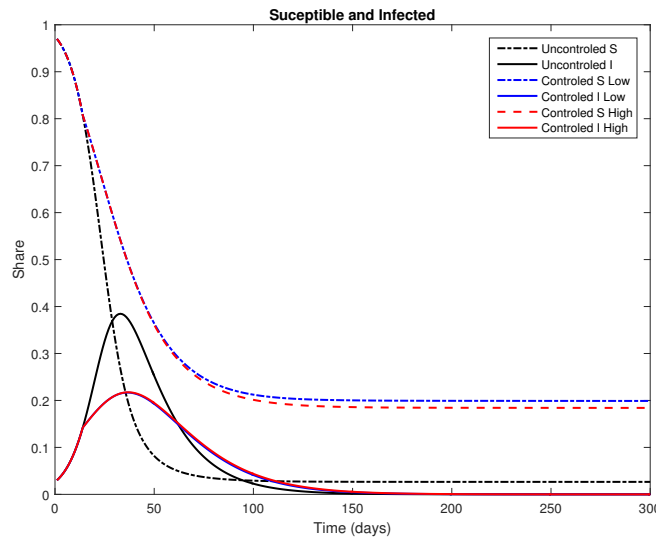


Figure 1: SIR in our model

Lockdown policy tend to be heterogeneous, which should be targeted to specific sector.

For the sake of simplifying, we divide the economy into two parts: necessary and unnecessary sector. Another self-evident issue is launched: will industrial structure affect the results? When we consider a lockdown policy related to the specific sector, the results will be different totally. Maintaining some parts of the society running is necessary, such as the health care system, power plant system, transportation system for commodity, and so forth. Those necessary sectors have to keep operating so that we can meet the basic demands. As Guerrieri et al. [2020], firstly, in one-sector economy, a pandemic's shock is negative (the drop in supply dominates). They prove a less obvious result: it also holds in incomplete market models that allow for heterogeneous agents, uninsurable income risk, and liquidity constraints, creating differences in marginal propensities to consume (MPC). In these models, there is a mechanism from income loss to demand decline. Although it makes the total demand decline larger than the total demand decline in typical agency cases, the decline is smaller than the output decline caused by a supply shock. Intuitively, the largest MPC of people who lose their income may be large, but only one means that their consumption decline is always a suppressed version of income loss. Then we turn to economies with multiple sectors. When the impact is concentrated in certain sectors, as in the case of a shutdown in response to an epidemic, there is more room for total spending to contract. Some goods are no longer available, which makes overall consumption less attractive. One explanation is that the suspension of production increases the shadow price of goods in the affected sectors, making current total consumption more expensive, thus inhibiting consumption. On the other hand, due to the unavailability of goods in some sectors, expenditures can be transferred to other sectors through alternative channels. In sectors not directly affected by closures, maintaining full employment depends on these two impacts' relative intensity.

The industrial structure is closely related to the problem of inequality in the economy. Because industries have different production characteristics, factor demand, and economic dependence, representative consumers in different industries will bring different profits and output. In other words, the industrial structure is the root cause of inequality in the economy.

Different industrial structure necessarily means different output value and wages. Under the condition of the incomplete market, people cannot freely choose the industry they are engaged in. From the perspective of each country's industrial structure, the demand for the non-essential production sector of the tertiary industry is also relatively large. Data show that the tertiary industry accounts for a very high proportion of both developed and underdeveloped countries. This is the inevitable result of economic development. In our model, the tertiary industry is the industry that needs to implement strict measures. Intuitively, the higher the tertiary industry, the more lockdowns it may bring, but this is not the case, and it is not an easy question to answer in our article. When we think about the optimal tradeoff between economic output, the value of life, it's not exactly what we would expect. So the inequality of social outcomes under an optimal lockdown is our main concern.

The cure for economic inequality may be fiscal and monetary policy. But during an epidemic, it is unemployment insurance in fiscal policy that can significantly influence individual decisions. An obvious idea is how fiscal policy can be used to address these distortions in the economy. We want to use such a fiscal tool to address inequality perfectly, without economic loss and loss of life. How would an optimal lockdown affect the effectiveness of unemployment insurance in our framework? One conclusion is that it might be counterintuitive.

Therefore, in our model, we present an optimal control model with heterogeneous production sectors for the COVID-19 epidemic. We adopt the model structure of [Alvarez et al. \[2020\]](#), a variation in the SIR epidemiology model reviewed and proposed by [Atkeson \[2020\]](#) to analyze the optimal lockdown policy. [Morton and Wickwire \[1974\]](#) promoted the origin edition of the planning problem where they construct a control scheme for the immunization of susceptibles in the Kermack-McKendrick epidemic model for a closed population. And they use the approach of dynamic programming and Pontryagin's Maximum Principle and allows one, for specific values of the cost and removal rates, to apply necessary and sufficient conditions for optimality and show that a one-switch candidate is an optimal control. In the remaining cases, they show that an optimal control, if it exists, has at most one switch. And

then [Hansen and Day \[2011\]](#) extend the existing work on the time-optimal control of the basic SIR epidemic model with a mass action contact rate. They provide analytic solutions for the model that minimizes the outbreak size (or infectious burden) under the assumption that there are limited resources and optimal control policies for an isolation only model, a vaccination only model and a combined isolation–vaccination model (or mixed model). Several conclusions are given by them: under certain circumstances, the optimal isolation only policy is not unique; further- more the optimal mixed policy is not merely a combination of the optimal isolation only policy and the optimal vaccination only policy. Some recent papers talk about the planning problem centrally, see [Acemoglu et al. \[2020\]](#); [Barro et al. \[2020\]](#); [Eichenbaum et al. \[2020\]](#); [Berger et al. \[2020\]](#); [Hall et al. \[2020\]](#); [Chari et al. \[2020\]](#); [Jones et al. \[2020\]](#) and [Richard Baldwin \[2020\]](#). The typical approach in the epidemiology literature is to study the dynamics of the pandemic, for infected, deaths, recovered, as functions of some exogenous chosen diffusion parameters, which are in turn related to various policies, such as the partial lockdown of schools, universities, businesses, and other measures of diffusion mitigation, and where the diffusion parameters are stratified by age and individual covariates.

We differ from these studies in two ways: first, we place the planning problem within the general equilibrium framework of the problems of utility and social planners. This economy is characterized by heterogeneous variables such as consumers, wages, prices, and debt. Risk aversion exists in the individual decision-making process. We construct an expected utility function that shows risk aversion in the choice between labor supply and consumption. For the economically productive part, we set up a linear technical production function. So we can get wages in a balanced economy. As far as the government is concerned, the fiscal deficit is the main source of the government’s fiscal deficit. On this basis we can discuss fiscal policy. Second, we consider the multisectoral model on this basis. We talked about how the structure of the industry has an impact on the economy. Given that we have to keep one part of the economy going, we have blocked off only one sector, the tertiary industry, in each country. Therefore, we can consider the problem of inequality in the economy due to sector heterogene-

ity. We have also demonstrated whether the unemployment insurance policies we are familiar with are effective in addressing economic inequality.

## 2 Social Planner's Optimal Lockdown Model

This section firstly employs the SIR model to model the population dynamics. We then define the expected utility for susceptible agents and fixed utility for those infected under the assumption that the probability of infection is proportional to lockdown policy. We employ an objective function of the social planner to compromise between total expected utility and death toll.

### 2.1 SIR Model

As in [Atkeson \[2020\]](#) and [Alvarez et al. \[2020\]](#), at any point in time  $t$ , the whole population  $N(t)$  is divided into those susceptible  $S(t)$ , those infected  $I(t)$  and those recovered  $R(t)$ , i.e.,

$$N(t) = S(t) + I(t) + R(t), \quad \forall t > 0. \quad (1)$$

The recovered category  $R(t)$  here includes individuals who have been infected, survived the disease, and are assumed to be immune to COVID-19 within a certain period of time. We normalize the initial population to  $N(0) = 1$  where only those alive population are considered. The social planner can control a fraction  $L(t)$  of the population, where  $L \leq 1$  allows us to meet a more actual situation to keep some significant departments and industries working on such as power plant, food supply vendors and groceries. The lockdown efficiency  $\theta$  measures the proportion that population cannot contact others freely. If  $\theta = 1$ , the lockdown policy completely yields its effects on population. But the actual scenario we never harbor the ability to control all people to move in different cities to curb the transmission of virus; so we take parameter  $\theta < 1$ .

Given the above setup, the law of dynamic of susceptible agents, infected agents, and total

population are

$$\dot{S} = -\beta S(t)(1 - \theta L(t))I(t)(1 - \theta L(t)), \quad (2)$$

$$\dot{I} = \beta S(t)(1 - \theta L(t))I(t)(1 - \theta L(t)) - \gamma I(t), \quad (3)$$

$$-\dot{N} = D(t) = \phi(I(t))I(t), \quad (4)$$

$$\phi(I(t)) = [\varphi + \kappa I]\gamma. \quad (5)$$

The parameter  $\beta$  represents the number that the susceptible agents contact the infected agents per unit time. We set the probability that people get infected from infected agents is 1 after contacting. For infected people, they can recover at the rate of  $\gamma$ . The death of disease  $D(t)$  is defined as product of rate of death per unit time  $\phi(I(t))$  and number of infected agents. The “case fatality rate” (CFR)  $\phi(I(t)) \in (0, 1)$  in [Alvarez et al. \[2020\]](#) is the rate of fatality of infected people. It appears that the CFR manifest the direct proportion with  $I(t)$ , which reflects the jam effects in health care system.

## 2.2 Agents’ Utility, One Sector Production and Equilibrium

We define the expected utility for susceptible agents and fixed utility for infected agents. The utility of each susceptible agent consists of two parts: maintaining the presented scenario, namely not be infected; and the possibility of being tracked into infected group. The utility formula is modeled as follows

$$E[u_s(c_t^s, n_t^s)] = p(L)u_s(c_t^s, n_t^s) + [1 - p(L)]u_i(c_t^i, n_t^i) \quad (6)$$

$$E[u_i(c_t^i, n_t^i)] = u_i(c_t^i, n_t^i) \quad (7)$$

where  $c_t^s$  and  $n_t^s$  are the consumption and labor supply of susceptible agents;  $c_t^i$  and  $n_t^i$  are for infected people. The probability function  $p(L)$  depends on direct proportion of lockdown policy  $L(t)$  and the effectiveness of lockdown policy  $\alpha$  with the form

$$p(L) = \alpha L(t). \quad (8)$$

The parameter  $\alpha$  takes values between 0 and 1. If  $\alpha = 1$ , the policy is fully effective of lockdown policy, however, some contacts may still happen even under a full economic lockdown, we set  $\alpha < 1$ .

Now we assume the consumption is inelastic during the period of pandemic, i.e. agents cannot make the optimal decision for  $c_t$  and  $n_t$ . Infected agents accept unemployment insurance  $b$ . Apparently, social planner is incentive to avoid the worse situation that become infected people for they will suffer from welfare loss. We constrain the consumption and labor supply as follows

$$c_t^s = L(t)w(t), \quad n_t^s = L(t) \quad (9)$$

and

$$c_t^i = b, \quad n_t^i = 0. \quad (10)$$

The remedy against the ongoing COVID-19 recession that is currently being debated is fiscal stimulus. To consider the effects of fiscal stimulus in our model, we introduce a stylized government sector. We assume that the government chooses paths of government spending  $G_t$ , lump-sum transfers (or taxes)  $T_t$  that can be targeted by sector, and government debt  $B_t$ , subject to the flow budget constraint. For simplifying the model, we set  $G_t = 0$ ,  $T_t = 0$ , and

$$(1 + r_{t-1})B_t = B_{t+1}, \quad (11)$$

$$B_t = I(t)b. \quad (12)$$

As [Guerrieri et al. \[2020\]](#), competitive firms produce the final good only from labor using the linear technology, so the wage can be same as price of product,

$$Y_t = N_t, \quad (13)$$

$$p_t = w_t. \quad (14)$$

Total demand comes from two parts: the consumption of susceptible agents  $S(t)c_t^s$  and of infected agents  $I(t)b$ ,

$$C_t = S(t)c_t^s + I(t)b. \quad (15)$$



Another side total supply, namely total labor supply  $N_t$  comes from only the susceptible people,

$$N_t = S(t)L(t). \quad (16)$$

Define market equilibrium is

$$N_t = C_t. \quad (17)$$

Under this scenario, the wage of economy is

$$w_t = \frac{SL - Ib}{S}. \quad (18)$$

### 2.3 Planning Problem

The planning problem is modified version of [Alvarez et al. \[2020\]](#) and [Acemoglu et al. \[2020\]](#). We employ an objective function of social planner. Assume that agents live forever, unless they die from the infection. The planner discounts all values at the rate  $r > 0$  and with probability  $\nu$  per unit of time both a vaccine or a cure appear, then all infected agents could be cured or get the immunity. Thus, the planning problem can be consist in maximizing the following present value

$$\max_{L(t)} \int_0^\infty e^{-(r+\nu)t} \left\{ S(t)u(c_t^s, n_t^s) + I(t)b - \chi_d D(t) \right\} dt \quad (19)$$

We get HJB equation:

$$\begin{aligned} (r + \nu)V(S, I) = \min_{L \in [0, L]} \left\{ S(t) \left[ (1 - \alpha L) \frac{SL - Ib}{S} + \alpha Lb \right] + I(t)b - \chi_d \phi(I(t))I(t) + \right. \\ \left. - \partial_S V(S, I) [-\beta S(t)I(t)(1 - \theta L(t))^2] \right. \\ \left. + \partial_I V(S, I) [\beta S(t)I(t)(1 - \theta L(t))^2 - \gamma I(t)] \right\} \end{aligned} \quad (20)$$

The domain of  $V(S, I)$  is  $S + I \leq 1$ . Following [Alvarez et al. \[2020\]](#), we will use the value function iteration method to solve this problem. Note the boundary of value function has an analytical formula.  $V(S, 0) = \frac{(1+\alpha bS)^2}{4\alpha(r+\nu)}$  and  $V(0, I) = \frac{a^2 b^2}{8\alpha\gamma+4\alpha(r+\nu)} I^2 + \frac{4\alpha\chi_d\phi+2ab}{4\alpha\gamma+4\alpha(r+\nu)} I + \frac{1}{4\alpha(r+\nu)}$ .

### 3 Two-sector Models

We introduce two kinds of market equilibrium mechanism. The key difference between these two systems is the tunnel of how the equilibrium is determined. Multisector is a basic fact of an economy that must maintain part of its production to meet basic needs. For simplicity's sake, let's split the economy into two sectors, essential and non-essential. The first system can be called dispersed equilibrium. The wages of the two sectors are set by the value of yield and consumption on these departments. For another one, the two sectors' wages are determined by a concentrated system (or so-called market): total output is linearly combined (completely substituted technology-based output function) by two intermediate firms. Given the market price index, we can solve the wages of the two departments.

#### 3.1 Supply Chain: Dispersed Equilibrium

As [Guerrieri et al. \[2020\]](#), we construct a two-sector model to explore the impact of the lockdown policy under different industrial structures. The economy is divided into necessary service and productive sector and unnecessary sector. Proportion of the two sectors is  $\phi$ . We still use linear technology in this part,

$$Y_{ti} = N_{ti} \tag{21}$$

where  $N_{ti}$  means that the labor is supplied by a specific part of total population. People cannot choose the sector they work on due to the skill sets are specific, which means people cannot flow inter-sector untetheredly,

$$Y_{t1} = \phi N_t, \quad Y_{t2} = (1 - \phi) N_t. \tag{22}$$

The probability of becoming infected is idiosyncratic for sectors

$$p_1(L) = \alpha_1 L(t), \quad p_2(L) = \alpha_2 L(t). \tag{23}$$

The Lockdown policy is also specific, we only limit the unnecessary part of the economy. It's self-evident because we have to maintain the flexibility of the necessary part to product for the

economy. Therefore, the labor supply for the two sectors are,

$$n_t^{s1} = L(t), \quad n_t^{s2} = 1. \quad (24)$$

And consumption are

$$c_t^{s1} = L(t)w_t^1, \quad c_t^{s2} = w_t^2 \quad (25)$$

where  $w_t^1$  is the wage for the sector one and  $w_t^2$  is for sector two. The partial equilibrium for the two-sector is idiosyncratic. We assume the total demand can be divided into two parts and is equal to the proportion of the sector  $\phi$ . Let  $x$  denote the fraction of sector two's output used as input in sector one. Total demand is

$$C_t = S_1 c_t^{s1} + S_2 c_t^{s2} + I(t)b. \quad (26)$$

The demand of sector 1 is,

$$\phi S_t L(t) = \phi S_t L(t) w_t^1 + \phi I(t)b \quad (27)$$

and the wage of sector 1,

$$w_t^1 = \frac{S_t L(t) - I(t)b}{S_t L(t)}. \quad (28)$$

For sector 2, we have

$$(1 - \phi) S_t (1 - x L(t)) = (1 - \phi) S_t w_t^2 + (1 - \phi) I(t)b, \quad (29)$$

$$w_t^2 = \frac{S_t (1 - x L(t)) - I(t)b}{S_t}. \quad (30)$$

The two sectors planning problem is

$$\max_{L(t)} \int_0^\infty e^{-(r+v)t} \left\{ S_1(t) u_1(c_t^{s1}, n_t^{s1}) + S_2(t) u_2(c_t^{s2}, n_t^{s2}) + I(t)b - \chi_d D(t) \right\} dt. \quad (31)$$

### 3.2 Intermediate Product: Concentrated Equilibrium

This section constructs an economic system with the final product department producing the final goods using two kinds of intermediate goods inputs from the necessary and unnecessary

sectors. Same as before, the intermediate production departments produce two kinds of goods,

$$\begin{aligned} Y_{1t} &= \phi N_{1t}, \\ Y_{2t} &= (1 - \phi) N_{2t}. \end{aligned} \quad (32)$$

Using completed substitute technology, the final production department creates final goods for the consumer,

$$Y_t = \xi_1 Y_{1t} + \xi_2 Y_{2t}. \quad (33)$$

Consumption is related to labor incomes,

$$c_t^{s1} = L(t)w_t^1, \quad c_t^{s2} = w_t^2.$$

Market clean conditions are

$$\begin{aligned} Y_t &= \xi_1 Y_{1t} + \xi_2 Y_{2t} = S_1 c_t^{s1} + S_2 c_t^{s2} + I(t)b, \\ \phi \xi_1 L(t) + (1 - \phi) \xi_2 &= S_1 c_t^{s1} + S_2 c_t^{s2} + I(t)b = S_1 L(t)w_{1t} + S_2 w_{2t} + I(t)b. \end{aligned} \quad (34)$$

Given price index  $P$ , final goods departments meet zero-profit condition,

$$PY_t - w_{1t}Y_{t1} - w_{2t}Y_{t2} = 0. \quad (35)$$

And we have

$$w_{2t} = \frac{P[\phi \xi_1 L(t) + (1 - \phi) \xi_2] - w_{1t} \phi L(t)}{(1 - \phi)}; \quad (36)$$

$$\phi \xi_1 L(t) + (1 - \phi) \xi_2 = S_1 L(t)w_{1t} + S_2 \frac{P[\phi \xi_1 L(t) + (1 - \phi) \xi_2]}{(1 - \phi)} - \frac{w_{1t} \phi}{(1 - \phi)} L(t) + I(t)b. \quad (37)$$

Solving the above equations, the equilibrium wage is determined as,

$$w_{1t} = \frac{\left[ \phi \xi_1 L(t) + (1 - \phi) \xi_2 - I(t)b - S_2 \frac{P[\phi \xi_1 L(t) + (1 - \phi) \xi_2]}{(1 - \phi)} \right]}{S_1 L(t) - \frac{\phi}{(1 - \phi)} L(t)}. \quad (38)$$

## 4 Numerical Method and Parameter Setting

We use value function iteration method to solve the HJB equations. In order to use this method, we have to get the difference form of the HJB equations. The process can be seen in the Appendix A.

As [Alvarez et al. \[2020\]](#), We use a value of statistical life of 20 times annual per capita GDP, which is in line with utilitarian values of life.

Table 1: Parameters

Parameter	Value	Definition
$\beta$	0.2	Daily increase of active cases if unchecked
$\gamma$	0.055	Daily rate of infected recovery (includes those that die)
$\varphi$	$0.01\gamma$	IFR: fatality per active case (per day)
$\kappa$	$0.05\gamma$	Implies a 3 percent fatality rate with 40 percent infected
$r$	0.05	Annual interest rate 5 percent
$\nu$	0.667	Prob rate vaccine + cure (exp. duration 1.5 years)
$\bar{L}$	0.9	1 - GPD share health, retail, government, utilities, and food mfg
$\theta$	0.5	Effectiveness of lockdown
$\chi$	0	Value of Statistical Life $20 \times w$

## 5 Numerical Results

We show the main quantitative results of the planning problem described in the previous section. Throughout our primary focus is on the controlled and uncontrolled scenario with comparing different risk aversion parameters  $\alpha$ , value of life  $\chi$  and unemployment insurance  $b$  to explore the optimal lockdown policies and changes in output, wages, and government deficits on this basis. We first analyze the one sector situation. And then move to the comparisons under different industrial structures to capture the optimal lockdown policy on the unnecessary sector and their corresponding outcomes.

## 5.1 Risk Aversion and Life Value

Risk aversion is an important feature of individual decision makers. When representative consumers take a hedging attitude towards unknown risks, it will directly affect individual decision making. In our model, where active unemployment is a consequence of this problem, policymakers have an incentive to choose initiatives when they consider the consequences of illness.

Presented in Figure 2, the optimal lockdown policy shows the monotonous decline after a short period of high lockdown rate. As the degree of risk aversion increases, the lockdown policy becomes stricter. Tighter lockdown policies have led to a decline in output, which ultimately fell by 2.5%. Increased risk aversion also leads to an obvious whole decline in wages. In both risk aversion scenarios, the wages appear to fall first and then rise. The fiscal deficit curves first increases and then decreases, and stricter lockdown policy leads to a whole increase in the fiscal deficits.

For the yield of the economy, the shock of the pandemic is totally negative for the economy. With the proportion  $I(t)$  increases the lost of the yield is also enhanced, from 2% to 2.5%.

In our baseline model, the wage is same as price index ( $w_t = p_t$ ), so we need only focus on the variation of wage of working population. We can get the conclusion from the numerical solution that the severity of pandemic is negative correlated with the price index(wage), which means the increasing of  $I(t)$  means the minimum of price decreases. In our results, it decreases from 0.981 to 0.977. And then, it will increase and converge to a fixed value. The reason for this can be explained by the fiscal policy. At the first stage, there are still many people who work on positions and accept the wage  $w_t$ . Pandemic shock decreases the wage at first. And then, fiscal policy exerts its ability to meet the requirement and demanding. Along with people who are infected and given unemployment insurance, the demanding increases again.

Fiscal deficit comes from the unemployment policy  $b$ . According the setting of the model, it should be concert with the trend of infected people. Deficit increases to 2.25% with the proportion  $I(t)$  enhances. And then, the same as wage and yield, decrease to a fixed value.

We set  $b = 0.05$  and  $b = 0.15$  as two comparison. We can get the conclusion that with benefit decreases the yield of economy will also decrease, but fiscal deficit decreases also, as we expected. And the minimum price also increase, which means that a more rigid lockdown policy will be used.

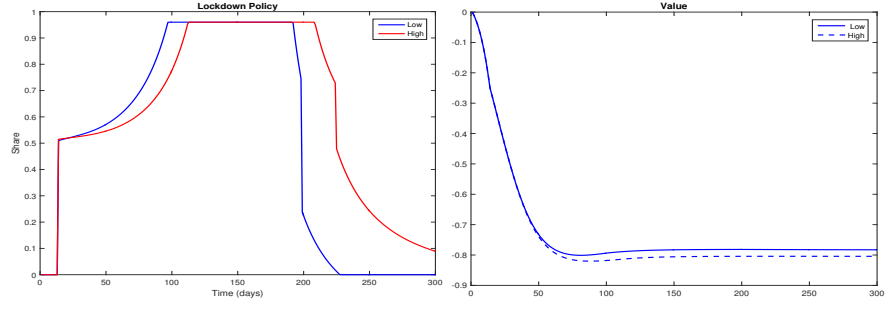
The value of life is an obvious concept, and as the value of life goes up, we tend to stick to a conservative lockdown policy. Presented in Figure 3, both of the optimal lockdown curves under the two scenarios show the monotonous decline and cross at date 100. As the value of life increases, the lockdown policy becomes stricter, resulting in an overall decline in output. The output curve with  $\chi_d = 1.5$  first falls and then rises. The wage curve with  $\chi_d = 1.5$  shows an obvious whole decline comparing to the curve with  $\chi_d = 0.6$ . Both of the wage curves appear to fall first and then rise. The fiscal deficit curves first increases and then decreases, and stricter lockdown policy leads to a whole increase in the fiscal deficits.

## 5.2 Industry structure and Inequality

From the point of view of the industrial structure of each country, the tertiary industry is also the demand of the non-essential production sector is greater. According to the data, the proportion of the tertiary industry in all countries exceeds 50%, no matter for developed countries or non-developed countries. Different industrial structure necessarily means different output value and wages. Under the condition of incomplete market, people cannot freely choose the sector they are engaged in. So what happens to the inequality of social output under the optimal lockdown?

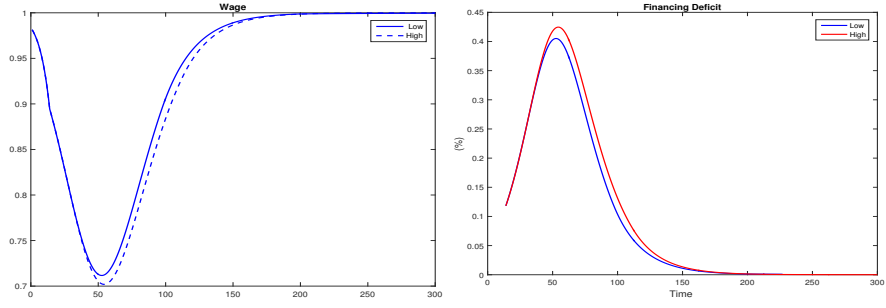
We have carefully compared the numerical results for dispersed equilibrium and concentrated equilibrium. We find that the two models show the similar conclusions, thus we will focus on the analysis on numerical results of dispersed equilibrium and leave the results of concentrated equilibrium in the appendix part.

In the new two-sector model we can see the difference between two sectors. In the sector 1, the part that is controlled by the government, the loss is less than sector 2. The total loss of the



(a) Lockdown

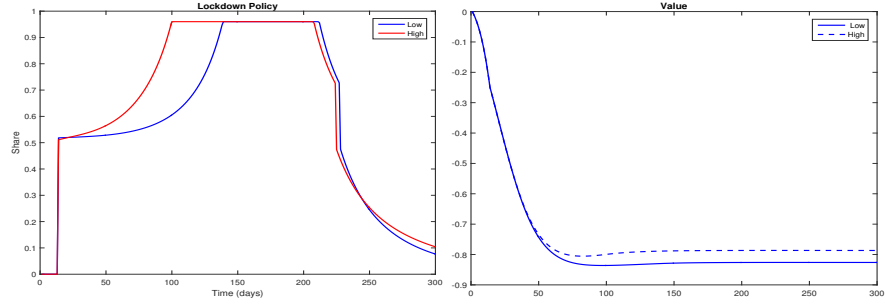
(b) Output



(c) Wage

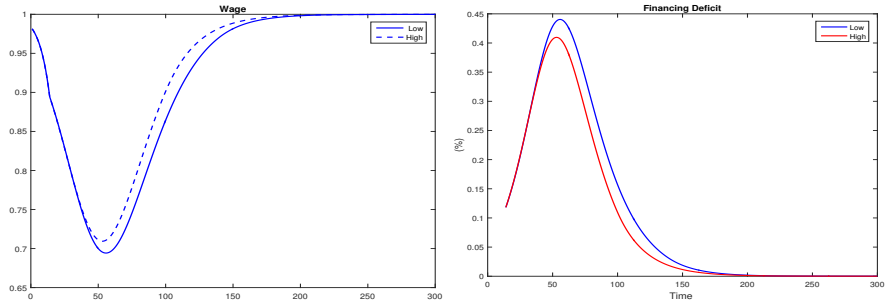
(d) Deficit

Figure 2: Risk aversion,  $\alpha = 0.01, \alpha = 0.02$



(a) Lockdown

(b) Output



(c) Wage

(d) Deficit

Figure 3: Value of Life,  $\chi_d = 1.5, \chi_d = 0.6$



economy depends on the industrial structure; we can see the loss is between the loss of sector 1 and sector 2. When the lockdown policy is less than before, the loss of the two sectors tend to converge to a 1%. Same as baseline model, price is same as wage. The wage of the sector 1 is less than sector 2 due to the lockdown policy and the degrading of demands.

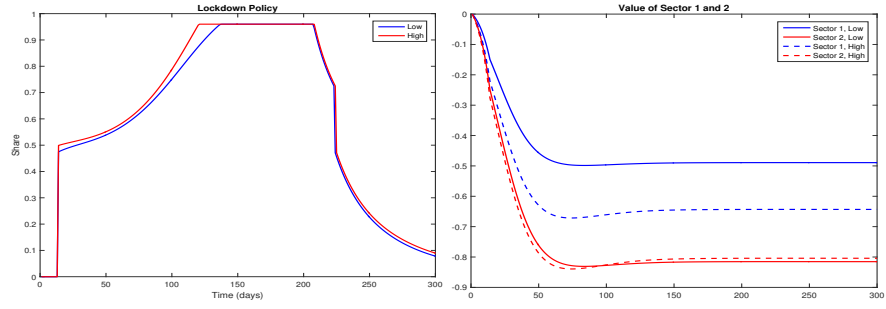
And then the policy of unemployment insure exerts the efficiency and promote the price of the sector 1 and sector 2. Compared to the former baseline model, the financial deficit is lower. As the increasing of infected people, the deficit manifests the trending of increasing. From then on, the deficit will decrease to a certain value.

What perspectives can these data can provide for us? Intuitively, inequality tends to rise significantly as the industrial structure becomes more differentiated. This proved to be the case as the wage gap grew significantly. The increase in non-essential sectors (sector 1) will lead to a decline in the lockdown to a certain extent, reflecting the trade-off between the economy and the value of life. In this case, losses in Department 1 are minimized from an output perspective. Therefore, the salary of department 1 will also be relatively minimized. These losses can be seen in debt, which has increased significantly as a result of increases in non-essential sectors.

### **5.3 Will unemployment insurance improve inequality in the economy?**

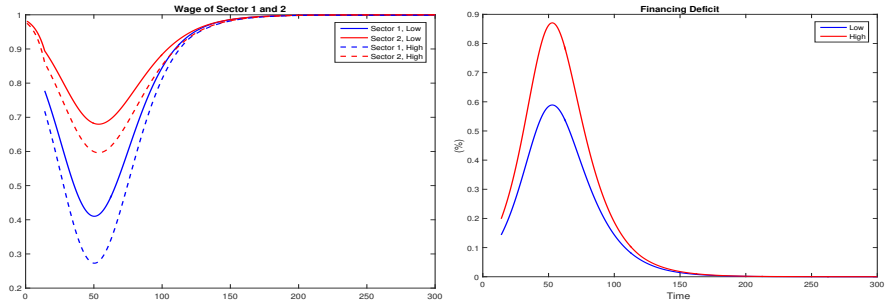
We've talked about inequality in the economy, and an obvious idea is how fiscal policy can address these distortions in the economy. But the Laffer curve and The Ricardian equivalence principle tell us that there is no such thing as a free lunch, which means that there is no perfect policy tool to solve this problem completely. In effect, when the central government or the Banks use various fiscal or monetary instruments, they are simply balancing the books or overdrawing their future wealth. How would an optimal lockdown affect the effectiveness of unemployment insurance in our framework?

The answer is that it has little effect, and it will not change the problem of inequality in the economy. The inequality in the economy comes from the different output value of the industry. When we choose to increase the intensity of unemployment insurance, the represen-



(a) Lockdown

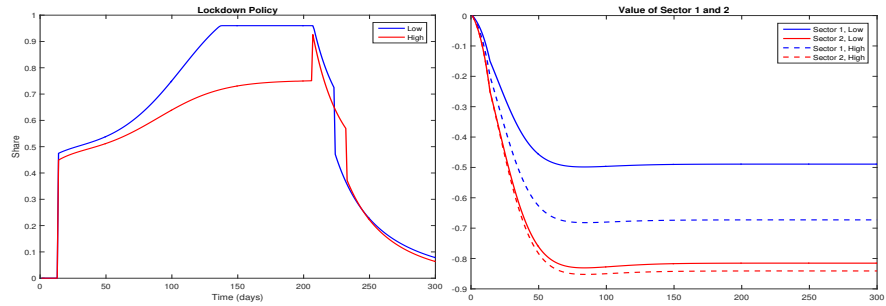
(b) Output



(c) Wage

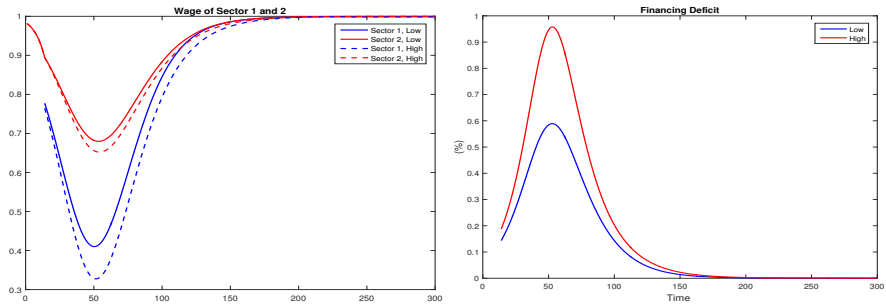
(d) Deficit

Figure 4: Unemployment insurance,  $b = 0.3, b = 0.5$



(a) Lockdown

(b) Output



(c) Wage

(d) Deficit

Figure 5: Industrial Structure,  $\phi = 0.5, \phi = 0.7$

tative consumers will choose voluntary unemployment. This consequence is not obvious, and depends on two effects. One is that a decline in the number of people in an industry leads to an increase in per capita welfare, and the other is that a decline in the number of people in employment leads to a serious decline in output. Which effect prevails will determine the effect of our policy.

As a result, the high income insurance policy increased the lockdown. It is obvious that social policy makers prefer to use unemployment insurance to reduce economic loss and loss of life value in policy optimization. This led to a significant increase in debt. From the perspective of output value, the loss of department 2 decreased a little and that of department 1 increased a lot. So, social inequality has gone down but the output loss has been greater.

## 6 Conclusion

We introduce a planning problem framework that includes utility, risk, and equilibrium, differentiated from others' planning problems. According to our analysis, we can get the relationship between lockdown and susceptible, infected population, and fiscal policy and lockdown. Another insight we provide for this arena is the substantial nexus between industrial structure and the critical index we mentioned above. Different industry organizations can lead to distinct outcomes for the whole economy.

When the industrial structure difference becomes larger, the degree of inequality will increase significantly because of the increased wage gap. The increase in non-essential sectors will lead to a decline in the lockdown to a certain extent, reflecting the trade-off between the economy and the value of life. In this case, losses in sector 1 are minimized from an output perspective. So wages in non-essential sectors will also be relatively minimized. These losses can be seen in debt, which has increased significantly due to increases in non-essential sectors. Unemployment insurance has a limited effect because of voluntary unemployment channels, which will not change the problem of inequality in the economy. One possible strategy is to

provide employee incentives to encourage people to join the necessary sectors. A reduction in the number of people in non-essential sectors would thus increase per-capita welfare more than a severe drop in output, resulting from a reduction in the number of people in employment. So, social inequality has gone down, but the output loss has been more significant. However, we still do not answer whether the debt will impact welfare in the long term. Since we are mainly considering short-term decisions, one possible strategy is to integrate long-term and short-term goals to measure this issue. Under the condition of an infinite time limit, the decision is time-homogeneous; In other words, starting at any time, our decision strategy is independent of time and related to other state variables. This may ignore the impact of variable debt accumulated over time. It is possible to approach this problem with a combination of short and long terms. It is not intuitive whether the short-term benefit increase is better or the future benefit loss from debt is more significant.

Nevertheless, there are still many uncharted fields we want to explore under this framework. The first nature is CRRA utility function, a typical utility function argued in many ex-works. It can reveal risk-aversion identity for inter-temporary decision making. Another problem is CES productive function, which can integrate various intermediate producers into one part flexibly. Considering the heterogeneous-agent nature about age, initial assets, in a word, these natures can describe the economy more realistically since these initial conditions affect the decision making under the pandemic scenario.

## **Acknowledgments**

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# A Appendix for Numerical Method

## A.1 One-sector Model

As [Alvarez et al. \[2020\]](#), we use value function iteration algorithm to solve this problem.

The utility of agents is

$$E[u_s(c_t^s, n_t^s)] = (1 - \alpha L)Lw + \alpha Lb \quad (39)$$

because we can calculate the equilibrium wage, the utility can be written as:

$$u_t^s = (1 - \alpha L) \frac{SL - Ib}{S} + \alpha Lb \quad (40)$$

Solve this social planner's problem:

$$\begin{aligned} & \max_{L(t)} \int_0^\infty e^{-(r+v)t} \left\{ S(t) \left[ (1 - \alpha L) \frac{SL - Ib}{S} + \alpha Lb \right] + I(t)b - \chi_d D(t) \right\} dt \\ & = \max_{L(t)} \int_0^\infty e^{-(r+v)t} \left\{ S(t) \left[ (1 - \alpha L) \frac{SL - Ib}{S} + \alpha Lb \right] + I(t)b + \chi_d \phi(I(t))I(t) \right\} dt \end{aligned} \quad (41)$$

The HJB equation of this problem is:

$$\begin{aligned} (r + v)V(S, I) = \min_{L \in [0, L]} & \left\{ S(t) \left[ (1 - \alpha L) \frac{SL - Ib}{S} + \alpha Lb \right] + I(t)b + \chi_d \phi(I(t))I(t) + \right. \\ & - \partial_S V(S, I) [-\beta S(t)I(t)(1 - \theta L(t))^2] \\ & \left. + \partial_I V(S, I) [\beta S(t)I(t)(1 - \theta L(t))^2 - \gamma I(t)] \right\} \end{aligned} \quad (42)$$

To calculate the partial difference  $\partial_S V(S, I)$  and  $\partial_I V(S, I)$ , we choose to  $V_S^-(i, j)$  and  $V_I^+(i, j)$

$$V_S^-(i, j) = \frac{V(S_i, I_j) - V(S_{i-1}, I_j)}{S_i - S_{i-1}} \quad (43)$$

$$V_I^+(i, j) = \frac{V(S_i, I_{j+1}) - V(S_i, I_j)}{I_{j+1} - I_j} \quad (44)$$

$$\begin{aligned} (r + v)V(S_i, I_j) = \min_{L \in [0, L]} & \left\{ S_i \left[ (1 - \alpha L) \frac{SL - Ib}{S} + \alpha Lb \right] + I_j b + \chi_d \phi(I_j)I_j \right. \\ & \left. + [\beta S_i I_j (1 - \theta L)^2] [V_I^+(i, j) - V_S^-(i, j)] - \gamma I_j V_I^-(i, j) \right\} \end{aligned} \quad (45)$$

We assume that  $S_i - S_{i-1} = I_{j+1} - I_j = \Delta$ .

$$V_I^+(i, j) - V_S^-(i, j) = \frac{1}{\Delta} [V(S_i, I_{j+1}) - V(S_i, I_j) - V(S_i, I_j) + V(S_{i-1}, I_j)] \quad (46)$$

$$[1 + (r + \nu)dt]V(S_i, I_j) = \min_{L \in [0, L]} \left\{ S_i \left[ (1 - \alpha L) \frac{SL - Ib}{S} + \alpha Lb \right] dt + \chi_d \phi(I_j) I_j dt \right. \\ \left. + \left[ \beta S_i I_j (1 - \theta L)^2 \right] dt [V_I^+(i, j) - V_S^-(i, j)] - \gamma I_j dt V_I^-(i, j) + V(S_i, I_j) \right\} \quad (47)$$

$$[1 + (r + \nu)dt]V(S_i, I_j) = \min_{L \in [0, L]} \left\{ S_i \left[ (1 - \alpha L) \frac{SL - Ib}{S} + \alpha Lb \right] dt + \chi_d \phi(I_j) I_j dt \right. \\ \left. + \left[ \beta S_i I_j (1 - \theta L)^2 \right] \frac{dt}{\Delta} [V(S_i, I_{j+1}) - 2V(S_i, I_j) + V(S_{i-1}, I_j)] \right. \\ \left. - \gamma I_j \frac{dt}{\Delta} [V(S_i, I_j) - V(S_i, I_{j-1})] + V(S_i, I_j) \right\} \quad (48)$$

$$[1 + (r + \nu)dt]V(S_i, I_j) = \min_{L \in [0, L]} \left\{ S_i \left[ (1 - \alpha L) \frac{SL - Ib}{S} + \alpha Lb \right] dt + \chi_d \phi(I_j) I_j dt \right. \\ \left. + [1 - (r + \nu)dt] \left\{ \frac{\beta S_i I_j (1 - \theta L)^2}{1 - (r + \nu)dt} \frac{dt}{\Delta} [V(S_i, I_{j+1}) - 2V(S_i, I_j) + V(S_{i-1}, I_j)] \right. \right. \\ \left. \left. - \frac{\gamma I_j}{1 - (r + \nu)dt} \frac{dt}{\Delta} [V(S_i, I_j) - V(S_i, I_{j-1})] + V(S_i, I_j) \right\} \right\} \quad (49)$$

$$V(S_i, I_j) = \min_{L \in [0, L]} \left\{ S_i \left[ (1 - \alpha L) \frac{SL - Ib}{S} + \alpha Lb \right] dt + \chi_d \phi(I_j) I_j dt \right. \\ \left. + [1 - (r + \nu)dt] \left[ 1 - 2 \frac{\beta S_i I_j (1 - \theta L)^2}{1 - (r + \nu)dt} \frac{dt}{\Delta} - \frac{\gamma I_j}{1 - (r + \nu)dt} \frac{dt}{\Delta} \right] V(S_i, I_j) \right. \\ \left. + [1 - (r + \nu)dt] \left[ \frac{\beta S_i I_j (1 - \theta L)^2}{1 - (r + \nu)dt} \frac{dt}{\Delta} \right] V(S_i, I_{j+1}) \right. \\ \left. + [1 - (r + \nu)dt] \left[ \frac{\beta S_i I_j (1 - \theta L)^2}{1 - (r + \nu)dt} \frac{dt}{\Delta} \right] V(S_{i-1}, I_j) \right. \\ \left. + [1 - (r + \nu)dt] \left[ \frac{\gamma I_j}{1 - (r + \nu)dt} \frac{dt}{\Delta} \right] V(S_i, I_{j-1}) \right\} \quad (50)$$

With respect the situation that we set the interval of different of direction on the discrete space.



$$\begin{aligned}
V(S_i, I_j) = & \min_{L \in [0, L]} \left\{ S_i \left[ (1 - \alpha L) \frac{SL - Ib}{S} + \alpha Lb \right] dt + \chi_d \phi(I_j) I_j dt \right. \\
& + [1 - (r + v)dt] \left\{ 1 - \frac{[\beta S_i I_j (1 - \theta L)^2]}{[1 - (r + v)dt]} \frac{dt}{\Delta_I} - \frac{[\beta S_i I_j (1 - \theta L)^2]}{[1 - (r + v)dt]} \frac{dt}{\Delta_S} - \frac{\gamma I_j}{[1 - (r + v)dt]} \frac{dt}{\Delta_I} \right\} V(S_i, I_j) \\
& + [1 - (r + v)dt] \frac{[\beta S_i I_j (1 - \theta L)^2]}{[1 - (r + v)dt]} \frac{dt}{\Delta_I} V(S_i, I_{j+1}) \\
& + [1 - (r + v)dt] \frac{[\beta S_i I_j (1 - \theta L)^2]}{[1 - (r + v)dt]} \frac{dt}{\Delta_S} V(S_{i-1}, I_j) \\
& \left. + [1 - (r + v)dt] \frac{\gamma I_j}{[1 - (r + v)dt]} \frac{dt}{\Delta_I} V(S_i, I_{j-1}) \right\}
\end{aligned} \tag{51}$$

On the edge of the space:

$$\begin{aligned}
V(S_i, I_j) = & \min_{L \in [0, L]} \left\{ S_i \left[ (1 - \alpha L) \frac{SL - Ib}{S} + \alpha Lb \right] dt + \chi_d \phi(I_j) I_j dt \right. \\
& + [1 - (r + v)dt] \left\{ 1 - \frac{[\beta S_i I_j (1 - \theta L)^2]}{[1 - (r + v)dt]} \frac{dt}{\Delta_S} - \frac{\gamma I_j}{[1 - (r + v)dt]} \frac{dt}{\Delta_I} \right\} V(S_i, I_j) \\
& + [1 - (r + v)dt] \frac{[\beta S_i I_j (1 - \theta L)^2]}{[1 - (r + v)dt]} \frac{dt}{\Delta_S} V(S_{i-1}, I_{j+k}) \\
& \left. + [1 - (r + v)dt] \frac{\gamma I_j}{[1 - (r + v)dt]} \frac{dt}{\Delta_I} V(S_i, I_{j-1}) \right\}
\end{aligned} \tag{52}$$

Before value function iteration, we have to determine the initial value of discrete space:

$$V(0, I) = \frac{a^2 b^2}{8\alpha\gamma + 4\alpha(r + v)} I^2 + \frac{4\alpha\chi_d\phi + 2ab}{4\alpha\gamma + 4\alpha(r + v)} I + \frac{1}{4\alpha(r + v)} \tag{53}$$

$$V(S, 0) = \frac{\left( \frac{(1 + abS)^2}{4\alpha} \right)}{r + v}; \tag{54}$$

And then we use FOC to find optimal policy  $L(t)$

$$L = \frac{(1 + \alpha bI + abS) - 2\theta[\beta SI] [V_I^+ - V_S^-]}{2\alpha - 2\theta^2[\beta SI] [V_I^+ - V_S^-]} \tag{55}$$

## A.2 Two-sector Model

Same as baseline model, we use value function iteration to find the solution of HJB equation:

The HJB equation in this part is:

$$\max_{L(t)} \int_0^\infty e^{-(r+v)t} \left\{ \phi S_t \left[ \frac{SL - Ib}{S} (1 - \alpha_1 L) + \alpha_1 Lb \right] + (1 - \phi) S_t \left[ \frac{S - Ib}{S} (1 - \alpha_2 L) + \alpha_2 Lb \right] + I(t)b - \chi_d D(t) \right\} dt \quad (56)$$

$$\begin{aligned} V(S_i, I_j) = \min_{L \in [0, L]} \{ & [(1 - \alpha_1 L) [S_t L(t) - I(t)b] + \alpha_1 Lb + (1 - \phi) [(1 - \alpha_2 L) [S_t - I(t)b] + \alpha_2 Lb] \\ & + I(t)b + \chi_d \phi(I_j) I_j] dt \\ & + [1 - (r + v)dt] \left\{ 1 - \frac{[\beta S_i I_j (1 - \theta L)^2]}{[1 - (r + v)dt]} \frac{dt}{\Delta_S} - \frac{\gamma I_j}{[1 - (r + v)dt]} \frac{dt}{\Delta_I} \right\} V(S_i, I_j) \\ & + [1 - (r + v)dt] \frac{[\beta S_i I_j (1 - \theta L)^2]}{[1 - (r + v)dt]} \frac{dt}{\Delta_S} V(S_{i-1}, I_{j+k}) \\ & + [1 - (r + v)dt] \frac{\gamma I_j}{[1 - (r + v)dt]} \frac{dt}{\Delta_I} V(S_i, I_{j-1}) \} \end{aligned} \quad (57)$$

The FOC of optimal policy L is:

$$L = \frac{\alpha_1 \phi I(t)b - \alpha_2 (1 - \phi) S_t + (1 - \phi) I(t)b \alpha_2 + \phi \alpha_1 b + (1 - \phi) \alpha_2 b + \phi S_t - 2\theta \beta S I [V^+ - V^-]}{2\alpha_1 \phi S_t - 2\theta^2 \beta S I [V^+ - V^-]} \quad (58)$$

## B Numerical Results for Baseline Model

### B.1 Benefit

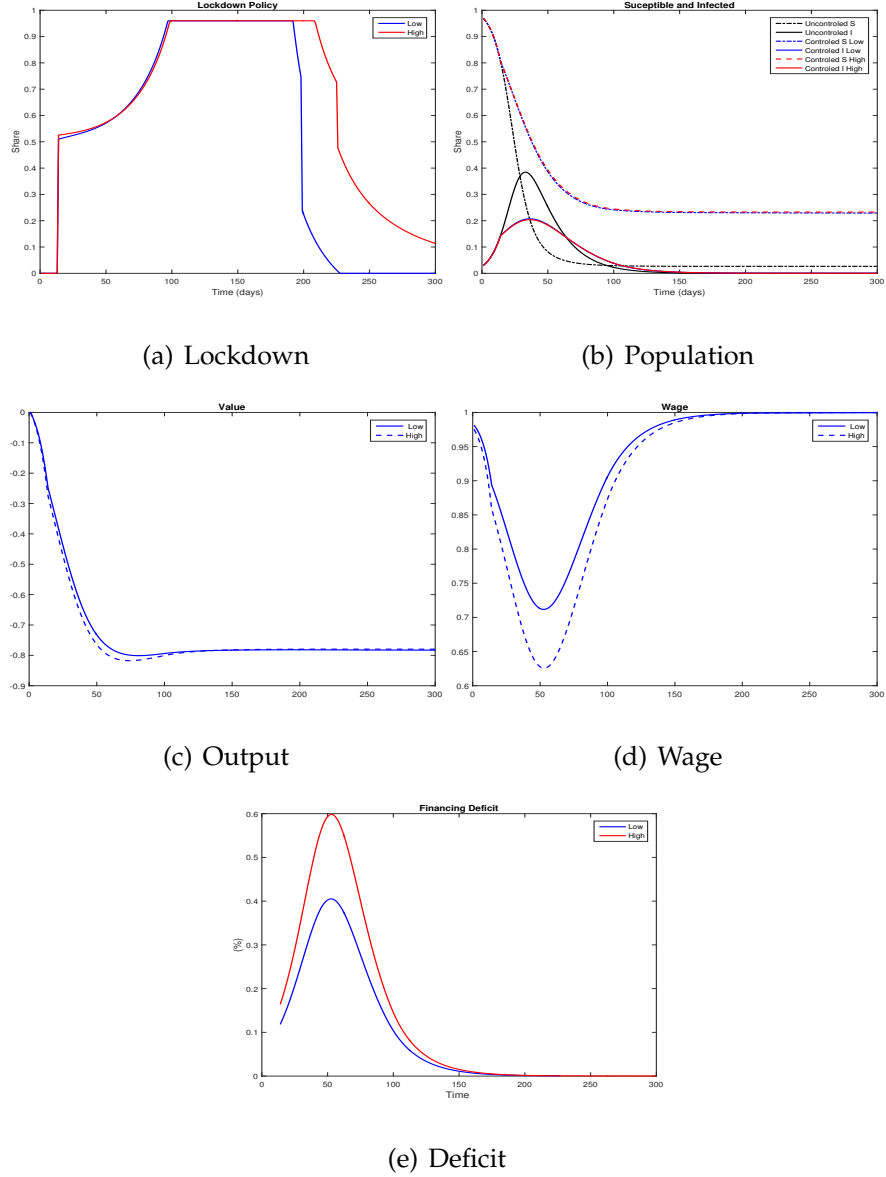


Figure 6: Unemployment Insurance,  $b = 0.3, b = 0.5$

### B.2 Risk aversion

It is obvious that the optimal lockdown policy is tighter when individuals have lower risk aversion. A high degree of risk aversion will drive individuals to actively avoid the risk of

infection, which also acts as an indirect lockdown. Therefore, necessary epidemic prevention propaganda can improve individual risk awareness and help control the epidemic, which .

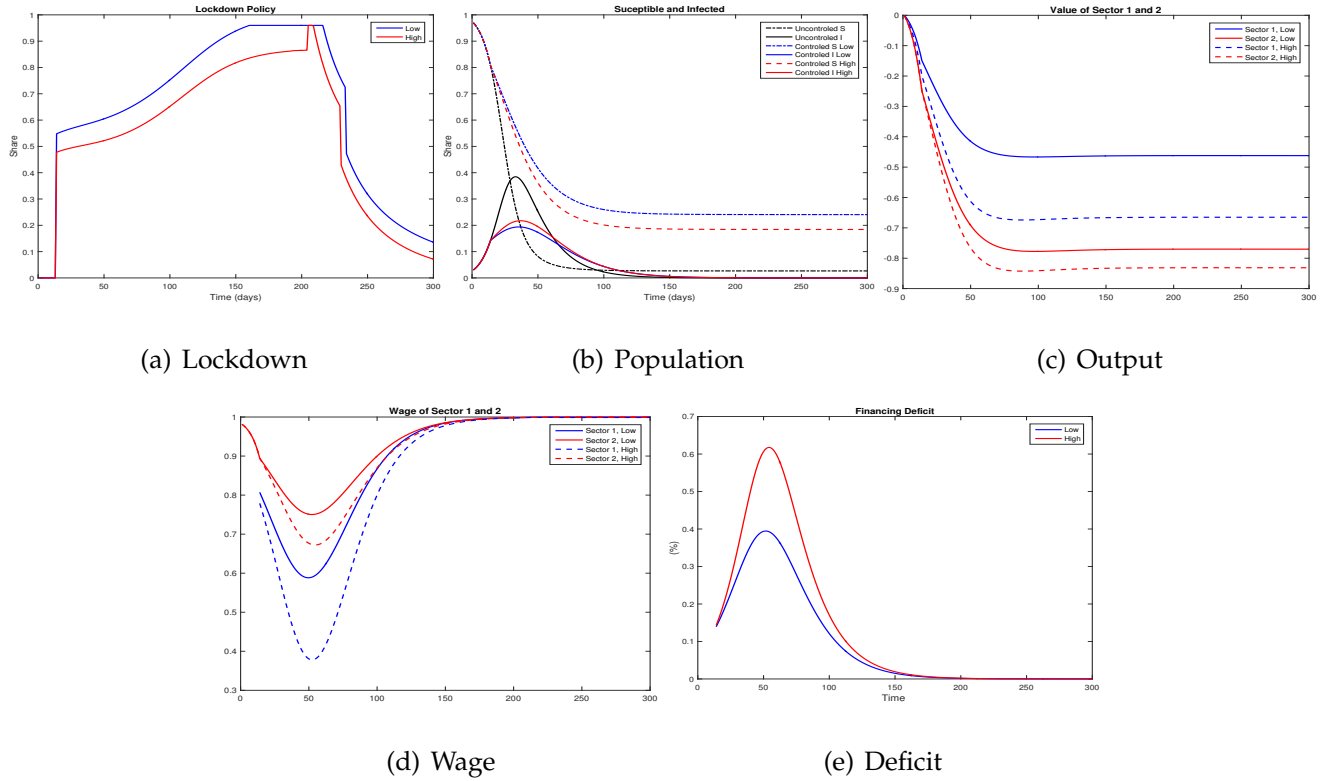
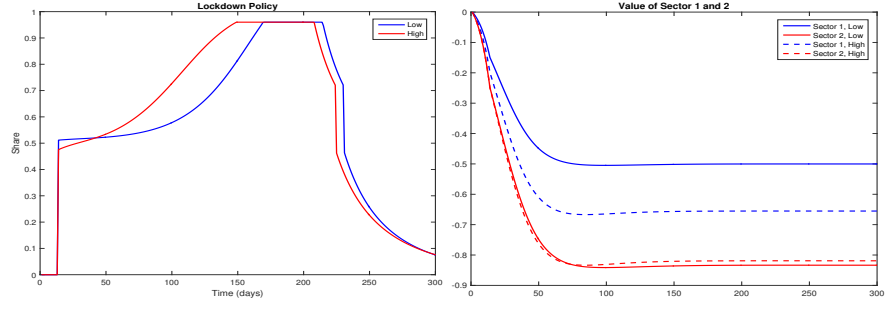


Figure 7: Risk aversion,  $\alpha_1^L = 0.01, \alpha_2 = 0.03$  and  $\alpha_1^H = 0.02, \alpha_2 = 0.03$

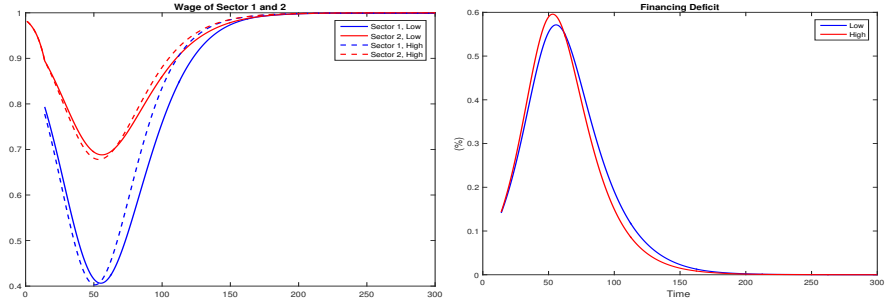
### B.3 Value of Life

When we increase the value of life, the lockdown become tighter. The lockdown policy flattens the spread curve. There is a jump down of the output in sector 1 for a higher value of life and stale for sector 2 under the two scenarios. The wages of sector 1 is lower due to lockdown policy. There is little difference between the two wage curves in each sector under the two sets of parameters. The fiscal deficit curves increase first and then decrease, with little difference under different parameters.



(a) Lockdown

(b) Output



(c) Wage

(d) Deficit

Figure 8: Value of Life,  $\chi_d^L = 0.6$  and  $\chi_d^H = 1.5$ ,

# C Numerical results for two sectors: Concentrate Equilibrium

## C.1 Infection Rates

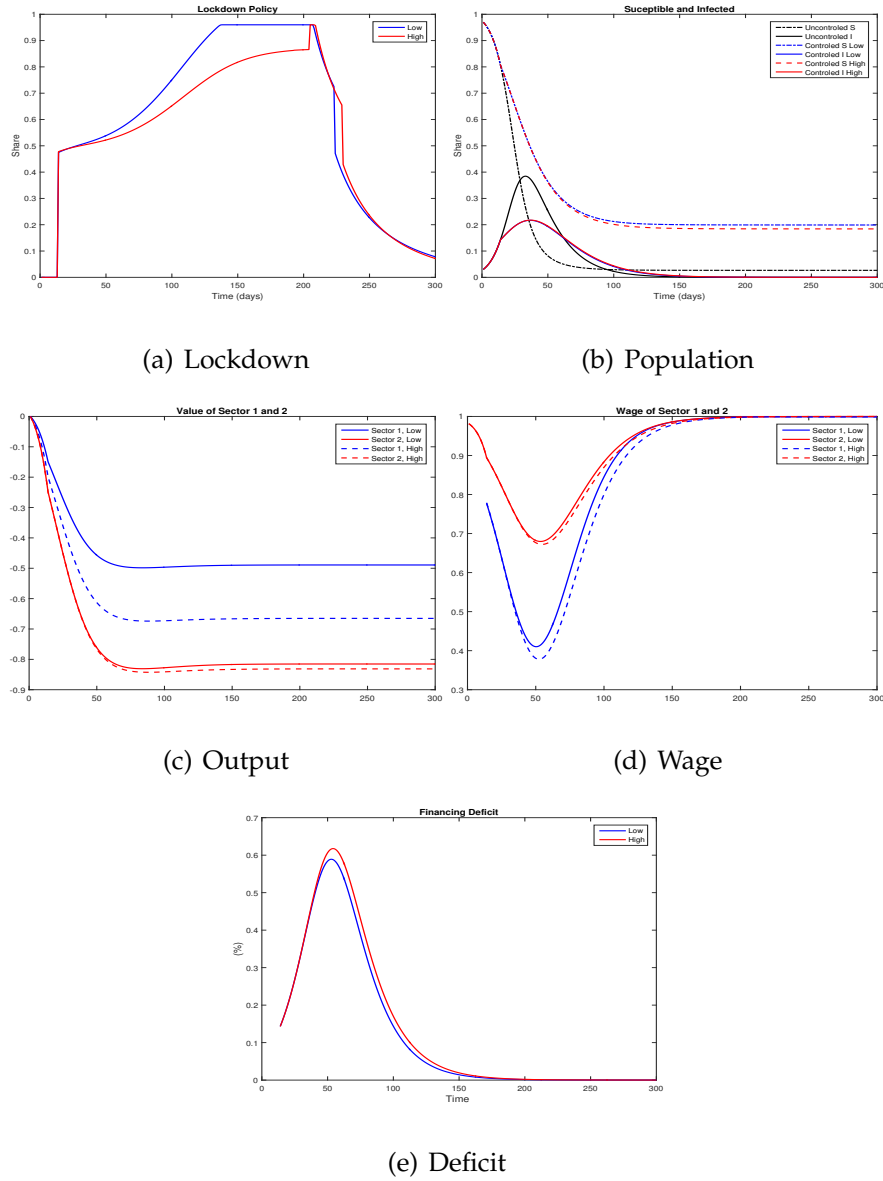


Figure 9: Infection Rates,  $\alpha_1 = 0.01, \alpha_2 = 0.03, \alpha_1 = 0.02, \alpha_2 = 0.03$

## C.2 Value of Life

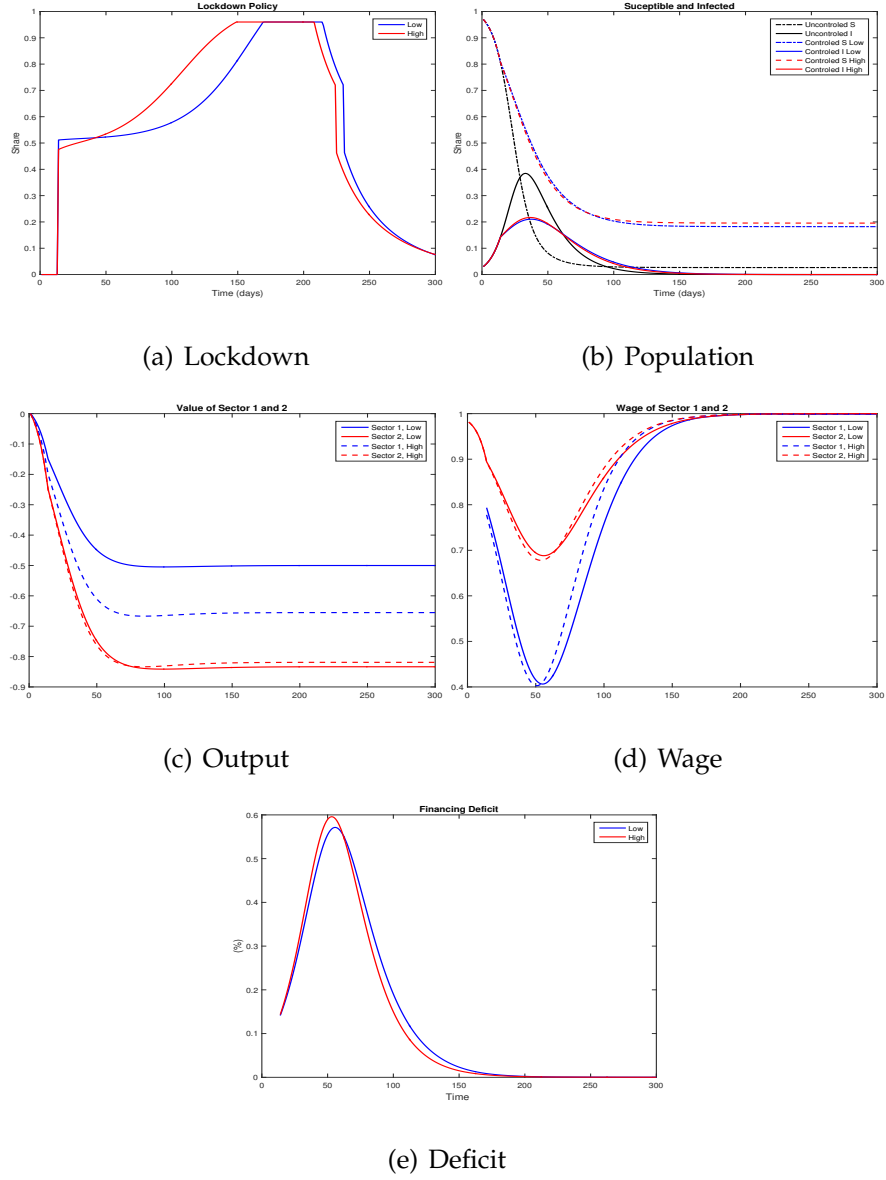


Figure 10: Value of Life,  $\chi_d = 1.5, \chi_d = 0.6$

### C.3 Benefit

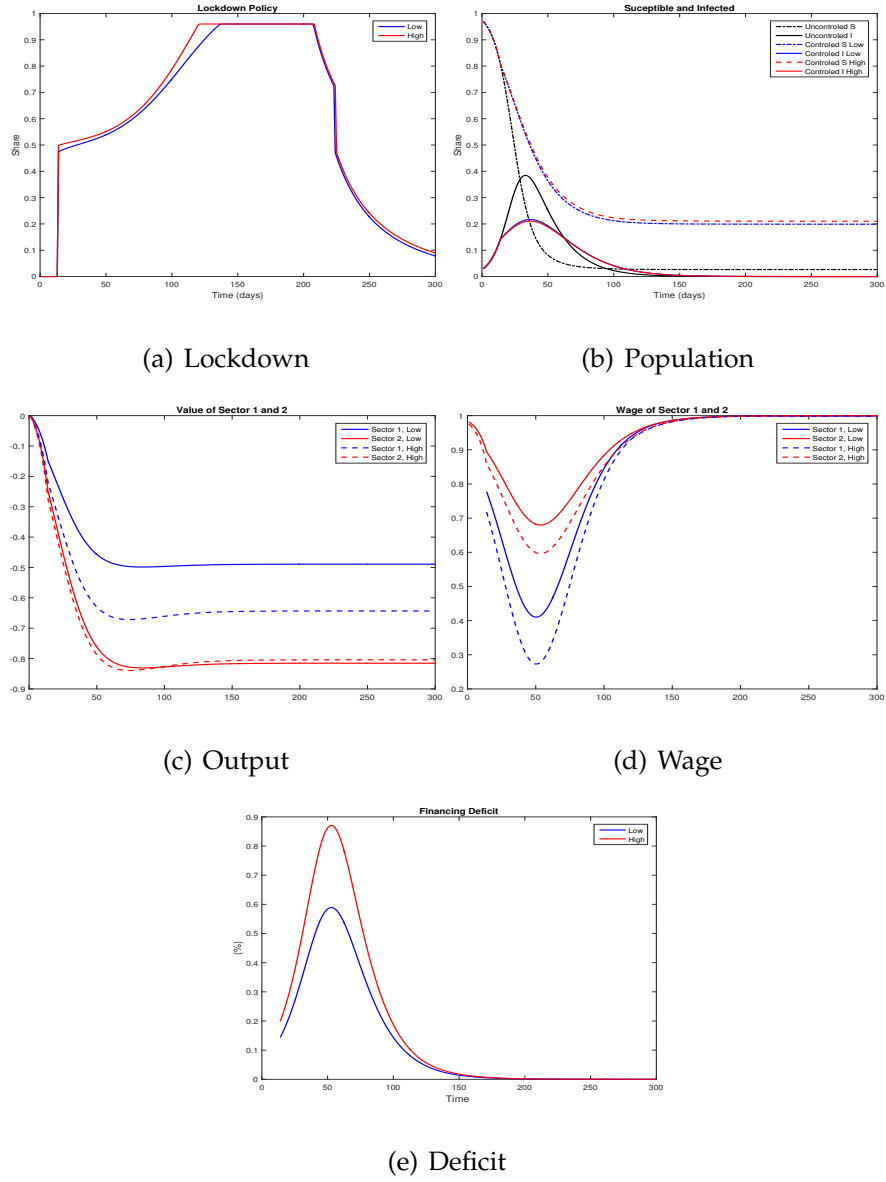


Figure 11: Benefit,  $b = 0.3, b = 0.5$



## C.4 Industrial Structure

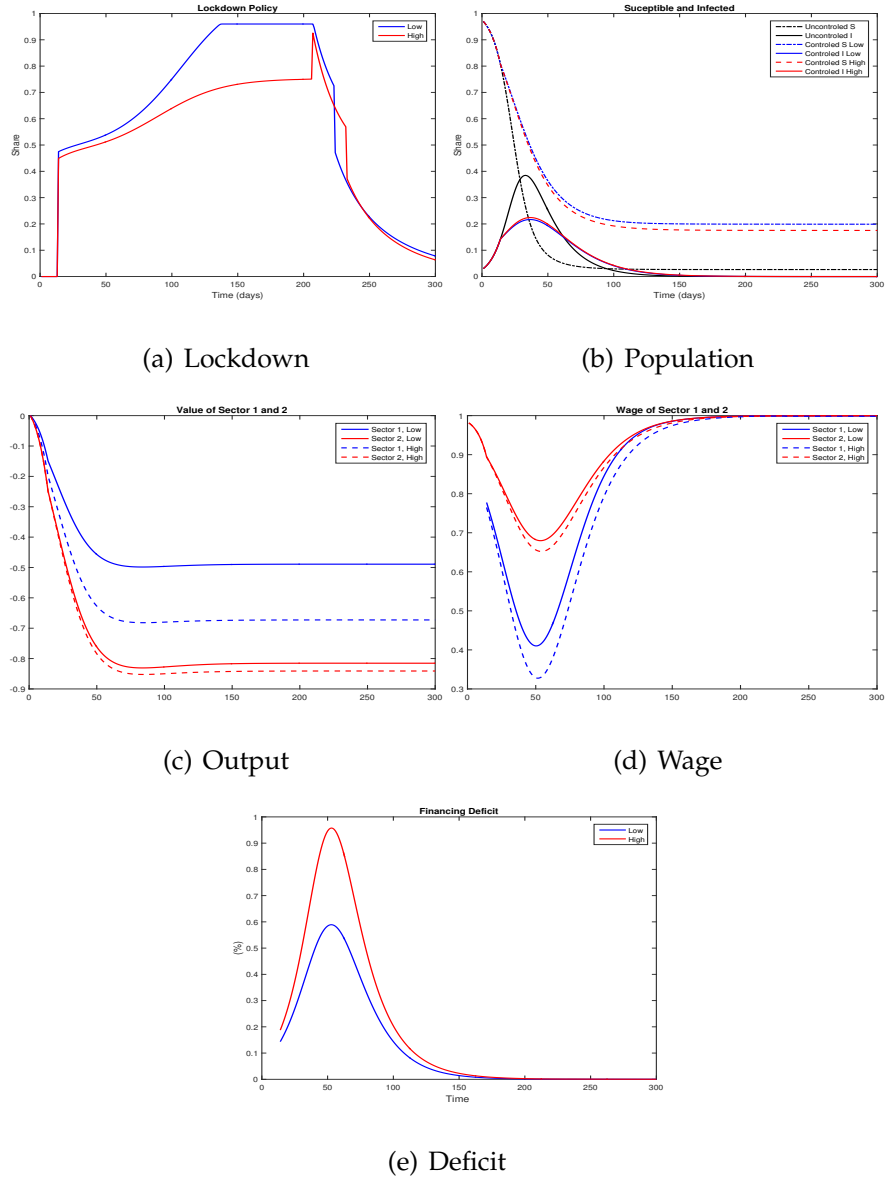


Figure 12: Industrial Structure,  $\phi = 0.5, \phi = 0.7$